

№1.

Урок 4. Часть 2.

$$a) \begin{cases} x_1 - 2x_2 = 1 \\ 3x_1 - 4x_2 = 7 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix} = -4 + 6 = 2$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 7 & -4 \end{vmatrix} = -4 + 14 = 10$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 \\ 3 & 7 \end{vmatrix} = 7 - 3 = 4$$

$$x_1 = \frac{\Delta_1}{\Delta} = 5 \quad ; \quad x_2 = \frac{\Delta_2}{\Delta} = 2$$

Ответ: (5 ; 2)

$$b) \begin{cases} 2x_1 - x_2 + 5x_3 = 10 \\ x_1 + x_2 - 3x_3 = -2 \\ 2x_1 + 4x_2 + x_3 = 1 \end{cases}$$

$$\Delta = \begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & -3 \\ 2 & 4 & 1 \end{vmatrix} = 2 \begin{vmatrix} 1 & -3 \\ 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} =$$

$$= 2(1 + 12) + (1 + 6) + 5(4 - 2) = 26 + 7 + 10 = 43$$

$$\Delta_1 = \begin{vmatrix} 10 & -1 & 5 \\ -2 & 1 & -3 \\ 1 & 4 & 1 \end{vmatrix} = 10 \begin{vmatrix} 1 & -3 \\ 4 & 1 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ 1 & 1 \end{vmatrix} + 5 \begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} =$$

$$= 10(1 + 12) + (-2 + 3) + 5(-8 - 1) = 130 + 1 - 45 = 86$$

$$\Delta_2 = \begin{vmatrix} 2 & 10 & 5 \\ 1 & -2 & -3 \\ 2 & 1 & 1 \end{vmatrix} = 2 \begin{vmatrix} -2 & -3 \\ 1 & 1 \end{vmatrix} - 10 \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} =$$

$$= 2(-2 + 3) - 10(1 + 6) + 5(1 + 4) = 2 - 70 + 25 = -43$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 10 \\ 1 & 1 & -2 \\ 2 & 4 & 1 \end{vmatrix} = 2 \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 10 \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} =$$

$$= 2(1+8) + (1+4) + 10(4-2) = 18 + 5 + 20 = 43$$

$$x_1 = \frac{\Delta_1}{\Delta} = 2; \quad x_2 = \frac{\Delta_2}{\Delta} = -1; \quad x_3 = \frac{\Delta_3}{\Delta} = 1$$

Отв.:  $X = (2; -1; 1)$

№2

a)  $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 9 & 12 \\ 3 & 26 & 30 \end{pmatrix}$

$$LUx = b$$

$$Ly = b$$

$$Ux = y$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 4 \\ 0 & 20 & 18 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix}$$

Отв.:  $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{pmatrix}$

8)  $\begin{pmatrix} 1 & 1 & 2 & 4 \\ 2 & 5 & 8 & 9 \\ 3 & 18 & 29 & 18 \\ 4 & 22 & 53 & 33 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 15 & 23 & 6 \\ 0 & 18 & 45 & 17 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 21 & 11 \end{pmatrix} \sim$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

Отв.:  $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 5 & 1 & 0 \\ 4 & 6 & 7 & 1 \end{pmatrix}$

$$\begin{aligned} \text{NS} \quad & \begin{cases} 2x_1 + x_2 + 3x_3 = 1 \\ 11x_1 + 7x_2 + 5x_3 = -6 \\ 9x_1 + 8x_2 + 4x_3 = -5 \end{cases} \quad \begin{matrix} L U x = b \\ y \end{matrix} \end{aligned}$$

$$\begin{aligned} U-? \quad & \begin{pmatrix} 2 & 1 & 3 \\ 11 & 7 & 5 \\ 9 & 8 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 3 \\ 22 & 14 & 10 \\ 18 & 16 & 8 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & -23 \\ 0 & 7 & -19 \end{pmatrix} \sim \\ & \sim \begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & -23 \\ 0 & 0 & \frac{104}{3} \end{pmatrix} = U \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 11 & 1 & 0 \\ 9 & \frac{7}{3} & 1 \end{pmatrix} \end{aligned}$$

$$-19 + 23 \cdot \frac{7}{3} = \frac{-57 + 161}{3} = \frac{104}{3}$$

$$Ly = b :$$

$$\begin{cases} y_1 = 1 \\ 11y_1 + y_2 = -6 \cdot 2 = -12 \rightarrow y_2 = -(11y_1 + 12) = -23 \\ 9y_1 + \frac{7}{3}y_2 + y_3 = -5 \cdot 2 = -10 \rightarrow y_3 = -10 - 9 + \frac{7}{3} \cdot 23 = \frac{104}{3} \end{cases}$$

$$Ux = y$$

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 1 \rightarrow 2x_1 = 1 - 0 - 3 = -2 \rightarrow x_1 = -1 \\ x_2 - 23x_3 = -23 \rightarrow x_2 = -23 + 23 = 0 \\ \frac{104}{3}x_3 = \frac{104}{3} \rightarrow x_3 = 1 \end{cases}$$

$$\text{OTB: } X = (-1; 0; 1)$$