

Q #4

$$4y'' + 4y' + 17y = 0$$

$$y(0) = 1$$

$$y'(0) = 2$$

$$4 \frac{d^2}{dx^2} y + 4 \frac{d}{dx} y + 17y = 0$$

$$\left(4 \frac{d^2}{dx^2} + 4 \frac{d}{dx} + 17\right) y = 0$$

$$4 \frac{d^2}{dx^2} + 4 \frac{d}{dx} + 17 = 0$$

$$\text{let } m = \frac{d}{dx}$$

$$4m^2 + 4m + 17 = 0$$

For Quadratic formula $[a=4, b=4, c=17]$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{16 - 272}}{8}$$

$$m = \frac{-4 \pm 16i}{8}$$

$$m = \frac{-1 \pm 2i}{2}$$

$$y_c = e^{(-1/2)x} [C_1 \cos(2)x + C_2 \sin(2)x]$$

$$y_c = e^{-x/2} [C_1 \cos 2x + C_2 \sin 2x]$$

$$y(0) = 1 \quad [x=0, y=1]$$

$$y = e^{-x/2} [C_1 \cos 2x + C_2 \sin 2x]$$

$$1 = e^{-(0)/2} [C_1 \cos 2(0) + C_2 \sin 2(0)]$$

$$1 = 1 [C_1 \cos 0 + C_2 \sin 0]$$

$$1 = C_1(1) + 0$$

$$\boxed{C_1 = 1}$$

$$y'(0) = 2$$

$$\frac{d}{dx} \text{ for } \left[y = e^{-x/2} [C_1 \cos 2x + C_2 \sin 2x] \right]$$

$$y' = e^{-x/2} \left[\underbrace{-2 \sin 2x + 2C_2 \cos 2x}_{\boxed{C_1=1}} + (\cos 2x + C_2 \sin 2x) e^{-x/2} \left(-\frac{1}{2} \right) \right]$$

$$\mathbf{2} = e^0 [-2 \sin 2(0) + 2C_2 \cos 2(0)] + [\cos 2(0) + C_2 \sin 2(0)] e^0 \left(-\frac{1}{2} \right)$$

$$\mathbf{2} = 1 [-2 \sin(0) + 2C_2 \cos(0)] + [\cos(0) + C_2 \sin(0)] (1) \left(-\frac{1}{2} \right)$$

$$\mathbf{2} = 0 + 2C_2(1) + [1 + 0] \left(-\frac{1}{2} \right)$$

$$\mathbf{2} + \frac{1}{2} = 2C_2$$

$$5/2 = 2C_2 \quad C_2 = 5/4$$

$$y = e^{-x/2} [C_1 \cos 2x + C_2 \sin 2x]$$

$$y = e^{-x/2} \left[(1) \cos 2x + \frac{5}{4} \sin 2x \right]$$

$$y = e^{-x/2} \left[\cos 2x + \frac{5}{4} \sin 2x \right]$$