ASSIGNMENT-1

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Section: C

Course: Linear Algebra

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Date: 20-03-2023

Q1: Consider the following linear System,

$$2x_1+3x_2-3x_3+x_4+x_5=7$$
$$3x_1+2x_3+3x_5=-2$$
$$2x_1+3x_2+2x_3+3x_5=7$$
$$x_1-3x_2+5x_3-x_4+2x_5=-9$$

Find the solution of the system by using Gauss-Elimination method.

$$\begin{bmatrix} 2 & 3 & -3 & 1 & 1 \\ 3 & 0 & 2 & 0 & 3 \\ 2 & 3 & 2 & 0 & 3 \\ 1 & -3 & 5 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 7 \\ -9 \end{bmatrix}$$

$$A \quad . \quad X \quad = \underline{b}$$

$$[A|b] = \left[\begin{array}{ccccc|c} 2 & +3 & -3 & 1 & 1 & 7 \\ 3 & 0 & 2 & 0 & 3 & -2 \\ 2 & 3 & 2 & 0 & 3 & 7 \\ 1 & -3 & 5 & -1 & 2 & -9 \end{array}\right]$$

$$R) \left[\begin{array}{ccccc|c} 1 & -3 & 5 & -1 & 2 & -9 \\ 3 & 0 & 2 & 0 & 3 & -2 \\ 2 & 3 & 2 & 0 & 3 & 7 \\ 2 & 3 & -3 & 1 & 1 & 7 \end{array}\right] R_{24}$$

$$R) \left[\begin{array}{ccccc|c} 1 & -3 & 5 & -1 & 2 & -9 \\ 0 & 9 & -13 & 3 & -3 & 25 \\ 0 & 9 & -8 & 2 & 1 & 25 \\ 0 & 9 & 13 & 3 & -3 & 25 \end{array}\right] \begin{array}{l} R_2-3R_1 \\ R_3-2R_1 \\ R_4-2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -3 & 5 & -1 & 2 & -9 \\ 0 & 9 & -13 & 3 & -3 & 25 \\ 0 & 0 & 5 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_3 - R_2 \\ R_4 - R_2 \end{array}$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -3 & 5 & -1 & 2 & -9 \\ 0 & 1 & -13/9 & 1/3 & -1/3 & 25/9 \\ 0 & 0 & 5 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] R_2/9$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -3 & 5 & -1 & 2 & -9 \\ 0 & 1 & -13/9 & 1/3 & -1/3 & 25/9 \\ 0 & 0 & 1 & -1/5 & 2/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] R_3/5$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 2/3 & 0 & 1 & -2/3 \\ 0 & 1 & -13/9 & 1/3 & -1/3 & 25/9 \\ 0 & 0 & 1 & -1/5 & 2/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] R_1 + 3R_2$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 2/15 & 11/15 & -2/3 \\ 0 & 1 & 0 & 2/45 & 11/45 & 25/9 \\ 0 & 0 & 1 & -1/5 & 2/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_1 - \frac{2}{3} R_3 \\ R_2 + \frac{13}{9} R_3 \end{array}$$

Let x_5 & $x_4 \in \mathbb{R}$

i.e $x_5 =$ arbitrary

$x_4 =$ arbitrary

$$x_3 - \frac{1}{5} x_4 + \frac{2}{5} x_5 = 0 \Rightarrow x_3 = \frac{1}{5} x_4 - \frac{2}{5} x_5$$

$$x_2 + \frac{2}{45} x_4 + \frac{11}{45} x_5 = \frac{25}{9} \Rightarrow x_2 = -\frac{2}{45} x_4 - \frac{11}{45} x_5 + \frac{25}{9}$$

$$x_1 + \frac{2}{15} x_4 + \frac{11}{15} x_5 = -\frac{2}{3} \Rightarrow x_1 = -\frac{2}{15} x_4 - \frac{11}{15} x_5 - \frac{2}{3}$$

Q2: Determine all values of a and b for which the following linear system has

$$x - 2y + 3z = 4$$
$$2x - 3y + az = 5$$
$$3x - 4y + 5z = 4b$$

(1) No Solution

(2) Infinite Solutions

(3) Unique Solution

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -3 & a \\ 3 & -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 4b \end{bmatrix}$$

$$A X = \underline{b}$$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & -3 & a & 5 \\ 3 & -4 & 5 & 4b \end{array}\right]$$

$$\underset{R}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & a-6 & -3 \\ 0 & 2 & -4 & 4b-12 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\underset{R}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & a-6 & -3 \\ 0 & 0 & -2a+8 & 4b-6 \end{array}\right] R_3 - 2R_2$$

(1) I.M.S if $a = 4$ & $b = \frac{3}{2}$

(2) No Solution if $a = 4$ & $b \neq \frac{3}{2}$ i.e for any value of "b" except $\frac{3}{2}$.

(3) $\forall a, b \in \mathbb{R}$ except $a = 4$ & $b = \frac{3}{2}$, we have unique solution.

: Find A^{-1} by elementary row operations, where

$$A = \begin{bmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 6 & 2 & 8 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 & 1 & 0 \\ 4 & -4 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 4 & -4 & 5 & 0 & 0 & 1 \\ -3 & 4 & 1 & 0 & 1 & 0 \\ 6 & 2 & 8 & 1 & 0 & 0 \end{array}\right] \quad R_{13}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 0 & 1 & 1 \\ -3 & 4 & 1 & 0 & 1 & 0 \\ 6 & 2 & 8 & 1 & 0 & 0 \end{array}\right] \quad R_1 + R_2$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 0 & 1 & 1 \\ 0 & 4 & +19 & 0 & 4 & 3 \\ 0 & 2 & -28 & 1 & -6 & -6 \end{array}\right] \quad \begin{matrix} R_2 + 3R_1 \\ R_2 - 6R_1 \end{matrix}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 0 & 1 & 1 \\ 0 & 2 & -28 & 1 & -6 & -6 \\ 0 & 4 & 19 & 0 & 4 & 3 \end{array}\right] \quad R_{23}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 0 & 1 & 1 \\ 0 & 2 & -28 & 1 & -6 & -6 \\ 0 & 0 & 75 & -2 & 16 & 15 \end{array}\right] \quad R_3 - 2R_2$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 0 & 1 & 1 \\ 0 & 1 & -14 & 1/2 & -3 & -3 \\ 0 & 0 & 75 & -2 & 16 & 15 \end{array}\right] \quad R_2/2$$

$$R \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 0 & 1 & 1 \\ 0 & 1 & -14 & 1/2 & -3 & -3 \\ 0 & 0 & 1 & -2/75 & 16/75 & 1/5 \end{array}\right] \quad R_3/75$$

$$R \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4/25 & -7/25 & -1/5 \\ 0 & 1 & 0 & 19/150 & -1/75 & -1/75 \\ 0 & 0 & 1 & -2/75 & 16/75 & 1/5 \end{array}\right] \quad \begin{array}{l} R_1 - 6R_3 \\ R_2 + 14R_3 \end{array}$$

$$= [\, I \mid A^{-1} \,]$$

So, $A^{-1} = \begin{bmatrix} 4/25 & -7/25 & -1/5 \\ 9/150 & -1/75 & -1/75 \\ -2/75 & 16/75 & 1/5 \end{bmatrix}$