

ASSIGNMENT 1

Date: _____

Ali-Ur-Rehman
G7267 (cs)

LINEAR ALGEBRA

MON TUE WED (12:30 to 1:25)

QUESTION 1

$$Q.1: A = \begin{vmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{vmatrix}, B = \begin{vmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ 1 & 3 & 3 \end{vmatrix}$$

- 1) $A+B$, 2) $A-B$, 3) $2A+3B$, 4) AB , 5) $3A-5B$, 6) BA

1) $A+B$

$$\text{Ans: } A+B = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{vmatrix}$$

2) $A-B$

$$A-B = \begin{vmatrix} 3 & -6 & -10 \\ -2 & 7 & 10 \\ 0 & -6 & -7 \end{vmatrix}$$

3) $2A+3B$

$$= \begin{vmatrix} 4 & -6 & -10 \\ -2 & 8 & 10 \\ 2 & -6 & -8 \end{vmatrix} + \begin{vmatrix} -3 & 9 & 15 \\ 3 & -9 & -15 \\ 3 & 9 & 9 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 & 5 \\ 1 & -1 & -5 \\ 5 & 3 & 1 \end{vmatrix}$$

QUESTION 1

4) AB

$$\begin{bmatrix} -2+3+5 & 6+9-15 & 10+15-15 \\ -3-12+15 & -3-12+15 & -5-20+15 \\ 1+4+5 & 5+15 & 5+15-12 \\ -1-3-4 & 3+9-12 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 0 & 10 \\ 9 & 0 & -10 \\ -8 & 0 & 8 \end{bmatrix}$$

5) 3A - 5B

$$\begin{array}{c|ccc} 6 & -9 & -15 & -5 & 15 & 25 \\ -3 & 12 & 15 & 5 & -15 & -25 \\ 3 & -9 & -12 & 5 & 15 & 15 \\ \hline \Rightarrow & 11 & -24 & -40 \\ & -8 & 27 & 40 \\ & -2 & -24 & -27 \end{array}$$

6) BA

$$\begin{bmatrix} -2-3+5 & 3+12-15 & 10+15-25 \\ 2+3+5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+12-15 & 5+15-20 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

QUESTION 2Determinants:

$$1) \begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix}$$

Ans:

$$1(28-30) - 0(21-25) + 2(18-20)$$

$$-2 - 4$$

$$\boxed{-6}$$

$$2) \begin{vmatrix} 6 & -6 & 6 \\ 2 & 4 & -6 \\ 10 & -5 & 5 \end{vmatrix}$$

$$6(20-30) + 6(10+60) + 6(-10-40)$$

$$6(-10) + 6(70) + 6(-50)$$

$$-60 + 420 - 300$$

$$\boxed{60}$$

$$3) \begin{vmatrix} 2 & 3 & -2 & 4 \\ 7 & 4 & -3 & 10 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{vmatrix}$$

$$2 \begin{vmatrix} 4 & -3 & 10 \\ 2 & 3 & 4 \\ 4 & 0 & 5 \end{vmatrix} - 3 \begin{vmatrix} 7 & -3 & 10 \\ 3 & 3 & 4 \\ -2 & 0 & 5 \end{vmatrix} - 2 \begin{vmatrix} 7 & 4 & 10 \\ 3 & 2 & 4 \\ -2 & 4 & 5 \end{vmatrix} + 4 \begin{vmatrix} 7 & 4 & -3 \\ 3 & 2 & 3 \\ -2 & 4 & 0 \end{vmatrix}$$

$$2(4(15-0)) + 3(10-16) + 10(0-12)$$

$$-3(7(15-0) + 3(15+8) + 10(0+6))$$

$$-2(7(10-16) - 4(15+8) + 10(12+4))$$

$$4(7(0-12) - 4(0+6) - 3(12+4))$$

$$2(60-18-120) - 3(105) + 69+60 - 2(-42-92+160) + 4(-84-24-48)$$

$$-156 - 702 - 52 - 624$$

$$\boxed{-1534}$$

Date: _____

QUESTION 2

$$4) \begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{vmatrix}$$

Ans:

