Applied Physics for Engineers (PHY121)

Electrostatics

LECTURE # 10

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A Dipole in an External Electric field

Electric Dipole in an External Electric Field

An electric dipole consists of two charges +q and -q, equal in magnitude but opposite in sign, separated by a fixed distance d. q is the "charge on the dipole."

Earlier, we calculated the electric field along the perpendicular bisector of a dipole (this equation gives the magnitude only).

$$E = \frac{qd}{4\pi\varepsilon_o r^3}.$$

Caution! This is not the general expression for the electric field of a dipole!

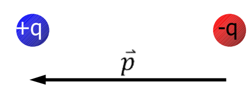
The electric field depends on the product qd. This is true in general.

q and d are parameters that characterize the dipole; we define the "dipole moment" of a dipole to be the vector

$$\vec{p} = q\vec{d},$$

caution: this p is not momentum!

where the direction of $\vec{p}$ (as well as $\vec{d}$) is from negative to positive (NOT away from +).



To help you remember the direction of p, this is on your equation sheet:

$$\boxed{\vec{p} = |q|d, \text{ from} - \text{to plus}}$$

A dipole in an uniform electric field experiences no net force, but probably experiences a torque.

To examine this behavior, we now consider such an abstract dipole in a uniform external electric field E , as shown in Fig. 1a. We assume that the dipole is a rigid structure that consists of two centers of opposite charge, each of magnitude q, separated by a distance d. The dipole moment $\vec{p}$ makes an angle θ with field E.

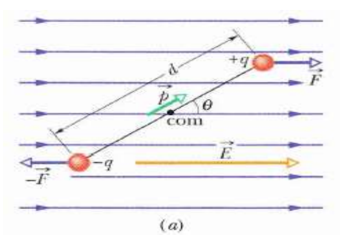
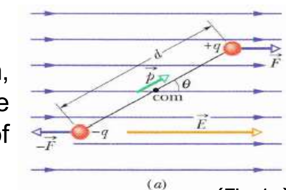


Fig.1a: An electric dipole in a uniform external electric field **E**. Two centers of equal but opposite charge are separated by distance *d*. The line between them represents their rigid connection.

- Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in Fig.1a) and with the same magnitude F = qE.
- Thus, because the field is uniform, the net force on the dipole from the field is zero and the center of mass of the dipole does not move.



(Fig.1a)

- However, the forces on the charged ends do produce a net torque τ on the dipole about its center of mass.
- The center of mass lies on the line connecting the charged ends, at some distance ‘x’ from one end and thus a distance ‘d – x’ from the other end.

From Eq. (τ = $rF\sin\theta$), we can write the magnitude of the net torque $\vec{\tau}$ as

$$\tau = Fx\sin\theta + F(d-x)\sin\theta = Fd\sin\theta \qquad \text{....(01)}$$

- we can also write the magnitude of $\vec{\tau}$ in terms of the magnitudes of the electric field E and the dipole moment p = qd.

To do so, we substitute qE for F and p/q for d in Eq. above, finding that the magnitude of τ is

$$\tau = pE\sin\theta \qquad \text{....(02)}$$

We can generalize this equation to vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \text{....(03)}$$

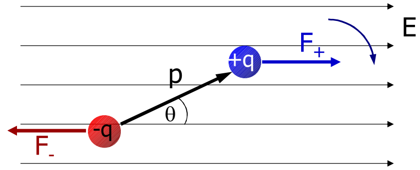
Vectors $\vec{p}$ and $\vec{E}$ are shown in Fig.1b. The torque acting on a dipole tends to rotate $\vec{p}$ (hence the dipole) into the direction of field $\vec{E}$, thereby reducing θ. In Fig.1, such rotation is clockwise.

Fig.1b: Field E causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗

As we discussed in previous chapter, we can represent a torque that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque. With that notation, the torque of Fig.1 is

$$\tau = -pE\sin\theta \qquad \text{....(04)}$$

Energy of an Electric Dipole in an External Electric Field



Potential energy can be associated with the orientation of an electric dipole in an electric field.

The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment $\vec{p}$ is lined up with the field $\vec{E}$ (then $\vec{\tau} = \vec{p} \times \vec{E} = 0$).

It has greater potential energy in all other orientations. Thus the dipole is like a pendulum, which has its least gravitational potential energy in its equilibrium orientation -- at its lowest point.

To rotate the dipole or the pendulum to any other orientation requires work by some external agent.

