COMSATS-University Islamabad, Lahore Campus.

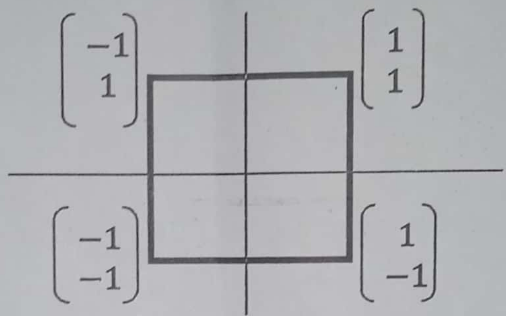
CUI, Defence Road, Off Raiwind Road, Lahore.

Terminal Exam – Spring 2023

Course Title:	Linear Algebra		Course Code:	MTH23	Credit Hours: 3(3,0)
Course	Dr. Maqsood Ahmad		Programme	BCS	
Semester:	3rd	Batch: SP22	Section: A, B, C	Date:	17-06-2023
Time	180Min		Maximum Marks:		100

Question 1: **(10+10)**

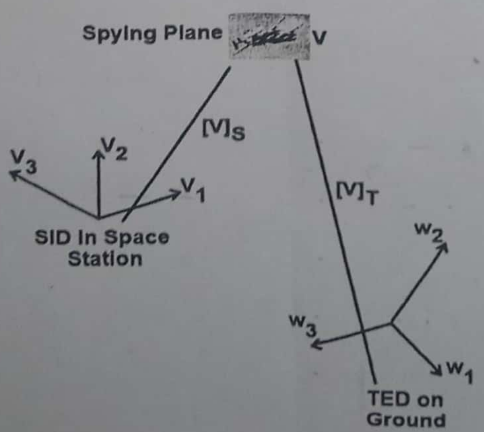
(a) Find eigenvalues and eigenvectors of matrix $\mathbf{A} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$. Redraw following figure of unit square using $L(X) = \mathbf{A}X$. What eigenvalues and eigenvectors predict about new figure.



(b) Find a 2×2 matrix A whose eigenvalues are 3 and 4, and the associated eigenvectors are $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, respectively.

Question 2: SID observes a suspicious object V (spying plane) in space from a space station and noted coordinates with reference basis $S = \{v_1 = [6 \quad 3 \quad 3]; v_2 = [4 \quad -1 \quad 3]; v_3 = [5 \quad 5 \quad 2]\}$ as $[\mathrm{V}]_S = [4 \quad -5 \quad 1]$. He communicated the coordinates of that object to his colleague TED on the ground having a reference basis $\mathrm{T} = \{\mathrm{w}_1 = [2 \quad 0 \quad 1];\ \mathrm{w}_2 = [1 \quad 2 \quad 0];\ \mathrm{w}_3 = [1 \quad 1 \quad 1]\}$.

Help TED to find the matrix from S to T basis and **USE THIS MATRIX** to compute the coordinates of object V with respect to T basis. **(10+10)**



(P.T.O)

(b) For the following matrix, find basis for row space that are **not rows** (vectors) of A and basis for column space that are columns (vectors) of A

$$A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 1 & -3 & 5 \\ 2 & 2 & -1 & 3 \end{bmatrix}$$

Question No 3: (a) Let $L: R^2 \rightarrow R^2$ be the linear transformation for which we know **(10+10)**

$$L\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix};\ L\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}.\text{ Find } L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right)$$

(b) Let $L: M_{22} \rightarrow M_{22}$ be the linear transformation defined by

$$L\left(\begin{bmatrix} a & c \\ b & d \end{bmatrix}\right) = \begin{bmatrix} a+b & b+c \\ a+d & b+d \end{bmatrix}$$

Prove that $\boldsymbol{Dim(Ker(L) + Dim\big(Range(L)\big) = Dim(M_{22})}$

Question No 4: (a) Use Gram-Schmidt process to find an orthonormal basis for **(15+5)**

vector space R^3 that contains the vectors $\begin{bmatrix} 2/3 \\ -2/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} 2/3 \\ 1/3 \\ -2/3 \end{bmatrix}$.

(b) Write vector $v = \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}$ as a linear combination of orthonormal basis found in part **(a).**

Question No 5: (a) A manufacturer makes three different types of electronic products **(10+10)**

A, B, and C. Each product must go through three processing machines: X, Y and Z. The products require the following times in machines X, Y and Z:

1. Product A requires 2 hours in machine X, 2 hours in machine Y and 1 hours in machine Z.

2. Product B requires 3 hours in machine X, 2 hours in machine Y and 1 hours in machine Z.

3. Product C requires 4 hours in machine X, 3 hours in machine Y and 1 hours in machine Z.

Machine X is available **80** hours per week, machine Y is available **60** hours per week and machine Z is available **25** hours per week. Since management does not want to keep the expensive machines X, Y and Z idle, it would like to know how many product to make so that the machines are fully utilized.

(b) Find a basis for the subspace W of vector space R_4 consisting of all vectors of the form

$$[a+c \quad a-b \quad b+c \quad b-a]$$