Variation of Parameter

Wronskian Method

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

$$y_c = c_1 \underbrace{\sin x}_{y_1} + c_2 \underbrace{\cos x}_{y_2}$$

$$y_p = u_1 y_1 + u_2 y_2$$

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$$y'' - 4y' + 4y = (x+1)e^{2x}$$

For y_c

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$m_1 = 2 \ ; m_2 = 2$$

Roots are Real and Same

$$y_c = c_1 \underbrace{e^{2x}}_{y_1} + c_2 \underbrace{x e^{2x}}_{y_2}$$

$y_1 = e^{2x}$

$y_1' = 2e^{2x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \Rightarrow \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$y_2 = x \cdot e^{2x}$

$y_2' = 2xe^{2x} + e^{2x}$

$$W = 2xe^{4x} + e^{4x} - 2xe^{4x}$$

$$W = e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & xe^{2x} \\ (x+1)e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$$W_1 = 0 - (x^2 + x)e^{4x}$$

$$W_1 = -(x^2 + x)e^{4x}$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} \Rightarrow \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & (x+1)e^{2x} \end{vmatrix}$$

$$W_2 = (x+1)e^{4x} - 0$$

$$W_2 = (x+1)e^{4x}$$

$$u_1' = \frac{W_1}{W} \Rightarrow \frac{-(x^2+x)e^{4x}}{e^{4x}} \Rightarrow -(x^2 + x)$$

$$u_2' = \frac{W_2}{W} \Rightarrow \frac{(x+1)e^{4x}}{e^{4x}} \Rightarrow x + 1$$

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$$u_1' = -x^2 - x$$

$$\int u_1'\,dx = -\int x^2\,dx - \int x\,dx$$

$$u_1 = -\frac{x^3}{3} - \frac{x^2}{2}$$

$$u_2' = x + 1$$

$$\int u_2'\,dx = \int x\,dx + \int 1\,dx$$

$$u_2 = \frac{x^2}{2} + x$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \left(-\frac{x^3}{3} - \frac{x^2}{2}\right)\left(e^{2x}\right) + \left(\frac{x^2}{2} + x\right)\left(x e^{2x}\right)$$

$$y = y_c + y_p$$

$$y = c_1 e^{2x} + c_2 x e^{2x} - \left(\frac{x^3}{3} + \frac{x^2}{2}\right) e^{2x} + \left(\frac{x^2}{2} + x\right)\left(x e^{2x}\right)$$

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