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Course:- Linear Algebra.

Submitted to:-

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Q 1 Show that

V = { The set of all symmetric matrices } forms a vector space or not:-

<u>Sol</u>

$$\underline{u} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} d & e \\ e & f \end{bmatrix}, \quad \underline{w} = \begin{bmatrix} g & h \\ h & i \end{bmatrix}.$$

Vector addition

(i) $\underline{u} + \underline{v} \in V$

$$= \begin{bmatrix} a & b \\ b & c \end{bmatrix} + \begin{bmatrix} d & e \\ e & f \end{bmatrix} \Rightarrow \begin{bmatrix} a+d & b+e \\ b+e & c+f \end{bmatrix} \in V$$

(ii) $\underline{u} + \underline{v} = \underline{v} + \underline{u}$

$$\underline{u} + \underline{v} = \begin{bmatrix} a+d & b+e \\ b+e & c+f \end{bmatrix} \Rightarrow \begin{bmatrix} d+a & e+b \\ e+b & f+c \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} d & e \\ e & f \end{bmatrix} + \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \underline{v} + \underline{u}$$

(iii) $\underline{u} + (\underline{v} + \underline{w}) = (\underline{u} + \underline{v}) + \underline{w}$

$$\underline{u} + (\underline{v} + \underline{w}) = \begin{bmatrix} a & b \\ b & c \end{bmatrix} + \left(\begin{bmatrix} d & e \\ e & f \end{bmatrix} + \begin{bmatrix} g & h \\ h & i \end{bmatrix} \right)$$

$$= \begin{bmatrix} a & b \\ b & c \end{bmatrix} + \begin{bmatrix} d+g & e+h \\ e+h & f+i \end{bmatrix}$$

$$\begin{bmatrix} a+d+g & b+e+h \\ b+e+h & c+f+i \end{bmatrix} \Rightarrow \begin{bmatrix} a+d & b+e \\ b+e & f+i \end{bmatrix} + \begin{bmatrix} g & h \\ h & i \end{bmatrix}$$

$= (u+v)+w$

$= R.H.S.$

(iv) $\underline{o} + \underline{v} = v + \underline{o} = v$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow \begin{bmatrix} 0+a & 0+b \\ 0+b & 0+c \end{bmatrix}$$

$$= \begin{bmatrix} a+0 & b+0 \\ b+0 & c+0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

(v) $\underline{u} + (-\underline{u}) = 0$

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} + \left(- \begin{bmatrix} a & b \\ b & c \end{bmatrix} \right) = \begin{bmatrix} a-a & b-b \\ b-b & c-c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Scalar multiplication

(vi) $K\underline{u} \in V$

$$K \begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow \begin{bmatrix} Ka & Kb \\ Kb & Kc \end{bmatrix} \in V$$

$\therefore$ The resultant Ku is in V

(vii) $K(\underline{u} + \underline{v}) = K\underline{u} + K\underline{v}$

$$K \begin{bmatrix} a+d & b+e \\ b+e & c+f \end{bmatrix} = \begin{bmatrix} ka+kd & kb+ke \\ kb+ke & kc+kf \end{bmatrix}.$$

$$\begin{bmatrix} ka & kb \\ kb & kc \end{bmatrix} + \begin{bmatrix} kd & ke \\ ke & kf \end{bmatrix}.$$

$\cancel{k}$ $k\underline{u} + k\underline{v}$

(viii) $(k+m)\underline{u} = k\underline{u} + m\underline{u}$

$$= k+m\begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow \begin{bmatrix} (k+m)a & (k+m)b \\ (k+m)b & (k+m)c \end{bmatrix}.$$

$$= \begin{bmatrix} ka + ma & kb + mb \\ kb + mb & kc + mc \end{bmatrix}.$$

$$= k\begin{bmatrix} a & b \\ b & c \end{bmatrix} + m\begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

$= k\underline{u} + m\underline{u}.$

(ix) $(km)\underline{u} = k(m\underline{u}) = m(k\underline{u}).$

$$km\begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow \begin{bmatrix} kma & kmb \\ kmb & kmc \end{bmatrix} = k\begin{bmatrix} ma & mb \\ mb & mc \end{bmatrix}.$$

$$= m\begin{bmatrix} ka & kb \\ kb & kc \end{bmatrix}.$$

(x) $1\underline{u} = \underline{u}$

$$1\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

Ans All the conditions are ~~sta~~ satified so V is a vector space.

Q2 $V = \{(x, y, z) \; x, y, z$ are real numbers & $x + y + z = 0\}$.

solution

let $\underline{u} = \{5, -2, -3\}$, $\underline{v} = \{4, -2, -2\}$, $w = \{+9, -3, -6\}$

Vector Addition:-

(i) $\underline{u} + \underline{v} \in V$

$\{5, -2, -3\} + \{4, -2, -2\}$.

$= \{5+4, -2-2, -3-2\}$.

$= \{9, -4, -5\} \in \underline{V}$

(ii) $\underline{u} + \underline{v} = \underline{v} + u$

$\{9, -4, -5\} \Rightarrow \{4+5, -2, -2, -2, -3\}$.

$\Rightarrow \{4, -2, -2\} + \{5, -2, -3\} \Rightarrow \underline{v} + \underline{u}$.

