Reaction Wheel Control Design Using Linear Quadratic Controller

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Abstract. This paper studies the design of active attitude control system of a nano-satellite in a single axis. In this paper, we consider dc motor based reaction wheel as an actuator, because of its pointing accuracy. However, the power consumption of the dc motor is often relatively large and needed to be optimized. Linear quadratic controller is supposed to have an ability to minimize power consumption and able to enhance the system performance. To show the advantage of this method, simulation result of attitude response, state trajectory, and trajectory of DC motor voltage are presented.

1. Introduction

Nano-satellite is a small sized satellite, weighted less than 10 Kg [1]. Nano-satellite has been widely used for various applications i.e mining exploration, disaster monitoring, educational purposes, remote sensing,etc [2]. One of previous researches on reaction wheel is reported in [3], in which Kalman Filter is implemented to enhance fault detection performance.

Active attitude control becomes necessary for nano-satellite to point its instrument (eg. camera) [4]. In this paper, reaction wheel is used as an actuator in active attitude control system. It is used as actuator due to its good pointing accuracy [5]. However, power consumption of DC motor in the reaction wheel is relatively large. Thus, optimal controller may become an answer for this circumstance.

Linear quadratic controller is an optimal control regulator, which poses quadratic cost function. The feedback gain of linear quadratic controller has to be determined, in order to minimized the cost function and optimizing system performance [6].The objective of this paper is to develop linear quadratic regulator for dc motor based reaction wheel. By using this controller, minimizing the power consumption of DC motor and reducing settling time of the system are done.

2. Controller design

The system consists of DC motor which is connected on the reaction wheel as its driver. As the reaction wheel begin to move, it controls the attitude of nano-satellite. The nano-satellite construction is illustrated by the following figure.
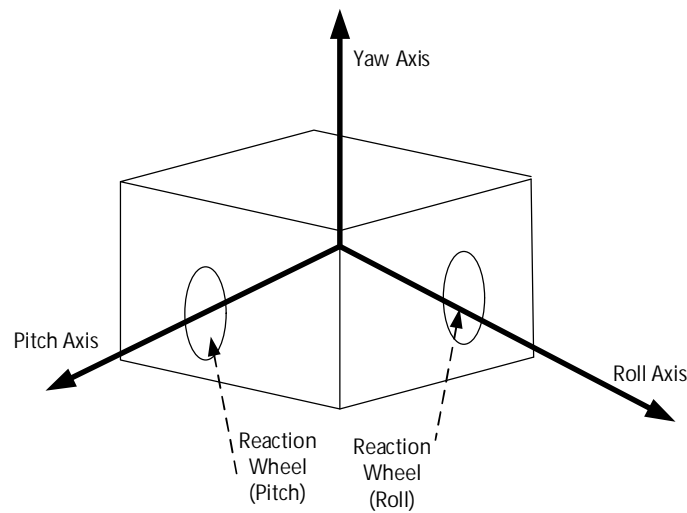
Fig. 1. Nano-satellite with reaction wheel construction

The used parameters to describe the dynamics of the reaction whell are as follows.

Table 1. Parameter description of Reaction wheel [5]

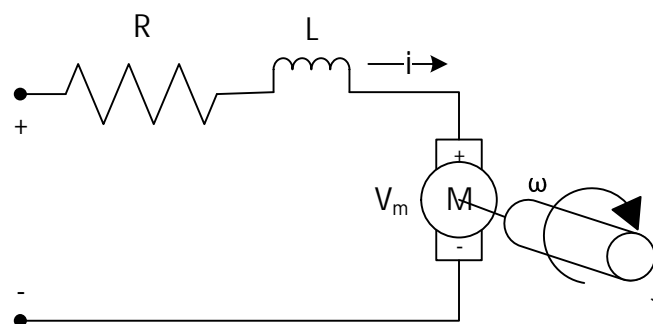| Parameter	Description	Value	Unit
I	Current of the dc motor on reaction wheel	-	A
ω	Angular speed of reaction wheel	-	Deg/s
θ	Angle of nano-satellite	-	Deg/s
R	Motor resistance	3.92	
L	Motor inductance	$150 \ 10^{-6}$	H
J	Inertia (including wheel)	$5.1181 \ 10^{-5}$	Kg m^2
B	Damping ratio	$1,1261 \ 10^{-6}$	Nm/(deg/s)
K_b	Back EMF constant	$2.7767 \ 10^{-4}$	Volt/(deg/s)
K_t	Torque constant	$15.9 \ 10^{-3}$	Nm/A
V	Voltage	-	Volt

