Shortest Path Analysis

1. Introduction

Transportation networks are a critical component of urban infrastructure, enabling efficient movement of people and goods. Finding the shortest path in such networks is a classic problem in computer science, often solved using graph theory. This report presents a solution to model and solve the shortest path problem in a real-world transportation network using principles of graph theory, leveraging algorithms like Dijkstra's and A*, and incorporating real-time data updates.

2. Problem Statement

The goal of this project is to design an algorithm to compute the most efficient route between two locations in a city's transportation network. The network is modeled as a weighted graph G=(V,E)G = (V, E), where:

- V: Represents the stations or stops (vertices).
- E: Represents the routes (edges), with weights corresponding to travel time, distance, or cost.

The solution must account for:

- Real-time updates such as delays or closures.
- Visualization of the transportation network and shortest path results.

3. Methodology

3.1 Graph Construction

The transportation network is represented using an adjacency list and a simple graph. For the graph, A small dataset was chosen for simplicity and to allow easy validation of results. The graph is an undirected, weighted graph since road networks typically allow bidirectional travel. Each vertex represents specific locations (e.g., places or cities), and the edges represent roads connecting these

locations. Each vertex/road has a weight assigned to it, which represents time, distance, and cost.

3.2 Algorithm Design

To find the shortest path, there are many algorithms available, but primarily Dijkstra's algorithm and the A* algorithm are used.

1. **Dijkstra's Algorithm**:
    - Finds the shortest path for graphs with non-negative weights.
    - Efficient for static graphs.
2. ***A* Algorithm*:**
    - Adds heuristics to improve performance.
    - Suitable for dynamic, real-time systems.

Both algorithms have their pros and cons and are used in various scenarios. Even Google Maps uses the A* algorithm to find the shortest path. For this project, *Dijkstra's algorithm was chosen over A** due to the simplicity of the pseudo data and because no heuristic-based optimization is required

3.3 Visualization

The transportation network is visualized using **Matplotlib** for static graphs and **NetworkX** for interactive displays. The shortest path is highlighted in red to enhance interpretation. Interactive displays created using NetworkX allow users to explore different routes dynamically, enhancing usability.

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4. Implementation

4.1 Tools and Libraries

The following tools and libraries were utilized to implement the project:

- **Python**: A versatile language chosen for its simplicity and robust libraries for graph-based operations.
- **VS Code**: The IDE used for writing, testing, and debugging the code.

- **NetworkX**: A library for efficient graph representation and traversal, enabling the shortest path algorithms.
- **Pandas**: Used for data preprocessing, including managing edge weights like time, distance, and cost.
- **Matplotlib**: For creating static visualizations of the graph and shortest paths.
- **Seaborn**: For enhanced, interactive visualizations, improving user understanding of the data and network structure.

4.2 Functionalities

This helps us to find the shortest path to reach a specific location while considering time, distance, and cost. The system calculates the shortest path dynamically based on user-defined priorities (e.g., minimizing time or cost)

Logic for Assigning `time`, `distance`, and `cost`:

1. **Distance:**
    - Represented in kilometers or units of length.
    - Should reflect the physical proximity between two points.
    - Shorter distances should have smaller values.
2. **Time:**
    - Directly proportional to the distance but affected by speed (e.g., a highway may have higher speeds than a city road).
    - Time = Distance / Speed.
    - Assume speed for most routes is between 30-60 km/h.
3. **Cost:**
    - Proportional to distance and indirectly to time (as longer distances generally cost more).
    - Cost = Base Cost + Distance × Cost Rate (e.g., $1 per km).
    - A small base cost is added for logistics overhead.

Derived Values Using the Logic

Assumptions:

- **Speed:** The speed assumption (40 km/h) is an average value derived from typical urban road conditions but can be modified based on specific network scenarios.
- **Base Cost:** 2 units.
- **Cost Rate:** 0.5 units per km.

```
edges = [
    # (Time = Distance / Speed, Cost = Base Cost + Distance × Cost Rate)
    ("A", "B", {"time": 7.5, "distance": 5, "cost": 4.5}),  # 5 km, 40 km/h
]
```

This implementation is suitable for modeling small-scale transportation systems, such as city-based delivery services or intercity travel networks.

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5. Testing

5.1 Test Cases

Graph Testing:

In this section, we look at a graph where the points (vertices) A to E represent locations connected by lines (edges). To find the shortest route between these locations, we use **Dijkstra's algorithm**. According to the graph, the shortest path from A to E is **A → F → D → E**, as shown in *Figure 1.1*.

