Mon Tue Wed Thu Fri Sat Sun

Date:

Higher Order Differential Equations

[Method of undetermined Coefficients)

$$[y = y_c + y_p]$$

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

For y_c

$$y'' + 4y' - 2y = 0$$

$$\frac{d^2}{dx^2}y + 4\frac{d}{dx}y - 2y = 0$$

$$\left(\frac{d^2}{dx^2} + 4\frac{d}{dx} - 2\right)y = 0$$

$$\frac{d^2}{dx^2} + 4\frac{d}{dx} - 2 = 0$$

let $\frac{d}{dx} = m$

$$m^2 + 4m - 2 = 0$$

$a = 1,\ b = 4,\ c = -2$

By using Quadraticformula

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{16 + 8}}{2}$$

$$m = \frac{-4 \pm \sqrt{4(6)}}{2}$$

$$m = \frac{-4 \pm 2\sqrt{6}}{2}$$

$$m = \frac{2(-2 \pm \sqrt{6})}{2}$$

$$m = -2 \pm \sqrt{6}$$

$m_1 = -2 + \sqrt{6},\ m_2 = -2 - \sqrt{6}$

Real & Distinct

$$y_c = c_1 e^{(-2+\sqrt{6})x} + c_2 e^{(-2-\sqrt{6})x}$$

For y_p

As $f(x) = 2x^2 - 3x + 6$

So $y = Ax^2 + Bx + C$

$$y' = 2Ax + B$$

$$y'' = 2A$$

Put values in equation

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

$$2A + 4(2Ax + B) - 2(Ax^2 + Bx + C) = 2x^2 - 3x + 6$$

$$2A + 8Ax + 4B - 2Ax^2 - 2Bx - 2C = 2x^2 - 3x + 6$$

Comparing Coefficients

for x^2	for x	constant
$-2A = 2$	$8A - 2B = -3$	$2A + 4B - 2C = 6$
$A = -1$	$-8 + 3 = 2B$	$-2 - 10 - 2C = 6$
	$B = -5/2$	$C = -9$

$$y = Ax^2 + Bx + C$$

$$y_p = -x^2 - \frac{5}{2}x - 9$$

$$y = y_c + y_p$$

$$y = c_1 e^{(-2+\sqrt{6})x} + c_2 e^{(-2-\sqrt{6})x} - x^2 - \frac{5}{2}x - 9$$

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$$y'' - y' + y = 2\sin 3x$$

Trignometric values → For y_p

$$y = A\cos 3x + B\sin 3x$$

Angle must be same

For y_c

$$m^2 - m + 1 = 0$$

Roots are

$$m = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

Roots are imagenery

$$y_c = e^{x/2}\left[C_1\cos\left(\frac{\sqrt{3}}{2}\right)x + C_2\sin\left(\frac{\sqrt{3}}{2}\right)x\right]$$

For y_p

$$y = A\cos 3x + B\sin 3x$$

$$y' = -3A\sin 3x + 3B\cos 3x$$

$$y'' = -9A\cos 3x - 9B\sin 3x$$

Put Values in Equation

$$y'' - y' + y = 2\sin 3x$$

$$-9A\cos 3x - 9B\sin 3x + 3A\sin 3x - 3B\cos 3x + A\cos 3x + B\sin 3x = 2\sin 3x$$

$$(-9A - 3B + A)\cos 3x + (-9B + 3A + B)\sin 3x = 2\sin 3x$$

Comparing coefficients

$$3(-8A - 3B = 0) \qquad , 8(-8B + 3A = 2)$$

$$-24A - 9B = 0 \qquad , 24A - 64B = 16$$

$$-24A - 9B = 0$$

$$24A - 64B = 16$$

$$-73B = 16$$

$$B = -16/73$$

$$-8A - 3B = 0$$

$$-8A = +3\left(\frac{-16}{73}\right)$$

$$A = \frac{48}{73(8)}$$

$$A = \frac{6}{73}$$

$$y_p = \frac{6}{73}\cos 3x - \frac{16}{73}\sin 3x$$

$$y = e^{x/2}\left[C_1\cos\left(\frac{\sqrt{3}}{2}\right)x + C_2\sin\left(\frac{\sqrt{3}}{2}\right)x\right] + \frac{6}{73}\cos 3x - \frac{16}{73}\sin 3x$$

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(a) $y'' - 5y' + 4y = 8 \cdot e^{x}$

$m^2 - 5m + 4 = 0$

$m_1 = 1$, $m_2 = 4$

Real & Distinct

$y_c = C_1 e^{x} + C_2 e^{4x}$

same

$f(x) = 8 \cdot e^{x}$

if same

For y_p → Exponential value must be same

$y = A \cdot x e^{x}$

(b) $y'' - 5y + 4y = 8 \cdot e^{2x}$

$m^2 - 5m + 4 = 0$

$m_1 = 1$, $m_2 = 4$

Real & Distinct

$y_c = c_1 e^{x} + c_2 e^{4x}$

Different Not same

$f(x) = 8 \cdot e^{2x}$

So, if Different

For y_p → Exponential value must be same

$y = A e^{2x}$

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$$y'' - 5y' + 4y = 8.e^{2x}$$

For y_c

$$m^2 - 5m + 4 = 0$$

$$m = 1, 4$$

Real & distinct

$$y_c = c_1 e^{x} + c_2 e^{4x}$$

For y_p

$$y = A.e^{2x}$$

$$y' = 2Ae^{2x}$$

$$y'' = 4A e^{2x}$$

Put values in equation

$$4Ae^{2x} - 10Ae^{2x} + 4A.e^{2x} = 8.e^{2x}$$

$$-2Ae^{2x} = 8e^{2x}$$

$$-2A = 8$$

$$A = -4$$

$$y_p = -4e^{2x}$$

$$y = y_c + y_p$$

$$y = c_1 e^{x} + c_2 e^{4x} - 4e^{2x}$$

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