VAR Model of Inflation and Unemployment Rates

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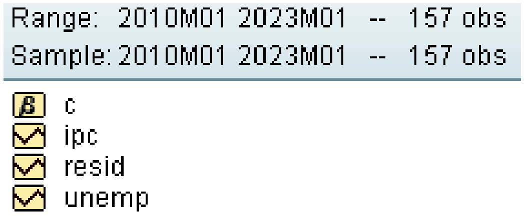
DATA

According to the National System of Statistical and Geographical Information (INEGI, 2023), inflation is the generalized and sustained increase of prices of goods and services of a country, and it is measured by indexes that reflect the percentage change in price of a pondered basket of goods and services such as the Consumer Price Index. On the other hand, the INEGI (2002) explains that the unemployment rate is that fraction of the Economically Active Population that does not have a formal job but has been actively looking for one in recent weeks. In macroeconomics, a phenomenon that is often discussed is the relationship that exists between inflation rates and unemployment rates. It has been stated in the past that there is a negative relationship among these variables, which is represented by the Phillips Curve (1958). Economists have found that it is difficult to lower inflation without increasing unemployment, and reduce unemployment without raising inflation, i.e., there is a bidirectional relationship between these variables. In fact, this is one of the main economic challenges governments and economic policy makers face in their labors.

Because unemployment appears to affect inflation since it lowers the demand for products and services, and inflation appears to affect unemployment since it makes it more difficult to pay employees' wages, a Vector Autoregressive (VAR) model comes to mind to explore the dynamics that take place among inflation and unemployment. VAR models are useful to study the joint behavior dynamics of multivariate time series that affect one another, have the same order of integration, and are not cointegrated. Additionally, through Impulse Response Functions (IRF) it is possible to analyze the dynamic response of each variable in the VAR system after a single shock in another one while all else remains constant. Likewise, with Variance Decomposition, it is possible to have a deeper insight on the sources of variation of a variable. That is, by decomposing the variance of each variable in the system into different components that come from shocks or innovations in the other variables in the system as well as the own variable that is analyzed.

Figure 1: Project Variables, Timeframe Range and Number of Observations



In this case, it is suspected that a bivariate VAR model can be estimated using the statistics software EViews. To carry this out, monthly data of inflation rates and unemployment rates in Mexico has been collected for the last 13 years. The timeframe of the data ranges from January 2010 to January 2023 (inclusive), giving a total of 157 observations. Both variables are written as percentages (for example, the first observation of inflation is 4.46, that is, 4.46%). The inflation rate variable has been named in EViews IPC because of its name in Spanish, *Índice de Precios al Consumidor*. The unemployment rate variable has been named UNEMP for simplicity. The data for these variables has been collected from the website of the National System of Statistical and Geographical Information or INEGI (https://www.inegi.org.mx/temas/inpc/ and https://www.inegi.org.mx/temas/empleo/). INEGI is an autonomous, known, and reliable source to obtain information pertaining Mexico's economic statistics. A visualization of the dataset is offered at the end of this document in the Appendix section.

THE MODEL

The popularity in usage of Vector Autoregressive models (especially for finance and economics) occurred since the 1980s through the work of economists Cristopher Sims and Lawrence Klein. Sims' paper, *Macroeconomics and Reality,* proved VAR models to be useful to analyze multivariate time series and make forecasts. It has since been used as a standard statistical tool to explore the dynamics of economic and financial systems as well as other phenomena in different areas. VAR models are useful when a variable is explained not only by its own lags but also by another variables and its respective lags. For a bivariate VAR (such as the one that is expected to be constructed in this project), two equations are ultimately obtained that can be used for static or dynamic forecasts. The general equations are provided below. When the required coefficients are obtained, they shall be used to construct these respective equations.

Equation 1:

$$Y_t = \alpha_1 + \phi_{11}Y_{t-1} + \cdots + \phi_{1p}Y_{t-p} + \beta_{11}X_{t-1} + \cdots + \beta_{1p}X_{t-p} + e_{1t}$$

Equation 2:

$$X_t = \alpha_2 + \phi_{21}Y_{t-1} + \cdots + \phi_{2p}Y_{t-p} + \beta_{21}X_{t-1} + \cdots + \beta_{2p}X_{t-p} + e_{2t}$$

When estimating a VAR model, the first thing that must be done is assessing whether the variables that will be included are integrated of the same order. That is, if both variables are stationary at level, first difference, or second difference. To evaluate this, Augmented Dickey-Fuller (ADF) tests are conducted. These statistic tests investigate whether there is presence of at least one unit root in a time series. If there is a unit root, the series is not stationary. The null hypothesis of the time series having a unit root can be rejected only if the obtained t-statistic is lower than the 5% confidence level critical value. There are three models that must be assessed one by one to check for unit roots; model 3 which takes into consideration the trend and the intercept of an OLS linear regression, model 2 which takes only the intercept or constant, and model 1 that does not take them into consideration. First, an ADF test is conducted for IPC with model 3 (trend and intercept) at level.

Figure 2: IPC ADF Test, Level, Model 3.

Null Hypothesis: IPC has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.389762	0.3834
Test critical values:	1% level	-4.018349	
	5% level	-3.439075	
	10% level	-3.143887	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(IPC)
Method: Least Squares
Date: 04/09/23 Time: 14:07
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
IPC(-1)	-0.054431	0.022777	-2.389762	0.0181
D(IPC(-1))	0.304628	0.077585	3.926380	0.0001
C	0.120939	0.091493	1.321835	0.1882
@TREND("2010M01")	0.001636	0.000750	2.180520	0.0308

R-squared	0.121529	Mean dependent var	0.019871
Adjusted R-squared	0.104076	S.D. dependent var	0.382748
S.E. of regression	0.362283	Akaike info criterion	0.832688
Sum squared resid	19.81865	Schwarz criterion	0.911228
Log likelihood	-60.53333	Hannan-Quinn criter.	0.864589
F-statistic	6.963208	Durbin-Watson stat	1.952073
Prob(F-statistic)	0.000203		

The t-statistic of the trend is 2.18. Since this is lower than the critical value of 2.79, obtained from the ADF critical values table, the trend is non-significant. It now follows to move to model 2 (model with intercept).

