# Computer Vision

Course Instructor:

Dr. M. Aksam Iftikhar

Slides credit: Dr. Mubarak Shah

# Filtering

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- Filtering/Smoothing/Removing Noise

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- Convolution/Correlation

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- Histogram

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- Filtering/Smoothing/Removing Noise
- Convolution/Correlation
- Image Derivatives
- Histogram
- Some Matlab Functions

# Images: General

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**Binary**

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Gray Scale

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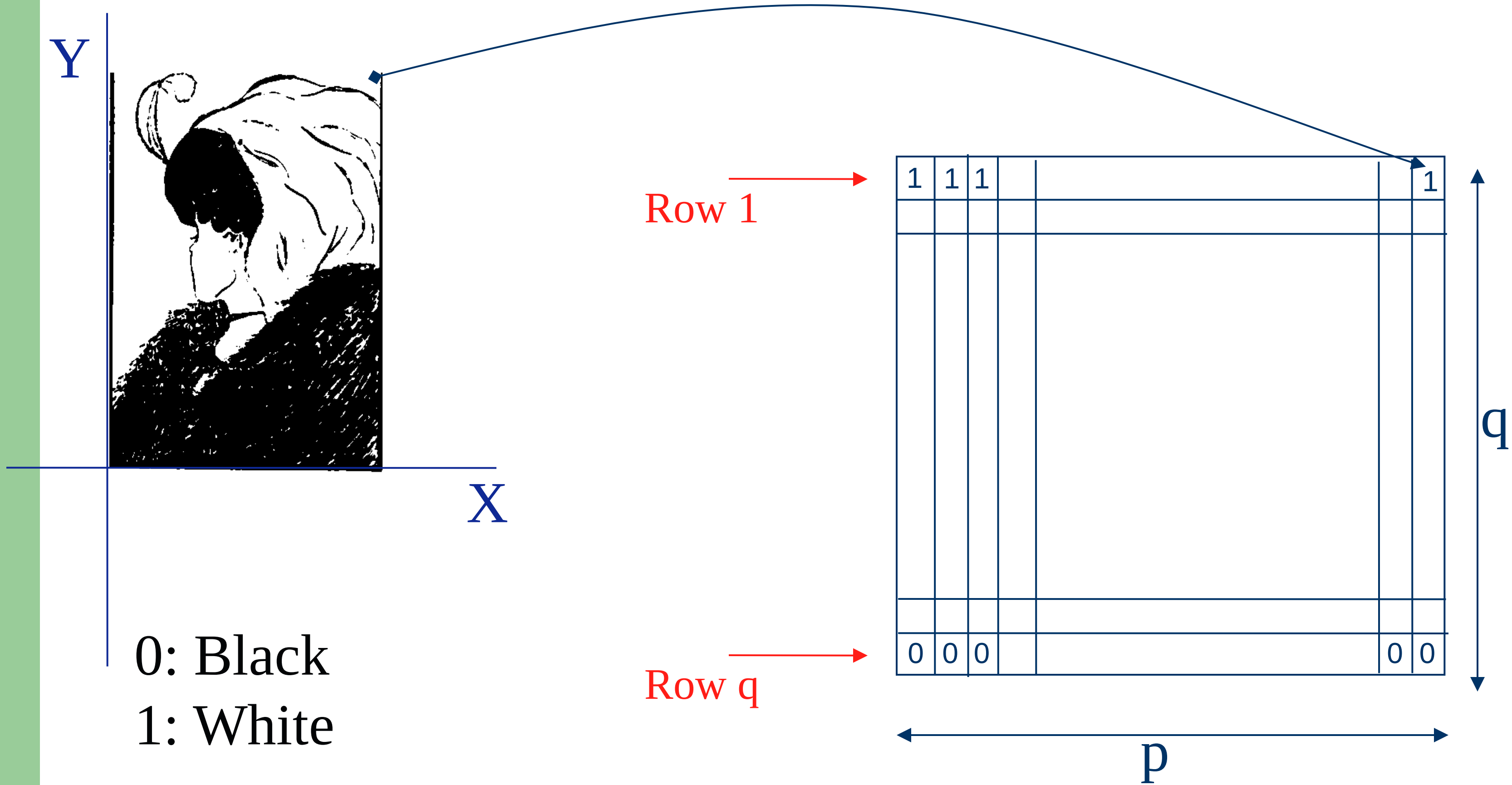
Binary

Gray Scale

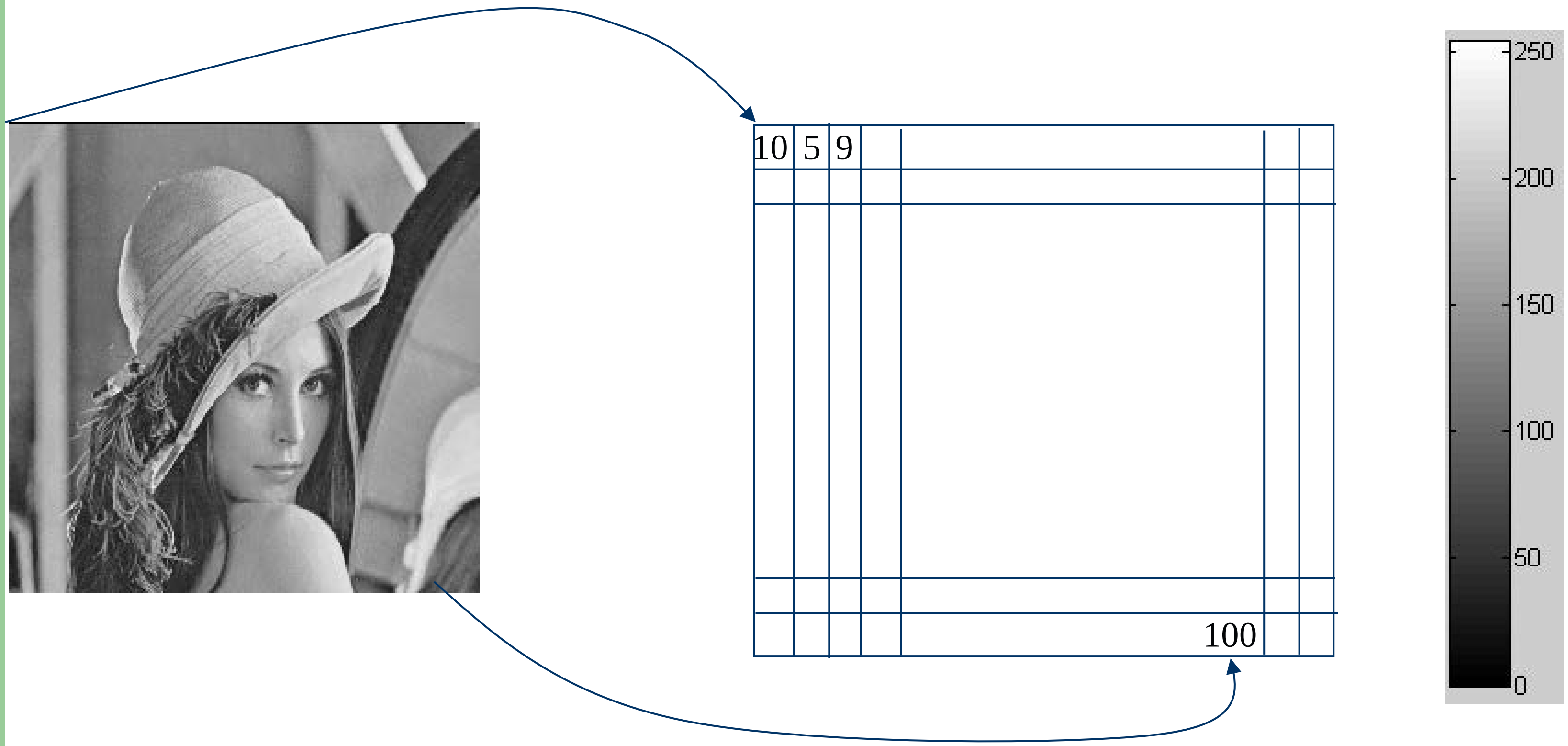
Color

# Binary Images

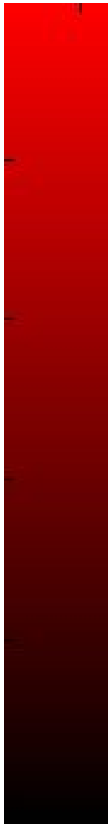



0: Black
1: White

# Gray Level Image



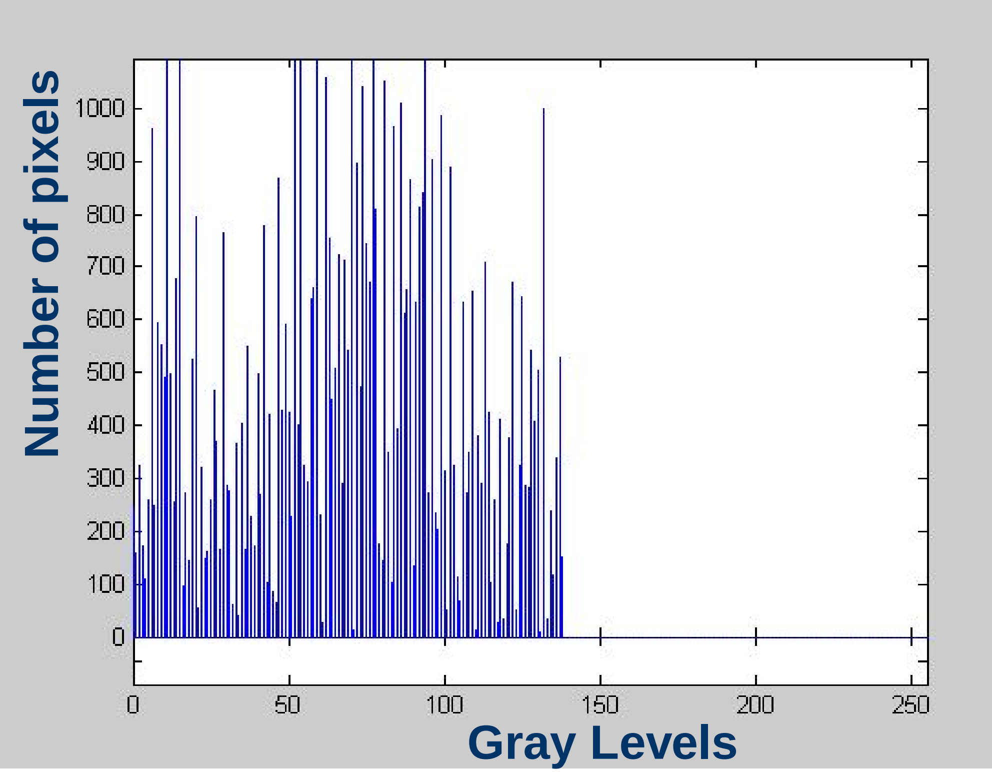
# Gray Scale Image



# Color Image
# Red, Green, Blue Channels

Phil Noble / AP

# Image Histogram




- Histogram captures the distribution of gray levels in the image.
- How frequently each gray level occurs in the image

# Histogram Code

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C Code:

```
for (i=0;i<m,i++)
    for (j=0;j<n,j++)
        hist[I[i,j]]++;
```

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for (i=0;i<m,i++)
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MATLAB: imhist(I)

# Image Noise

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- Light Variations

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- Camera Electronics

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- Camera Electronics
- Surface Reflectance

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- Noise is random, it occurs with some probability
- It has a distribution

# Image Noise

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$$\hat{I}(x,y) = I(x,y) + n(x,y)$$ Additive noise

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Multiplicative noise

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$$\hat{I}(x,y) = I(x,y) \times n(x,y)$$

Multiplicative noise

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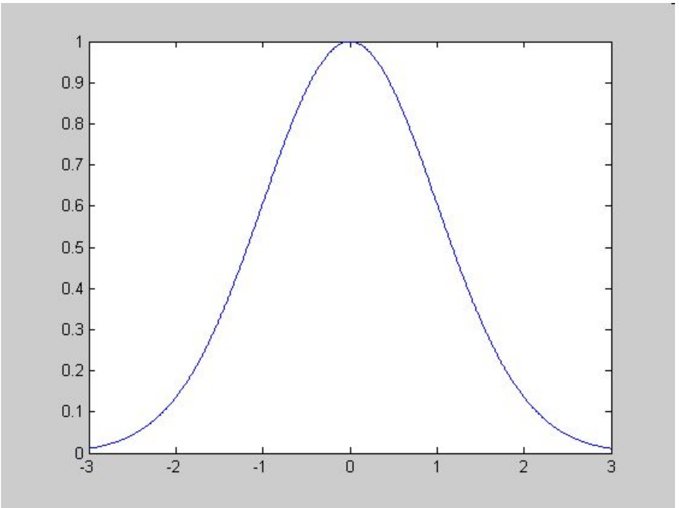
# Gaussian Noise

$$n(x,y) \approx g(n) = e^{\frac{-n^2}{2\sigma^2}}$$

$g(n)$




*n*

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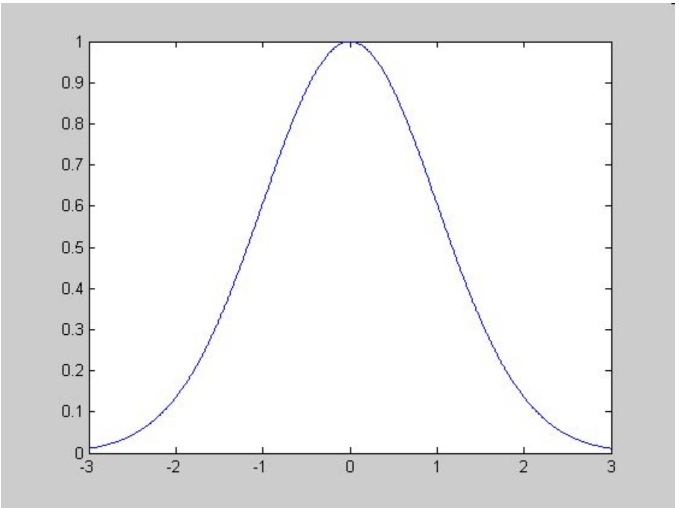


$n$

Probability Distribution
*n* is a random variable

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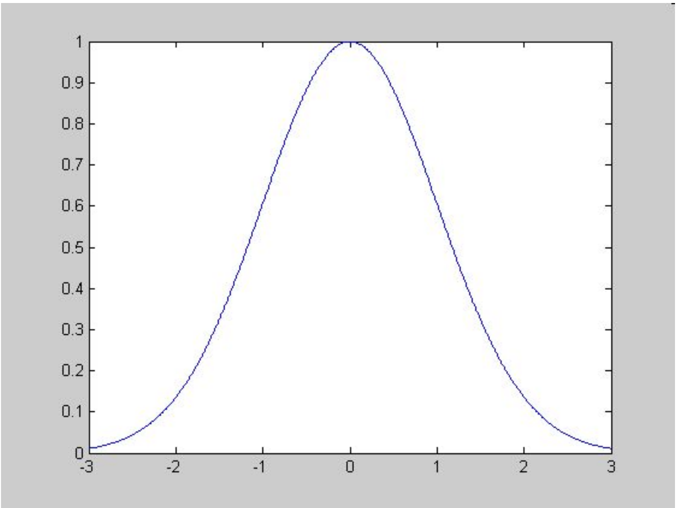
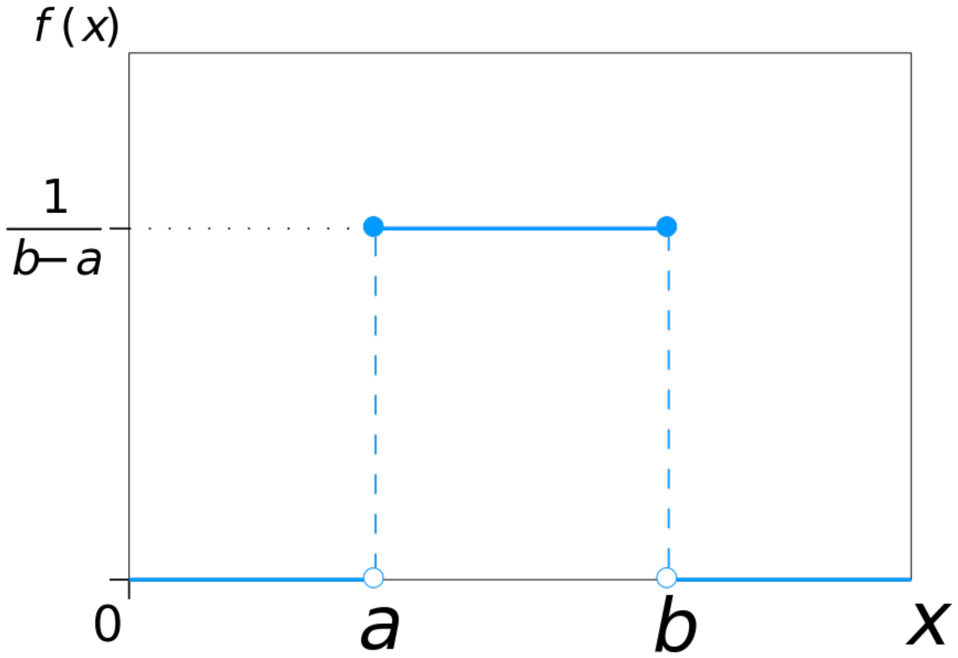



*n*

Probability Distribution
*n* is a random variable

# Uniform Distribution



# Salt and Pepper Noise

- Each pixel is randomly made black or white with a uniform probability distribution

➔

# Image filtering

- Image filtering: compute function of local neighborhood at each position

Slide Credit: James Hays, Cornell U.

# Image filtering

- Image filtering: compute function of local neighborhood at each position
- Really important!
  - Enhance images
    - Denoise, resize, increase contrast, etc.

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  - Enhance images
    - Denoise, resize, increase contrast, etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.

# Image filtering

- Image filtering: compute function of local neighborhood at each position
- Really important!
  - Enhance images
    - Denoise, resize, increase contrast, etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching

Slide Credit: James Hays, Cornell U.

