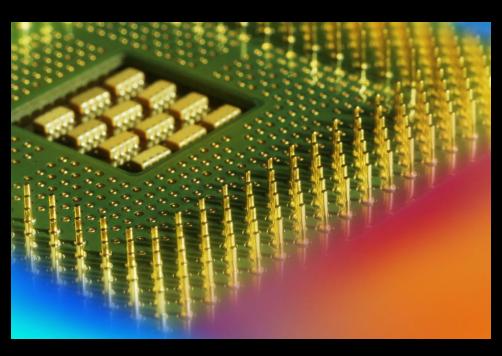
Digital Fundamentals

Tenth Edition

Floyd





Half-Adder

Basic rules of binary addition are performed by a **half adder**, which has two binary inputs (*A* and *B*) and two binary outputs (Carry out and Sum).

The inputs and outputs can be summarized on a truth table.

Inp	uts	Outputs							
Α	В	Cout	Σ						
0	0	0	0						
0	1	0	1						
1	0	0	1						
1	1	1	0						

The logic symbol and equivalent circuit are:

$$\Sigma = A \oplus B = A\overline{B} + \overline{A}B$$

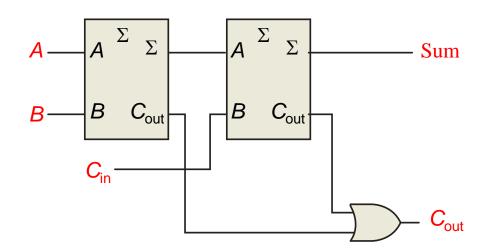
$$A \longrightarrow C_{\text{out}} = AB$$



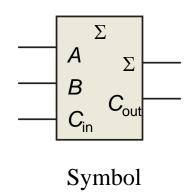
Full-Adder

By contrast, a **full adder** has three binary inputs (*A*, *B*, and Carry in) and two binary outputs (Carry out and Sum). The truth table summarizes the operation.

A full-adder can be constructed from two half adders as shown:



Inp	outs		Outputs					
Α	В	C_{ln}	C_{out}	Σ				
0	0	0	0	0				
0	0	1	0	1				
0	1	0	0	1				
0	1	1	1	0				
1	0	0	0	1				
1	0	1	1	0				
1	1	0	1	0				
1	1	1	1	1				





Full-Adder

Truth Table

Α	В	Cin	Cout	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

 $C_{\rm in}$ = input carry, sometimes designated as CI

 $C_{\text{out}} = \text{output carry}$, sometimes designated as CO

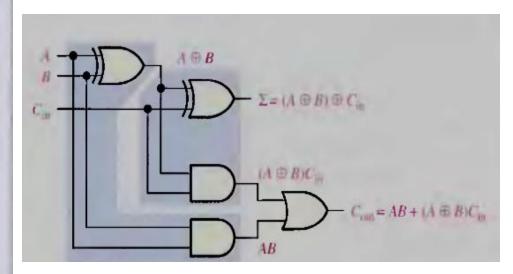
 $\Sigma = sum$

A and B = input variables (operands)

Equation

$$\Sigma = (A \oplus B) \oplus C_{\rm in}$$

$$C_{\text{out}} = AB + (A \oplus B)C_{\text{in}}$$

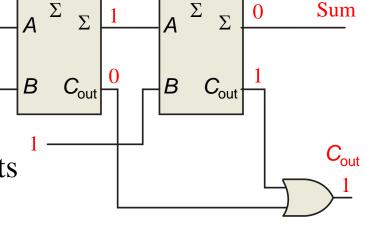


Summary

Full-Adder

Example

For the given inputs, determine the intermediate and final outputs of the full adder.



Solution

The first half-adder has inputs of 1 and 0; therefore the Sum =1 and the Carry out =0.

The second half-adder has inputs of 1 and 1; therefore the Sum = 0 and the Carry out = 1.

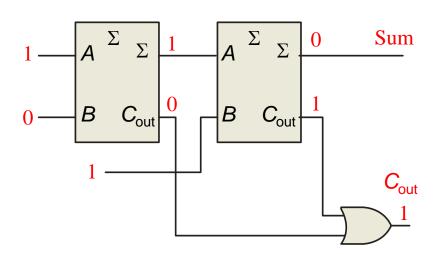
The OR gate has inputs of 1 and 0, therefore the final carry out = 1.



Full-Adder

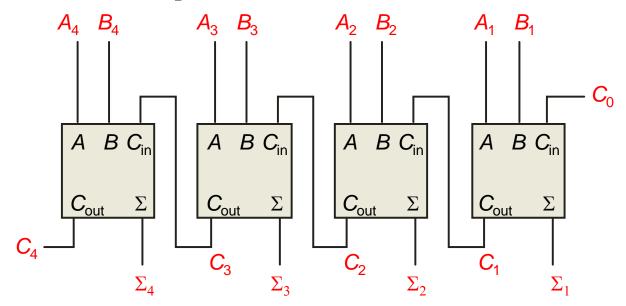
Notice that the result from the previous example can be read directly on the truth table for a full adder.

