PHY121-Applied Physics for Engineers

Current, Resistance and Ohm's law

LECTURE # 15

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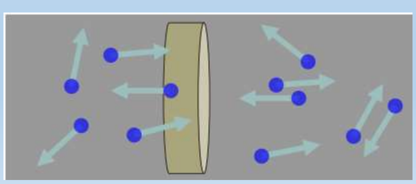
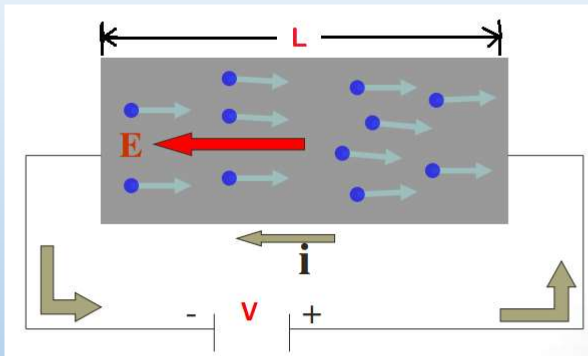
Electric Current

- Rate of flow of electric charge from one region to another is called electric current $I = Q/t$
- In metals, like copper and aluminum some of electrons are free to move
- These free electrons move randomly in all directions
- There is no net flow of charge in any direction and hence no current
- There is no force on electrons and no net flow of charges



- Let us consider a battery of potential difference **V**, is connected across the ends of a conductor having length **L**
- An electric field **E = V/L** is established in the conductor



- This electric field E acts on the electrons and gives them a net motion in the direction opposite to E
- An electric current "**i**" has been established when a net charge **dq** passes through any surface in a time interval **dt**

$$i = {}^{dq}/_{dt}$$

- The net charge that passes through the surface in any interval is found by integrating the current $q = \int i \, dt$
- The electric current i is same for all cross-sections of a conductor, even though the cross-sectional area may be different at different points
- The direction of current is the direction that positive charges would move, even if the actual charge carriers are negative
- Even though we assign direction to current but is a scalar quantity, the arrow that we draw is just to show the sense of charge flow
- Electric circuits are means for conveying energy from one place to another place
- Electric potential is transferred from a source (battery) to a device where the energy is either stored or converted to another form i.e. sound, heat or light etc..
- Electric circuits transport energy without moving any of its parts

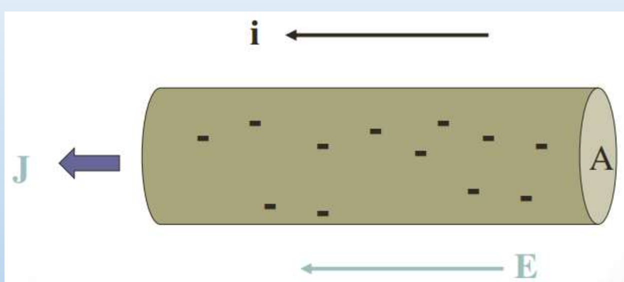
Current Density

- Current i is a characteristic of a particular conductor and is a macroscopic quantity like, the mass, volume and length of an object
- The related microscopic quantity is the current density J and can be defined as
- The number of subatomic particles per unit time crossing a unit area in a designated plane perpendicular to the direction of movement of the particles
- It is a vector and is characteristic of a point inside the conductor, while current is the characteristic of the conductor as a whole
- The magnitude of current density is simply electric current per unit area i.e.

$$J = {}^{i}/_{A} \qquad \text{....(1)}$$

- Its unit is ampere/m^2

- J will be oriented in the direction that a positive charge carrier would move in that direction

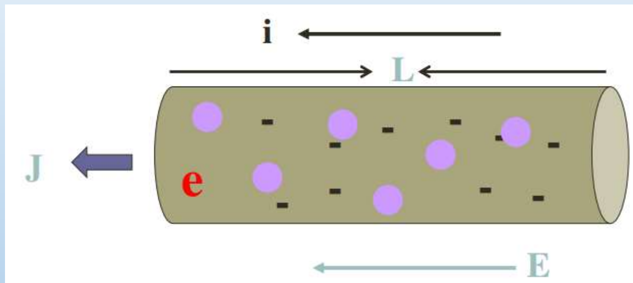


- Thus current can also determined integrating the current density over the surface or current is the flux of the vector J over the surface A

$$i = \int \vec{J} . \vec{dA} \qquad \text{....(2)}$$

- Above equation clearly shows that current is a scalar because it is the scalar product of two vectors