# Image Derivatives & Averages

# Definitions

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- Derivative: Rate of change
  - *Speed* is a rate of change of a *distance*
  - *Acceleration* is a rate of change of *speed*

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- Derivative: Rate of change
  - *Speed* is a rate of change of a *distance*
  - *Acceleration* is a rate of change of *speed*
- Average (Mean)
  - Dividing the sum of $N$ values by $N$

# Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

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$$v = \frac{ds}{dt}$$ speed

$$a = \frac{dv}{dt}$$ acceleration

# Examples: Analytic Derivatives

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$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

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$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = \cos x + (-1)e^{-x}$$

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# Finite Difference

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Backward difference

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$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$

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Backward difference

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Forward difference

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Forward difference

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Forward difference

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Central difference

# Example

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$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

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$$f'(x) = 0 \quad 5 \quad -5 \quad 0 \quad 15 \quad -5 \quad 0 \quad 0$$

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$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

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Derivative Masks

# Example

$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

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$$f''(x) = 0 \quad 5 \quad 10 \quad 5 \quad 15 \quad -20 \quad 5 \quad 0$$

## Derivative Masks

| | |
|---|---|
| Backward difference | [-1 1] |
| Forward difference | [1 -1] |
| Central difference | [-1 0 1] |

# Derivatives in 2 Dimensions

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Given function $f(x, y)$

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Gradient vector $$\nabla f(x, y) = \begin{bmatrix} \dfrac{\partial f(x, y)}{\partial x} \\ \dfrac{\partial f(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

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Gradient magnitude $$\left| \nabla f(x, y) \right| = \sqrt{f_x^2 + f_y^2}$$

Gradient direction $$\theta = \tan^{-1} \frac{f_x}{f_y}$$

# Derivatives of Images

Derivative masks

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$$f_x \Rightarrow \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

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# Correlation

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$$f \otimes h = \sum_k \sum_l f(k,l)\, h(k,l)$$

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$$f \otimes h = \sum_k \sum_l f(k,l)h(k,l)$$

$f$ = Image

$h$ = Kernel

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$$f \otimes h = \sum_{k} \sum_{l} f(k,l) h(k,l)$$

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$h$ = Kernel

$f$

| | | |
|---|---|---|
| $f_1$ | $f_2$ | $f_3$ |
| $f_4$ | $f_5$ | $f_6$ |
| $f_7$ | $f_8$ | $f_9$ |

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$\otimes$

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$$f \qquad \qquad h$$

$$\begin{array}{|c|c|c|} \hline f_1 & f_2 & f_3 \\ \hline f_4 & f_5 & f_6 \\ \hline f_7 & f_8 & f_9 \\ \hline \end{array} \otimes \begin{array}{|c|c|c|} \hline h_1 & h_2 & h_3 \\ \hline h_4 & h_5 & h_6 \\ \hline h_7 & h_8 & h_9 \\ \hline \end{array}$$

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$\otimes$

$h$

| | | |
|---|---|---|
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$\otimes$

$h$

| | | |
|---|---|---|
| $h_1$ | $h_2$ | $h_3$ |
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$$\begin{aligned} f \otimes h = {} & f_1 h_1 + f_2 h_2 + f_3 h_3 \\ & + f_4 h_4 + f_5 h_5 + f_6 h_6 \\ & + f_7 h_7 + f_8 h_8 + f_9 h_9 \end{aligned}$$

# Convolution

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$$f * h = \sum_k \sum_l f(k,l)h(-k,-l)$$

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# Convolution

$$f * h = \sum_k \sum_l f(k,l)\,h(-k,-l)$$

$f$ = Image

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$X - flip$

$h$

| | | |
|---|---|---|
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$h$

| | | |
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| $h_7$ | $h_8$ | $h_9$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_1$ | $h_2$ | $h_3$ |

$Y - flip$

| | | |
|---|---|---|
| $h_9$ | $h_8$ | $h_7$ |
| $h_6$ | $h_5$ | $h_4$ |
| $h_3$ | $h_2$ | $h_1$ |

# Convolution

$$f * h = \sum_k \sum_l f(k,l)\,h(-k,-l)$$

$f$ = Image

$h$ = Kernel

$h$

| | | |
|---|---|---|
| $h_1$ | $h_2$ | $h_3$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_7$ | $h_8$ | $h_9$ |

$X - flip$

| | | |
|---|---|---|
| $h_7$ | $h_8$ | $h_9$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_1$ | $h_2$ | $h_3$ |

$Y - flip$

| | | |
|---|---|---|
| $h_9$ | $h_8$ | $h_7$ |
| $h_6$ | $h_5$ | $h_4$ |
| $h_3$ | $h_2$ | $h_1$ |

$f$

| | | |
|---|---|---|
| $f_1$ | $f_2$ | $f_3$ |
| $f_4$ | $f_5$ | $f_6$ |
| $f_7$ | $f_8$ | $f_9$ |

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| | | |
|---|---|---|
| $h_1$ | $h_2$ | $h_3$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_7$ | $h_8$ | $h_9$ |

$X - flip$

| | | |
|---|---|---|
| $h_7$ | $h_8$ | $h_9$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_1$ | $h_2$ | $h_3$ |

$Y - flip$

$f$

| | | |
|---|---|---|
| $f_1$ | $f_2$ | $f_3$ |
| $f_4$ | $f_5$ | $f_6$ |
| $f_7$ | $f_8$ | $f_9$ |

*

| | | |
|---|---|---|
| $h_9$ | $h_8$ | $h_7$ |
| $h_6$ | $h_5$ | $h_4$ |
| $h_3$ | $h_2$ | $h_1$ |

# Convolution

$$f * h = \sum_k \sum_l f(k,l)\, h(-k,-l)$$

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$h$

| | | |
|---|---|---|
| $h_1$ | $h_2$ | $h_3$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_7$ | $h_8$ | $h_9$ |

$X - flip$

| | | |
|---|---|---|
| $h_7$ | $h_8$ | $h_9$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_1$ | $h_2$ | $h_3$ |

$Y - flip$

$f$

| | | |
|---|---|---|
| $f_1$ | $f_2$ | $f_3$ |
| $f_4$ | $f_5$ | $f_6$ |
| $f_7$ | $f_8$ | $f_9$ |

*

| | | |
|---|---|---|
| $h_9$ | $h_8$ | $h_7$ |
| $h_6$ | $h_5$ | $h_4$ |
| $h_3$ | $h_2$ | $h_1$ |

$$\begin{aligned} f * h = {} & f_1 h_9 + f_2 h_8 + f_3 h_7 \\ & + f_4 h_6 + f_5 h_5 + f_6 h_4 \\ & + f_7 h_3 + f_8 h_2 + f_9 h_1 \end{aligned}$$

# Correlation and Convolution

- Convolution is associative

# Correlation and Convolution

- Convolution is associative

$$F*(G*I)=(F*G)*I$$

# Averages

- Mean

# Averages

- Mean

$$I = \frac{I_1 + I_2 + \ldots I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

# Averages

- Mean

$$I = \frac{I_1 + I_2 + \ldots I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

- Weighted mean

# Averages

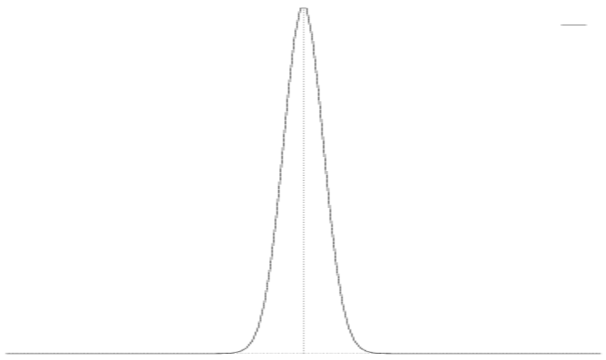
- Mean

$$I = \frac{I_1 + I_2 + \ldots I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

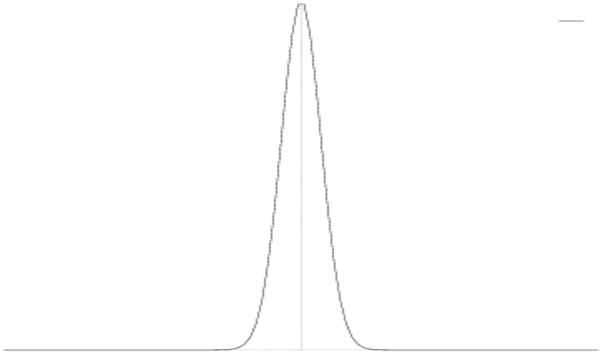
- Weighted mean

$$I = \frac{w_1 I_1 + w_2 I_2 + \ldots + w_n I_n}{n} = \frac{\sum_{i=1}^{n} w_i I_i}{n}$$