Figure 3: IPC ADF Test, Level, Model 2.

Null Hypothesis: IPC has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.496525	0.5329
Test critical values:	1% level	-3.472813	
	5% level	-2.880088	
	10% level	-2.576739	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(IPC)
Method: Least Squares
Date: 04/09/23 Time: 14:10
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
IPC(-1)	-0.030067	0.020091	-1.496525	0.1366
D(IPC(-1))	0.306063	0.078534	3.897184	0.0001
C	0.144126	0.091989	1.566784	0.1192

R-squared	0.093868	Mean dependent var	0.019871
Adjusted R-squared	0.081945	S.D. dependent var	0.382748
S.E. of regression	0.366731	Akaike info criterion	0.850787
Sum squared resid	20.44270	Schwarz criterion	0.909692
Log likelihood	-62.93601	Hannan-Quinn criter.	0.874713
F-statistic	7.873001	Durbin-Watson stat	1.943102
Prob(F-statistic)	0.000558		

The t-statistic of the intercept 1.57 is lower than the critical value 2.54. The intercept is non-significant. It is necessary to move now to model 1 (model with no intercept and no trend).

Figure 4: IPC ADF Test, Level, Model 1.

Null Hypothesis: IPC has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.038836	0.6683
Test critical values:	1% level	-2.579967	
	5% level	-1.942896	
	10% level	-1.615342	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(IPC)
Method: Least Squares
Date: 04/09/23 Time: 14:13
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
IPC(-1)	-0.000251	0.006475	-0.038836	0.9691
D(IPC(-1))	0.285404	0.077787	3.669051	0.0003

R-squared	0.079234	Mean dependent var	0.019871
Adjusted R-squared	0.073216	S.D. dependent var	0.382748
S.E. of regression	0.368470	Akaike info criterion	0.853905
Sum squared resid	20.77285	Schwarz criterion	0.893175
Log likelihood	-64.17764	Hannan-Quinn criter.	0.869856
Durbin-Watson stat	1.934650		

It is now time to assess whether there is presence of a unit root in the time series IPC. The t-statistic is -0.039. Since this is higher than the critical value at a 5% confidence level -1.943 (see first table above), it is possible to say that IPC contains at least one unit root. Therefore, IPC is not stationary at level. It is necessary to do this same process testing IPC at first difference.

Figure 5: IPC ADF Test, First Difference, Model 3.

Null Hypothesis: D(IPC) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.350052	0.0000
Test critical values:	1% level	-4.018349	
	5% level	-3.439075	
	10% level	-3.143887	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(IPC,2)
Method: Least Squares
Date: 04/09/23 Time: 14:18
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(IPC(-1))	-0.726275	0.077676	-9.350052	0.0000
C	-0.045837	0.060076	-0.762979	0.4467
@TREND("2010M01")	0.000757	0.000664	1.139463	0.2563

R-squared	0.365208	Mean dependent var	-0.001806
Adjusted R-squared	0.356856	S.D. dependent var	0.458693
S.E. of regression	0.367855	Akaike info criterion	0.856908
Sum squared resid	20.56821	Schwarz criterion	0.915813
Log likelihood	-63.41039	Hannan-Quinn criter.	0.880834
F-statistic	43.72426	Durbin-Watson stat	1.933755
Prob(F-statistic)	0.000000		

Starting with model 3 (trend and intercept), it is visible that the t-statistic of the trend, 1.14, is lower than the critical value 2.79. This means the trend is non-significant and that it is necessary to move to model 2 (intercept).

Figure 6: IPC ADF Test, First Difference, Model 2.

Null Hypothesis: D(IPC) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.272673	0.0000
Test critical values:	1% level	-3.472813	
	5% level	-2.880088	
	10% level	-2.576739	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(IPC,2)
Method: Least Squares
Date: 04/09/23 Time: 14:21
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(IPC(-1))	-0.716979	0.077322	-9.272673	0.0000
C	0.013736	0.029623	0.463686	0.6435

R-squared	0.359786	Mean dependent var	-0.001806
Adjusted R-squared	0.355601	S.D. dependent var	0.458693
S.E. of regression	0.368213	Akaike info criterion	0.852511
Sum squared resid	20.74390	Schwarz criterion	0.891781
Log likelihood	-64.06958	Hannan-Quinn criter.	0.868461
F-statistic	85.98247	Durbin-Watson stat	1.933641
Prob(F-statistic)	0.000000		

The t-statistic of the intercept (0.46) is lower than the critical value (2.54). Therefore, the constant is non-significant, and it is necessary to move to model 1.

Figure 7: IPC ADF Test, First Difference, Model 1.

Null Hypothesis: D(IPC) has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.284969	0.0000
Test critical values:	1% level	-2.579967	
	5% level	-1.942896	
	10% level	-1.615342	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(IPC,2)
Method: Least Squares
Date: 04/09/23 Time: 14:27
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(IPC(-1))	-0.714950	0.077001	-9.284969	0.0000

R-squared	0.358886	Mean dependent var	-0.001806
Adjusted R-squared	0.358886	S.D. dependent var	0.458693
S.E. of regression	0.367274	Akaike info criterion	0.841012
Sum squared resid	20.77305	Schwarz criterion	0.860647
Log likelihood	-64.17841	Hannan-Quinn criter.	0.848987
Durbin-Watson stat	1.934508		

Testing the presence of at least one unit root in the series IPC, it is found that the t-statistic of -9.28 is smaller than the critical value at a 5% confidence level, which means the IPC series is stationary at first difference. That is, IPC is I(1).