$$\begin{array}{c|ccc} 1 & 1 & -1 & 1 \\ \hline 3 & 0 & -3 \\ -7 & 3 & 1 \end{array} + 2 \begin{array}{c|ccc} 0 & -1 & 1 \\ \hline 1 & 0 & -3 \\ 0 & 3 & 1 \end{array} + 3 \begin{array}{c|ccc} 0 & 1 & 1 \\ \hline 1 & 3 & -3 \\ 0 & -7 & 1 \end{array} + 4 \begin{array}{c|ccc} 0 & 1 & -1 \\ \hline 1 & 3 & 0 \\ 0 & -7 & 3 \end{array}$$

$$1(1(0+9)+1(3-21)+1(9+0))$$

$$2(0(0+9)+1(1-0)+1(3-0))$$

$$3(0(3-21)-1(1-0)+1(-7-0))$$

$$4(0(9-0)-1(3-0)-1(-7-0))$$

$$1(9+18+9)+2(0+1+3)+3(0-1-7)+4(0-3+7)$$

$$0+8-24+16$$

$$D = 0$$

DATE _____

ASSIGNMENT

DAY M T W T F S S

2-0

QUESTION: 5:

$$1) \quad 2x_1 + x_2 + 2x_3 = 1$$

$$x_1 + 2x_2 + x_3 = 8$$

$$3x_1 + x_2 - x_3 = 2$$

Ans:

Using Gauss elimination method.

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 8 \\ 3 & 1 & -1 & 2 \end{array} \right] \xrightarrow{\text{Row operations}}$$

⇒ System of equation

$$x_1 + 2x_2 + x_3 = 1$$

$$+ x_2 = 17/3$$

$$R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 8 \\ 2 & 1 & 2 & 1 \\ 3 & 1 & -1 & 2 \end{array} \right] \xrightarrow{\text{Row operations}}$$

put values of x_2 and x_3
in eq (1).

$$R_2 \rightarrow 2R_1 - R_2, R_3 \rightarrow 3R_1 - R_3$$

$$x_1 + 2 \cdot 17 - 19 = 8$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 8 \\ 0 & 3 & 0 & 17 \\ 0 & 5 & 4 & 22 \end{array} \right]$$

$$13 - 12$$

$$\cancel{x_1 + 39} - 8$$

$$4$$

$$3R_2$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 8 \\ 0 & 1 & 0 & 17/3 \\ 0 & 5 & 4 & 22 \end{array} \right]$$

$$x_1 + 34 - 19 = 8$$

$$3 - 12$$

$$x_1 + \frac{39}{4} = 8$$

$$R_3 \rightarrow 5R_2 - R_3$$

$$x_1 = -\frac{7}{4}$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 8 \\ 0 & 1 & 0 & 17/3 \\ 0 & 0 & -4 & 19/3 \end{array} \right]$$

$$\{x_1, x_2, x_3\} = \left\{ -\frac{7}{4}, \frac{17}{3}, -\frac{19}{12} \right\}$$

$$R_3 \div -4$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 8 \\ 0 & 1 & 0 & 17/3 \\ 0 & 0 & 1 & -19/12 \end{array} \right]$$

DATE _____

PRACTICE

DAY

Q-S \Rightarrow

Same question with gauss-jordan method.

