PHY121 Applied Physics for Engineers

Magnetism (part-3)

By
Muhammad Kaleem Ullah
Lecturer
Department of Physics

Outlines

1. Hall Effect
2. Biot-Savart law
3. Magnetic field due to Long Straight Wire
4. Force between two Parallel Wires
5. Ampere's law
6. Solenoids

Hall Effect

- In 1879, Edwin H. Hall, a 24-year-old graduate student at the Johns Hopkins University, showed that the drifting conduction electrons in a copper wire can also be deflected by a magnetic field.
- This Hall effect allows to find out whether the charge carriers in a conductor are positively or negatively charged.
- The number of such carriers per unit volume of the conductor can also be measured.
- Fig.1.a shows a copper strip of width ***d***, carrying a current ***i*** whose conventional direction is from the top of the figure to the bottom.
- The charge carriers are electrons and, as we know they drift (with drift speed v_D) in the opposite direction, from bottom to top.

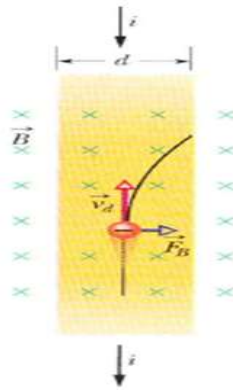


Fig.1.a

Hall Effect

- At the instant shown in Fig.1.a, an external magnetic field $\vec{B}$, pointing into the plane of the figure, has just been turned on.

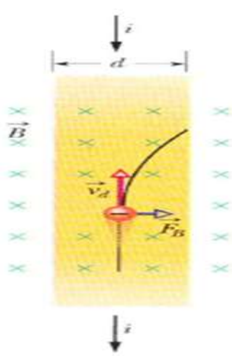


Fig.1.a

- From Eq. ($\overrightarrow{F_B} = q(\vec{v} \times \vec{B})$) we see that a magnetic deflecting force $\overrightarrow{F_B}$ will act on each drifting electron, pushing it toward the right edge of the strip.
- As time goes on, electrons move to the right, mostly piling up on the right edge of the strip, leaving uncompensated positive charges in fixed positions at the left edge.
- The separation of positive and negative charges produces an electric field E within the strip, pointing from left to right in Fig.1.b.
- This field exerts an electric force $\overrightarrow{F_E}$ on each electron, tending to push it to the left.

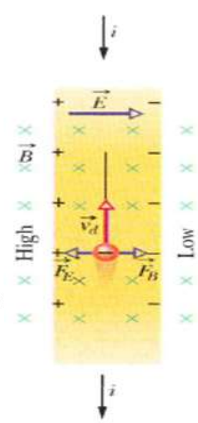


Fig.1.b

Hall Effect

- An equilibrium quickly develops in which the electric force on each electron builds up until it just cancels the magnetic force.
- When this happens, as Fig.1.b shows, the force due to $\vec{B}$ and the force due to $\vec{E}$ are in balance.
- The drifting electrons then move along the strip toward the top of the page at velocity $\vec{v_D}$, with no further collection of electrons on the right edge of the strip and thus no further increase in the electric field $\vec{E}$.
- A Hall potential difference **V** is associated with the electric field across strip width ***d***.
- From Eq. ($E = -\Delta V/\Delta s$), the magnitude of that potential difference is

$$V = Ed \qquad \dots (1)$$

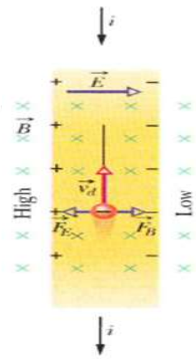


Fig.1.b

Hall Effect

When the electric and magnetic forces are in balance, then

$$F_B = F_E$$

$$eE = ev_D B \qquad \dots (2)$$

As the drift speed v_D is

$$v_D = \frac{J}{ne} = \frac{i}{neA} \qquad \dots (3)$$

in which ***J*** (**= *i*/A**) is the current density in the strip, ***A*** is the cross-sectional area of the strip, and ***n*** is the number density of charge carriers (their number per unit volume). In Eq.2, substituting for ***E*** with Eq.1 and substituting for v_D with Eq.3, we obtain

$$n = \frac{Bi}{Vle} \qquad \dots (4)$$

in which ***l*** (***= A/d***) is the thickness of the strip. With this equation we can find ***n*** from measurable quantities.

Biot – Savart Law

➢Biot - Savart law is used to calculate the magnetic field due to a current carrying conductor.

➢According to this law, the magnitude of the magnetic field at any point P due to a small current element $I.ds$ (I = current through the element, ds = length of the element) is,

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

Symbol μ_0 is a constant, called the permeability constant, whose value is

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}.$$

In vector form

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{\mathrm{r}}}{r^2} \qquad \text{(Biot–Savart law).}$$

In vector notation, $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\cdot\frac{\overrightarrow{idl}\times\vec{r}}{r^3}$

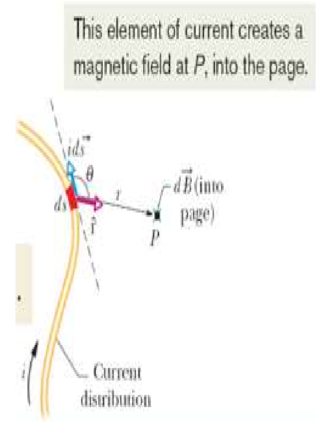


Fig. 29-1 A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P. The green × (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

Magnetic Field due to a Long Straight Wire:

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}.$$

→ $B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta\,ds}{r^2}.$

where

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}.$$

→ $$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{(s^2 + R^2)^{3/2}} = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2 + R^2)^{1/2}}\right]_0^\infty = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire).}$$

$$B = \frac{\mu_0 i}{4\pi R} \qquad \text{(semi-infinite straight wire).}$$

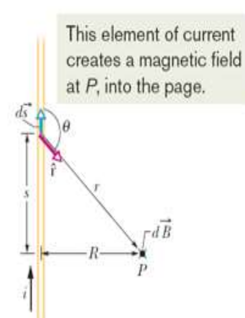
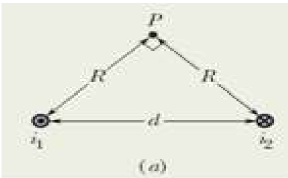


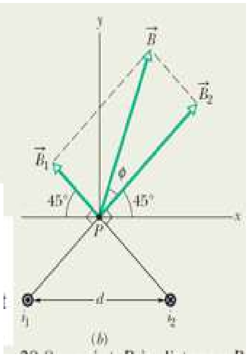
Fig. 29-5 Calculating the magnetic field produced by a current i in a long straight wire. The field $d\vec{B}$ at P associated with the current-length element $i\,d\vec{s}$ is directed into the page, as shown.

Example, Magnetic field off to the side of two long straight currents:

Figure 29-8*a* shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point *P*? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.



The two currents create magnetic fields that must be added as vectors to get the net field.



Finding the vectors: In Fig. 29-8*a*, point *P* is distance *R* from both currents i_1 and i_2. Thus, Eq. 29-4 tells us that at point *P* those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-8*a*, note that the base angles (between sides *R* and *d*) are both 45°. This allows us to write $\cos 45° = R/d$ and replace *R* with $d \cos 45°$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

Adding the vectors: We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point *P*, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. 29-8*b*, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. The Pythagorean theorem then gives us

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)}\sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\sqrt{(15\ \text{A})^2 + (32\ \text{A})^2}}{(2\pi)(5.3 \times 10^{-2}\ \text{m})(\cos 45°)}$$

$$= 1.89 \times 10^{-4}\ \text{T} \approx 190\ \mu\text{T}. \qquad \text{(Answer)}$$

The angle ϕ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. 29-8*b* follows from

$$\phi = \tan^{-1}\frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\ \text{A}}{32\ \text{A}} = 25°.$$

The angle between the direction of $\vec{B}$ and the *x* axis shown in Fig. 29-8*b* is then

$$\phi + 45° = 25° + 45° = 70°. \qquad \text{(Answer)}$$

Force Between Two Parallel Wires:

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

The field due to *a* at the position of *b* creates a force on *b*.

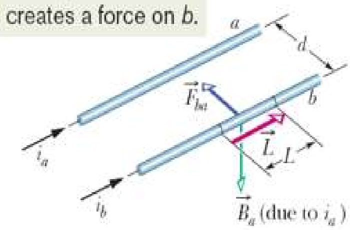


Fig. 29-9 Two parallel wires carrying currents in the same direction attract each other. $\vec{B}_a$ is the magnetic field at wire *b* produced by the current in wire *a*. $\vec{F}_{ba}$ is the resulting force acting on wire *b* because it carries current in $\vec{B}_a$.

Parallel currents attract each other, and antiparallel currents repel each other.

Ampere's Law:

- It states that the line integral of the magnetic field (vector B) around any closed path or circuit is equal to μ_0 (permeability of free space) times the total current (I) flowing through the closed circuit.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).}$$

Only the currents encircled by the loop are used in Ampere's law.

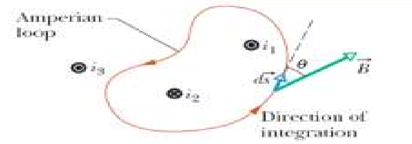


Fig. 29-11 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

This is how to assign a sign to a current used in Ampere's law.

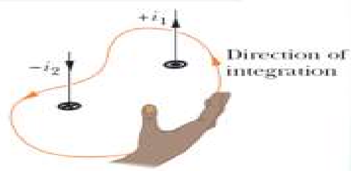


Fig. 29-12 A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-11.

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

Solenoids

Fig. 29-16 A solenoid carrying current i.

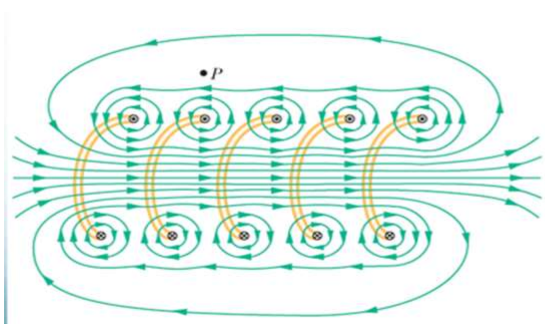
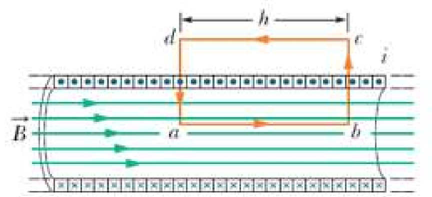


Fig. 29-17 A vertical cross section through the central axis of a "stretched-out" solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid's axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

Solenoids:

Fig. 29-19 Application of Ampere's law to a section of a long ideal solenoid carrying a current i. The Amperian loop is the rectangle $abcda$.



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}.$$

$i_{\text{enc}} = i(nh)$. Here n be the number of turns per unit length of the solenoid

$$Bh = \mu_0 inh$$

$$B = \mu_0 in \qquad \text{(ideal solenoid).}$$

Example, The field inside a solenoid:

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$B = \mu_0 in$$

$$= (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.57\ \text{A})\frac{5 \times 850\ \text{turns}}{1.23\ \text{m}}$$

$$= 2.42 \times 10^{-2}\ \text{T} = 24.2\ \text{mT}. \qquad \text{(Answer)}$$

END OF LECTURE