	Inp	outs		Outp	uts
	Α	В	C_{ln}	$C_{ m out}$	Σ
	0	0	0	0	0
	0	0	1	0	1
	0	1	0	0	1
	0	1	1	1	0
	1	0	0	0	1
<	1	0	1	1	0
	1	1	0	1	0
	1	1	1	1	1





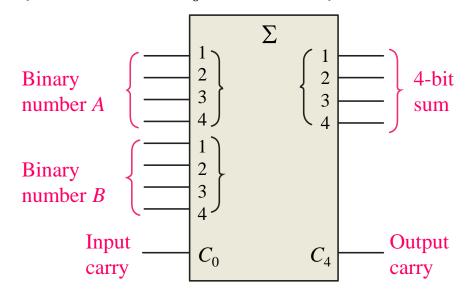
Full adders are combined into parallel adders that can add binary numbers with multiple bits. A 4-bit adder is shown.



The output carry (C_4) is not ready until it propagates through all of the full adders. This is called *ripple carry*, delaying the addition process.



The logic symbol for a 4-bit parallel adder is shown. This 4-bit adder includes a carry in (labeled (C_0) and a Carry out (labeled C_4).



The 74LS283 is an example. It features *look-ahead carry*, which adds logic to minimize the output carry delay. For the 74LS283, the maximum delay to the output carry is 17 ns.

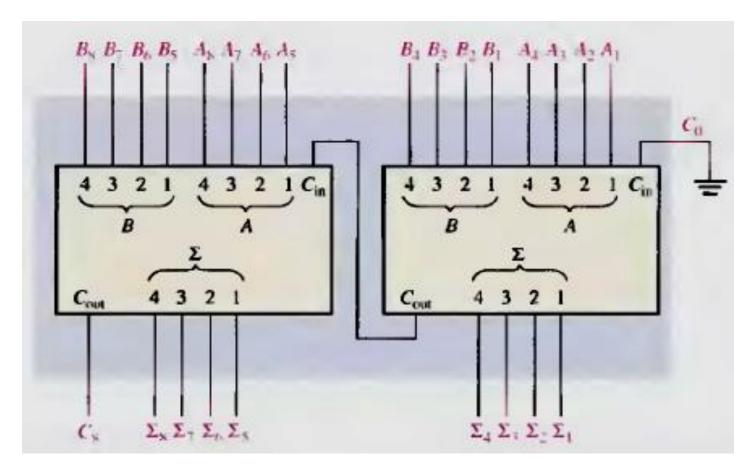


Truth Table for a 4-Bit Parallel Adder

C1	A,	Bn	Σ_n	C,
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	t	0	0	1
1	ı	1	1	1

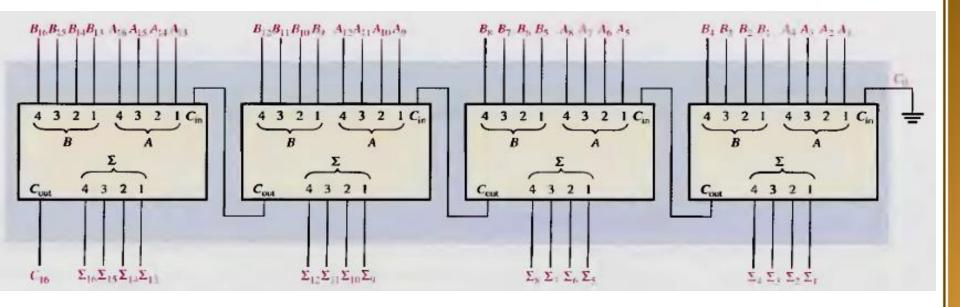


Cascading of two 4-bit adders to form an 8-bit adder





Cascading of four 4-bit adders to form a 16-bit adder



Show how two 74LS283 adders can be connected to form an 8-bit parallel adder. Show output bits for the following 8-bit input numbers:

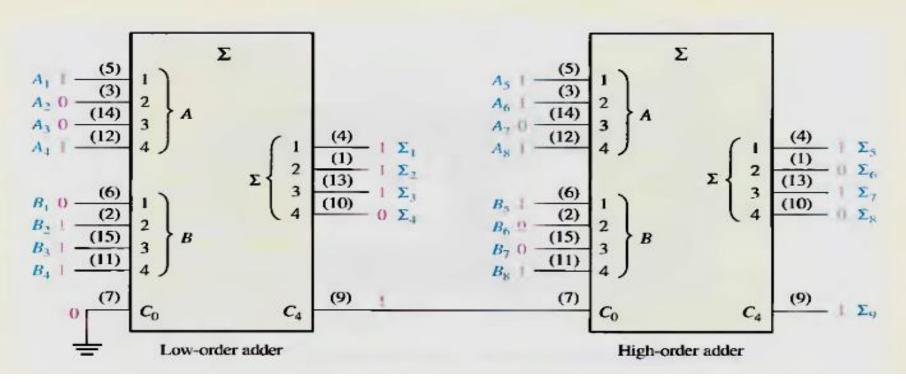
$$A_8A_7A_6A_5A_4A_3A_2A_1 = 10111001$$
 and $B_8B_7B_6B_5B_4B_3B_2B_1 = 10011110$

Solution

Two 74LS283 4-bit parallel adders are used to implement the 8-bit adder. The only connection between the two 74LS283s is the carry output (pin 9) of the low-order adder to the carry input (pin 7) of the high-order adder, as shown in Figure 6–13. Pin 7 of the low-order adder is grounded (no carry input).

