

# An adaptive controller for cooperative teleoperation system with time-varying delay and formation

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**Abstract**—In contemporary technology, cooperative teleoperation systems play a crucial role for scientists and factories engaged in intricate tasks. This paper presents a novel adaptive synchronization method tailored for a single-master and three-slave teleoperation system. This new method achieves acceptable tracking despite time-varying formation, modeling uncertainty, environment torque, and time-varying delay. Furthermore, the Lyapunov stability analysis guarantees the stability of the system. The adaptive synchronization method has been demonstrated in analytical simulations as an effective way to control cooperative teleoperation systems.

**Keywords**—Cooperative Teleoperation, Adaptive Controller, Synchronization, Modeling Uncertainty

## I. INTRODUCTION

The intersection of robotics, internet connectivity, and the imperative deployment of robotic systems in critical scenarios has given rise. The transformative impact of these advancements resonates across diverse domains, encompassing applications in rescue missions, space exploration [1], rehabilitation [2], surgeries [3], robotic assistance [4], biped robots [5], and multi-agent systems [6]. As society continues to embrace the capabilities of robots in addressing complex challenges, the need for sophisticated teleoperation systems becomes increasingly apparent. One of the critical milestones in the trajectory of teleoperation systems is the advent of commercially available solutions, notable among them being the da Vinci surgical system [7] and the Sina teleoperation system [8].

The genesis of teleoperation systems can be traced back to the mid-1940s, marked by the conception of a teleoperation setup comprising a slave robot and a master robot [9]. Alfi et al. [10] used a proportional controller against model mismatches. