Applied Physics for Engineers (PHY121)

Electric Potential and Electric Potential Energy

LECTURE # 09

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Outlines

1. Electric Potential
2. Equipotential Surfaces
3. Electric Potential Energy

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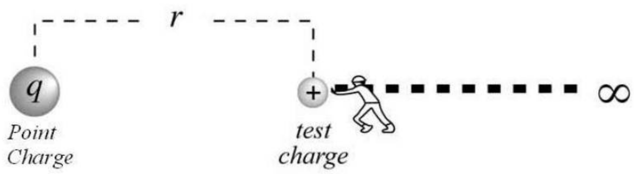
1.4 Electric Potential, V

Electric potential, *V* is defined as the work done per unit charge to bring a test charge (positive charge) from infinity (∞) to a point in an electric field produced by a source point charge, Q.



$$V = \frac{W_{\infty \mapsto r}}{q_0}$$

Electric potential, *V* is a scalar quantity

SI Unit is J C^{-1} or Volt (V).

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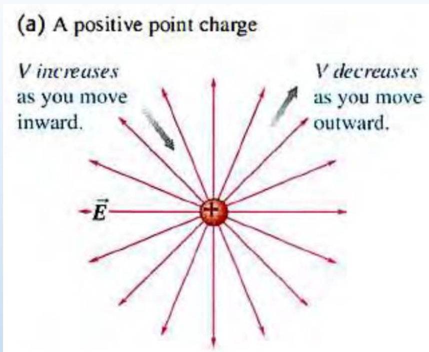
V **can also be written in terms of k, Q and r which is given by :**

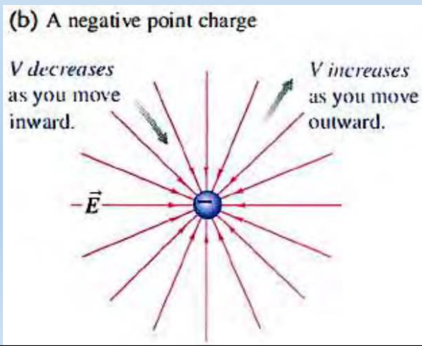
$$V = k\frac{Q}{r}$$

If the electric potential at a point in an electric field is *V*, then a charge q placed at that point will have electric potential energy, U of :

$$U = qV$$

Note:
- **The total electric potential at a point in space is equal to the algebraic sum of the constituent potentials at that point.**
- **In the calculation of *U and V, the sign of the charge* must be substituted in the related equations.**





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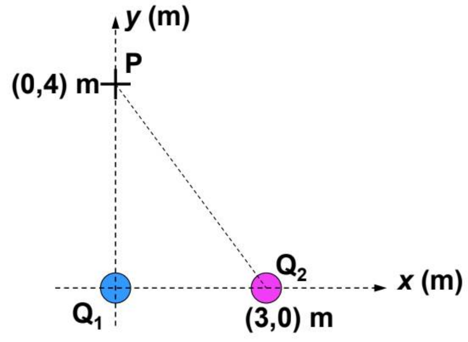
Example 7 :

A point charge q_1 = +5.0 μC is at the origin and a point charge q_2 = -2.0 μC is on the x-axis at (3,0) m as in figure below.



Find the total electric potential due to these charges at point P.

Solution :

V at P due to each charge can be calculated from ;

$$V = k\frac{Q}{r}$$

The total *V* is the scalar sum of these two potentials.

$$V_1 = k\frac{Q_1}{r_1}$$

$$= (9\times10^9)\left(\frac{+5.0\times10^{-6}}{4.0}\right)$$

$$= 1.125\times10^4\ V$$

$$V_2 = k\frac{Q_2}{r_2}$$

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Include sign of charge in the substitution.

$$= (9\times10^9)\left(\frac{-2.0\times10^{-6}}{5.0}\right)$$

$$= -0.36\times10^4\ V$$

Therefore,

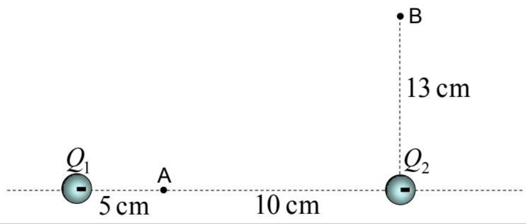
$V_p = V_1 + V_2$

$= (+1.125 \times 10^4) + (-0.36 \times 10^4)$

= **7650 V**

FOLLOW UP EXERCISE

Two point charges, Q_1= –40 μC and Q_2= –30 μC are separated by a distance of 15 cm as shown in Figure below.



