COMSATS University Islamabad

Terminal Examination – Semester FA 22

Course Title:	Statistics and Probability Theory			Course	MTH262	Credit Hours:	3(3,0)
Course Instructor/s:	Dr. Tajammal Hussain, Dr. Aamir Sana Ullah, Dr. Mian Muhammad Farooq, & Dr.Irfan Aslam			Program:	BCS & BSE		
Semester:	3rd & 4th	Batch:	BCS & BSE	Section:	A, B, C	Date:	Feb 01, 2023
Time Allowed:	02 Hour				Maximum Marks:	100	
Student's Name:	[redacted]				Reg.	CIIT/[redacted]	

Important Instructions / Guidelines:

- Attempt all questions.

Question 1: (15)

A candy company distributes boxes of chocolates with a mixture of creams, toffees, and nuts coated in both light and dark chocolate. For a randomly selected box, let X and Y, respectively, be the proportion of the light and dark chocolates that are creams and suppose that the joint distribution function is

$$f(x,y) = \begin{cases} \dfrac{2(2x+3y)}{5} & for\ 0 \le x \le 1, \\ & 0 \le y \le 1 \\ 0 & elsewhere \end{cases}$$

i) Prove that $f(x,y)$ is a valid joint probability density function.

ii) Find Expected values for each X and Y.

iii) Are X and Y independent to each other?

iv) Find P $(0 < X < \frac{1}{2} \ / Y = 0.5)$.

v) Find covariance between X and Y, and interpret it.

Question 2: (10)

A traffic control engineer reports that 75% of the vehicles passing through a check point are from within the state. Assume that 10 vehicles arrive at checkpoint independently. What is the probability that;

a) At most 2 vehicles are from within state?

b) At least 3 vehicles are from within state?

Question 3: (10)

An electric company currently experiences on the average 3 monthly line failures on 50 miles of supply line. Assuming that failure rate follows a Poisson distribution, then what is the probability that:

a) There would be exactly 3 failures on 50 miles of transmission line?

b) There would be at least 2 failures on 100 miles of transmission line?

Question 4: (10)

To avoid detection at customs, a traveler has placed 6 narcotic tablets in a bottle containing 2 vitamin pills that are similar in appearances. If the customs official selected 3 of the tablets at random for analysis, what is the probability that the traveler will be arrested for illegal possession of narcotics? Also, justify the use of hyper- geometric in this case.

Question 5: (15)

Oil drilling company ventures into various locations, and their success or failure is independent from one location to another. Suppose the probability of a successful oil exploration at a specific location is 0.35.

i. Write down the assumptions of probability distribution associated with the given statement.
ii. What is the probability that 5^{th} successful drilling is actually made on 12^{th} attempted drilling.
iii. What is the probability that for 3 successful drillings at-most 6 attempts are required?
iv. What is the probability that 1st successful drillings will occur on 7^{th} attempt?

Question 6: (10)

In a biomedical study with rats, a dose-response investigation is used to determine the effect of the dose of a toxicant on their survival time. The toxicant is one that is frequently discharged into the atmosphere from jet fuel. For a certain dose of the toxicant the study determines that the survival time, in weeks, has a Gamma distribution with $\alpha = 3, and\ \beta = 6$. What is the probability that a rat survives no longer than 50 weeks?

Question 7: (10)

The average lifetime of a certain type of small motor is 10 years with a standard deviation of 2 years. Assume that the lifetime of the motors follows a normal probability distribution. Then

i. What is the probability that a randomly selected motor will survive for more than 13 years?
ii. What is the probability that a randomly selected motor will survive from 8 to 12 years?
iii. If the manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 3% of the motors that fail, how long a guarantee should he offer?

Question 8: (10)

Suppose a production line operates with a mean filling weight of 16 ounces per container. Since over- or under-filling can be dangerous, a quality control inspector samples 30 items to determine whether or not the filling weight has to be adjusted. The sample revealed a mean of 16.32 ounces. From past data, the standard deviation is known to be 0.8 ounces. Using a 0.10 level of significance, can it be concluded that process mean not equal to 16 ounces?

Question 9: (10)

The thrust of an engine (y) is a function of exhaust temperature(x) in ^{0}F when other important variables are held constant. Consider the following data.

Temperature (x)	1760	1652	1485	1390	1820	1665	1550	1700	1270
Thrust of engine (y)	4300	4650	3200	3150	4950	4010	3810	4500	3008

i. Fit a simple linear regression line to predict thrust of an engine (y) for exhaust temperature (x).
ii. Estimate the thrust of an engine (y) at 1680 ^{0}F temperature.
iii. Calculate a measure about the goodness of fit ($R^{2)}$ for the fitted regression line, and interpret its result.