7.2 A RESISTOR IN AN AC CIRCUIT

- Figure 7.1 shows a circuit consisting of a resistor of resistance R and an ac emf source.

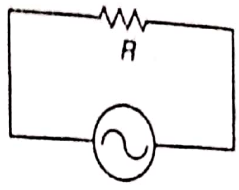

Figure 7.1 A resistor R connected to an ac source.

Suppose the ac voltage across the terminals of the source varies sinusoidally with time according as:

$$\boxed{V = V_m \sin(\omega t)} \quad (7.1)$$

where V_m is the *amplitude* (or *peak value*) of the alternating voltage, and ω is the angular frequency at which the voltage changes.

To find the instantaneous current i in the resistor, we apply Kirchhoff's loop rule to the circuit:

$$V - iR = 0$$

or

$$V_m \sin(\omega t) - iR = 0$$

or

$$i = \frac{V_m}{R} \sin(\omega t) \quad (7.2)$$

The maximum (or peak) current is given by

$$\boxed{I_m = \frac{V_m}{R}} \quad (7.3)$$

Equation (7.2) can be rewritten as:

$$\boxed{i = I_m \sin(\omega t)} \quad (7.4)$$

At any instant, the voltage across the resistor is:

$$\boxed{V_R = iR} \quad (7.5)$$

Using the expression for current given in Eq. (7.4), we obtain

$$V_R = iR = I_m R \sin(\omega t)$$

or

$$\boxed{V_R = V_m \sin(\omega t)} \quad (7.6)$$

Equations (7.4) and (7.6) indicate that both the current and voltage vary in time according to the function sin (ωt), reaching their maximum values at the same time, as shown in Figure 7.2. We say that

In an ac circuit, the current through a resistor is in phase with the voltage across its terminals.

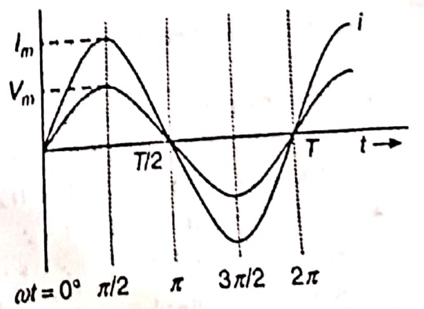

Figure 7.2 The voltage v and current I across a resistor are in phase.

EXAMPLE 7.1

A 20-Ω resistor is connected across an ac emf source having a frequency of 50 Hz and an emf amplitude of 12 V. Find (i) the amplitude of the current in the circuit, and (ii) the current in the circuit at 0.5 s.

Solution Here $R = 20\ \Omega$, $f = 50$ Hz, $V_m = 12$ V

The angular frequency is $\omega = 2\pi f = 314.16\ \text{rads}^{-1}$

(i) The amplitude of the current is:

$$I_m = \frac{V_m}{R} = \frac{12\ \text{V}}{20\ \Omega} = 0.6\ \text{A}$$

(ii) The current through the resistor is in phase with the voltage across it, and is given by

$$i(t) = I_m \sin(\omega t)$$

At time $t = 0.5$ s, we have

$$i(0.5\ \text{s}) = (0.6\ \text{A}) \sin(314.16\ \text{rads}^{-1} \times 0.5\ \text{s}) = 0.23\ \text{A}$$

7.3 ROOT MEAN SQUARE VALUE OF CURRENT AND VOLTAGE

- We consider the sinusoidal voltage and current given by

$$v = V_m \sin(\omega t) \tag{7.7}$$

$$i = I_m \sin(\omega t) \tag{7.8}$$

Both v and i are positive for one half of the cycle and negative in the other half, as shown in Figure 7.3. Consequently, the average value of v (or i) for a complete cycle (or any number of complete cycles) is zero.

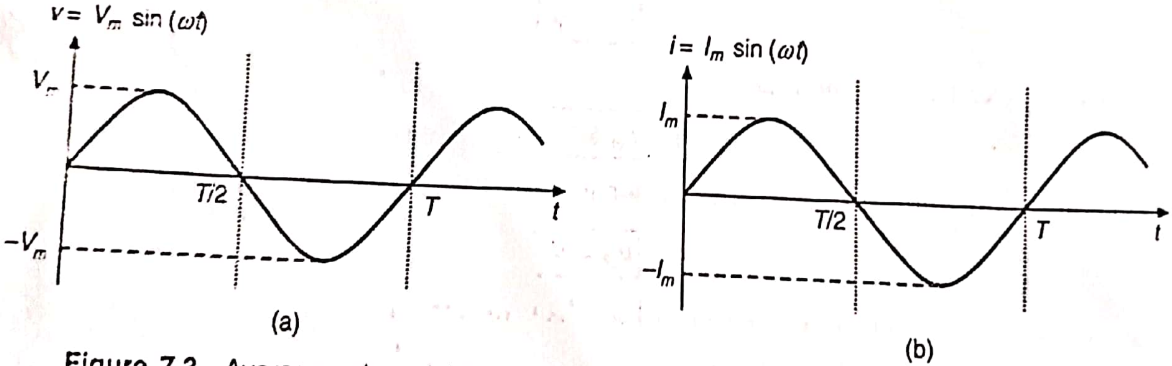

Figure 7.3 Average value of (a) voltage, (b) current, over a complete cycle is zero.

