In The Name of God

Model Predictive Control

Fall Semester of 2024

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Exercise 3

Tube MPC

Analytical and Simulation Questions

Question 1

A schematic diagram of a hydraulic actuator in contact with the environment is shown in Fig 1. In some study, the well-known model of manipulator-sensor-environment is coupled with the nonlinear hydraulic actuator dynamics. The sensor with stiffness k_s and damping d_s connects the actuator piston, represented by the mass m_a, to the environment. The environment is represented by mass m_e, damping d_e, and stiffness k_e. The dynamic equations are:

$$\begin{aligned} m_a\ddot{x} &= f_a - d_s(\dot{x} - \dot{x}_e) - d\dot{x} - k_s(x - x_e), \\ m_e\ddot{x}_e &= d_s(\dot{x} - \dot{x}_e) - d_e\dot{x}_e - k_s(x - x_e) - k_e x_e, \\ f &= k_s(x - x_e). \end{aligned} \tag{1}$$

Linearization of this model leads to uncertainty in the system parameters. The linear transfer function between measured contact force, $F(s)$, and control voltage, $U(s)$, is written as:

$$\frac{F(s)}{U(s)} = \frac{\frac{k_{sp}}{\tau s+1} \cdot K_s k_e (A_i + A_o)}{(K_p + Cs)(m_a s^2 + ds + k_e) + (A_i^2 s + A_o^2 s)} \tag{2}$$

This equation is now considered as a parametrically uncertain system. For example, the uncertainty ranges in K_s and K_p reflect variations in the operating point, supply pressure, and, in part, the orifice area gradient. The variations in the environmental stiffness and damping of the system are included in parameters k_e and d, respectively. Furthermore, the uncertainty in C represents the changes in the fluid bulk modules and the volumes of fluid trapped at the sides of the actuator. The uncertainty in the valve characteristic is captured in variations of and $k_s p$. All these parameters are known to have an effect on the system's stability. The range of these uncertainties is as shown in Table 1.

Table 1: Parameter ranges pertaining to the linear transfer function

Parameter	**Range**
k_e	50 - 100 (kN/m)
K_s	0.25 - 0.5 (m^3/Pa.s)
K_p	0 - 5×10^{-12} (m^2/s)
C	10^{-11} - 3×10^{-11} (m^3/Pa)
d	600 - 800 (N/m/s)
m_a	19.9 - 20.1 (kg)
A_i	0.00193 - 0.00213 (m^2)
A_o	0.00144 - 0.0016 (m^2)
k_{sp}	0.0011 - 0.0013 (m/V)
τ	30 - 40 (ms)

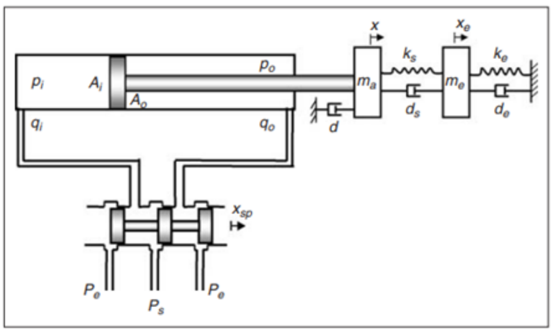

Figure 1: Schematic model of the hydraulic actuator interacting with the environment

- Calculate the state space of this transformation function manually. (Hint: use minimal realization)
- Considering the uncertainty of the parameters, design a linear MPC that the output of this system tracks the reference value and explain the result. (Consider the reference input value as 5)
- Design the controller of Part 2 assuming that a disturbance is added to the system as follows: $d = 0.1 \times sin\,(t)$
- Considering the uncertainty of the parameters and additive disturbance, design a tube MPC for this system and explain the result.
- Reduce the overshoot of the controlled system as much as possible by applying constraints to the controller in Part 4.

