Mon Tue Wed Thu Fri Sat Sun

CH # 12

Date: 08-12-2023

Partial Differential Equations
Separation of Variable Method.

Heat Equation:

Q #1:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

(∂x: distance / length; ∂t: time; heat)

$$u(x,t) = X(x)T(t)$$

$$\frac{\partial u}{\partial x} = X'(x)\cdot T(t)$$

$$\frac{\partial u}{\partial t} = X(x)\cdot T'(t)$$

put values in equation

$$X'(x).T(t) = 2.X(x).T'(t) + X(x).T(t)$$

$$X'T = 2XT' + XT$$

$$X'T - XT = 2XT'$$

$$T(X'-X) = 2XT'$$

$$\frac{X'-X}{X} = \frac{2T'}{T}$$

$$\frac{X'}{X} - \frac{X}{X} = \frac{2T'}{T}$$

$$\frac{X'}{X} - 1 = \frac{2T'}{T}$$

$$\int\frac{X'}{X} - \int 1 = 2\int\frac{T'}{T}$$

$$\ln X - x = 2\ln T$$

$$\ln X - X = 2\ln T$$

Let Both sides = "k"

$\ln x - x = k$	$2\ln T = k$
$\ln X = k + x$	$\ln T = k/2$
$e^{\ln x} = e^{k+x}$	$e^{\ln T} = e^{k/2}$
$X = ke^x$	$T = e^{k/2}$

As

$$u(x,t) = X(x).T(t)$$

$$u(x,t) = ke^x \cdot e^{k/2}$$