Applied Physics for Engineers (PHY121)

Electrostatics

LECTURE # 5

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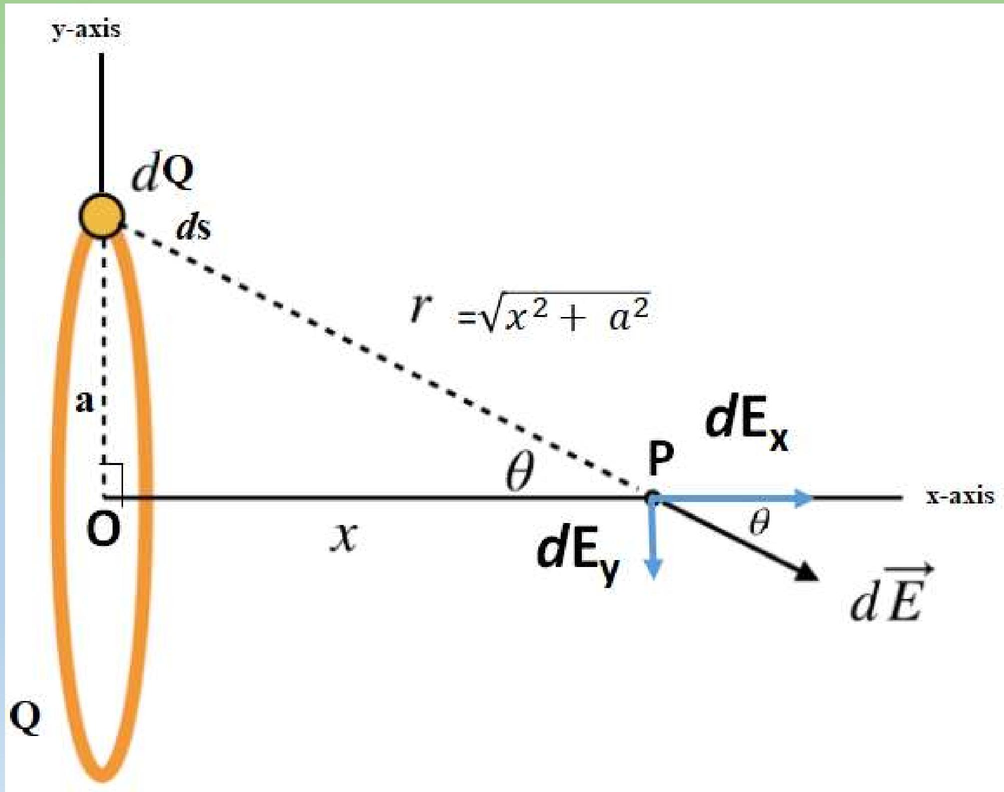
Outlines

1. Electric field due to Ring of Charge

2. Electric field due to Disk of Charge

Field of a Ring of Charge

A ring-shaped conductor with radius ‘a’ carries a total charge Q uniformly distributed around it (see Fig). Find the electric field at a point P that lies on the axis of the ring at a distance ‘x’ from its center.



$$dE = \frac{1}{4\pi\epsilon_o} \cdot \frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_o} \cdot \frac{dQ}{x^2 + a^2}$$

$$dE_x = dE\cos\theta \qquad dE_y = dE\sin\theta$$

So,

$$E_y = \int dE_y = 0$$

$$dE_x = dE\cos\theta = \frac{1}{4\pi\epsilon_o} \cdot \frac{dQ}{x^2 + a^2} \cdot \frac{x}{\sqrt{x^2 + a^2}}$$

$$dE_x = \frac{1}{4\pi\epsilon_o} \cdot \frac{xdQ}{\left(x^2 + a^2\right)^{3/2}}$$

To find the total x-component E_x of the field at P, we integrate this expression over all segments of ring:

$$E_x = \int dE_x =$$

$$\int \frac{1}{4\pi\epsilon_o} \cdot \frac{xdQ}{\left(x^2 + a^2\right)^{3/2}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{x}{\left(x^2 + a^2\right)^{3/2}} \int dQ$$

Finally,

$$\boxed{\vec{E} = E_x\hat{\imath} = \frac{1}{4\pi\epsilon_o} \cdot \frac{Qx}{\left(x^2 + a^2\right)^{3/2}}\,\hat{\imath}}$$