Calculate

a. the electric potential at point A and describe the meaning of the answer,

b. the electric potential at point B.

Answer : (a) $V_A = -9.9\times10^6\ \text{V}$

(b) $V_B = -3.886\times10^6\ \text{V}$

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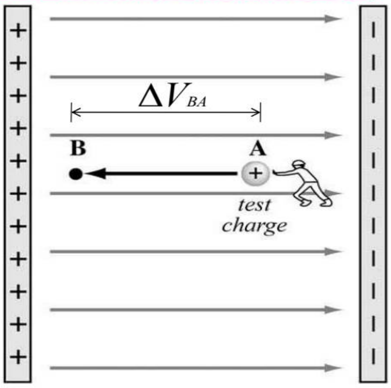
Electric Potential Difference, ΔV



Potential difference between two points A & B in an electric field is the work done (or required) in bringing a positive test charge from point A to point B in an electric field per unit charge.

$$\Delta V_{BA} = \frac{W_{A\rightarrow B}}{q}$$

where :

$$\Delta V_{BA} = V_B - V_A \quad (\ * V_{final} - V_{initial})$$

The electric field is a conservative field. The work done to bring a charge from one point to another point in an electric field is independent of the path.

If the value of work is positive – work is required (we need external force to move the charge).

If the value of work done is negative – no work required (or work done by the electric force itself, no external force is needed to move the charge).

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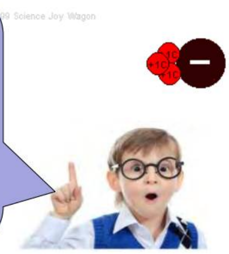
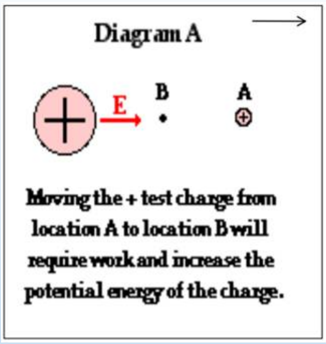
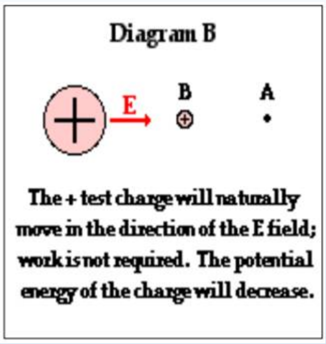
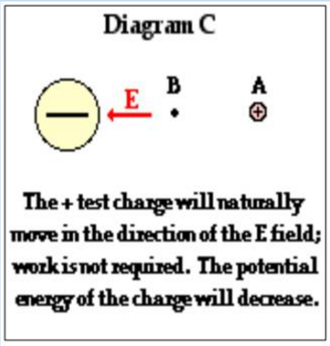
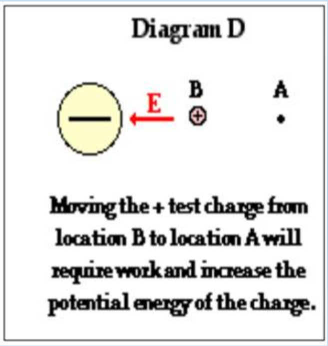
CAUTION !
In the calculation of *U, W* and *V*, the sign of the charge must be substituted in the related equations.











Example 8 :

Points A and B are at distances of 2.0 cm and 3.0 cm respectively from a point charge Q = –100 μC.

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Determine ;

(a) the electric potentials at A and B,

(b) the work required in moving a point charge q = +2.0 μC, from A to B.