- However, the direction of the current has nothing to do with its heating effect on the resistor in a circuit. Because what matters is the collision between electrons and the fixed atoms of the resistor that results in an increase in the temperature of the resistor.
- To express it quantitatively, the power delivered (*Joule heating*) to a resistor, $p = i^2R$, is proportional to the square of the current. So, it makes no difference whether the current is direct or alternating. As the current varies sinusoidally with frequency f, the power rises to a maximum and falls to zero with a frequency $2f$. We can always define a quantity called the **average power over a cycle** which is not zero.

- Since $p \propto i^2$, the average power is proportional to the average value of the square of the current. The square root of the average value of the square of the current is called the **root mean square (rms) current.**

The rms value of a current is equal to the magnitude of a steady direct current that would produce the same heating effect.

It is the rms current which is of importance for calculating average power in an ac circuit.

To find the value of I_{rms} in terms of I_m

Squaring the expression $i = I_m \sin(\omega t)$, we have

$$i^2 = I_m^2 \sin^2(\omega t)$$

The average value of i^2 over a complete cycle is:

$$<i^2> = <I_m^2 \sin^2(\omega t)> = I_m^2 <\sin^2(\omega t)> \qquad (7.9)$$

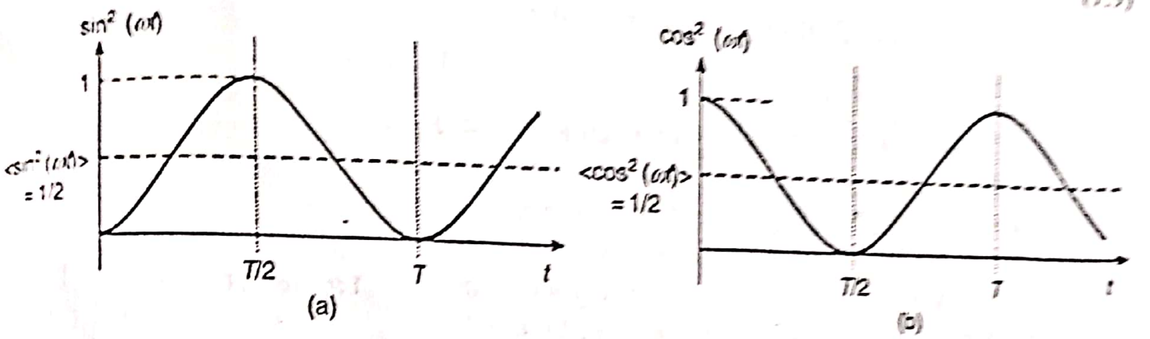

Figure 7.4 Average of (a) $\sin^2(\omega t)$, (b) $\cos^2(\omega t)$, is 1/2.

We know that the time average of $\sin^2(\omega t)$ is equal to the time average of $\cos^2(\omega t)$ when taken over a complete cycle:

$$<\sin^2(\omega t)> = <\cos^2(\omega t)>$$

Using this relation in the trigonometric identity $\sin^2(\omega t) + \cos^2(\omega t) = 1$, we obtain

$$<\sin^2(\omega t)> + <\cos^2(\omega t)> = 2<\sin^2(\omega t)> = 1$$

or

$$<\sin^2(\omega t)> = \frac{1}{2}$$

Eq. (7.9) can then be written as:

$$<i^2> = \frac{I_m^2}{2}$$

From definition, $I_{rms} = \sqrt{<i^2>}$, so that

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m} \qquad (7.10)$$

Equation (7.10) says that an alternating current i whose maximum value is I_m will produce the same heating effect in a resistor as a direct current of $I_{rms} = I_m/\sqrt{2}$. Figure 7.5 shows the level of I_{rms} in a plot of the current i in a resistor versus time.

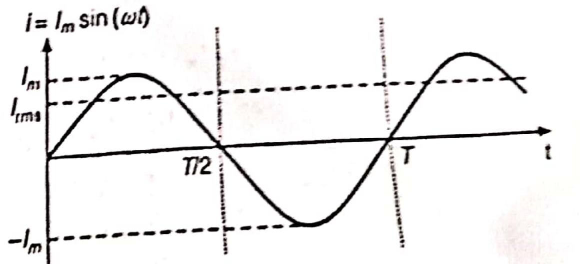

Figure 7.5 The rms current I_{rms} is related to the peak current I_m by $I_{rms} = I_m/\sqrt{2} = 0.707 I_m$

EXAMPLE 7.2

A 40 Ω resistor is connected across an ac emf source having emf amplitude of 120 V. Find the rms voltage across the resistor and the average power dissipated in it.

