

# FLUCTUATION MICROSCOPIES

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# FLUCTUATION MICROSCOPIES

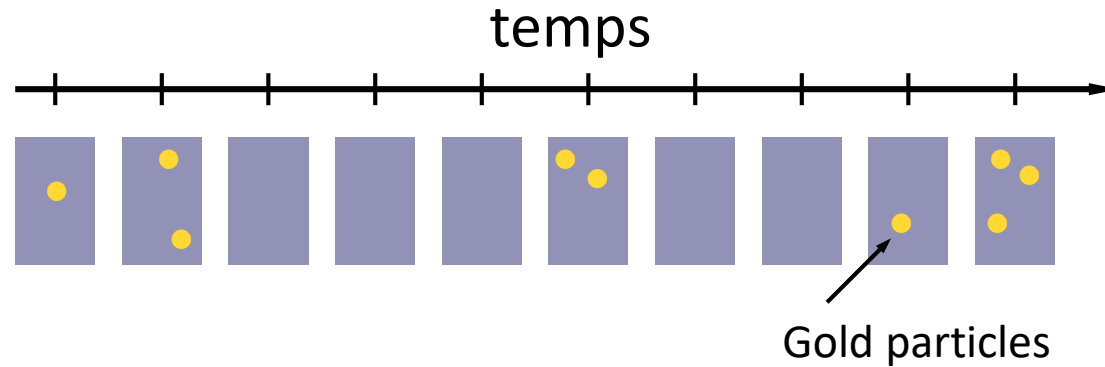
- INTRODUCTION
- PRINCIPLE OF FCS
- AUTOCORRELATION
- PHOTON COUNTING HISTOGRAM
- CROSS CORRELATION
- EXPERIMENTAL DETAILS
- NUMBER & BRIGHTNESS
- PRACTICAL PART

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# History : 1<sup>st</sup> measurement of correlation spectroscopy

(Svedberg & Inouye, 1911) *Occupancy Fluctuation*



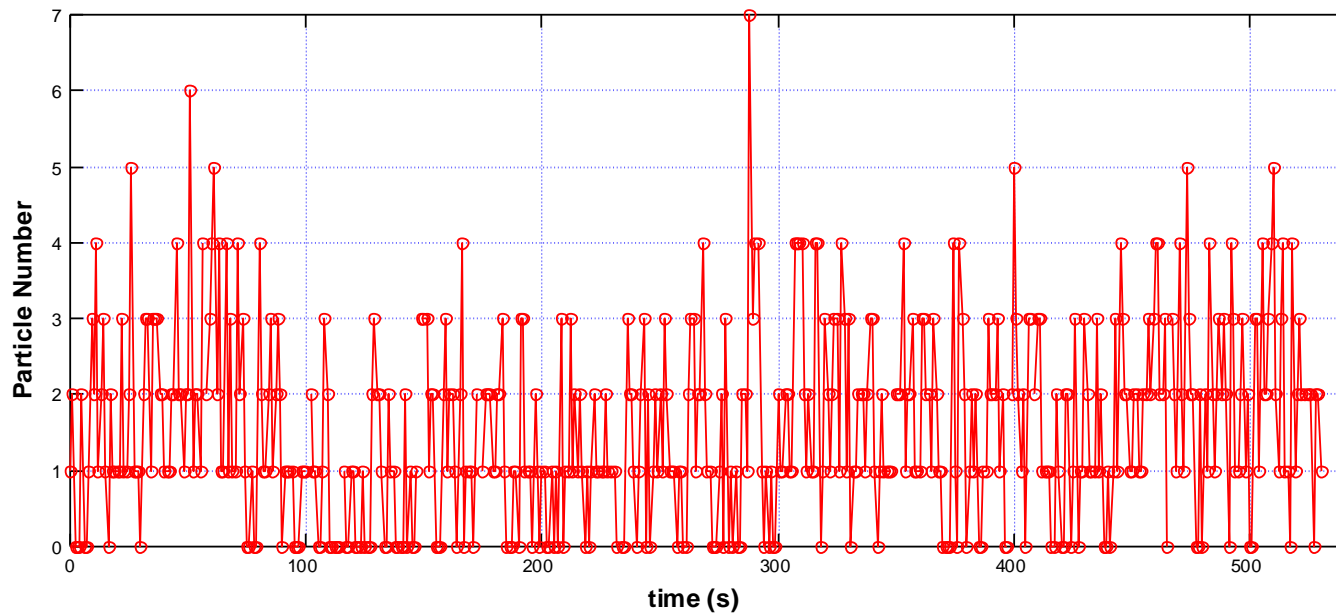
1200020013241231021111311251110233133322111224221226122142345241141311423  
100100421123123201111000111\_211001320000010011000100023221002110000201001  
\_333122000231221024011102\_12221122310001103311102101100101030113121210101  
21111211\_1000322101230201212132111011002331224211000120301010022173441010  
1002112211444421211440132123314313011222123310121111222412231113322132110  
000410432012120011322231200\_253212033233111100210022013011321131200101314  
322112211223234422230321421532200202142123232043112312003314223452134110  
412322220221

Svedberg and Inouye, *Zeitschr. F. physik. Chemie* 1911, **77**:145

Visual counting of visible gold particles, as seen through an “ultramicroscope”

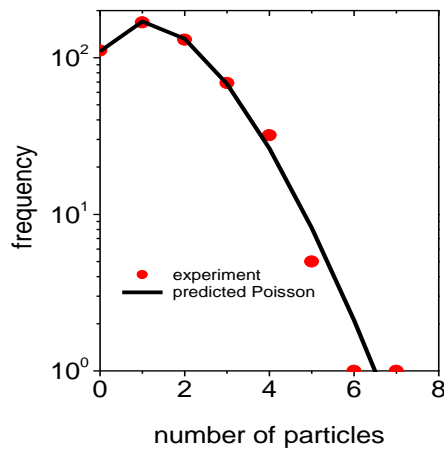


Need for statistical analysis



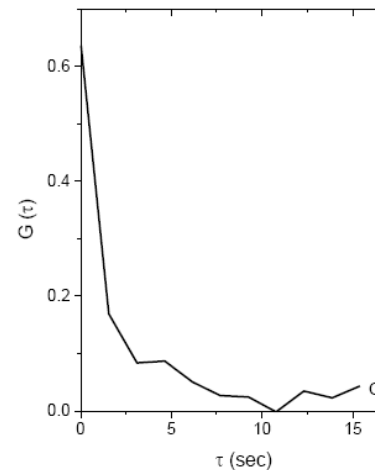
**Histogram of particles numbers**

**\*Autocorrelation**



- **Poisson law**

$$\langle N \rangle = 1.55$$

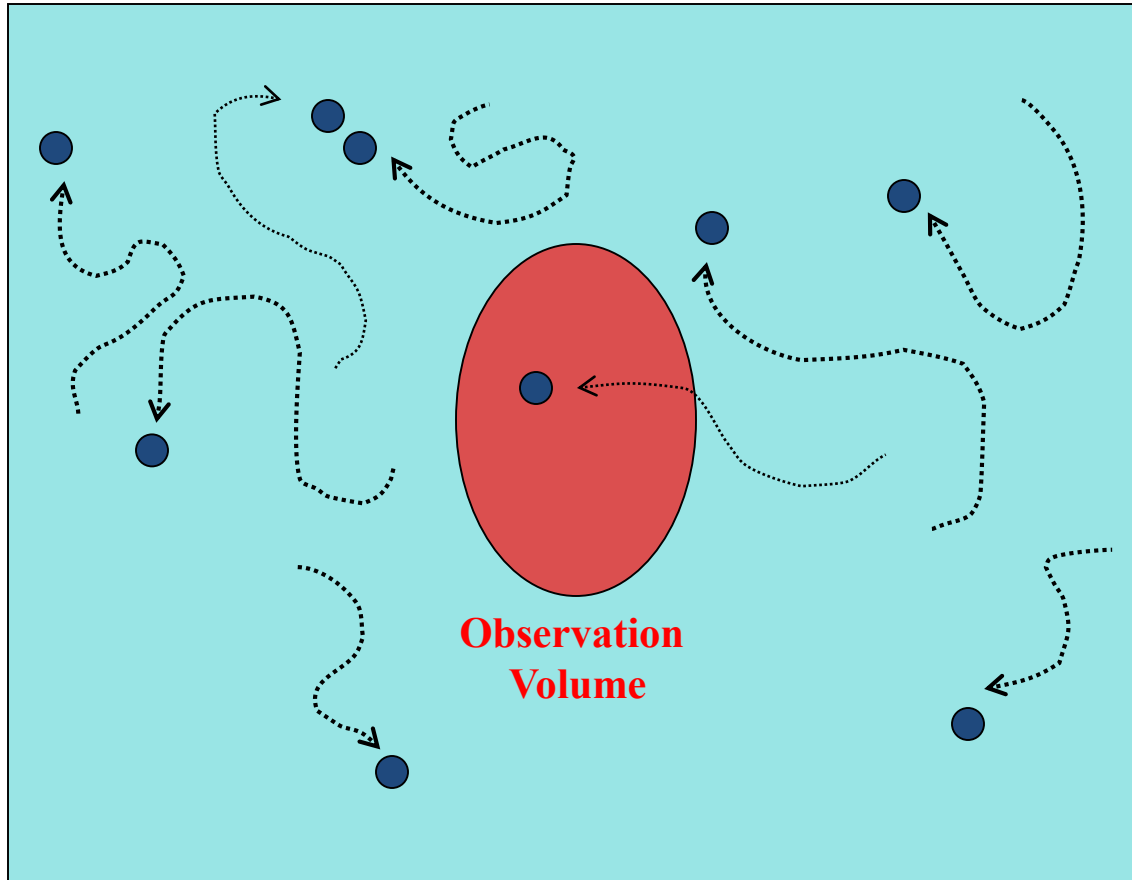


- **Today : easy to calculate**

$$\langle N \rangle = \frac{1}{G(0)} = 1.56$$

Poisson law : describes independant events, occurring in a fixed time lag, with a fixed average frequency

# Observation of fluctuations



Sample Space

**What can we observe ?**

**1. Speed of the movement**

**2. Particles concentration**

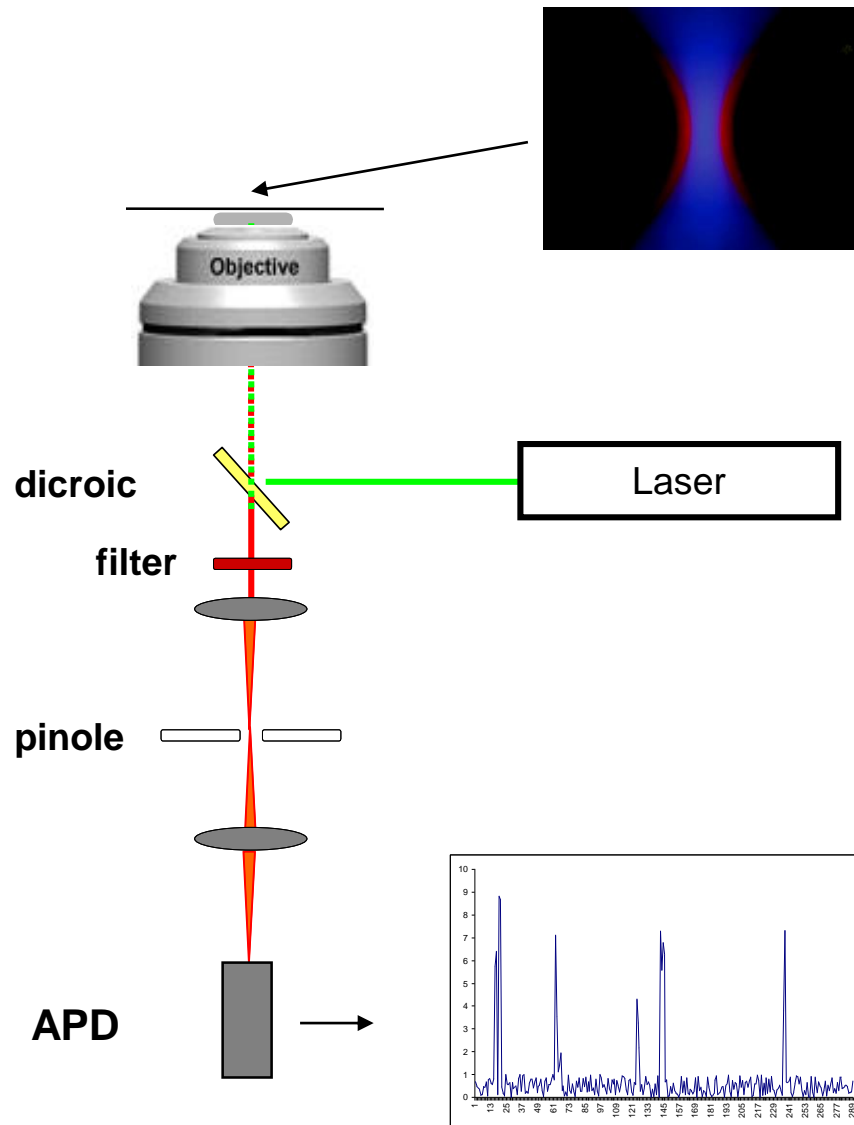
**3. Change of state, and thus of the signal, of the particles**

**→ Particles interactions**

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# Observing fluctuations : fluorescence microscopy

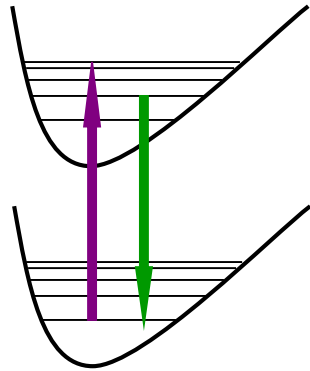




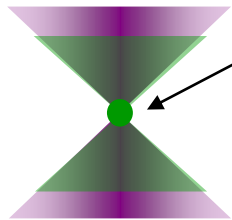
# The observation volume

## One- & Two-Photon Excitation.

### 1 - Photon



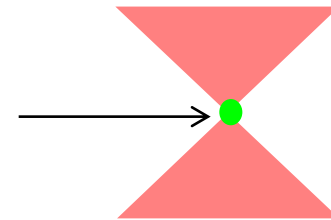
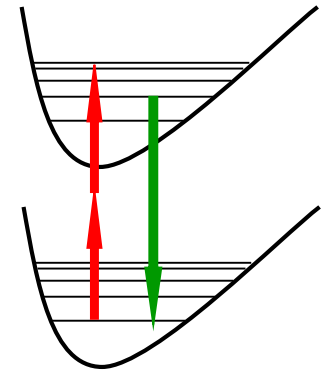
Defined by the size of the pinole, the wavelength, the magnification, and the numerical aperture



