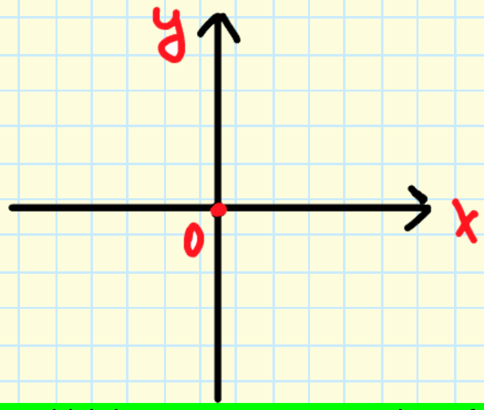
A relation is an expression, which defines the relationship between two variables and may have different outputs for an input.

A function is a relation, which can have only one output for an input.

A relation: {(3, 10), (3, 5), (4, 6), (7,10)} – one input has two different outputs.

A function: {(5, 7), (6, 3), (8, 3), (7, 7)} – no same input for different outputs.

The Cartesian plane is a two-dimensional plane formed by the intersection of x-axis and y-axis. The origin of the plane is located on the intersection point of two axis.

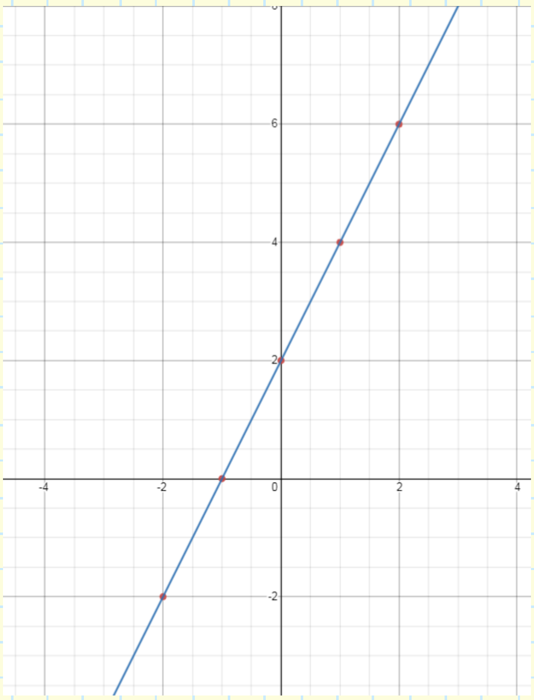


A linear function is an equation which is a constant or a product of a constant and a variable. The form of a linear function: $y = mx + b$, where x is an input variable and y is an output variable; m is a constant called gradient (or slope) and b is a constant called y-intercept (or vertical axis intercept).

A set of solutions for the different inputs of a linear functions forms the straight line in the Cartesian plane.

1) Consider a linear function $y = 2x + 2$. Draw the graph of the function in the plane.

x	*y= 2x+2*
2	*6*
1	*4*
0	*2*
-1	*0*
-2	*-2*

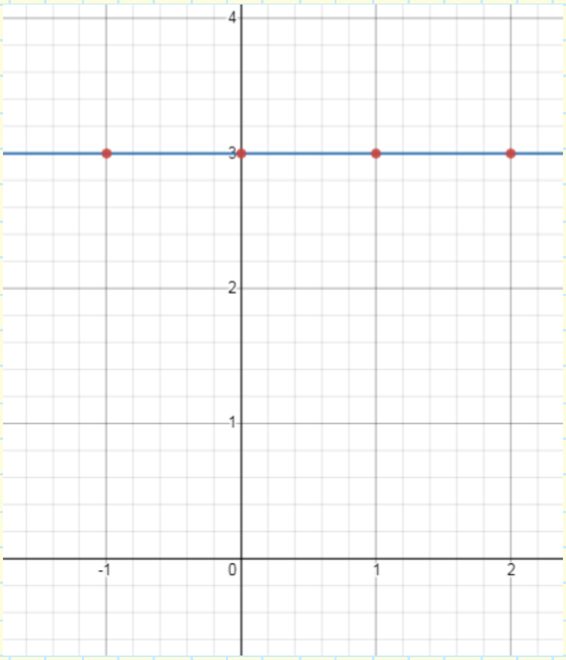


Making the table of results of outputs for different inputs gives us the set of outputs. Drawing the points of pairs such as (x, y) in the plane gives us the graph of the linear function $y = 2x + 2$

Note that any linear function can be illustrated on the plane by two points, since any straight line is formed by two points.

