Mon Tue Wed Thu Fri Sat Sun

Date:

Non-Autonomous Differential Equations

Bernoulli's Equation

$$\frac{dy}{dx} + P(x) \cdot y = f(x) \cdot y^n$$

Question:

$$x \cdot \frac{dy}{dx} + y = x^2 y^2$$

Dividing by coefficient

$$\frac{dy}{dx} + \frac{1}{x} y = x y^2$$

Put values of y & $\frac{dy}{dx}$

$$\left(-\frac{1}{v^2}\right)\frac{dv}{dx} + \frac{1}{x}\left(\frac{1}{v}\right) = x\left(\frac{1}{v^2}\right)$$

Multiplying "$-v^2$"

$$\frac{dv}{dx} + \left(-\frac{v}{x}\right) = -x$$

$$\frac{dv}{dx} - \frac{1}{x} v = -x$$

$$P(x) = -\frac{1}{x}, \quad f(x) = -x$$

$$I.F = e^{-\int \frac{1}{x} dx} \Rightarrow e^{\ln(x)^{-1}} \Rightarrow \frac{1}{x}$$

$$\frac{d}{dx}[I.F \cdot y] = (I.F) f(x)$$

$$\frac{d}{dx}\left[\frac{1}{x} \cdot v\right] = \left(\frac{1}{x}\right)(-x)$$

$$\int \frac{d}{dx}\left[\frac{1}{x} \cdot v\right] = -\int 1 \, dx$$

$$\frac{1}{x}\left(\frac{1}{y}\right) = -x + c$$

$$c = \frac{1}{xy} + x$$

$$v = y^{(1-n)}$$

$f(x) = x^2 y^2$ as n

$$v = y^{1-2}$$

$$v = y^{-1}$$

$$v = \frac{1}{y}$$

$$y = \frac{1}{v}$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{d}{dv}(v^{-1}) \cdot \frac{dv}{dx}$$

$$\frac{dy}{dx} = -v^{-2} \cdot \frac{dv}{dx}$$

$$\frac{dy}{dx} = -\left(\frac{1}{v^2}\right)\frac{dv}{dx}$$

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