Department Of Computer Science, CUI Lahore Campus

CSC102 - Discrete Structures

By

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Course Outline

- Sets
  - Set Operations
  - Inclusion-Exclusion Principle of Sets
  - Set Identities
  - Membership Tables

Set Operations

- Two sets can be combined in many different ways.

- Set operations can be used to combine sets.
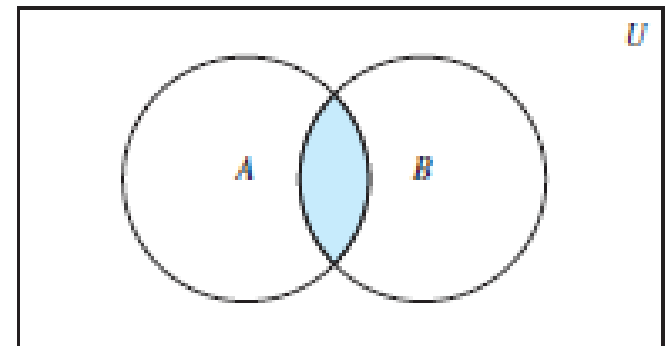
Union

- Let A and B be sets.

- The **union** of A and B, denoted by **A**∪**B**, is the set containing those elements that are either in A or in B, or in both.

- A ∪ B = {x | x ∈ A ∨ x ∈ B}
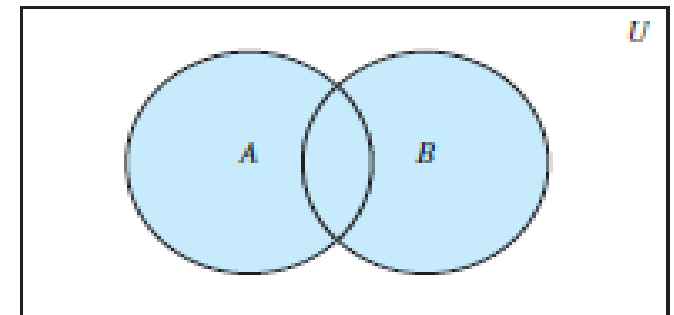
$A \cup B$ is shaded.

Intersection

- Let A and B be sets.

- The **intersection** of A and B, denoted by **A ∩ B**, is the set containing those elements in both A and B.

- A ∩ B = {x | x ∈ A ∧ x ∈ B}

$A \cap B$ is shaded.

Union (example)

- Let A = {1,2,3}
    B = {2,4,6,8}
    A ∪ B = {1,2,3,4,6,8}

- Let A = {x | x ∈ **Z** ∧ x is even}
    B = {x | x ∈ **Z** ∧ x is odd}
    A ∪ B = **Z**

Intersection (example)

- Let A = {1,2,3}

    B = {2,4,6,8}

    A ∩ B = { 2 }

- Let A = **Z**

    B = {x |x ∈ **Z** ^ x is odd}

    A ∩ B = {x | x ∈ **Z** ^ x is odd}

Disjoint Sets

- Two sets are called **disjoint** if their intersection is empty.

- Let A = {x | x ∈ **Z** ∧ x is even}
      B = {x | x ∈ **Z** ∧ x is odd}
      A ∩ B = Ø

The Cardinality of the Union of Sets

- **Principle of inclusion-exclusion**
- $|A \cup B| = |A| + |B| - |A \cap B|$
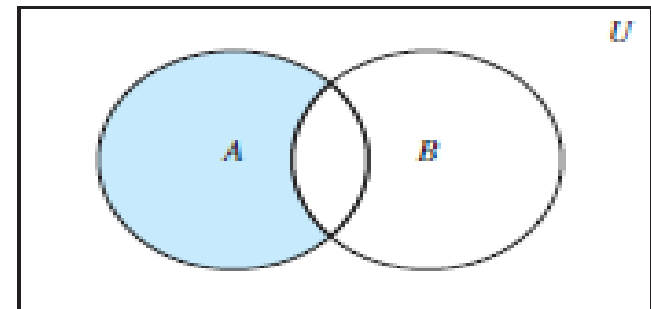
The Cardinality of the Union of Sets

- Find $|A \cup B| =$?

