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Course:- Calculus and Analytical geometry (MT1003)
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Assignment #4

Q:-1 Use Integration by Parts to Prove Reduction Formula.

$$\int (\ln x)^n dx = x \ln^n x - n\int (\ln x)^{n-1} dx.$$

Sol $= \int \ln(x)^n \, dx$

Let $u = (\ln x)^n$, $dv = dx$.

$\frac{du}{dx} = n(\ln \cancel{x})^{n-1} \frac{1}{x}$, $v = x$.

$du = n(\ln)^{n-1}$

By using reduction Formula.

$= uv - \int v du.$

$= (\ln x)^n x - \int x . n(\ln)^{n-1} dx.$

$= \ln(x)^n x - n\int (\ln x)^{n-1} dx.$

$\int (\ln x)^n = x(\ln x)^n - n\int (\ln x)^{n-1} dx.$

Hence Proved

Question #2

Sol

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

$$\therefore \quad V_{avg} = \frac{1}{b-a}\int_a^b v(r).$$

$$V_{avg} = \frac{1}{R-0}\int_0^R \frac{P}{4\eta l}(R^2 - r^2)\,dr$$

$$= \frac{1}{R}\cdot\frac{P}{4\eta l}\int_0^R R^2 - r^2\,dr$$

$$= 1/R \cdot \frac{P}{4\eta l}\left[R^2\int_0^R 1\,dr - \int_0^R r^2\,dr\right]$$

$$= \frac{P}{4R\eta l}\left[R^2 r - \frac{r^3}{3}\right]_0^R$$

$$= \frac{P}{4R\eta l}\cdot\left[R^2 \cdot R - \frac{R^3}{3}\right] - 0$$

$$= \frac{P}{4R\eta l}\left[\frac{2R^3}{3}\right] = \frac{2PR^{\cancel{3}\,2}}{\cancel{12}_{6}\cancel{R}\eta l} = \frac{PR}{6\eta l}$$

$$V_{avg} = \frac{PR}{6\eta l}$$

Q:-3 Determine whether the following integral is convergent or divergent:-

$$\int_2^{\infty} \frac{dv}{v^2+2v-3}$$

Sol

$$\int_2^{\infty} \frac{dv}{v^2+2v-3}$$

$$\lim_{t \to \infty} \int_2^{t} \frac{1}{v^2+3v-v-3}\, dv$$

$$= \int_2^{t} \frac{1}{v(v+3)-1(v+3)}\, dv$$

$$= \int_2^{t} \frac{1}{(v-1)(v+3)}\, dv$$

By using Partial Fraction.

$$\int_2^{t} \frac{1}{(v-1)(v+3)} = \int \frac{A}{v-1} + \frac{B}{v+3}$$

$$1 = A(v+3) + B(v-1)$$

$$1 = Av + 3A + Bv - B$$

$$1 = 3A - B + (A+B)v$$

comparing coficents.

$$3A - B = 1$$

$$A + B = 0$$

Adding (i) and (iii)

$$3A - B = 1$$
$$A + B = 0$$
$$4A = 1$$

$\boxed{A = 1/4}$ put in -(iii)

$1/4 + B = 0 \Rightarrow \boxed{B = -1/4}$

$$\int_2^t \frac{1}{4(v-1)} + \frac{1}{4(v+3)}$$

$$1/4 \int_2^t \frac{1}{v-1} dv - \frac{1}{4} \int_2^t \frac{1}{v+3} dv$$

$$\lim_{t \to \infty} 1/4 \left[\ln|v-1| - \ln|v+3| \right]_2^t$$

$$= \frac{1}{4} \left[\left[\ln(t-1) - \ln|t+3| \right] - \left[\ln|2-1| - \ln|2+3| \right] \right]$$

$$\lim_{t \to \infty} 1/4 \left[\frac{\ln|t-1|}{\ln|t+3|} + \ln(5) \right] \quad \because \ln(1) = 0$$

Applying L'Hopital rule.

$$\frac{1}{4} \left(\left[\ln\left(\frac{1}{1}\right) + \ln(5) \right] \right.$$

$$= \frac{1}{4} \ln(5)$$

The derivative is convergent

Find the Area of region bounded by parabola $y=x^2$. The tangent line to Parabola at $(1,1)$ and the x- axis:

<u>Sol</u> $y=x^2$.

The slope of line is given by derivative of $y=x^2$.

$$\frac{dy}{dx} = 2x^{2-1}$$
$$= 2x$$

Put $x=1$

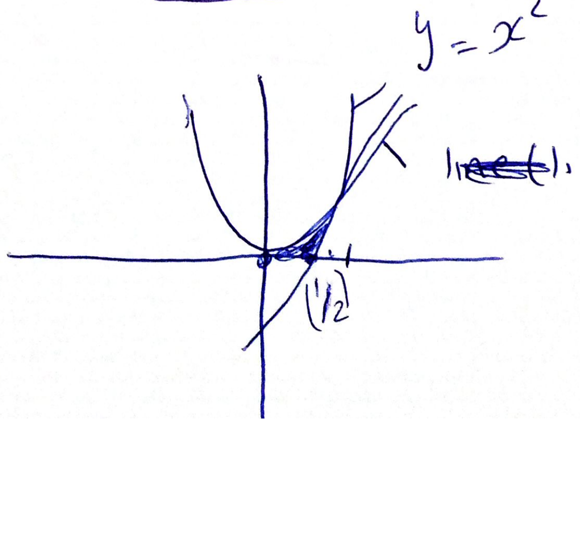
$$\text{slop} = 2(1)$$
$$= 2.$$

equation of line $= 2x+b$.

$$y = 2x+b.$$

Put Point $x=1, y=1$.

$$b = 1-(2).$$
$$\boxed{b = -1}$$



For x intercept $y = 0$.

$$2x - 1 = 0$$

$$\boxed{x = 1/2}$$

$$\int_0^1 x^2\,dx - \int_{1/2}^1 (2x-1)\,dx.$$

$$\left[\frac{x^3}{3}\right]_0^1 - \left[x^2 - x\right]_{1/2}^1.$$

$$= \left[1/3 - 0/3\right] - \left[1^2 - 1 - \left(\left(1/2\right)^2 - \frac{1}{2}\right)\right]$$

$$= 1/3 - 1/4$$

$$= \frac{4-3}{12} = 1/12.$$

Area bounded by region is

$$\boxed{A = 1/12}$$