- Electric field exerts a force "**–eE**" on the electrons as they move within the conductor
- Electrons collide with the atoms, ions, lattice ……
- Thus there is no net acceleration



- Energy transfer from accelerating electron to the vibrating atoms because of the collision
- The electron acquire a constant average velocity called drift velocity (**v_d**) in the direction opposite to **E**

- We can calculate the magnitude of that drift velocity from the current density ***J***
- Let **L** be length and **A** be the cross-section area of the conductor
- Thus volume of the conductor be **V = AL**
- **n** be the number of free electrons per unit volume
- The number of free electrons in the wire will be **= nAL**
- So charge will be **q = (nAL)e**, where e is the charge on single electron
- This much amount of charges passes through the wire in time ***t***

$$t = L/v_d$$

- where current is

$$i = q/t$$

$$i = \frac{nALe}{L/v_d} = nAev_d$$

$$=> \quad v_d = \frac{i}{nAe}$$

$$=> \quad v_d = \frac{J}{ne} \qquad \because J = {}^{i}/_{A}$$

$$\mathbf{j = - ne}\ v_d \qquad \text{....(3)}$$

- Following is the expression for vector current density $\vec{J}$ that includes the direction of the drift velocity,

$$\vec{J} = nq\overrightarrow{v_d}$$

- If **q** is positive, $\overrightarrow{v_d}$ is in the same direction as $\vec{E}$; if **q** is negative, $\overrightarrow{v_d}$ is opposite to $\vec{E}$. In either case, $\vec{J}$ is in the same direction as $\vec{E}$.

Example1

An 18-gauge copper wire (the size usually used for lamp cords) has a nominal diameter of 1.02 mm. This wire carries a constant current of 1.67 A to a 200-watt lamp. The density of free electrons is 8.5×10^{28} electrons per cubic meter. Find the magnitudes of (a) the current density and (b) the drift velocity.

Solution

(a) The cross-sectional area is

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.02 \times 10^{-3}\ \text{m})^2}{4} = 8.17 \times 10^{-7}\ \text{m}^2$$

The magnitude of the current density is

$$J = \frac{I}{A} = \frac{1.67\ \text{A}}{8.17 \times 10^{-7}\ \text{m}^2} = 2.04 \times 10^{6}\ \text{A/m}^2$$

(b) Solving Eq. for the drift velocity magnitude v_d, we find

$$v_d = \frac{J}{n|q|}$$

$$= \frac{2.04 \times 10^6 \text{ A/m}^2}{(8.5 \times 10^{28} \text{ m}^{-3})|-1.60 \times 10^{-19} \text{ C}|}$$

$$= 1.5 \times 10^{-4} \text{ m/s} \quad = 0.15 \text{ mm/s}$$

Resistance:

- The electrical resistance of a circuit component or device is defined as the ratio of the voltage applied to the electric current which flows through it, $R = V/i$ the resistance of a conductor between the two points by applying the potential difference ***V*** between those points and measure the current ***i*** that results.

Conductivity:

- The electrons in a conducting materials are accelerated by the electric field ***E***, Thus their drift velocity v_d is proportional to the electric field ***E***
- Which means that the current density ***j*** is proportional to the electric field ***E***

$$\vec{J} \propto \vec{E}$$

$$\vec{J} = \sigma\vec{E} \qquad \text{....(4)}$$

- The proportionality σ constant is called the electrical conductivity of the material. It is not property of the particular sample of the material

- In SI units, the unit of conductivity is siemen/meter = S/m

  where 1 Siemen = ampere/volt

- Inverse of the conductivity is called resistivity, which is also the characteristic of materials

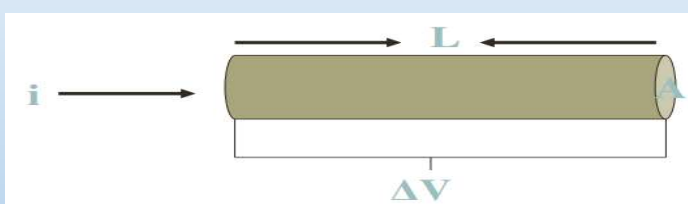
$$\rho = {}^{1}/_{\sigma}$$

$$=> \quad \vec{j} = \frac{\vec{E}}{\rho} \qquad \text{....(5)}$$

- The units of resistivity is ohm-meter i.e. Ω-m.
- It must be noted that the resistivity is independent of the magnitude and direction of applied electric field.