$\sim 1 \text{ um}^3$

Defined by the wavelength and the numerical aperture

### 2 - Photon



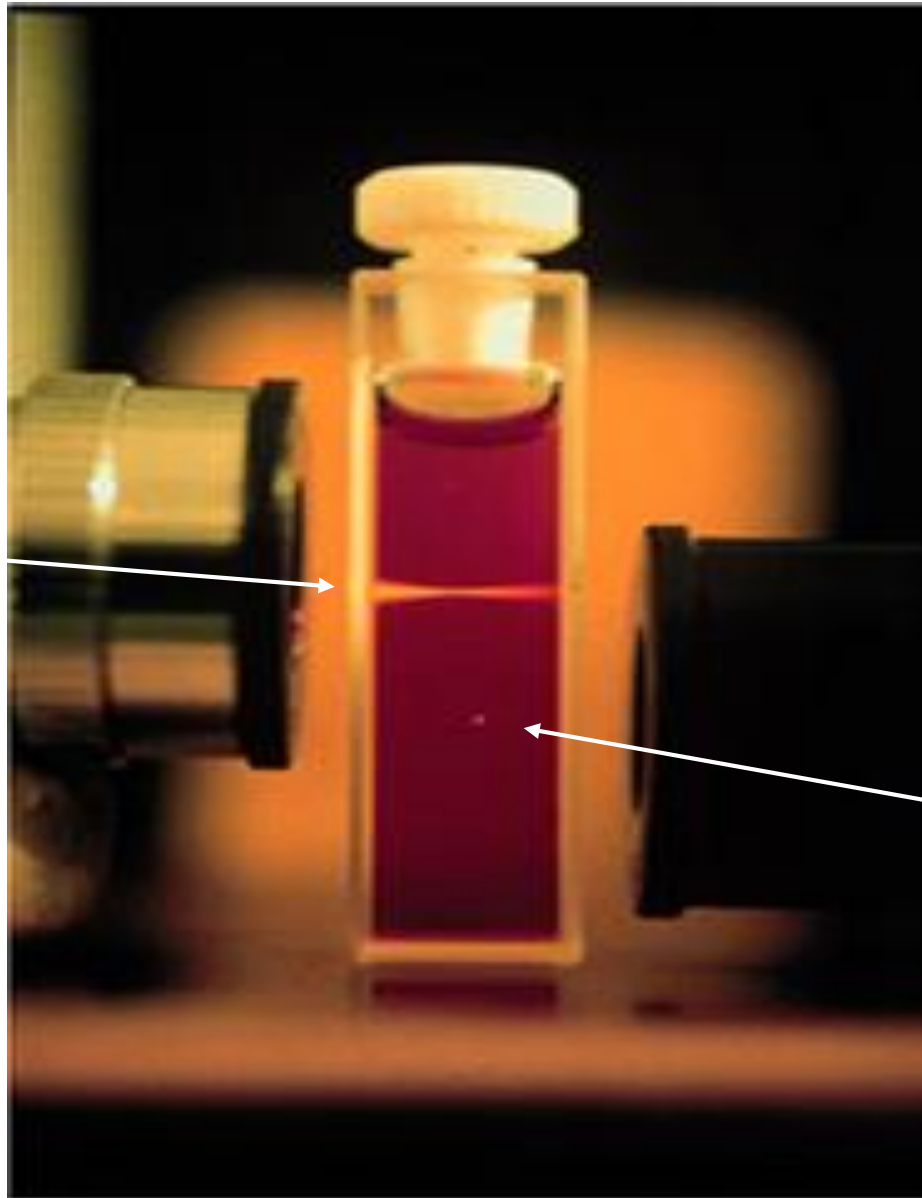
1-photon



Need a pinhole to  
define a small volume

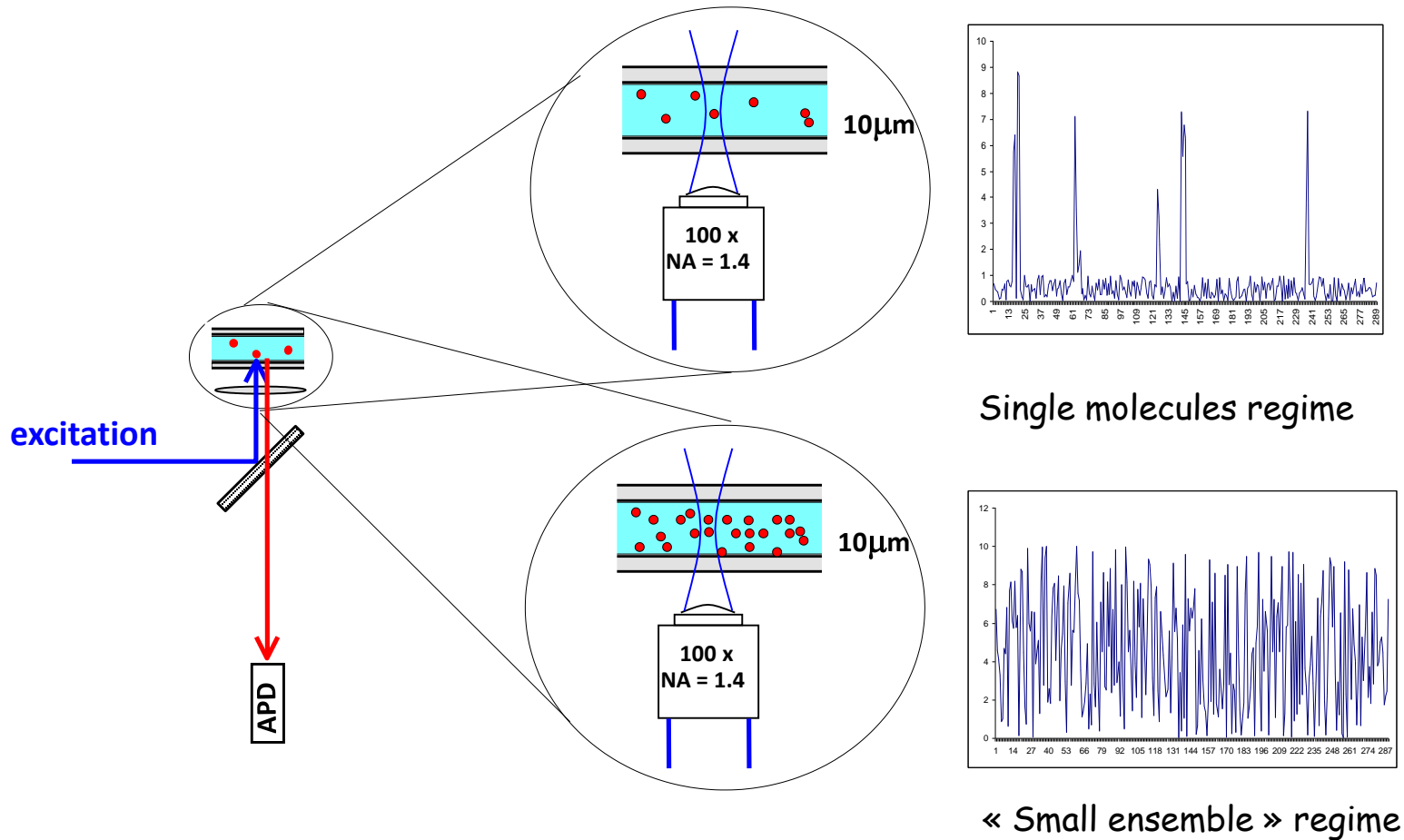


2-photon



Brad Amos  
MRC, Cambridge, UK

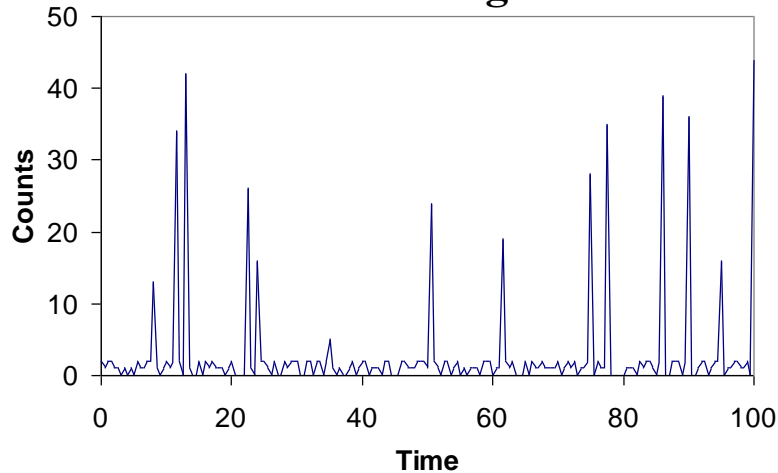
# Fluorescence correlation spectroscopy



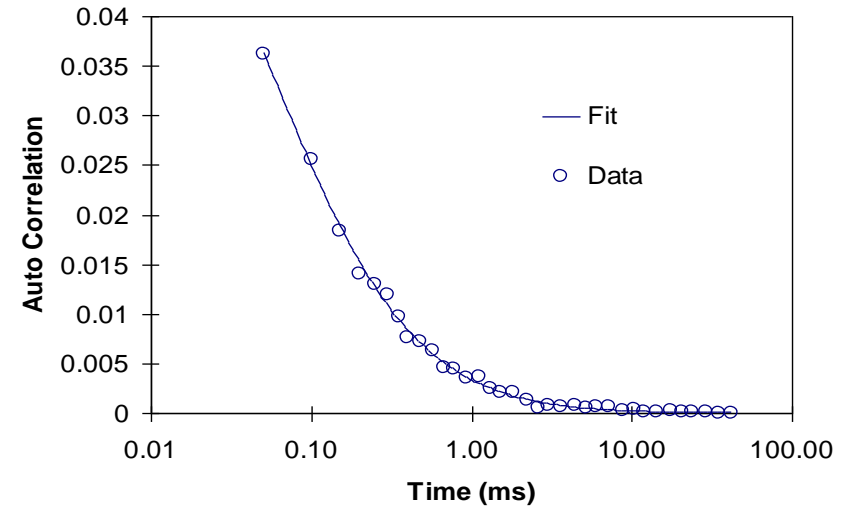
Using FCS, we will analyze these intensity fluctuations

# Data Treatment & Analysis

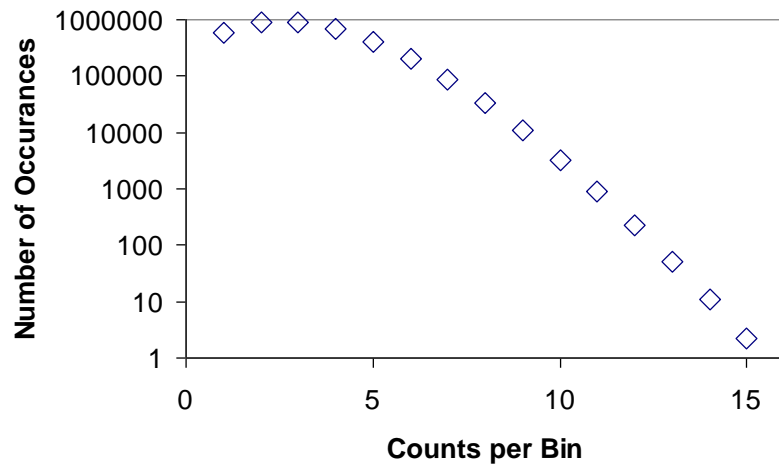
## Time Histogram



## Autocorrelation



## Photon Counting Histogram (PCH)



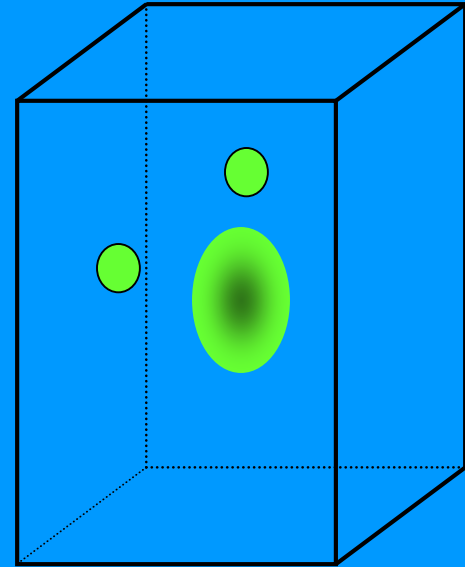
**Autocorrelation Parameters:**  
 **$G(0)$  &  $k_{\text{action}}$**

**PCH Parameters:  $\langle N \rangle$  &  $\underline{\epsilon}$**

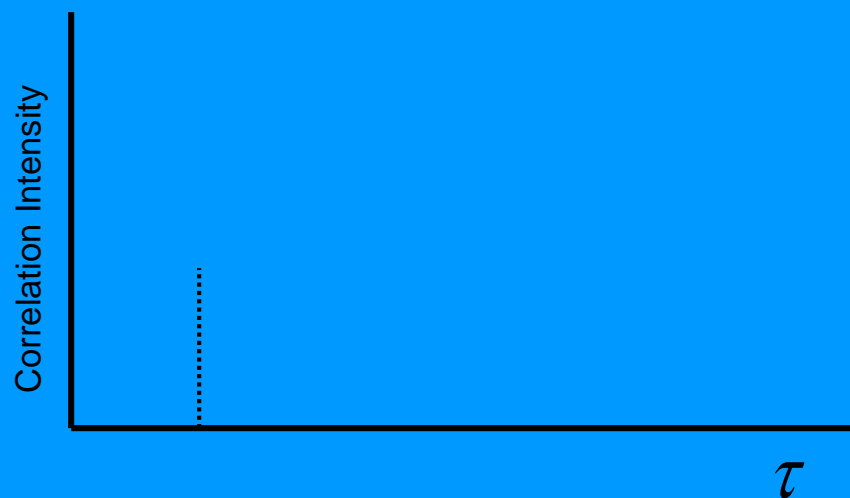
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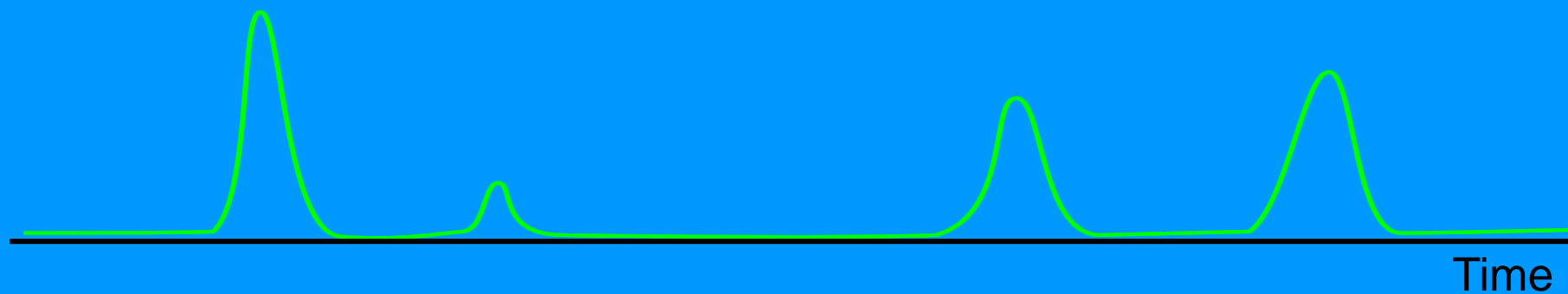
## Introduction to correlation function.



