The gambler's ruin Part 1 (two-player)

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Recursion

Model

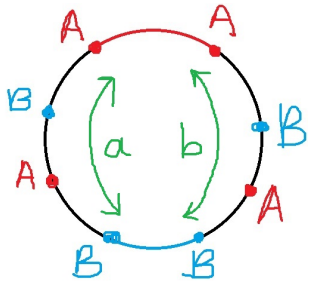

Figure 1: two-player with $k = 8$ and initial value of a and b for players

Definition

$$t_a := expected\ \ duration\ \ of\ \ game\ \ starting\ \ from\ \ a$$

$$t_a = \frac{1}{2}t_{a-1} + \frac{1}{2}t_{a+1} + 1$$

$$\Rightarrow t_{a+1} = 2t_a - t_{a-1} - 2$$

$$\Rightarrow t_a = 2t_{a-1} - t_{a-2} - 2$$

Initial values

mine

$$\begin{cases} t_0 = 0 \\ t_k = 0 \end{cases}$$

yours

$$\begin{cases} t_0 = 0 + \frac{2}{k}t_1 + 1 \\ X_k = 0 \end{cases}$$

Solving recurrence

mine

$$r^2 - 2r + 1 = 0$$

$$\Rightarrow t_a^{(h)} = \alpha_1 + \alpha_2 n$$

yours

$$-2\alpha^2 + \alpha + 2 = 0$$
$$\Rightarrow \alpha = \frac{-1 \pm \sqrt{1+16}}{-4}$$

Proof. □