The sum of the two 8-bit numbers is

$$\Sigma_9 \Sigma_8 \Sigma_7 \Sigma_6 \Sigma_5 \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 = 1010101111$$



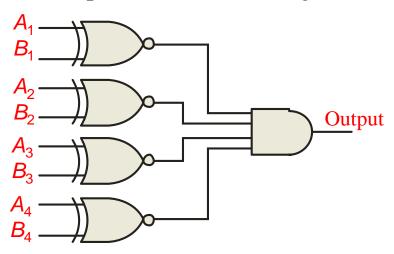


The function of a comparator is to compare the magnitudes of two binary numbers to determine the relationship between them. In the simplest form, a comparator can test for equality using XNOR gates.

Example Solution

How could you test two 4-bit numbers for equality?

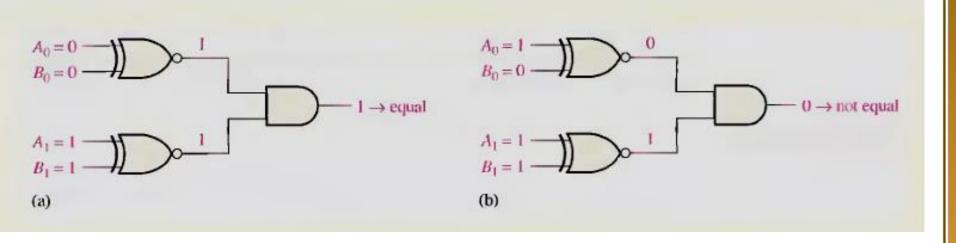
AND the outputs of four XNOR gates.



EXAMPLE

Apply each of the following sets of binary numbers to the comparator inputs in Figure 6–21, and determine the output by following the logic levels through the circuit.

(a) 10 and 10 (b) 11 and 10

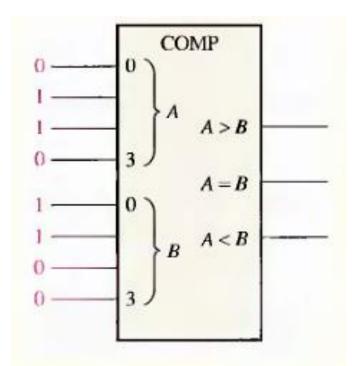


- Solution (a) The output is 1 for inputs 10 and 10, as shown in
 - (b) The output is 0 for inputs 11 and 10, as shown in

EXAMPLE

Determine the A = B, A > B, and A < B outputs for the input numbers shown on the

comparator in Figure

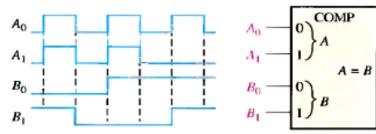


Solution The number on the A inputs is 0110 and the number on the B inputs is 0011. The A > B output is HIGH and the other outputs are LOW.

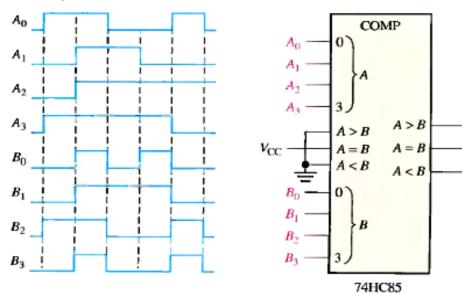
EXAMPLE

Comparators

11. The waveforms in Figure 6–79 are applied to the comparator as shown. Determine the output (A = B) waveform.

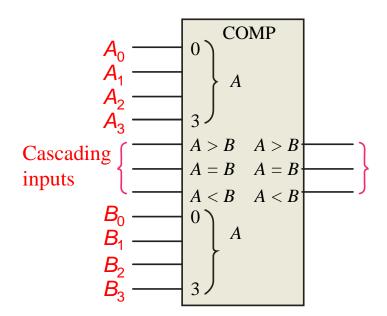


For the 4-bit comparator in Figure 6–80, plot each output waveform for the inputs shown. The outputs are active-HIGH.





IC comparators provide outputs to indicate which of the numbers is larger or if they are equal. The bits are numbered starting at 0, rather than 1 as in the case of adders. Cascading inputs are provided to expand the comparator to larger numbers.

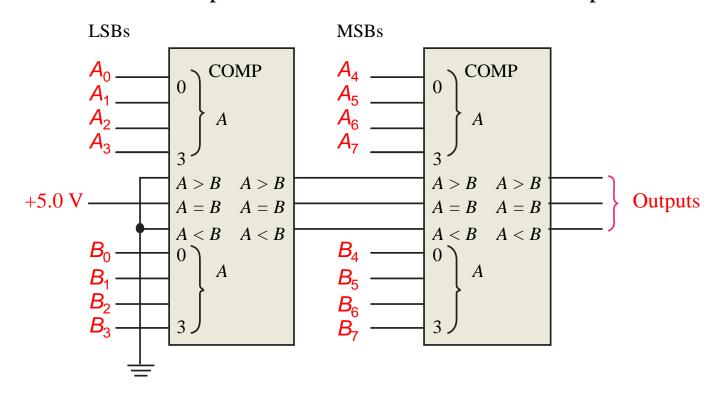


Outputs

The IC shown is the 4-bit 74LS85.

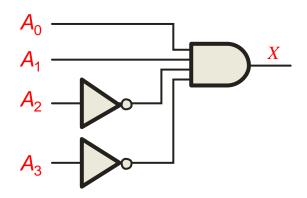


IC comparators can be expanded using the cascading inputs as shown. The lowest order comparator has a HIGH on the A = B input.