Because IPC is I(1), o be able to make a VAR model with UNEMP and IPC, UNEMP must be I(1) as well. That is, they must be integrated of the same order. The same process that was carried out to define the order of integration of IPC will now be carried out for UNEMP.

<u>Figure 8: UNEMP ADF Test, Level, Model 3.</u>

Null Hypothesis: UNEMP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.013190	0.5892
Test critical values:	1% level	-4.018349	
	5% level	-3.439075	
	10% level	-3.143887	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNEMP)
Method: Least Squares
Date: 04/09/23 Time: 14:33
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UNEMP(-1)	-0.079863	0.039670	-2.013190	0.0459
D(UNEMP(-1))	-0.434457	0.073113	-5.942262	0.0000
C	0.407537	0.211278	1.928914	0.0556
@TREND("2010M01")	-0.001130	0.000644	-1.755435	0.0812

R-squared	0.247568	Mean dependent var	-0.014303
Adjusted R-squared	0.232619	S.D. dependent var	0.262739
S.E. of regression	0.230160	Akaike info criterion	-0.074614
Sum squared resid	7.999037	Schwarz criterion	0.003926
Log likelihood	9.782588	Hannan-Quinn criter.	-0.042713
F-statistic	16.56086	Durbin-Watson stat	2.044542
Prob(F-statistic)	0.000000		

Starting with model 3 (trend and intercept) at level, it is possible to see that the t-statistic of the trend, -1.75, is smaller than the critical value 2.79. The trend is therefore non-significant. Model 2 (intercept) shall now be inspected.

Figure 9: UNEMP ADF Test, Level, Model 2.

Null Hypothesis: UNEMP has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.032195	0.7410
Test critical values:	1% level	-3.472813	
	5% level	-2.880088	
	10% level	-2.576739	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNEMP)
Method: Least Squares
Date: 04/09/23 Time: 14:37
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UNEMP(-1)	-0.026458	0.025633	-1.032195	0.3036
D(UNEMP(-1))	-0.460868	0.072037	-6.397696	0.0000
C	0.090863	0.110730	0.820579	0.4132

R-squared	0.232213	Mean dependent var	-0.014303
Adjusted R-squared	0.222110	S.D. dependent var	0.262739
S.E. of regression	0.231731	Akaike info criterion	-0.067315
Sum squared resid	8.162278	Schwarz criterion	-0.008410
Log likelihood	8.216917	Hannan-Quinn criter.	-0.043389
F-statistic	22.98574	Durbin-Watson stat	2.060055
Prob(F-statistic)	0.000000		

The t-statistic of the constant is 0.82. This is lower than the critical value 2.54, so the constant is non-significant. It is required to move now to model 1.

Figure 10: UNEMP ADF Test, Level, Model 1.

Null Hypothesis: UNEMP has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.327661	0.1700
Test critical values:	1% level	-2.579967	
	5% level	-1.942896	
	10% level	-1.615342	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNEMP)
Method: Least Squares
Date: 04/09/23 Time: 14:41
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UNEMP(-1)	-0.005725	0.004312	-1.327661	0.1863
D(UNEMP(-1))	-0.472155	0.070636	-6.684372	0.0000

R-squared	0.228811	Mean dependent var	-0.014303
Adjusted R-squared	0.223771	S.D. dependent var	0.262739
S.E. of regression	0.231483	Akaike info criterion	-0.075798
Sum squared resid	8.198436	Schwarz criterion	-0.036528
Log likelihood	7.874356	Hannan-Quinn criter.	-0.059848
Durbin-Watson stat	2.071365		

Testing for unit roots in UNEMP at level, since the t-statistic, -1.33, is higher than the critical value at a 5% confidence level, -1.94, it is possible to state that the UNEMP series is not stationary at level. Same as before, it is now necessary to move to the first difference.

Figure 11: UNEMP ADF Test, First Difference, Model 3.

Null Hypothesis: D(UNEMP) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.75944	0.0000
Test critical values:	1% level	-4.018349	
	5% level	-3.439075	
	10% level	-3.143887	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNEMP,2)
Method: Least Squares
Date: 04/09/23 Time: 14:45
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(UNEMP(-1))	-1.474661	0.071036	-20.75944	0.0000
C	-0.011044	0.037899	-0.291404	0.7711
@TREND("2010M01")	-0.000136	0.000417	-0.326339	0.7446

R-squared	0.739278	Mean dependent var	0.001496
Adjusted R-squared	0.735847	S.D. dependent var	0.452294
S.E. of regression	0.232460	Akaike info criterion	-0.061031
Sum squared resid	8.213736	Schwarz criterion	-0.002126
Log likelihood	7.729865	Hannan-Quinn criter.	-0.037105
F-statistic	215.4977	Durbin-Watson stat	2.074317
Prob(F-statistic)	0.000000		

Because the t-stat of the trend is -0.33, and this is lower than the critical value 2.79, it is concluded that the trend is not significant. Next step is to evaluate model 2 (intercept).

Figure 12: UNEMP ADF Test, First Difference, Model 2.

Null Hypothesis: D(UNEMP) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.81874	0.0000
Test critical values:	1% level	-3.472813	
	5% level	-2.880088	
	10% level	-2.576739	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNEMP,2)
Method: Least Squares
Date: 04/09/23 Time: 14:48
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(UNEMP(-1))	-1.474522	0.070827	-20.81874	0.0000
C	-0.021800	0.018651	-1.168870	0.2443

R-squared	0.739095	Mean dependent var	0.001496
Adjusted R-squared	0.737390	S.D. dependent var	0.452294
S.E. of regression	0.231780	Akaike info criterion	-0.073233
Sum squared resid	8.219491	Schwarz criterion	-0.033963
Log likelihood	7.675585	Hannan-Quinn criter.	-0.057283
F-statistic	433.4200	Durbin-Watson stat	2.073121
Prob(F-statistic)	0.000000		

Once again, the t-stat of the constant, -1.17, is lower than the critical value 2.54. Therefore, the constant is non-significant. It is necessary to move to model 1 (no intercept and no trend).

