

$$E_{CNDO/2} = \underbrace{\frac{1}{2} \sum_{\mu\nu} P_{\mu\nu}^\alpha (H_{\mu\nu} + F_{\mu\nu}^\alpha)}_{\equiv E^\alpha} + \underbrace{\frac{1}{2} \sum_{\mu\nu} P_{\mu\nu}^\beta (H_{\mu\nu} + F_{\mu\nu}^\beta)}_{\equiv E^\beta} + V_{nnn}$$

$$\begin{aligned} \frac{\partial E^\alpha}{\partial R_A} &= \frac{1}{2} \frac{\partial}{\partial R_A} \left(\sum_{\mu \in A} \sum_{\nu \in B \setminus A} + \sum_{\nu \in A} \sum_{\mu \in B \setminus A} + \sum_{\mu \in A, \nu = \mu} + \sum_{\mu \in B \setminus A, \nu = \mu} \right) P_{\mu\nu}^\alpha (H_{\mu\nu} + F_{\mu\nu}^\alpha) \\ &= \frac{1}{2} \left[\sum_{\mu \in A} \sum_{\nu \in B \setminus A} P_{\mu\nu}^\alpha \left(\frac{\partial H_{\mu\nu}}{\partial R_A} + \frac{\partial F_{\mu\nu}^\alpha}{\partial R_A} \right) + \sum_{\nu \in A} \sum_{\mu \in B \setminus A} P_{\mu\nu}^\alpha \left(\frac{\partial H_{\mu\nu}}{\partial R_A} + \frac{\partial F_{\mu\nu}^\alpha}{\partial R_A} \right) \right. \\ &\quad \left. + \sum_{\mu \in A, \nu = \mu} P_{\mu\mu}^\alpha \left(\frac{\partial H_{\mu\mu}}{\partial R_A} + \frac{\partial F_{\mu\mu}^\alpha}{\partial R_A} \right) + \sum_{\mu \in B \setminus A, \nu = \mu} P_{\mu\mu}^\alpha \left(\frac{\partial H_{\mu\mu}}{\partial R_A} + \frac{\partial F_{\mu\mu}^\alpha}{\partial R_A} \right) \right] \end{aligned}$$

$$\frac{\partial F_{\mu\mu}^\alpha}{\partial R_A} = \sum_{B \setminus A} (P_{BB} - Z_B) \frac{\partial \gamma_{AB}}{\partial R_A} \quad \mu \in A$$

$$\frac{\partial H_{MM}}{\partial R_A} = - \sum_{B \setminus A} Z_B \frac{\partial \gamma_{AB}}{\partial R_A} \quad \mu \in A$$

$$\begin{aligned} \frac{\partial F_{\mu\nu}}{\partial R_A} &= \frac{1}{2} (\beta_A + \beta_B) \frac{\partial S_{\mu\nu}}{\partial R_A} - P_{\mu\nu}^\alpha \frac{\partial \gamma_{AB}}{\partial R_A} \quad \mu, \nu \text{ belong to different atoms} \\ \frac{\partial H_{\mu\nu}}{\partial R_A} &= \frac{1}{2} (\beta_A + \beta_B) \frac{\partial S_{\mu\nu}}{\partial R_A} \end{aligned}$$

π -term: extract all terms relate to $\frac{\partial S_{\mu\nu}}{\partial R_A}$

$$\begin{aligned} \frac{1}{2} \sum_{\mu \in A} \sum_{\nu \in B \setminus A} P_{\mu\nu}^\alpha \left[\frac{1}{2} (\beta_A + \beta_B) + \frac{1}{2} (\beta_A + \beta_B) \right] \frac{\partial S_{\mu\nu}}{\partial R_A} \\ + \frac{1}{2} \sum_{\nu \in A} \sum_{\mu \in B \setminus A} P_{\mu\nu}^\alpha \left[\frac{1}{2} (\beta_A + \beta_B) + \frac{1}{2} (\beta_A + \beta_B) \right] \frac{\partial S_{\mu\nu}}{\partial R_A} \end{aligned}$$

$$= \sum_{\mu \in A} \sum_{\nu \in B \setminus A} P_{\mu\nu}^\alpha (\beta_A + \beta_B) \frac{\partial S_{\mu\nu}}{\partial R_A}$$

$\because \nu, \mu$ are dummy variables

\therefore switch them won't influence anything.

Similar for E^β : $\sum_{\mu \in A} \sum_{\nu \in B \setminus A} P_{\mu\nu}^\beta (\beta_A + \beta_B)$

$$\Rightarrow \sum_{\mu \in A} \sum_{\nu \in B \setminus A} X_{\mu\nu} \frac{\partial S_{\mu\nu}}{\partial R_A} = \sum_{\mu \in A} \sum_{\nu \in B \setminus A} \underbrace{(P_{\mu\nu}^\alpha + P_{\mu\nu}^\beta)}_{\equiv P_{\mu\nu}^{+\alpha\beta}} (\beta_A + \beta_B) \frac{\partial S_{\mu\nu}}{\partial R_A}$$

$$\Rightarrow X_{\mu\nu} = P_{\mu\nu}^{+\alpha\beta} (\beta_A + \beta_B) \quad \equiv P_{\mu\nu}^{+\alpha\beta}$$