Solution Here $R = 40\ \Omega$, $V_m = 120$ V

The rms voltage across the resistor is:

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{120\text{ V}}{\sqrt{2}} = 87.85\text{ V}$$

The average power dissipated in the resistor is:

$$\bar{P}_R = I_{rms}^2 R = \frac{V_{rms}^2}{R} = \frac{(84.85\text{ V})^2}{40\ \Omega} = 180\text{ W}$$

7.4 PHASOR DIAGRAMS

- The phase relationship between the current and voltage in an ac circuit can be easily determined by using a technique which involves *phasors.*
- In this technique, voltage and current that varies sinusoidally with time are represented by rotating vectors **V** and **I** called **phasors,** whose lengths represent the maximum voltage V_m and maximum current I_m. These phasors rotate anticlockwise in a Cartesian plane with angular frequency ω.
- The diagrams containing phasors are called **phasor diagrams.**
- Figure 7.8(a) shows the phasor diagram for an ac resistor circuit.
  - ♦ The phasors **V** and **I** are collinear as they rotate because the voltage and current are in phase.
  - ♦ At time t, the phasors make an angle ωt with the positive x-axis.
  - ♦ The projections of the phasors **V** and **I** on the y-axis give the instantaneous voltage v and current i respectively at a given time.
  - ♦ As the angle ωt increases with time t, the phasors rotate in anticlockwise direction, and the curves $v = V_m \sin(\omega t)$ and $i = I_m \sin(\omega t)$, shown in Figure 7.8(b), are traced out by the projections of the phasors **V** and **I** on the y-axis.

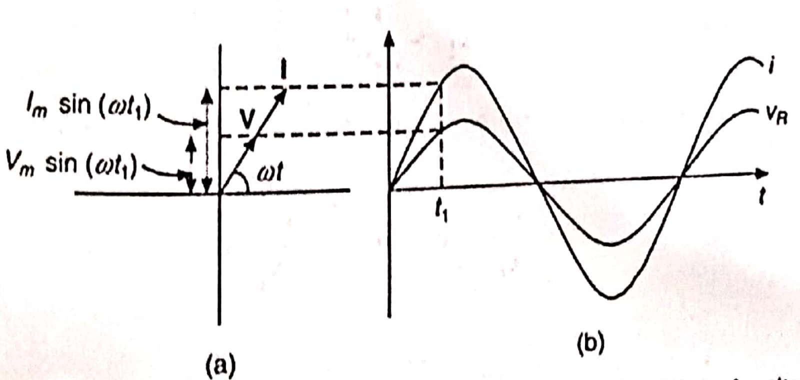

Figure 7.8 (a) Phasor diagram for a purely resistive ac circuit. (b) Plot of voltage v_R and current i versus time can be generated from the phasor diagram. The instantaneous voltage $V_m \sin(\omega t_1)$ and current $I_m \sin(\omega t_1)$ at a particular time t_1 are shown.

7.5 A CAPACITOR IN AN AC CIRCUIT

We consider an ac circuit consisting of a capacitor of capacitance C and an ac source, as shown in Figure 7.9.

Suppose the instantaneous voltage across the source is:

$$v = V_m \sin(\omega t) \tag{7.15}$$

If q be the charge on the capacitor at any time t, then the instantaneous voltage across the capacitor is:

$$v_C = \frac{q}{C} \tag{7.16}$$

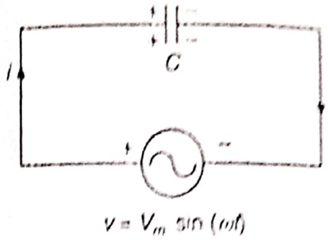

Figure 7.9 A capacitor connected to an ac source.

Applying loop rule to the circuit, we have

$$v - v_C = 0$$

Thus,

$$\boxed{v_C = V_m \sin(\omega t)} \tag{7.17}$$

and

$$q = Cv_C = CV_m \sin(\omega t) \tag{7.18}$$

The instantaneous current in the circuit is equal to the rate of change of the capacitor charge q:

$$i = \frac{dq}{dt} = \omega CV_m \cos(\omega t) = \omega CV_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

or

$$\boxed{i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)} \tag{7.19}$$

where $I_m = \omega CV_m$ is the amplitude of the oscillating current.

Capacitive reactance

Rewriting the expression $I_m = \omega CV_m$ as:

$$I_m = \frac{V_m}{(1/\omega C)}$$

We can now define a quantity X_C, called the **capacitive reactance** of the capacitor, as:

$$\boxed{X_C = \frac{1}{\omega C}} \tag{7.20}$$

so that the amplitude of the current in a purely capacitive circuit can now be written in the same form as that for a resistor:

$$\boxed{I_m = \frac{V_m}{X_C} \quad \text{or} \quad I_{rms} = \frac{V_{rms}}{X_C}} \tag{7.21}$$

The capacitive reactance X_C

- like resistance, is measured in ohms.
- limits the amplitude of the alternating current in a manner similar to that of a resistance in a dc circuit.