<u>Resistivity:</u>

Consider a conductor of length **L** having cross section area A, let **ΔV** is the potential applied across the two ends



There is a uniform electric field **E** and current density **j** produced in the conductor

$$E = \frac{\Delta V}{L} \qquad j = \frac{i}{A} \qquad \text{....(6)}$$

From eqs (5) and (6), we have

$$\rho = \frac{E}{j}$$

$$\rho = \frac{\Delta V / L}{i / A}$$

where $\Delta V / i = R$, Thus

$$R = \rho \frac{L}{A} \qquad \text{....(7)}$$

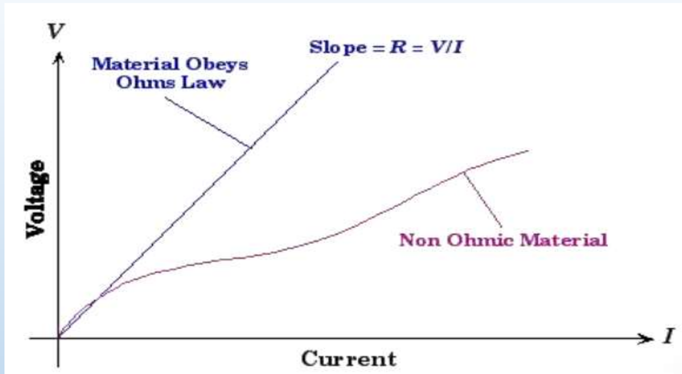
The resistance of a conductor is independent of the magnitude and sign of the applied potential, it is the property of specific specimen of the sample.

Ohm's law:

- The current ***i*** passing through a conductor is directly proportional to the applied potential difference **V**

$$V = iR \qquad \text{....(8)}$$

- If we plot current verses voltage for a conductor, the ratio **V/i** is always constant, which is called resistance
- Thus the resistance of a device is constant and is independent of the potential difference across it.
- A conducting device obeys ohm's law if the resistance between any pair of points is independent of the magnitude and polarity of the applied potential difference



- If a conductor obeys Ohm's law its ***V*** versus ***i*** graph is linear and such conductors/devices are called Ohmic.
- If a conductor does not obey Ohm's law its ***V*** versus ***i*** graph is not linear and is called non-Ohmic like a pn junction diode.

- The statement v = iR is not Ohm's law statement but it is a general definition of the resistance of a conductor whether it obeys Ohm's law or not.
- The microscopic equivalent of the statement ***V = iR*** is

$$\mathbf{E} = \rho \mathbf{j}$$

Example2

The 18-gauge copper wire in Example1 has a diameter of 1.02 mm and a cross-sectional area of 8.20×10^{-7} m^2. It carries a current of 1.67 A. Find (a) the electric-field magnitude in the wire; (b) the potential difference between two points in the wire 50.0 m apart; (c) the resistance of a 50.0-m length of this wire.

Solution

(a) the resistivity of copper is $1.72 \times 10^{-8}\ \Omega \cdot \text{m}$.
Hence,

$$E = \rho J = \frac{\rho I}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(1.67\ \text{A})}{8.20 \times 10^{-7}\ \text{m}^2} = 0.0350\ \text{V/m}$$

(b) The potential difference is given by

$$V = EL = (0.0350\ \text{V/m})(50.0\ \text{m}) = 1.75\ \text{V}$$

(c) the resistance of a 50.0-m length of this wire is

$$R = \frac{V}{I} = \frac{1.75\ \text{V}}{1.67\ \text{A}} = 1.05\ \Omega$$

Or

$$R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(50.0\ \text{m})}{8.20 \times 10^{-7}\ \text{m}^2} = 1.05\ \Omega$$