Figure 13: UNEMP ADF Test, First Difference, Model 1.

Null Hypothesis: D(UNEMP) has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.76138	0.0000
Test critical values:	1% level	-2.579967	
	5% level	-1.942896	
	10% level	-1.615342	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNEMP,2)
Method: Least Squares
Date: 04/09/23 Time: 14:51
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(UNEMP(-1))	-1.469555	0.070783	-20.76138	0.0000

R-squared	0.736765	Mean dependent var	0.001496
Adjusted R-squared	0.736765	S.D. dependent var	0.452294
S.E. of regression	0.232056	Akaike info criterion	-0.077246
Sum squared resid	8.292889	Schwarz criterion	-0.057611
Log likelihood	6.986598	Hannan-Quinn criter.	-0.069271
Durbin-Watson stat	2.064027		

Testing for unit roots, it is visible that the t-statistic, -20.76, is lower than the critical value at a 5% confidence level, -1.94. The series does not contain a unit root at first difference. Therefore, UNEMP is stationary at first difference, or, I(1).

Since UNEMP and IPC are both I(1), one of the two main assumptions of the VAR model has been satisfied. Now, it is required to test whether UNEMP and IPC are cointegrated. If they are not, it is possible to proceed to create the VAR model using the variables $\Delta\mathrm{UNEMP}$ and $\Delta\mathrm{IPC}$ (or as expressed in EViews, d(UNEMP) and d(IPC)). If they are cointegrated, it is not possible to make a VAR model using UNEMP and IPC; a Vector Error Correction Model should be used instead.

To find if UNEMP and IPC are cointegrated or not, it is necessary to evaluate if the residuals of a simple regression of the variables are stationary. If the residuals are not stationary at level, UNEMP and IPC are not cointegrated and it would be possible to move on to make the VAR model. Below is the result of the simple OLS linear regression $UNEMP_t = c + \alpha IPC_t + e_t$.

Figure 14: Simple Linear Regression $\boldsymbol{UNEMP_t = c + \alpha IPC_t + e_t}$ statistics.

Dependent Variable: UNEMP
Method: Least Squares
Date: 04/09/23 Time: 15:08
Sample: 2010M01 2023M01
Included observations: 157

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.306465	0.160419	33.07876	0.0000
IPC	-0.241658	0.034644	-6.975439	0.0000

R-squared	0.238915	Mean dependent var	4.248834
Adjusted R-squared	0.234005	S.D. dependent var	0.750078
S.E. of regression	0.656477	Akaike info criterion	2.008799
Sum squared resid	66.79914	Schwarz criterion	2.047732
Log likelihood	-155.6907	Hannan-Quinn criter.	2.024611
F-statistic	48.65675	Durbin-Watson stat	0.177005
Prob(F-statistic)	0.000000		

Saving the residuals of this regression and applying an ADF test at level with model 1 (no trend and no intercept), the following results are obtained.

Figure 15: Residuals of the Linear Regression ADF Test.

Null Hypothesis: RESID01 has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.109443	0.0339
Test critical values:	1% level	-2.579967	
	5% level	-1.942896	
	10% level	-1.615342	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RESID01)
Method: Least Squares
Date: 04/09/23 Time: 15:10
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RESID01(-1)	-0.067415	0.031959	-2.109443	0.0365
D(RESID01(-1))	-0.347434	0.075025	-4.630932	0.0000

R-squared	0.166418	Mean dependent var	-0.009501
Adjusted R-squared	0.160970	S.D. dependent var	0.276403
S.E. of regression	0.253182	Akaike info criterion	0.103400
Sum squared resid	9.807444	Schwarz criterion	0.142670
Log likelihood	-6.013511	Hannan-Quinn criter.	0.119351
Durbin-Watson stat	2.002087		

This time, the same critical values as with the previous ADF tests must not be employed. This time, it is necessary to use the table of Engle and Yoo (1987) to evaluate whether the residual series is stationary or not. First, however, it is necessary to make a linear interpolation of the table's values because EViews suggests a lag of 1 (see "Lag Length" above) and the table only contains the values for 0 and 4 lags.

$$p = 0 \rightarrow \; -3.37$$

$$p = 1 \rightarrow \; x$$

$$p = 4 \rightarrow \; -3.17$$

$$\frac{1-0}{4-0} = \frac{x-(-3.37)}{-3.17-(-3.37)}$$

$$\frac{1}{4} = \frac{x + \; 3.37}{0.2}$$

$$x = \; -3.32$$

$$\therefore p = 1 \rightarrow -3.32$$

It is found that for 1 lag, the critical value is -3.32. The t-statistic of the residual series is -2.11. Because -2.11 is greater than -3.32, the residual series is not stationary. Therefore, UNEMP and IPC are not cointegrated and a VAR model can be estimated.

Figure 16: Lag Order Selection Criteria.

