Higher Order D.E 20-10-23

Linear Equations with constant Coefficient

$$2y'' - 5y' - 3y = 0$$

$$2\frac{d^2}{dx^2}y - 5\frac{dy}{dx} - 3y = 0$$

$$\left(2\frac{d^2}{dx^2} - 5\frac{d}{dx} - 3\right)y = 0$$

$$2\frac{d^2}{dx^2} - 5\frac{d}{dx} - 3 = 0$$

Let $\frac{d}{dx} = m$

$$2m^2 - 5m - 3 = 0$$

$$2m^2 - 6m + 1m - 3 = 0$$

$$2m(m-3) + 1(m-3) = 0$$

$$(m-3)(2m+1) = 0$$

$m - 3 = 0$	$2m + 1 = 0$
$m = 3$	$m = -1/2$

(if) Roots are Real / Distinct (unique) → Different

$m_1 = 3$, $m_2 = -1/2$

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y_c = c_1 e^{3x} + c_2 e^{-x/2}$$

H.O.D.E

$$y'' - 10y' + 25y = 0$$

$$\frac{d^2}{dx^2}y - 10\frac{d}{dx}y + 25y = 0$$

$$\left(\frac{d^2}{dx^2} - 10\frac{d}{dx} + 25\right) y = 0$$

$$\frac{d^2}{dx^2} - 10\frac{d}{dx} + 25 = 0$$

let $m = \frac{d}{dx}$

$$m^2 - 10m + 25 = 0$$

$$m^2 - 5m - 5m + 25 = 0$$

$$m(m-5) - 5(m-5) = 0$$

$$(m-5)(m-5) = 0$$

$m-5=0$	$m-5=0$
$m=5$	$m=5$

(if) Real & same [Roots]

$$y_c = c_1 e^{m_1 x} + c_2 x e^{m_2 x} + c_3 x^2 e^{m_3 x} + c_4 x^3 e^{m_4 x} \cdots$$

For 2 points

$$y_c = c_1 e^{m_1 x} + c_2 x e^{m_2 x}$$

$$y_c = c_1 e^{5x} + c_2 x e^{5x}$$

H.O.D.E

Question #3

$$y'' + 4y' + 7y = 0$$

$$\frac{d^2}{dx^2}y + 4\frac{d}{dx}y + 7y = 0$$

$$\left(\frac{d^2}{dx^2} + 4\frac{d}{dx} + 7\right)y = 0$$

$$\frac{d^2}{dx^2} + 4\frac{d}{dx} + 7 = 0$$

let $\left[\frac{d}{dx} = m\right]$

$$m^2 + 4m + 7 = 0$$

For Quadratic formula $[a=1, b=4, c=7]$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{16-28}}{2}$$

$$m = \frac{-4 \pm \sqrt{-12}}{2}$$

$$m = \frac{-4 \pm 2\sqrt{3}i}{2}$$

$$m = -2 \pm \sqrt{3}i$$

Roots are Imagenery

$$y_c = e^{\alpha x}\left[C_1 \cos \beta x + C_2 \sin \beta x\right]$$

$$y_c = e^{-2x}\left[C_1 \cos(\sqrt{3})x + C_2 \sin(\sqrt{3})x\right]$$

If points (Roots) are more than 2 so we add same terms for different root values

As Roots are

$$m = -2 \pm \sqrt{3}\, i$$

↓ Real part ↓ Imagenery part