Solution :

(a) $V_A = \dfrac{kQ}{r_A}$

$$= (9\times10^9)\left(\frac{-100\times10^{-6}}{0.02}\right)$$

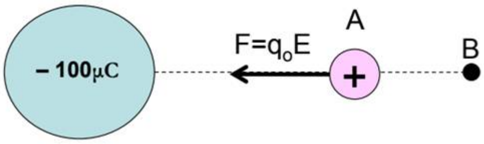
$$= -4.5\times10^7\ V$$

$$V_B = \frac{kQ}{r_B}$$

$$= (9\times10^9)\left(\frac{-100\times10^{-6}}{0.03}\right)$$

$$= -3.0\times10^7\ V$$

(b) the work required is given by :



$$W_{AB} = q\Delta V_{BA}$$

$$= q(V_B - V_A)$$

$$= (+2\times10^{-6})[(-3.0)-(-4.5)]\times10^7$$

$$= 30\ J$$

- Work is done on the system, W positive
- Potential energy, U increase

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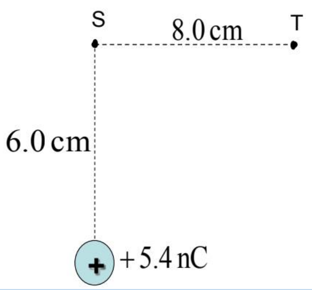
FOLLOW UP EXERCISE

(1) Two points, S and T are located around a point charge of +5.4 nC as shown in Figure below



Calculate

a. the electric potential difference between points S and T,

b. the work done in bringing a charge of –1.5 nC from point T to point S.

Answer : $V_{ST} = 324\ V$ $W_{TS} = -4.86\times10^{-7}\ J$

(2) A test charge q_O =+2.3 μC is placed 20 cm from a point charge Q. A work done of 25 joule is required in bringing the test charge q_O to a distance 15 cm from the charge Q.

Determine

a. the potential difference between point 15 cm and 20 cm from the point charge, Q,

b. the value of charge Q,

c. the magnitude of the electric field strength at point 10 cm from the charge Q.

(Given k = 9×10^9 N m² C⁻²)

Answer

(a) $V_{BA} = 1.09\times10^7\ V$

(b) $Q = +7.27\times10^{-4}\ C$

(c) $E = 6.54\times10^8\ N\,C^{-1}$

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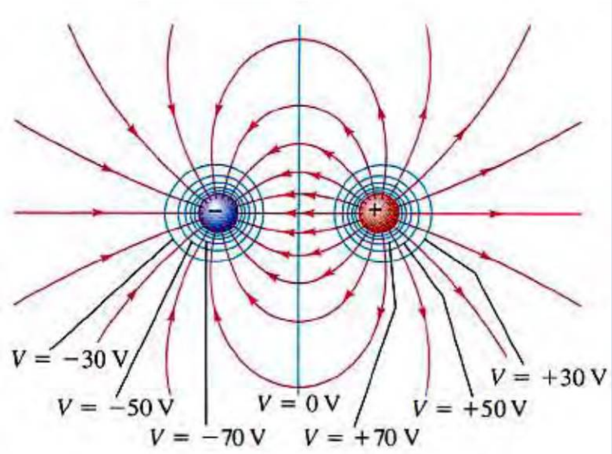
Equipotential Lines and Surfaces

Equipotential line or surface is a line or surface on which **all points** are at the **same potential**.



The blue lines represent the equipotential surfaces (lines). Red line with arrow head represents electric field lines.

<u>The important properties of equipotential surfaces are</u>

(1) The **electric field** at every point on an equipotential surface is **perpendicular to** the **surface**.

(2) The **electric field points in the direction of <u>decreasing</u> electric potential.**

(3) The surfaces are **closer together where the electric field is stronger,** and farther apart where the field is weaker

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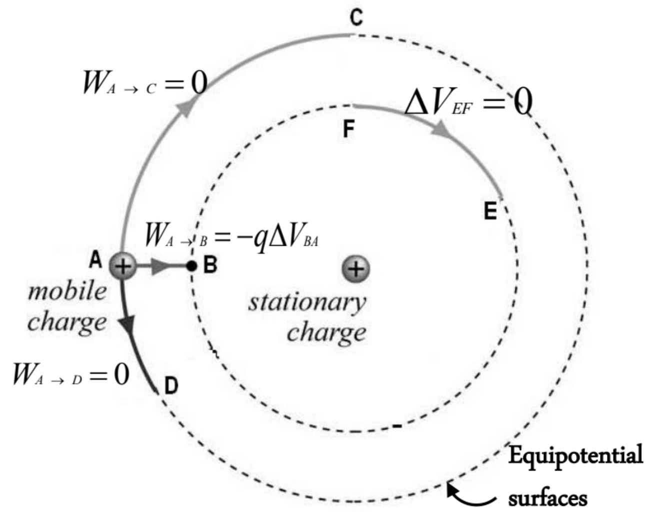
(4) No two equipotential surfaces can intersect each other. An equipotential surface is normal to the electric field. If two equipotential surfaces intersect each other then at the point of intersection there will **be** two directions of electric field, which is impossible.



