Department Of Computer Science, CUI Lahore Campus

CSC102 - Discrete Structures

By

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Lecture Outline

- Rules of Inference
  - What is an argument and argument form?
  - Rules of inference
  - Proof using rules of Inference
  - Fallacy

Rules of Inference

- Rules of inference are templates for constructing valid arguments.
- Rules of inference are basic tools for establishing the truth of statements.
- Valid means that the conclusion of the argument must follow from the truth of the preceding statements of the argument.

Argument

- An **Argument** is a sequence of propositions.

1. If you have the current password, then
you can log onto the network.”
2. “You have a current password.”
Therefore,
3. “You can log onto the network.”

Premises and Conclusion

1. If you have the current password, then you can log onto the network."
2. "You have a current password."

Premises

Therefore,

3. "You can log onto the network."

Conclusion

- All but the final proposition are called **premises.**
- The final proposition is called the **conclusion.**

Valid Arguments

1. If you have the current password, then you can log onto the network.”
2. “You have a current password.”

Premises true

Therefore,

3. “You can log onto the network.”

Conclusion true

- An argument is **valid** if the truth of all premises implies that conclusion is true.

Valid Arguments

- An **Argument form** in propositional logic is sequence of compound propositions involving propositions variables. An argument form is valid if no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.

Valid Arguments

1. If you have the current password, then you can log onto the network."
2. "You have a current password."

Therefore,

3. "You can log onto the network."

Valid Arguments (Argument Form)

1. If you have the current password, then you can log onto the network.”

2. “You have a current password.”

 Therefore,

3. “You can log onto the network.”

Argument form:

1. If p, then q
2. p

therefore

3. q

$p \to q$ Hypothesis 1

p Hypothesis 2

$\therefore q$ Conclusion

Modus Ponens

- The tautology $(p \wedge (p \rightarrow q)) \rightarrow q$ is basis of modus ponens.

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore\ q \end{array}$$

Modus Ponens

- The tautology $(p \wedge (p \rightarrow q)) \rightarrow q$ is basis of modus ponens.
- In particular, modus ponens tells us that if a conditional statement and the hypothesis of this conditional statement are both true, then the conclusion must also be true.

Modus Ponens Example

- Assume you are given the following two statements:
  - “you are in this class”
  - “if you are in this class, you will get a grade”

- Let p = “you are in this class”
- Let q = “you will get a grade”

Modus Ponens Example

- Assume you are given the following two statements:
  - “you are in this class”
  - “if you are in this class, you will get a grade”

- Let p = “you are in this class”
- Let q = “you will get a grade”

- By Modus Ponens, you can conclude that “you will get a grade”.

Modus Ponens Example

- Consider the following statements
  - “If it snows today, then we will stay at home”
  - “It is snowing today”

- Let p = “it is snowing today”
- Let q =“we will stay at home”

Modus Ponens Example

- Consider the following statements
  - “If it snows today, then we will stay at home”
  - “It is snowing today”

- Let p = “it is snowing today”
- Let q =“we will stay at home”

- By Modus Ponens, you can conclude that “We will stay at home”.

Modus Tollens

- Assume that we know: $\neg q$ and $p \rightarrow q$
  - Recall that $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- Thus, we know $\neg q$ and $\neg q \rightarrow \neg p$
- We can conclude $\neg p$

$$\begin{array}{rl} & \neg q \\ & \underline{p \rightarrow q} \\ \therefore & \neg p \end{array}$$

Modus Tollens Example

- Assume you are given the following two statements:
  - “you will not get a grade”
  - “if you are in this class, you will get a grade”

- Let p = “you are in this class”
- Let q = “you will get a grade”

Modus Tollens Example

- Assume you are given the following two statements:
  - “you will not get a grade”
  - “if you are in this class, you will get a grade”

- Let p = “you are in this class”
- Let q = “you will get a grade”

- By Modus Tollens, you can conclude that you are not in this class

Addition & Simplification

- Addition: If you know that p is true, then $p \vee q$ will ALWAYS be true i.e. $p \to p \vee q$

$$\frac{p}{\therefore\ p \vee q}$$

- Simplification: If $p \wedge q$ is true, then p will ALWAYS be true i.e. $p \wedge q \to p$

$$\frac{p \wedge q}{\therefore\ p}$$

Addition Example

- Assume you are given the following statements:
- “ It is below freezing now. Therefore, it is either below freezing or raining now”

- Let p = “It is below freezing now”.
- Let q = “It is raining now”.

- Then this argument is of the form

$$\begin{array}{rl} & p \\ \hline \therefore & p \vee q \end{array}$$

Simplification Example

- Assume you are given the following statements:
- " It is below freezing and raining now. Therefore, it is below freezing now"

- Let p = "It is below freezing now".
- Let q = "It is raining now".

- This argument is of the form

$$\begin{array}{rl} & p \wedge q \\ \hline \therefore & p \end{array}$$

Hypothetical Syllogism

- If $p \rightarrow q$ is true, and $q \rightarrow r$ is true, then $p \rightarrow r$ must be true

$$
\begin{array}{l}
p \rightarrow q \\
q \rightarrow r \\
\hline
\therefore p \rightarrow r
\end{array}
$$

Hypothetical Syllogism Example

- Assume you are given the following statements:
- If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

Hypothetical Syllogism Example

- Assume you are given the following statements:
- If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

- Let p = “It is raining today,”
- Let q = “We will not have a barbecue today,”
- Let r =“We will have a barbecue tomorrow.”

