Mon Tue Wed Thu Fri Sat Sun

Date:

Exact Differential Equations

$$\overbrace{2xy}^{\partial f/\partial x}\,dx + \overbrace{(x^2-1)}^{\partial f/\partial y}\,dy = 0$$

$M = 2xy$, $N = x^2 - 1$

If this Equation satisfies $\left[\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}\right]$, then it is an Exact Differential Equ.

Step # 1 : $\dfrac{\partial M}{\partial y} = 2x(1) = 2x$

$$\frac{\partial N}{\partial x} = \frac{d}{dx}(x^2-1) = 2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$2x = 2x$$

∴ This Equation holds the property of Exact Differential Equation

Step #2 :

$$M = 2xy$$

$$\frac{\partial f}{\partial x} = 2xy$$

$$\int \frac{d}{dx} f = \int 2xy\,dx$$

$$f = 2y \int x\,dx$$

$$f = 2y\left(\frac{x^2}{2} + c\right)$$

$$f = 2y\left(\frac{x^2}{2}\right) + c$$

$$f = x^2 y + c$$

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"C" is constant of another function $g(y)$

$$f = x^2y + g(y) \rightarrow \text{equ (A)}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2y) + g'(y)$$

$$\frac{\partial f}{\partial y} = x^2\frac{\partial}{\partial y}(y) + g'(y)$$

$$\frac{\partial f}{\partial y} = x^2(1) + g'(y)$$

$$\frac{\partial f}{\partial y} = x^2 + g'(y)$$

Step#3 : $\frac{\partial f}{\partial y} = x^2 + g'(y)$

$$x^2 - 1 = x^2 + g'(y)$$

$$g'(y) = -1$$

Step # 4: $\int (g'(y))dy = -\int 1 dy$

$$g(y) = -y \longrightarrow \text{equ (B)}$$

Put equ(B) in equ(A)

$$f = x^2y + g(y)$$

$$f = x^2y + g(y)$$

$$f(x,y) = x^2y - y$$

$$C = x^2y - y \qquad \because C = f(x,y)$$

$$C = y(x^2 - 1)$$

$$y = \frac{C}{x^2-1}$$

Ex : 2.4

Question # (11) (Exact Differential Equations)

$$\left(y\ln(y) - e^{-xy}\right)dx + \left(\frac{1}{y} + x\ln y\right)dy = 0$$

If this equation satisfies the property $\left[\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}\right]$, so It is an Exact Differential equation otherwise vice versa.

$$M = y\ln y - e^{-xy}$$

{'x' is uses as constant}

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left\{y\ln y - e^{-xy}\right]$$

$$\frac{\partial M}{\partial y} = \frac{d}{dy}(y.\ln y) - \frac{d}{dy}(e^{-xy})$$

$$\frac{\partial M}{\partial y} = \left[y.\frac{d}{dy}(\ln y) + (\ln y)\frac{d}{dy}(y)\right] - \frac{d}{dy}\left[e^{-xy}\right]$$

$$\frac{\partial M}{\partial y} = \left[\cancel{y}\left[\frac{1}{\cancel{y}}\right] + \ln y(1)\right] - e^{-xy}(-x)$$

$$\frac{\partial M}{\partial y} = 1 + \ln y - xe^{-xy}$$

$$N = \frac{1}{y} + x\ln y$$

$$\frac{\partial N}{\partial x} = \frac{d}{dx}\left[\frac{1}{y} + x\ln y\right]$$

$$\frac{\partial N}{\partial x} = \frac{d}{dx}\left(\frac{1}{y}\right) + \frac{d}{dx}(x\ln y)$$

$$\frac{\partial N}{\partial x} = 0 + \ln y.\frac{d}{dx}(x)$$

$$\frac{\partial N}{\partial x} = \ln y\,(1)$$

$$\frac{\partial N}{\partial x} = \ln(y)$$

As this equation is not satisfying the property $\left[\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}\right]$, so this equation is

not an Exact Differential Equation.

Here

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$ [Not an Exact Differential Equation]

Exercise # 2.4

Question(1): (Exact Differential Equations):

$(2x-1)dx + (3y+7)dy = 0$

if This equation satisfies $\left[\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}\right]$, then it is an Exact Differential Equation

$M = 2x-1$ $\qquad$ $N = 3y+7$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left[2x-1\right]$ $\qquad$ $\frac{\partial N}{\partial x} = \frac{d}{dx}\left[3y+7\right]$

$\frac{\partial M}{\partial y} = 0$ $\qquad$ $\frac{\partial N}{\partial x} = 0$

So, this equations holds the property $\left[\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}\right]$ so it is an Exact Differential Equation.

Solution:

$M = 2x-1$

$\frac{\partial f}{\partial x} = 2x-1$

Taking Integral on both sides

$\int \frac{\partial f}{\partial x} = \int (2x-1)\,\partial x$

$f = 2\int x\,dx - \int 1\,dx$

$f = \cancel{2}\left(\frac{x^2}{\cancel{2}}\right) - x + g(y)$

$f = x^2 - x + g(y) \longrightarrow equ\ 1$

$\frac{\partial f}{\partial y} = 0 - 0 + g'(y)$

$3x+7 = g'(y)$

$g'(y) = 3x+7$

$$g'(y) = 3x+7$$

Taking Integral on both sides

$$\int [g'(y)]\,dy = 3x\int 1dy + 7\int 1.dy$$

$$\int [g'(y)]\,dy = 3xy + 7y$$

$$g(y) = (3x+7)y \longrightarrow \text{equ } ②$$

Put equ ② in equ ①

$$f = x^2 - x + g(y)$$

$$f(x,y) = x^2 - x + (3x+7)y$$

$$\therefore C = f(x,y)$$

$$C = x^2 - x + (3x+7)y$$

$$\frac{C + x - x^2}{(3x+7)} = y$$

$$\boxed{y = \frac{C + x - x^2}{3x+7}}$$