PHY121 Applied Physics for Engineers

Electrostatics (Cont...)

LECTURE # 12

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Potential due to Point Charge

To find the potential V due to a single point charge q,

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

where r is the distance from the point charge q to the point at which the potential is evaluated. If q is positive, the potential that it produces is positive at all points; if q is negative, it produces a potential that is negative everywhere. In either case, V is equal to zero at $r = \infty$, an infinite distance from the point charge. Note that potential, like electric field, is independent of the test charge q_0 that we use to define it.

Potential due to a collection of Point Charge

Similarly, to find the potential due to a collection of point charges:

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad \text{(potential due to a collection of point charges)}$$

In this expression, r_i is the distance from the ith charge, q_i, to the point at which V is evaluated.

Potential due to a Continuous Distribution of Charge

When we have a continuous distribution of charge along a line, over a surface, or through a volume, we divide the charge into elements dq, and the sum in Eq.(above) becomes an integral:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \quad \text{(potential due to a continuous distribution of charge)}$$

where r is the distance from the charge element dq to the field point where we are finding V.

Finding Electric Potential from Electric Field

The force $\vec{F}$ on a test charge q_0 can be written as $\vec{F} = q_0\vec{E}$, so the work done by the electric force as the test charge moves from a to b is given by

$$W_{a\rightarrow b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

If we divide this by q_0 and compare the result with Eq. ($\frac{W_{a\rightarrow b}}{q_0} = V_a - V_b$) , we find

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi \, dl \quad \text{(potential difference as an integral of } \vec{E}\text{)}$$

Note:

Above equation shows that the unit of potential difference $(1\ \text{V})$ is equal to the unit of electric field $(1\ \text{N/C})$ multiplied by the unit of distance $(1\ \text{m})$. Hence the unit of electric field can be expressed as 1 *volt per meter* $(1\ \text{V/m})$, as well as $1\ \text{N/C}$:

$$1\ \text{V/m} = 1\ \text{volt/meter} = 1\ \text{N/C} = 1\ \text{newton/coulomb}$$

In practice, the volt per meter is the usual unit of electric-field magnitude.

Electron Volts (eV)

The magnitude e of the electron charge can be used to define a unit of energy that is useful in many calculations with atomic and nuclear systems.

When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

$$U_a - U_b = q(V_a - V_b) = qV_{ab}$$

If the charge q equals the magnitude e of the electron charge, 1.602×10^{-19} C, and the potential difference is $V_{ab} = 1$ V, the change in energy is

$$U_a - U_b = (1.602 \times 10^{-19}\,\text{C})(1\,\text{V}) = 1.602 \times 10^{-19}\,\text{J}$$

This quantity of energy is defined to be 1 **electron volt** (1 eV):

$$1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$$

The multiples meV, keV, MeV, GeV, and TeV are often used.

Problem#01 (Electric Force and Electric Potential)

A proton (charge $+e = 1.602 \times 10^{-19}$ C) moves in a straight line from point a to point b inside a linear accelerator, a total distance $d = 0.50$ m. The electric field is uniform along this line, with magnitude $E = 1.5 \times 10^7\,\text{V/m} = 1.5 \times 10^7\,\text{N/C}$ in the direction from a to b. Determine (a) the force on the proton; (b) the work done on it by the field; (c) the potential difference $V_a - V_b$.

Solution:

(a) The force on the proton is in the same direction as the electric field, and its magnitude is

$$F = qE = (1.602 \times 10^{-19}\,\text{C})(1.5 \times 10^7\,\text{N/C}) = 2.4 \times 10^{-12}\,\text{N}$$

(b) The force is constant and in the same direction as the displacement, so the work done on the proton is

$$W_{a\to b} = Fd = (2.4 \times 10^{-12}\,\text{N})(0.50\,\text{m}) = 1.2 \times 10^{-12}\,\text{J}$$
$$= (1.2 \times 10^{-12}\,\text{J})\frac{1\,\text{eV}}{1.602 \times 10^{-19}\,\text{J}}$$
$$= 7.5 \times 10^6\,\text{eV} = 7.5\,\text{MeV}$$

(c) The angle ϕ between the constant field $\vec{E}$ and the displacement is zero, so

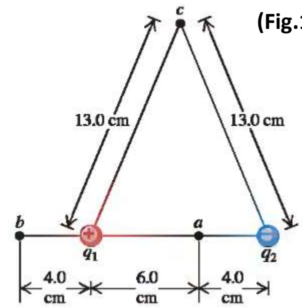
$$V_a - V_b = \int_a^b E\cos\phi\, dl = \int_a^b E\, dl = E\int_a^b dl$$

The integral of dl from a to b is just the distance d, so we again find

$$V_a - V_b = Ed = (1.5 \times 10^7\,\text{V/m})(0.50\,\text{m}) = 7.5 \times 10^6\,\text{V}$$

Problem#02 (Potential due to Two Point Charges)

An electric dipole consists of two point charges, $q_1 = +12\text{ nC}$ and $q_2 = -12\text{ nC}$, placed 10 cm apart (Fig. 1). Compute the potentials at points a, b, and c by adding the potentials due to either charge.