Solution:

- Let $A = \{1,2,3\}$

$B = \{2,3,4\}$

$A \cap B = \{2,3\}$

$A \cup B = \{1,2,3,4\}$

- $|A| = 3 \qquad |B| = 3 \qquad |A \cap B| = 2$
- $|A \cup B| = 4$

Difference

- Let A and B be sets.

- The **difference** of A and B, denoted by **A - B**, is the set containing those elements that are in A but not in B. (also called **complement of B with respect to A**).

- A **-** B = $\{x \mid x \in A \land x \notin B\}$

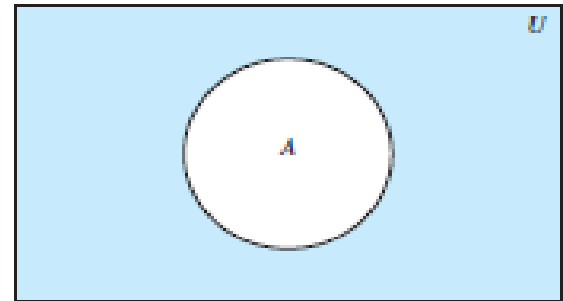$A - B$ is shaded.

Difference (example)

- Let A = {1,2,3}

    B = {2,4}

    A – B = {1,3}

- Let A = **Z**

    B = { x | x ∈ **Z** ^ x is odd }

    A – B = { x | x ∈ **Z** ^ x is even }

Complement

- Let U be the universal set and A be a set.

- The **complement** of A, denoted by $\overline{A}$, is the complement of A with respect to U (which is U - A).

- $\overline{A} = \{\, x \mid x \notin A \,\}$

$\overline{A}$ is shaded.

Complement (example)

- Let A = { a, b, c, d } and

     U is the set of English alphabet

     $\overline{A}$ = { e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z }

- Let A = { x | x ∈ **Z** ∧ x is odd } and

     U is **Z**

     $\overline{A}$ = { x | x ∈ **Z** ∧ x is even }

Summary Set Operations

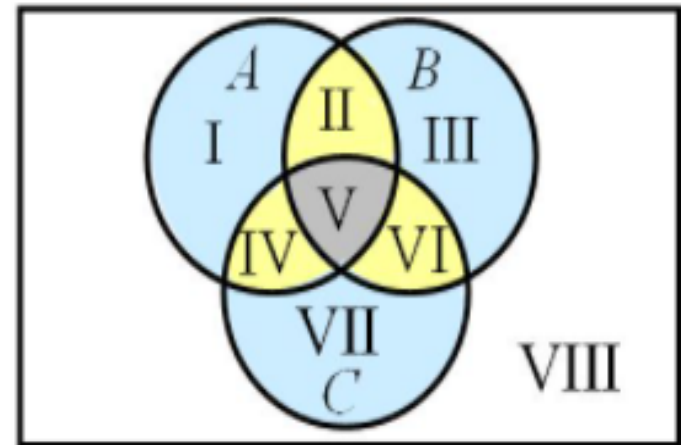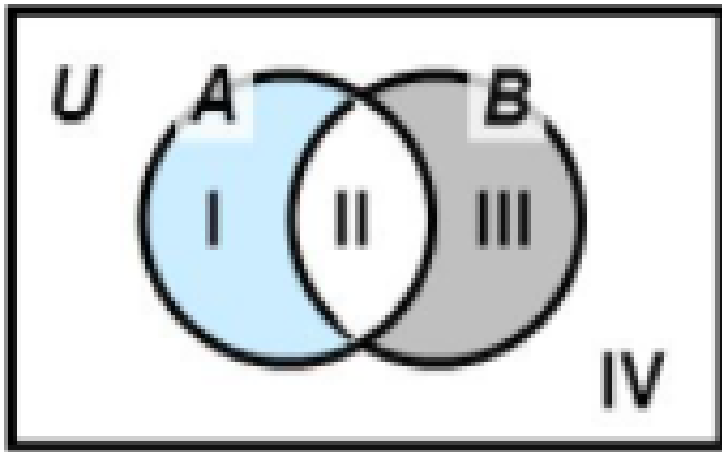| Operation	Notation
Union	$A \cup B = \{x \mid x \in A \lor x \in B\}$
Intersection	$A \cap B = \{x \mid x \in A \land x \in B\}$
Difference	$A - B = \{x \mid x \in A \land x \notin B\}$
Complement (U - A)	$\overline{A} = \{x \mid x \notin A\}$

