Applied Physics for Engineers (PHY121)

Electrostatics (Continued)

LECTURE # 8

Instructor
Muhammad Kaleem Ullah
Lecturer
Department of Physics
CUI, Lahore Campus.

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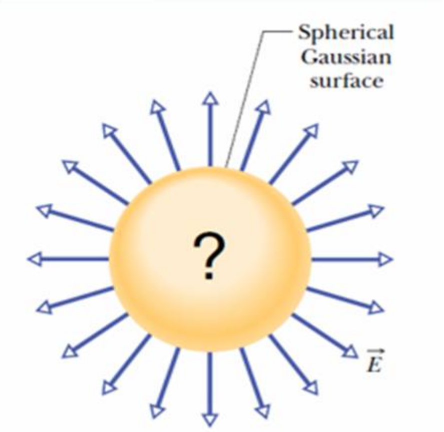
Outlines

1. Electric Flux
2. Gauss' Law and its Applications



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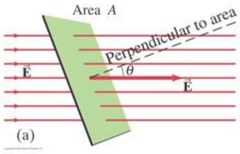
Electric Flux

- The electric flux refers to the electric field lines that penetrate a given surface

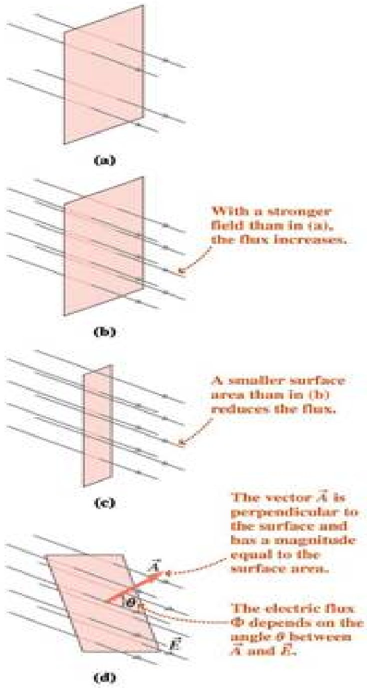


- For a given uniform electric field E passing through an area A, the electric flux is defined as:

$$\Phi_E = EA\cos\theta$$

- Here $\vec{A}$ is a **vector** whose magnitude is the surface area A and whose orientation is normal to the surface.

– The electric flux Φ through a flat surface in a uniform electric field depends on the **field strength** E, the **surface area** A, and the **angle** θ between the field and the normal to the surface.



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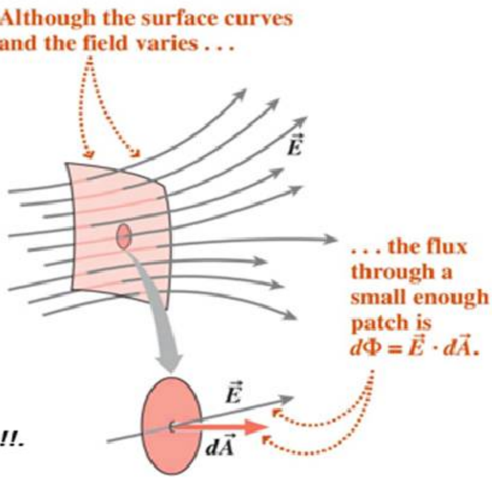
Electric flux with curved surfaces and nonuniform fields

- When the surface is curved or the field is nonuniform, we calculate the flux by dividing the surface into small patches $d\vec{A}$, so small that each patch is essentially flat and the field is essentially uniform over each.
  - We then sum the fluxes $d\Phi = \vec{E}\cdot d\vec{A}$ over each patch.
  - In the limit of infinitely many infinitesimally small patches, the sum becomes a **surface integral**:

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

The circle on the integration sign simply means the surface is CLOSED!!.



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Flux of a Non-uniform Electric Field

Here we have an arbitrary (asymmetric) Gaussian surface immersed in a non-uniform electric field.

The surface has been divided up into small squares each of area ΔA, small enough to be considered flat.

We represent each element of area with a vector area $\Delta \underline{A}$ and magnitude ΔA.