# Gaussian Filter

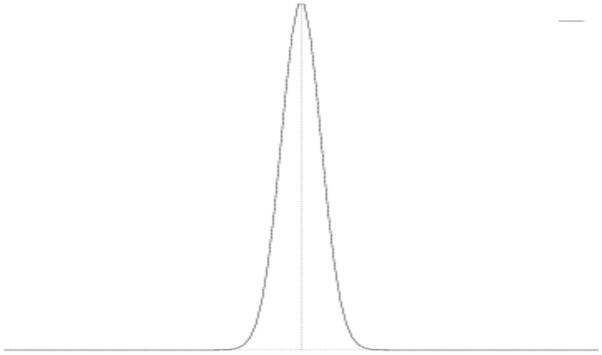


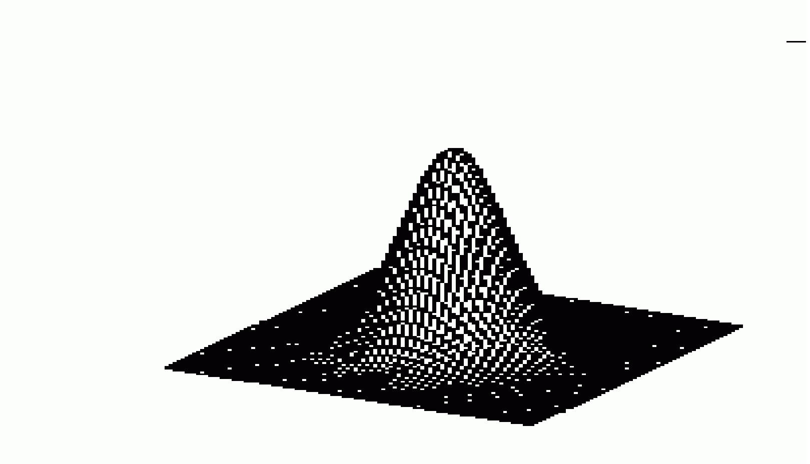
# Gaussian Filter



$$g(x) = e^{\frac{-x^2}{2o^2}}$$

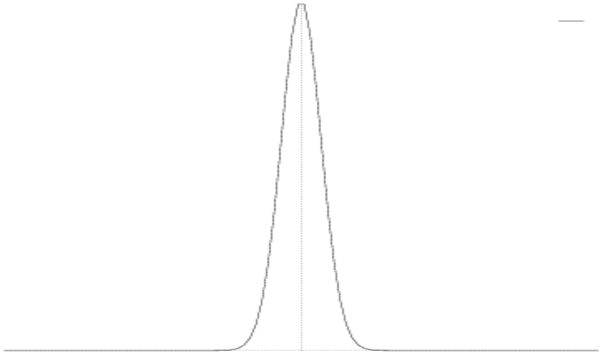
# Gaussian Filter

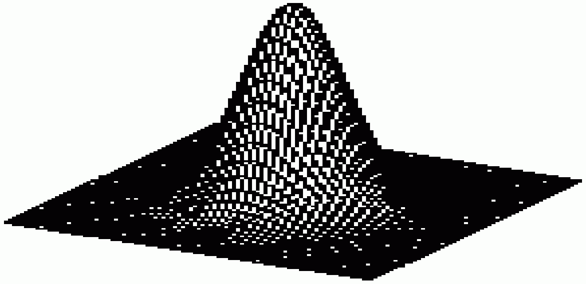




$$g(x) = e^{\frac{-x^2}{2o^2}}$$

# Gaussian Filter

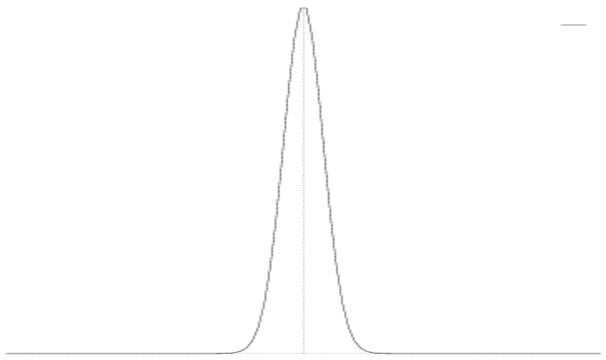


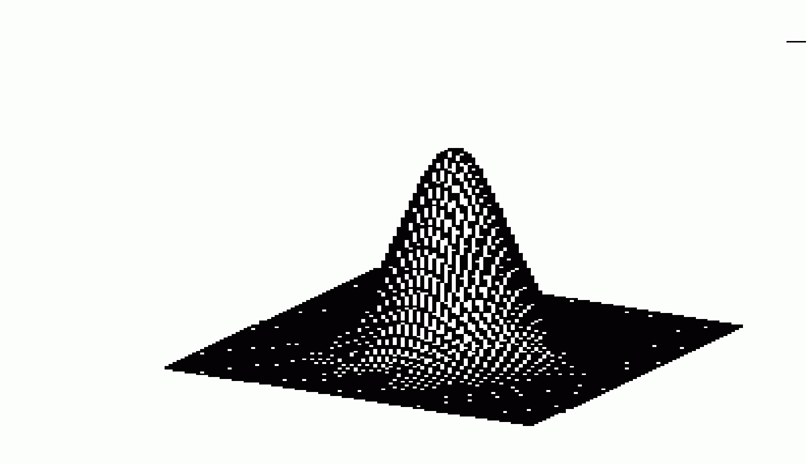


$$g(x) = e^{\frac{-x^2}{2o^2}}$$

$$g(x, y) = e^{\frac{-(x^2+y^2)}{2o^2}}$$

# Gaussian Filter

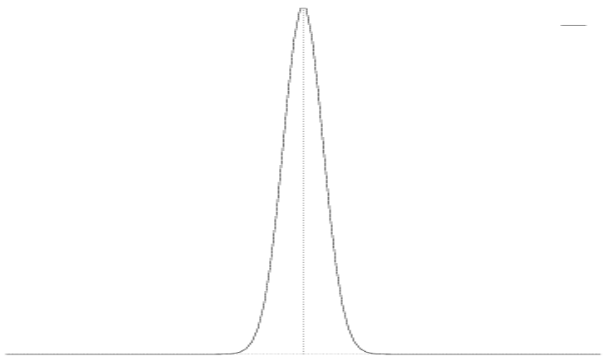


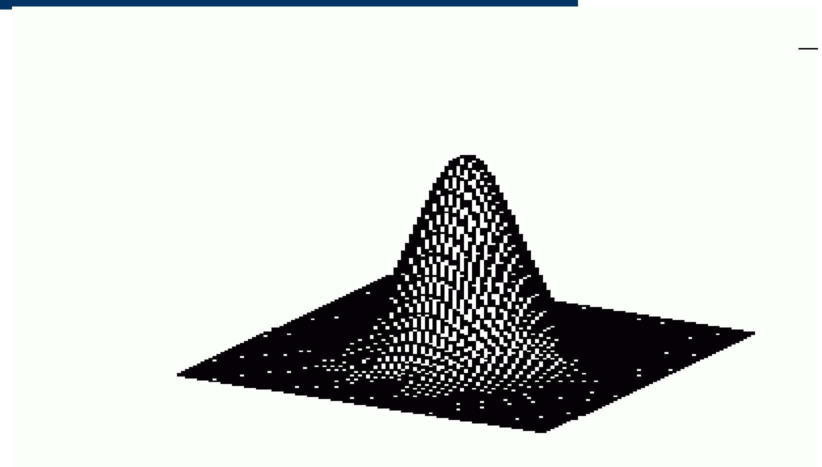


$$g(x) = e^{\frac{-x^2}{2o^2}} \qquad g(x,y) = e^{\frac{-(x^2+y^2)}{2o^2}}$$

$$g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$

# Gaussian Filter





$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

$$g(x, y) = e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$

$$\sigma = 1$$

# Properties of Gaussian

# Properties of Gaussian

- Most common natural model

# Properties of Gaussian

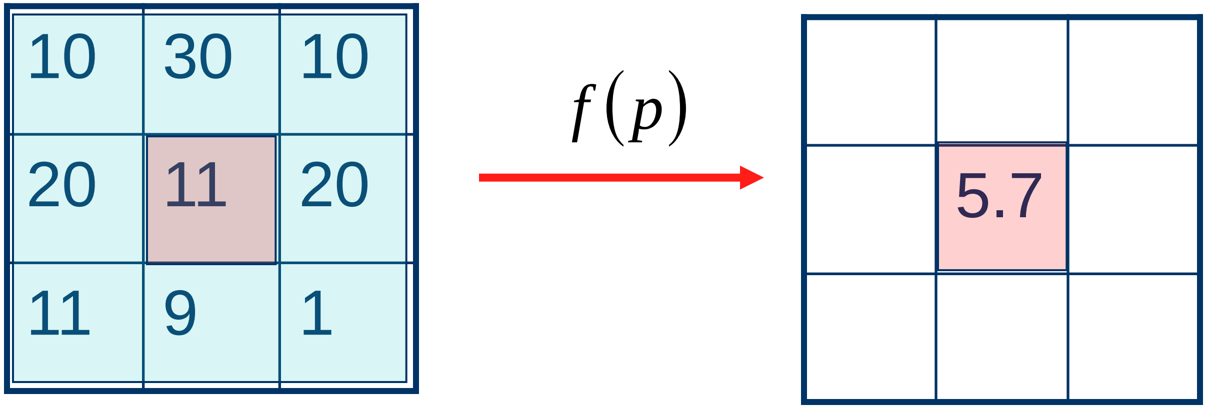
- Most common natural model
- Smooth function, it has infinite number of derivatives