- One end of an aluminum wire whose diameter is *2.5 mm* is welded to one end of copper wire whose magnitude is *1.8 mm*. The composite wire carries I of *1.3A*. What is the current density in each wire?
- A strip of silicon, of cross sectional width *w = 3.2 mm* and thickness *d = 250 µm*, carries a current of *190 mA*. The silicon is n-type semiconductor, having doped. The doping has the effect of greatly increasing *n*, the number of charge carriers per unit volume as compared with value for pure silicon. In this case $n = 8.0\times10^{21}\ m^{-3}$ *(a)* what is current density *(b)* what is drift speed?
- A rectangular block of iron has dimensions *1.2 cm×1.2 cm×158 cm (a)* what is the resistance of the block measured between the two square ends? *(b)* What is the resistance between two opposing rectangular faces? The resistivity of iron at room temperature is $9.68\times10^{-8}\ \Omega m$.
- A steel trolley-car rail has a cross sectional area of $56\ cm^2$. What is the resistance of *11km* of rail? The resistivity of the steel is $3.0\times10^{-7}\ \Omega m$.
- Using the *470 Ω* resistor in series with the LED, how much current will be able to flow with a *5V* source?

P1

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m^2. What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig.1) ?

Solution

We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020\text{ m})^2 = 9.424 \times 10^{-6}\text{ m}^2.$$

As we know

$$i = JA'$$

$$= (2.0 \times 10^5\text{ A/m}^2)(9.424 \times 10^{-6}\text{ m}^2)$$

$$= 1.9\text{ A}.$$

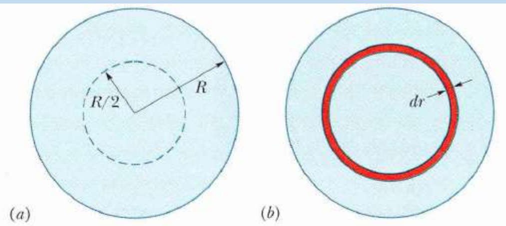


Fig.1 (a) Cross section of a wire of radius R. (b) A thin ring has width dr and circumference $2\pi r$, and thus a differential area $dA = 2\pi r\,dr$.

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m^4 and r is in meters. What now is the current through the same outer portion of the wire?

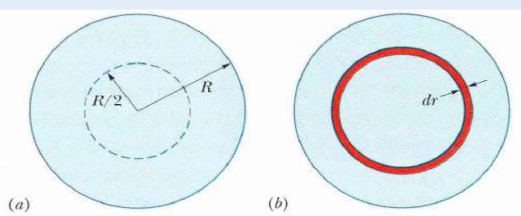


Fig.1 (a) Cross section of a wire of radius R. (b) A thin ring has width dr and circumference $2\pi r$, and thus a differential area $dA = 2\pi r\,dr$.

KEY IDEA Because the current density is not uniform across a cross section of the wire, we must resort to Eq. ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$.

Calculations: The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J\,dA \cos 0 = J\,dA.$$

We need to replace the differential area dA with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because J is given as a function of r) is the area $2\pi r\,dr$ of a thin ring of circumference $2\pi r$ and width dr (Fig.1.b) . We can then integrate with r as the variable of integration. Equation then gives us

$$i = \int \vec{J} \cdot d\vec{A} = \int J\,dA$$

$$= \int_{R/2}^{R} ar^2\, 2\pi r\,dr = 2\pi a \int_{R/2}^{R} r^3\,dr$$

$$= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R}$$

$$= \frac{\pi a}{2}\left[R^4 - \frac{R^4}{16}\right]$$

$$= \frac{15}{32}\pi a R^4$$

$$= \frac{15}{32}\pi(3.0 \times 10^{11}\ \text{A/m}^4)(0.0020\ \text{m})^4 = 7.1\ \text{A}.$$

(Answer)

P2

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900\ \mu\text{m}$ when it has a uniform current $i = 17\ \text{mA}$? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

Solution

KEY IDEAS

1. The drift speed v_d is related to the current density $\vec{J}$ and the number n of conduction electrons per unit volume according to Eq. 26-7, which we can write as $J = nev_d$.
2. Because the current density is uniform, its magnitude J is related to the given current i and wire size by Eq. ($J = i/A$, where A is the cross-sectional area of the wire).
3. Because we assume one conduction electron per atom, the number n of conduction electrons per unit volume is the same as the number of atoms per unit volume.