NOTE

The **potential difference** between any two points on an equipotential surface is **zero**.

Hence, **no work** is **required** to move a charge at constant speed along an equipotential surface.

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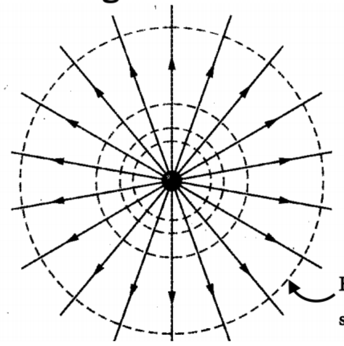
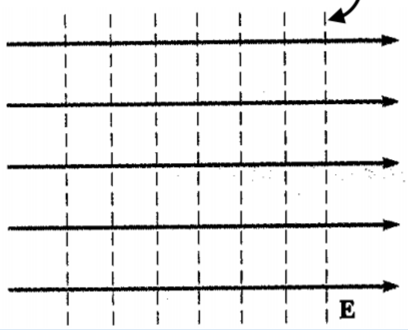
Electric field lines and cross sections of equipotential surfaces for

(a) A point charge



(b) A uniform field



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Relationship between uniform E and potential difference.

Consider a uniform electric field is produced by a pair of flat metal plates, one at which is earthed and the other is at a potential of +V as shown in Figure (a)

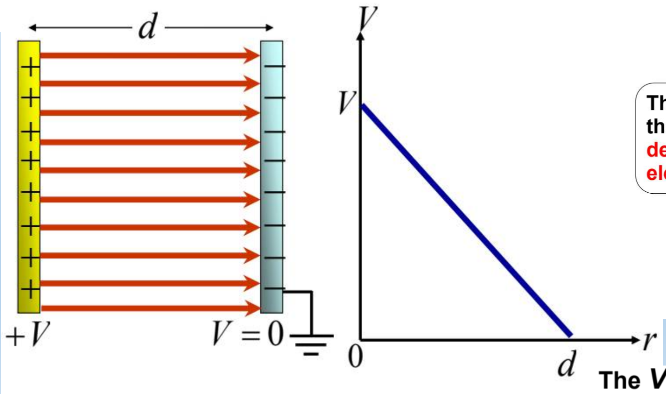

Figure (a) **Figure (b)**

The V against r graph for pair of flat metal plates can be shown in Figure (b)

The graph is a straight line with negative constant gradient, thus

$$E = \frac{\Delta V}{\Delta r} = \frac{(0 - V)}{(d - 0)}$$

$$\boxed{E = -\frac{V}{d}}$$

For uniform E such as in capacitor.

The negative sign indicates that the value of electric potential decreases in the direction of electric field.

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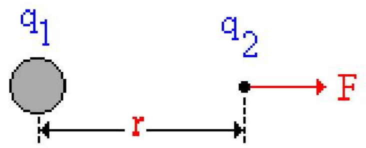
Electric Potential Energy, U of two point charges



$$U = \frac{k\, q_1 q_2}{r}$$

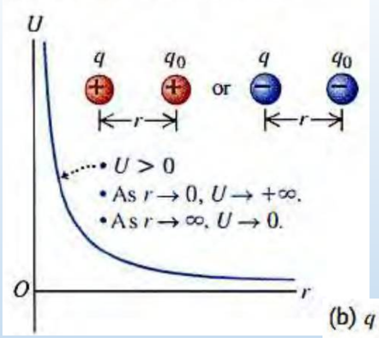
where

- k – Coulomb constant
- q_1 – charge 1
- q_2 – charge 2
- r – distance between q_1 & q_2

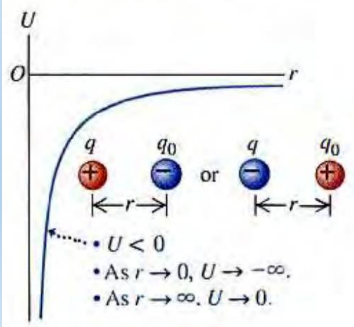
U is +ve if the charges q_1 & q_2 have the same sign, –ve if they have opposite sign.