2) *The graph of the linear function* $y = 3$

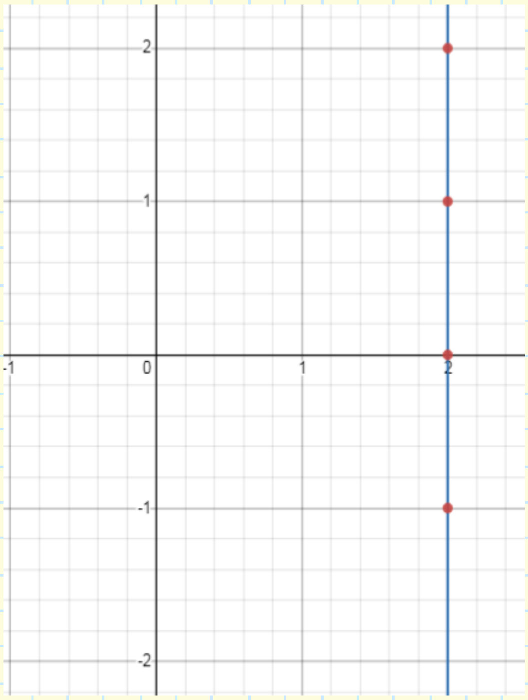
x	*y=3*
2	*3*
1	*3*
0	*3*
-1	*3*



For a graph which function is a constant an input is always the same and equals to the constant. In this case every point on the graph can be written as (x, 3), where x is any real number.

3) *Consider the equation x=2. Draw the graph of the equation. Is it a function?*

x	*y=3*
2	*-1*
2	*0*
2	*1*
2	*2*



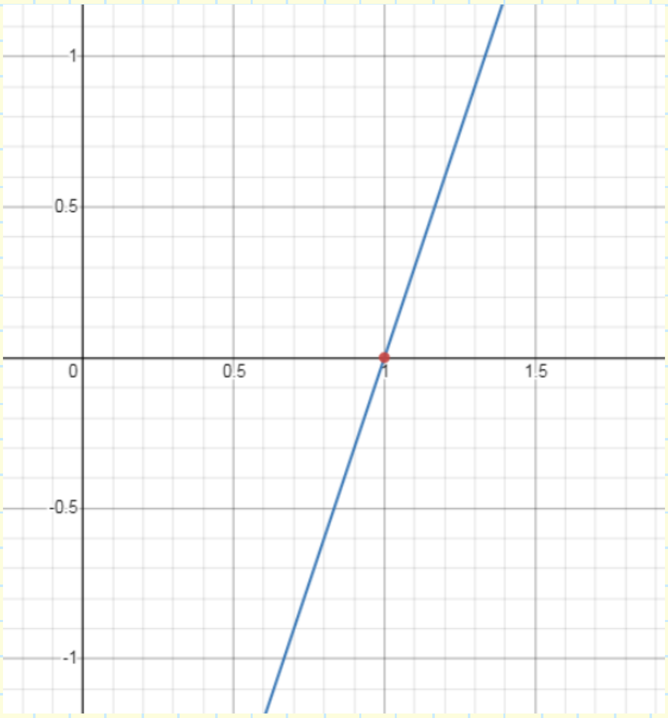
The graph consists of a set of the same inputs with the same outputs. Since for one input there are multiple outputs x=2 is not a function. So, any equation such that x=constant is not a function.

The slope (gradient) of a line is a coefficient that describes the direction and the steepness of the line. Direction – a function increases, decreases, horizontal, vertical. Steepness – the rate of change of a function.

Slope can be calculated using the formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, where m is the slope, Δy - change of y-coordinates, Δx – change of x-coordinates and (x_1, y_1), (x_2, y_2) are the points on the line.

Zeroes (roots or x-intercept) of a function are the values of x, when y=0. They can be determined algebraically using the equation of a function or graphically using the graph of a function.

