CMSC 5718   Introduction to Computational Finance

Assignment 3: Performance of Asset Allocation Strategies
(16% of total grade)

Instructions

1) Submit a copy of your work by **uploading to Blackboard on or before March 29, 2024, 11:59pm.** The report, supporting worksheets/programs should be submitted as a zipped file. The **file name** of the zipped file should include your surname and have the following format, **e.g. ChauKL_Assign1**.   [If uploading to Blackboard is not successful, you may consider sending a email to kalokchau@cuhk.edu.hk, but submission through Blackboard is preferred.]

2) Late submission of one week or less will attract a penalty of 20% of the assignment mark. *Submission will not be accepted after April 5, 2024, 11:59pm.*

3) You can either submit your work **individually** or work together in a **group of two or three students**. Students in the same group will get the same assignment grade. Please state the name(s) and student number(s) clearly in the report. However, if you are submitting as a group, you only need to submit one copy of the report with one of the names in the file name (described in (1) above).

4) *__Please observe the university's plagiarism guidelines.__*

Introduction

In a research report ("the Report"),[1] the authors use historical data between January 2000 to June 2020 to illustrate the superior performance of risk-parity based strategies. Given the substantially different market environment since 2021 (especially the rapid rise in interest rates and the poor performance of the US fixed income market compared to the US equity market), we want to conduct a simplified test to confirm whether some of the conclusions in section II of the paper (p.2 to 11) are still valid by partially extending the study using data from 2021 to 2023.

We use the following US Exchange Traded Funds (ETFs) as proxies for the major asset classes:

    a.   SPDR S&P 500 ETF Trust (SPY) – US Equity market
    b.   iShares US Treasury Bond ETF (GOVT) – US Government Bond market
    c.   iShares S&P GSCI Commodity-Indexed Trust (GSG) – Commodities market
    d.   Grayscale Bitcoin Trust (GBTC) – Bitcoin [for the bonus question]

Note: The prices in the spreadsheet are already adjusted to take into account of dividends paid. There is no need to consider the effect of dividends and other distributions, and other transaction costs are ignored in this study. Fractional shares are allowed in this study.

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[1] Liu, B., P. Brzenk and T. Cheng, "Indexing Risk Parity Strategies," *S&P Dow Jones Indices Research Report*, October 2020.

1.   Preparing the data

In each calculation below, price data for SPY, GOVT and GRG refer to the following range:

Year 2021	31/12/2020 to 31/12/2021 (both dates inclusive)
Year 2022	31/12/2021 to 30/12/2022 (both dates inclusive)
Year 2023	30/12/2022 to 29/12/2023 (both dates inclusive)
Year 2021 to 2023	31/12/2020 to 29/12/2023 (both dates inclusive)

2.   Portfolio Return, volatlity and risk contribution: Two-asset Portfolios (35 marks)

The calculations below aim to reproduce results similar to Exhibit 4 and 7 in the Report.

With an initial capital of $1,000,000, two-asset portfolios are constructed consisting of SPY and GOVT, starting with a 0/100 equity/bond mix, with additional portfolios created in 10% weight increments, resulting in 11 total portfolios. [The first portfolio is 0% SPY and 100% GOVT, the second portfolio is 10% SPY and 90% GOVT, and so on.] The data for **Year 2021 to 2023** are used in the analysis. The portfolios should be re-balanced at the end of each calendar year. [*Refer to the Pricing Notes section for details of the calculations*].

(a) Construct the portfolios and calculate the daily portfolio values.
(b) For each portfolio, calculate the annualized standard deviation (i.e. portfolio risk) and the portfolio return *using the answer in (a)*, and hence work out the return/risk ratio.
(c) For each portfolio, calculate the risk contribution ratio of each asset.
(d) Plot the results similar to Exhibit 4 and Exhibit 7 of the report (just show the graphs; no need to reproduce the legends).
(e) From your results, do you agree with the comments in pages 6 and 7 of the Report?

