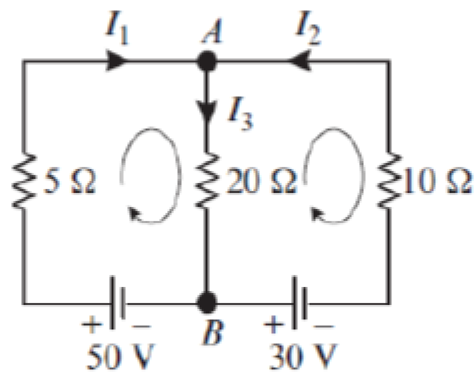
Solution Quiz No 1: ***Class SP22-BCS-A & C***

Q1: Determine currents I_1, I_2 and I_3 in the circuit below using Gauss-Elimination method.



Solution Using the assigned directions for the currents, Kirchhoff's current law provides one equation for each node:

Node	Current In		Current Out
A	$I_1 + I_2$	=	I_3
B	I_3	=	$I_1 + I_2$

	Voltage Rises	Voltage Drops
Left Inside Loop	50	$5I_1 + 20I_3$
Right Inside Loop	$30 + 10I_2 + 20I_3$	0
Outside Loop	$30 + 50 + 10I_2$	$5I_1$

These conditions can be rewritten as

$$\begin{aligned} 5I_1 \qquad\qquad + 20I_3 &= 50 \\ 10I_2 + 20I_3 &= -30 \\ 5I_1 - 10I_2 \qquad\qquad &= 80 \end{aligned}$$

Now mathematical model of above circuit is

$$I_1 + I_2 - I_3 = 0$$

$$5I_1 + 20I_3 = 50$$

$$10I_2 + 20I_3 = -30$$

$$5I_1 - 10I_2 = 80$$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 5 & 0 & 20 & 50 \\ 0 & 10 & 20 & -30 \\ 5 & -10 & 0 & 80 \end{array}\right]$$

Apply row operations yourself (I am using linear algebra toolkit)

$$\sim = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$I_1 = 6\,; \quad I_2 = -5; \quad I_3 = 1$$

Question 2: Write matrices A and A^{-1}as a product of elementary matrices.

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ 1 & 0 & 3 \end{bmatrix}$$

Solution:-

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right] R_3 - R_1$$

$$R\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] R_3 - R_2 \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] R_1 + R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 3 & -2 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \begin{matrix} R_1 - 2R_3 \\ R_2 + R_3 \end{matrix}$$

We Know $A^{-1} = E_5E_4E_3E_2E_1$ **and** $A = E_1{}^{-1}E_2{}^{-1}E_3{}^{-1}E_4{}^{-1}E_5{}^{-1}$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} R_3 - R_1 \ \ and\ E_1{}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} R_3 - R_2 \;\; and \;\; E_2{}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R_1 + R_2 \;\; and \;\; E_3{}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R_1 - 2R_3 \;\; and \;\; E_4{}^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} R_2 + R_3 \;\; and \;\; E_5{}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$