Linear Differential Equation

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

$\downarrow$

(Non standard form of Linear Differential Equation)

(a) Homogeneous Differential Equations

$$\frac{dy}{dx} + 3y = 0$$

(b) Non Homogeneous Differential Equation

$$\frac{dy}{dx} + 3y = x$$

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Solve

Homogeneous Linear Differential Equations

$$\frac{dy}{dx} - \underset{\uparrow}{\overset{p(x)}{3}}y = \underset{\uparrow}{\overset{f(x)}{0}}$$

Step # 1 : Dividing by the coefficient a_1 from $(a_1(x))$

Here coeficient is "1"

Step # 2 : Integrating factor

$$I.F = e^{\int p(x)dx}$$

$$\frac{dy}{dx} - \overset{p(x)}{3}y = 0$$

$$I.F = e^{\int(-3)dx}$$

$$I.F = e^{(-3)\int 1 \cdot dx}$$

$$I.F = e^{-3x+c}$$

$$I.F = e^{-3x}.e^{c}$$

$$I.F = c.e^{-3x}$$

$$I.F = e^{-3x} \qquad \therefore c = 1$$

Step #3 : $\frac{d}{dx}[(I.F)y] = (I.F)f(x)$

$$\frac{d}{dx}[e^{-3x}.y] = e^{-3x}(0)$$

$$\frac{d}{dx}[e^{-3x}.y] = 0$$

Step #4: Taking "$\int$" on both sides

$$\int \frac{d}{dx}[e^{-3x}.y] = \int 0$$

$$e^{-3x}.y = 0 + C$$

$$y = C.e^{3x}$$

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Non Homogeneous Linear Differential Equation

$$x\frac{dy}{dx} - 4y = x^6 . e^x$$

Step # 1: Dividing by coefficient "x"

$$\frac{dy}{dx} - \frac{4}{x}y = x^5 . e^x$$

$$p(x) = -4x^{-1}$$

$$f(x) = x^5 . e^x$$

Step #2 : Integerating factor

$$I.F = e^{\int p(x)dx} \Rightarrow e^{\int(-4x^{-1})dx}$$

$$I.F = e^{-4\int(\frac{1}{x})dx} \Rightarrow e^{-4\ln x + C}$$

$$I.F = e^{-4\ln x} . e^c$$

$$I.F = c . e^{-4\ln x}$$

$$I.F = e^{\ln x^{-4}} \quad \because c = 1$$

$$I.F = x^{-4}$$

Step # 3: $\frac{d}{dx}[(I.F)y] = (I.F)f(x)$

$$\frac{d}{dx}[x^{-4}.y] = (x^{-4})(x^5.e^x)$$

$$\frac{d}{dx}[x^{-4}.y] = x.e^x$$

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Step # 4: Taking "$\int$" on both sides

$$\int \frac{d}{dx}\left[x^{-4}\cdot y\right] = \int \underset{A}{x}\cdot \underset{B}{e^{x}}\, dx$$

$$x^{-4}\cdot y = x\int(e^{x}) - \int\int(e^{x})\cdot\frac{d}{dx}(x)\, dx$$

$$x^{-4}\cdot y = xe^{x} - \int(e^{x})(1)\, dx$$

$$x^{-4}\cdot y = xe^{x} - e^{x} + C$$

$$y = x^{5}e^{x} - x^{4}\cdot e^{x} + x^{4}C$$

$$y = x^{5}e^{x} - x^{4}\cdot e^{x} + C$$

Product Rule of Integration

Inverse
Logs
Algebric
Trignometric
Exponential

$$\int A\cdot B\, dx = A\int B - \int\int B\, dx\cdot A'$$

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Solve

$$(x^2-9)\frac{dy}{dx} + xy = 0$$

Step #1: Dividing by coefficient of $(a_1(x))$

$$\frac{dy}{dx} + \frac{x}{x^2-9}y = 0$$

$$P(x) = \frac{x}{x^2-9} \quad , \quad f(x) = 0$$

Step #2: $I.F = e^{\int P(x)dx}$

$$I.F = e^{\int\left(\frac{x}{x^2-9}\right)dx}$$

$$I.F = e^{\frac{1}{2}\int\frac{2x}{x^2-9}dx} \Rightarrow e^{\frac{1}{2}\ln(x^2-9)}$$

$$I.F = e^{\ln(x^2-9)^{1/2}}$$

$$I.F = (x^2-9)^{1/2} = \sqrt{x^2-9}$$

Step # 3: $\frac{d}{dx}\left[I.F(y)\right] = I.F(f(x))$

$$\frac{d}{dx}\left[(\sqrt{x^2-9})y\right] = (\sqrt{x^2-9})(0)$$

$$\frac{d}{dx}\left[(\sqrt{x^2-9})y\right] = 0$$

Step #4: Taking Integral "$\int$" on both sides

$$\int\frac{d}{dx}\left[(\sqrt{x^2-9})y\right] = \int 0\,dx$$

$$(\sqrt{x^2-9})y = 0 + C$$

$$(\sqrt{x^2-9})y = C$$

$$y = \frac{C}{\sqrt{x^2-9}}$$

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Solve

$$\frac{dy}{dx} + y = x$$

Step # 1:

$p(x) = 1$, $f(x) = x$

Step # 2:

$$I.F = e^{\int p(x)dx}$$

$$I.F = e^{\int 1 dx}$$

$$I.F = e^{x}$$

Step # 3:

$$\frac{d}{dx}\left[e^{x}.y\right] = (e^{x})(x)$$

Step # 4: Taking integrals on both sides

$$\int \frac{d}{dx}\left[e^{x}.y\right] = \int \overset{A}{x} . \overset{B}{e^{x}} dx$$

$$e^{x}.y = x\int e^{x}dx - \int\left(\left(\int e^{x}\right)\left(\frac{d}{dx}(x)\right)\right).dx$$

$$e^{x}.y = xe^{x} - \int (e^{x})(1)dx$$

$$e^{x}.y = xe^{x} - e^{x} + c \longrightarrow equ(A)$$

Initial Value Problems

$$y(0) = 4$$

$$x = 0 \quad , \quad y = 4$$

Put values in equ(A)

$$e^{0}.(4) = (0)e^{0} - e^{0} + c$$

$$4 = c - 1$$

$$\boxed{c = 5}$$

$$e^{x}\cdot y = x\cdot e^{x} - e^{x} + C$$

$$e^{x}\cdot y = x\cdot e^{x} - e^{x} + 5$$

$$y = x - 1 + 5e^{-x} \longrightarrow B$$

$$\frac{dy}{dx} = 1 - 0 + 5e^{-x}(-1)$$

$$\frac{dy}{dx} = 1 - 5e^{-x} \longrightarrow C$$

Check

$$\frac{dy}{dx} + y = x$$

$$(1-5e^{-x}) + (x-1+5e^{-x}) = x$$

$$\cancel{1} - \cancel{5e^{-x}} + x - \cancel{1} + \cancel{5e^{-x}} = x$$

$$x = x$$