ASSIGNMENT #2

Date: 06/09/22

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Batch :- BS(AI)

Submitted To :- Prof. Askar Ali

Subject :- Linear Algebra (MT-1004).

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Q:- Solve the linear system by Gauss-Jordan elimination.

$$x_1 + 2x_2 - 3x_3 = 6$$
$$2x_1 - x_2 + 4x_3 = 2$$
$$x_1 - x_2 + x_3 = 3$$

Sol Augmented matrix is

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 1 & -1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix}.$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 1 \\ 1 & - & 1 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & 5 & -10 & 11 \\ 0 & 3 & -4 & 3 \end{array}\right] \begin{array}{l} \\ \therefore 2R_1 - R_2 \\ \therefore R_1 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & 11/5 \\ 0 & 3 & -4 & 3 \end{array}\right] \therefore 1/5\, R_2$$

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$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 8/5 \\ 0 & 1 & -2 & 11/5 \\ 0 & 3 & -4 & 3 \end{array}\right] \quad \therefore R_1 - 2R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 8/5 \\ 0 & 1 & -2 & 11/5 \\ 0 & 0 & 2 & -18/5 \end{array}\right] \quad \therefore R_3 - 3R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 8/5 \\ 0 & 1 & -2 & 11/5 \\ 0 & 0 & 1 & -9/5 \end{array}\right] \quad \therefore 1/2\, R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 17/5 \\ 0 & 1 & 0 & -7/5 \\ 0 & 0 & 1 & -9/5 \end{array}\right] \quad \begin{array}{l} \therefore R_1 - R_3 \\ \therefore R_2 + 2R_3 \end{array}$$

$x_1 = 17/5 \qquad x_2 = -7/5 \qquad x_3 = -9/5$

★——★——★

2] $2x_1 + 2x_2 + 2x_3 = 4$

$-2x_1 + 5x_2 + 2x_3 = 1$

$8x_1 + x_2 + 4x_3 = 11$

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Sol Augmented matrix is

$$A=\begin{bmatrix}2 & 2 & 2\\ -2 & 5 & 2\\ 8 & 1 & 4\end{bmatrix} \quad X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad B=\begin{bmatrix}4\\ 1\\ 11\end{bmatrix}$$

$$=\left[\begin{array}{ccc|c}2 & 2 & 2 & 4\\ -2 & 5 & 2 & 1\\ 8 & 1 & 4 & 11\end{array}\right]$$

$$=\left[\begin{array}{ccc|c}1 & 1 & 1 & 2\\ -2 & 5 & 2 & 1\\ 8 & 1 & 4 & 11\end{array}\right] \quad \therefore \tfrac{1}{2}R_1$$

$$=\left[\begin{array}{ccc|c}1 & 1 & 1 & 2\\ 0 & 7 & 4 & 5\\ 8 & 1 & 4 & 11\end{array}\right] \quad \therefore 2R_1 + R_2$$

$$=\left[\begin{array}{ccc|c}1 & 1 & 1 & 2\\ 0 & 7 & 4 & 5\\ 0 & -7 & -4 & 5\end{array}\right] \quad \therefore R_3 - 8R_1.$$

$$=\left[\begin{array}{ccc|c}1 & 1 & 1 & 2\\ 0 & 1 & 4/7 & 5/7\\ 0 & -7 & -4 & 5\end{array}\right] \quad \therefore \tfrac{1}{7}R_2$$

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$$= \begin{bmatrix} 1 & 0 & 3/7 & 9/7 \\ 0 & 1 & 4/7 & 5/7 \\ 0 & -7 & -4 & -5 \end{bmatrix} \quad \therefore R_1 - R_2$$

$$= \begin{bmatrix} 1 & 0 & 3/7 & 9/7 \\ 0 & 1 & 4/7 & 5/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \therefore 7R_2 - R_3$$

$\Rightarrow x_1 + 3/7 x_3 = 9/7$ —(i)

$\Rightarrow x_2 + 4/7 x_3 = 5/7$ —(ii)

$x_3 =$ Free or t

put $x_3 = t$ in (i) & (ii)