- is inversely proportional to both the capacitance C and the angular frequency ω. Figure 7.10 shows a plot of X_C versus ω, for a fixed value of the capacitance C.
- behaves differently (Figure 7.10) in an ac circuit as compared with resistance (Figure 7.6).
- approaches zero as the angular frequency ω becomes very large, indicating that a capacitor offers little opposition to the ac current.
- becomes infinitely large as ω approaches zero (i.e., direct current), and a capacitor provides so much opposition to the motion of charges that there is no current.

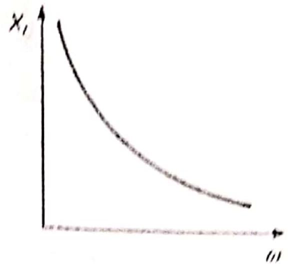

Figure 7.10 *Capacitive reactance is inversely proportional to the angular frequency ω.*

Phasor diagram

Comparing Eq. (7.17) with Eq. (7.19), we note that

In an ac circuit, the current through a capacitor leads the voltage across its terminals by $\pi/2$.

This result is represented by the phasor diagram of Figure 7.11(a), which shows that the phasors **V** and **I** remain perpendicular while rotating in the same direction.

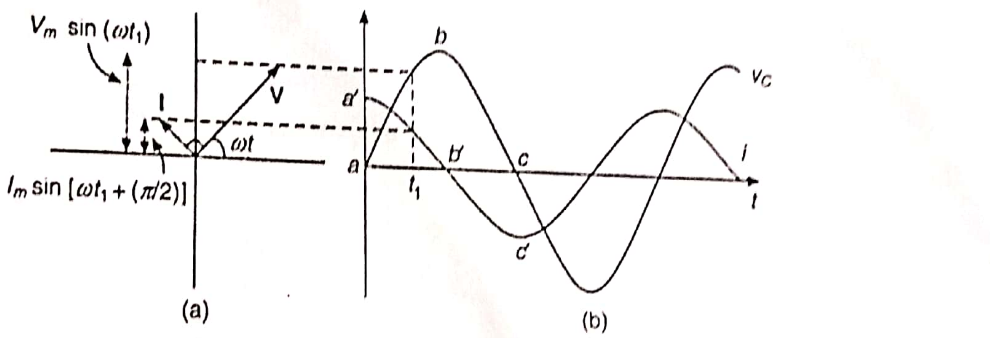

Figure 7.11 (a) Phasor diagram for a purely capacitive circuit. (b) The current i leads the voltage v_C by $\pi/2$.

Figure 7.11(b) shows graphs of the ac voltage and current versus time for a circuit that contains only a capacitor. As the voltage increases from a to b, the charge on the capacitor increases and reaches its full value at b. However, the current, representing the rate of flow of charge, has a maximum positive value at the start of the charging process at a'. It is a maximum because there is no charge on the capacitor at the start and hence no capacitor voltage to oppose the source voltage. When the capacitor is fully charged at b, the capacitor voltage has a magnitude equal to that of the source and completely opposes the source voltage. The result is that the current decreases to zero at b'. While the capacitor voltage decreases from b to c, the charges flow out of the capacitor in a direction opposite to that of the charging current, as indicated by the negative current from b' to c'. Thus, voltage and current are one-quarter wave cycle out of step.

Power delivered to a capacitor

- The fact that the current and voltage for a capacitor are $\pi/2$ out of phase has an important consequence in the calculation of electric power, defined as the product of the two. For the time interval between points a and b (or a' and b') in Figure 7.11(b), both current and voltage are positive. Therefore, the power is also positive, meaning that the source is delivering energy to the capacitor during this period. However, for the period between b and c (or b' and c'), the current is negative while the voltage remains positive, and hence the power is negative. This means that the capacitor is returning energy to the source during this period. Thus, the power alternates between positive and negative values for equal periods of time. In other words, *the capacitor alternately absorbs and releases energy*. As a result, the average power supplied by the source is zero, and a capacitor does not dissipate energy in an ac circuit. We can show this result quantitatively as follows:

The instantaneous power supplied to the capacitor is:

$$p_C = vi = [V_m \sin(\omega t)] \cdot [I_m \cos(\omega t)] = \frac{V_m I_m}{2} \sin(2\omega t)$$

so that the average power supplied in one complete cycle is:

$$\bar{P}_C = <\frac{V_m I_m}{2} \sin(2\omega t)> = \frac{V_m I_m}{2} <\sin(2\omega t)>$$

or

$$\boxed{\bar{P}_C = 0} \qquad (7.22)$$

since the time average of sin $(2\omega t)$ over a complete cycle is zero.