as we know,

$$= \left[\begin{array}{cccc} 1 & 2 & 1 & 8 \\ 0 & 1 & 0 & 17/3 \\ 0 & 0 & 1 & -19/12 \end{array} \right]$$

$$R_1 \rightarrow R_1 - R_3$$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 2 & 0 & -4/3 \\ 0 & 1 & 0 & 17/3 \\ 0 & 0 & 1 & -19/12 \end{array} \right]$$

$$R_1 \rightarrow 2R_2 - R_1$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 77/3 \\ 0 & 1 & 0 & 17/3 \\ 0 & 0 & 1 & -19/12 \end{array} \right]$$

$$x_1 = 77/3$$

$$x_2 = 17/3$$

$$x_3 = -19/12$$

LINEAR ALGEBRA

DATE _____

ASSIGNMENT

DAY M T W T F S S

Q. 5

GAUSS ELIMINATION

$$2) \quad 2x_1 + 8x_2 + 2x_3 = 14$$

$$2x_1 + 6x_2 - x_3 = 13$$

$$2x_1 - x_2 + 2x_3 = 15$$

Ans:

$$\left[\begin{array}{ccc|c} 2 & 8 & 2 & 14 \\ 6 & 6 & -1 & 13 \\ 2 & -1 & 2 & 15 \end{array} \right]$$

$$R_1 \div 2$$

$$\left[\begin{array}{cccc} 1 & 4 & 1 & 6 \\ 0 & 1 & 0 & -0.3 \\ 0 & -1 & 2 & 15 \end{array} \right]$$

$$R_3 \div -7$$

$$\left[\begin{array}{cccc} 1 & 4 & 1 & 6 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & 1 & 4.05 \end{array} \right]$$

$$R_2 \rightarrow 6R_1 - R_2 \quad \& \quad 2R_1 - R_3$$

$$\left[\begin{array}{cccc} 1 & 4 & 1 & 6 \\ 0 & 18 & 7 & 23 \\ 0 & 9 & 0 & -3 \end{array} \right]$$

$$x_1 + 4x_2 + x_3 = 6$$

$$+ x_2 = -0.3$$

$$+ x_3 = 4.05$$

$$R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc} 1 & 4 & 1 & 6 \\ 0 & 9 & 0 & -3 \\ 0 & 18 & 7 & 23 \end{array} \right]$$

$$x_1 + 4(-0.3) + 4.05 = 6$$

$$x_1 = 6 - 2.85$$

$$x_1 = 3.15$$

$$R_2 \div 9$$

$$\left[\begin{array}{cccc} 1 & 4 & 1 & 6 \\ 0 & 1 & 0 & -0.3 \\ 0 & 18 & 7 & 23 \end{array} \right]$$

$$\{x_1, x_2, x_3\} = \{3.15, -0.3, 4.05\}$$

$$R_3 \rightarrow 18R_2 - R_3$$

$$\left[\begin{array}{cccc} 1 & 4 & 1 & 6 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & -1 & -28.4 \end{array} \right]$$

DATE _____

DA

 $\Rightarrow Q-S$

Same question with gauss jordan
as we know,

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 86 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & 1 & -2.86 \\ \hline & & & 4.05 \end{array} \right]$$

$$R_1 \rightarrow R_3 - R_1 \quad \& \quad R_1 \rightarrow R_2 - R_1$$

$$\left[\begin{array}{ccc|c} -1 & -3 & 0 & -1.95 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & 1 & 4.05 \end{array} \right]$$

$$R_1 \rightarrow 4R_2 + R_1$$

$$\left[\begin{array}{ccc|c} -1 & 0 & 0 & -3.15 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & 1 & 4.05 \end{array} \right]$$

$$-R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3.15 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & 1 & 4.05 \end{array} \right]$$

$$x_1 = 3.15$$

$$x_2 = -0.3$$

$$x_3 = 4.05$$

$$\{x_1, x_2, x_3\} = \{3.15, -0.3, 4.05\}$$

QUESTION 5

3) $5x_1 + 4x_3 + 2x_4 = 3$

$$x_1 - x_2 + 2x_3 + x_4 = 1$$

$$4x_1 + x_2 + 2x_3 = 1$$

$$x_1 + x_2 + x_3 + x_4 = 0$$

Ans: GAUSS ELIMINATION:

$$\left| \begin{array}{cccc|c} 5 & 4 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right|$$

