Mon Tue Wed Thu Fri Sat Sun

Date:

Legendre's Equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \underbrace{12}_{(3\times4)}y = 0$$

$(3\times4) \longrightarrow$ multiplication of consecutive Num

$P_0(x) = 1-x^2$, $P_1(x) = -2$, $P_2(x) = 12$

$1-x^2=0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

singular points $= \pm 1$

$\ldots\ldots -3, -2, 0, 2, 3, \ldots\ldots$ Ordinary points

Let [Power Series]

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0x^0 + a_1x^1 + a_2x^2 + \ldots\ldots$$

$$dy/dx = y' = \sum_{n=1}^{\infty} a_n(n)x^{n-1}$$

$$d^2y/dx^2 = y'' = \sum_{n=2}^{\infty} a_n(n)(n-1)x^{n-2}$$

As Equation is

$$(1-x^2)y'' - 2xy' + 12y = 0$$

Put the values

$$(1-x^2)\left(\sum_{n=2}^{\infty} n(n-1)a_nx^{n-2}\right) - 2x\left(\sum_{n=1}^{\infty} na_nx^{n-1}\right) + 12\left(\sum_{n=0}^{\infty} a_nx^n\right) = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_nx^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_nx^n - 2\sum_{n=1}^{\infty} na_nx^n + 12\sum_{n=0}^{\infty} a_nx^n = 0$$

only $\boxed{\text{Put } n=n+2}$ [Power "x^n" is same]

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_nx^n - 2\sum_{n=1}^{\infty} na_nx^n + 12\sum_{n=0}^{\infty} a_nx^n = 0$$

$$2a_2 + 6a_3x + \sum_{n=2}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_nx^n - 2a_1x - 2\sum_{n=2}^{\infty} na_nx^n + 12a_0 + 12a_1x + 12\sum_{n=2}^{\infty} a_nx^n = 0$$

Comparing Coefficients

$$(n+2)(n+1)a_{n+2} - n(n-1)a_n - 2na_n + 12a_n = 0$$

$$(n+2)(n+1)a_{n+2} = n(n-1)a_n + 2na_n - 12a_n$$

$$a_{n+2} = \frac{(n(n-1)+2n-12)}{(n+2)(n+1)}a_n$$

$2a_2 + 12a_0 = 0$

$a_2 = -6a_0$

$6a_3 + 12a_1 - 2a_1 = 0$

$a_3 = -5/3\, a_1$