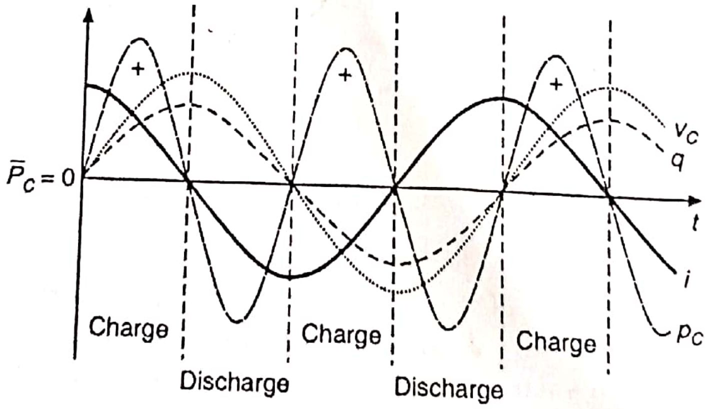

Figure 7.12 Average of the power p_C delivered to a capacitor over a complete cycle is $\bar{P}_C = 0$.

EXAMPLE 7.3

A 12-μF capacitor is connected across an ac emf source having a frequency of 15 Hz and an emf amplitude of 25 V. Find the current in the circuit at 0.24 s.

Solution Here, $C = 12 \times 10^{-6}$ F, $f = 15$ Hz, $V_m = 25$ V

The angular frequency is $\omega = 2\pi f = 97.25$ rads^{-1}

The capacitive reactance is:

$$X_C = \frac{1}{\omega C} = \frac{1}{(94.25 \text{ rads}^{-1})(12 \times 10^{-6} \text{ F})} = 8.84 \times 10^2\ \Omega$$

The amplitude of the current in the circuit is:

$$I_m = \frac{V_m}{X_C} = \frac{25\text{ V}}{8.84 \times 10^2\ \Omega} = 0.028\text{ A}$$

The current through the capacitor leads the voltage across it by $\pi/2$, and is given by

$$i(t) = I_m \sin(\omega t + \pi/2)$$

At time $t = 0.24$ s, we have

$$i(0.24\text{ s}) = (0.028\text{ A}) \sin(97.25\text{ rad s}^{-1} \times 0.24\text{ s}) = 0.011\text{ A}$$

7.6 AN INDUCTOR IN AN AC CIRCUIT

We consider an ac circuit consisting only of an inductor of inductance L connected to an ac source, as shown in Figure 7.13. Suppose the ac voltage across the source is:

$$V = V_m \sin(\omega t) \tag{7.23}$$

The changing current output of the ac source produces a back emf in the coil of magnitude

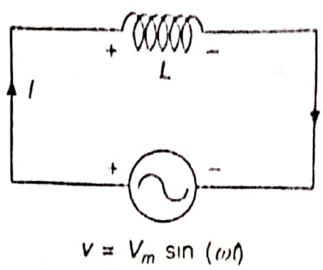

Figure 7.13 An inductor connected to an ac source.

$$V_L = L\frac{di}{dt} \tag{7.24}$$

Applying Kirchhoff's loop rule to the circuit, we have

$$V - V_L = 0$$

Thus,

$$\boxed{V_L = V_m \sin(\omega t)} \tag{7.25}$$

or

$$L\frac{di}{dt} = V_m \sin(\omega t)$$

or

$$\frac{di}{dt} = \frac{V_m}{L}\sin(\omega t)$$

Integrating both sides with respect to time,

$$\int \frac{di}{dt}dt = \int \frac{V_m}{L}\sin(\omega t)\,dt$$

We obtain

$$i = -\frac{V_m}{\omega L}\cos(\omega t) + C$$

where C is an integration constant. If $i = 0$ at $\omega t = \pi/2$, then $C = 0$, so that

$$i = -\frac{V_m}{\omega L}\cos(\omega t)$$

or

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \tag{7.26}$$

where the amplitude of the oscillating current is written as $I_m = V_m/\omega L$.

Inductive reactance

From the expression $I_m = V_m/\omega L$, we see that the quantity ωL plays the same role of resistance and is called the **inductive reactance** of the inductor, denoted by X_L, that is,

$$\boxed{X_L = \omega L} \tag{7.27}$$

so that the amplitude of the current in a purely inductive circuit can now be written in the same form as that for a resistor:

$$\boxed{I_m = \frac{V_m}{X_L} \quad \text{or} \quad I_{rms} = \frac{V_{rms}}{X_L}} \tag{7.28}$$

The inductive reactance X_L

- like resistance, is measured in ohms.
- limits the amplitude of the alternating current in a manner similar to that of a resistance in a dc circuit.
- is directly proportional to both the inductance L and the angular frequency ω. Figure 7.14 shows the variation of X_L with frequency ω for a fixed value of L.
- unlike resistance, decreases linearly as the frequency decreases.
- becomes zero as ω approaches zero (i.e., direct current), signifying that an inductor does not oppose direct current at all.