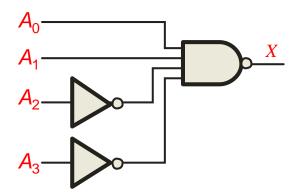




A **decoder** is a logic circuit that detects the presence of a specific combination of bits at its input. Two simple decoders that detect the presence of the binary code 0011 are shown. The first has an active HIGH output; the second has an active LOW output.



Active HIGH decoder for 0011

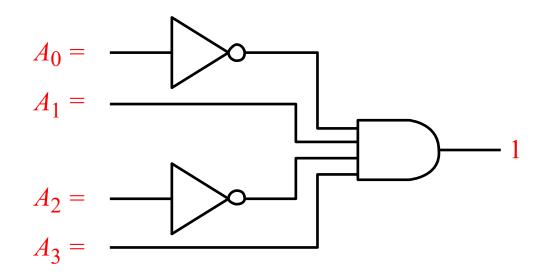


Active LOW decoder for 0011



Question

Assume the output of the decoder shown is a logic 1. What are the inputs to the decoder?

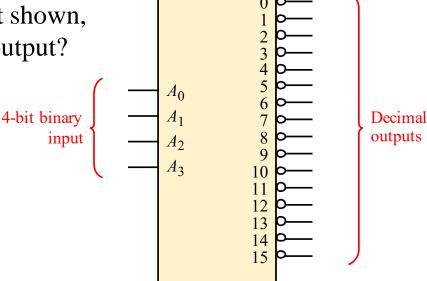




IC decoders have multiple outputs to decode any combination of inputs. For example the binary-to-decimal decoder shown here has 16 outputs – one for each combination of binary inputs.

Question

For the input shown, what is the output?



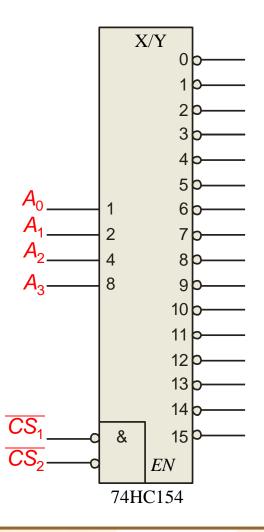
Bin/Dec

Decoding functions and truth table for a 4-line-to-16-line (1-of-16) decoder with active-LOW outputs.

DECIMAL	BINARY INPUTS DECODING								11011	YE			O	UT	PUT	S				91	
DIGIT	A ₃	Az	A_1	A_0	FUNCTION	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	$\overline{A}_3\overline{A}_2\overline{A}_1\overline{A}_0$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	$\overline{A}_3\overline{A}_2\overline{A}_1A_0$	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	j	0	$\overline{A}_3\overline{A}_2A_1\overline{A}_0$	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	1	1	$\overline{A}_3\overline{A}_2A_1A_0$	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
4	0	1	0	0	$\overline{A}_3 A_2 \overline{A}_1 \overline{A}_0$	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
5	0	1	0	1	$\overline{A}_3 A_2 \overline{A}_1 A_0$	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
6	0	1	1	0	$\overline{A}_3A_2A_1\overline{A}_0$	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
7	0	1	1	1	$\overline{A}_3A_2A_1A_0$	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
8	1	0	0	0	$A_3\overline{A}_2\overline{A}_1\overline{A}_0$	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
9	1	0	0	1	$A_3\overline{A}_2\overline{A}_1A_0$	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
10	1	0	ı	O	$A_3\overline{A}_2A_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
11	1	0	1	1	$A_3\overline{A}_2A_1A_0$	1	1	1	1	1	1	1	1	ı	1	1	0	1	1	1	1
12	1	1	0	0	$A_3A_2\overline{A}_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	ı	1	1	0	1	1	1
13	1	1	0	1	$A_3A_2\overline{A}_1A_0$	1	1	1	1	1	t	1	1	1	1	1	1	1	0	l	1
14	1	1	1	0	$A_3A_2A_1\overline{A}_0$	1	1	1	1	I	1	1	1	1	1	1	1	I	1	0	1
15	1	ı	1	1	$A_3A_2A_1A_0$	1	1	1	1	1	1	i	1	1	1	1	ı	1	1	1	0

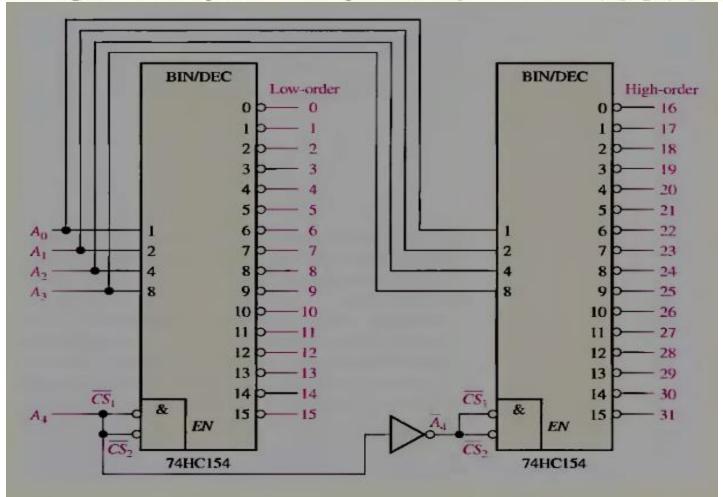


A specific integrated circuit decoder is the 74HC154 (shown as a 4-to-16 decoder). It includes two active LOW chip select lines which must be at the active level to enable the outputs. These lines can be used to expand the decoder to larger inputs.