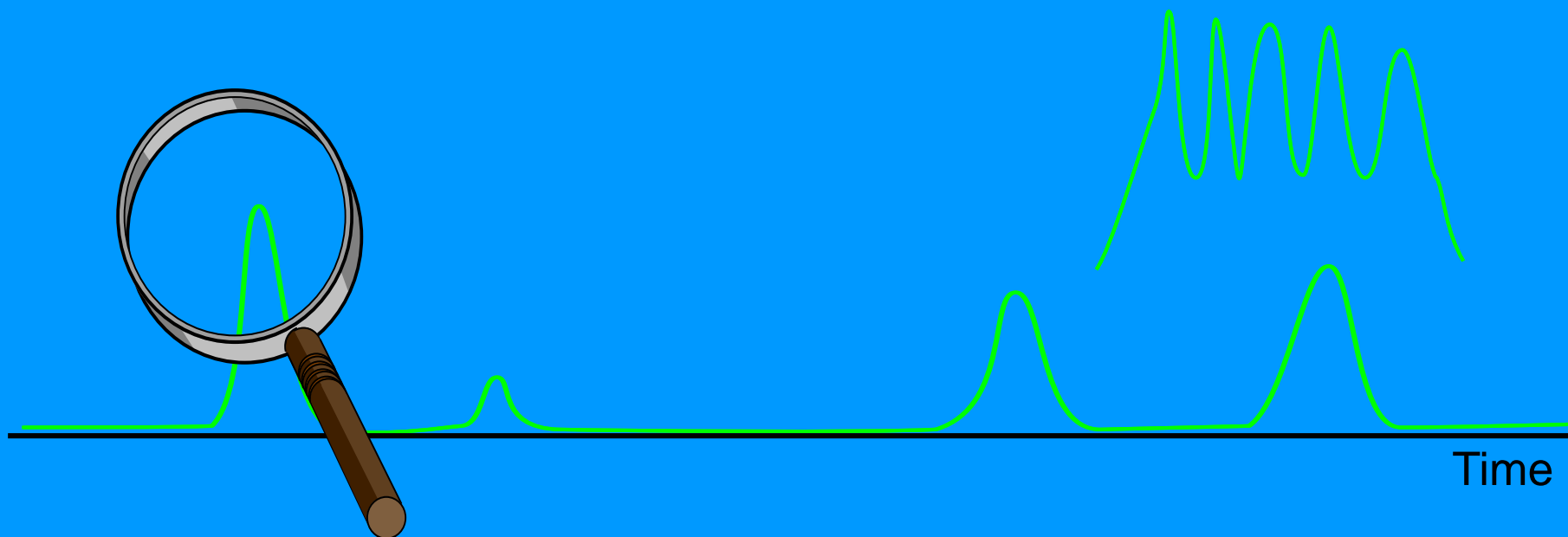
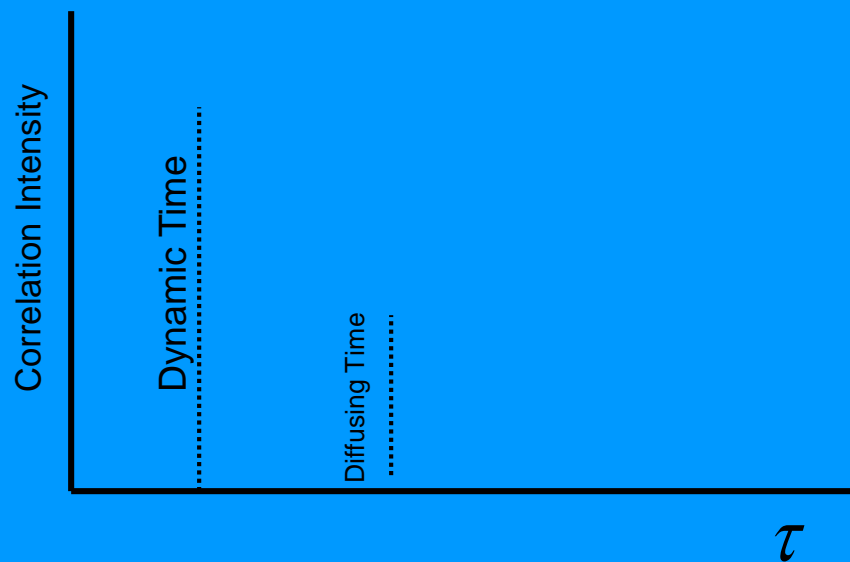
# AutoCorrelation. Diffusion only, No Dynamics. “Donor-Donor”



$$G(\tau) = \frac{\langle F(t + \tau) \cdot F(t) \rangle}{\langle F \rangle^2}$$



# AutoCorrelation. Diffusion and Dynamics. “Donor-Donor”





## 2 definitions for the autocorrelation function

$$G(\tau) = \frac{\langle \delta F(t) \delta F(t + \tau) \rangle}{\langle F \rangle^2}$$

$$\delta F(t) = F(t) - \langle F(t) \rangle$$

Decays to 0 for  $\tau \rightarrow \infty$

$$G(\tau) = \frac{\langle F(t + \tau) \cdot F(t) \rangle}{\langle F \rangle^2}$$

Decays to 1 for  $\tau \rightarrow \infty$

Factors influencing the fluorescence signal:

$$F(t) = \kappa Q \int d\mathbf{r} W(\mathbf{r}) C(\mathbf{r}, t)$$

$\kappa Q$  = quantum yield and detector sensitivity (how bright is our probe). This term could contain the fluctuation of the fluorescence intensity due to internal processes

$W(\mathbf{r})$  describes our observation volume

$C(\mathbf{r}, t)$  is a function of the fluorophore concentration over time. This is the term that contains the “physics” of the diffusion processes

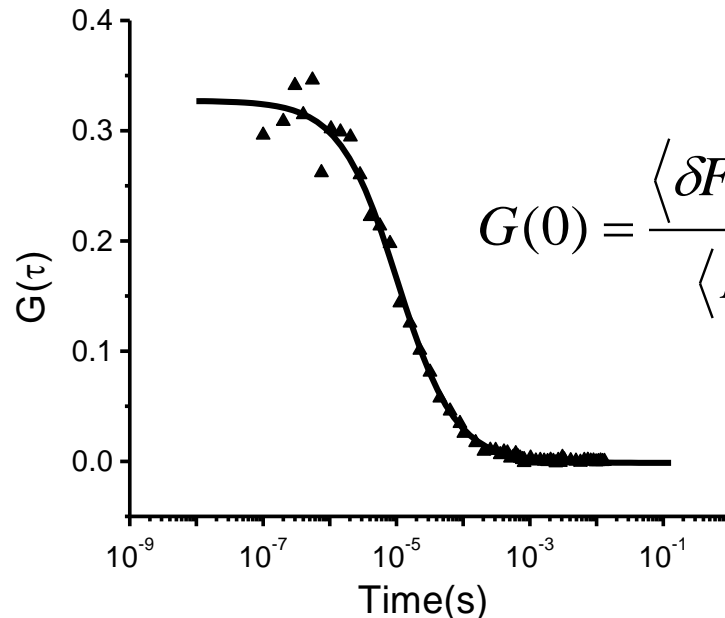
## $G(0) = 1/Nb$ of observed particles

In a poissonian systme the variance of the number of events is equal to its mean  
(here the average numer of observed particles)

$$\langle N \rangle = \text{Variance}_N = \sigma_N^2$$

$F = \varepsilon.N$  , avec  $\varepsilon$  = molecular brightness

$$G(\tau) = \frac{\langle \delta F(t) \delta F(t + \tau) \rangle}{\langle F(t) \rangle^2}$$



$$G(0) = \frac{\langle \delta F(t)^2 \rangle}{\langle F \rangle^2} = \frac{\langle (F(t) - \langle F(t) \rangle)^2 \rangle}{\langle F \rangle^2} = \frac{\sigma_F^2}{\langle F \rangle^2} = \frac{\varepsilon^2 \cdot \sigma_N^2}{\varepsilon^2 \cdot \langle N \rangle^2}$$

$$G(0) = \frac{\text{Variance}}{\langle N \rangle^2} = \frac{1}{\langle N \rangle}$$

## About the observation volume

$$G(0) = \frac{1}{\langle N \rangle} = \frac{1}{\langle C \rangle V_{eff}} = \frac{\gamma}{\langle C \rangle V_{PSF}}$$

$V_{eff}$  = effective volume

$V_{PSF}$  = volume of the psf

$\gamma$  = shape factor, that takes into account the non-uniformity of the observation volumes. Practically,  $\gamma < 1$

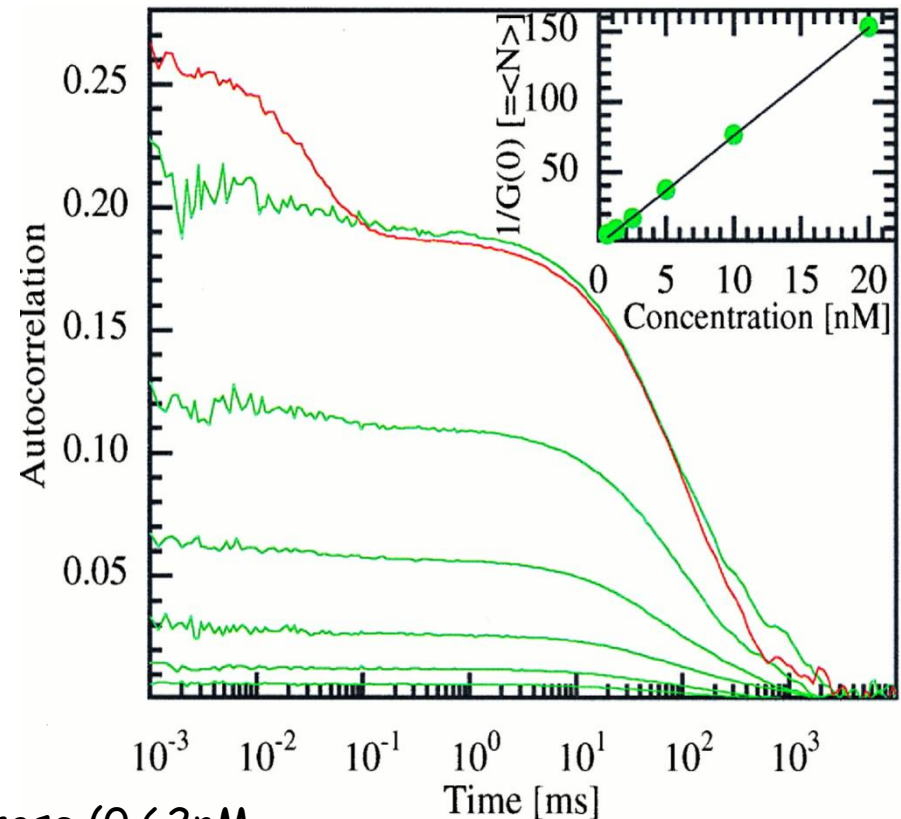
$$\gamma = \frac{1}{2\sqrt{2}} \quad \text{3D Gaussian volume}$$

$$\gamma = \frac{3}{4\pi^2} \quad \text{2D Gaussian – Lorentzien volume}$$

# Effect of the concentration

For a single species, no photobleaching

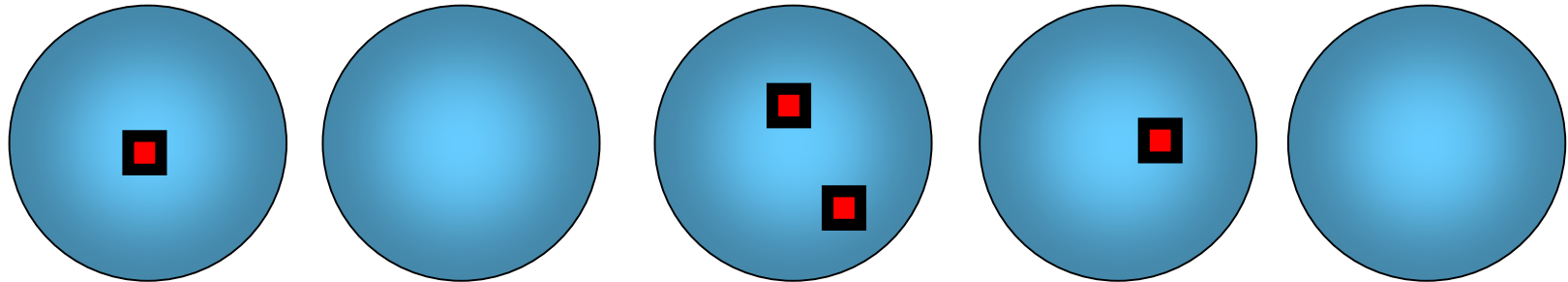
$$G(0) = \frac{1}{\langle N \rangle}$$



Rhodamine 6G in 70% sucrose (0.62 nM,  
1.25 nM, 2.5 nM, 5 nM, 10 nM, 20 nM)  
(green curves)

0.62 nM with a 70 times higher  
excitation power (red curve)

# G(0), Particle Brightness and Poisson Statistics



1 0 0 0 0 0 0 0 0 2 0 1 1 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 0 1 0 0

Time

Average = 0.275

Variance = 0.256

$$\langle N \rangle \propto \frac{\text{Average}^2}{\text{Variance}} = \frac{0.275^2}{0.256} = 0.296$$

Lets increase the particle brightness by 4x:

4 0 0 0 0 0 0 0 0 8 0 4 4 4 0 0 0 0 0 0 4 0 0 0 0 0 0 0 4 0 4 0 0 0 4 0 0 4 0 0

Average = 1.1

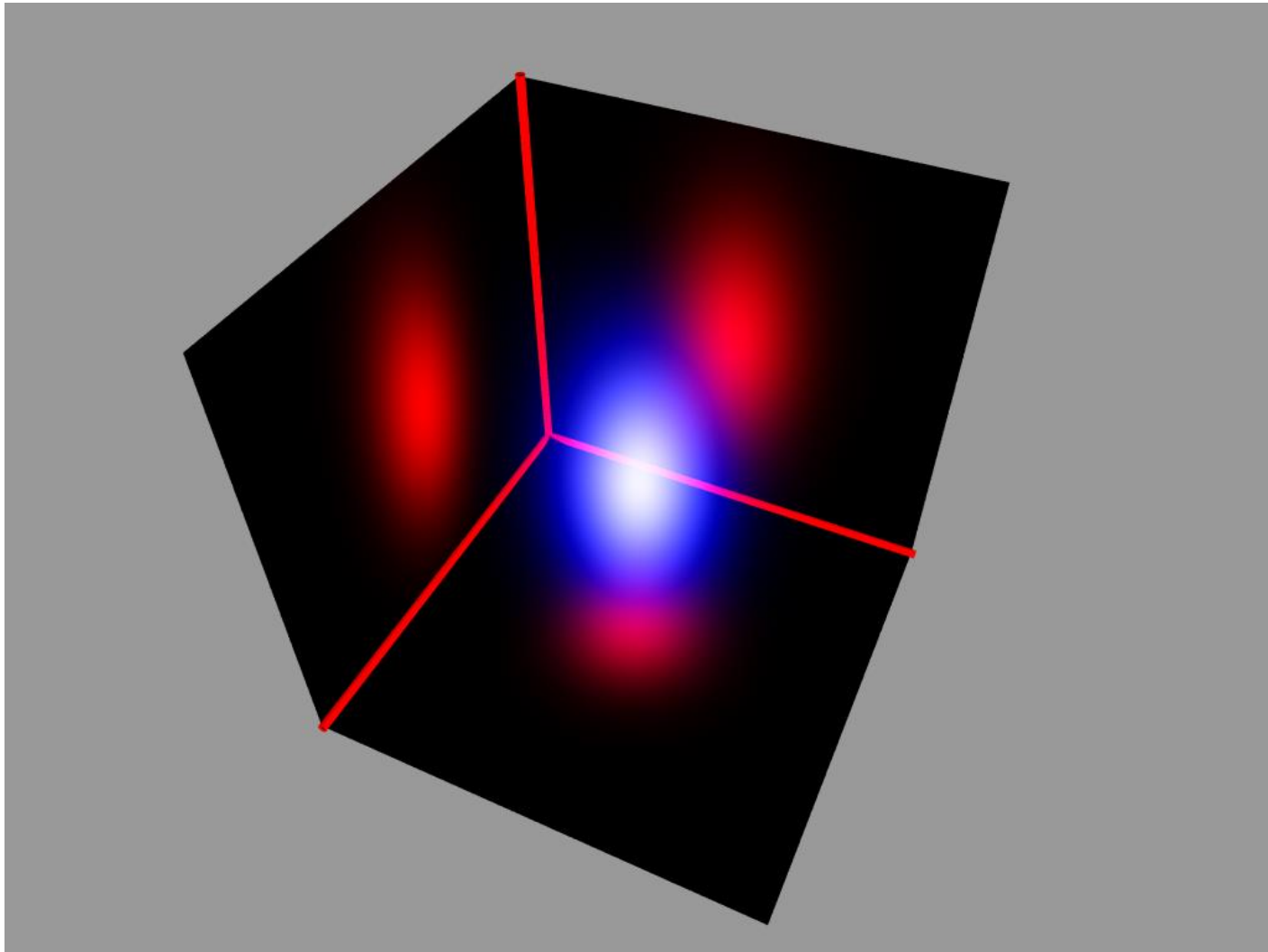
Variance = 4.09

$$\langle N \rangle \propto 0.296$$

$$F = \varepsilon \cdot N$$

Mais  $\varepsilon \times 4$

**What about the excitation (or observation) volume shape?**



# Volume shape and autocorrelation

## 2-dimensional Gaussian excitation volume:

$$G(\tau) = \frac{\gamma}{N} \left( 1 + \frac{4D\tau}{w_{2DG}^2} \right)^{-1}$$

For 2photon excitation,  $8D\tau$

## 3-dimensional Gaussian excitation volume:

$$G(\tau) = \frac{\gamma}{N} \left( 1 + \frac{4D\tau}{w_{3DG}^2} \right)^{-1} \left( 1 + \frac{4D\tau}{z_{3DG}^2} \right)^{-1/2}$$