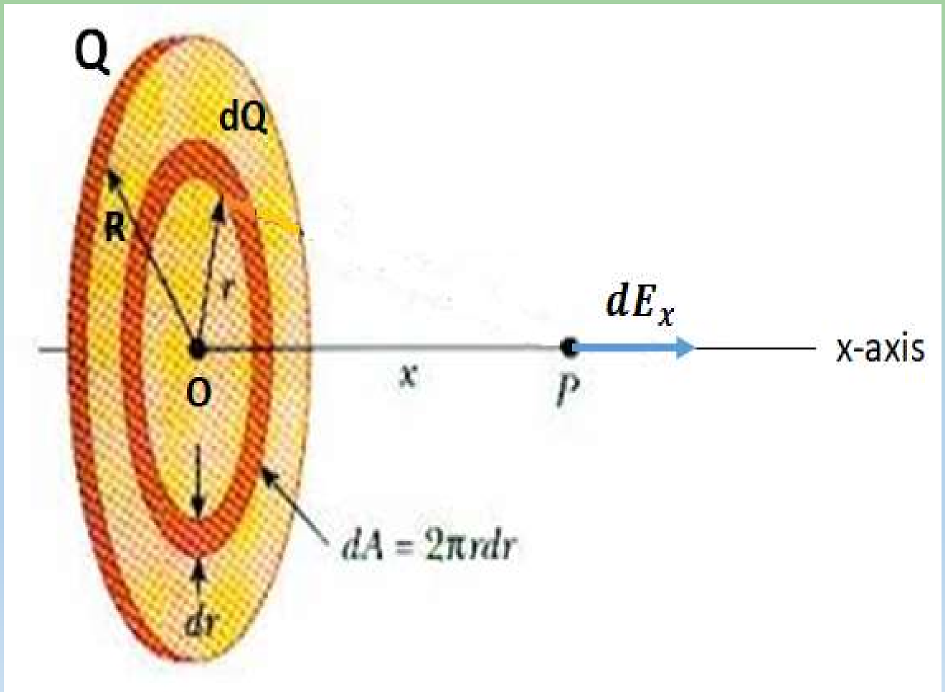
<u>At x = 0, E=0</u>

And when x>>a then

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \cdot \frac{Q}{x^2}\,\hat{\imath}$$

Field of a uniformly Charged Disk

"Find the electric field caused by a disk of radius R with a uniform positive surface charge density (charge per unit area) σ, at a point along the axis of the disk a distance 'x' from its center. Assume that x is positive."



Surface charge density = $\sigma = dQ/dA$

dQ = σ.dA = σ (2πrdr) or, dQ = 2πσrdr

The field component dE_x at point P due to ring of charge dQ is

$$dE_x = \frac{1}{4\pi\epsilon_o}\cdot\frac{xdQ}{(x^2+r^2)^{3/2}} = \frac{1}{4\pi\epsilon_o}\cdot\frac{x(2\pi\sigma r dr)}{(x^2+r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r from r = 0 to r = R:

$$E_x = \int_0^R \frac{1}{4\pi\epsilon_o}\cdot\frac{x(2\pi\sigma r dr)}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{2\epsilon_o}\int_0^R \frac{rdr}{(x^2+r^2)^{3/2}}$$

Take, $\int_0^R \frac{rdr}{(x^2+r^2)^{3/2}}$ from the above expression

Let,

z = $x^2 + r^2$

=> dz = 2rdr

=> dz/2 = rdr Putting in above expression, we have

$$= \frac{1}{2}\int_0^R \frac{dz}{(z)^{3/2}} = \frac{1}{2}\int_0^R (z)^{-3/2} dz = \frac{1}{2}\left|\frac{z^{-1/2}}{-1/2}\right|_0^R$$

$$= -\left|z^{-1/2}\right|_0^R = -\left|\frac{1}{z^{1/2}}\right|_0^R = -\left|\frac{1}{(x^2+r^2)^{1/2}}\right|_0^R = -\left[\frac{1}{\sqrt{(x^2+R^2}} - \frac{1}{x}\right] = = \left[\frac{1}{x} - \frac{1}{\sqrt{(x^2+R^2}}\right]$$

Finally,

$$E_x = \frac{\sigma x}{2\epsilon_o}\left[\frac{1}{x} - \frac{1}{\sqrt{(x^2+R^2}}\right] = \frac{\sigma}{2\epsilon_o}\left[1 - \frac{1}{\sqrt{(R^2/x^2)+1}}\right]$$

when R>>x, then the term $1/\sqrt{R^2/x^2+1}$ in above eq. becomes negligibly small, and we get

$$E = \frac{\sigma}{2\epsilon_o}$$

Hence, the electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet.

<u>END OF LECTURE</u>