y term: extract all terms relate to $\frac{\partial Y_{AB}}{\partial R_A}$ in $\frac{\partial E^A}{\partial R_A}$

$$\begin{aligned}
 & -\frac{1}{2} \sum_{\mu \in A} \sum_{\nu \in B \neq A} P_{\mu\nu}^\alpha P_{\mu\nu}^\alpha \frac{\partial Y_{AB}}{\partial R_A} - \frac{1}{2} \sum_{\nu \in A} \sum_{\mu \in B \neq A} P_{\mu\nu}^\alpha P_{\mu\nu}^\alpha \frac{\partial Y_{AB}}{\partial R_A} \\
 & + \frac{1}{2} \sum_{\mu \in A} P_{\mu\mu}^\alpha \sum_{B \neq A} (-Z_B + P_{BB}^{tot} - Z_B) \frac{\partial Y_{AB}}{\partial R_A}, \\
 & + \frac{1}{2} \sum_{\mu \in B \neq A} P_{\mu\mu}^\alpha \sum (-Z_A + P_{AA}^{tot} - Z_A) \frac{\partial Y_{AB}}{\partial R_A} \\
 = & - \sum_{\mu \in A} \sum_{\nu \in B \neq A} P_{\mu\nu}^\alpha P_{\mu\nu}^\alpha \frac{\partial Y_{AB}}{\partial R_A} \\
 & + \frac{1}{2} \sum_{\mu \in A} \sum_{B \neq A} P_{\mu\mu}^\alpha P_{BB}^{tot} \frac{\partial Y_{AB}}{\partial R_A} - \sum_{\mu \in A} \sum_{B \neq A} Z_B P_{\mu\mu}^\alpha \frac{\partial Y_{AB}}{\partial R_A} \\
 & + \frac{1}{2} \sum_{\mu \in B \neq A} \sum P_{\mu\mu}^\alpha P_{AA}^{tot} \frac{\partial Y_{AB}}{\partial R_A} - \sum_{\mu \in B \neq A} \sum Z_A P_{\mu\mu}^\alpha \frac{\partial Y_{AB}}{\partial R_A}
 \end{aligned}$$

Similarly, for E^P :

$$\begin{aligned}
 & - \sum_{\mu \in A} \sum_{\nu \in B \neq A} P_{\mu\nu}^\beta P_{\mu\nu}^\beta \frac{\partial Y_{AB}}{\partial R_A} \\
 & + \frac{1}{2} \sum_{\mu \in A} \sum_{B \neq A} P_{\mu\mu}^\beta P_{BB}^{tot} \frac{\partial Y_{AB}}{\partial R_A} - \sum_{\mu \in A} \sum_{B \neq A} Z_B P_{\mu\mu}^\beta \frac{\partial Y_{AB}}{\partial R_A} \\
 & + \frac{1}{2} \sum_{\mu \in B \neq A} \sum P_{\mu\mu}^\beta P_{AA}^{tot} \frac{\partial Y_{AB}}{\partial R_A} - \sum_{\mu \in B \neq A} \sum Z_A P_{\mu\mu}^\beta \frac{\partial Y_{AB}}{\partial R_A}
 \end{aligned}$$

\Rightarrow sum them up:

$$\begin{aligned}
 \sum_{B \neq A} Y_{AB} \frac{\partial Y_{AB}}{\partial R_A} &= \left[- \sum_{\mu \in A} \sum_{\nu \in B \neq A} (P_{\mu\nu}^\alpha P_{\mu\nu}^\alpha + P_{\mu\nu}^\beta P_{\mu\nu}^\beta) \right. \\
 &+ \frac{1}{2} \sum_{\mu \in A} \sum_{B \neq A} P_{\mu\mu}^{tot} P_{BB}^{tot} - \sum_{\mu \in A} \sum_{B \neq A} Z_B P_{\mu\mu}^{tot} \\
 &+ \frac{1}{2} \sum_{\mu \in B \neq A} \sum_{A \neq B} P_{\mu\mu}^{tot} P_{AA}^{tot} - \sum_{\mu \in B \neq A} \sum_{A \neq B} Z_A P_{\mu\mu}^{tot} \Big] \frac{\partial Y_{AB}}{\partial R_A} \\
 &= \sum_{B \neq A} \left[- \sum_{\mu \in A} \sum_{\nu \in B} (P_{\mu\nu}^\alpha P_{\mu\nu}^\alpha + P_{\mu\nu}^\beta P_{\mu\nu}^\beta) + P_{AA}^{tot} P_{BB}^{tot} \right. \\
 &\quad \left. - Z_B P_{AA}^{tot} - Z_A P_{BB}^{tot} \right] \frac{\partial Y_{AB}}{\partial R_A}
 \end{aligned}$$

$$\Rightarrow Y_{AB} = - \sum_{\mu \in A} \sum_{\nu \in B} (P_{\mu\nu}^\alpha P_{\mu\nu}^\alpha + P_{\mu\nu}^\beta P_{\mu\nu}^\beta) + P_{AA}^{tot} P_{BB}^{tot} - Z_B P_{AA}^{tot} - Z_A P_{BB}^{tot}$$