# Power Series

## Chebyshev's Equation

$$(1-x^2)y'' - xy' + 4y = 0$$

$n^2 = (2)^2 \rightarrow$ chebyshev's D.E

$P_0(x) = (1-x^2)$ , $P_1(x) = -x$ , $P_2(x) = 4$

$P_0(x) = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x = \pm 1$ [singular Points]

$\ldots\ldots, -3, -2, 0, 2, 3, \ldots\ldots$ [ordinary points]

Let [Power Series Method]

$$y = \sum_{n=0}^{\infty} a_n x^n \Rightarrow a_0x^0 + a_1x^1 + a_2x^2 \ldots\ldots\ldots$$

$$dy/dx = y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$d^2y/dx^2 = y'' = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

Put values in equation

$$(1-x^2)y'' - xy' + 4y = 0$$

$$(1-x^2)\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - x\sum_{n=1}^{\infty} n a_n x^{n-1} + 4\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^n - \sum_{n=1}^{\infty} n a_n x^n + 4\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_n x^n - \sum_{n=1}^{\infty} n a_n x^n + 4\sum_{n=0}^{\infty} a_n x^n = 0 \quad [x^n \rightarrow \text{same}]$$

$$2a_2 + 6a_3x + \sum_{n=2}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_n x^n - a_1x - \sum_{n=2}^{\infty} n a_n x^n + 4a_0 + 4a_1x + 4\sum_{n=2}^{\infty} a_n x^n = 0 \quad \left[\sum_{n=2}^{\infty} \downarrow \text{same}\right]$$

Comparing Coefficients

| | | |
|---|---|---|
| $(n+2)(n+1)a_{n+2} - n(n-1)a_n - na_n + 4a_n = 0$ | $6a_3 - a_1 + 4a_1 = 0$ | $2a_2 + 4a_0 = 0$ |
| $(n+2)(n+1)a_{n+2} = n(n-1)a_n + na_n - 4a_n$ | $6a_3 - 3a_1 = 0$ | $2a_2 = -4a_0$ |
| $a_{n+2} = \dfrac{[n(n-1)+n-4]a_n}{(n+2)(n+1)}$ | $a_3 = \frac{1}{2}a_1$ | $a_2 = -2a_0$ |