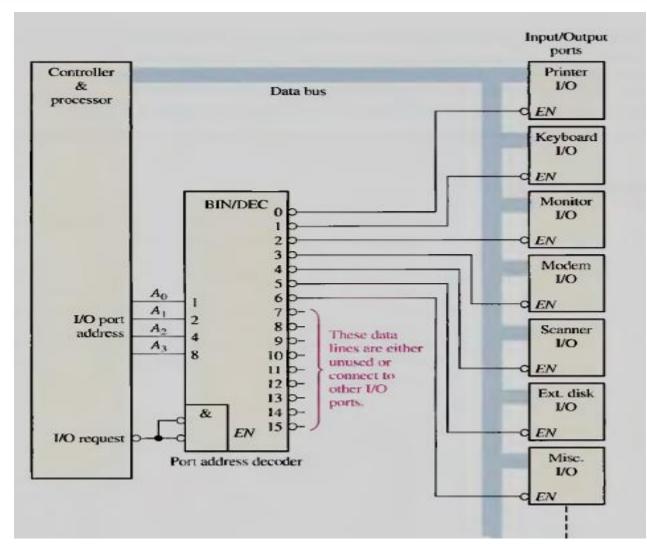


Question

A certain application requires that a 5-bit number be decoded. Use 74HC154 decoders to implement the logic. The binary number is represented by the format $A_4A_3A_2A_1A_0$.



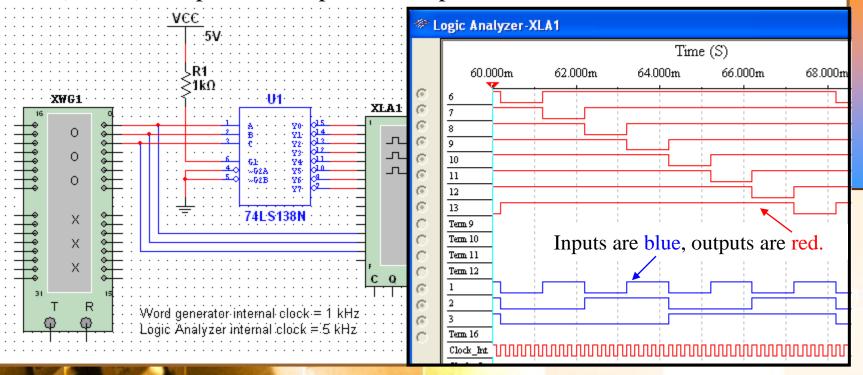
EXAMPLE



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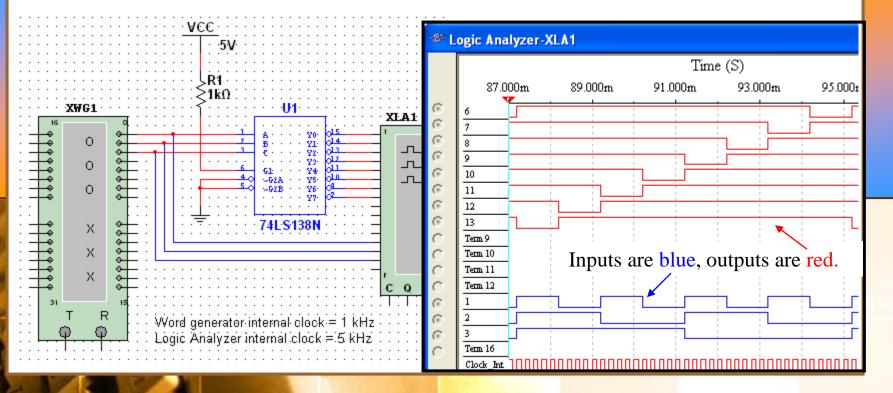
The 74LS138 is a 3-to-8 decoder with three chip select inputs (two active LOW, one active HIGH). In this Multisim circuit, the word generator (XWG1) is set up as an up counter. The logic analyzer (XLA1) compares the input and outputs of the decoder.



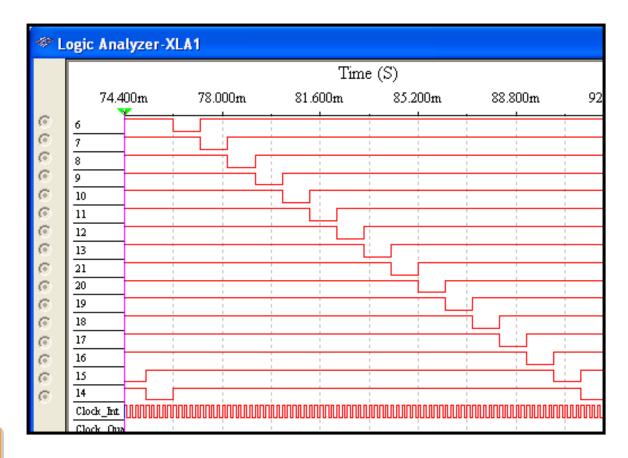


Question

How will the waveforms change if the word generator is configured as a down counter instead of an up counter?





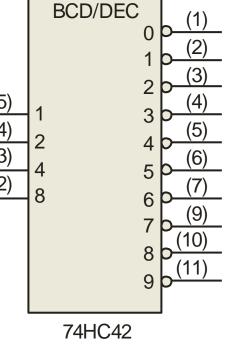


Question

Is the word generator set as an up counter or a down counter? (The least significant decoder output at the top). It is an up counter.