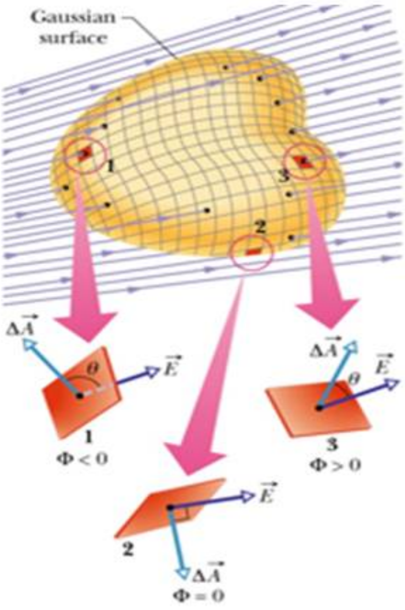
Each vector $\Delta \underline{A}$ is perpendicular to the Gaussian surface and directed outwards

Electric field $\underline{E}$ may be assumed to be constant over any given square

Vectors $\Delta \underline{A}$ and $\underline{E}$ for each square make an angle θ with each other

Now we could estimate that the flux of the electric field for this Gaussian surface is

$$\Phi = \Sigma\, \underline{E} \bullet \Delta \underline{A}$$



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CHECKPOINT: Gaussian cube of face area A is immersed in a uniform electric field $\underline{E}$ that has positive direction along z axis.

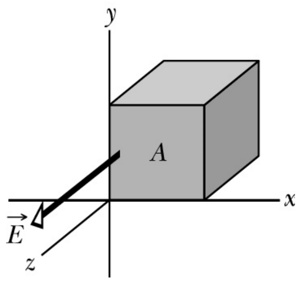
In terms of E and A, what is the flux through .....

..the front face (in the xy plane)?
- A. +EA
- B. 0
- C. -EA

..the rear face?
- A. +EA
- B. 0
- C. -EA

..the top face?
- A. +EA
- B. 0
- C. -EA

..the whole cube?
- A. +EA
- B. 0
- C. -EA



Answers:
(a) +EA
(b) –EA
(c) 0
(d) 0

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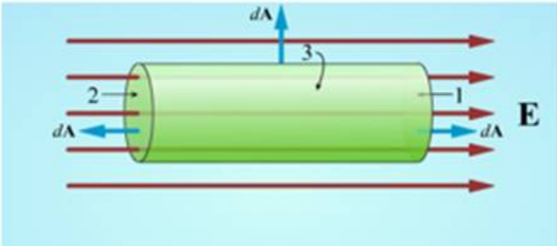
Electric Flux

What is the electric flux of this cylinder?



$$\Phi_E = \sum \Phi = \Phi_1 + \Phi_2 + \Phi_3$$

$E = \text{constant}, \ A_1 = A_2$

$$\Phi_E = EA_1 \cos 0 + EA_2 \cos 180 + EA_3 \cos 90$$

$$\Phi_E = EA_1(1) + EA_2(-1) + 0$$

$$\Phi_E = 0$$

What does this tell us?

This tells us that there are NO sources or sinks INSIDE the cylindrical object.

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Gauss' Law

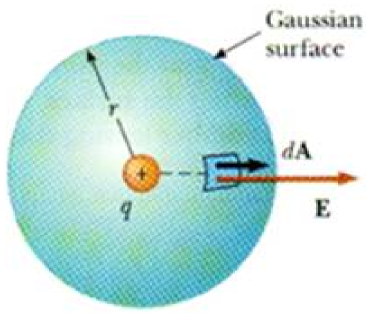
The electric flux (flow) is in direct proportion to the charge that is enclosed within some type of surface, which we call Gaussian.

Mathematically,

$$\varepsilon_0 \Phi = q_{\text{enc}} \quad \text{(Gauss' law).}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad \text{(Gauss' law).}$$

where ε_0 is called constant of proportionality.



The net charge q_{enc} is the algebraic sum of all the enclosed positive and negative charges, and it can be positive, negative, or zero.

The electric field at the surface is due to all the charge distribution, including both that inside and outside the surface.

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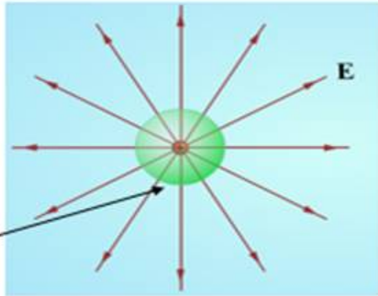
Gauss' Law – How does it work?

Consider a POSITIVE POINT CHARGE, ***Q***.

Step 1 – Is there a source of symmetry?

Yes, it is spherical symmetry!

