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Date:

[Partial Differential Equations]
[Separation of variable Method]

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + U$$

heat

Distance (length)

time

single derivative w.r.t "t"

single derivative with.r.t "x"

Solution:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + U$$

$U(x,t) = X(x).T(t)$

$\partial u/\partial x = X'(x).T(t)$

$\partial u/\partial t = X(x).T'(t)$

Put values in equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + U$$

$X'(x).T(t) = 2(X(x).T'(t)) + X(x).T(t)$

$X'T = 2XT' + XT$

$X'T - XT = 2XT'$

$(X' - X)T = 2XT'$

$$\frac{X'-X}{X} = 2\frac{T'}{T}$$

$$\frac{X'}{X} - \frac{X}{X} = 2\frac{T'}{T}$$

$$\frac{X'}{X} - 1 = 2\frac{T'}{T}$$

$$\int \frac{X'}{X} - \int 1 = 2\int \frac{T'}{T}$$

$\ln X - X = 2\ln T$

$\ln X - X = 2\ln T$

K-method

Let Both sides = "K"

$\ln X - X = K$	$2\ln T = K$
$\ln X = K + X$	$\ln T = K/2$
$e^{\ln X} = e^{K+X}$	$e^{\ln T} = e^{K/2}$
$X = e^{K+x}$	$T = e^{K/2}$
$X = e^{K}.e^{x}$	
$X = Ke^{x}$	

As

$U(x,t) = X(x).T(t)$

$U(x,t) = Ke^{x}.e^{K/2}$

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Difference b/w Heat & Wave Equation

Heat Equation

Thermal diffusivity ← α ← c^2

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

u → heat/temperature

u → dependent variable

function of both time(t) and spatial coordinates(......) depending on dimensionality of the problem

t → time

x → space/Distance/spatial/length

$\frac{\partial u}{\partial t}$ → single derivative w.r.t "t" [time]

$\frac{\partial^2 u}{\partial x^2}$ → ~~Double~~ second derivative w.r.t "x" [Spatial/space/Distance/length]

Wave Equation

c^2 → (+ive) speed of wave determining the behavior of wave propagation.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

u → wave function describes the displacement or amplitude of a wave as a function of both time and space.

t → time

x → [space/spatial/Distance/Length]

$\frac{\partial^2 u}{\partial t^2}$ → double derivative w.r.t. "t" [time]

$\frac{\partial^2 u}{\partial x^2}$ → double second derivative w.r.t. "x" [spatial]

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Heat Equation

$$\frac{\partial u}{\partial t} = c^2 . \frac{\partial^2 u}{\partial x^2}$$

$U(x,t) = X(x).T(t)$

$\frac{\partial u}{\partial t} = X(x).T'(t)$ [first derivative w.r.t. "t"]

$\frac{\partial u}{\partial x} = X'(x).T(t)$

$\frac{\partial^2 u}{\partial x^2} = X''(x).T(t)$ [second derivative w.r.t. "x"]

Put values in equation

$$\frac{\partial u}{\partial t} = c^2 . \frac{\partial^2 u}{\partial x^2}$$

$X(x).T'(t) = c^2[X''(x).T(t)]$

$\frac{T'}{T} = c^2 \frac{X''}{X}$

Let both sides = "k"

$c^2 \frac{X''}{X} = k$ & $\frac{T'}{T} = k$

K-Method

$c^2 \frac{X''}{X} = k$

$c^2 X'' = kX$

$c^2 X'' - kX = 0$

$X'' - \frac{k}{c^2} X = 0$

$[D^2 - \lambda]X = 0$

$D^2 - \lambda = 0$

$D = \pm\sqrt{\lambda}$

$X(x) = C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda} x}$

$\frac{T'(t)}{T(t)} = k$

$\int \frac{T'}{T} dt = \int k \, dt$

$\ln|T| = kt$

$e^{\ln(T)} = e^{kt}$

$T_{(t)} = e^{kt}$

$U(x,t) = X(x).T(t)$

$U(x,t) = [C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda}}] e^{kt}$

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Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}$$

second derivative w.r.t "t"

second derivative w.r.t "x"

$U(x,t) = X(x) \cdot T(t)$

$\frac{\partial U}{\partial t} = X(x) \cdot T'(t)$

$\frac{\partial^2 U}{\partial t^2} = X(x) \cdot T''(t)$

$\frac{\partial U}{\partial x} = X'(x) \cdot T(t)$

$\frac{\partial^2 U}{\partial x^2} = X''(x) \cdot T(t)$

Put values in equation

$$\frac{\partial^2 U}{\partial t^2} = c^2 \cdot \frac{\partial^2 U}{\partial x^2}$$

$X(x) \cdot T''(t) = c^2 [X''(x) \cdot T(t)]$

$\frac{T''}{T} = c^2 \frac{X''}{X}$

Let both sides = "K"

$\frac{c^2 X''}{X} = K$ & $\frac{T''}{T} = K$

K-Method

$\frac{c^2 X''}{X} = K$

$c^2 X'' = KX$

$c^2 X'' - KX = 0$

$X'' - \frac{K}{c^2} X = 0$

$X'' - \lambda X = 0$

$[D^2 - \lambda] X = 0$

$D^2 - \lambda = 0$

$D^2 = \lambda$

$D = \pm \sqrt{\lambda}$

$X(x) = C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda} x}$

$\frac{T''(t)}{T(t)} = K$

$T'' = KT$

$T'' - KT = 0$

$(D^2 - K) T = 0$

$D^2 - K = 0$

$D^2 = K$

$D = \pm \sqrt{K}$

$T(t) = C_3 e^{\sqrt{K} t} + C_4 e^{-\sqrt{K} t}$

$U(x,t) = X(x) \cdot T(t)$

$U(x,t) = [C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda} x}] \cdot [C_3 e^{\sqrt{K} t} + C_4 e^{-\sqrt{K} t}]$