**Triplet :**

$$\left( 1 + \frac{T}{1-T} e^{\frac{-\tau}{\tau_T}} \right)$$

*..where  $T$  is the triplet state amplitude and  $\tau_T$  is the triplet lifetime.*

## Orders of magnitude (for 1 $\mu\text{M}$ solution, small molecule, water)

Volume	Device	Size( $\mu\text{m}$ )	Molecules	Time
milliliter	cuvette	10000	$6 \times 10^{14}$	$10^4$
microliter	plate well	1000	$6 \times 10^{11}$	$10^2$
nanoliter	microfabrication	100	$6 \times 10^8$	1
picoliter	typical cell	10	$6 \times 10^5$	$10^{-2}$
femtoliter	confocal volume	1	$6 \times 10^2$	$10^{-4}$
attoliter	nanofabrication	0.1	$6 \times 10^{-1}$	$10^{-6}$



# The Effects of Particle Size on the Autocorrelation Curve

## Diffusion Constants

300  $\mu\text{m}^2/\text{s}$

90  $\mu\text{m}^2/\text{s}$

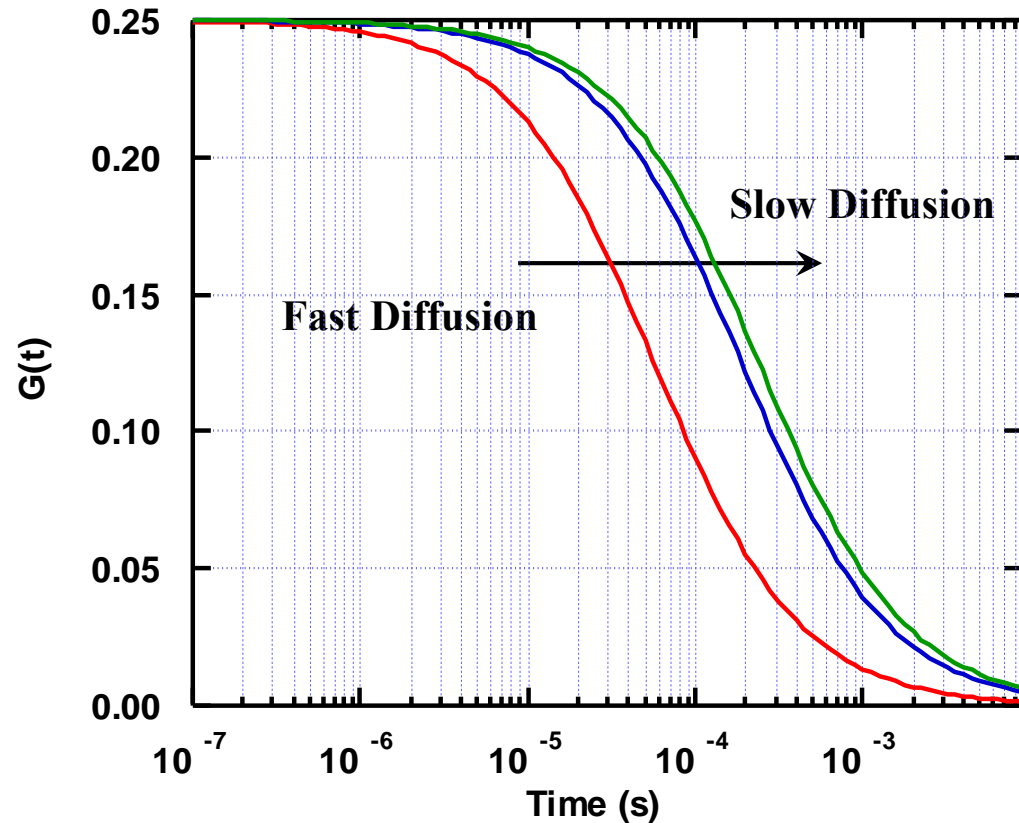
71  $\mu\text{m}^2/\text{s}$

## Stokes-Einstein Equation:

$$D = \frac{k \cdot T}{6 \cdot \pi \cdot \eta \cdot r}$$

and

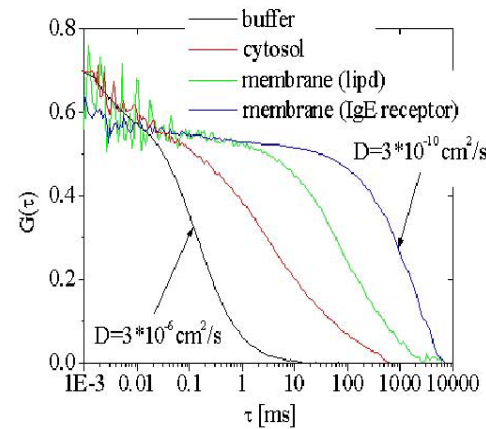
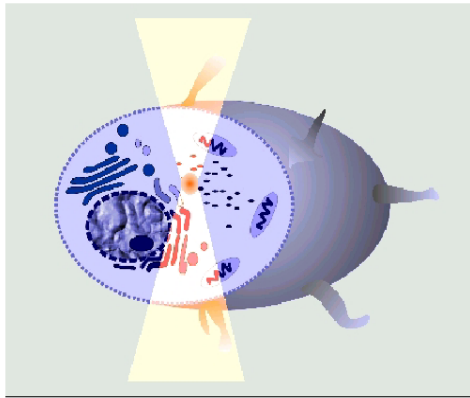
$$MW \propto \text{Volume} \propto r^3$$



Monomer  $\rightarrow$  Dimer

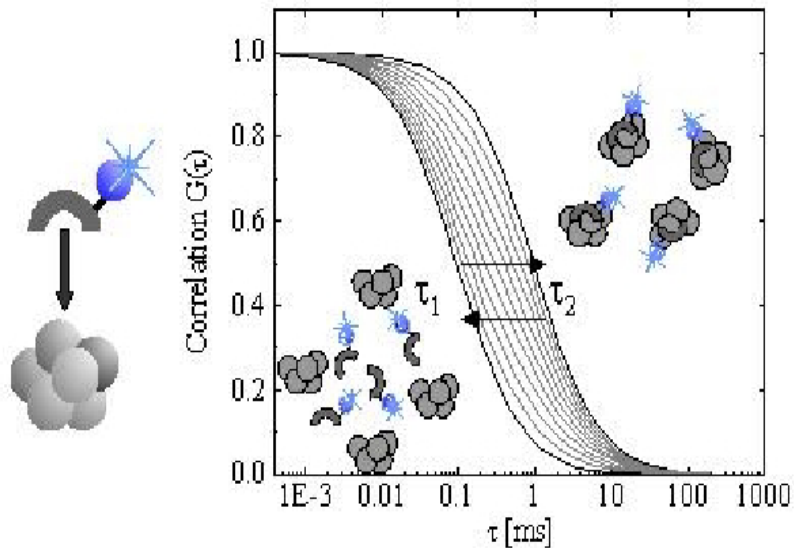
Only a change in  $D$  by a factor of  $2^{1/3}$ , or 1.26

# Mobility of a fluorescent macromolecule

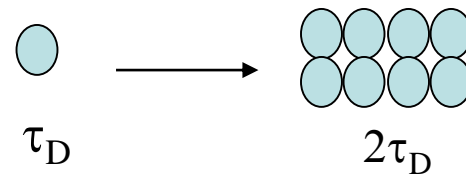


Rhodamine dye in different environments

# Measuring molecular interactions



$$\text{Diffusion time} = f(\text{MW})^{1/3}$$

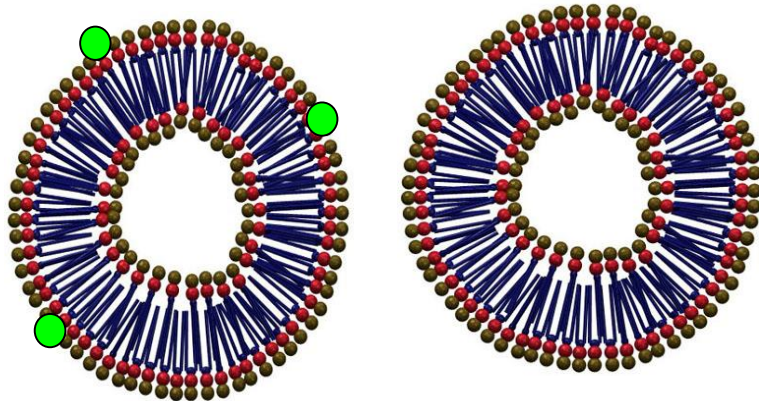


# Measurement of the interaction between ezrin and Large Unilamellar Vesicles (LUV)

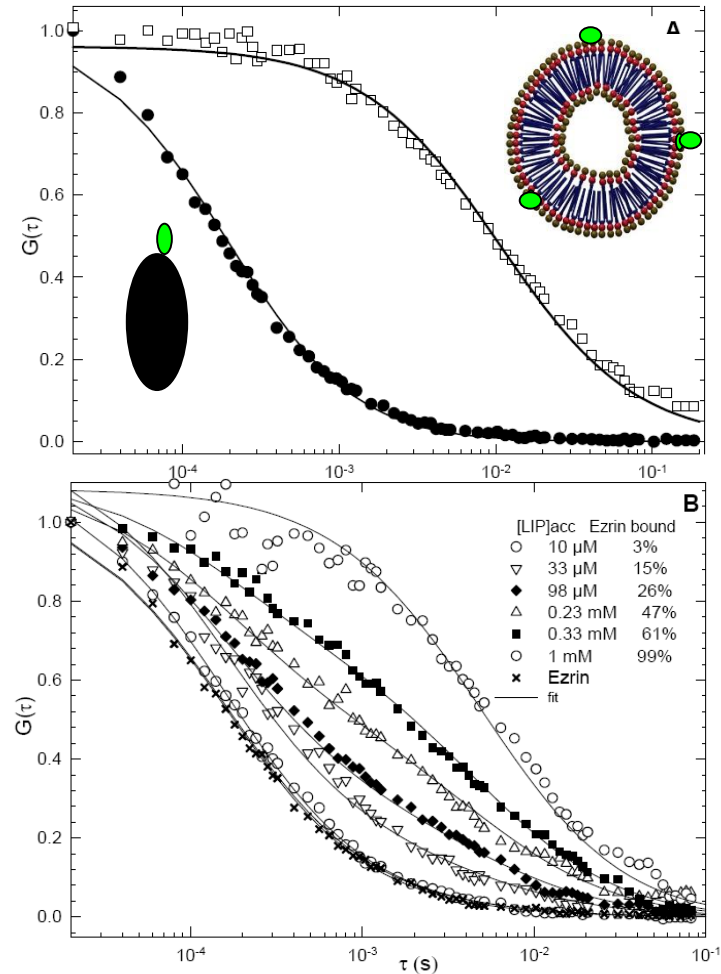
Collaboration with Pr. Picard (UM2)

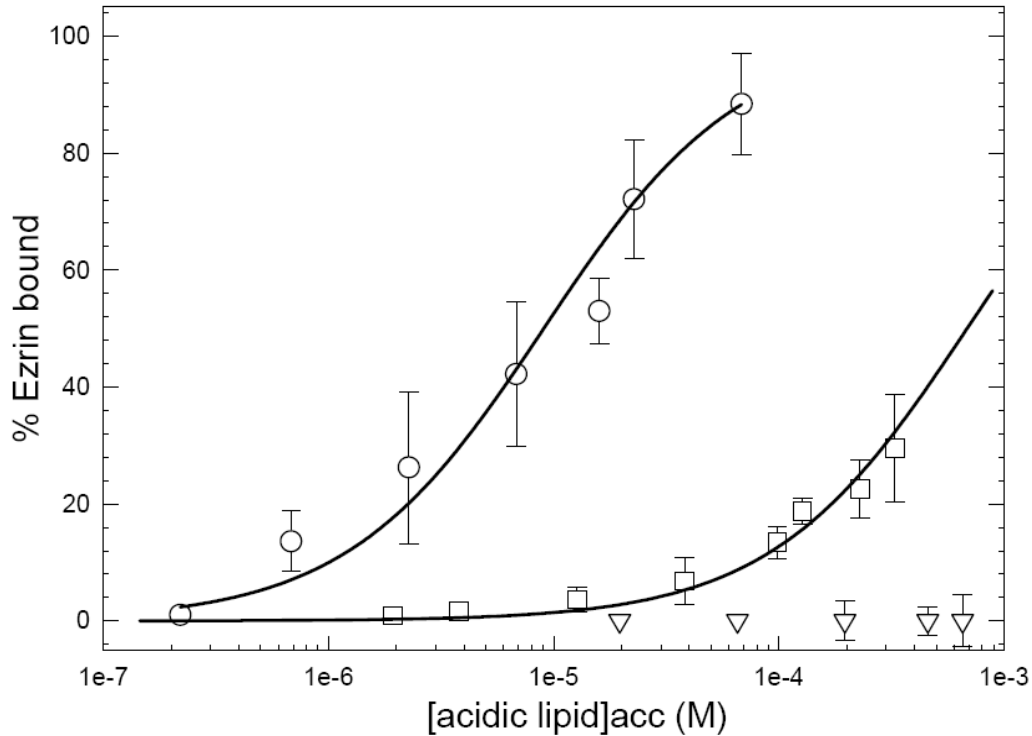


Ezrin : protein interacting with actin and phospholipids  
Specific labeling with Alexa488



LUV, labeled or unlabeled





- Titration curve allowing the determination of the affinity between ezrin and LUV with different lipid compositions
- Demonstration of préférential binding of ezrin to  $PIP_2$ -containing vesicles

# Multiple Species

Case 1: Species vary by a difference in diffusion constant,  $D$ .

*Autocorrelation function can be used:*

$$G(\tau)_{sample} = \sum_{i=1}^M f_i^2 \cdot G(0)_i \cdot \left(1 + \frac{8D\tau}{w_{2DG}^2}\right)^{-1} \quad (2D\text{-Gaussian Shape})$$

!  $f_i$  is the fractional fluorescence intensity of species  $i$ .

$$G(0)_{sample} = \sum f_i^2 \cdot G(0)_i$$

$G(0)_{sample}$  is no longer  $\gamma/N$  !