VAR Lag Order Selection Criteria
Endogenous variables: D(UNEMP) D(IPC)
Exogenous variables: C
Date: 04/09/23 Time: 15:34
Sample: 2010M01 2023M01
Included observations: 148

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-81.05520	NA	0.010531	1.122368	1.162870	1.138824
1	-55.32137	50.42439*	0.007851*	0.828667*	0.950176*	0.878036*
2	-52.38729	5.669919	0.007966	0.843071	1.045586	0.925353
3	-48.43475	7.531178	0.007971	0.843713	1.127233	0.958906
4	-46.72493	3.211693	0.008223	0.874661	1.239187	1.022767
5	-43.62595	5.737310	0.008326	0.886837	1.332369	1.067856
6	-39.33079	7.835752	0.008296	0.882849	1.409386	1.096779
7	-36.22331	5.585067	0.008401	0.894910	1.502453	1.141753
8	-35.31561	1.606870	0.008765	0.936697	1.625246	1.216453

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Inspecting the VAR order selection criteria, it is seen that all the information criteria (LR, FPE, AIC, SC, and HQ) agree that 1 lag should be used to carry out the VAR model. This means that the equations will have the following structure:

Equation 1:

$$\Delta UNEMP_t = \alpha_{UNEMP} + \phi_{UNEMP1} * \Delta UNEMP_{t-1} + \beta_{IPC1} * \Delta IPC_{t-1} + e_{UNEMPt}$$

Equation 2:

$$\Delta IPC_t = \alpha_{IPC} + \phi_{UNEMP1} * \Delta UNEMP_{t-1} + \beta_{IPC1} * \Delta IPC_{t-1} + e_{IPCt}$$

Below is the estimation of the VAR(1) model using UNEMP and IPC both at first difference.

Figure 17: Vector Autoregressive Model Estimation.

Vector Autoregression Estimates
Date: 04/09/23 Time: 15:37
Sample (adjusted): 2010M03 2023M01
Included observations: 155 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	D(UNEMP)	D(IPC)
D(UNEMP(-1))	-0.475392	0.120492
	(0.07105)	(0.11252)
	[-6.69053]	[1.07089]
D(IPC(-1))	-0.019635	0.285542
	(0.04883)	(0.07732)
	[-0.40213]	[3.69297]
C	-0.021388	0.015585
	(0.01873)	(0.02966)
	[-1.14193]	[0.52546]
R-squared	0.227653	0.087402
Adj. R-squared	0.217490	0.075395
Sum sq. resids	8.210756	20.58856
S.E. equation	0.232418	0.368037
F-statistic	22.40131	7.278767
Log likelihood	7.757991	-63.48705
Akaike AIC	-0.061393	0.857897
Schwarz SC	-0.002488	0.916802
Mean dependent	-0.014303	0.019871
S.D. dependent	0.262739	0.382748

Determinant resid covariance (dof adj.)	0.007313
Determinant resid covariance	0.007033
Log likelihood	-55.68875
Akaike information criterion	0.795984
Schwarz criterion	0.913794
Number of coefficients	6

With the previous estimates, it is possible to construct the two equations (which could be used for forecasts) of the VAR(1) model.

Equation 1:

$$\Delta UNEMP_t = -0.021388 - 0.475392 * \Delta UNEMP_{t-1} - 0.019635 * \Delta IPC_{t-1} + e_{UNEMPt}$$

Equation 2:

$$\Delta IPC_t = 0.015585 + 0.120492 * \Delta UNEMP_{t-1} + 0.285542 * \Delta IPC_{t-1} + e_{IPCt}$$

Equation 1 implies that for every additional percentage point in the change of unemployment rate one month before, the current change of unemployment rate would lower 0.475392 percentage points. This could suggest that unemployment in Mexico tends to have a natural decrease as time goes by and that the unemployment rate does not severely resent increases in the past. Equation 1 also suggests that for every additional percentage point in the change of inflation rate one month before, the current change of unemployment rate would lower 0.019635 percentage points. This is consistent with the theory; as inflation increases, employers find it more difficult to pay inflation-adjusted wages to their employees and may have to let some of them go in the near future, increasing unemployment rates. The intercept of Equation 1 could be hinting that the natural rate of unemployment tends to be low.

Equation 2 on the other hand, yields different results. All the coefficients are positive. Equation 2 suggests that for every additional percentage point in the change in unemployment one month before, it corresponds an increase in the change of current inflation rate of 0.120492 percentage points. This goes against macroeconomic theory. As unemployment increases, the demand for goods and services is expected to decrease, and with it, inflation as well. This can be perhaps explained by the fact that the dataset includes a black-swan event (the COVID-19 pandemic) that could be creating some noise because of the increase in unemployment it created added to the high inflation rates Mexico has been experiencing in recent years mainly due to political instability. Additionally, Equation 2 suggests that for every additional percentage point in change in the inflation rate one month before it corresponds an increase of 0.285542 percentage points to the change of the current inflation rate. Inflation is known for not being easy to bring back to a certain level since there are a lot of factors that affect it, because of this, inflation can resent increases in the past and keep naturally increasing unless strong measures are taken against it.

The two previous paragraphs are related to the interpretation of the coefficients in the VAR model assuming they were significant. However, inspecting the t-statistics (in []), it is seen that none of the coefficients are significant as all their corresponding t-statistics are all lower than the 5% significance critical value, 1.96. This suggests that the effect unemployment has on inflation and the effect that inflation has on unemployment in Mexico is insignificant. Consequently, it is expected that the Impulse Response Functions and Variance Decomposition analyses will show that shocks in one variable are not followed by significant responses on the other, and that the composition of the variance is mostly attributed to the own variable under study.

IMPULSE RESPONSE FUNCTIONS (IRFs)

IRFs show how a perturbation, innovation, or shock in one variable in the VAR system affects the other variables (and itself) over time. They are useful in the field of economics because they can offer a forecasting insight on what will happen in future periods after a macroeconomic perturbation takes place. Economists can use this as a base for their policy making decisions according to what the economy needs.

To obtain the IRFs, the variables have been Cholesky-ordered d(IPC) d(UNEMP) (IPC is shocked, UNEMP has a response) because while it is true that in economic theory both variables influence each other, shocks on inflation can have a more direct effect on unemployment than shocks on unemployment on inflation. Below are the impulse response values to a shock of one standard deviation and their standard errors over the following 10 months. The graphs below them visually present these responses.