# Properties of Gaussian

- Most common natural model
- Smooth function, it has infinite number of derivatives
- It is Symmetric
- There are cells in eyes that perform Gaussian filter-
ing

# Filtering

- Modify pixels based on some function of the neighborhood



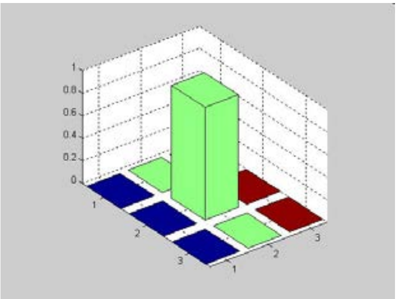
# Linear Filtering

- The output is the linear combination of the neighborhood pixels

$$\begin{bmatrix}1 & 3 & 0\\ 2 & 10 & 2\\ 4 & 1 & 1\end{bmatrix} \otimes \begin{bmatrix}1 & 0 & -1\\ 1 & 0.1 & -1\\ 1 & 0 & -1\end{bmatrix} = \begin{bmatrix} & & \\ & 5 & \\ & & \end{bmatrix}$$

Image Kernel Filter Output

# Filtering Examples



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| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

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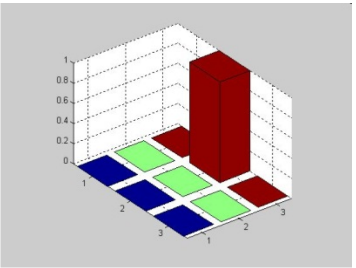
# Filtering Examples

*



| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

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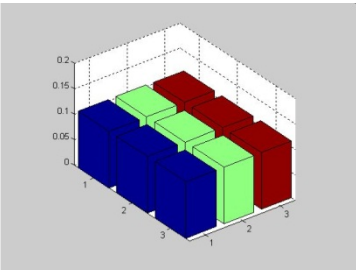
# Filtering Examples

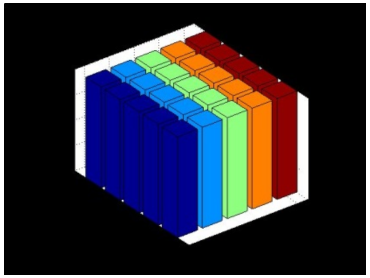



$$* \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$$=$$

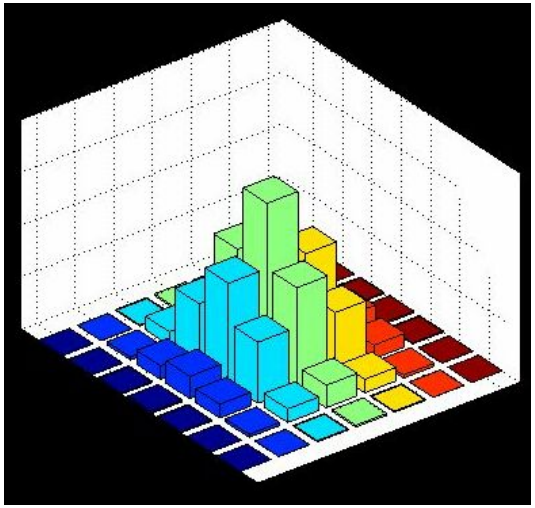
# Filtering Examples



$$* \frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} =$$

# Filtering Gaussian

*



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# Gaussian vs. Averaging

Gaussian Smoothing

Smoothing by Averaging

# Noise Filtering

After Averaging

After additive Gaussian Noise

After Gaussian Smoothing

# Example: box filter

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

# Image filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

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$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

# Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[.,.]$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

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$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

# Image filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

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$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

# Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

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$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

# Image filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$$f[.,.]$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$$h[.,.]$$

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| | 0 | 10 | 20 | 30 | | | | | |
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$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

# Image filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

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| | 0 | 10 | 20 | 30 | 30 | | | | |
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$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

# Image filtering

$$g[\cdot,\cdot] \quad \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
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$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k, n+l]$$

Credit: S. Seitz

# Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | ? | | | |
| | | | | | | | | | |
| | | | | 50 | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

# Image filtering

$g[\cdot\,,\cdot\,]$ $\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 60 | 90 | 90 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 10 | 20 | 30 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$$h[m,n] = \sum_{k,l} g[k,l]\; f[m+k, n+l]$$

Credit: S. Seitz

# Box Filter

What does it do?

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

# Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood

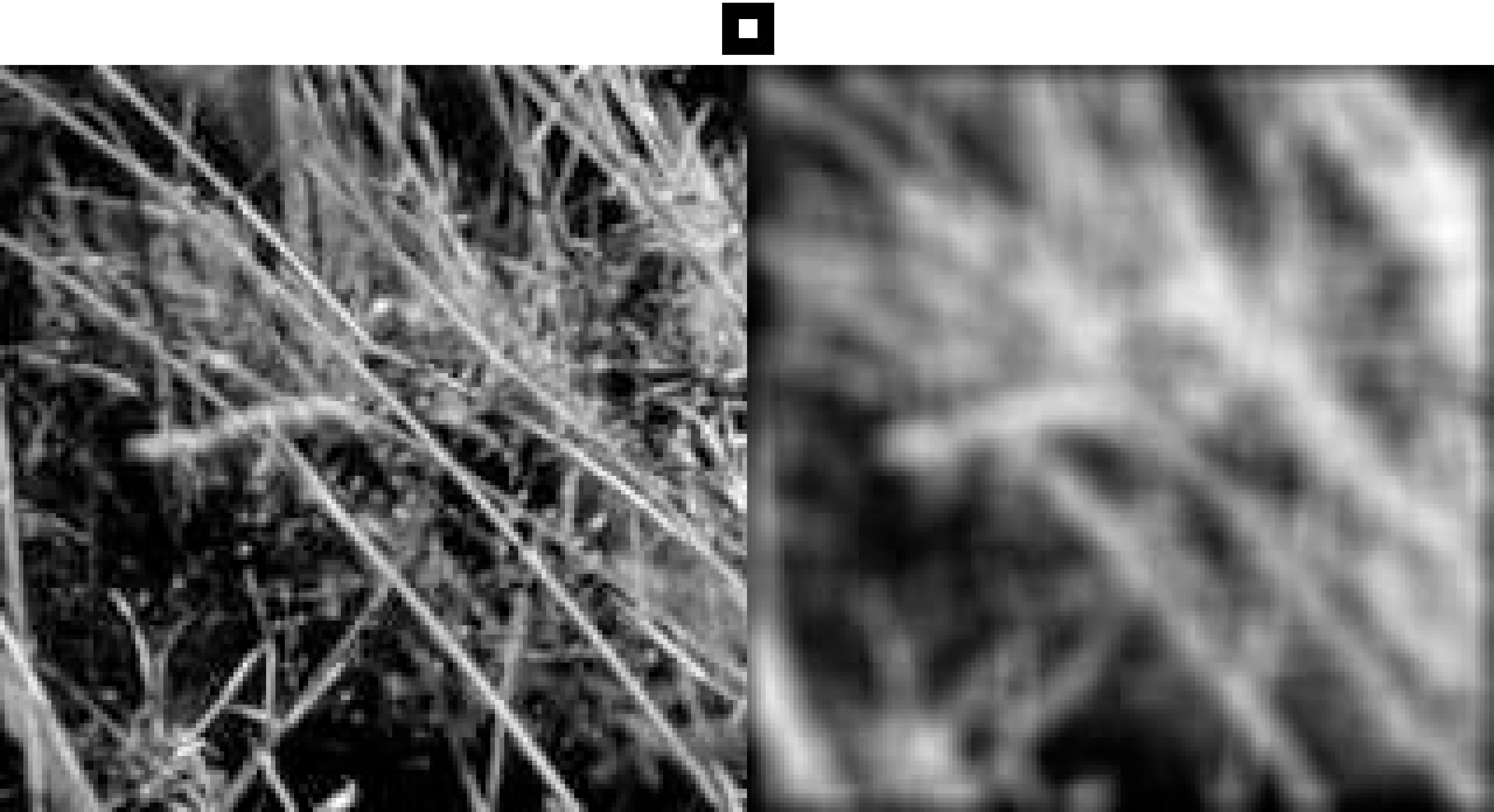
- Achieve smoothing effect (remove sharp features)

$$g[\cdot\,,\cdot\,]$$

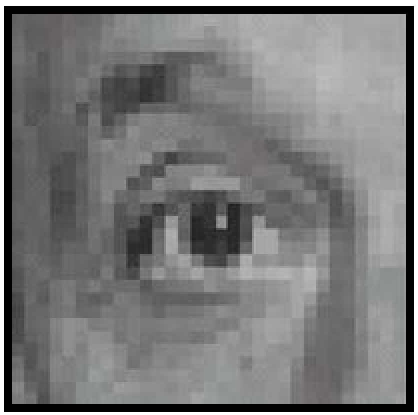
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