Inclusion-Exclusion Principle of Sets

2 and 3 - Set Venn Diagram

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

2 - Set Venn Diagram

- **Region I** represents the elements in set A that are not in set B.
- **Region II** represents the elements in both sets A and B. **Region III** represents the elements in set B that are not in set A.
- **Region IV** represents the elements in the universal set that are in neither set A nor set B.
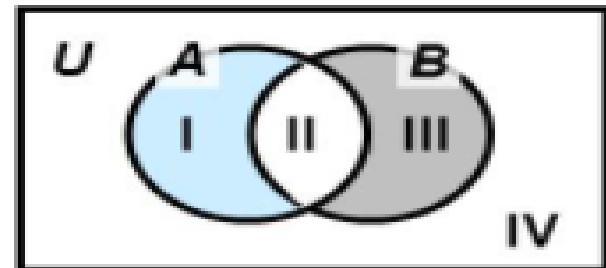
Example

Suppose a list *A* contains the 30 students in a mathematics class, and a list *B* contains the 35 students in an English class, and suppose there are 20 names on both lists.

Find the number of students:

- only on list *A*
- only on list *B*
- on list *A* or *B* (or both)
- on exactly one list.
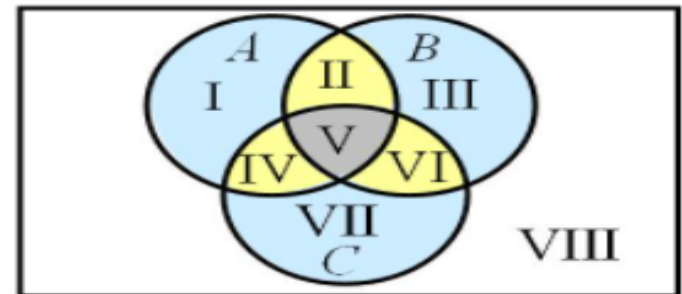
Example

Solution:

- 30 − 20 = 10 names are only on list *A*.

- 35 − 20 = 15 are only on list *B*.

- $|A \cup B| = |A| + |B| − |A \cap B| = 30 + 35 − 20 = 45$.

- 10 + 15 = 25 names are only on one list; that is,
  $|A \oplus B| = 25$.

3 - Set Venn Diagram

- **Region I** represents the elements in set A but not in set B or set C.
- **Region II** represents the elements in set A and set B but not in set C.
- **Region III** represents the elements in set B but not in set A or set C.
- **Region IV** represents the elements in sets A and C but not in set B.
- **Region V** represents the elements in sets A, B, and C.
- **Region VI** represents the elements in sets B and C but not in set A.
- **Region VII** represents the elements in set C but not in set A or set B.
- **Region VIII** represents the elements in the universal set U, but not in set A, B, or C.

Example

Consider the following data for 120 mathematics students at a college concerning the languages French, German, and Russian:

65 study French,　　45 study German,

42 study Russian ,　20 study French and German,

25 study French and Russian,

15 study German and Russian.

8 study all three languages.

Determine how many students study exactly 1 language course and fill the correct numbers of students in each eight region of Venn diagram shown in figure.