Fig. 2. DC motor schematic circuit [5]

The electrical dynamic equation of the DC motor above can be derived by applying Kirchoff's voltage law and written as bellow [5]

$$V = -V + V_R + V_L + V_M = 0 \tag{1}$$

$$iRa + L\frac{di}{dt} + K_b\omega = V \tag{2}$$

$$\frac{di}{dt} = (-iRa - K_b\omega + V)/L \tag{3}$$

The mechanical dynamic equations of DC motor movement is

$$\omega = J\frac{d\omega}{dt} + iK_t - b\omega = 0 \tag{4}$$

$$\frac{d\omega}{dt} = (iK_t - b\omega)/J \tag{5}$$

The angular rate movement of nano-satellite is assumed to be proportional with the movement of the DC motor, hence the equation becomes

$$\frac{d\theta}{dt} = K_a\omega \tag{6}$$

The state space equation for attitude of the nano-satellite is

$$\dot{x} = Ax + Bu, u = -kx \tag{7}$$

$$\dot{x} = Ax + B(-kx) = (A - BK)x \tag{8}$$

$$y = Cx \tag{9}$$

Refer to equation (3), (5), and (6); equation (7) and (9) become

$$\begin{vmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \end{vmatrix} = \begin{vmatrix} \dot{i} \\ \dot{\omega} \\ \dot{\theta} \end{vmatrix} = \begin{vmatrix} -R/L & -Kb/L & 0 \\ Kt/J & -b/J & 0 \\ 0 & K_a & 0 \end{vmatrix}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}u \tag{10}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} \tag{11}$$

The performance index, which needs to be minimized is

$$J = \int_0^\infty x^T Qx + u^T Ru\, dt, Q = C^T C, R = 1 \tag{12}$$

$$J = \int_0^\infty \theta^2 + u^T Ru\, dt, \tag{13}$$

The gain of linear quadratic controller could be determined after acquiring the state space equation and the performance objective for the attitude of the nano-satellite by using the following ricatti equation.

$$PA + A^TP - PBR^{-1}B^TP + Q = 0 \tag{14}$$

Where the feedback gain is

$$k = R^{-1}B^TP \tag{15}$$

Given ratio between angular speed of nano-satellite and angular speed of DC motor is 0.002, hence the state space of this system becomes

$$\begin{vmatrix} \dot{i} \\ \dot{\omega} \\ \dot{\theta} \end{vmatrix} = \begin{vmatrix} -26133.333 & -1.8444444 & 0 \\ 310.660642 & -0.0220027 & 0 \\ 0 & 0.002 & 0 \end{vmatrix}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} 6666.7 \\ 0 \\ 0 \end{bmatrix}u \tag{16}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} \tag{17}$$

By solving ricatti equation, we obtain feedback gain as bellow

$$k = \begin{bmatrix} 93.159\,10^{-6} & 0.0078 & 1 \end{bmatrix} \tag{18}$$

As the feedback gain applied, the augmented matrix of A from state space matrix becomes

$$A = \begin{vmatrix} -26133.954 & -54,08968 & -6666.7 \\ 310.660642 & -0.0220027 & 0 \\ 0 & 0.00277 & 0 \end{vmatrix} \tag{19}$$

The eigen value of this system are -26133.3 , -0.33+0.33i, and -0.33 - 0.33 i. Since the real parts of eigen value are negatives, we can conclude that this system is asymptotically stable.