(iii) $\underline{u} + (\underline{v} + \underline{w}) = (\underline{u} + \underline{v}) + \underline{w}$

$\{5, -2, -3\} + (\{4, -2, -2\} + \{9, -3, -6\})$.

$\{5, -2, -3\} + \{13, -5, -8\}$.

$= \{18, -7, -11\}$.

$\Rightarrow \{5+4+9, -2, -2, -3, -3, -2, -6\}$

$= \{5+4, -2+2, -3-2\} + \{9, -3, -6\}$

$= (\underline{u} + \underline{v}) + \underline{w}$

(iv) $\underline{0} + \underline{u} = \underline{u} + \underline{0} = \underline{u}$

$\{0,0,0\} + \{5,-2,-3\} \Rightarrow \{5+0, -2+0, -3+0\}$

$\Rightarrow \{0+5, 0+(-2), 0+(-3)) = \{5, -2, -3\}$.

(v) $\underline{u} + (-\underline{u}) = \underline{0}$

$\{5,-2,-3\} + \{-5, -(-2), -(-3)\}$.

$= \{5-5, -2+2, -3+3\} = \{0, 0, 0\} \Rightarrow \underline{0}$

Scalar multiplication:-

(vi) $K\underline{u} \in V$

let $k=2$

$2 \times \{5,-2,-3\}$

$= \{10, -4, -6\} \in V$

(vii) $k(\underline{u}+\underline{v}) = k\underline{u}+k\underline{v}$

$2(9,-4,-5) \Rightarrow (18,-8,-10)$.

$\{2((4+5), 2(-2,-2, 2(-2,-3)\}$.

$2(4,-2,-2) + 2(5,-2,-3) \Rightarrow 2(5,-2,-3)+2(4,-2,-$

$k\underline{u} + k\underline{v}$

(viii) $(k+m)\underline{u} = k\underline{u}+m\underline{u}$

let $k=2$, $m=3$

$(2+3)(5,-2,-3) \Rightarrow 5(5,-2,-3)$.

$= (25,-10,-15)$

$= ((2+3)5, (2+3)(-2); (2+3)(-3))$.

$2(5,-2,-3) + 3(5,-2,-3)$.

$k\underline{u} + m\underline{u}$.

(ix) $(km)\underline{u} = k(m\underline{u}) = m(k\underline{u})$.

$2(3)(5,-2,-3) = 2(15,-6,-9) = k(m\underline{u})$

$= 3(10,-4,-6) = m(k\underline{u})$.

(x) $1\underline{u} = \underline{u}$

$1(5,-2,-3)$

$=(5,-2,-3)$

· As all the axioms are satisfied So, V is a vector Space.

Q:3 $V = \{$ The set of All 2×2 matrix of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. $\}$.

Sol

Let $\underline{u} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $\underline{v} = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$ $\underline{w} \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix}$.

Vector Addition:-

(i) $\underline{u} + \underline{v} \in V$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} a+c & 0+0 \\ 0+0 & b+d \end{bmatrix} \in \underline{V}.$$

(ii) $\underline{u} + \underline{v} = \underline{v} + \underline{u}$.

$$\begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} = \begin{bmatrix} c+a & 0 \\ 0 & d+b \end{bmatrix}$$

$$= \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \Rightarrow \underline{v} + \underline{u}.$$

(iii) $\underline{u} + (\underline{v} + \underline{w}) = (\underline{u} + \underline{v}) + \underline{w}$.

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \left(\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix} \right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c+e & 0 \\ 0 & d+f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+c+e & 0 \\ 0 & b+d+f \end{bmatrix} \Rightarrow \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} + \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix}.$$

$$\Rightarrow (\underline{u} + \underline{v}) + \underline{w}.$$

(iv) $\underline{0} + \underline{U} = \underline{U} + 0 = \underline{U}$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \underline{U}$$

(v) $\underline{U} + (-U) = \underline{0}$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix} = \begin{bmatrix} a-a & 0 \\ 0 & b-b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Sacalar multiplication:-

(vi) $K\underline{U} \in V$.

$$k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix} \in V.$$

(vii) $K(\underline{U} + \underline{V}) = k\underline{U} + k\underline{V}$

$$k\begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} = \begin{bmatrix} ka+kc & 0 \\ 0 & kb+kd \end{bmatrix}.$$

$$\Rightarrow k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + k\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = K\underline{U} + K\underline{V}.$$

(viii) $(K+m)\underline{U} = K\underline{U} + m\underline{U}$

$$\begin{bmatrix} (k+m)a & 0 \\ 0 & (k+m)b \end{bmatrix} = \begin{bmatrix} ka+ma & 0 \\ 0 & kb+mb \end{bmatrix}$$

$$= k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + m\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = K\underline{U} + m\underline{U}$$

(ix) $(km)\underline{v} = k(m\underline{v}) = m(k\underline{v})$.

$km\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \Rightarrow k\begin{bmatrix} ma & 0 \\ 0 & mb \end{bmatrix} \Rightarrow k(m\underline{v})$.

$\Rightarrow m\begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix} = m(k\underline{v})$.

x $1U = U$

$1\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \underline{U}$.

As all the axioms are satisfied So, V is a vector space.

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