Example 1. *A battery of e.m.f. 6 volts and internal resistance 5 ohm is joined in parallel with another of e.m.f. 10 volts and internal resistance 1 ohm and the combination sends a current through an external resistance 12 ohms. Calculate the current through each battery.*

Sol. Applying Kirchhoff's II law to the mesh *ABFEA* (Fig. 6.8),

$$I_1 \times 5 + (I_1 + I_2) \times 12 = 6$$

or $$17I_1 + 12I_2 = 6 \qquad ...(1)$$

Applying Kirchhoff's II law to the mesh *CDEFC*

$$I_2 \times 1 + (I_1 + I_2) \times 12 = 10$$

or $$12I_1 + 13I_2 = 10 \qquad ...(2)$$

Solving (1) and (2), we get

$$I_1 = -\frac{6}{11} A, \; I_2 = \frac{14}{11} A.$$

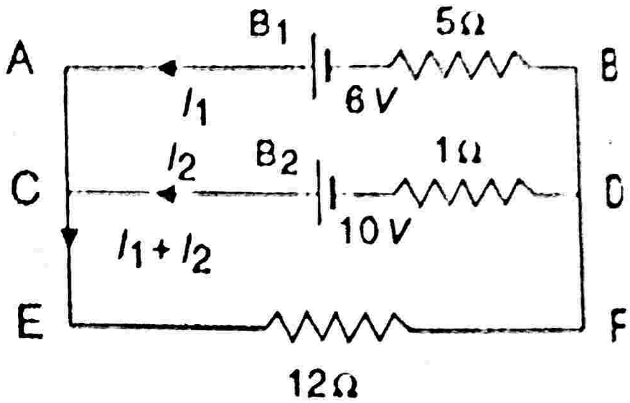

Fig. 6.8

Problem 1.3 *Two point charges of equal magnitude repel each other with a force of 1.5 N when separated by 0.04 m. Find the magnitude of the charge on each.*

Solution Here $|q_1| = |q_2| = q$, $F = 1.5$ N, $r = 0.04$ m

Then
$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

Thus,
$$q = \sqrt{\frac{Fr^2}{(1/4\pi\varepsilon_0)}} = \sqrt{\frac{(1.5)(0.04)^2}{(9\times 10^9)}}\ \text{C} = 5.16 \times 10^{-7}\ \text{C}$$

Problem 1.4 *The force of repulsion that two like charges exert on each other is 2.0 N. What will be the force if the distance between the charges is increased to six times its original value?*

Solution Here
$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 2.0\ \text{N}$$

If the distance is increased to $6r$, the force becomes
$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(6r)^2} = \frac{1}{36}F_1 = \frac{1}{36}(2.0\ \text{N}) = 5.6 \times 10^{-2}\ \text{N}$$

Problem 1.5 *An alpha particle consists of two protons and two neutrons bound together. What is the repulsive force between two alpha particles at a distance of* 10^{-15} m, *comparable to the sizes of nuclei?*

Solution An alpha particle has charge
$$q = 2e = 2(1.60 \times 10^{-19}\ \text{C}) = 3.20 \times 10^{-19}\ \text{C}$$

The repulsive force is:
$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = (9 \times 10^9)\frac{(3.20\times 10^{-19})^2}{(10^{-15})^2}\ \text{N} = 922\ \text{N}$$

Problem 1.6 *Two particles, with identical positive charges and a separation of* 0.02 m, *are released from rest. Immediately after the release, particle 1 has an acceleration* $\mathbf{a}_1$ *whose magnitude is* $2.5 \times 10^3\ \text{ms}^{-2}$; *while particle 2 has an acceleration* $\mathbf{a}_2$ *whose magnitude is* $4.2 \times 10^3\ \text{ms}^{-2}$. *Particle 1 has a mass of* 3.0×10^{-6} kg. *Find the mass of particle 2 and the charge on each particle.*

Solution Here $r = 0.02$ m, $m_1 = 3.0 \times 10^{-6}$ kg, $a_1 = 2.5 \times 10^3\ \text{ms}^{-2}$, $a_2 = 4.2 \times 10^3\ \text{ms}^{-2}$