3. Simulation result and discussion

Attitude response of the nano-satellite is illustrated on Fig. 3. It shows that attitude of the nano-satellite converges to 0 degress, as it reaches its steady state condition. According to fig.2, it shows that system with linear quadratic controller reaches its steady state condition, faster than system without using state feedback controller.
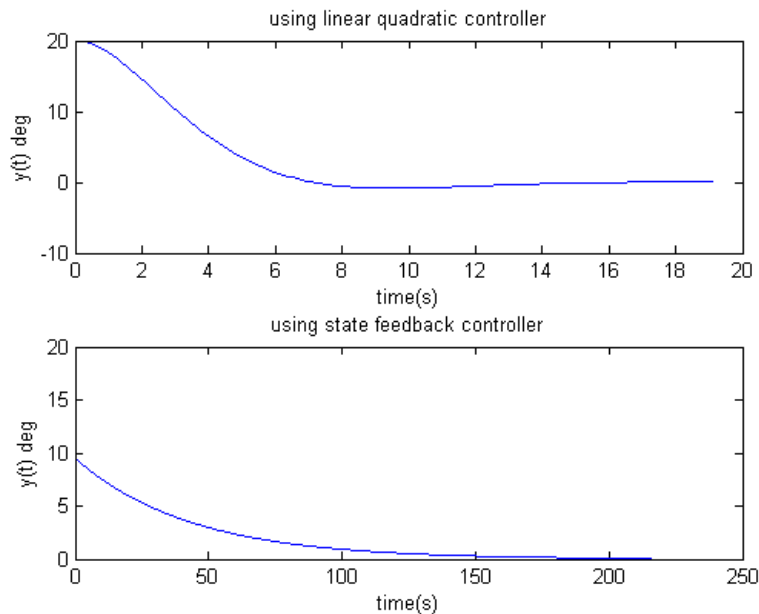

Fig. 3 Attitude response of nano-satellite

States trajectory of this system is illustrated in fig.4, x1 represents current of the dc motor on the reaction wheel, x2 represents angular speed of the reaction wheel, and x3 represents the angle of nano-satellite. It shown that this system achieves its steady state within 20 seconds, proving that linear quadratic controller has significantly enhance the performance of nano-satellite.
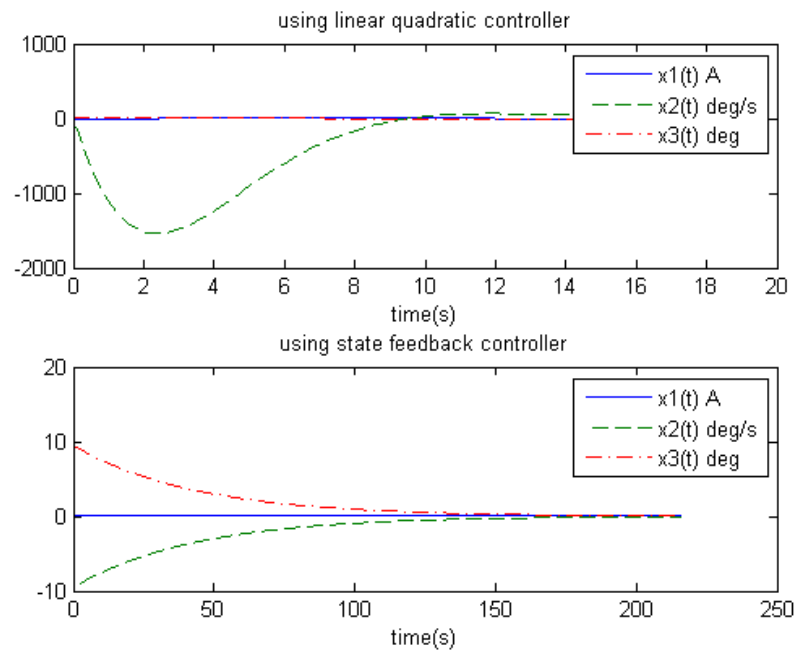
Fig. 4. States trajectory