Question 2

Problem Description

Consider the mass-spring-damper system depicted in Figure 2:

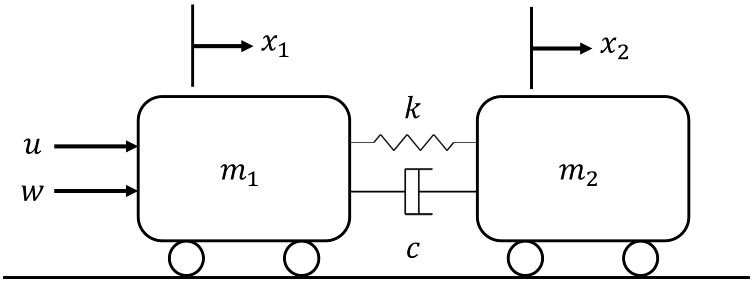

Figure 2: Mass-spring-damper system

represented by the following equations:

$$\begin{aligned}
\dot{x}_1 &= x_3, \\
\dot{x}_2 &= x_4, \\
\dot{x}_3 &= \frac{1}{m_1} k(-x_1 + x_2) + c(-x_3 + x_4) + u + w, \\
\dot{x}_4 &= \frac{1}{m_2} (k(x_1 - x_2) + c(x_3 - x_4)).
\end{aligned} \tag{3}$$

and the parameter values for this system are given in Table 2:

Table 2: Parameter Values

Parameter	Value
m_1	10000
m_2	8000
k	500000
c	5000
x_0	$\begin{bmatrix} 0 & 0 & 2 & 0.5 \end{bmatrix}$

Where:

- x_1, x_2: Positions of the two masses.
- x_3, x_4: Velocities of the two masses ($x_3 = \dot{x}_1$, $x_4 = \dot{x}_2$).
- m_1, m_2: Masses of the two bodies.
- k: Spring constant describing the stiffness of the spring connecting the two masses.
- c: Damping coefficient representing the damping effect of damper between the two masses.

- u: Control input applied to the system.
- w: External disturbance affecting the first mass.

The velocity reference is:

- **5** in Parts **A** and **B**.
- **3** in all other Parts.

Part A: Linear MPC Design

1. Design a **Linear MPC** for the given system.
2. Tune the controller to achieve stable operation.
3. Plot and analyze:
   - States of the system.
   - Control inputs.

Part B: Constraints on the Linear MPC

B1: Soft Constraints

1. Incorporate the following soft constraint into the Linear MPC design:

$$x_4 < 1.5.$$

2. Plot and analyze:
   - States of the system with soft constraints.
   - Control inputs with soft constraints.

B2: Hard Constraints

1. Modify the constraint in part **B1** to a **hard constraint**.
2. Compare the results with those of **B1**.
3. Plot and analyze:
   - States of the system with hard constraints.
   - Control inputs with hard constraints.

Part C: Disturbance Rejection

1. Apply the following disturbance to the system:

$$D = 2\sin(4\pi t).$$

2. Evaluate the performance of the **Linear MPC** under disturbance.
   - Plot the states.
   - Plot the control inputs.
3. Design a **Tube MPC** to handle the disturbance.
   - Use a PID controller for designing the Tube MPC and tune its coefficients using a PID tuner.
   - Plot and compare:
     - States of the system.
     - Control inputs.

Part D: Uncertainty Handling

1. Consider uncertainties in the system:
   - 10% uncertainty for m_1 and m_2.
   - 20% uncertainty for k and c.

   Hint: Simulate the uncertainties using a sine wave in Simulink.
2. Evaluate the performance of **Linear MPC** and **Tube MPC** under uncertainties. Check:
   - States.
   - Control inputs.

   for both controllers.

Notes on Submission

- Submit a single PDF with analytical solutions, simulation reports, and explanations.
- Include MATLAB and Simulink files in a folder named `Simulation`.
- Use LaTeX for a 10% bonus and include source files in a folder named `LaTeX`.
- Submit a single ZIP file named `HW#-StudentID-StudentName`.
- Late submissions will incur a 10% deduction per day.