**Case 2: Species vary by a difference in brightness**  
**assuming that  $D_1 \approx D_2$**

The quantity  $G(0)$  becomes the only parameter to distinguish species,  
but we know that:

$$G(0)_{sample} = \sum f_i^2 \cdot G(0)_i$$

**The autocorrelation function is not suitable  
for analysis of this kind of data without additional information.**

We need a different type of analysis



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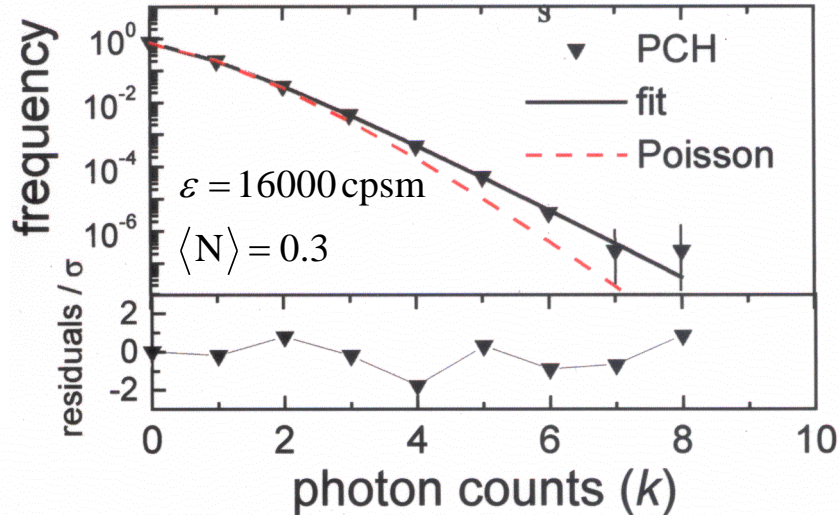
# Photon Counting Histogram (PCH)

**Aim:** To resolve species from differences in their molecular brightness

**Molecular brightness  $\varepsilon$  :** The average photon count rate of a single fluorophore

**PCH:** probability distribution function  $p(k)$

where  $p(k)$  is the probability of observing  $k$  photon counts



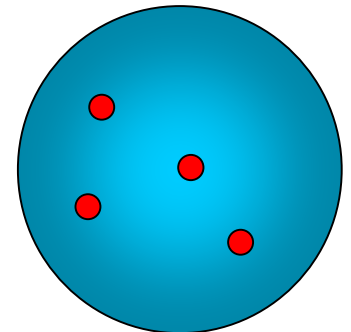
**Single Species:**

$$p(k) = PCH(\varepsilon, \langle N \rangle)$$

Note: PCH is Non-Poissonian!

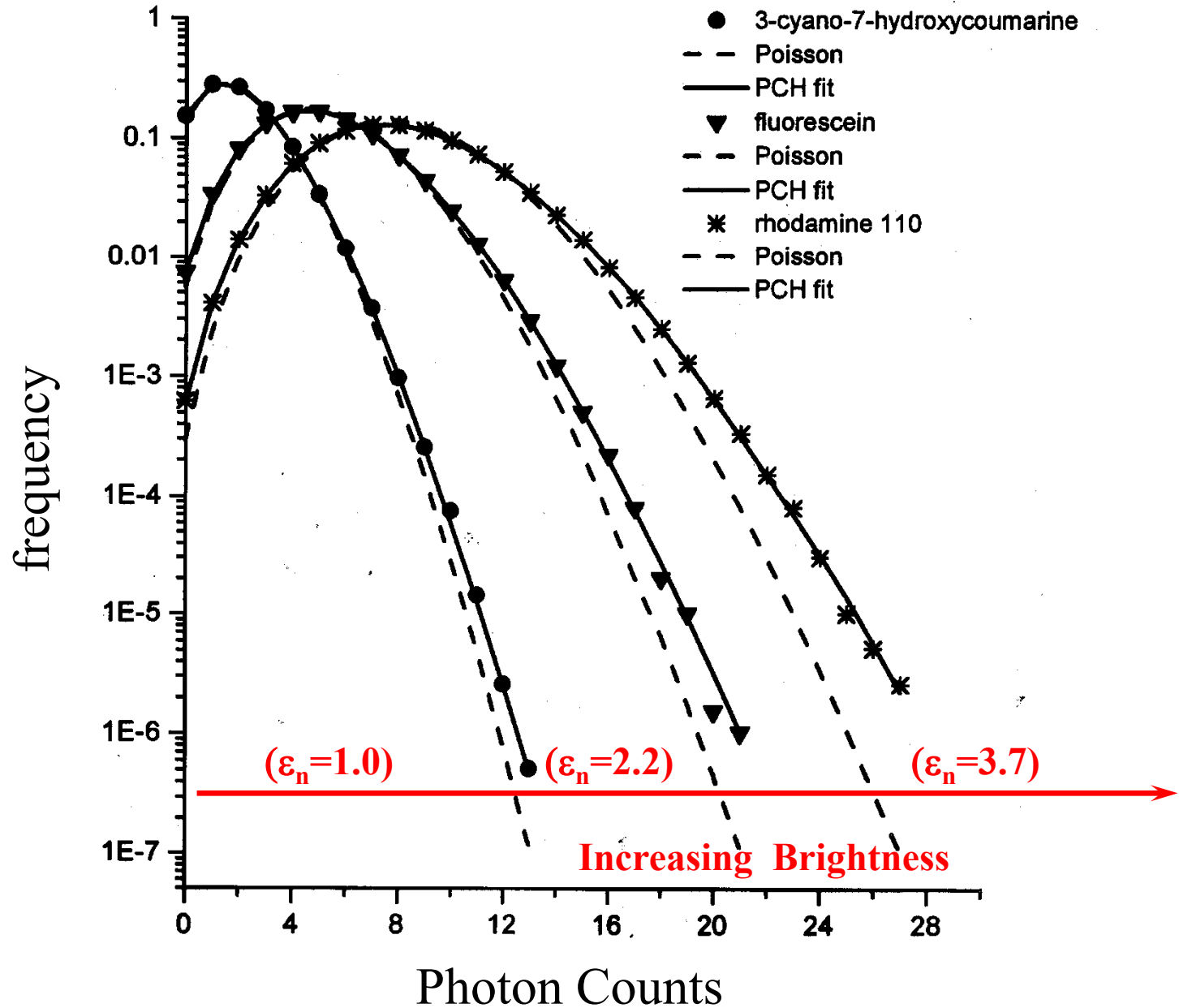
## Sources of Non-Poissonian Noise

- Detector Noise
- Diffusing Particles in an Inhomogeneous Excitation Beam\*
- Particle Number Fluctuations\*
- Multiple Species\*



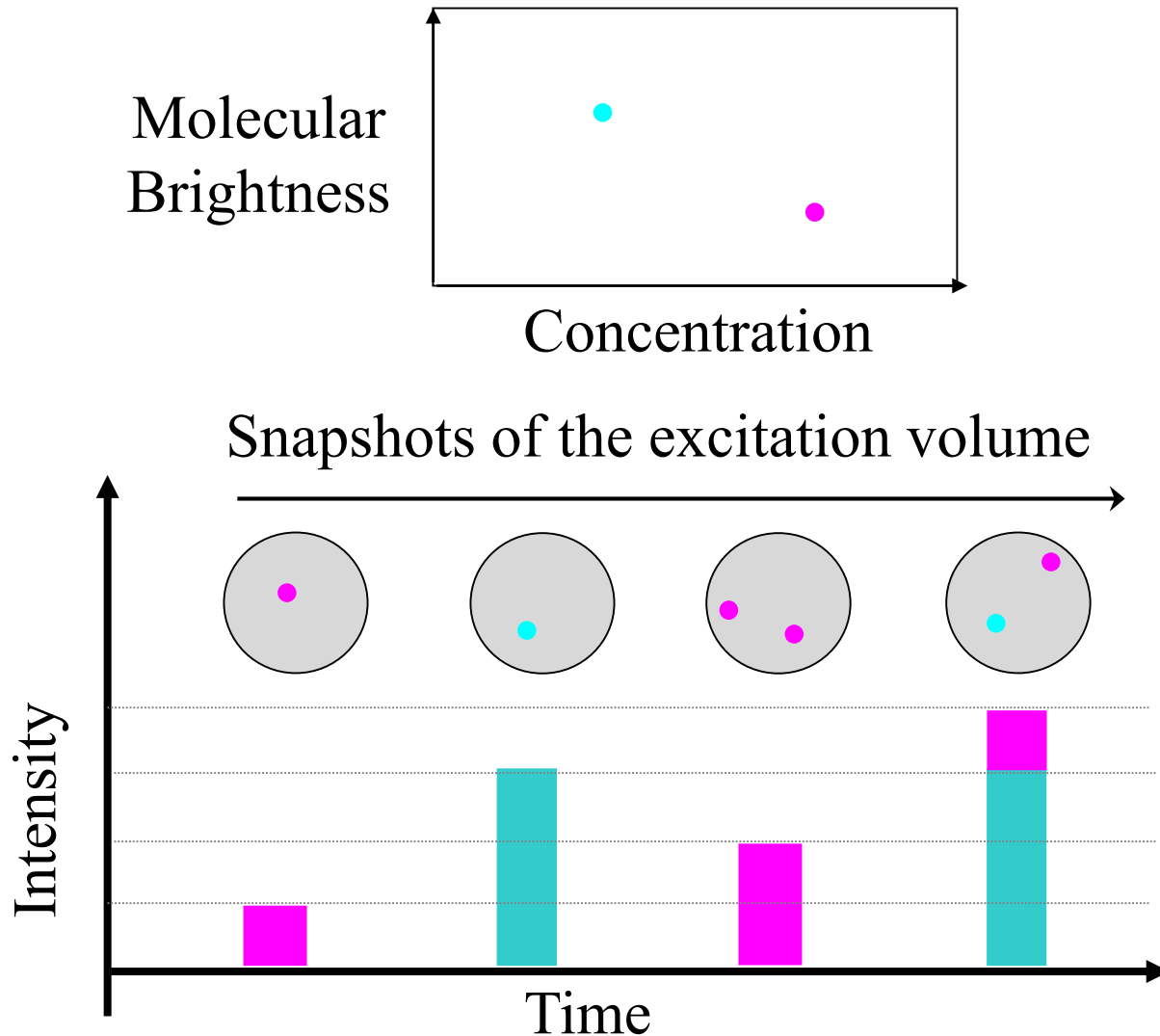


# PCH Example: Differences in Brightness



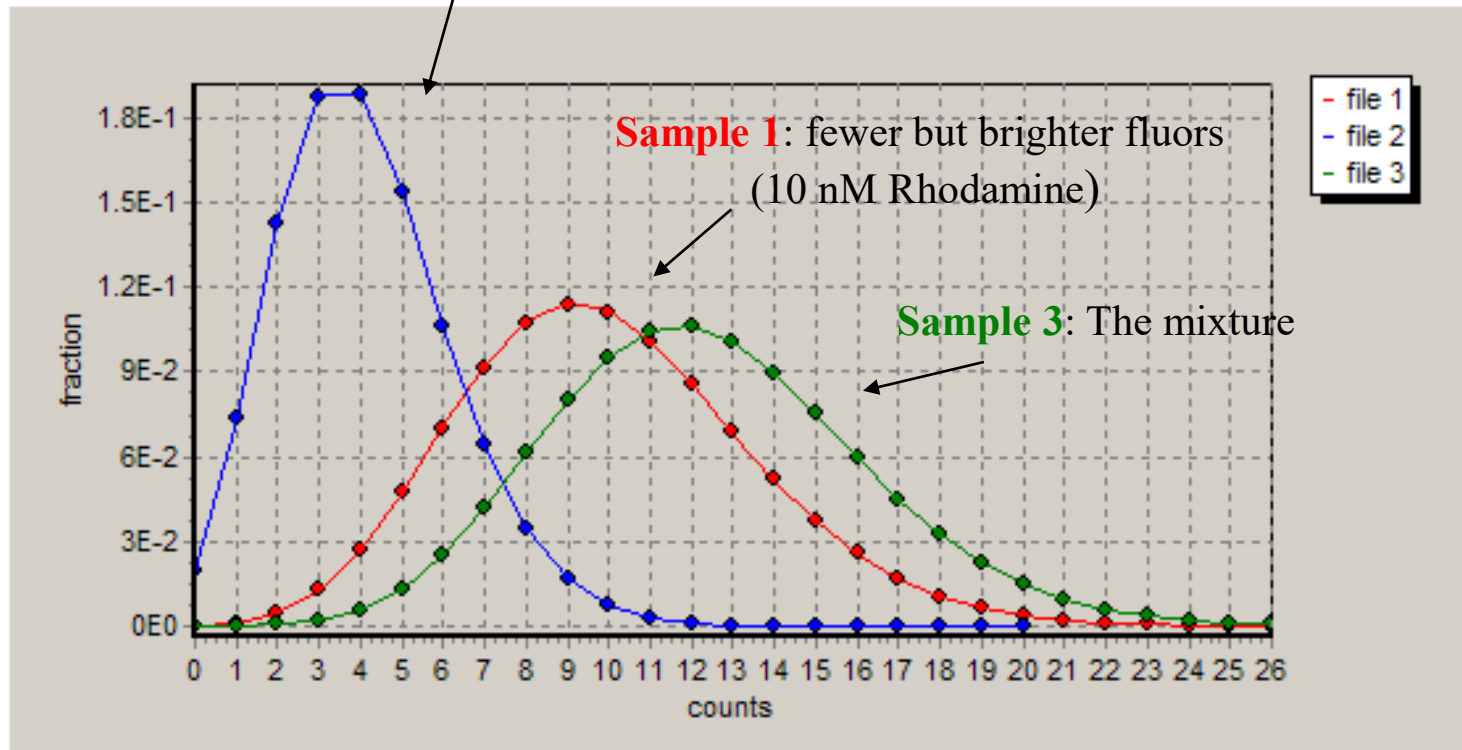
# Photon Counting Histogram: Multispecies

Binary Mixture:  $p(k) = PCH(\varepsilon_1, \langle N_1 \rangle) \otimes PCH(\varepsilon_2, \langle N_2 \rangle)$



# Photon Counting Histogram: Multispecies

**Sample 2:** many but dim (23 nM fluorescein at pH 6.3)

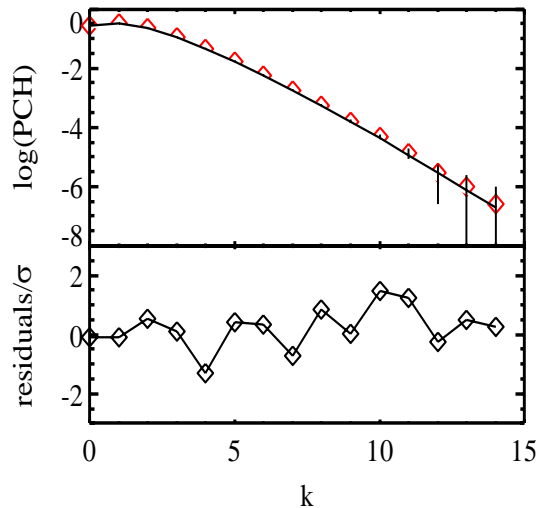


The occupancy fluctuations for each specie in the mixture becomes a convolution of the individual specie histograms. The resulting histogram is then broader than expected for a single species.