BCD-to-decimal decoders accept a binary coded decimal input and activate one of ten possible decimal digit indications.



Example

Assume the inputs to the 74HC42 decoder are the sequence 0101, 0110, 0011, and 0010. Describe the output.

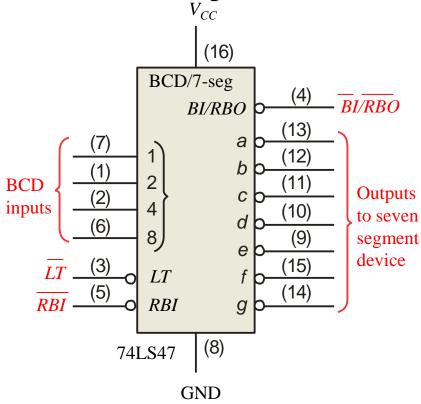
Solution

All lines are HIGH except for one active output, which is LOW. The active outputs are 5, 6, 3, and 2 in that order.



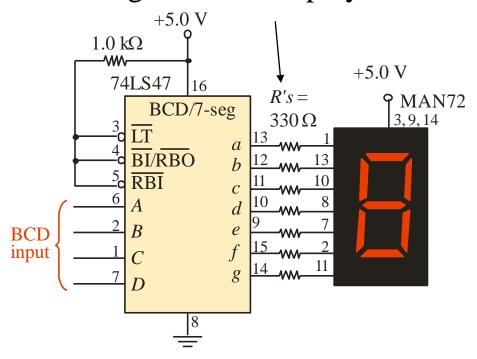
Another useful decoder is the 74LS47. This is a BCD-to-seven segment display with active LOW outputs.

The *a-g* outputs are designed for much higher current than most devices (hence the word driver in the name).



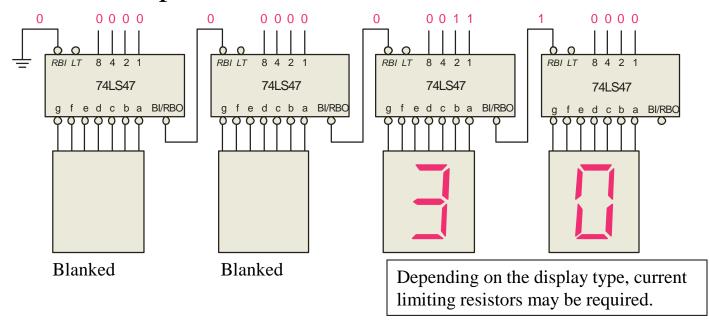


Here the 7447A is an connected to an LED seven segment display. Notice the current limiting resistors, required to prevent overdriving the LED display.



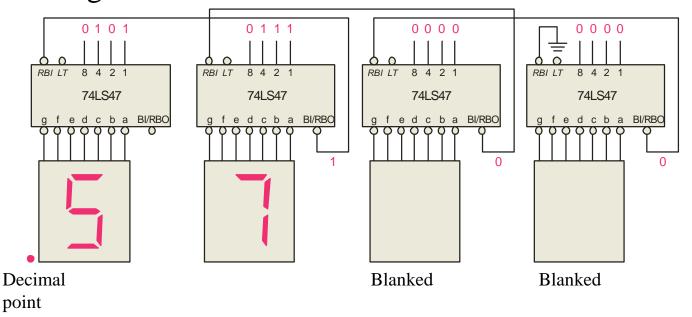


The 74LS47 features leading zero suppression, which blanks unnecessary leading zeros but keeps significant zeros as illustrated here. The $\overline{BI/RBO}$ output is connected to the \overline{RBI} input of the next decoder.





Trailing zero suppression blanks unnecessary trailing zeros to the right of the decimal point as illustrated here. The \overline{RBI} input is connected to the $\overline{BI/RBO}$ output of the following decoder.

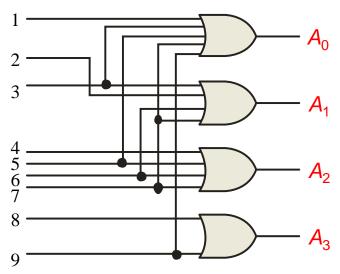




Encoders

An **encoder** accepts an active logic level on one of its inputs and converts it to a coded output, such as BCD or binary.

The decimal to BCD is an encoder with an input for each of the ten decimal digits and four outputs that represent the BCD code for the active digit. The basic logic diagram is shown. There is no zero input because the outputs are all LOW when the input is zero.





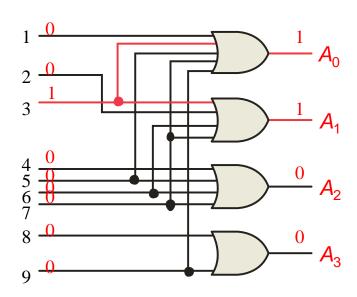
Encoders

Example

Show how the decimal-to-BCD encoder converts the decimal number 3 into a BCD 0011.

Solution

The top two OR gates have ones as indicated with the red lines. Thus the output is 0111.