# Smoothing with box filter



# Practice with linear filters



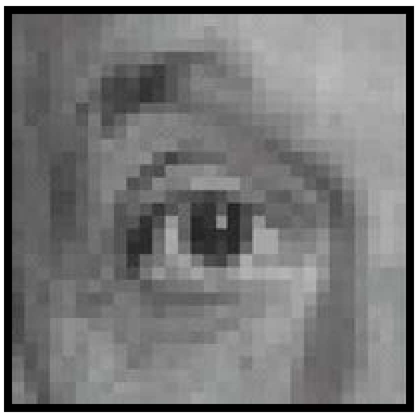
Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

**?**

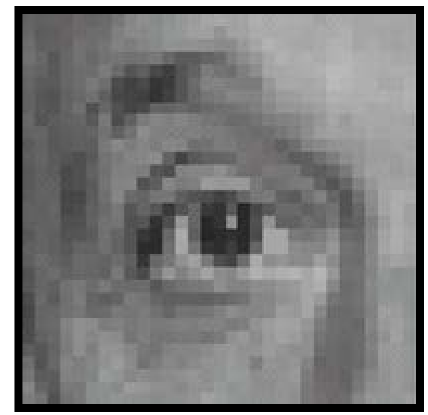
Source: D. Lowe

# Practice with linear filters



Original

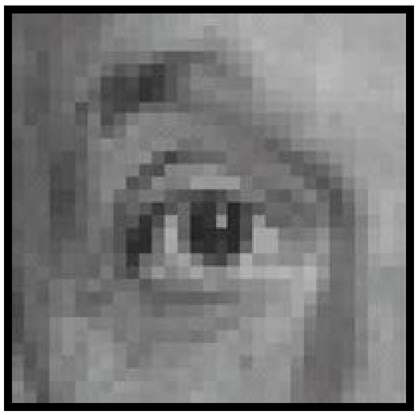
| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |



Filtered
(no change)

Source: D. Lowe

# Practice with linear filters



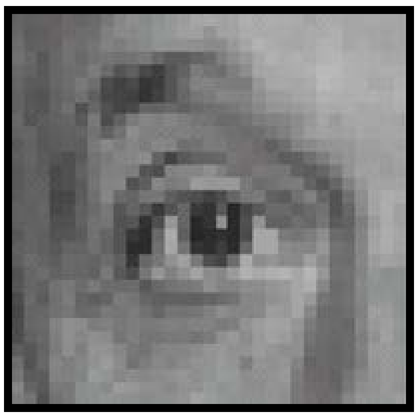
Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

**?**

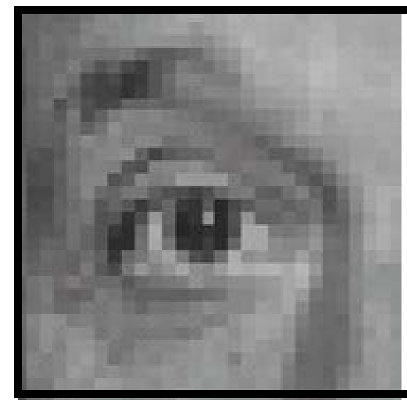
Source: D. Lowe

# Practice with linear filters



Original

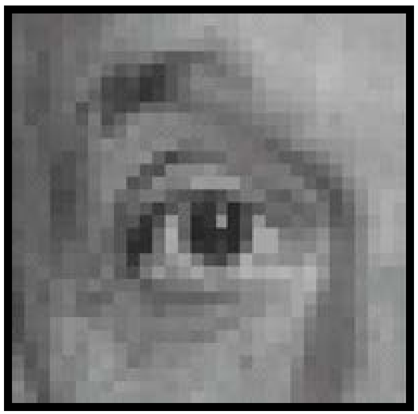
| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |



Shifted left
By 1 pixel

Source: D. Lowe

# Practice with linear filters



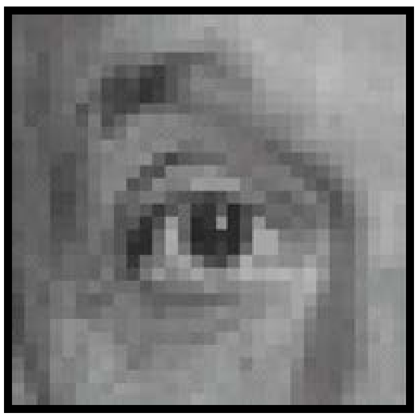
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

(Note that filter sums to 1)

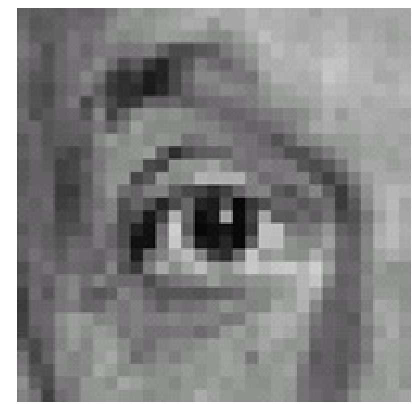
Source: D. Lowe

# Practice with linear filters



Original

$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

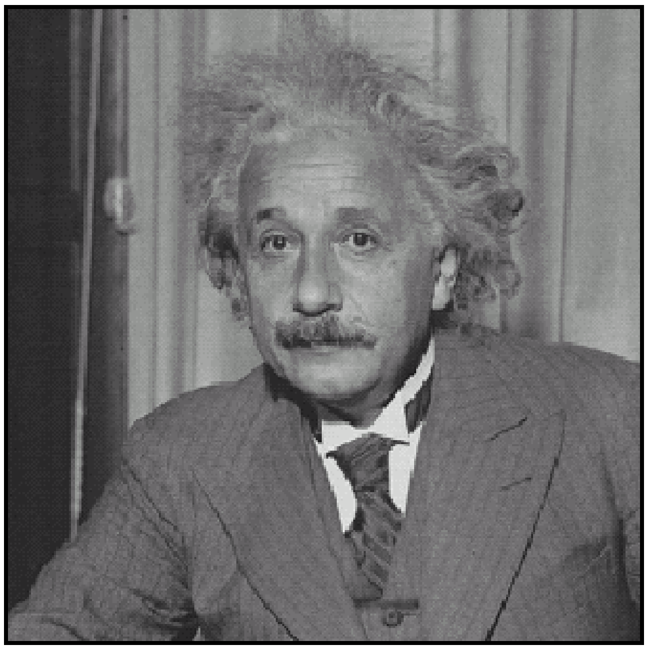


**Sharpening filter**

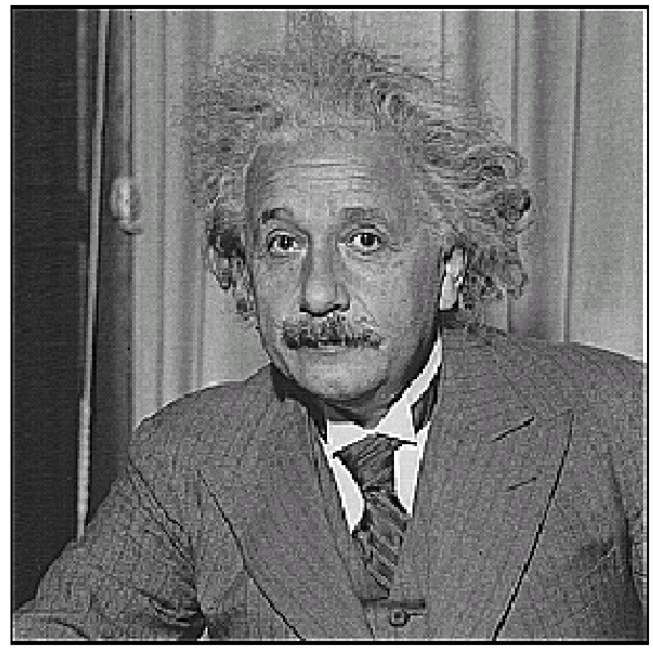
- Accentuates differences with local average