Figure 18: Responses to Shocks for 10 Months.

Response of D(UNEMP):

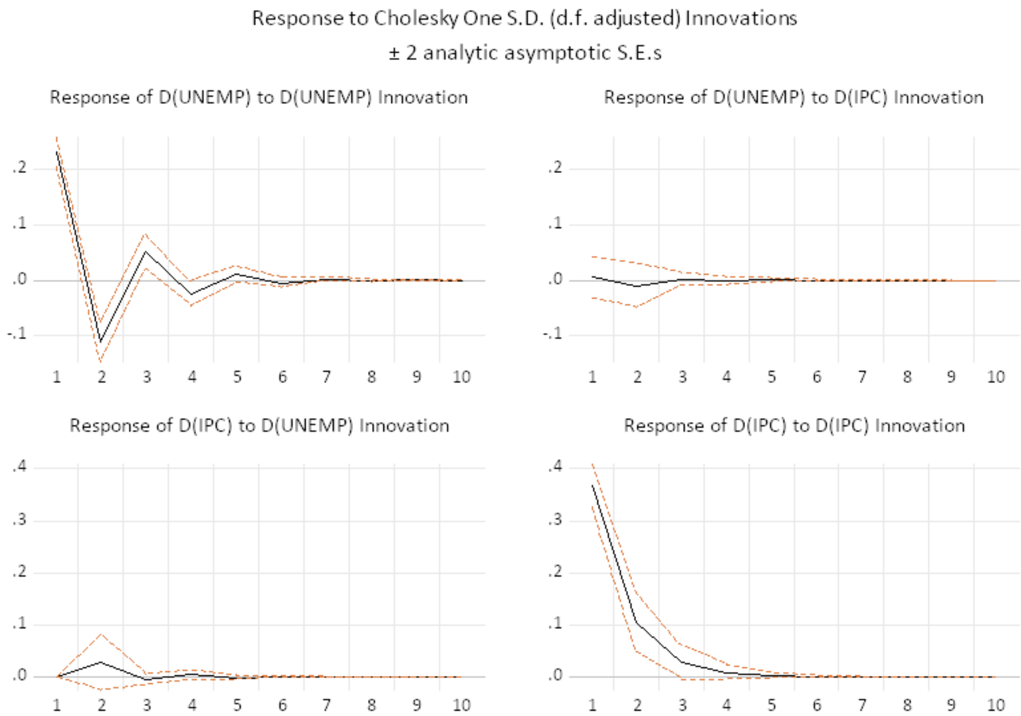
Period	D(UNEMP)	D(IPC)
1	0.232358	0.005300
	(0.01320)	(0.01867)
2	-0.110461	-0.009746
	(0.01766)	(0.02006)
3	0.051962	0.002557
	(0.01601)	(0.00550)
4	-0.024598	-0.001785
	(0.01123)	(0.00364)
5	0.011601	0.000680
	(0.00706)	(0.00144)
6	-0.005483	-0.000367
	(0.00416)	(0.00077)
7	0.002588	0.000160
	(0.00236)	(0.00035)
8	-0.001223	-7.94E-05
	(0.00130)	(0.00018)
9	0.000577	3.65E-05
	(0.00070)	(8.4E-05)
10	-0.000273	-1.75E-05
	(0.00037)	(4.1E-05)

Response of D(IPC):

Period	D(UNEMP)	D(IPC)
1	0.000000	0.368037
	(0.00000)	(0.02090)
2	0.027997	0.105729
	(0.02619)	(0.02919)
3	-0.005315	0.029016
	(0.00568)	(0.01658)
4	0.004743	0.008593
	(0.00456)	(0.00687)
5	-0.001609	0.002239
	(0.00179)	(0.00266)
6	0.000938	0.000721
	(0.00102)	(0.00091)
7	-0.000393	0.000162
	(0.00049)	(0.00033)
8	0.000200	6.55E-05
	(0.00026)	(0.00010)
9	-9.03E-05	9.13E-06
	(0.00013)	(3.8E-05)
10	4.38E-05	7.00E-06
	(6.7E-05)	(1.2E-05)

Cholesky One S.D. (d.f. adjusted)
Cholesky ordering: D(IPC) D(UNEMP)
Standard errors: Analytic

Figure 19: Graphs of the Responses to Shocks for 10 Months.



Response of d(UNEMP) to d(UNEMP) shock: The responses of UNEMP to shocks of one standard deviation on UNEMP shift direction and diminish in magnitude in every month. At the beginning, a shock in the unemployment causes it to rise 23.23% in the next month, then it makes it decrease 11.05% in the second month, then increase again 5.20% in the third month and so on.

Response of d(UNEMP) to d(IPC) shock: Inflation does not seem to have a very strong effect over unemployment. The most significant change occurs in the second month after the shock in inflation when unemployment decreases 0.97%, but still, this is very low. Additionally, as the standard error bars are never on the same sign of the vertical axis, this would suggest that the effect of a shock to UNEMP is not significant to how IPC behaves at any future month.

Response of d(IPC) to d(UNEMP) shock: The effect of a shock in unemployment to inflation also appears to be negligible. In the first period inflation has a 0.00% response to this shock; it remains unaltered. In the second month it rises 2.80% and the effect proceeds to die out in the following months. Similar to the previous graph, the standard error bars are never on the same side which suggests the effect is never significant. Same as in the Equation 2 interpretation, this is inconsistent with the macroeconomic theory. A sudden increase in unemployment should reduce inflation in theory. This again, could be due to the noise in the dataset created by the simultaneous increase of unemployment and inflation rates that started in June of 2020 due to COVID-19.

Response of d(IPC) to d(IPC) shock: A sudden increase in inflation will drive it up in the following month by 36.80% but this effect will lower as the months pass, and by the third one it will become insignificant (standard error bars on opposite signs from the third month onward).