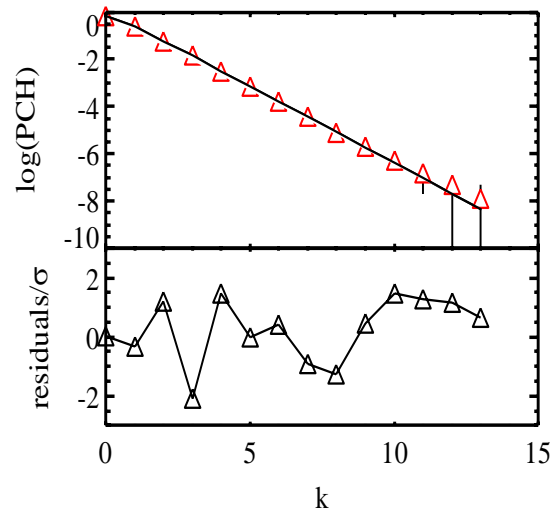
# Resolve a protein mixture with a brightness ratio of **two**

## Alcohol dehydrogenase labeling experiments

### **Singly** labeled proteins



### Mixture of singly or **doubly** labeled proteins

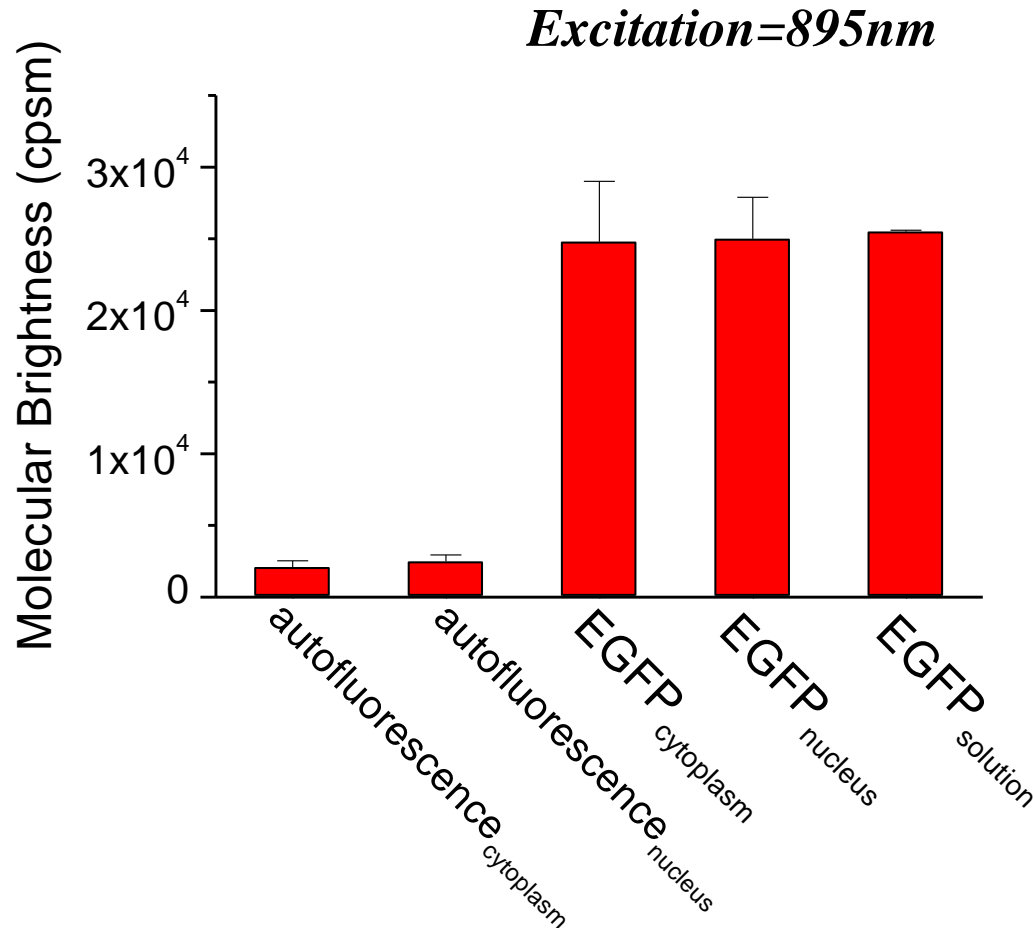


*Both species have  
**same***

- color
- fluorescence lifetime
- diffusion coefficient
- polarization

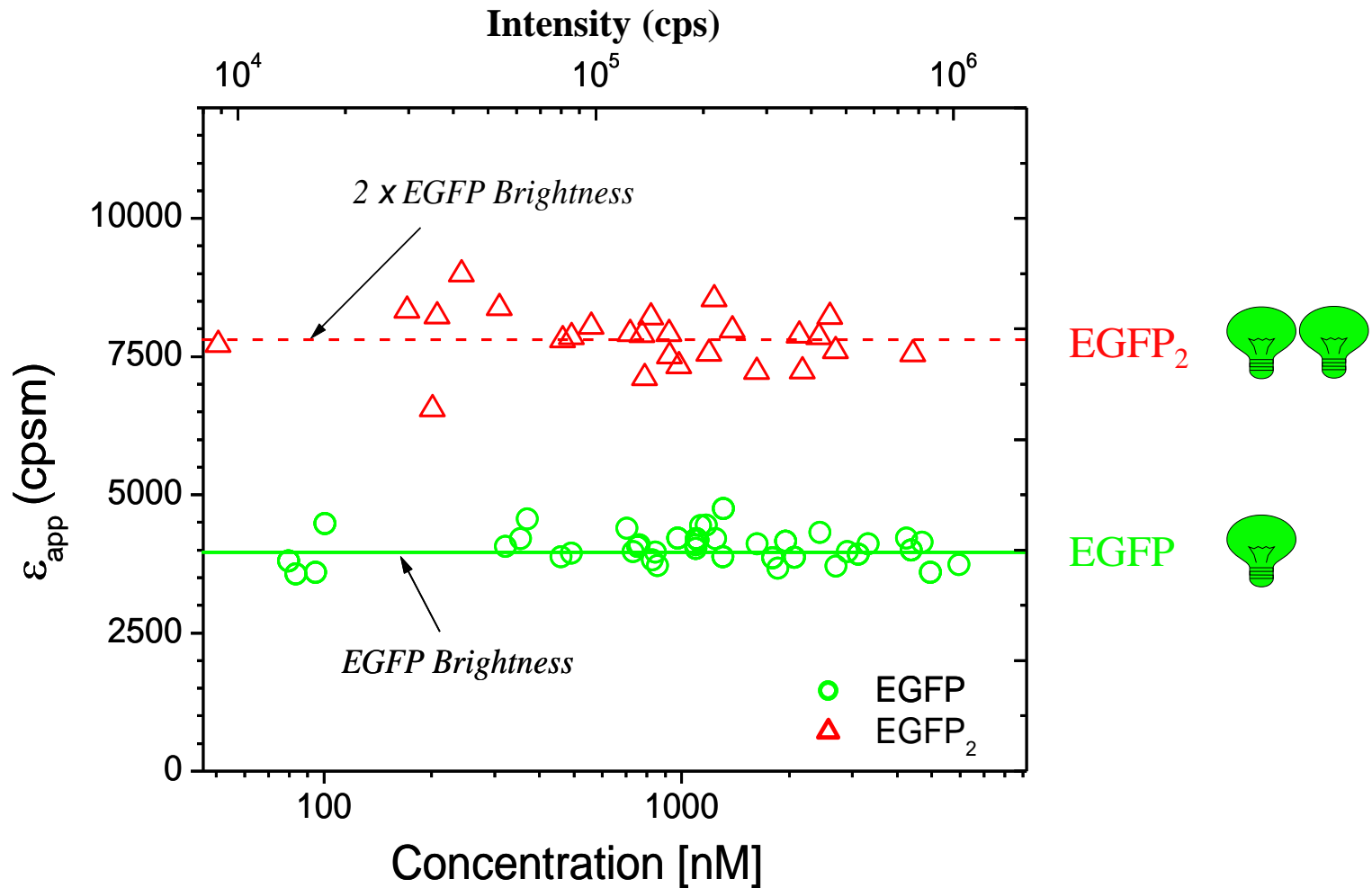
c	$\varepsilon_1$ kcpsm	$\bar{N}_1$	$\varepsilon_2$ kcpsm	$\bar{N}_2$
<b>Sample A</b>	$26.2^{+0.19}_{-0.18}$	$0.540^{+0.004}_{-0.004}$	-----	-----
<b>Sample B</b>	$25.1^{+0.6}_{-1.2}$	$0.155^{+0.007}_{-0.002}$	$56^{+10}_{-10}$	$0.006^{+0.008}_{-0.003}$

# PCH in cells: Brightness of EGFP



*The molecular brightness of EGFP is a factor **ten** higher than that of the autofluorescence in HeLa cells*

# Brightness and Stoichiometry

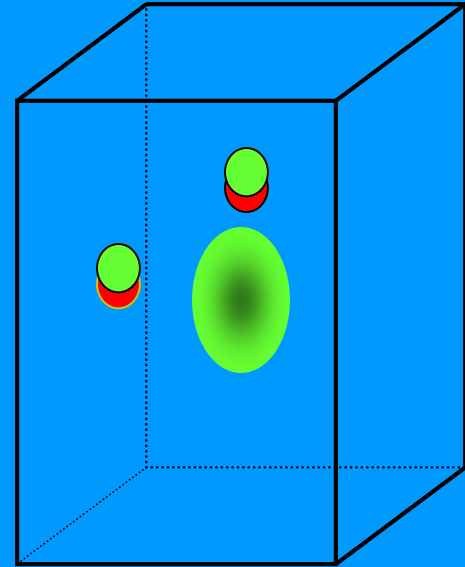


**Brightness of EGFP<sub>2</sub> is twice the brightness of EGFP**

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# Introduction to correlation function.

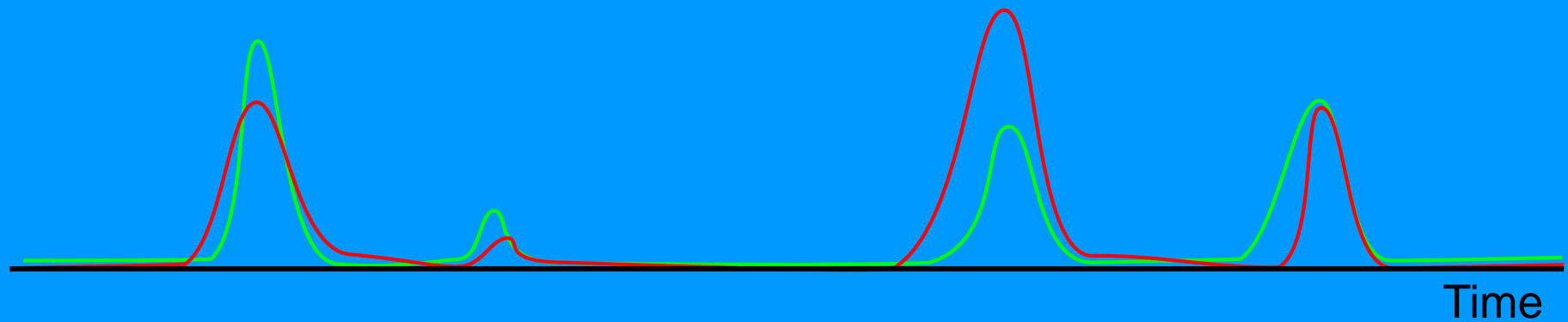


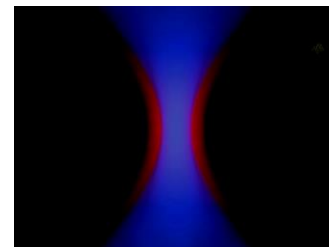
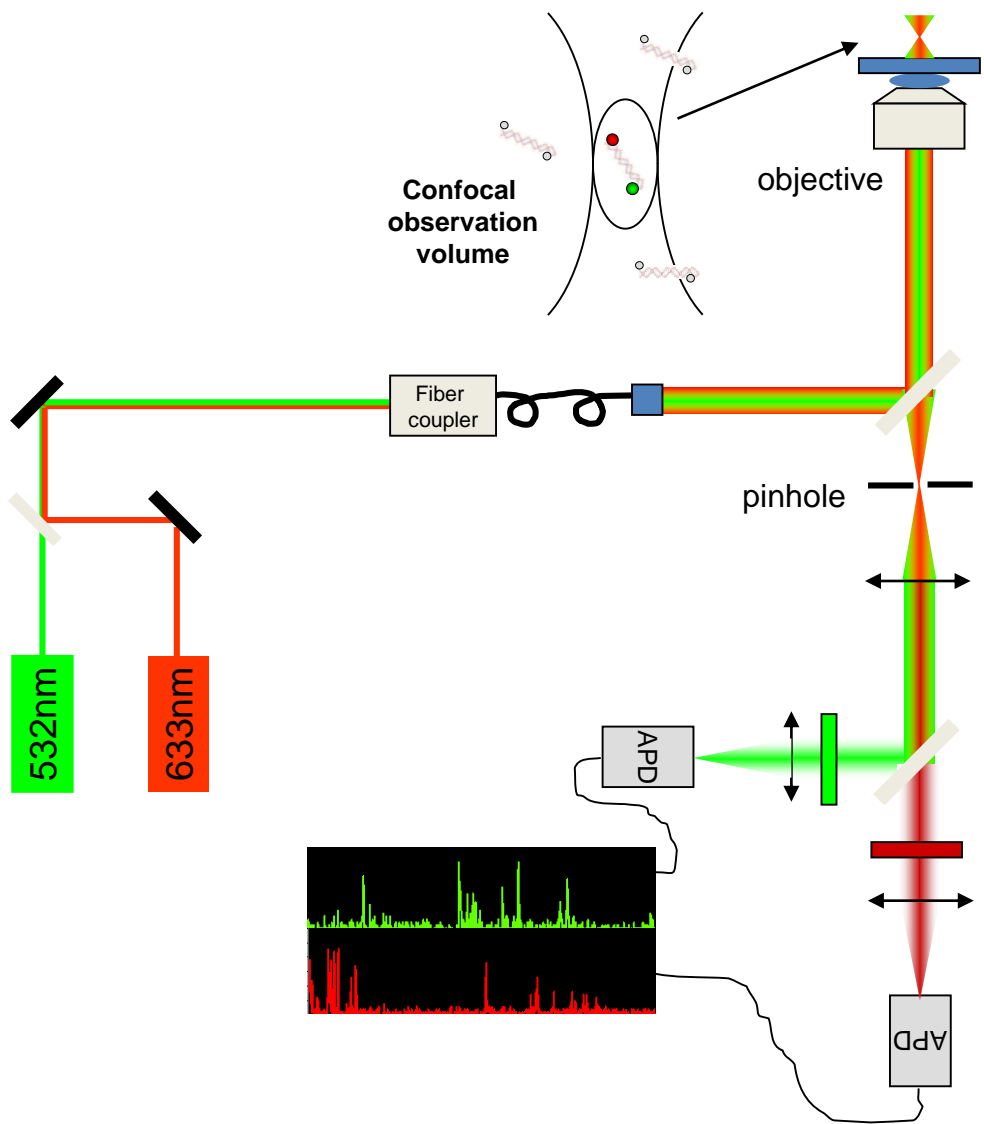
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Time