Example
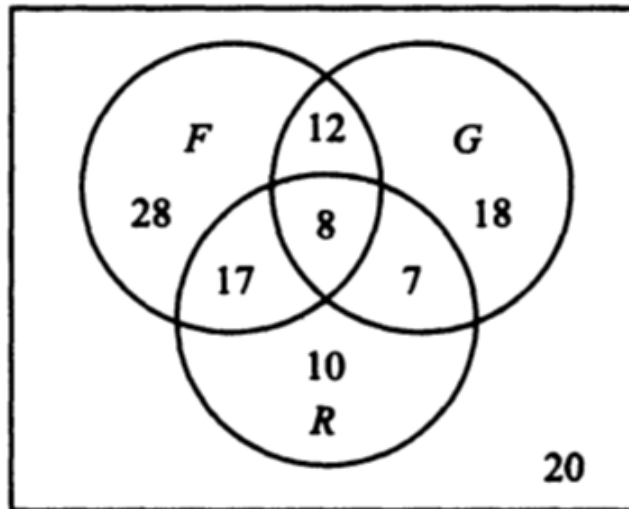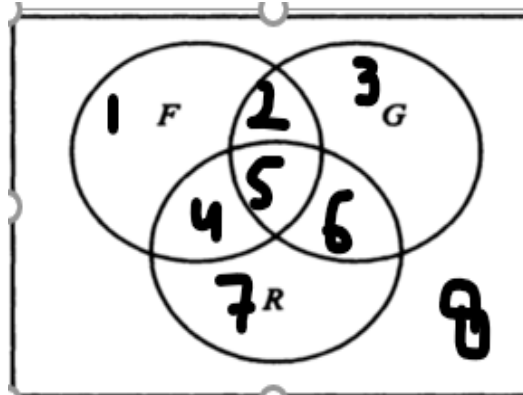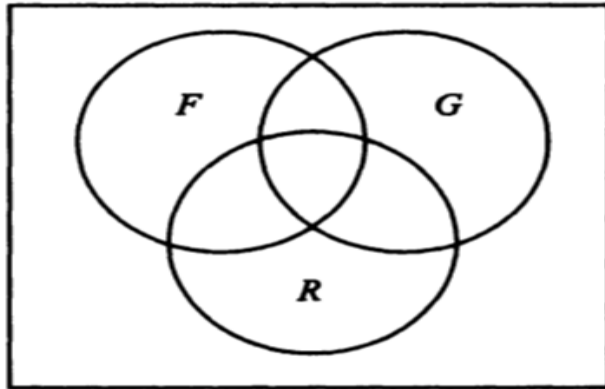
$$U = 120$$
$$F = 65$$
$$G = 45$$
$$R = 42$$
$$F \cap G = 20$$
$$F \cap R = 25$$
$$G \cap R = 15$$
$$F \cap G \cap R = 8$$

$$R1 - |F| = 65 - 12 - 8 - 17 = 28$$
$$R3 - |G| = 45 - 12 - 8 - 7 = 18$$
$$R7 - |R| = 42 - 17 - 8 - 7 = 10$$
$$R2 - |F \cap G| = 20 - 8 = 12$$
$$R4 - |F \cap R| = 25 - 8 = 17$$
$$R6 - |G \cap R| = 15 - 8 = 7$$
$$R5 - |F \cap G \cap R| = 8$$
$$|F \cup G \cup R| = 65 + 45 + 42 - 20 - 25 - 15 + 8 = 100$$
$$R8 - |U| - |F \cup G \cup R| = 120 - 100 = 20$$