$$R_1 \leftrightarrow R_2$$

$$\left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & 1 \\ 5 & 4 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right|$$

$$R_2 \rightarrow 5R_1 - R_2 \text{ & } R_3 \rightarrow 4R_1 - R_3 \text{ & } R_4 \rightarrow R_1 - R_4$$

$$\left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & 1 \\ 0 & -9 & 6 & 3 & 2 \\ 0 & 0 & -5 & 6 & 3 \\ 0 & -2 & 1 & 0 & 1 \end{array} \right|$$

$$R_2 \div -9$$

$$\left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & \frac{-2}{9} & \frac{1}{3} & \frac{2}{9} \\ 0 & 0 & -5 & 6 & 3 \\ 0 & -2 & 1 & 0 & 1 \end{array} \right|$$

$$R_4 \rightarrow -2R_1 + R_2$$

$$\left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & -2 & -1 & -2/9 \\ 0 & 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right|$$

Date:

⇒

GUASS JORDAN

1	-1	2	1	1
0	1	-2/3	1/3	2/9
0	0	1	6	4
0	0	0	0	7

$$R_2 \rightarrow R_2 - \frac{2}{3}R_3 \text{ & } R_1 \rightarrow 2R_3 - R_1$$

1	-1	0	1	1
0	1	0	1/2	12/7
0	0	1	3/6	-8/3
0	0	0	0	7

QUESTION 6

Date: _____

INVERSION:

$$1) \quad 2x + 3y + 3z = 5$$

$$x - 2y + z = 4$$

$$3x - y - 2z = 3$$

Ans:

$$\therefore A^{-1} = \frac{1}{|A|} \text{Adj}(A) \quad \therefore x = A^{-1} \cdot B$$

$|A|$

$$A = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} \quad |A| = 5 \quad B = \begin{vmatrix} 5 \\ 4 \\ 3 \end{vmatrix}$$

$$|A| = 2(-1+6) - 3(-2-3) + 3(-1+6) \\ = 2(5) - 3(-5) + 3(5) \\ = 10 + 15 + 15 \Rightarrow |A| = 40$$

$$\text{Adj}(A) = \begin{vmatrix} -2 & 1 & 1 & 1 & -2 \\ -1 & -2 & 3 & -2 & 3 & -1 \\ 3 & 3 & 2 & 3 & 2 & 3 \\ -1 & -2 & 3 & -2 & 3 & -1 \\ 3 & 3 & 2 & 3 & 2 & 3 \\ -2 & 1 & 1 & 1 & 3 & -1 \end{vmatrix}$$

$$\text{Adj}(A) = \begin{vmatrix} 5 & -5 & 5 \\ -3 & -13 & -11 \\ 9 & -1 & -11 \end{vmatrix}$$

$$A^{-1} = \frac{1}{40} \begin{vmatrix} 5 & -5 & 5 \\ -3 & -13 & -11 \\ 9 & -1 & -11 \end{vmatrix}$$

$x =$

$$x = \begin{vmatrix} 1/8 & -1/8 & 1/8 \\ 3/40 & -13/40 & -11/40 \\ 9/40 & -1/40 & -11/40 \end{vmatrix} \cdot \begin{vmatrix} 5 \\ 4 \\ 3 \end{vmatrix}$$

QUESTION 6

Date:

\Rightarrow

$$2) \quad x_1 + 3x_2 + 3x_3 = 5$$

$$x_1 - 2x_2 + x_3 = -4$$

$$3x_1 - x_2 - 2x_3 = 3$$

Ans:

$$\therefore X = A^{-1}B \quad \because A^{-1} = 1/|A| \times \text{adj}(A)$$

$$A = \begin{vmatrix} 1 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}, B = \begin{vmatrix} 5 \\ -4 \\ 3 \end{vmatrix}$$

