Assignment #1

Part I - Decision Tree

1. The decision of going skiing depends on three features: snow, weather, and season. All variables are binary: **Skiing** (yes or no), **Snow** (fresh or frosted), **Weather** (windy or sunny), **Season** (low or high). The following table shows a data set that contains 10 data points.

No.	Snow	Weather	Season	Skiing
1	fresh	sunny	low	yes
2	fresh	sunny	high	yes
3	frosted	windy	high	no
4	fresh	windy	high	no
5	fresh	windy	low	yes
6	frosted	windy	low	no
7	frosted	sunny	high	yes
8	fresh	sunny	high	yes
9	fresh	windy	high	yes
10	frosted	sunny	low	no

Now what you need to do is to carry out the first split for growing a decision tree model. Please use information gain based on Gini impurity as the criteria for splitting. **Note:** Your calculation will be stopped at the point you figure out which variable, **Snow**, **Weather**, or **Season**, you decide to use as the root node. You must write out all the intermediate steps.

Answer:

1. Skiing
    a. Yes = 6; No =4
    b. Gini Impurity = 1-(6/10)^2-(4/10)^2=0.48
2. Snow as root node
    a. Fresh 6/10 : Yes = 5; No = 1
        i. Gini Impurity = 1-(5/6)^2-(1/6)^2=0.2778
    b. Frosted 4/10 : Yes = 1; No = 3

        i. Gini Impurity = 1-(1/4)^2-(3/4)^2=0.3750
    c. Weighted average impurity of Snow = 0.2778*(6/10)+0.3750*(4/10)=0.31668
    d. Information gain = 0.48-0.31668=0.16332
3. Weather as root node
    a. Sunny 5/10 : Yes = 4; No = 1
        i. Gini Impurity = 1-(4/5)^2-(1/5)^2=0.32
    b. Windy 5/10 : Yes = 2; No = 3
        i. Gini Impurity = 1-(2/5)^2-(3/5)^2=0.48
    c. Weighted average impurity of Weather = 0.32*(5/10)+0.48*(5/10)=0.4
    d. Information gain = 0.48-0.4=0.08
4. Season as root node
    a. Low 4/10 : Yes = 2; No = 2
        i. Gini Impurity = 1-(2/4)^2-(2/4)^2=0.5
    b. High 6/10 : Yes = 4; No = 2
        i. Gini Impurity = 1-(4/6)^2-(2/6)^2=0.4444
    c. Weighted average impurity of Weather = 0.5*(4/10)+0.4444*(6/10)=0.46664
    d. Information gain = 0.48-0.46664=0.01336
5. Conclusion: using SNOW as our leaf node will result in a larger information gain.

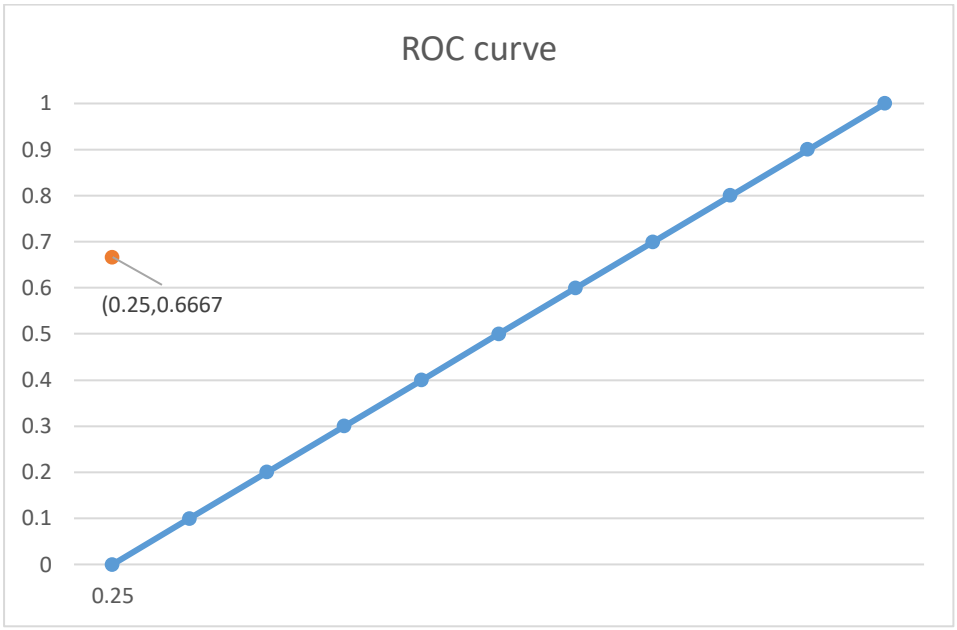
Part II – Evaluation Measures

1. Given the following confusion matrix (obtained by using the default decision threshold of 0.5 for the probability estimates of a classifier on all the test examples), answer the subsequent questions.

		Predicted +	Predicted -
Actual	+	4	2
	-	1	3

1) What is the value of accuracy? What are the values of TPR (true positive rate), FPR (false positive rate)? Draw this point on the ROC graph.

1. **The value of accuracy = (4+3)/(4+1+2+3) = 0.7**
2. **TRP (true positive rate) = TP/(TP+FN)=4/(4+2)=0.6667**
3. **FPR (false positive rate) = FP/(FP+TN)=1/(1+3)=0.25**



2) If the cost of false positive prediction = $1 and cost of false negative prediction = $5, what is the total cost? How to adjust decision threshold (i.e., increase or decrease the decision threshold of classifying into positive class) to lower the total cost in this case?

	Predicted (+)	Predicted (-)
Actual (+)	$0	$5
Actual (-)	$1	$0

False positive prediction cost = 1* $1 = $1
False negative prediction cost = 2 *$5 = $10
Total cost = $1 + $10 =$11
Conclusion: The cost of a false-negative prediction is much higher than the cost of a false-positive prediction. This means that the cost of a positive prediction is lower than the cost of a negative prediction. Therefore, lowering the threshold (allow more prediction to be positive) can reduce the total cost.