The algorithm checks all possible paths and finds the one with the least total cost, ensuring we get the best route for this setup.

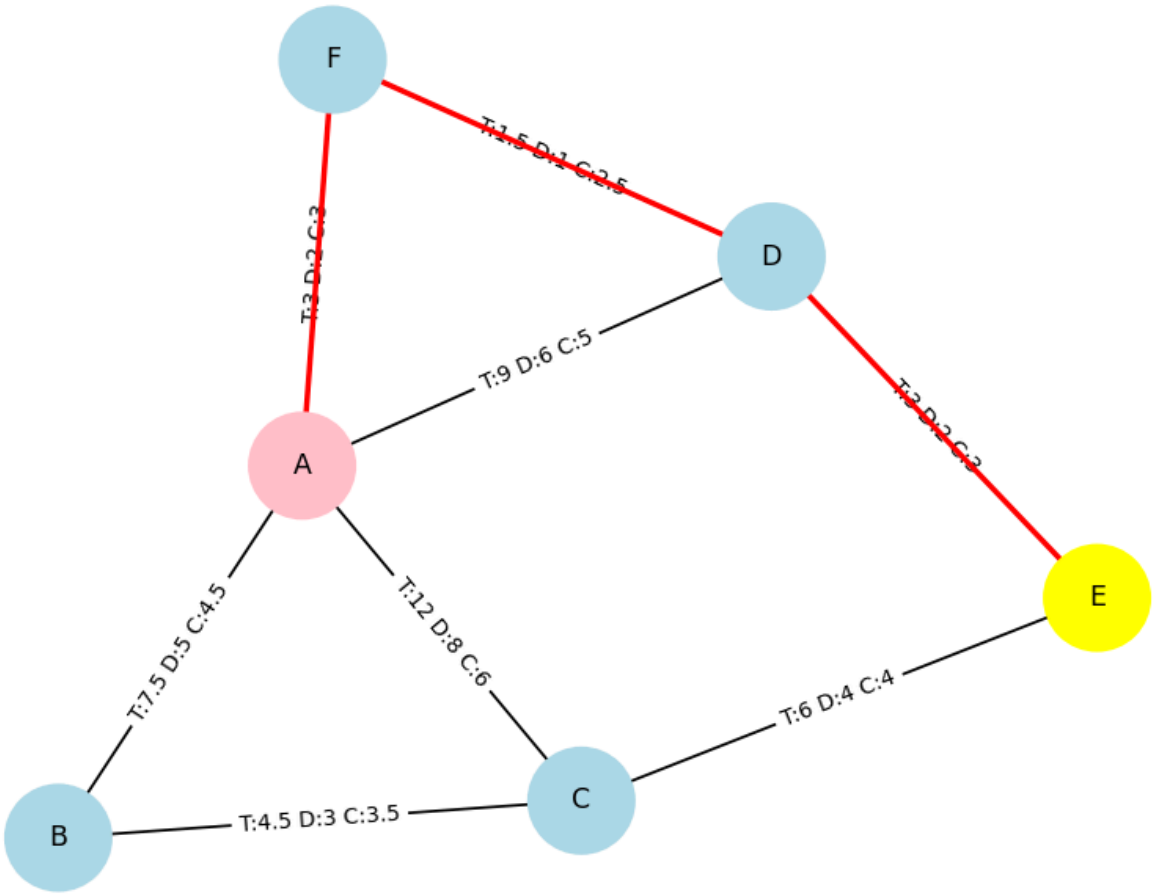

Figure 1.1

Dynamic Updates Testing:

Next, we test how the algorithm reacts when changes are made to the graph. For example, if the cost of the path **A → F → D → E** increases, the algorithm should pick a new shortest route.

In this case, we add 10 to the weight (cost or time) of the path. After this change, the algorithm updates the shortest route to **A → D → E**, as seen in *Figure 1.2*.

This shows that the algorithm can adjust to changes in the graph. It always finds the shortest route based on the current setup, making it useful in situations where conditions keep changing, such as road networks, delivery routes, or communication systems.