$= x_1 + 3/7 t = 9/7$

$\Rightarrow x_1 = 9/7 - 3/7 t$

$x_2 + 4/7 t = 5/7 \Rightarrow x_2 = 5/7 - 4/7 t$

$x_1 = 9/7 - 3/7 t$

$x_2 = 5/7 - 4/7 t$

$x_3 = t.$ Ans

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3] $3x - y + z + 7w = 13$
$-2x + y - z - 3w = -9$
$-2x + y \quad\quad - 7w = -8$

Sol Writing in matrix form.

$$A = \begin{bmatrix} 3 & -1 & 1 & 7 \\ -2 & 1 & -1 & -3 \\ -2 & 1 & 0 & -7 \end{bmatrix} \quad B = \begin{bmatrix} 13 \\ -9 \\ -8 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$= \left[\begin{array}{cccc|c} 3 & -1 & 1 & 7 & 13 \\ -2 & 1 & -1 & -3 & -9 \\ -2 & 1 & 0 & -7 & -8 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ -2 & 1 & -1 & -3 & -9 \\ -2 & 1 & 0 & -7 & -8 \end{array}\right] \therefore 1/3\, R_1$$

$$= \left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1/3 & -1/3 & 5/3 & -1/3 \\ -2 & 1 & 0 & -7 & -8 \end{array}\right] \therefore 2R_1 + R_2$$

$$= \left[\begin{array}{cccc|c} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1/3 & -1/3 & 5/3 & -1/3 \\ 0 & 1/3 & 2/3 & -7/3 & 2/3 \end{array}\right] \therefore 2R_1 + R_3$$

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$$= \begin{bmatrix} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 1/3 & 2/3 & -7/3 & 2/3 \end{bmatrix} \quad \therefore 3(R_2)$$

$$= \begin{bmatrix} 1 & -1/3 & 1/3 & 7/3 & 13/3 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 0 & -1 & 4 & -1 \end{bmatrix} \quad \therefore 1/3(R_2) - R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 0 & -1 & 4 & -1 \end{bmatrix} \quad \therefore R_1 + 1/3\, R_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & -1 & 5 & -1 \\ 0 & 0 & 1 & -4 & 1 \end{bmatrix} \quad \therefore -(R_3)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -4 & 1 \end{bmatrix} \quad \therefore R_2 + R_3$$

$\Rightarrow x_1 + 4x_4 = 4 \quad \Rightarrow x_1 = 4 - 4x_4$

$\Rightarrow x_2 + x_4 = 0 \quad \Rightarrow x_2 = -x_4$

$\Rightarrow x_3 - x_4 = 1 \quad \Rightarrow x_3 = 1 + x_4$

$\therefore x_4 = t \quad \Rightarrow \quad x_4 = t$

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4) $-2y + 3x = 3$
$3x + 6y - 3z = -2$
$6x + 6y + 3z = 4$

Sol Convert into Matrix form.

$$A = \begin{bmatrix} 3 & -2 & 0 \\ 3 & 6 & -3 \\ 6 & 6 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 3 & -2 & 0 & 3 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 4 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -2/3 & 0 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 4 \end{array}\right] \quad 1/3\, R_1$$

$$= \left[\begin{array}{ccc|c} 1 & -2/3 & 0 & 1 \\ 0 & -8 & 3 & 5 \\ 0 & -10 & -3 & 2 \end{array}\right] \quad \begin{array}{l} \therefore 3R_1 - R_2 \\ \therefore 6R_1 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 1 \\ 0 & 1 & -3/8 & -5/8 \\ 0 & 0 & -27/4 & -17/4 \end{array}\right] \quad \begin{array}{l} \therefore 2/3\, R_2 + R_1 \\ \therefore 10 R_2 + R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 1 \\ 0 & 1 & -3/8 & -5/8 \\ 0 & 0 & 1 & 17/27 \end{array}\right] \quad \therefore -4/27\, R_3$$

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$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 20/27 \\ 0 & 1 & 0 & -7/8 \\ 0 & 0 & 1 & 17/27 \end{array}\right] \begin{array}{l} \therefore 1/4\,R_3 + R_1 \\ \\ \therefore 3/8\,R_3 + R_2 \end{array}$$

$x = 20/27$

$y = -7/8$

$z = 17/27$ Ans.