You then draw a shape in such a way as to obey the symmetry and ***ENCLOSE*** the charge. In this case, we enclose the charge within a sphere. This surface is called a **GAUSSIAN SURFACE**.



Step 2 – What do you know about the electric field at all points on this surface?

It is constant.

The "E" is then brought out of the integral.

$$E\oint da = \frac{q_{enc}}{\varepsilon_o}$$

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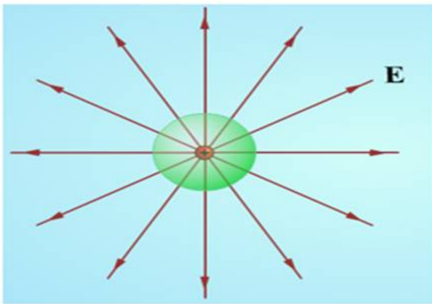
Step 3 – Identify the area of the Gaussian surface?

In this case, summing each and every dA gives us the surface area of a sphere.

$$E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_o}$$



Step 4 – Identify the charge enclosed?

The charge enclosed is Q!

$$E(4\pi r^2) = \frac{Q}{\varepsilon_o} \quad \rightarrow \quad E = \frac{Q}{4\pi r^2 \varepsilon_o}$$

This is the equation for a POINT CHARGE!

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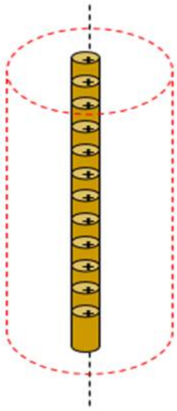
Gauss' Law and cylindrical symmetry

Consider a line(or rod) of charge that is very long (infinite)

We can **ENCLOSE** it within a **CYLINDER**. Thus our Gaussian surface is a cylinder.



$$E\oint da = \frac{q_{enc}}{\varepsilon_o} \longleftarrow \textbf{Gauss' law}$$

$$E(2\pi rL) = \frac{q_{enc}}{\varepsilon_o}$$

$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_o}$$

RECALL : $\lambda = \frac{Q}{L}$

$Q = \lambda L = q_{enc}$

$A_{cylinder} = 2\pi rL$

This is the same equation we got doing extended charge distributions.

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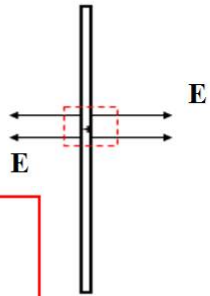
Gauss' Law for insulating sheets

A charge is distributed with a uniform charge density over an infinite plane INSULATING thin sheet. Determine E outside the sheet.

For an insulating sheet the charge resides INSIDE the sheet. Thus there is an electric field on BOTH sides of the plane.



Now, applying Gauss' law

$$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o} \rightarrow \mathbf{2}\ E\oint da = \frac{q_{enc}}{\varepsilon_o}$$

$$EA + EA = \frac{Q}{\varepsilon_o} \rightarrow 2EA = \frac{Q}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

RECALL : $\sigma = \frac{Q}{A}$

$Q = \sigma A = q_{enc}$

This is the same equation we got doing extended charge distributions.

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Gauss' Law for conducting sheets

A charge is distributed with a uniform charge density over an infinite thick *conducting* sheet. Determine E outside the sheet.

For a thick conducting sheet, the charge exists on the surface only

Now, applying Gauss' law

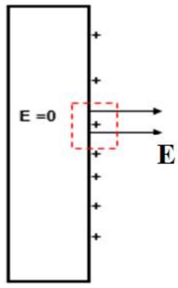
$$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$$

$$EA = \frac{Q}{\varepsilon_o}$$

$$EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{\varepsilon_o}$$

$$\boxed{RECALL: \quad \sigma = \frac{Q}{A} \qquad Q = \sigma A = q_{enc}}$$



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In summary

Whether you use electric charge distributions or Gauss' Law you get the SAME electric field functions for symmetrical situations.

$$E = \frac{Q}{4\pi\varepsilon_o r^2} \rightarrow dE = \frac{dq}{4\pi\varepsilon_o r^2}$$

$$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$$

Function	Point, hoop, or Sphere (Volume)	Disk or Sheet (AREA) "insulating and thin"	Line, rod, or cylinder (LINEAR)
Equation	$E = \frac{Q}{4\pi\varepsilon_o r^2}$	$E = \frac{\sigma}{2\varepsilon_o}$	$E = \frac{\lambda}{2\pi\varepsilon_o r}$

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END OF LECTURE