(Fig.1)

Solution: To find V at each point, we do the *algebraic* sum in

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

At point a the potential due to the positive charge q_1 is

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = (9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{12\times10^{-9}\text{ C}}{0.060\text{ m}}$$
$$= 1800\text{ N}\cdot\text{m/C}$$
$$= 1800\text{ J/C} = 1800\text{ V}$$

and the potential due to the negative charge q_2 is

$$\frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} = (9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-12\times10^{-9}\text{ C})}{0.040\text{ m}}$$
$$= -2700\text{ N}\cdot\text{m/C}$$
$$= -2700\text{ J/C} = -2700\text{ V}$$

The potential V_a at point a is the sum of these:

$$V_a = 1800\text{ V} + (-2700\text{ V}) = -900\text{ V}$$

By similar calculations you can show that at point b the potential due to the positive charge is $+2700$ V, the potential due to the negative charge is -770 V, and

$$V_b = 2700\text{ V} + (-770\text{ V}) = 1930\text{ V}$$

At point c the potential due to the positive charge is

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = (9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{12\times10^{-9}\text{ C}}{0.13\text{ m}} = 830\text{ V}$$

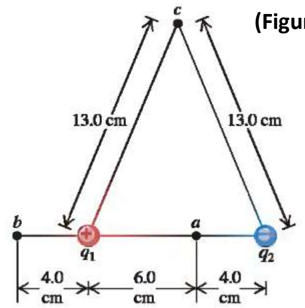
The potential due to the negative charge is -830 V, and the total potential is zero:

$$V_c = 830\text{ V} + (-830\text{ V}) = 0$$

Problem#03 (Potential and Potential Energy)

Compute the potential energy associated with a point charge of $+4.0\ \text{nC}$ if it is placed at points a, b, and c in Figure.

(Figure)



Solution:

At point a,

$$U_a = qV_a = (4.0 \times 10^{-9}\ \text{C})(-900\ \text{J/C}) = -3.6 \times 10^{-6}\ \text{J}$$

At point b,

$$U_b = qV_b = (4.0 \times 10^{-9}\ \text{C})(1930\ \text{J/C}) = 7.7 \times 10^{-6}\ \text{J}$$

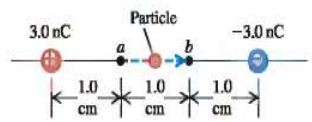
At point c,

$$U_c = qV_c = 0$$

All of these values correspond to U and V being zero at infinity.

Problem#04 (moving through a potential difference)

In **Figure** a dust particle with mass $m = 5.0 \times 10^{-9}\ \text{kg} = 5.0\ \mu\text{g}$ and charge $q_0 = 2.0\ \text{nC}$ starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?



The particle moves from point a to point b; its acceleration is not constant.

Solution:

SET UP: Only the conservative electric force acts on the particle, so mechanical energy is conserved:

$$K_a + U_a = K_b + U_b$$

EXECUTE: For this situation, $K_a = 0$ and $K_b = \frac{1}{2}mv^2$. We get the potential energies (U) from the potentials (V) using Eq. **(above)** $U_a = q_0V_a$ and $U_b = q_0V_b$.

Substituting these into the energy-conservation equation and solving for v, we find

$$0 + q_0V_a = \frac{1}{2}mv^2 + q_0V_b$$

$$v = \sqrt{\frac{2q_0(V_a - V_b)}{m}}$$

We calculate the potentials just as we did in **problem 02 earliar**

$$V_a = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{3.0 \times 10^{-9}\ \text{C}}{0.010\ \text{m}} + \frac{(-3.0 \times 10^{-9}\ \text{C})}{0.020\ \text{m}}\right) = 1350\ \text{V}$$

$$V_b = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{3.0 \times 10^{-9}\ \text{C}}{0.020\ \text{m}} + \frac{(-3.0 \times 10^{-9}\ \text{C})}{0.010\ \text{m}}\right) = -1350\ \text{V}$$

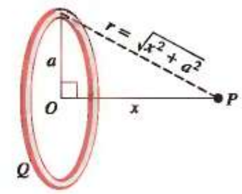
$$V_a - V_b = (1350\ \text{V}) - (-1350\ \text{V}) = 2700\ \text{V}$$

Finally,

$$v = \sqrt{\frac{2(2.0 \times 10^{-9}\,\text{C})(2700\,\text{V})}{5.0 \times 10^{-9}\,\text{kg}}} = 46\,\text{m/s}$$

Problem#05 (Ring of Charge)

Electric charge is distributed uniformly around a thin ring of radius a, with total charge Q (Fig.). Find the potential at a point P on the ring axis at a distance x rrom the center of the ring.



All the charge in a ring of charge Q is the same distance r from a point P on the ring axis.

Solution:

SET UP: Figure shows that it's far easier to find V on the axis by using the infinitesimal-segment approach. That's because all parts of the ring (that is, all elements of the charge distribution) are the same distance r from point P.

EXECUTE: Figure shows that the distance from each charge element dq on the ring to the point P is $r = \sqrt{x^2 + a^2}$. Hence we can take the factor $1/r$ outside the integral in Eq. (23.16), and

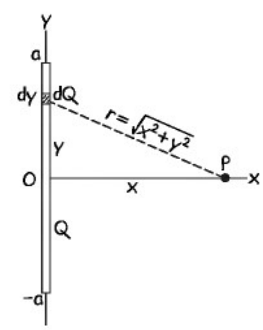
$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2 + a^2}}\int dq$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$$

Potential is a *scalar* quantity; there is no need to consider components of vectors in this calculation, as we had to do when we found the electric field at P. So the potential calculation is a lot simpler than the field calculation.

Problem#06 (A line of Charge)

Electric charge Q is distributed uniformly along a line or thin rod of length $2a$. Find the potential at a point P along the perpendicular bisector of the rod at a distance x from its center.



Solution:

EXECUTE: The element of charge dQ corresponding to an element of length dy on the rod is given by $dQ = (Q/2a)dy$. The distance from dQ to P is $\sqrt{x^2 + y^2}$, and the contribution dV that it makes to the potential at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2 + y^2}}$$

To get the potential at P due to the entire rod, we integrate dV over the length of the rod from $y = -a$ to $y = a$:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2 + y^2}}$$

You can look up the integral in a table. The final result is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\ln\left(\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right)$$

END OF LECTURE