Example

- Total number of students exactly registered in one course = 28+18+10=56
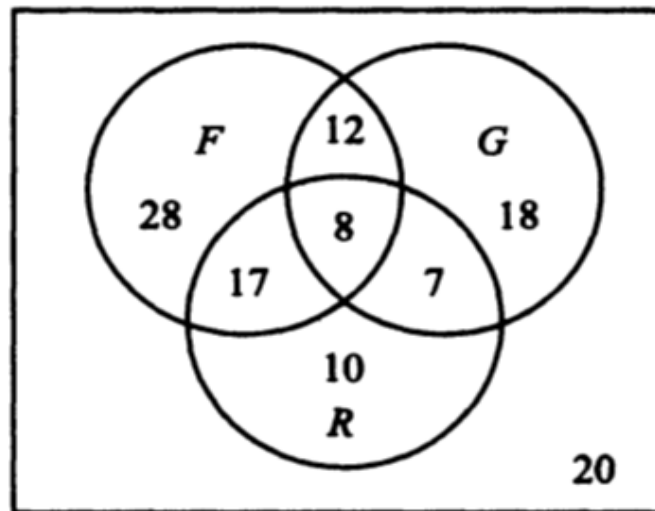
Set Identities

$A \cup \emptyset = A$ Identity Laws
$A \cap U = A$

$A \cup U = U$ Domination Laws
$A \cap \emptyset = \emptyset$

$A \cup A = A$ Idempotent Laws
$A \cap A = A$

$\overline{(\overline{A})} = A$ Complementation Law

$A \cup \overline{A} = U$ Complement Laws
$A \cap \overline{A} = \emptyset$

Set Identities

$A \cup B = B \cup A$ Commutative Laws

$A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$ Associative Laws

$A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup (A \cap B) = A$ Absorption Laws

$A \cap (A \cup B) = A$

Set Identities

$\overline{A \cup B} = \overline{A} \cap \overline{B}$ De Morgan's Law

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Distributive Law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A - B = A \cap \overline{B}$ Difference Law

How to Prove a Set Identity

- Four methods:

  - Use membership tables
  - Use the basic set identities
  - Prove each set is a subset of each other
  - Use set builder notation and logical equivalences

What is a membership table

- Membership tables show all the combinations of sets an element can belong to
  - 1 means the element belongs, 0 means it does not
- Consider the following membership table:

A	B	A ∪ B	A ∩ B	A - B
1	1	1	1	0
1	0	1	0	1
0	1	1	0	0
0	0	0	0	0

Membership Table

A	B	B-A	$\overline{A}$	$\overline{B}$	$\overline{A \cap B}$	$\overline{A} \cup \overline{B}$
1	1	0	0	0	0	0
1	0	0	0	1	1	1
0	1	1	1	0	1	1
0	0	0	1	1	1	1

Membership Table

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Distributive Law

A	B	C	B ∩ C	A ∪ (B ∩ C)	A ∪ B	A ∪ C	(A ∪ B) ∩ (A ∪ C)
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

Set Identities (example)

- Show $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

Set Identities (example)

- Show $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

Solution:

$$\overline{A \cup (B \cap C)}$$
$$= \overline{A} \cap (\overline{B \cap C}) \qquad \text{(By DeMorgan's Law)}$$
$$= \overline{A} \cap (\overline{B} \cup \overline{C}) \qquad \text{(By DeMorgan's Law)}$$
$$= \overline{A} \cap (\overline{C} \cup \overline{B}) \qquad \text{(By Commutative Law)}$$
$$= (\overline{C} \cup \overline{B}) \cap \overline{A} \qquad \text{(By Commutative Law)}$$

Set Identities

$$A \cup (B - A) = A \cup B$$

$$L.H.S = A \cup (B - A)$$

$$= A \cup (B \cap \overline{A}) \qquad\qquad \text{Difference Law}$$

$$= (A \cup B) \cap (A \cup \overline{A}) \qquad \text{Distribution Law}$$

$$= (A \cup B) \cap \bigcup \qquad\qquad \text{Complement Law}$$

$$= A \cup B \qquad\qquad\qquad \text{Identity Law}$$

$$= R.H.S$$

Exercise Questions

Chapter # 2

Topic # 2.2

Question # 1, 2, 3,4,5,6,15,16,17,18,

19,20,21,22,23,24,25