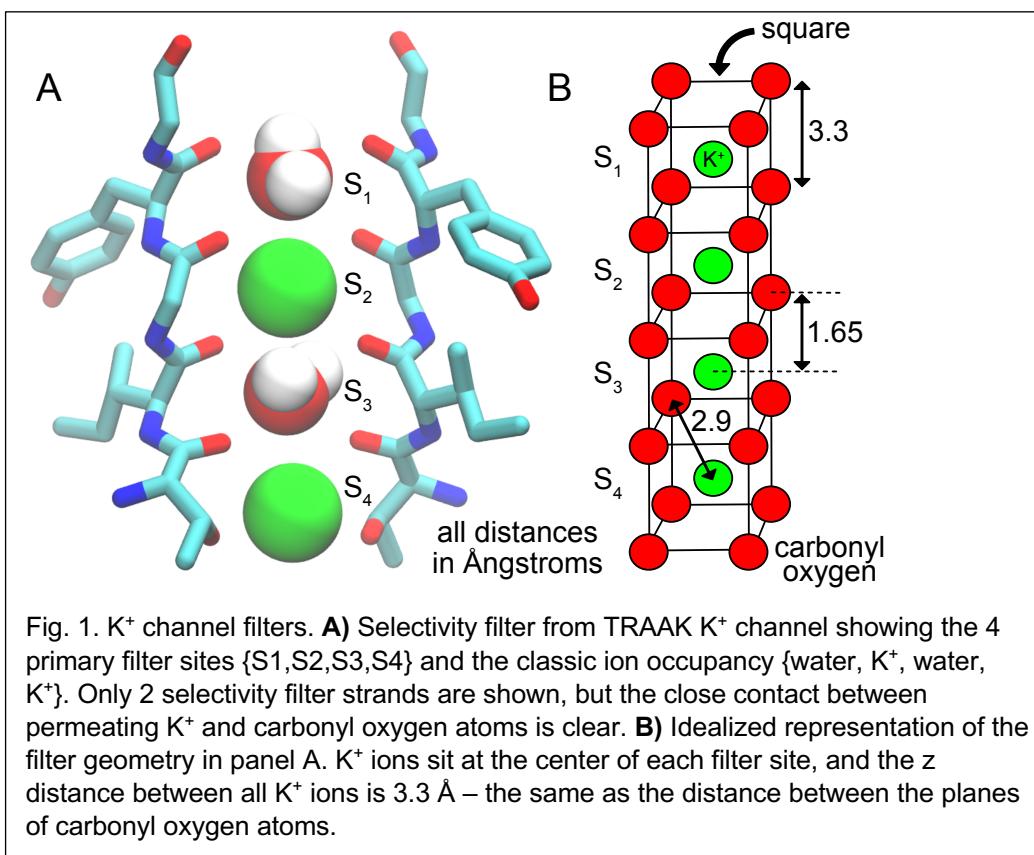


1. Energetic cost of hard knock on. The dominant view of K^+ permeation through K^+ channels is that ions are separated by water, in part because the cost of having two K^+ next to each other is thought to be too high (the latter is known as the hard-knock configuration). Let us estimate these notions with a simple electrostatic model shown in Figure 1.

- Simulations by DeGroot and those in the Grabe lab suggest that water rarely enters the filter, and ions often occupy sites next to each other. Use Coulomb's law and the geometry in Fig. 1B to determine the energy of the following configurations: {0, K^+ , 0, K^+ } and {0, K^+ , K^+ , 0}, where a 0 indicates nothing in a site, K^+ indicates a +1 cation, and the positions are { S_1 , S_2 , S_3 , S_4 }. Assume that the entire space has a uniform dielectric value of 4, and the oxygen atoms have a partial negative charge of -0.51 fundamental charge units (e). Ignore the desolvation energy required to move the ions into the filter. Also ignore interaction energies between all protein atoms. What is the energy difference between the classic configuration and the hard-knock configuration.

It is very likely that you will want to write a short Python or Matlab program to do this.

- For two ions next to each other, what is the most favorable configuration? Use symmetry arguments to reduce your calculations. Again ignore interaction energies between protein atoms.
- Add in the carbon atoms. They have a charge of +0.51, and assume that they are in the same plane as the oxygen atoms, but each is 3.7 Å away from the closest K^+ . What are the new energies for all of the configurations in question 1b? Ignore interaction energies between all protein atoms.



electrostatic notes

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The relevant equations

The permittivity of free space is $\epsilon = 1$ in CGS units. In addition $k_B T$ has the following value at 298 K:

$$k_B T = 1.3807 \times 10^{-16} \left[\frac{\text{erg}}{\text{K}} \right] \cdot 298 \text{ [K]} = 4.114 \times 10^{-14} \text{ [erg]}$$

In an infinite medium the solution to the point charge is:

$$\phi = \frac{q}{\epsilon r}$$

The charge of a single proton is 4.803×10^{-10} stat-coulomb. Therefore, the energy of fundamental charges separated by Angstroms is:

$$U = 2.307 \times 10^{-11} \frac{1}{\epsilon r} \text{ [erg]} = \frac{560.40}{\epsilon r} \text{ [k}_B T\text{]}$$

where multiplication by 10^{-8} accounts for the change from centimeters to Å. So that the energy of two protons separated by 5 Å in a protein with dielectric $\epsilon = 10$ is about 11.208 $k_B T$. APBS outputs energies in kJ/mol. The conversion to $k_B T$ at 298.15 K is:

$$1 \text{ [k}_B T\text{]} = 2.47817 \left[\frac{\text{kJ}}{\text{mol}} \right] = 0.5923 \left[\frac{\text{Kcal}}{\text{mol}} \right] \quad \text{AND} \quad 4.184 \left[\frac{\text{kJ}}{\text{mol}} \right] = 1 \left[\frac{\text{Kcal}}{\text{mol}} \right]$$

where $1 \text{ [J]} = 10^{-7} \text{ [erg]}$, $k = 1.3807 \times 10^{-16} \text{ [erg/K]}$, and a $\text{mole} = 6.02 \times 10^{23}$. Now lets look at this particular energy as determined by an APBS calculation.

The APBS manual does not say how the electrostatic energy is calculated. Looking in the code one finds that there are several ways of calculating the energy. The most common mode is as follows:

$$U = \frac{1}{2} \int_{\text{all space}} \phi(x) \rho(x) dV$$

This is a particularly nice definition to use since the integral of the electric field over all space will require addressing what the field is doing outside of the domain of interest. For this same reason,