So, the expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ in Fig.1 is 90°.

If the dipole is free to rotate, the electric field does work to rotate the dipole.

We then can find the potential energy U of the dipole at any other value of θ with Eq. ($\Delta U = -W$) by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.

With the aid of Eq. ($W=\int \tau d\theta$) and eq.4, we find that the potential energy U at any angle θ is

$$U = -W = -\int_{90^o}^{\theta} \tau d\theta = \int_{90^o}^{\theta} pEsin\theta d\theta \qquad \text{....(05)}$$

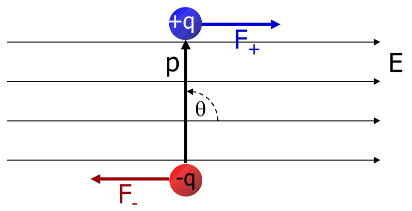
Evaluating the integral leads to

$$U = -pEcos\theta \qquad \text{....(06)}$$

We can generalize this equation to vector form as

$$U = -\vec{p} \, . \, \vec{E} \quad \text{(potential energy of a dipole)} \qquad \text{....(07)}$$

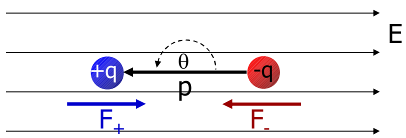
$$U_{dipole} = -pE\cos\theta$$



With this definition, U is zero* when $\theta=\pi/2$.

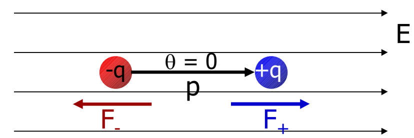
*Remember, zero potential energy does not mean minimum potential energy!

$$U_{dipole} = -pE\cos\theta$$



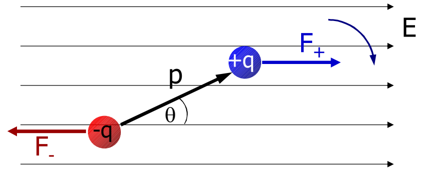
U is maximum when cosθ=-1, or $\theta=\pi$ (a point of unstable equilibrium*).

$$U_{dipole} = -pE\cos\theta$$



U is minimum when cosθ=+1, or $\theta=0$ (stable equilibrium*).

$$U_{dipole} = -pE\cos\theta$$



With this definition, U is zero when $\theta=\pi/2$.

U is maximum when $\cos\theta=-1$, or $\theta=\pi$ (a point of unstable equilibrium).

U is minimum when $\cos\theta=+1$, or $\theta=0$ (stable equilibrium).

It is "better" to express the dipole potential energy as

$$\boxed{U_{dipole} = -\vec{p}\cdot\vec{E}.}$$

Recall that the unit of energy is the joule, which is a N·m, but is not the same as the N·m of torque!

When a dipole rotates from an initial orientation θ_i to another orientatron θ_f, the work W done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i) \qquad \text{....(08)}$$

where U_f and U_i are calculated with Eq.7. If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,

$$W_a = -W = (U_f - U_i) \qquad \text{....(09)}$$

PROBLEM

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2 x 10^{-30} C.m.

(a) How far apart are the molecule's centers of positive and negative charge?

SOLUTION

There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)\,(d)$$

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ C.m}{(10)(1.6 \times 10^{-19}\ C)}$$

$$= 3.9 \times 10^{-12}\ m$$

$$= 3.9\ pm$$

(b) If the molecule is placed in an electric field of 1.5 x 10^4 N/C, what maximum torque can the field exert on it?

SOLUTION

The torque on a dipole is maximum when the angle θ between $\vec{p}$ and $\vec{E}$ is 90°. Therefore,

$$\tau = pEsin\theta$$

$$= (6.2 \times 10^{-30}\ C.m)(1.5 \times 10^4\ N/C)(sin 90^\circ)$$

$$= 9.3 \times 10^{-26}\ N.m$$

(c) How much work must an external agent do to rotate this molecule by 180º in this field, starting from its fully aligned position, for which θ=0?

SOLUTION

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

From eq.09, we find

$$W_a = U_{180^o} - U_o$$

$= (-pE\cos 180^o)\ (-pE\cos 0^o)$

$= 2pE = (2)\ (6.2 \times 10^{-30}\ C.m)(1.5 \times 10^{4}\ N/C)$

$= 1.9 \times 10^{-25}$ J