

## Solve by power Series

$$y'' + xy = 0$$

$$P_0(x) = 1$$

$$P_1(x) = 0$$

$$P_2(x) = x$$

standard form

$$P_0(x) \frac{d^2 y}{dx^2} + P_1(x) \frac{dy}{dx} + P_2(x) y = 0$$

we get at point  $x = 0$

$$P_0(x) = 1$$

$$P_0(0) = 1 \neq 0$$

we will use Power Series so solution will be

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$\frac{dy}{dx} = \sum_{n=1}^{\infty} a_n n x^{n-1}$$

$$\frac{d^2 y}{dx^2} = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

Put the values

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + x \left( \sum_{n=0}^{\infty} a_n x^n \right) = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=1}^{\infty} a_{n-1} x^n = 0$$

$$2a_2 + \sum_{n=1}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=1}^{\infty} a_{n-1} x^n = 0$$

$$2a_2 + \sum_{n=1}^{\infty} (n+2)(n+1)a_{n+2} x^n + \sum_{n=1}^{\infty} a_{n-1} x^n = 0$$

$$2a_2 = 0$$

$$a_2 = 0$$

$$(n+2)(n+1)a_{n+2} + a_{n-1} = 0$$

$$(n+2)(n+1)a_{n+2} = -a_{n-1}$$

$$a_{n+2} = -\frac{1}{(n+2)(n+1)} a_{n-1}$$