$$|A| = 1(4+1) - 3(-2-3) + 3(-1+6)$$

$$|A| = 5 + 15 + 15$$

$$|A| = 35$$

$$\text{adj}(A) = \begin{vmatrix} -2 & 1 & 1 & 1 & -2 \\ -1 & -2 & 3 & -2 & 3 & -1 \\ 3 & 3 & 1 & 3 & 1 & 3 \\ -1 & -2 & 3 & -2 & 3 & -1 \\ 3 & 3 & 1 & 3 & 1 & 3 \\ -2 & 1 & 1 & 1 & 1 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -5 & 5 \\ -3 & -11 & -10 \\ 9 & -2 & -5 \end{vmatrix}$$

$$A^{-1} = \frac{1}{35} \begin{vmatrix} 5 & -5 & 5 \\ -3 & -11 & -10 \\ 9 & -2 & -5 \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} 1/7 & -1/7 & 1/7 \\ -3/35 & -11/35 & -10/35 \\ 9/35 & -11/35 & -1/7 \end{vmatrix}$$

$\times \rightarrow$

$$X = \begin{vmatrix} 1/7 & -1/7 & 1/7 & 5 \\ -3/35 & -11/35 & -10/35 & -4 \\ 9/35 & -11/35 & -1/7 & 3 \end{vmatrix}$$

QUESTION 6

3)

$$3x + y + 2z = 1$$

$$3x + 2y + z = 7$$

$$2x + y + 3z = 2$$

Ans:

$$\because X = A^{-1}B \quad \therefore A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$A = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}, B = \begin{vmatrix} 1 \\ 7 \\ 2 \end{vmatrix}$$

$$|A| = 1(6-1) - 1(9-2) + 2(3-4)$$

$$|A| = 5 - 8 - 2 = -5$$

$$\text{adj}(A) = \begin{vmatrix} 2 & 1 & 3 & 1 & 3 & 2 \\ 1 & 3 & 2 & 3 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 1 \\ 1 & 3 & 2 & 3 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 1 & 3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & 7 & -1 \\ 1 & -1 & -1 \\ -3 & -5 & -1 \end{vmatrix}$$

$$A^{-1} = \frac{1}{-5} \begin{vmatrix} 5 & 7 & -1 \\ 1 & -1 & -1 \\ -3 & -5 & -1 \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} -1 & -7/5 & 1/5 \\ -1/5 & 1/5 & 1/5 \\ 3/5 & 1 & 1/5 \end{vmatrix}$$

$$X = \begin{vmatrix} -1 & -7/5 & 1/5 & 1 \\ -1/5 & 1/5 & 1/5 & 7 \\ 3/5 & 1 & 1/5 & 2 \end{vmatrix}$$

QUESTION 4

i) $A = \begin{vmatrix} 1 & -3 & -c \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{vmatrix}$

Ans:

To show is periodic with 2.

$$\Rightarrow A^2 = I$$

$$A \times A = A^2 = \begin{vmatrix} -2 & -9 & -15 \\ 9 & 13 & 9 \\ -4 & -c & -3 \end{vmatrix}$$

$$A^2 \neq I = \begin{vmatrix} -2 & -9 & -15 \\ 9 & 13 & 9 \\ -4 & -c & -3 \end{vmatrix}$$

not periodic.

ii) $\begin{vmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{vmatrix}$

Ans

To show nilpotent,

$$A^2 = 0$$

$$A \times A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$A^2 = 0 \quad \text{Proved.}$$

QUESTION 8

$$1) \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

Ans:

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

$$R_2 \rightarrow 2R_1 - R_2, \& R_3 \rightarrow 3R_1 - R_3$$

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & -3 & 4 & 0 \\ 0 & -7 & 7 & 0 \end{bmatrix}$$

$$R_2 \div -3$$

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 1 & -4/3 & 0 \\ 0 & -7 & 7 & 0 \end{bmatrix}$$

$$R_3 \rightarrow 7R_2 + R_3$$

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 1 & -4/3 & 0 \\ 0 & 0 & -28/3 & 0 \end{bmatrix}$$

Since value is not

change

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$