4) Consider the graph of a function below. Determine roots of the function.



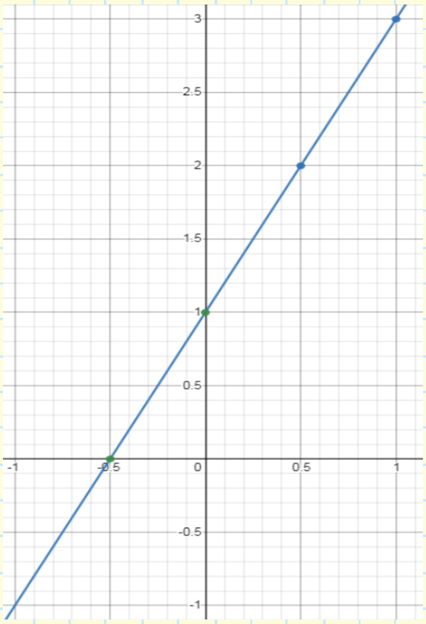
On the given graph line crosses the x-axis at the point (0,1). Any point of linear function such that y=0 is a zero and x-intercept of a function. So, the root is x=1.

5) Consider the equation of a function to be $y = 4x + 10$. Determine coordinates of x-intercept of the function algebraically.

Note that x-intercept is always in the form of (x, 0). So, $y = 4x + 10 = 0$. Solving the equation $4x + 10 = 0$ gives us $x = -2.5$. So, the coordinates of x-intercept are (-2.5, 0).

y-intercept is a point at which a line crosses the y-axis. It corresponds to the points, where x=0 and the point is in the form (0, x). It also can be determined algebraically and graphically, like x-intercept.

6) Consider the graph of a linear function below. Determine the slope, x-intercept and y-intercept. Write down the equation of the function.



Take two points on the line: (1, 3), (0.5, 2). Then using the formula for slope to determine coefficient m for the equation of the function: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{1-0.5} = 2$. So, the slope of the function is 2 and m = 2.

Looking on the graph the line intersects x-axis at the point (-0.5, 0) and y-axis at the point (0, 1). So, root of the function will x = - 0.5, y-intercept is the point (0, 1) and coefficient b = 1

Combining everything will give us the equation of the function:
$y = 2x + 1$

Two straight lines of two different functions will never meet if they are perpendicular. Lines are parallel if they have the same gradient.

7) Consider function $y = \frac{1}{2}x$*. State two different functions parallel to it.*

The function has the slope $m = \frac{1}{2}$*. Considering functions of the same gradient will give us 1)* $y = \frac{1}{2}x + 4$ *and 2)* $y = \frac{1}{2}x - 3$*. All of the three lines are parallel.*

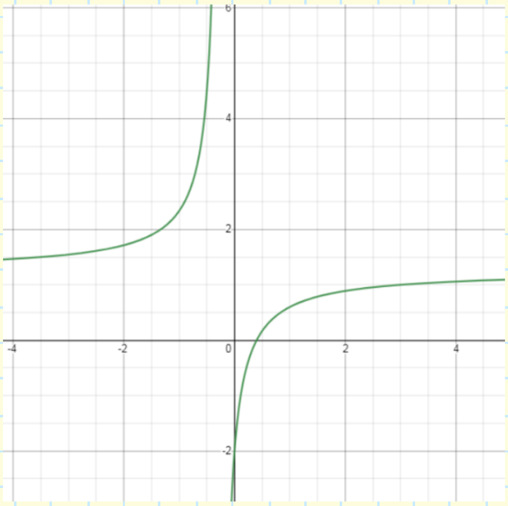
Two straight lines are perpendicular if they meet at a right angle. For any two linear functions their lines will be perpendicular if the product of their gradients is -1.

8) Consider two functions $y = 4x + 2$ *and* $y = 6 - \frac{1}{4}x$*. Prove that they are perpendicular.*

The first function has the slope $m = 4$ *and the second function has the slope* $m = \frac{-1}{4}$*. Multiplying slopes of the functions will give us* $4 \times \frac{-1}{4} = -1$*. So, the lines are perpendicular, since the product of their gradients is -1*

Rational functions are the functions in the form $(x) = \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomials.