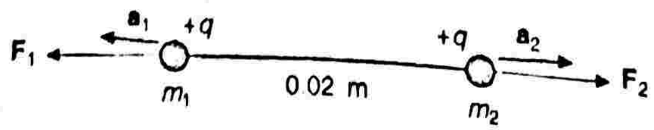

Figure 1.37

Magnitude of force on particle 1 = Magnitude of force on particle 2 = F (say)

Then $$F = m_2 a_2 = m_1 a_1 = (3.0 \times 10^{-6})(2.5 \times 10^{3})\ \text{N} = 7.5 \times 10^{-3}\ \text{N}$$

which gives

$$m_2 = \frac{7.5 \times 10^{-3}\ \text{N}}{a_2} = \frac{7.5 \times 10^{-3}}{4.2 \times 10^{3}}\ \text{kg} = 1.8 \times 10^{-6}\ \text{kg}$$

Also $$F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2}$$

or $$q = \sqrt{\frac{Fr^2}{(1/4\pi\varepsilon_0)}} = \sqrt{\frac{(7.5 \times 10^{-3})(0.02)^2}{(9 \times 10^{9})}}\ \text{C} = 1.83 \times 10^{-8}\ \text{C}$$

Thus, $q = \sqrt{(1/4\pi\varepsilon_0)} \ldots \sqrt{9 \times 10^{\ldots}}$

Problem 1.11 *A charge of 2×10^{-9} C and a charge of -4×10^{-9} C are separated by a distance of 0.1 m. Find the position at which a third charge of 6×10^{-9} C can be placed so that the net electrostatic force on it is zero.*

Solution

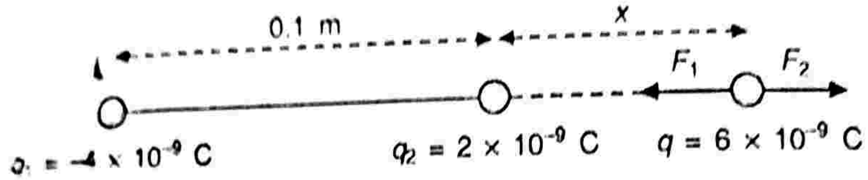

Figure 1.42

Suppose the third charge q is placed at a distance x from q_2, on the opposite side of q_1 (Figure 1.42).

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q|}{r_1^2} = (9 \times 10^9)\frac{(4\times10^{-9})(6\times10^{-9})}{(x+0.1)^2}\ \text{N}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_2||q|}{r_2^2} = (9 \times 10^9)\frac{(2\times10^{-9})(6\times10^{-9})}{x^2}\ \text{N}$$

Since $F_1 = F_2$, we have

$$(9 \times 10^9)\frac{(4\times10^{-9})(6\times10^{-9})}{(x+0.1)^2} = (9 \times 10^9)\frac{(2\times10^{-9})(6\times10^{-9})}{x^2}$$

or

$$\frac{2}{(x+0.1)^2} = \frac{1}{x^2}$$

or

$$x^2 - (0.2)x - 0.01 = 0$$

or

$$x = \frac{0.2 \pm \sqrt{(0.2)^2 - 4(-0.01)}}{2}\ \text{m} = 0.1\ \text{m} \pm 0.14\ \text{m}$$

The acceptable value of x is 0.24 m.

Note: The other value of x, namely −0.04 m, corresponds to the case when the charges q_1 and q_2 are of the same sign.

Problem 1.12 *A charge of 2.0 μC is located at a place where there is an electric field that points due east and has a magnitude of 2000 NC⁻¹. What are the magnitude and direction of the force acting on the charge?*

Solution The force on the charge is given by

$$\mathbf{F} = q\mathbf{E}$$

Here $q = 2.0\ \mu C = 2.0 \times 10^{-6}\ C$, $E = 2000\ NC^{-1}$

The force **F** has magnitude $(2.0 \times 10^{-6}\ C)(2000\ NC^{-1}) = 4.0 \times 10^{-3}\ N$, and is directed towards east.