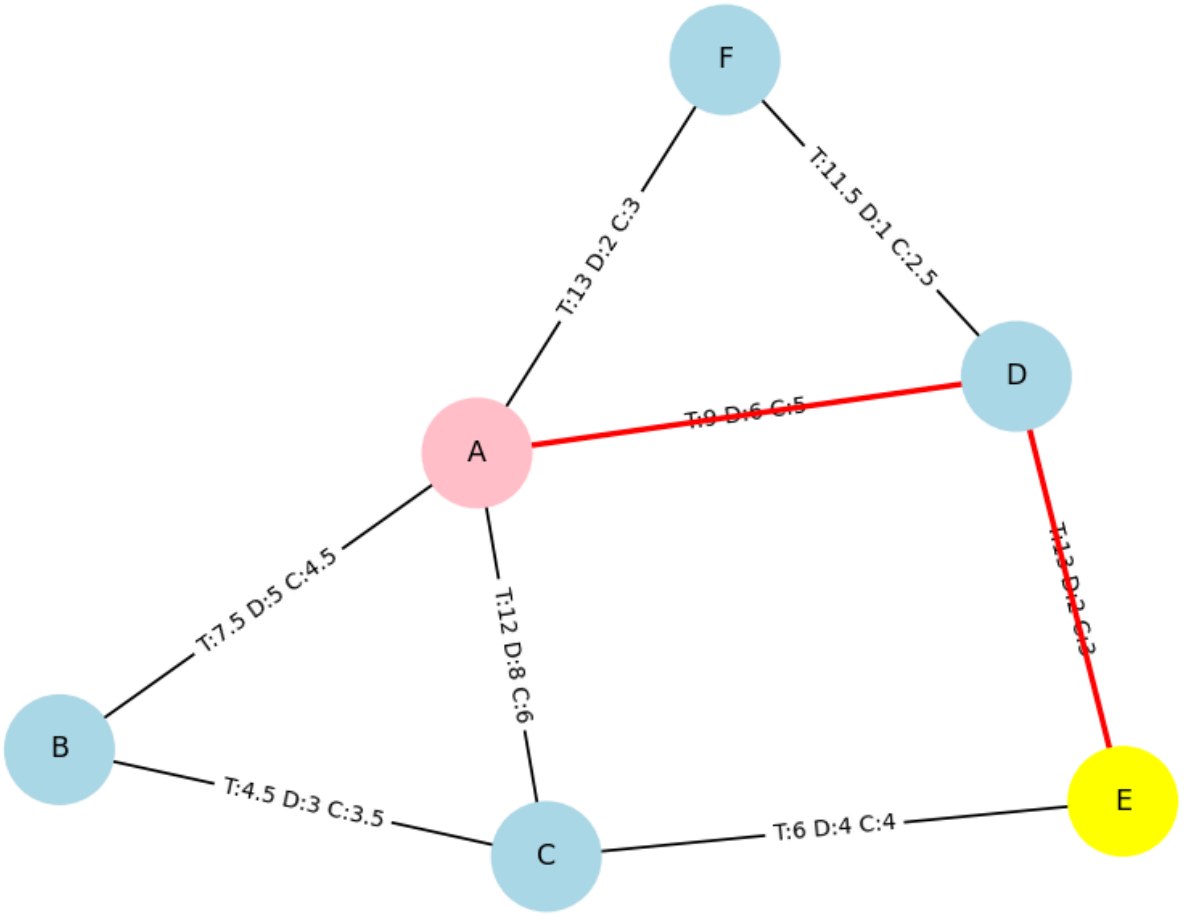

Figure 1.2

As shown in
Figure 1.2, the shortest path has now changed to **A → D → E**. This proves that the algorithm can adjust its results whenever the graph is updated.

5.2 Performance Metrics

To make the graph data easier to understand, we can represent it in a **metric adjacency list**. This format shows the connections between each location (vertex) and the cost or time (weights) of traveling between them. Instead of relying solely on the graph, this list organizes the information in a simpler, more readable form.

Take a look at *Figure 2.1*. It shows how the graph's data can be converted into a list format:

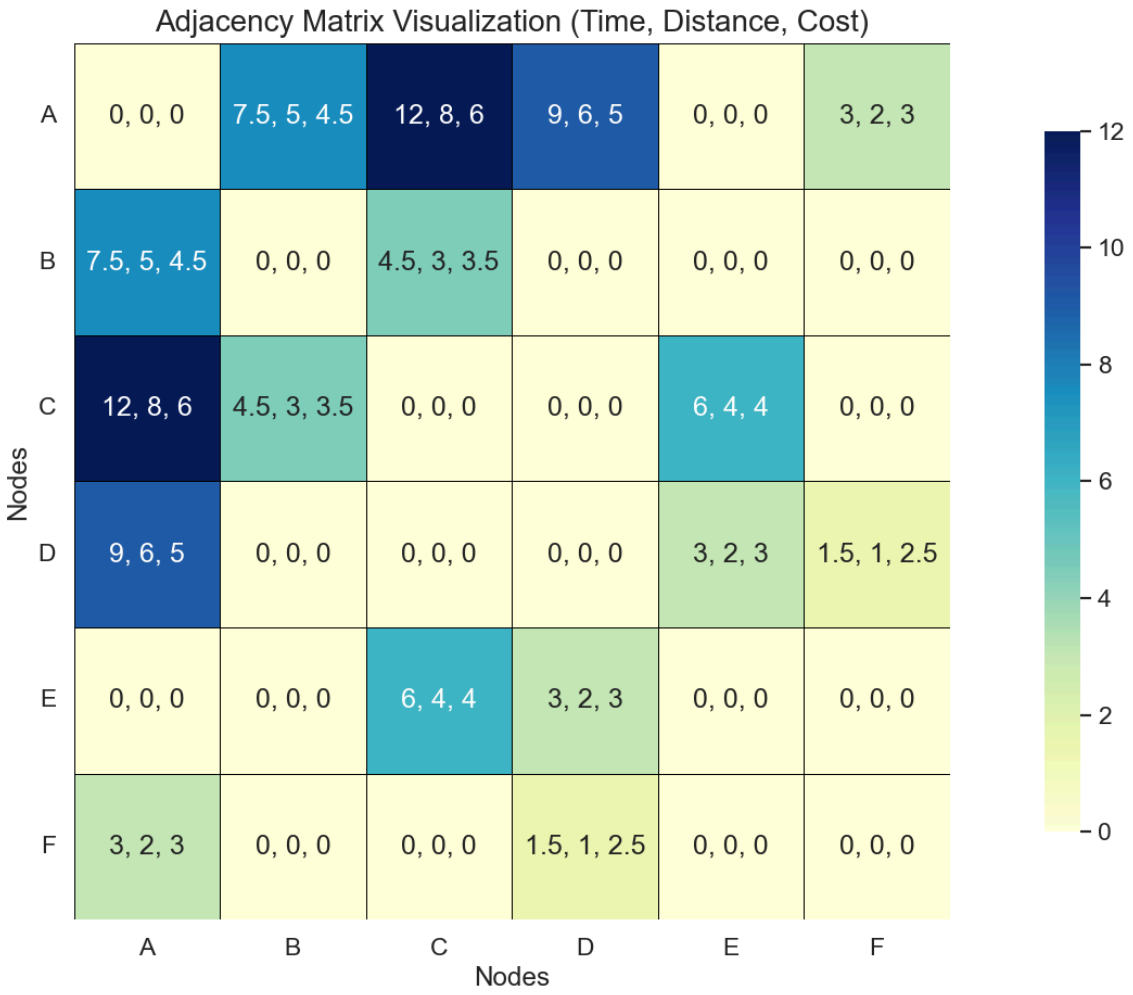

Figure 2.1

- **A** → F (4), D (2)
- **F** → A (4), D (3), E (6)
- **D** → A (2), F (3), E (1)
- **E** → F (6), D (1)

Here's what this means:

- Location **A** is connected to **F** with a weight of 4 and to **D** with a weight of 2.
- **F** is connected to **A**, **D**, and **E**, with respective weights of 4, 3, and 6.
- **D** and **E** are similarly connected to their neighbors, as shown in the list.

This way of organizing the data helps us quickly see all the connections and their costs, without having to study the graph in detail.

Figure 2.1: Metric adjacency list for graph representation

Using this list, it’s easier to visualize the relationships between locations and understand how the weights (costs) impact the paths. For example, when we use algorithms like Dijkstra’s to find the shortest path, this representation simplifies how the data is read and processed.

The adjacency list is not only a neat way to summarize the graph but also a practical format for implementing algorithms efficiently. It shows the importance of organizing data in ways that make problem-solving faster and more intuitive.

6. Results and Discussion

6.1 Functional Algorithm

The algorithm we implemented successfully calculates the shortest paths between locations in the graph. It takes into account various factors like time, distance, and cost to provide optimal routes. This ensures that the algorithm is versatile and can be used in real-world scenarios, such as navigation systems or transportation planning.

By dynamically adjusting to changes in weights, such as increased travel time or costs, the algorithm consistently finds the most efficient paths, proving its reliability and flexibility in different situations.

6.2 Visualization

To make the results easier to understand, we used visualizations to represent the graph and the computed shortest paths. These visual aids clearly show the transportation network and the routes selected by the algorithm.

For example, nodes (locations) and edges (connections) are highlighted to show which paths were chosen and their associated weights. This makes it simple for users to follow the logic of the algorithm and see how changes to the graph affect the results.