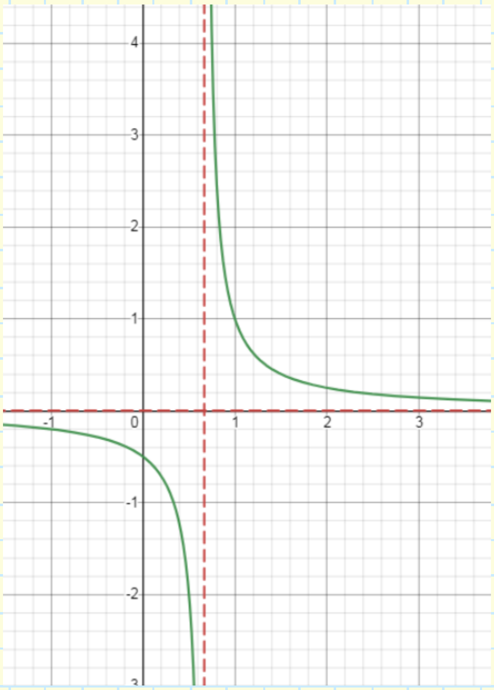
$f(x) = \frac{2}{3x+1}$



$f(x) = \frac{5x-2}{4x+1}$

- An assymptote is a straight line that approaches to a curve but does not intersect
- A rational function has both horizontal and a vertical assymptote.
- A vertical assymptote refers to the restrictions for the domain of a function. Vertical assymptotes can be determined by the values restricted in the denominator and usually denoted as the equation for the x: $x = const$.
- A horizontal assymptote refers to the restrictions for the range of a function.
- If the highest degree of the denominator is greater, than the highest degree of nominator, then the function has the horizontal assymptote in the form of a linear function: $y = 0$
- If the highest degree of the denominator is equal to the highest degree of nominator, then the function has the horizontal assymptote in the form of a linear function: $y = const$. Equation of the horizontal assymptote can be determined using long division of the nominator and denominator.

1) Consider the function $f(x) = \frac{1}{3x-2}$. State the domain and the range of the function.



The rational function $f(x)$ has the domain $3x-2 \neq 0 \to x \neq \frac{2}{3}$.

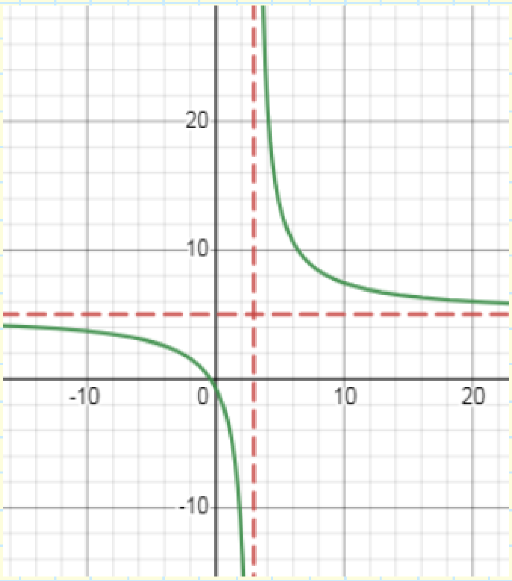
Since degree of denominator > degree of nominator, then the horizontal assymptote of the function will be $y = 0$ and the restriction for the range: $y \neq 0$

2) Consider the function $f(x) = \frac{5x+2}{x-3}$. Determine horizontal and vertical assymptotes of the function.

The rational function $f(x)$ has the domain $x-3 \neq 0 \to x \neq 3$. So the equation for the vertical assymptote will be $x = 3$.

$$\begin{array}{r|l} 5x+2 & x-3 \\ \hline -\;5x-15 & 5 \\ \cline{1-1} 17 & \end{array}$$

Using the long division the function can be rewritten in the form $f(x) = 5 + \frac{17}{x-3}$. So, the equation of the horizontal assymptote will be $y = 5$