3.   Equal-Weight Portfolio and Equal-Risk-Contribution Portfolio (65 marks)

The calculations below aim to reproduce results similar to Exhibits 8,9 and 10 in the Report.

(a) Equal-Weight Portfolio
(i) For **each year** (**2021**, **2022**, and **2023**), construct an equal weight portfolio with an initial capital of $1,000,000 (33.33% for each of SPY, GOVT and GSG) that runs for one year. Calculate the portfolio's value on each day.
(ii) Calculate the return, the standard deviation of return, and the return/risk ratio for each portfolio in each year based on the portfolio values calculated in (i).
(iii) Calculate the risk contribution ratio of each asset in each year.
(iv) Plot the results similar to Exhibit 8, for years 2021, 2022 and 2023.

(b) Equal-Risk-Contribution Portfolio
(i) For **year 2022**, calculate the annualized standard deviations of returns for each asset and the correlation coefficient between each pair of assets.
(ii)Use the answers from (i) to construct an equal-risk-contribution portfolio. From the weights of the assets, use the portfolio formula to calculate the portfolio standard deviation with the data of year 2022. The weights of these assets will be applied to **year 2023** for further analysis below.

[Note: the results will be different from the results shown in Exhibit 9; without setting the level of overall risk, each asset class's contribution will not be above 100%]

(c) Comparison
(i) For **year 2023**, apply the weights obtain in (ii) and generate the daily portfolio values. From these values, calculate the portfolio return, standard deviation of return, and the return/risk ratio. Compare the answers with the equal-weight portfolio obtained in Q3(a)(ii) (as in 1-year profile in exhibit 10).
(ii) Compare your findings to the comment at the beginning of p.11 of the Report,

Bonus question (maximum 4 marks for parts(i) to (iii), maximum 2 marks for part(iv))

GBTC is an ETF that tracks the performance of Bitcoin. We would like to include this asset in an equal-risk-contribution portfolio.
(i) Use the data in **year 2022** to calculate:
- the annualized standard deviation of return of GBTC;
- the correlation coefficient between GBTC/SPY, GBTC/GOV, GBTC/GRG.
(ii) Construct an equal-risk-contribution portfolio based on the parameters calculated. These are the weights of the portfolio as of 30/12/2022.
(iii) use these weights and apply to **year 2023**. Compare your answers with the two portfolios obtained in Q3(c).
(iv) repeat (i) to (iii) above using data in **year 2021** and apply to **year 2022**.

Pricing Notes

1) Example of portfolio construction procedure: year end portfolio value is V, asset prices PA and PB on 31/12/2020. To construct a portfolio with weight x% in asset A and (1-x)% in asset B, the number of units of assets A and B to be held in year 2021 are given by V*x/PA and V*(1-x)/PB respectively. The number of units would be kept constant, until the next re-balancing date (e.g. 31/12/2021). If the value of the portfolio becomes Y on that day, the new number of units of assets A and B become Y*x/PA and Y*(1-x)/PB respectively, to be applied to the data from 3/1/2022 onward.

2) If the portfolio is not re-balanced constantly, the formulas used for calculating the risk contribution ratios of each asset are not exact, and may not sum to 100%.

3) Use the formula in Topic 2-2, slide 23 to calculate the total risk contribution and the risk contribution ratio of each asset. The term "marginal contribution MCi" in the Report (e.g. the formula in p.21) corresponds to "total risk contribution TRi" in Topic 2-2, which is not the same as MRi in Topic 2-2.

4) A proper comparison of the equal-weight and the equal-risk-contribution portfolios should introduce some constraints (c.f. Endnote 6 of the Report). In the current assignment question, for Q2(d) and Q3(i), no target has been set for the portfolio risk, and the distribution of risk contribution of each asset may be different from the figures in Exhibit 7 and Exhibit 8 of the Report.