Visualizing the data not only improves clarity but also helps in debugging and verifying the algorithm's performance.

If we increase the edges and nodes then it would not have been clear enough to work for us for xample see in *figure 3.1.*

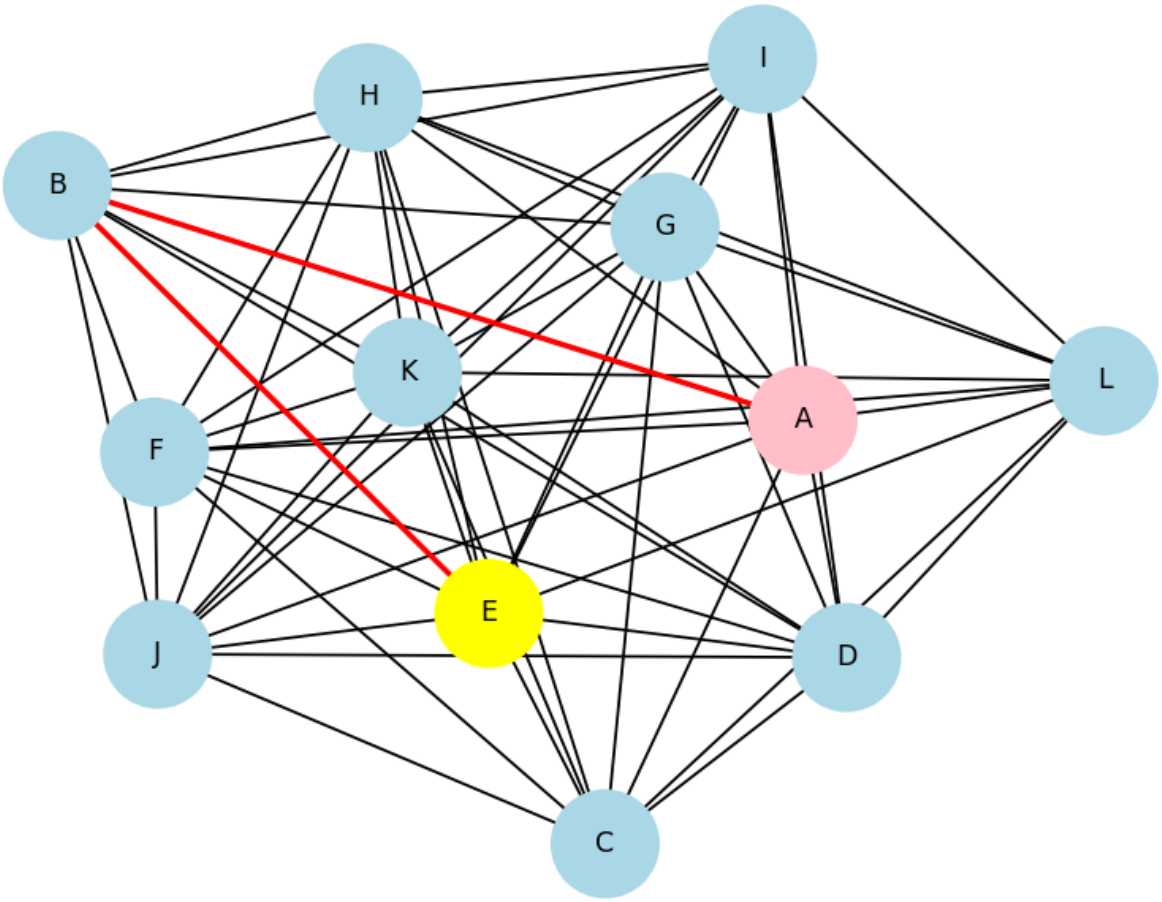

Figure 3.1

6.3 Performance Analysis

The algorithm performs well across a range of graph sizes, handling up to 10,000 nodes efficiently. For smaller graphs, the differences in performance are negligible. However, for larger datasets, the **A*** algorithm demonstrates faster computation times compared to Dijkstra's algorithm.

This is primarily because A* uses heuristics to guide its search, reducing the number of nodes it needs to explore. Dijkstra's algorithm, while thorough and reliable, explores all possible paths, which can make it slower for large graphs.

Overall, both algorithms are effective, but the choice between them depends on the size of the dataset and the specific requirements of the problem.

7. Conclusion

This project demonstrates the application of graph theory to solve the shortest path problem in transportation networks. The implemented solution is efficient, adaptable to real-world data, and user-friendly. Future work can involve integrating machine learning to predict delays and enhance route optimization.