# CrossCorrelation. Diffusion only, No Dynamics. “Donor-Acceptor”

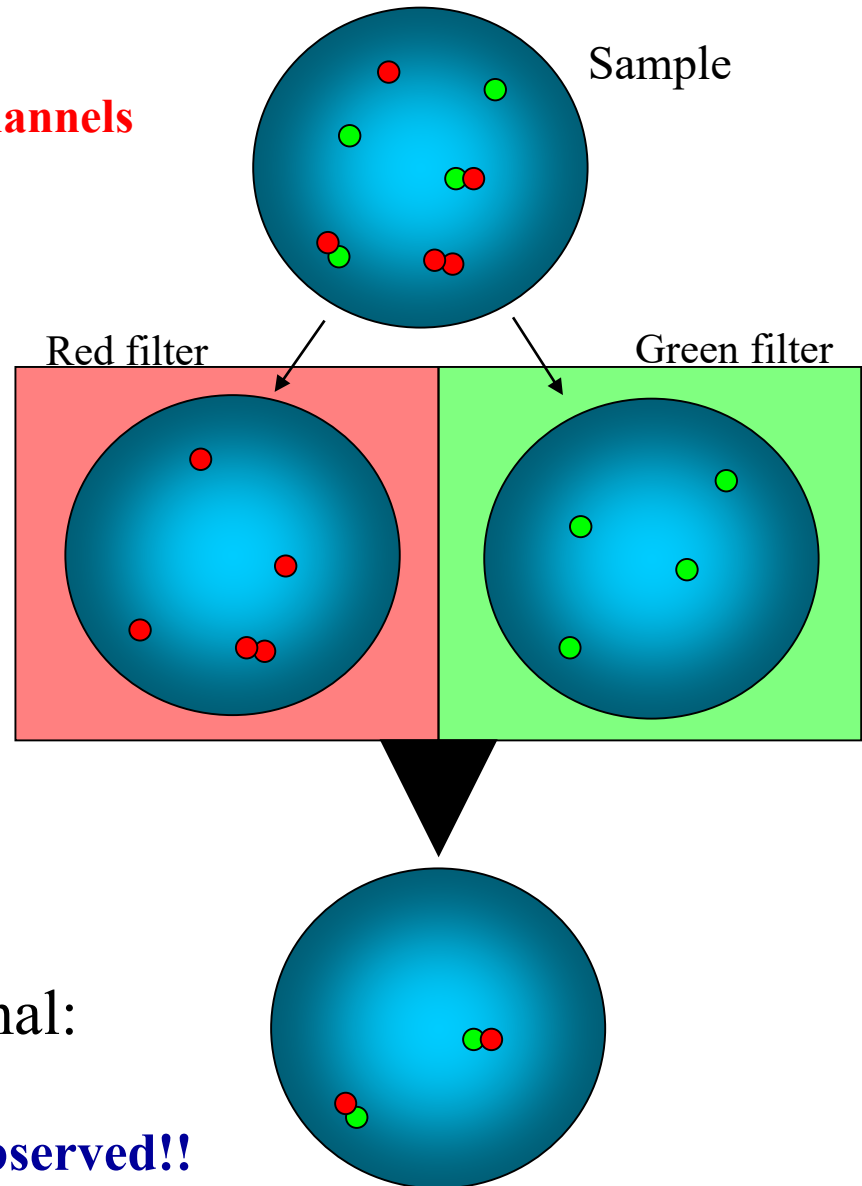




# Two-Color Cross-correlation

**The cross-correlation  
ONLY if particles are observed in both channels**

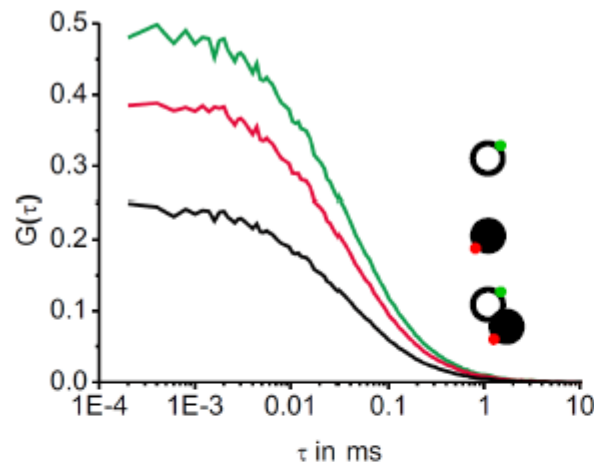
Each detector observes  
particles with a particular color



The cross-correlation signal:

**Only the green-red molecules are observed!!**

$$G_X(\tau) = \frac{\langle \delta F_1(t) \cdot \delta F_2(t + \tau) \rangle}{\langle F_1(t) \rangle \langle F_2(t) \rangle}$$



- $n_1$ : number of green molecules
- $n_2$ : number of red molecules
- $n_{12}$ : number of green-red molecules

$$g_1(0) = \frac{1}{n_1 + n_{12}}, \quad g_2(0) = \frac{1}{n_2 + n_{12}}, \quad g_{\times}(0) = \frac{n_{12}}{(n_1 + n_{12})(n_2 + n_{12})}$$

$$\frac{g_{\times}(0)}{g_2(0)} = \frac{n_{12}}{n_1 + n_{12}} \rightarrow \text{fraction of green in complex with red}$$

$$\frac{g_{\times}(0)}{g_1(0)} = \frac{n_{12}}{n_2 + n_{12}} \rightarrow \text{fraction of red in complex in complex with green}$$

# Cross-correlation Calculations

One uses the same fitting functions you would use for the standard autocorrelation curves.

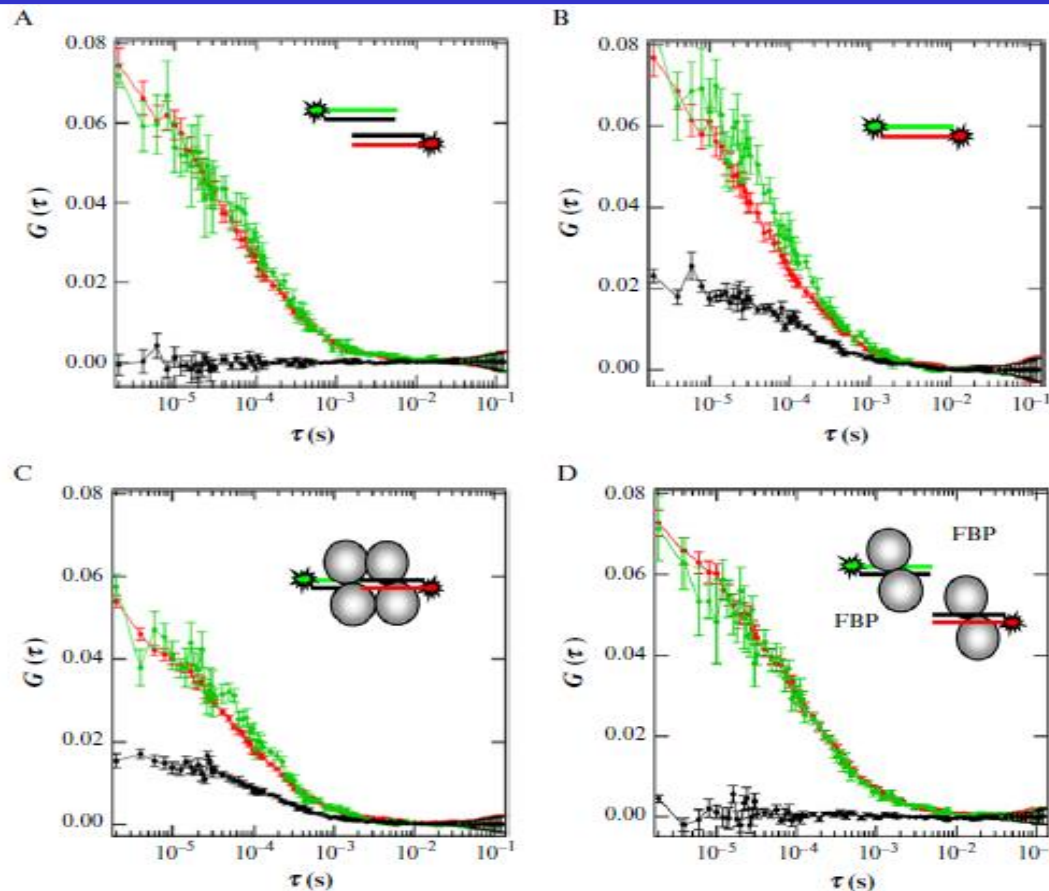
**Thus, for a 3-dimensional Gaussian excitation volume one uses:**

$$G_{12}(\tau) = \frac{\gamma}{N_{12}} \left( 1 + \frac{8D_{12}\tau}{w^2} \right)^{-1} \left( 1 + \frac{8D_{12}\tau}{z^2} \right)^{-1/2}$$

**$G_{12}$  is commonly used to denote the cross-correlation and  $G_1$  and  $G_2$  for the autocorrelation of the individual detectors.** Sometimes you will see  $G_x(0)$  or  $C(0)$  used for the cross-correlation.

# In vitro interaction between the *CGGR* repressor and its operator

Nathalie Declerck (CBS)



FBP induces the dissociation of the *CGGR* tetramer from its operator

FBP: fructose-1,6-bis-phosphate

# FLUCTUATION MICROSCOPIES

- INTRODUCTION
- PRINCIPLE OF FCS
- AUTOCORRELATION
- PHOTON COUNTING HISTOGRAM
- CROSS CORRELATION
- **EXPERIMENTAL DETAILS**
- NUMBER & BRIGHTNESS
- PRACTICAL PART

# Experimental details

- Background
- Photobleaching
- Size and shape of the observation volume
- Cross-talk between detection channels (FCCS)
- Afterpulsing



# Background

- Constant, uncorrelated (B)  
→ Diminishes the value of  $G(0)$

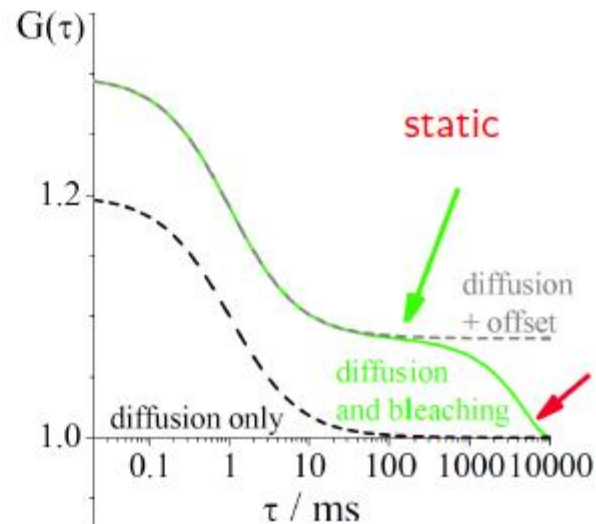
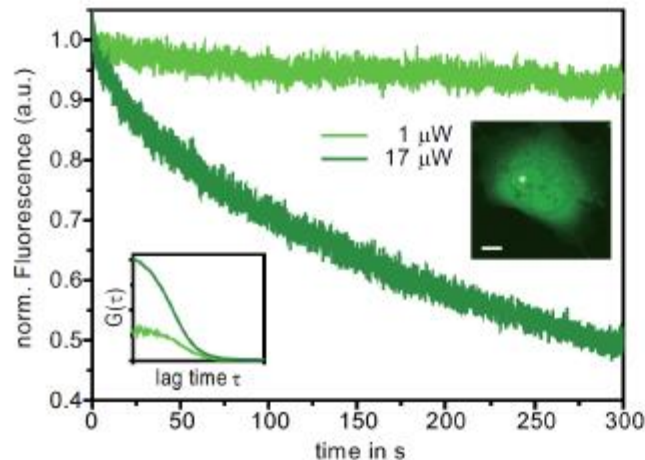
$$g'(\tau) = g(\tau) \left( \frac{\langle F \rangle}{\langle F \rangle + B} \right)^2$$

- Correlated (arising from diffusing molecules)  
→ Add a new component to the analysis

$$G(\tau) = \sum_{i=1}^M f_i^2 \cdot G(0)_i \cdot \left( 1 + \frac{8D\tau}{w_{2DG}^2} \right)^{-1} \quad (2D\text{-Gaussian, 2PE})$$

# Photobleaching

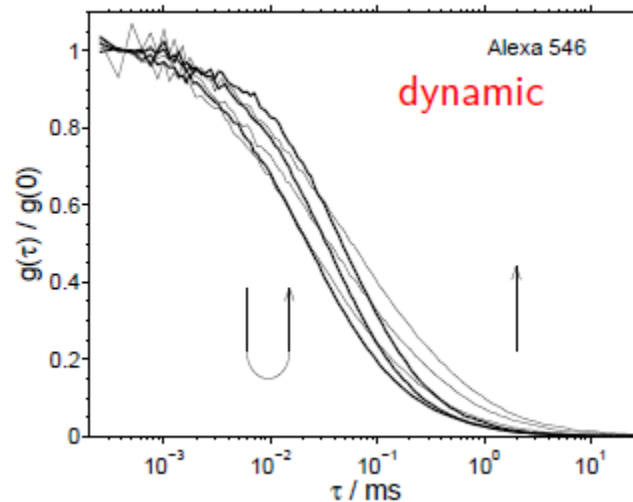
- Static (photobleaching of quasi immobile molecules)



# Photobleaching

- Dynamic

→ Diminishes the apparent diffusion coefficient

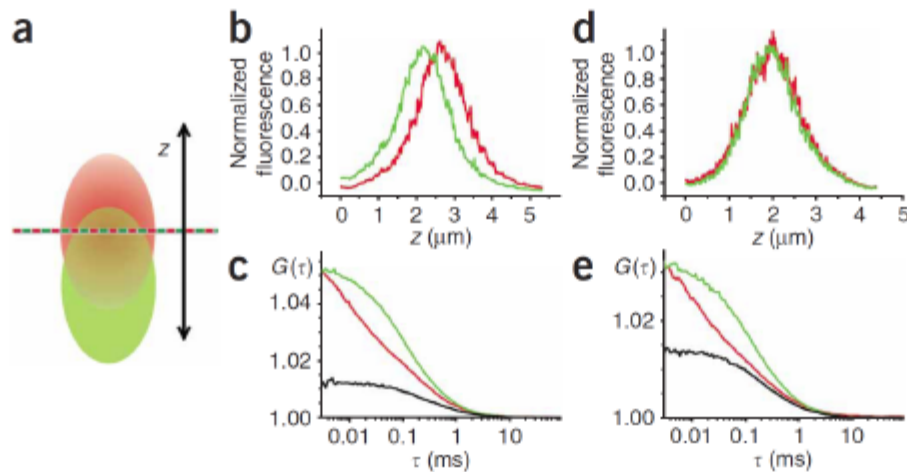


# Size and shape of the observation volume

**FCS:** distortions of the volume size can affect:

- measured diffusion coefficient  $D$  via  $r_0$  in  $D = r_0^2/\tau_D$
- measured concentration via changed  $V_{\text{eff}}$ :  $g(0) = 1/cV_{\text{eff}}$

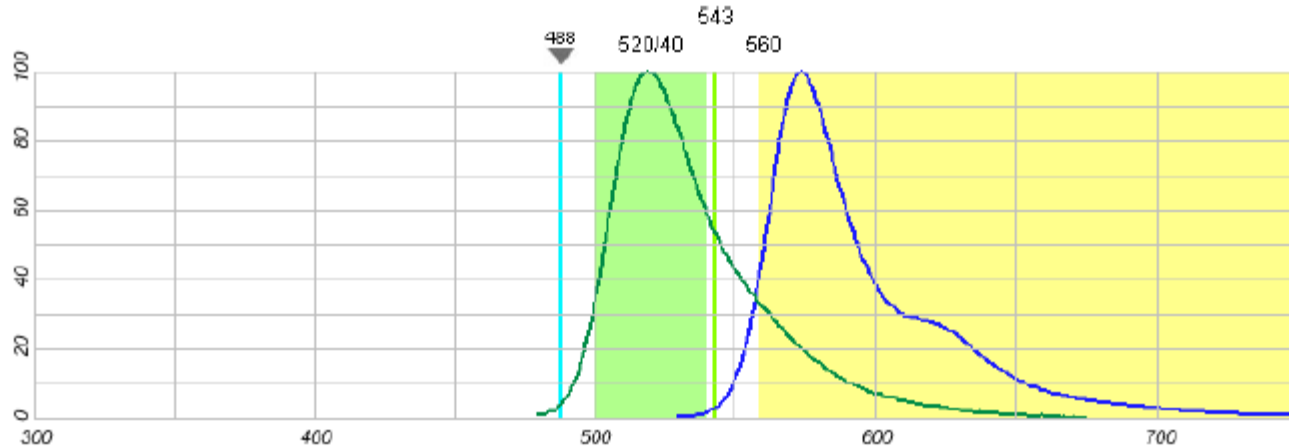
**FCCS:** in addition to the above, the volume overlap affects  $g_{\times}(\tau)$