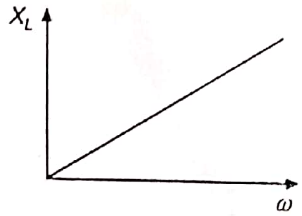

Figure 7.14 Inductive reactance is directly proportional to the angular frequency ω.

Phasor diagram

Comparing Eq. (7.25) with Eq. (7.26), we note that

In an ac circuit, the current through an inductor lags the voltage across its terminals by $\pi/2$.

This result is represented by the phasor diagram of Figure 7.15(a). The instantaneous voltage and instantaneous current across the inductor as functions of time are shown in Figure 7.15(b). At a maximum or minimum on the current graph, the current does not change with time, so the

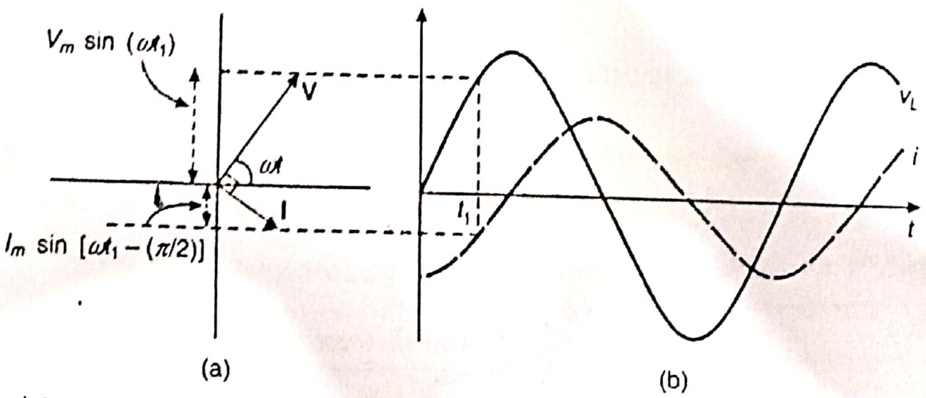

Figure 7.15 (a) Phasor diagram for a purely inductive circuit. (b) The current i lags the voltage v_L by $\pi/2$.

voltage generated by the inductor to oppose a change in the current is zero. At the points on the current graph where the current is zero, the graph is at its steepest, and the current has the largest rate of increase or decrease. Correspondingly, the voltage generated by the inductor to oppose a change in the current has the largest positive or negative value. Thus, voltage and current are not in phase, but are one-quarter of a wave cycle out of phase.

Power delivered to an inductor

- The fact that the current and voltage for an inductor are $\pi/2$ out of phase leads to the same result of zero average power as discussed in the case of capacitor. The power again alternates between positive and negative values for equal periods of time. Consequently, on the average, the power is zero and an inductor uses no energy in an ac circuit. The result is quantitatively shown as follows:

The instantaneous power supplied to the inductor is:

$$p_L = vi = [V_m \sin(\omega t)] \cdot [-I_m \cos(\omega t)] = -\frac{V_m I_m}{2} \sin(2\omega t)$$

so that, the average power over one complete cycle is:

$$\bar{P}_L = <-\frac{V_m I_m}{2} \sin(2\omega t)> = -\frac{V_m I_m}{2} <\sin(2\omega t)>$$

or

$$\boxed{\bar{P}_L = 0} \quad (7.29)$$

Since the time average of sin $(2\omega t)$ over a complete cycle is zero.

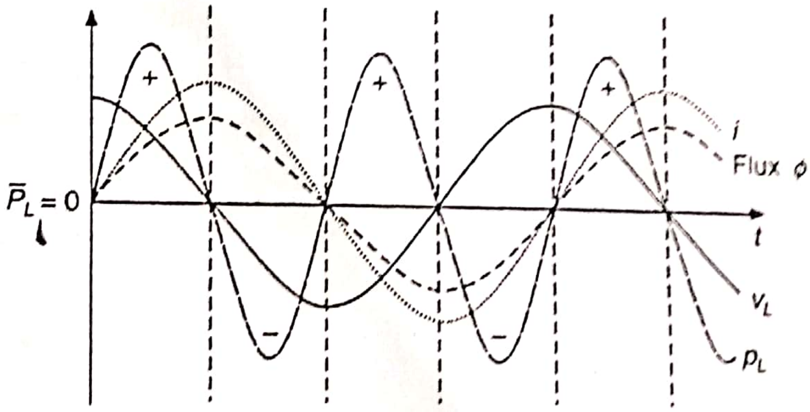

Figure 7.16 Average power delivered to an inductor over a complete cycle is zero.

- The relationships between the current and voltage for ac circuits containing only a resistor, inductor, or capacitor are given in Table 7.1.