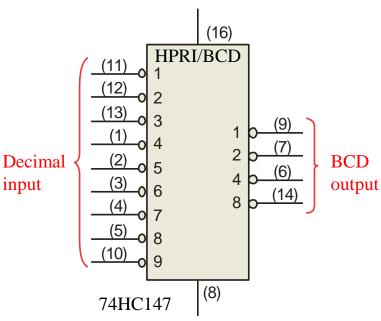




Encoders

The 74HC147 is an example of an IC encoder. It is has ten active-LOW inputs and converts the active input to an active-LOW BCD output. V_{cc}

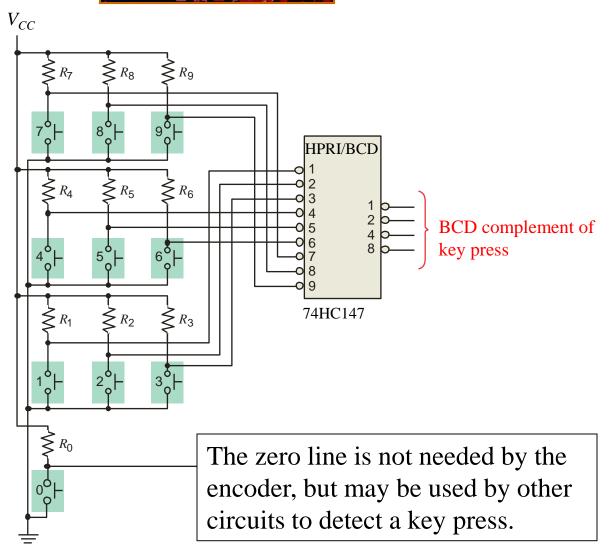
This device is offers additional flexibility in that it is a **priority encoder**. This means that if more than one input is active, the one with the highest order decimal digit will be active.



The next slide shows an application ...

Encoders

Keyboard encoder





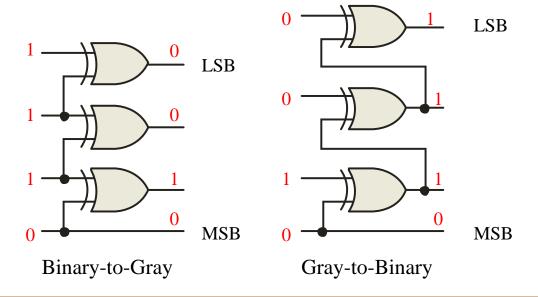
Code converters

There are various code converters that change one code to another. Two examples are the four bit binary-to-Gray converter and the Gray-to-binary converter.

Example

Show the conversion of binary 0111 to Gray and back.

Solution





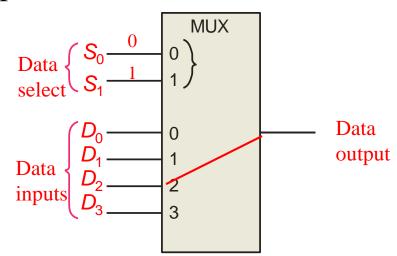
Multiplexers

A multiplexer (MUX) selects one data line from two or more input lines and routes data from the selected line to the output. The particular data line that is selected is determined by the select inputs.

Two select lines are shown here to choose any of the four data inputs.

Question

Which data line is selected if $S_1S_0 = 10$? D_2

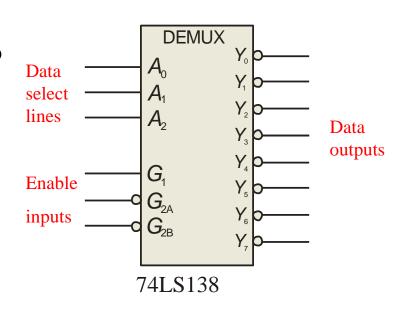




Demultiplexers

A demultiplexer (DEMUX) performs the opposite function from a MUX. It switches data from one input line to two or more data lines depending on the select inputs.

The 74LS138 was introduced previously as a decoder but can also serve as a DEMUX. When connected as a DEMUX, data is applied to one of the enable inputs, and routed to the selected output line depending on the select variables. Note that the outputs are active-LOW as illustrated in the following example...



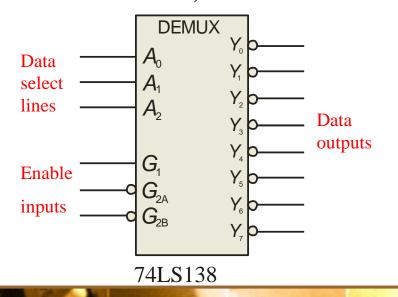
Summary

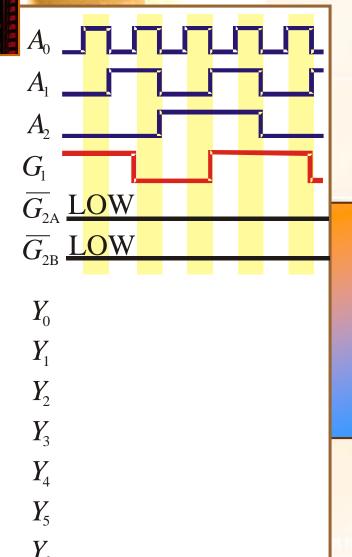
Demultiplexers

Example Solution

Determine the outputs, given the inputs shown.

The output logic is opposite to the input because of the active-LOW convention. (Red shows the selected line).





 Y_7

Selected Key Terms

Full-adder

A digital circuit that adds two bits and an input carry bit to produce a sum and an output carry.

Cascading

Connecting two or more similar devices in a manner that expands the capability of one device.

Ripple carry

A method of binary addition in which the output carry from each adder becomes the input carry of the next higher order adder.

Look-ahead carry

A method of binary addition whereby carries from the preceding adder stages are anticipated, thus eliminating carry propagation delays.