Source: D. Lowe

# Sharpening



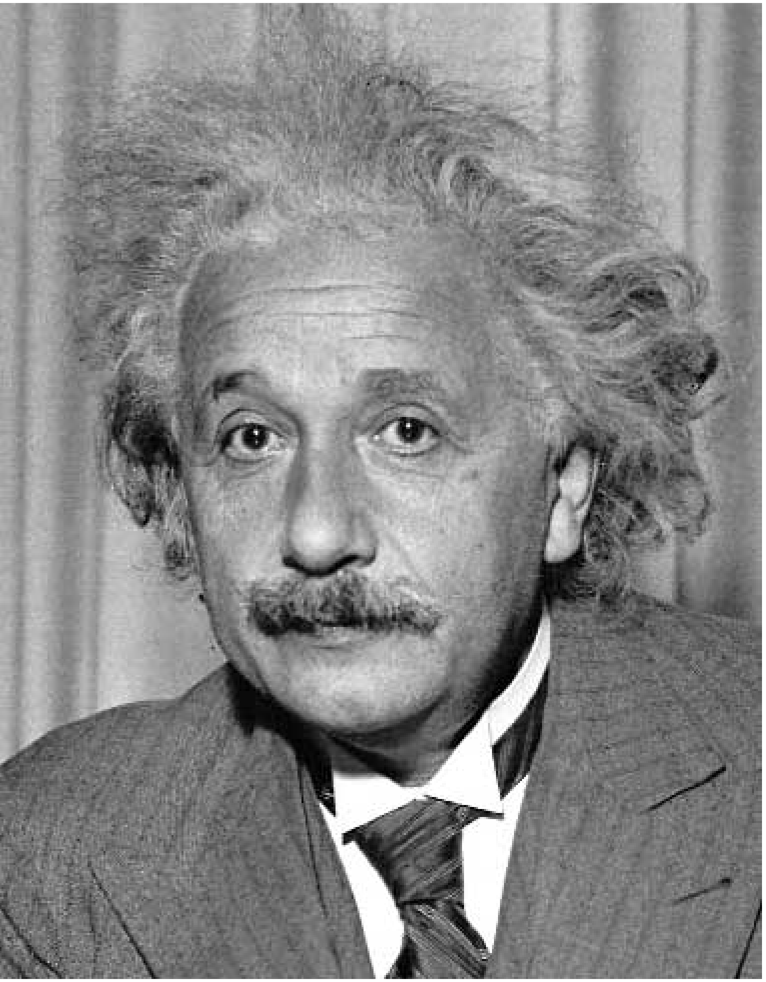
before



after

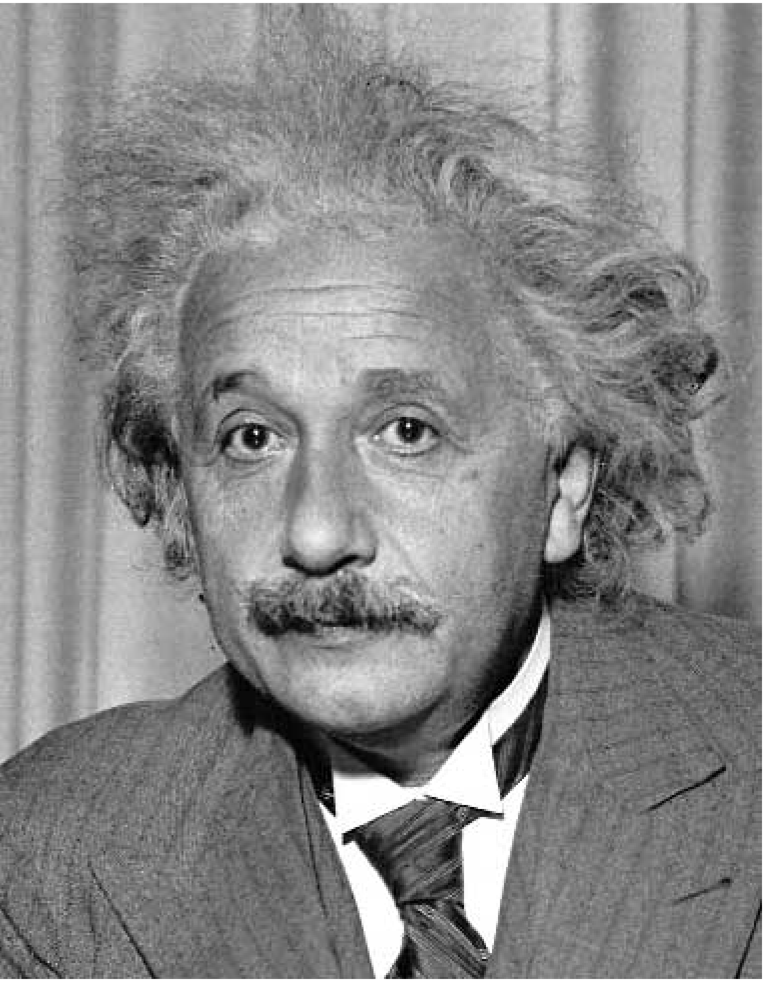
Source: D. Lowe

# Other filters



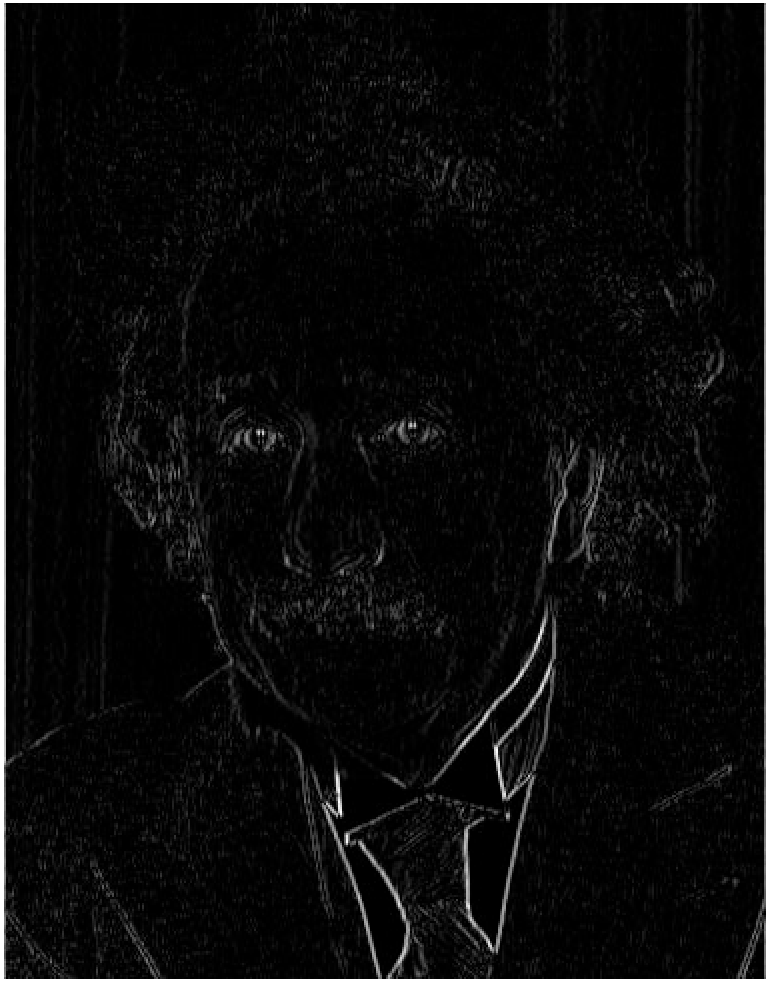
| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

# Other filters



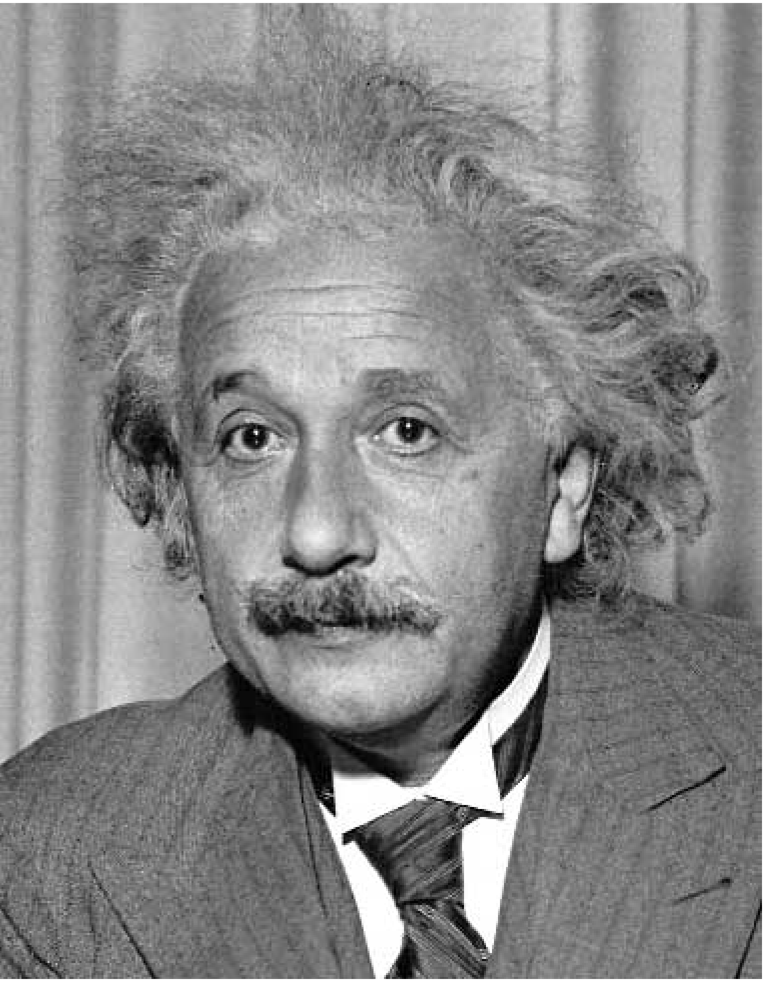
| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

