Homogeneous D.Es

$$(x^2+y^2)dx + (x^2-xy)dy = 0$$

Substitute $y = u\cdot x$ & $u = \frac{y}{x}$

$$dy = u\cdot dx + x\cdot du$$

Put values of y and dy

$$(x^2+u^2x^2)dx + (x^2 - x(ux))(\cancel{e}\, udx + xdu) = 0$$

$$\cancel{x^2(1+u^2)}$$

$$x^2dx + \cancel{u^2x^2dx} + ux^2dx + x^3du - \cancel{u^2x^2dx} - ux^3du = 0$$

$$x^2dx + ux^2dx + x^3du - ux^3du = 0$$

$$x^2(1+u)dx - x^3(u-1)du = 0$$

$$x^2(1+u)dx = x^3(u-1)du$$

$$\frac{x^2}{x^3}dx = \frac{u-1}{u+1}du$$

$$\frac{1}{x}dx = \left(1-\frac{2}{u+1}\right)du$$

$$\int \frac{1}{x}dx = \int\left(1-\frac{2}{u+1}\right)du$$

$$\ln x = \int 1du - 2\int \frac{1}{u+1}du$$

$$= \frac{u-1}{u+1}$$

$$= \frac{u-1+1-1}{u+1}$$

$$= \frac{(u+1)-2}{u+1}$$

$$= \frac{u+1}{u+1} - \frac{2}{u+1}$$

$$= 1 - \frac{2}{u+1}$$

$$\ln x = u - 2\ln(u+1) + c$$

$$\ln x = \frac{y}{x} - 2\ln\left(\frac{y}{x}+1\right) + c$$

$$\ln x + 2\ln\left(\frac{y}{x}+1\right) = \frac{y}{x} + c$$

$$\ln(x)\left(\frac{y}{x}+1\right)^2 = \frac{y}{x} + c$$

$$c = \ln(x)\left(\frac{y}{x}+1\right)^2 - \frac{y}{x}$$