VARIANCE DECOMPOSITION

Variance decomposition allows researchers to see the different components that make up the variance of a variable. It signals which portions of the variance of a variable are explained by shocks in other variables or in itself. The same Cholesky ordering has been kept, d(IPC) d(UNEMP). Below are the results of the variance decompositions of unemployment and inflation in addition to the graph that represents them.

Figure 20: Variance Decompositions for 10 Months.

Variance Decomposition of D(UNEMP):

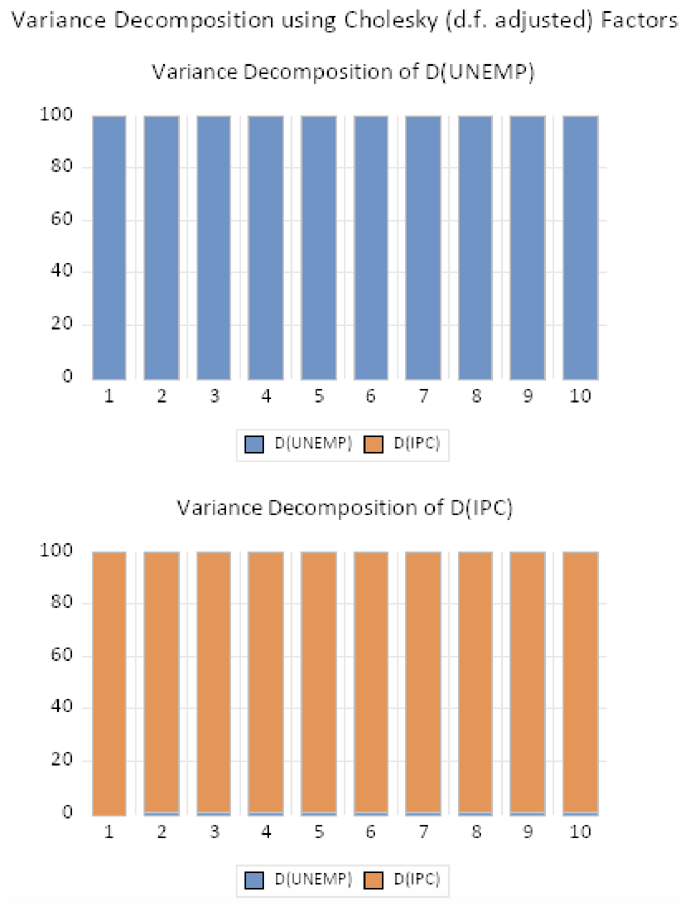
Period	S.E.	D(UNEMP)	D(IPC)
1	0.232418	99.94800	0.051995
2	0.257516	99.81441	0.185586
3	0.262719	99.81222	0.187782
4	0.263874	99.80928	0.190719
5	0.264130	99.80899	0.191013
6	0.264187	99.80888	0.191124
7	0.264200	99.80886	0.191142
8	0.264203	99.80885	0.191147
9	0.264203	99.80885	0.191148
10	0.264204	99.80885	0.191148

Variance Decomposition of D(IPC):

Period	S.E.	D(UNEMP)	D(IPC)
1	0.368037	0.000000	100.0000
2	0.383945	0.531733	99.46827
3	0.385076	0.547666	99.45233
4	0.385201	0.562474	99.43753
5	0.385211	0.564190	99.43581
6	0.385213	0.564778	99.43522
7	0.385213	0.564882	99.43512
8	0.385213	0.564908	99.43509
9	0.385213	0.564914	99.43509
10	0.385213	0.564915	99.43508

Cholesky One S.D. (d.f. adjusted)
Cholesky ordering: D(IPC) D(UNEMP)

Figure 21: Graphs of Variance Decompositions for 10 Months.



By looking at the graph, it is possible to see that the variance of past changes in unemployment explain current changes in unemployment more than past changes in inflation do. Likewise, it would appear that the variance of changes in inflation can be better explained by past changes in inflation than by past changes in unemployment.

Looking at the tables, it is possible to confirm this by obtaining that, within our bivariate system, across all periods, an unemployment shock explains the fluctuations of unemployment in more than a 99%, while an inflation shock explains them in less than a 1%. The same happens with inflation. An unemployment shock does not explain inflation fluctuations to even a 1%. Therefore, it could be said that shocks in inflation are insignificant to explain unemployment fluctuations and shocks in unemployment are insignificant to explain inflation. This is consistent with the non-significant coefficients obtained for the VAR equations (Figure 17).

The turnout of this investigation was not as expected. Macroeconomic theory would suggest that inflation and unemployment negatively affect each other. However (and added to the previously discussed noise in the dataset), this may be explained by the fact that the theory clearly states that this relationship is given only under the assumption that all else remains constant. In practice, by contrary, all else does not remain constant. Different variables that also affect inflation and unemployment are constantly shifting, causing them to rise or diminish as well. The VAR model specification could benefit from adding other variables that affect inflation and unemployment such as aggregate demand and supply, and those related to fiscal and monetary policy such as government spending, subsidies, tax cuts, and interest rates. Nevertheless, this project was extremely useful to understand through practice how a Vector Autoregressive model can be used to obtain useful information about how variables in the system interact with each other.