U is proportional to $\frac{1}{r}$.

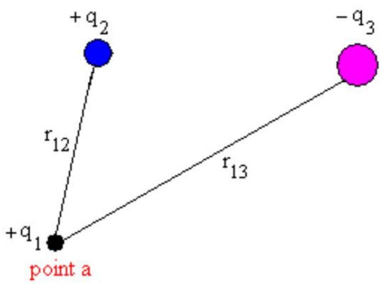
(a) q and q_0 have the same sign.



(b) q and q_0 have opposite signs.



Electric Potential Energy, U of a system of point charges

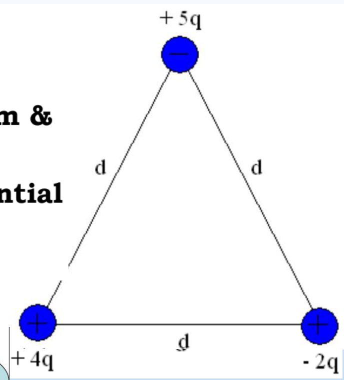


The potential energy associated with the charge q_1 at point a is the sum of the potential energy of interaction for each pair of charges.

$$U = k\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

(Algebraic sum with the sign of charge included)

Example 9



Take distance d = 14.0 cm & charge q = 150 nC

What is the electric potential energy of this system of charges ?

Solution

Include sign of in the substitu

Include sign of charge in the substitution.

$$U = k\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

$$= k\left[\frac{+4q - 2q}{d} + \frac{+4q + 5q}{d} + \frac{+5q - 2q}{d}\right]$$

$$= k\left[\frac{-8q^2}{d} + \frac{20q^2}{d} + \frac{-10q^2}{d}\right]$$

$$= k\left[\frac{+2q^2}{d}\right]$$

$$= 8.99\times10^9\left[\frac{+2(150\times10^{-9})^2}{0.14}\right]$$

$$\boxed{U = 2.9\times10^{-3}\,\text{J}}$$

Change in Electric Potential Energy, ΔU between 2 points in electric field

When a charged particle moves in an electric field, the field exerts a force that can do work on the particle.

When a charged particle moves from a point where the potential energy is U_A to a point where it is U_B , the change in potential energy is $\Delta U = U_B - U_A$ and the work, $W_{A\rightarrow B}$ done by the external force to move the charge is

$$W_{A\rightarrow B} = \Delta U$$

$$W_{A\rightarrow B} = (U_B - U_A)$$

$$q(V_B - V_A) = U_B - U_A$$

$$\boxed{q\Delta V_{BA} = \Delta U_{BA}}$$

Note:
B : Final ; A : Initial

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Example 10

When an alpha particle which has charge 2e moves between 2 points with potential difference of 1000 V, the change in potential energy is

$$\Delta U = q\Delta V$$

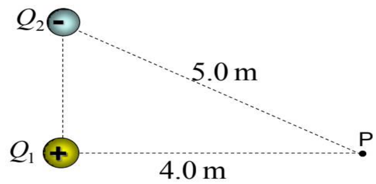
$$= qV_{ab} = 2e(1000)$$

$$= 2(1.6\times10^{-19})(1000)$$

$$\boxed{\Delta U = 3.2\times10^{-16}\ \text{J}}$$

FOLLOW UP EXERCISE

(1) Two point charges, Q_1= +2.0 μC and Q_2= –6.0 μC, are placed 4.0 m and 5.0 m from a point P respectively as shown in Figure below.



a) Calculate the electric potential at P due to the charges. { ans : –6300 }

b) If a charge Q_3= +3.0 μC moves from infinity to P, determine the change in electric potential energy for this charge. { ans : –1.89×10^{-2} }

c) When the charge Q_3 at point P, calculate the electric potential energy for the system of charges. { ans : –5.49×10^{-2} }

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END OF LECTURE

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