Trajectory of control action of this system is shown in fig.5, illustrating used voltage to drive DC motor.
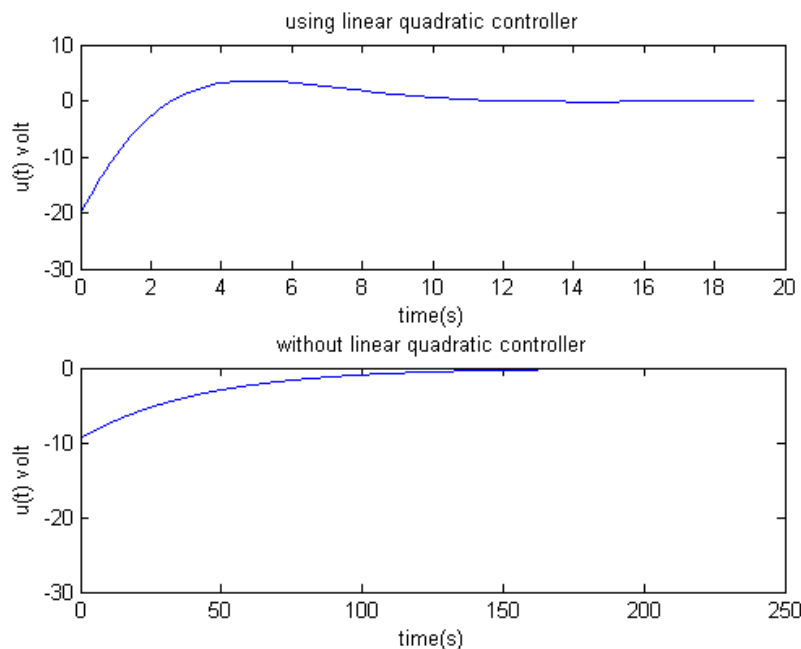

Fig. 5. Control action trajectory

Comparison of power consumption of DC motor between using and without using linear quadratic controller is illustrated in fig. 6. It shows that power consumption of DC motor is extremely larger, when state feedback controller is applied to this system. It can be infered that linear quadratic controller could minimize power consumption of DC motor.
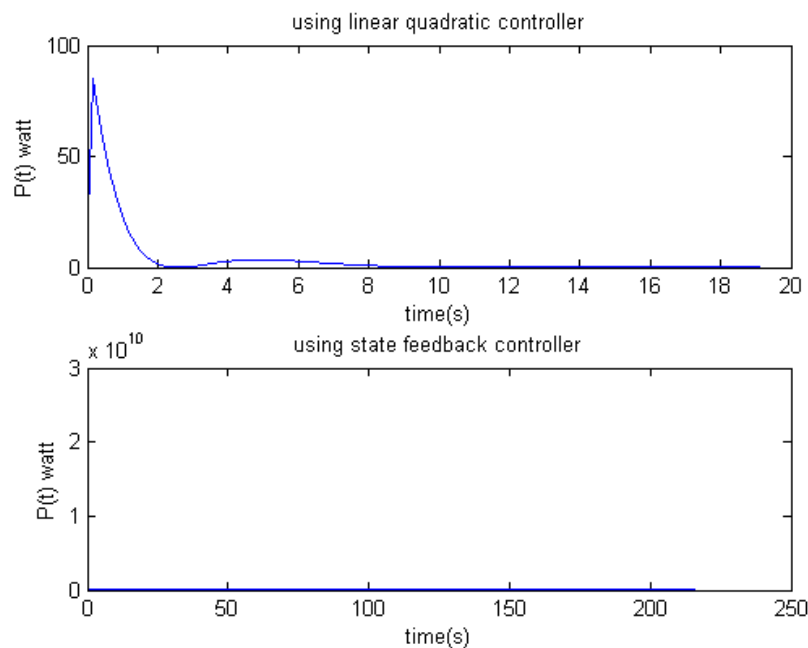
Fig. 6. Power consumption trajectory

4. Conclusion

We present controller design of nano-satellite attitude using linear quadratic controller. The simulation result shows that the attitude control can be obtained. Linear quadratic controller is designed to minimize the power consumption of the DC motor, which is given by performance index. It also shown that system with linear quadratic controller achieve its steady state condition, more than 180 seconds faster compared to system with state feedback controller. Simulation results are provided to demonstrate the effectiveness and efficiency of the controller.

References
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