National University
of Computer & Emerging Sciences Peshawar Campus

Program: BS (SE & AI)
Semester: Fall-2022
Course: MT1003-Calculus & Analytical Geometry

Examination: Assignment # 02
Total Marks: 10, Weightage: 2.5
Date of Submission: 22 / 11 / 2022

Note: Attempt all questions.

Q1. Let $f(x) = \sqrt[3]{x}$.

    (a) If $a \neq 0$, use equation $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ to find $f'(a)$.

    (b) Show that $f'(0)$ does not exist.

Q2. Use definition of derivative to show that $\dfrac{d}{dx}(\sin x) = \cos x.$

Q3. A rock thrown vertically upward from the surface of the moon at a velocity of $24 \ m/\sec$ reaches a

    height of $s = 24t - 0.8t^2 \ m$ in $t \sec$.

    (a) Find the rock's velocity and acceleration at time t.

    (b) How long does it take the rock to reach its highest point?

    (c) How high does the rock go?

    (d) How long does it take to reach half its maximum height?

Q4. Find an equation of tangent line to the **witch of Maria Agnesi** i.e $y = \dfrac{1}{1 + x^2}$ at $(-1, \dfrac{1}{2})$. Also find

    equation of normal line at $(-1, \dfrac{1}{2})$.

Q5. Let $f(x) = |4x + 5|$. Compute the left hand derivative and right derivative at $x = -\dfrac{5}{4}$.

    Does $f'\left(\dfrac{-5}{4}\right)$ exist ?

Q6. Find an equation of tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at the point (x_0, y_0).

Q7. Find the point(s) on the graph of the following functions where the tangent line is horizontal:

    (a) $h(x) = (x - 2)(x^2 - x - 11)$ (b) $g(x) = \sin x$ (c) $f(x) = \cos x.$

Q8. Use Implicit Differentiation to find $\dfrac{dy}{dx}$ if $x^4 + x^2 y^3 - y^5 = 2x + 1.$

Q9. Find the derivative of $y = \dfrac{\sqrt[3]{x^4 + 6x^2}\,(8x + 3)^5}{(2x^2 + 7)^{2/3}}.$

Q10. (a) Find the 50th derivative of $y = \cos(2x).$

(b) Find the 1000th derivative of $y = xe^{-x}$.

Q11. Find derivatives of the following functions:

(a) $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$

(b) $y = x^{\sin x}$

(c) $y = x^{x^{x}}$

(d) $y = \sin\left(\ln\left(\dfrac{1}{x}\right)\right)$

(e) $y = \left(\sin^{-1}(x) + \ln(\ln(x^2))\right)^5$

(f) $y = (1 + 2^{\sin x})^2$

(g) $y = (1 + \log_4(x^2))^5$

(h) $\dfrac{d^{101}}{dx^{101}}(x^{100})$

The End