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APPENDIX

Date	IPC	UNEMP
2010/01	4.46	5.412093791527
2010/02	4.83	5.112132068491
2010/03	4.97	5.148934641362
2010/04	4.27	5.457441578861
2010/05	3.92	5.251889106409
2010/06	3.69	5.137006992015
2010/07	3.64	5.293886492263
2010/08	3.68	5.015294979221
2010/09	3.7	5.28666908463
2010/10	4.02	5.332095628577
2010/11	4.32	5.309140273941
2010/12	4.4	5.511198943136
2011/01	3.78	5.031715947409
2011/02	3.57	5.261911924004
2011/03	3.04	5.110606742284
2011/04	3.36	5.199242279531
2011/05	3.25	5.366311144431
2011/06	3.28	5.555860543987
2011/07	3.55	5.205172484915
2011/08	3.42	5.289033423659
2011/09	3.14	5.080700129975
2011/10	3.2	4.874350780876

2011/11	3.48	5.056712788404
2011/12	3.82	5.023154187495
2012/01	4.05	4.631663675354
2012/02	3.87	5.287876149234
2012/03	3.73	5.019194610298
2012/04	3.41	4.908619243709
2012/05	3.85	4.77494617305
2012/06	4.34	4.819326654915
2012/07	4.42	4.745887078072
2012/08	4.57	4.942268721628
2012/09	4.77	4.59948662222
2012/10	4.6	4.912709904738
2012/11	4.18	5.141098209095
2012/12	3.57	4.898195593553
2013/01	3.25	5.165825988067
2013/02	3.55	4.808038161528
2013/03	4.25	4.97503929463
2013/04	4.65	5.018222043958
2013/05	4.63	4.967944799304
2013/06	4.09	5.036459468133
2013/07	3.47	4.861596736194
2013/08	3.46	4.821751707879
2013/09	3.39	4.978124572307
2013/10	3.36	4.895264648375
2013/11	3.62	4.567463746352
2013/12	3.97	4.737595936507
2014/01	4.48	4.888530276393
2014/02	4.23	4.775032690298
2014/03	3.76	5.328804777432
2014/04	3.5	4.925488375229
2014/05	3.51	4.970041402519
2014/06	3.75	4.838715759076
2014/07	4.07	5.201401865745
2014/08	4.15	4.851568249362
2014/09	4.22	4.789449809325
2014/10	4.3	4.655394979375
2014/11	4.17	4.635106731584
2014/12	4.08	4.149330804144
2015/01	3.07	4.37493180498
2015/02	3	4.446168984031
2015/03	3.14	4.279133159478
2015/04	3.06	4.354716993745
2015/05	2.88	4.431219273392
2015/06	2.87	4.41998392793
2015/07	2.74	4.470709785413
2015/08	2.59	4.388915676709
2015/09	2.52	4.192252595561
2015/10	2.48	4.437505968229
2015/11	2.21	4.044422560241
2015/12	2.13	4.299957409233
2016/01	2.61	4.159052928027
2016/02	2.87	4.274580509555
2016/03	2.6	4.147269383134
2016/04	2.54	3.826754417458
2016/05	2.6	3.995256379594
2016/06	2.54	3.921462793706
2016/07	2.65	3.796626118353
2016/08	2.73	3.760530112421

2016/09	2.97	3.842213500133
2016/10	3.06	3.583078881276
2016/11	3.31	3.575722335844
2016/12	3.36	3.624656461533
2017/01	4.72	3.533619429896
2017/02	4.86	3.458021944247
2017/03	5.35	3.544040576837
2017/04	5.82	3.480385341876
2017/05	6.16	3.548830015606
2017/06	6.31	3.277721551178
2017/07	6.44	3.249499116527
2017/08	6.66	3.330649977067
2017/09	6.35	3.338708510392
2017/10	6.37	3.445369191286
2017/11	6.63	3.490280743099
2017/12	6.77	3.347457581248
2018/01	5.55	3.289568256712
2018/02	5.34	3.291649009149
2018/03	5.04	3.272680087284
2018/04	4.55	3.394032307184
2018/05	4.51	3.247385180926
2018/06	4.65	3.359852710888
2018/07	4.81	3.295713018151
2018/08	4.9	3.269736870646
2018/09	5.02	3.331417561497
2018/10	4.9	3.205116912151
2018/11	4.72	3.33663302818
2018/12	4.83	3.55954799616
2019/01	4.37	3.464729114664
2019/02	3.94	3.413680640907
2019/03	4	3.6257376423
2019/04	4.41	3.503904460026
2019/05	4.28	3.559063091483
2019/06	3.95	3.544221015691
2019/07	3.78	3.529340633572
2019/08	3.16	3.484668546431
2019/09	3	3.518954292121
2019/10	3.02	3.618901459423
2019/11	2.97	3.531152519591
2019/12	2.83	3.077137011107
2020/01	3.24	3.656420581402
2020/02	3.7	3.65116119791
2020/03	3.25	3.296623748276
2020/04	2.15	4.712113850416
2020/05	2.84	4.289244829442
2020/06	3.33	5.458410118296
2020/07	3.62	5.024079534746
2020/08	4.05	4.904290862999
2020/09	4.01	4.790434898578
2020/10	4.09	4.582297975003
2020/11	3.33	4.519840262519
2020/12	3.15	4.029458016413
2021/01	3.54	4.537568453159
2021/02	3.76	4.526181296187
2021/03	4.67	4.426930054486
2021/04	6.08	4.707903592975
2021/05	5.89	4.09951872373
2021/06	5.88	3.9911243772

2021/07	5.81	4.098116215391
2021/08	5.59	4.00242294022
2021/09	6	3.913655957563
2021/10	6.24	3.824452039669
2021/11	7.37	3.827448888404
2021/12	7.36	3.752504458788
2022/01	7.07	3.572096880339
2022/02	7.28	3.838853603715
2022/03	7.45	3.409197298871
2022/04	7.68	3.084445576291
2022/05	7.65	3.360672505373
2022/06	7.99	3.324403351807
2022/07	8.15	3.205780262527
2022/08	8.7	3.250087307506
2022/09	8.7	3.122354816135
2022/10	8.41	3.179290551324
2022/11	7.8	2.976560763091
2022/12	7.82	2.963236660844
2023/01	7.91	2.89514009344