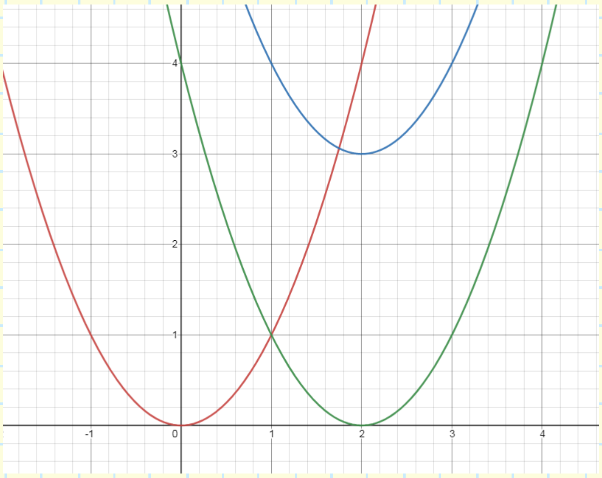
Function transformations are the operations, which involve translation, reflection, and dilation of the graphs of given functions.

Translation is a movement of functions across the plane in a fixed distance and a given direction.

Translation of function f(x):

- $y = f(x) + c$. If $c > 0$ then graph moves up. If $c < 0$ then graph moves down.
- $y = f(x + c)$. If $c < 0$ then graph moves right. If $c > 0$ then graph moves left.

1) Consider the function $f(x) = x^2$. Describe the transformations to move the graph of the function $f(x)$ onto the graph of the function $g(x) = (x - 2)^2 + 3$. Sketch both graphs.



First, the function $f(x) = x^2$ has been translated two units to the right, forming function $f(x - 2) = (x - 2)^2$. Then, it has been translated 3 units up forming function $f(x - 2) + 3 = g(x) = (x - 2)^2 + 3$.

Reflection is the flipping of a function over the line of reflection

Reflection of function f(x):

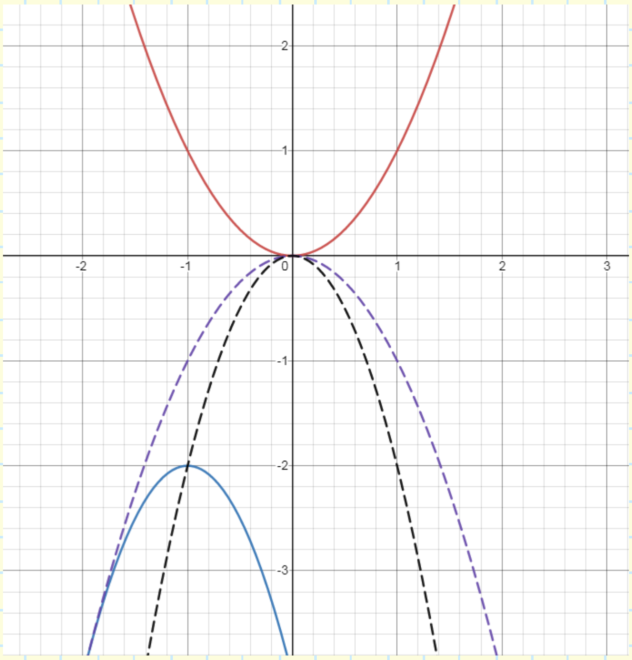
- $y =- f(x)$. Reflection about x-axis.
- $y = f(- x)$. Reflection about y-axis.

Dilation is a stretching or shrinking of the original function.

Dilation of function f(x):

- $y = cf(x)$. If $c > 1$, then the graph stretches in the y-direction. If $0 < c < 1$, then the graph shrinks in the y-direction
- $y = f(cx)$. If $c > 1$, then the graph compresses in the x-direction. If $0 < c < 1$, then the graph stretches in the x-direction

2) Consider the function $f(x) = x^2$. Describe the transformations to move the graph of the function $f(x)$ onto the graph of the function $g(x) =- 2(x + 1)^2 - 2$. Sketch both graphs.



First, graph of $f(x) = x^2$ has been reflected of x-axis and transformed to $-f(x) =- (x)^2$. Then new function $-f(x)$ has been dilated by the scale factor $\frac{1}{2}$ forming new function $-2f(x) =- 2(x)^2$. After that new function has been translated 1 unit to the left and 2 units down transforming into $-2f(x+1)-2 = g(x) =- 2f(x+1)^2 - 2$.