— use objective correction collar for coverslip thickness correction

NB : Using 2photon excitation, the excitation volumes are identical

# Spectral cross talk (FCCS)

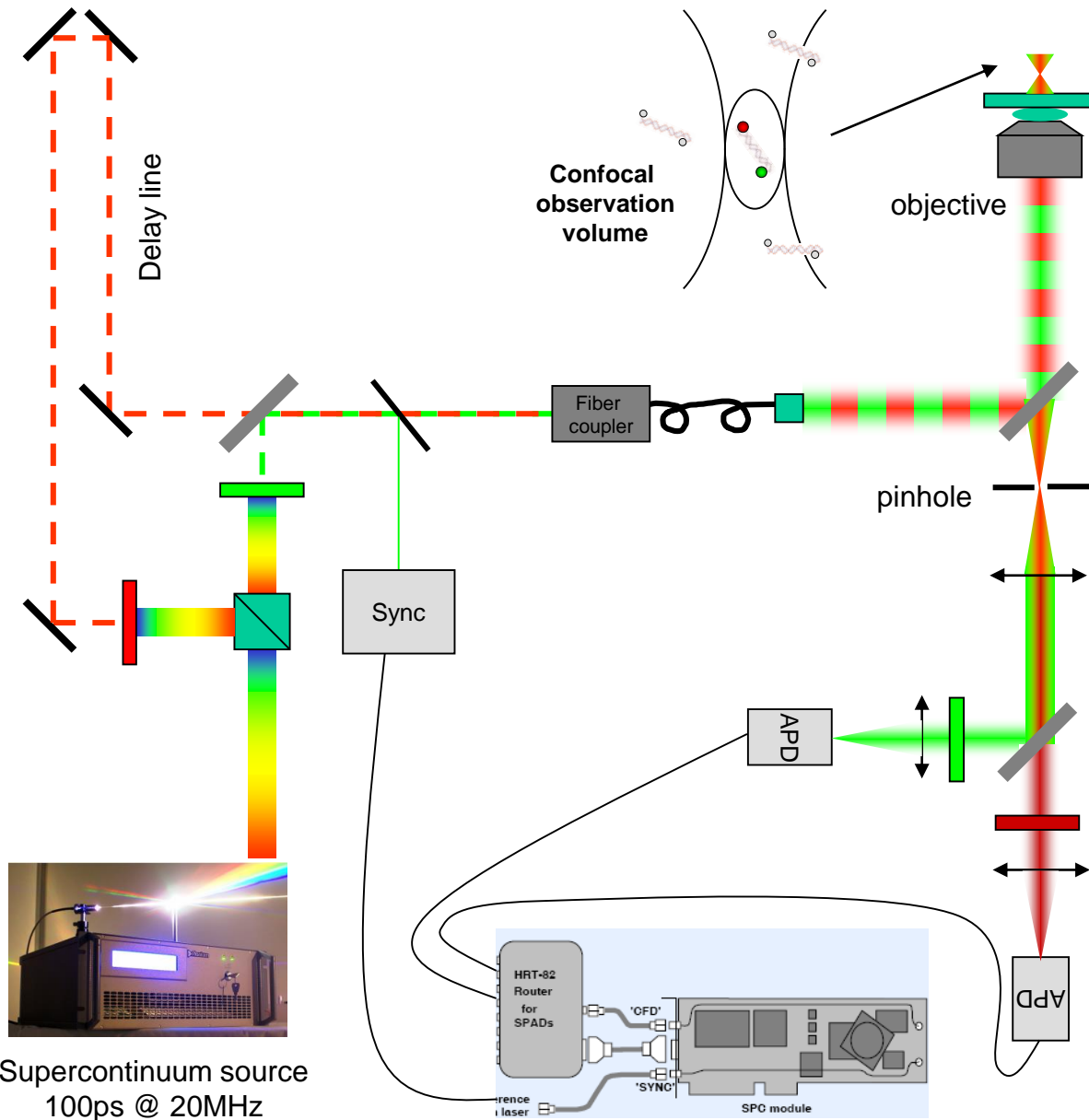


Detection of the « green » fluorophore in the « red » detection channel : → Artifactual cross correlation

- Use spectrally distinct fluorophores
- Use alternated laser excitation (nsALEX / PIE)

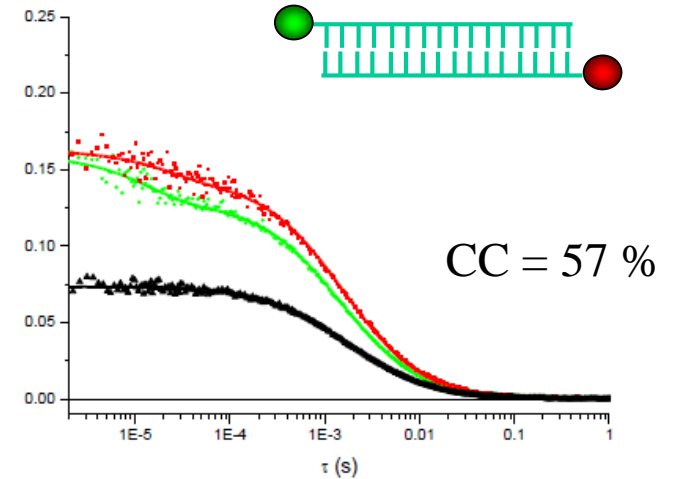
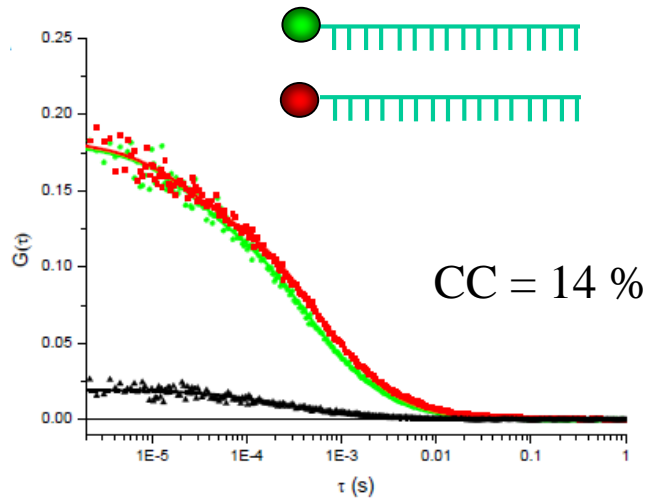
ALEX : Alternating laser excitation, PIE : Pulsed interleaved excitation

# PIE / nsALEX with a supercontinuum source

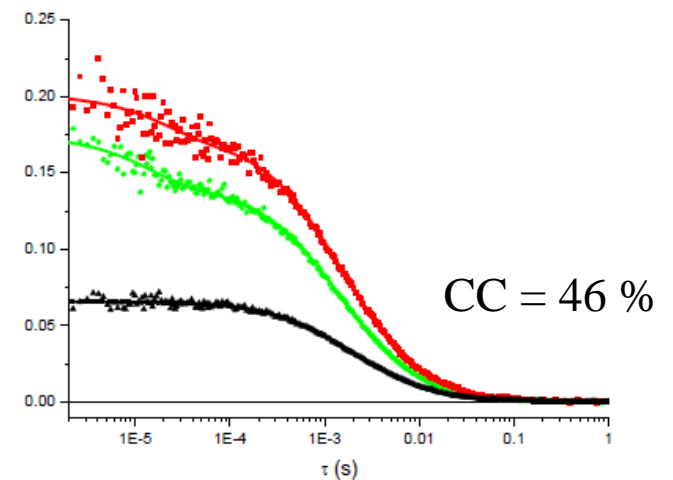
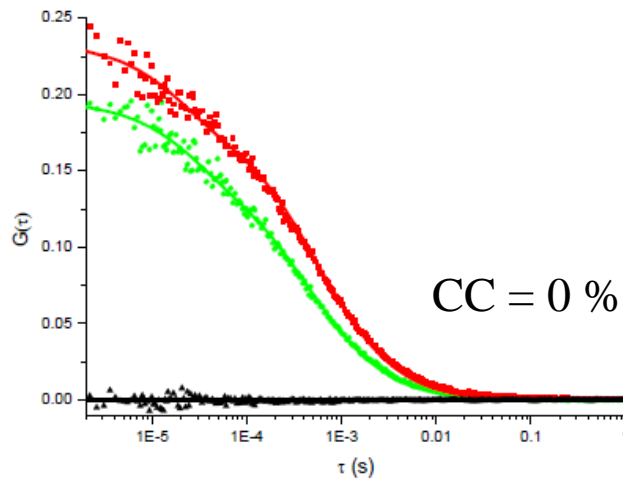


# Removal of crosstalk in FCCS

Regular  
FCCS

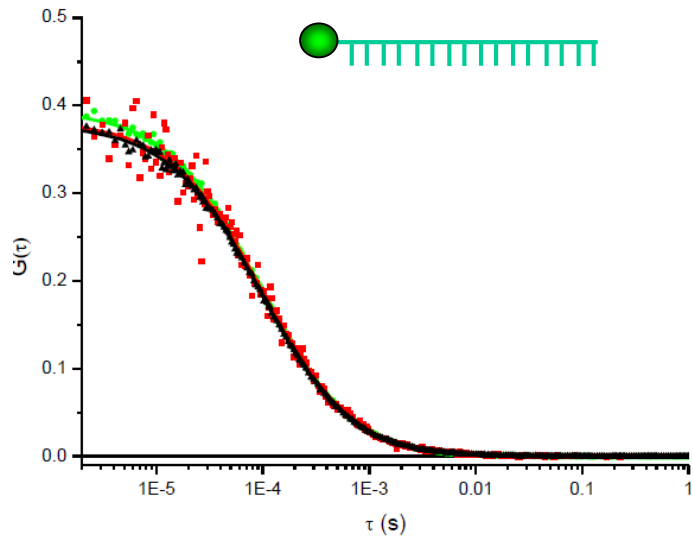


PIE  
FCCS



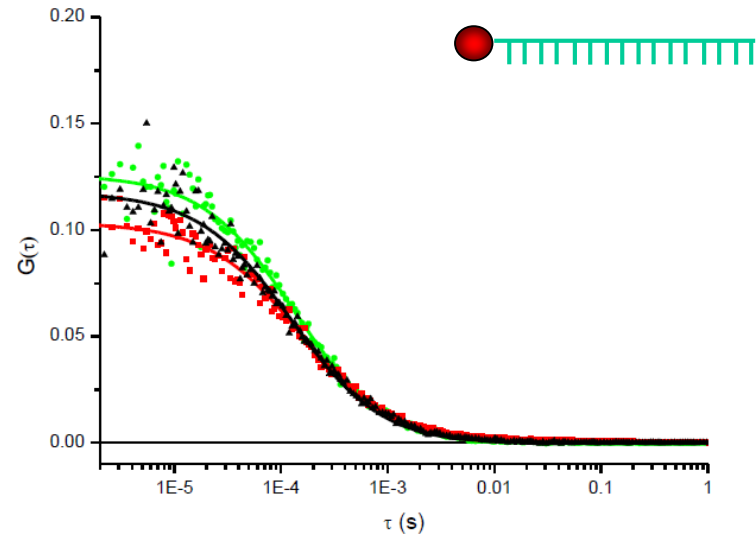
- Green AC
- Red AC
- Green Red CC

# Overlap of the excitation and detection volumes



Green  
excitation

- Green detection AC
- Red detection AC
- Green Red CC

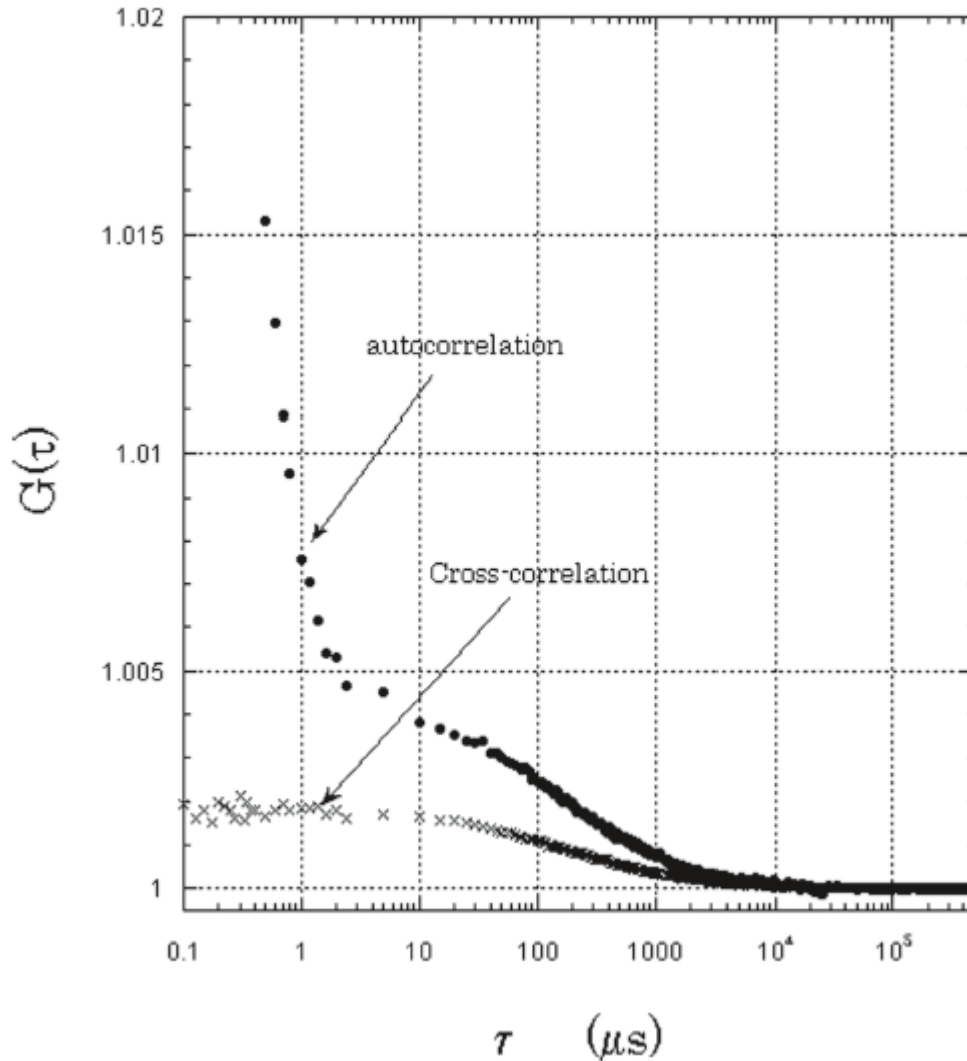


Red  
detection

- Green excitation AC
- Red excitation AC
- Green Red CC



# Afterpulsing



Autocorrelation :  
One detector

Cross correlation :  
Two spectrally equivalent detectors  
50/50 beam splitter