Separable Variable Method

$$(1+x)dy - ydx = 0$$

$$(1+x)\,dy = y\,dx$$

$$\frac{dy}{dx} = f(x).g(y)$$

$$\frac{1}{y}dy = \frac{1}{(1+x)}dx$$

Taking "$\int$" on both sides

$$\int \frac{1}{y}dy = \int \frac{1}{1+x}dx$$

$$\ln|y| = \ln|1+x| + \ln C$$

$$\ln y = \ln(1+x)c \quad [\text{Log Rule}]$$

Taking Antilog on both sides

$$y = (1+x)c \rightarrow (A)$$

Check

$$(1+x)dy = ydx$$

$$\frac{dy}{dx} = \frac{y}{1+x}$$

$$\frac{d}{dx}(1+x) = \frac{y}{(1+x)} \rightarrow (A)$$

$$1 = y/(1+x)$$

$$y = (1+x)$$

Initial Value Problems [I.V.P]

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$y(4) = -3 \leftarrow \text{I.V.P}$$

$$\underset{x}{\downarrow} \qquad \underset{y}{\downarrow}$$

$$\int y.dy = -\int x dx$$

$$y^2/2 = -x^2/2 + C$$

$$y^2 = -x^2 + 2C$$

$$x^2 + y^2 = 2C$$

put Given [I.V.P]

$$(4)^2 + (-3)^2 = 2C$$

$$25 = 2C$$

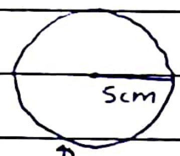
$$\boxed{C = 12.5}$$

$$x^2 + y^2 = 2C$$

$$x^2 + y^2 = 2(12.5)$$

$$x^2 + y^2 = 25$$

$$x^2 + y^2 = 5^2$$



equation of Circle

Separable Variable Method

$$\frac{dy}{dx} = y^2 - 4$$

$$\int \frac{dy}{(y^2-4)} = \int 1.dx$$

$$\int \frac{1}{y^2-4} dy = \int 1.dx$$

$$\int \frac{1}{(y+2)(y-2)} dy = \int 1.dx$$

$$\int \left[\frac{-1}{4(y+2)} + \frac{1}{4(y-2)} \right] dy = \int 1.dx$$

$$\int \left(\frac{-1}{4(y+2)} \right) dy + \int \left(\frac{1}{4(y-2)} \right) dy = \int 1 dx$$

$$-\frac{1}{4} \int \frac{1}{y+2} dy + \frac{1}{4} \int \frac{1}{y-2} dy = \int 1 dx$$

$$-\frac{1}{4}[\ln(y+2)] + \frac{1}{4}[\ln(y-2)] = x + c$$

Applying log Property

Partial Fraction Decomposition

$$\frac{1}{(y+2)(y-2)} = \frac{A}{y+2} + \frac{B}{y-2}$$

$$1 = A(y-2) + B(y+2)$$

As $(y+2=0)$ so $(y=-2) \rightarrow$ Put

$$1 = A(-2-2) + 0$$

$$1 = -4A$$

$$\boxed{A = -1/4}$$

As $[y-2=0]$ so $[y=2] \rightarrow$ Put

$$1 = A[2-2] + B[2+2]$$

$$1 = 4B$$

$$\boxed{B = 1/4}$$

Put values of A & B

$$\frac{1}{(y+2)(y-2)} = -\frac{1}{4(y+2)} + \frac{1}{4(y-2)}$$

$$\ln \left| \frac{y-2}{y+2} \right|^{1/4} = x + c$$

Taking Antilog on both sides

$$\left(\frac{y-2}{y+2} \right)^{1/4} = e^{x+c}$$

$$\left(\frac{y-2}{y+2} \right)^{1/4} = e^{x} . e^{c}$$

$$\left(\frac{y-2}{y+2} \right)^{1/4} = c . e^{x}$$