Selected Key Terms

Decoder A digital circuit that converts coded information into a familiar or noncoded form.

Encoder A digital circuit that converts information into a coded form.

Priority An encoder in which only the highest value input *encoder* digit is encoded and any other active input is ignored.

Multiplexer A circuit that switches digital data from several input (MUX) lines onto a single output line in a specified time sequence.

Demultiplexer A circuit that switches digital data from one input line (DEMUX) onto a several output lines in a specified time Floyd, Digital Fundamentals, 10th ed

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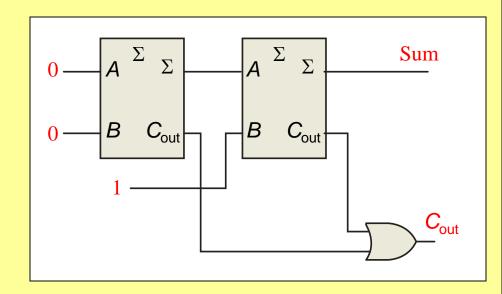
1. For the full-adder shown, assume the input bits are as shown with A = 0, B = 0, $C_{in} = 1$. The Sum and C_{out} will be

a. Sum =
$$0 C_{out} = 0$$

b. Sum =
$$0 C_{out} = 1$$

c. Sum = 1
$$C_{out} = 0$$

d. Sum =
$$1 C_{out} = 1$$



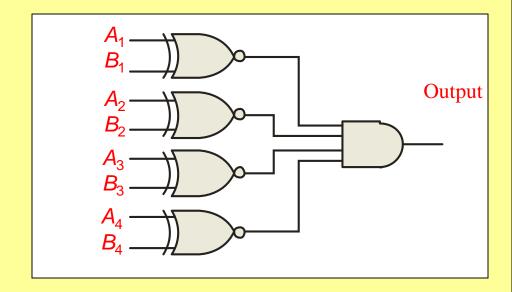
2. The output will be LOW if

a.
$$A < B$$

b.
$$A > B$$

c. both a and b are correct

$$d. A = B$$

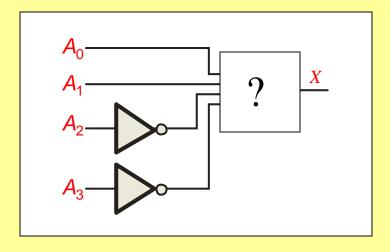




- 3. If you expand two 4-bit comparators to accept two 8-bit numbers, the output of the least significant comparator is
 - a. equal to the final output
 - b. connected to the cascading inputs of the most significant comparator
 - c. connected to the output of the most significant comparator
 - d. not used

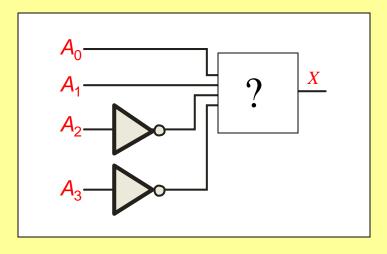
4. Assume you want to decode the binary number 0011 with an active-LOW decoder. The missing gate should be

- a. an AND gate
- b. an OR gate
- c. a NAND gate
- d. a NOR gate



5. Assume you want to decode the binary number 0011 with an active-HIGH decoder. The missing gate should be

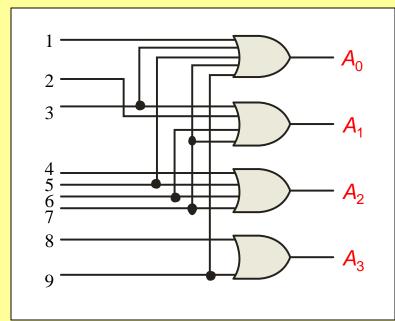
- a. an AND gate
- b. an OR gate
- c. a NAND gate
- d. a NOR gate





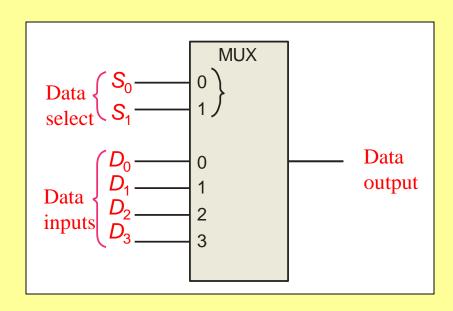
- 6. The 74138 is a 3-to-8 decoder. Together, two of these ICs can be used to form one 4-to-16 decoder. To do this, connect
 - a. one decoder to the LSBs of the input; the other decoder to the MSBs of the input
 - b. all chip select lines to ground
 - c. all chip select lines to their active levels
 - d. one chip select line on each decoder to the input MSB

- 7. The decimal-to-binary encoder shown does not have a zero input. This is because
 - a. when zero is the input, all lines should be LOW
 - b. zero is not important
 - c. zero will produce illegal logic levels
 - d. another encoder is used for zero



8. If the data select lines of the MUX are $S_1S_0 = 11$, the output will be

- a. LOW
- b. HIGH
- c. equal to D_0
- d. equal to D_3





- 9. The 74138 decoder can also be used as
 - a. an encoder
 - b. a DEMUX
 - c. a MUX
 - d. none of the above



10. The 74LS280 can generate even or odd parity. It can also be used as

- a. an adder
- b. a parity tester
- c. a MUX
- d. an encoder

Answers:

- 1. c 6. d
- 2. c 7. a
- 3. b 8. d
- 4. c 9. b
- 5. a 10. b