Calculations: Let us start with the third idea by writing

$$n = \begin{pmatrix}\text{atoms}\\ \text{per unit}\\ \text{volume}\end{pmatrix} = \begin{pmatrix}\text{atoms}\\ \text{per}\\ \text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\ \text{per unit}\\ \text{mass}\end{pmatrix}\begin{pmatrix}\text{mass}\\ \text{per unit}\\ \text{volume}\end{pmatrix}.$$

The number of atoms per mole is just Avogadro's number N_A ($= 6.02 \times 10^{23}\ \text{mol}^{-1}$). Moles per unit mass is the inverse of the mass per mole, which here is the molar mass M of copper. The mass per unit volume is the (mass) density ρ_{mass} of copper. Thus,

$$n = N_A\left(\frac{1}{M}\right)\rho_{\text{mass}} = \frac{N_A\rho_{\text{mass}}}{M}.$$

$$n = \frac{(6.02 \times 10^{23}\ \text{mol}^{-1})(8.96 \times 10^{3}\ \text{kg/m}^3)}{63.54 \times 10^{-3}\ \text{kg/mol}}$$

$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

or

$$n = 8.49 \times 10^{28}\ \text{m}^{-3}.$$

Next let us combine the first two key ideas by writing

$$\frac{i}{A} = nev_d.$$

Substituting for A with πr^2 ($= 2.54 \times 10^{-6}\ \text{m}^2$) and solving for v_d, we then find

$$v_d = \frac{i}{ne(\pi r^2)}$$

$$= \frac{17 \times 10^{-3}\ \text{A}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})(2.54 \times 10^{-6}\ \text{m}^2)}$$

$$= 4.9 \times 10^{-7}\ \text{m/s}, \qquad \text{(Answer)}$$

which is only 1.8 mm/h, slower than a sluggish snail.

P3

Fig.2

A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 2). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions 1.2 cm × 1.2 cm) and (2) two rectangular sides (with dimensions 1.2 cm × 15 cm)?

Solution

Calculations: For arrangement 1, we have $L = 15$ cm $= 0.15$ m and

$$A = (1.2\ \text{cm})^2 = 1.44 \times 10^{-4}\ \text{m}^2.$$

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(0.15\ \text{m})}{1.44 \times 10^{-4}\ \text{m}^2} = 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega.$$

Similarly, for arrangement 2, with distance $L = 1.2$ cm and area $A = (1.2\ \text{cm})(15\ \text{cm})$, we obtain

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(1.2 \times 10^{-2}\ \text{m})}{1.80 \times 10^{-3}\ \text{m}^2} = 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega.$$

P4

What is the mean free time τ (average time τ between collisions) between collisions for the conduction electrons in copper?

Solution

Calculations: That equation gives us

$$\tau = \frac{m}{ne^2\rho}.$$

We take the value of n, the number of conduction electrons per unit volume in copper, from P 2

The denominator then becomes

$$(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.69 \times 10^{-8}\ \Omega\cdot\text{m})$$
$$= 3.67 \times 10^{-17}\ \text{C}^2\cdot\Omega/\text{m}^2 = 3.67 \times 10^{-17}\ \text{kg/s},$$

where we converted units as

$$\frac{\text{C}^2\cdot\Omega}{\text{m}^2} = \frac{\text{C}^2\cdot\text{V}}{\text{m}^2\cdot\text{A}} = \frac{\text{C}^2\cdot\text{J/C}}{\text{m}^2\cdot\text{C/s}} = \frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass m, we then have

$$\tau = \frac{9.1 \times 10^{-31}\ \text{kg}}{3.67 \times 10^{-17}\ \text{kg/s}} = 2.5 \times 10^{-14}\ \text{s}. \quad \text{(Answer)}$$