Table 7.1 Relationships between Current and Voltage for ac Circuits

Instantaneous values	*Maximum values*	*rms values*	*Phase relations*
$v_R = iR$	$(V_R)_m = I_m R$	$(V_R)_{rms} = I_{rms} R$	v and i are in phase
$v_C = iX_C$	$(V_C)_m = I_m X_C$	$(V_C)_{rms} = I_{rms} X_C$	i leads v by $\pi/2$
$v_L = iX_L$	$(V_L)_m = I_m X_L$	$(V_L)_{rms} = I_{rms} X_L$	i lags v by $\pi/2$

EXAMPLE 7.4

A sinusoidal emf soukce $v = (20\ V) \sin [(32\pi\ rads^{-1})t]$ is connected across a 0.14-H inductor. Find the current in the circuit at 0.45 s.

Solution Here $L = 0.14$ H, $\omega = 32\pi\ rads^{-1}$, $V_m = 20$ V

The angular frequency is $\omega = 2\pi f = 97.25\ rads^{-1}$

The inductive reactance is:

$$X_L = \omega L = (32\pi\ rads^{-1})(0.14\ H) = 14.07\ \Omega$$

The amplitude of the current in the circuit is:

$$I_m = \frac{V_m}{X_L} = \frac{20\ V}{14.07\ \Omega} = 1.421\ A$$

The current through the capacitor leads the voltage across it by $\pi/2$, and is given by

$$i(t) = I_m \sin(\omega t + \pi/2)$$

At time $t = 0.45$ s, we have

$$i\ (0.45\ s) = (1.421\ A) \sin (32\pi\ rads^{-1} \times 0.45\ s) = 1.009\ A$$

7.7 LCR SERIES CIRCUIT

RL: circuit

We consider a circuit containing a resistor of resistance R, an inductor of inductance L, and a capacitor of capacitance C connected in series across an ac source, as shown in Figure 7.17.

We suppose that the source voltage is:

$$v = V_m \sin(\omega t) \qquad (7.30)$$

Our aim is to determine the instantaneous current i in the circuit and its phase relationship to the applied ac voltage. We will do this by using following two methods:

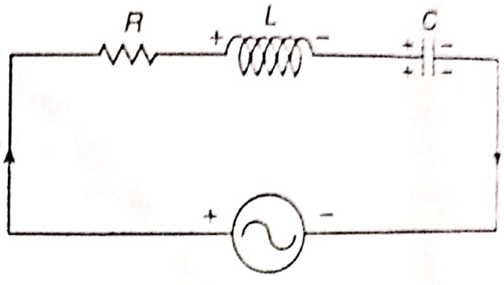

Figure 7.17 An *LCR* series circuit connected to an ac source.

7.7.1 Phasor-diagram Solution

The instantaneous current has the same value at all points of the circuit. We assume it to vary sinusoidally with time as:

$$i = I_m \sin(\omega t + \phi) \qquad (7.31)$$

where ϕ is the phase difference between the voltage across the source and the current in the circuit.

We will now determine the maximum current I_m and phase angle ϕ using phasor diagram of the circuit.

The instantaneous voltages across the three elements have the following phase relations to the instantaneous current:

(i) The instantaneous voltage across the resistor v_R is in phase with the instantaneous current i.

(ii) The instantaneous voltage across the inductor v_L leads the current i by $\pi/2$.
(iii) The instantaneous voltage across the capacitor v_C lags the current i by $\pi/2$.

Applying Kirchhoff's loop rule to the circuit, we get

$$v - v_R - v_L - v_C = 0$$

or

$$v = v_R + v_L + v_C \tag{7.32}$$

The phasor diagram for the circuit is shown in Figure 7.18(a). The current i in the circuit is represented by the current phasor **I**, of length equal to the maximum value of the current I_m. Its projection on the vertical axis, at any instant, gives the instantaneous current i. The voltage phasor $\mathbf{V}_R$, in phase with the current phasor **I**, represents the voltage across the resistor. The projection of $\mathbf{V}_R$ at any instant on the vertical axis equals v_R, with its maximum value given by

$$(V_R)_m = I_m R$$

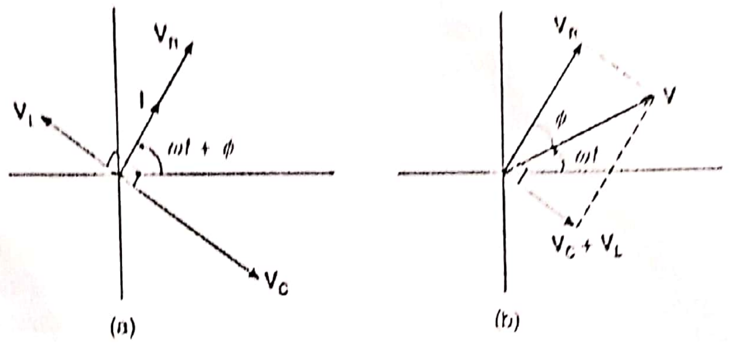

Figure 7.18 (a) Phasor diagram for an *LCR* series circuit. (b) Phasors $\mathbf{V}_R$, $\mathbf{V}_L$ and $\mathbf{V}_C$ are added vectorially to get the phasor **V**.

