Applied Physics for Engineers (PHY121)

Electrostatics

LECTURE # 3

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The Electric Field due to an Electric Dipole

Electric Dipole

"The configuration of the two equal and opposite charges separated by a distance is called an electric dipole".

The magnitude of the electric dipole moment 'p' is given by

$$\mathbf{p = qd} \qquad \text{........ (1)}$$

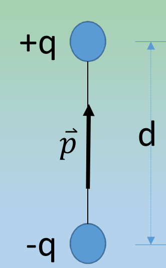


Fig.1

It turns out that this quantity behaves like a vector. We define the vector electric dipole moment $\vec{p}$ to have magnitude '**qd**' and a direction pointing from the **–ve** charge to the **+ve** charge along the line joining the two charges. **Fig.1** shows an electric dipole and its vector dipole moment.

We now calculate the electric field $\vec{E}$ of the dipole at a point '**P**' a distance '**x**' along the perpendicular bisector of the dipole, as shown in **Fig.2**.

The **+ve** and **–ve** charges set up electric fields $\vec{E}_+$ and $\vec{E}_-$ respectively. The magnitudes of these two fields at '**P**' are equal because the point '**P**' is equidistance from the +ve and –ve charges. **Fig.2** also shows the direction of $\vec{E}_+$ and $\vec{E}_-$, determined by their signs of the charges at point '**P**'.
The resultant electric field at '**P**' is determined by

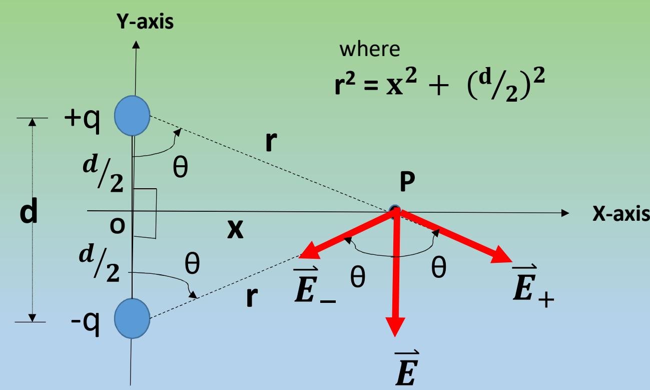


Fig.2

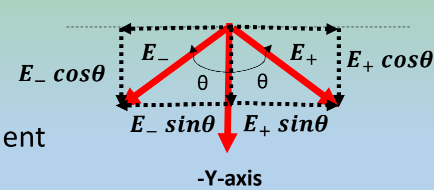
$$\vec{E} = \vec{E}_+ + \vec{E}_- \quad \text{........ (2)}$$

The magnitudes of the fields from each charge are given by

$$E_+ = E_- = \frac{1}{4\pi\epsilon_o} \cdot \frac{q}{x^2 + (d/2)^2} \quad \text{........ (3)}$$

Because the fields E_+ and E_- have equal magnitudes and lie at equal angles θ w.r.t. the y-direction as shown in Fig.2. The x-component of the resultant field $\vec{E}$ is

$$E_+ \sin\theta - E_- \sin\theta = 0$$



The resultant field $\vec{E}$, therefore has only y-component of magnitude

$$E = E_+ \cos\theta + E_- \cos\theta = 2E_+ \cos\theta \quad \text{........ (4)}$$

We can find out the value of $cos\theta$ from Fig.2

$$cos\theta = \frac{d/2}{r} = \frac{d/2}{\sqrt{x^2 + (d/2)^2}} \qquad \text{........ (5)}$$

Hence, using eq.3 and eq.5 in eq.4, we have

$$E = 2E_+ \, cos\theta \; = 2\,\frac{1}{4\pi\epsilon_o} \cdot \frac{q}{x^2 + (d/2)^2} \cdot \frac{d/2}{\sqrt{x^2 + (d/2)^2}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{qd}{\left[x^2 + (d/2)^2\right]^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{\left[x^2 + (d/2)^2\right]^{3/2}} \qquad \text{........ (6)}$$

Eq.6 gives the magnitude of the electric field at point '**P**' due to the electric dipole.

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \cdot \frac{\vec{\boldsymbol{p}}}{\left[x^2 + (d/2)^2\right]^{3/2}}\,\hat{r} \qquad \text{........ (7)}$$

where **eq.7** gives the vector form of electric field intensity of the electric dipole at point '**P**' and $\hat{\boldsymbol{r}} = \vec{r}/\boldsymbol{r}$ is the direction of the vector electric field intensity $\vec{\boldsymbol{E}}$ of the electric dipole.

Often, we observe the field of an electric dipole at point 'P' whose distance 'x' from the dipole is very large as compared with the separation 'd'. In this case we can simplify the dipole field somewhat by making use of the binomial expansion, i.e.

$$(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}\,y^2 + \cdots \qquad \text{........ (8)}$$

Using eq.6 again as

$$E = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{\left[x^2 + (d/2)^2\right]^{3/2}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{x^3} \cdot \frac{1}{\left[1 + (d/2x)^2\right]^{3/2}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{x^3} \cdot \left[1 + \; (d/2x)^2\right]^{-3/2}$$

Now apply the binomial expansion as given in eq.8 to the factor in brackets of the above equation, which gives

$$E = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{x^3} \cdot \left[\mathbf{1} + \left(-\frac{\mathbf{3}}{\mathbf{2}}\right) \left(\mathbf{d}/_{\mathbf{2x}}\right)^{\mathbf{2}} + \cdots\right] \qquad \text{........ (9)}$$

For **x>>d** it is sufficient to keep only the first term in the brackets **(the 1)**, and so we find an expression for the magnitude of the electric field due to a dipole at distant points in its median plane:

$$E = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{x^3} \qquad \text{........ (10)}$$

As we can see from **eq.10**, the field at distant points varies with the distance '**r**' from the dipole as **$1/r^3$**. This is the characteristic result for the electric dipole field. The field varies more rapidly with the distance than the **$1/r^2$** dependence characteristic of a point charge.

END OF LECTURE