Problem 1.13 *Find the magnitude and direction of the electric field at a distance of 0.02 m from a proton.*

Solution Here $q = +1.6 \times 10^{-19} C$, $r = 0.02$ m

The electric field is directed radially outward from the proton and has magnitude

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = (9 \times 10^9)\frac{1.6 \times 10^{-19}}{(0.02)^2}\ NC^{-1} = 3.6 \times 10^{-6}\ NC^{-1}$$

Problem 1.14 *An electric field with a magnitude of 220 NC⁻¹ exists at a spot that is 0.25 m away from a point charge. At a place that is 0.40 m away from this charge, what is the electric field strength?*

Solution The magnitude of the electric field at a distance r from a point charge q is:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Here $E_1 = 220\ NC^{-1}$, $r_1 = 0.25$ m, $r_2 = 0.40$ m, $E_2 = ?$

Then
$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1^2}, \quad E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2^2}$$

Thus,
$$\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}$$

or
$$E_2 = \frac{E_1 r_1^2}{r_2^2} = \frac{(220)(0.25)^2}{(0.40)^2}\ NC^{-1} = 85.94\ NC^{-1}$$

Problem 1.15 *A 2.0 μC point charge is placed in an external uniform electric field of 2.5 × 10³ NC⁻¹. At what distance from the charge is the net electric field zero?*

Solution The net electric field is zero at a distance r from the 2.0-μC charge where the electric field produced by the charge has a magnitude of $2.5 \times 10^3\ NC^{-1}$, and is directed opposite to the external field.

From $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, we have

$$r = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q}{E}} = \sqrt{(9 \times 10^9)\frac{2.0 \times 10^{-6}}{2.5 \times 10^3}}\ \text{m} = 2.68\ \text{m}$$

Problem 1.16 An electron is accelerated by using a constant electric field of magnitude 200 NC⁻¹. Determine the acceleration of the electron.

Solution Here $q = -1.6 \times 10^{-19}$ C, $E = 200$ NC^{-1}, $m = 9.1 \times 10^{-31}$ kg

The force **F** (= q**E**) on the electron is directed opposite to the field and has magnitude

$$F = (1.6 \times 10^{-19})(200)\ \text{N} = 3.2 \times 10^{-17}\ \text{N}$$

The acceleration **a** (= **F**/m) of the electron is directed opposite to the field and has magnitude

$$a = \frac{3.2 \times 10^{-17}}{9.1 \times 10^{-31}}\ \text{ms}^{-2} = 3.5 \times 10^{13}\ \text{ms}^{-2}$$

Problem 1.17 What is the strength of the electric field that would give a proton an acceleration equal to that of gravity?

Solution Here $q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg, $a = 9.80$ ms^{-2}

The strength of the electric field is given by

$$E = \frac{ma}{q} = \frac{(1.67 \times 10^{-27})(9.80)}{1.60 \times 10^{-19}}\ \text{NC}^{-1} = 1.02 \times 10^{-7}\ \text{NC}^{-1}$$

Problem 1.18 A small drop of water is suspended motionless in air by a uniform electric field that is directed upward and has a magnitude of 5820 NC^{-1}. The mass of the water drop is 2.40 × 10^{-9} kg. (a) Is the excess charge on the water drop positive or negative? (b) How many excess electrons or protons reside on the drop?

Solution Here $E = 5820$ NC^{-1}, $m = 2.40 \times 10^{-9}$ kg

The excess charge q on the water drop is positive.

Under the equilibrium condition, the upward-directed electrostatic force on the water drop balances the weight of the drop. That is,

$$qE = mg$$

or

$$q = \frac{mg}{E} = \frac{(2.40 \times 10^{-9}\ \text{kg})(9.80\ \text{ms}^{-2})}{5820\ \text{NC}^{-1}} = 4.04 \times 10^{-12}\ \text{C}$$

The excess number of protons on the drop $n = \frac{q}{e} = \frac{4.04 \times 10^{-12}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 2.53 \times 10^7$

Problem 1.19 A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A(0, 0, –15 cm) and B(0, 0, +15 cm), respectively. What are the total charge and electric dipole moment of the system?