The phasor $\mathbf{V}_L$, leading the current phasor **I** by $\pi/2$, represents the voltage across the inductor. Its projection on the vertical axis at any instant equals v_L, with its maximum value given by

$$(V_L)_m = I_m X_L$$

The phasor $\mathbf{V}_C$, lagging the current phasor **I** by $\pi/2$, represents the voltage across the capacitor. Its projection on the vertical axis at any instant equals v_C, with its maximum value given by

$$(V_C)_m = I_m X_C$$

We now add the voltage phasors $\mathbf{V}_R$, $\mathbf{V}_L$ and $\mathbf{V}_C$ vectorially, as shown in Figure 7.18(b), to get the phasor **V** representing the net voltage across the circuit. The right triangle in this figure gives the following equations for the maximum value of the net voltage and the phase angle:

$$\boxed{V_m = \sqrt{(V_R)_m^2 + [(V_C)_m - (V_L)_m]^2}} \tag{7.33}$$

$$\boxed{\tan\phi = \frac{(V_C)_m - (V_L)_m}{(V_R)_m}} \tag{7.34}$$

We can express Eq. (7.33) in the form of Ohm's law using the relations $(V_R)_m = I_m R$, $(V_L)_m = I_m X_L$, and $(V_C)_m = I_m X_C$ as:

$$V_m = I_m \sqrt{R^2 + (X_C - X_L)^2} \tag{7.35}$$

It is convenient to define a parameter called the **impedance** Z of the circuit as:

$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}} \tag{7.36}$$

so that Eq. (7.35) becomes

$$V_m = I_m Z$$

or

$$\boxed{I_m = \frac{V_m}{Z} \quad \text{or} \quad I_{rms} = \frac{V_{rms}}{Z}} \tag{7.37}$$

The unit of impedance is one ohm. Equation (7.37) can be regarded as a generalized form of Ohm's law applied to a series ac circuit. It can be noted that the current in the circuit depends upon R, L, C as well as the frequency ω. We can also write Eq. (7.34) in terms of R, X_L and X_C as:

$$\boxed{\tan\phi = \frac{X_C - X_L}{R}} \tag{7.38}$$

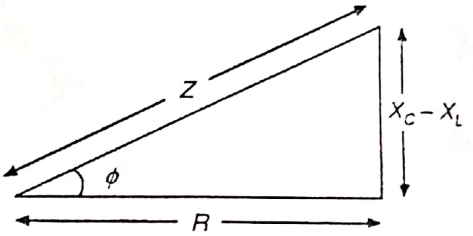

Figure 7.19 Impedance diagram.

Equations (7.36) and (7.38) are represented graphically in Figure 7.19. This is called **impedance diagram** which is a right triangle with the impedance Z as it hypotenuse. The quantity $X_L - X_C$ is called the **reactance** of the circuit, denoted by X.

Figures 7.18 and 7.19 have been constructed for a circuit in which $X_C > X_L$. In this case, the phase angle ϕ is positive, and the current leads the voltage across the source. Figure 7.20 shows graphs of voltage and current versus time for a LCR series circuit when $X_C > X_L$.

If $X_L > X_C$, ϕ is negative, and the current lags the voltage across the source.

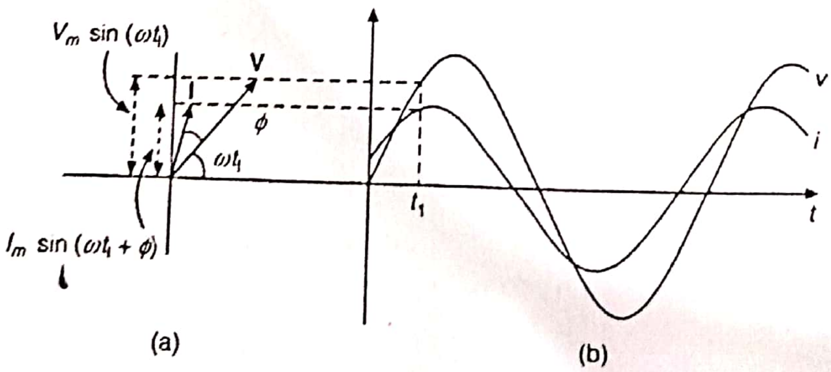

Figure 7.20 (a) Phasor diagram of **V** and **I**, (b) Plot of voltage v and current i versus time for an LCR series circuit, when $X_C > X_L$.

7.7.2 Analytical Solution

Applying Kirchhoff's loop rule to the circuit, we have

$$V - V_R - V_L - V_C = 0 \tag{7.39}$$