Solution Quiz No 1: ***Class SP22-BCS-B***

Question 1: Decode the encrypted message **NUH ZCR**, where encryption is applied by following matrix

$$A = \begin{bmatrix} 3 & -6 & 2 \\ 0 & 1 & 0 \\ -2 & 4 & -2 \end{bmatrix}$$

Solution 1: First compute A^{-1} using row operations

$$[A|I] = \left[\begin{array}{ccc|ccc} 3 & -6 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -2 & 4 & -2 & 0 & 0 & 1 \end{array}\right]$$

Apply row operations to get reduced row echelon form (I am using linear algebra toolkit)

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -3/2 \end{array}\right] = [I|A^{-1}]$$

$$\begin{bmatrix} N \\ U \\ H \end{bmatrix} = \begin{bmatrix} 14 \\ 21 \\ 8 \end{bmatrix} \text{ and } = \begin{bmatrix} Z \\ C \\ R \end{bmatrix} = \begin{bmatrix} 26 \\ 3 \\ 18 \end{bmatrix}$$

$$A^{-1}\begin{bmatrix} N \\ U \\ H \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & -3/2 \end{bmatrix}\begin{bmatrix} 14 \\ 21 \\ 8 \end{bmatrix} = \begin{bmatrix} 64 \\ 21 \\ -26 \end{bmatrix} = \begin{bmatrix} 12 \\ 21 \\ 26 \end{bmatrix} = \begin{bmatrix} L \\ C \\ Z \end{bmatrix}$$

$$A^{-1}\begin{bmatrix} Z \\ C \\ R \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & -3/2 \end{bmatrix}\begin{bmatrix} 26 \\ 3 \\ 18 \end{bmatrix} = \begin{bmatrix} 50 \\ 3 \\ -53 \end{bmatrix} = \begin{bmatrix} 24 \\ 3 \\ 25 \end{bmatrix} = \begin{bmatrix} X \\ C \\ Y \end{bmatrix}$$

Question 2: Find all values of a for which resulting linear system has

(a) No solution (b) Unique solution (c) Infinite many solution

$$\begin{aligned} x + y \quad\quad + z &= 3 \\ x + 2y + \quad\quad z &= 2 \\ x + y + (a^2 - 8)z &= a \end{aligned}$$

Solution 2: $[A|b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & a^2-8 & a \end{array}\right] \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & a^2-9 & a-3 \end{array}\right]$

If $a^2 - 9 = 0$and $a - 3 \neq 0$implies No solution;

Now$a^2 = 9 \rightarrow a = \pm 3$; when $a = -3$, then

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

$0x + 0y + 0z = -6$, not acceptable. Hence No solution at $a = -3$

Now take$a = 3$, then

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0x + 0y + 0z = 0$, (This expression signals about “Infinite many solutions”)

Hence

For$a = 3$ given system has Infinite many solution.

Unique Solution: For all values of $a \in R$ other than ± 3 system has Unique solution.