In The Name of God

Model Predictive Control

Fall Semester of 2024

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Exercise 4

Hybrid MPC and Explicit MPC

Analytical and Simulation Questions

Question 1

Problem Description

- Consider the Two-DOF quarter-car model with an active suspension depicted in Figure 1:

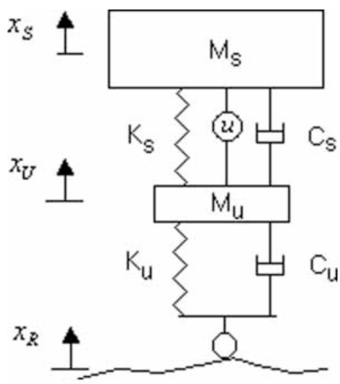

Figure 1: Two-DOF quarter-car model with an active suspension

The state-space model of the system is as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{c_S}{M_S} & -\frac{k_S}{M_S} & \frac{c_S}{M_S} \\ 0 & 1 & 0 & -1 \\ -\frac{k_U}{M_U} & \frac{c_S}{M_U} & \frac{k_S+k_U}{M_U} & -\frac{c_S+c_U}{M_U} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \frac{c_S c_U}{M_S M_U} \\ -\frac{c_U}{M_U} \\ (\frac{c_U}{M_U})(\frac{k_U}{c_U} - \frac{c_S}{M_U} - \frac{c_U}{M_U}) \end{bmatrix} d + \begin{bmatrix} 0 \\ \frac{1}{M_S} \\ 0 \\ -\frac{1}{M_U} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u$$

Table 1 presents the parameter values:

Table 1: Vehicle Parameter Values

Parameter	Value
Vehicle Mass (M_S)	2500 kg
Unsprung Mass (M_U)	320 kg
Suspension Stiffness (k_S)	80,000 N/m
Tire Stiffness (k_U)	500,000 N/m
Suspension Damping (c_S)	350 Ns/m
Tire Damping (c_U)	15,020 Ns/m

- Incorporate the first disturbance signal (see Figure 2) as the disturbance d.

Part A: Design of Linear MPC with the First Disturbance

- Choose prediction horizon N_p and control horizon N_c.
- Choose weights Q which penalizes deviations in state variables, and R which penalizes control effort.
- Apply constraints on control inputs (u) and states (x) as necessary.

Implementation

- Discretize the continuous-time state-space model.
- Simulate the system using MATLAB with the disturbance d.

Analysis

- Evaluate system performance: root-mean-square error (RMSE) of x_1 and x_3, and control effort (u).
- Compare with a baseline (e.g., uncontrolled system or PID controller).

Part B: Tube MPC with the First Disturbance

Motivation for Tube MPC

- Address model uncertainties or external disturbances.
- Ensure robust performance and constraint satisfaction.

Tube MPC Design

- Use Linear MPC as a nominal controller.
- Augment with a PID controller to define a robust invariant tube around the nominal trajectory.
- Define tightened constraints and weights to ensure the nominal trajectory remains feasible under disturbance d.

Implementation

- Include the given disturbance (Figure 2) in the analysis.
- Simulate the Tube MPC with MATLAB/Simulink, observing how the control effort u compensates for d.

Analysis

- Compare performance metrics, control effort u) against Linear MPC.
- Highlight improvements in robustness and disturbance rejection.

Part C: Explicit MPC and Explicit Tube MPC with the First Disturbance

Explicit MPC

- **Tube MPC Design:**
  - Design a Explicit MPC controller using the parameters obtained in part A.
- **Implementation:**
  - Implement the explicit MPC controller in simulation.
  - Evaluate the performance of the Explicit MPC controller with the given disturbance (Figure 2).
- **Analysis:**
  - Compare the performance of Explicit MPC with Linear MPC in part A.

Explicit Tube MPC

- **Tube MPC Design:**
  - Design a robust tube MPC controller using the techniques discussed in Part B.
- **Implementation:**

- Implement the explicit tube MPC controller in simulation.
- Evaluate the performance of the explicit tube MPC controller with the given disturbance (Figure 2) .

- **Analysis:**
  - Compare the performance of explicit tube MPC with standard tube MPC.

Figures

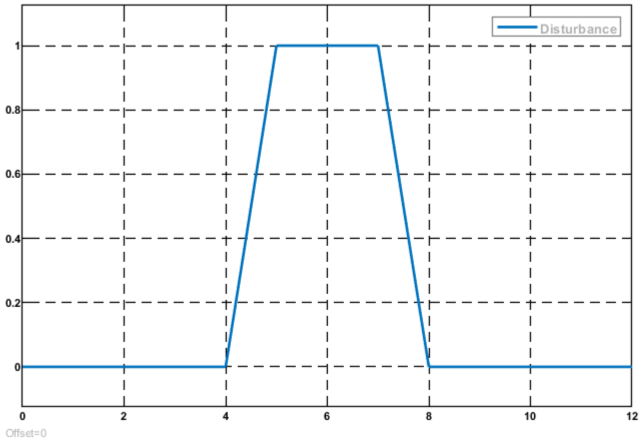

Figure 2: Disturbance Signal

Hint

- For the stability of system you should change and tune the MPC blocks weights.
- For the range of states in Explicit MPC, you should consider 8 states.

Question 2

Problem Description

Design a Hybrid Model Predictive Controller (MPC) for a two-state switched dynamical system, named the **Two-State Switched Oscillator**. The system has the following dynamics:

1. **Continuous Dynamics (Two Modes):**
   - **Mode 1:**

$$\dot{x}(t) = \begin{bmatrix} 0 & 2 \\ -2 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

- **Mode 2:**

$$\dot{x}(t) = \begin{bmatrix} 0 & 4 \\ -4 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u(t)$$

2. **Switching Conditions:**
   - The mode switches from **Mode 1 to Mode 2** when $x_1(t) > 5$.
   - The mode switches from **Mode 2 to Mode 1** when $x_1(t) < -5$.

3. **Output Equation:**

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

4. **Constraints:**
   - State constraints: $-30 \le x_1(t) \le 30$
   - Control constraints: $-30 \le u(t) \le 30$

5. **Weights:**
   - Output weight: $Q = 5$
   - Control weight: $R = 0.1$

6. **Initial states:**
   - Use the initial states $x(0) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$.

7. **Reference Signal:** The reference signal is a sinusoidal trajectory defined as:

$$r(t) = 20 \sin(\frac{2\pi}{20} t)$$

This represents the desired output trajectory that the system should track.

Tasks

1. **Hybrid MPC Problem Formulation:**
   - Incorporate the state and control constraints.

2. **Performance Analysis:**
   - Simulate the closed-loop system with the Hybrid MPC controller for 50 seconds.
   - Plot:
     - Output trajectory $y(t)$
     - Control input $u(t)$
     - Mode switch signal $S(t)$ over time.

- Reference signal $r(t)$ alongside the output $y(t)$.

3. **Discussion:**

- Discuss the behavior of the system under Hybrid MPC, particularly focusing on:
  - How the controller manages switching between modes.
  - How the controller handles constraints and tracks the sinusoidal reference.

Notes on Submission

- Submit a single PDF with analytical solutions, simulation reports, and explanations.
- Include MATLAB and Simulink files in a folder named `Simulation`.
- Use LaTeX for a 10% bonus and include source files in a folder named `LaTeX`.
- Submit a single ZIP file named `HW#-StudentID-StudentName`.
- Late submissions will incur a 10% deduction per day.