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19)

$$x_1 + 2x_2 - 3x_3 = 6$$
$$2x_1 - x_2 + 4x_3 = 1$$
$$x_1 - x_2 + x_3 = 3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 1 \\ 1 & -1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & 5 & -10 & 11 \\ 0 & 3 & -4 & 3 \end{array}\right] \quad \therefore 2R1 - R2$$
$$\therefore R1 - R3.$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & 11/5 \\ 0 & 3 & -4 & 3 \end{array}\right] \quad \therefore \frac{1}{5} R2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8/5 \\ 0 & 1 & -2 & 11/5 \\ 0 & 3 & -4 & 3 \end{array}\right] \quad R1 - 2R2.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8/5 \\ 0 & 1 & -2 & 11/5 \\ 0 & 0 & 2 & -18/5 \end{array}\right] \quad R3 - 3R2.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8/5 \\ 0 & 1 & -2 & 11/5 \\ 0 & 0 & 1 & -9/5 \end{array}\right] \quad \frac{1}{2} R3.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8/5 \\ 0 & 1 & -2 & 11/5 \\ 0 & 0 & 1 & -9/5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17/5 \\ 0 & 1 & 0 & -7/5 \\ 0 & 0 & 1 & -9/5 \end{array}\right] \quad \therefore R_1 - R_3$$

$$\therefore R_2 + 2R_3$$

$$x_1 = 17/5 \quad x_2 = -7/5 \quad , x_3 = -9/5$$

Ans.

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2d) $2x_1 + 2x_2 + 2x_3 = 4$
$-2x_1 + 5x_2 + 2x_3 = 1$
$8x_1 + x_2 + 4x_3 = 11$

Sol)
$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & 4 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & 11 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & 11 \end{array}\right] \quad \tfrac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 7 & 4 & 5 \\ 8 & 1 & 4 & 11 \end{array}\right] \quad 2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 7 & 4 & 5 \\ 0 & -7 & -4 & -5 \end{array}\right] \quad \cancel{8R_1 + R_3}. \quad R_3 - 8R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 4/7 & 5/7 \\ 0 & -7 & -4 & -5 \end{array}\right] \quad \tfrac{1}{7}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3/7 & 9/7 \\ 0 & 1 & 4/7 & 5/7 \\ 0 & -7 & -4 & -5 \end{array}\right] \quad \cancel{R_2 + 1}\ R_1 - R_2.$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & 3/7 & 9/7 \\ 0 & 1 & 4/7 & 5/7 \\ 0 & -7 & -4 & -5 \end{array}\right].$$

$7R_2 - R_3$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3/7 & 9/7 \\ 0 & 1 & 4/7 & 5/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 3/7 x_3 = 9/7 \Rightarrow x_1 = 9/7 - \left(\frac{3}{7} x_3\right)$.

$x_2 + 4/7 x_3 = 5/7 \quad x_2 = 5/7 - \left(4/7 x_3\right)$.

$x_3 =$ free or $x_3 = t$.

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21) $3x - y + z + 7w = 13$
$-2x + y - z - 3w = -9$
$-2x + y \quad -7w = -8$

$$\left[\begin{array}{cccc|c} 3 & -1 & 1 & 7 & 13 \\ -2 & 1 & -1 & -3 & -9 \\ -2 & 1 & 0 & -7 & -8 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ -2 & 1 & -1 & -3 & -9 \\ -2 & 1 & 0 & -7 & -8 \end{array}\right] \quad 1/3\ R_1.$$

$$\left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1/3 & -1/3 & 5/3 & -1/3 \\ -2 & 1 & 0 & -7 & -8 \end{array}\right] \quad 2R_1 + R_2.$$

$$\left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1/3 & -1/3 & 5/3 & -1/3 \\ 0 & 1/3 & 2/3 & -7/3 & 2/3 \end{array}\right] \quad 2R_1 + R_3.$$

$$\left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 1/3 & 2/3 & -7/3 & 2/3 \end{array}\right] \quad 3(R_2).$$

$$\left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 0 & -1 & 2 & -1 \end{array}\right] \quad 1/3\ R_2 - R_3.$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 0 & -1 & 4/3 & -1 \end{array}\right]$$

$1/3\,R_2 \to R_1$

$R_1 + 1/3\,R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 0 & 1 & -4/3 & 1 \end{array}\right]$$

$-(R_3)$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 13/3 & 0 \\ 0 & 0 & 1 & -4/3 & 1 \end{array}\right]$$

$R_2 \to R_2 + R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 13/3 & 0 \\ 0 & 0 & 1 & -4/3 & 1 \end{array}\right]$$

$x_1 + 4x_4 = 4 \Rightarrow x_1 = 4 - 4x_4$

$x_2 + 13/3\,x_4 = 0$ $\qquad x_2 + x_4 = 0 \qquad x = -x_4$

$x_3 + 4/3\,x_4 = 1$ $\qquad x_3 - x_4 = 1 \qquad x_3 = 1 + x_4$

$x_4 = t \qquad x_4 = \text{free} \qquad x_4 = \text{free}$

$x_3 = 1 - 2/3\,t$

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22] $3x - 2y = 3$

$3x + 6y - 3z = -2$.

$6x + 6y + 3z = 4$.

$$\left[\begin{array}{ccc|c} 3 & -2 & 0 & 3 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 4 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & -2/3 & 0 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 4 \end{array}\right] \quad 1/3\ R_1.$$

$$\left[\begin{array}{ccc|c} 1 & -2/3 & 0 & 1 \\ 0 & -8 & +3 & 5 \\ 0 & -10 & -3 & 2 \end{array}\right] \quad \begin{array}{l} 3R_1 - R_2. \\ \\ 6R_1 - R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2/3 & 0 & 1 \\ 0 & 1 & -3/8 & -5/8 \\ 0 & -10 & -3 & 2 \end{array}\right] \quad -\frac{1}{8}\ R_2.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 7/12 \\ 0 & 1 & -3/8 & -5/8 \\ 0 & 0 & -27/4 & -17/4 \end{array}\right] \quad \begin{array}{l} 2/3\ R_2 + R_1. \\ \\ 10\ R_2 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 7/12 \\ 0 & 1 & -3/8 & -5/8 \\ 0 & 0 & -27/4 & -17/4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 7/12 \\ 0 & 1 & -3/8 & -5/8 \\ 0 & 0 & 1 & 17/27 \end{array}\right] \quad -4/27\ R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 20/27 \\ 0 & 1 & 0 & -7/8 \\ 0 & 0 & 1 & 17/27 \end{array}\right] \quad \begin{array}{l} +1/4\ R_3 + R_1 \\ 3/8\ R_3 + R_2 \end{array}$$

Solution set

$x = 20/27$

$y = -7/8$

$z = 17/27$