Hypothetical Syllogism Example

- Assume you are given the following statements:
- If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

- Let p = "It is raining today,"
- Let q = "We will not have a barbecue today,"
- Let r ="We will have a barbecue tomorrow."

- Then this argument is of the form

$$\begin{array}{r} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Exercise

- What rule of inference is used in each of these arguments?

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

   Addition

b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

   Simplification

c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

   Modus Ponens

Exercise

d) If it snows today, then university will close. The university is not closed today. Therefore, it did not snow today.

Modus Tollens

d) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

Hypothetical Syllogism

Steps for Proof

- Identify the atomic propositions and represent using propositional variable.
- Make the argument from
- Proof using rules of inference

Example Proof

- We have the hypotheses:
  - “It is not sunny this afternoon and it is colder than yesterday”
  - “We will go swimming only if it is sunny”
  - “If we do not go swimming, then we will take a canoe trip”
  - “If we take a canoe trip, then we will be home by sunset”
- Can it lead to the conclusion that “we will be home by sunset”?
- $((\neg s \wedge c) \wedge (m \to s) \wedge (\neg m \to t) \wedge (t \to h)) \to h$???
  - Where
    - s = “It is sunny this afternoon”; c = “it is colder than yesterday”
    - m = “We will go swimming”; t = “we will take a canoe trip”
    - h = “we will be home by sunset”

Example of Proof

$$\begin{array}{ll} H1: & \neg s \wedge c \\ H2: & m \rightarrow s \\ H3: & \neg m \rightarrow t \\ H4 & t \rightarrow h \\ \hline C: & \therefore h \end{array}$$

1. $\neg s \wedge c$ — 1st hypothesis
2. $\neg s$ — Simplification using step 1
3. $m \rightarrow s$ — 2nd hypothesis
4. $\neg m$ — Modus tollens using steps 2 & 3
5. $\neg m \rightarrow t$ — 3rd hypothesis
6. t — Modus ponens using steps 4 & 5
7. $t \rightarrow h$ — 4th hypothesis
8. h — Modus ponens using steps 6 & 7

Example Proof

- "If it does not rain or it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on"
  - $(\neg r \vee \neg f) \rightarrow (s \wedge d)$
- "If the sailing race is held, then the trophy will be awarded"
  - $s \rightarrow t$
- "The trophy was not awarded"
  - $\neg t$
- Can you conclude: "It rained"?

$$\begin{array}{ll} \text{H1:} & (\neg r \vee \neg f) \rightarrow (s \wedge d) \\ \text{H2:} & s \rightarrow t \\ \text{H3:} & \neg t \\ \hline \text{C:} & \therefore r \end{array}$$

Example of Proof

$$
\begin{array}{ll}
\text{H1:} & (\neg r \vee \neg f) \rightarrow (s \wedge d) \\
\text{H2:} & s \rightarrow t \\
\text{H3:} & \neg t \\
\hline
\text{C:} & \therefore r
\end{array}
$$

1.	$\neg t$	3rd hypothesis
2.	$s \rightarrow t$	2nd hypothesis
3.	$\neg s$	Modus tollens using steps 1 & 2
4.	$(\neg r \vee \neg f) \rightarrow (s \wedge d)$	1st hypothesis
5.	$\neg(s \wedge d) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of step 4
6.	$(\neg s \vee \neg d) \rightarrow (r \wedge f)$	DeMorgan's law and double negation law
7.	$\neg s \vee \neg d$	Addition from step 3
8.	$r \wedge f$	Modus ponens using steps 6 & 7
9.	r	Simplification using step 8

Fallacies

- Several common fallacies arise in incorrect arguments.
- The proposition $[(p \rightarrow q) \wedge q] \rightarrow p$ is not a tautology, because it is false when p is false and q is true.
- There are many incorrect arguments that treat this as a tautology
- This type of incorrect reasoning is called the **fallacy of affirming the conclusion**

Example

- Consider the following argument
- If you do the every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics

Therefore

You did every problem in this book

Let p = “you do the every problem in this book”

q = “you learned discrete mathematics”

$$\begin{array}{l} p \rightarrow q \\ q \\ \hline \therefore\ p \end{array}$$

Summary: Rules of Inference

Rules of Inference	Tautology	Name
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism

Summary: Rules of Inference

Rules of Inference	Tautology	Name
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Example Proof

- We have hypotheses

  "If you send me an e-mail message, then I will finish writing the program,"

  "If you do not send me an e-mail message, then I will go to sleep early,"

  "If I go to sleep early, then I will wake up feeling refreshed"

  The conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

- Let p = "You send me an e-mail message,"
- q = "I will finish writing the program,"
- r = "I will go to sleep early,"
- s = "I will wake up feeling refreshed."

$$\begin{array}{ll} \text{H1:} & p \rightarrow q \\ \text{H2:} & \neg p \rightarrow r \\ \text{H3:} & r \rightarrow s \\ \hline \text{C:} & \therefore \neg q \rightarrow s \end{array}$$

Example Proof

$$
\begin{array}{ll}
\text{H1:} & p \to q \\
\text{H2:} & \neg p \to r \\
\text{H3:} & r \to s \\
\hline
\text{C:} & \therefore \neg q \to s
\end{array}
$$

1. $p \to q$ — 1st Hypothesis
2. $\neg q \to \neg p$ — Contrapositive of 1
3. $\neg p \to r$ — 2nd Hypothesis
4. $\neg q \to r$ — Hypothetical syllogism using 2 and 3
5. $r \to s$ — 3rd Hypothesis
6. $\neg q \to s$ — Hypothetical syllogism using 4 and 5