Sobel



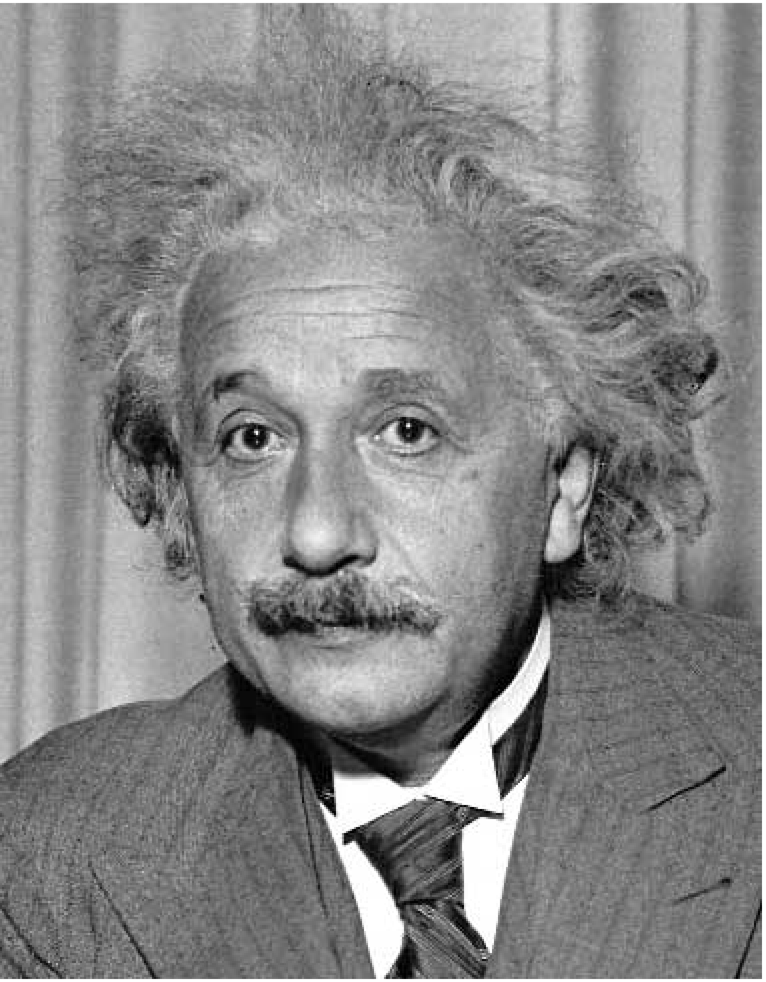
Vertical Edge
(absolute value)

# Other filters



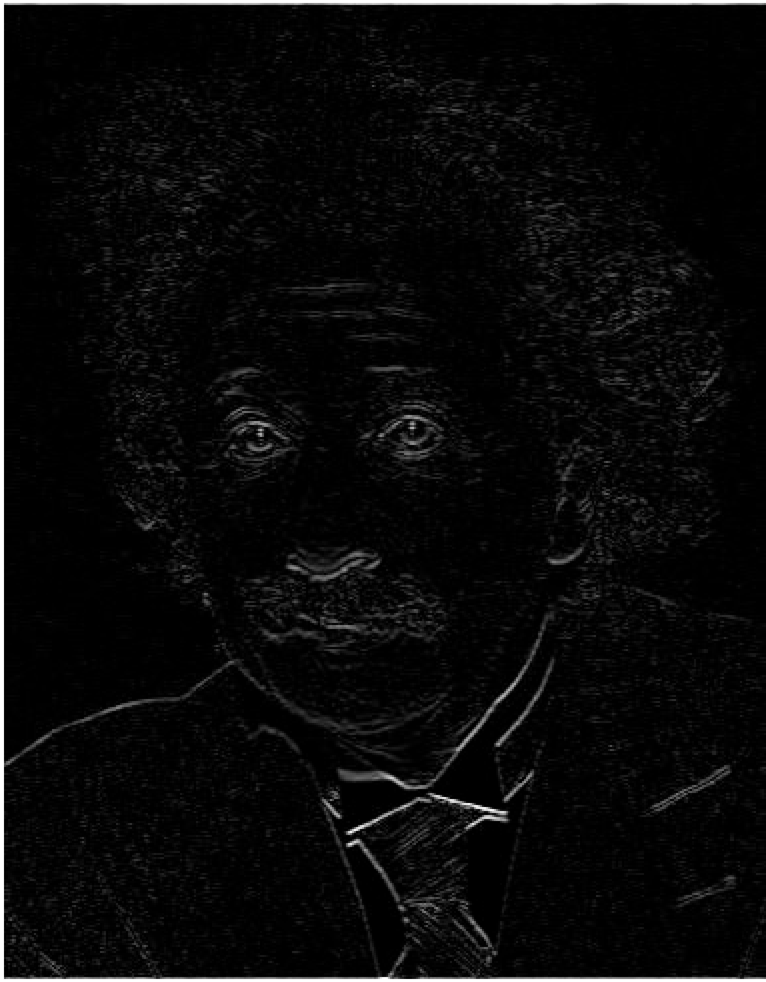
| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

# Other filters



| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Sobel

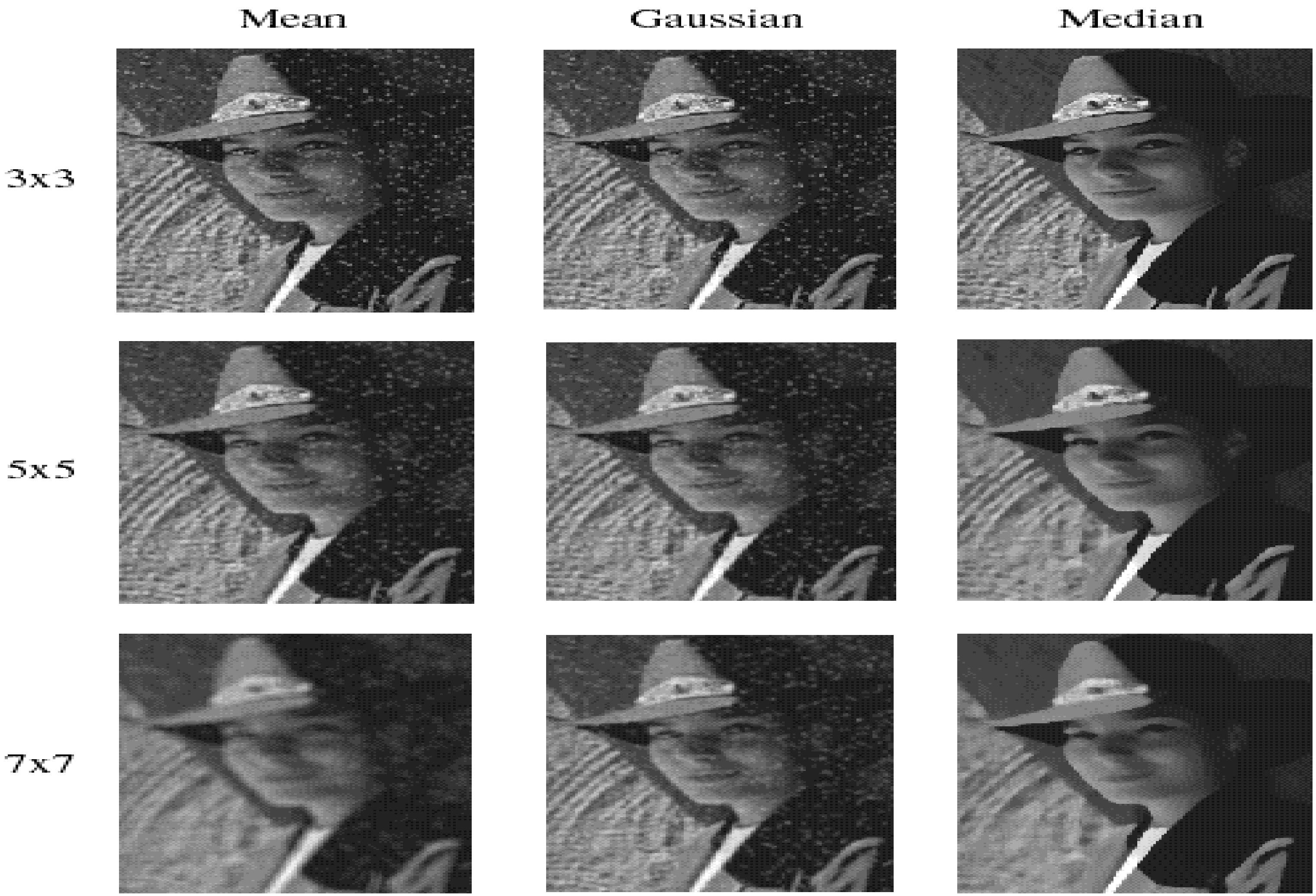


Horizontal Edge
(absolute value)

# Median filters

- A **Median Filter** operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

# Comparison: salt and pepper noise






Slide by Steve Seitz

# Practical matters

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

Source: S. Marschner

# Practical matters

- What about near the edge?
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Source: S. Marschner

# Reading Material

- Mubarak Shah, "Fundamentals of Computer Vision".
  - Chapter, 2

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- Mubarak Shah, "Fundamentals of Computer Vision".
  - Chapter, 2
- Richard Szeliski, "Computer Vision: Algorithms and Applications".
  - Section 3.2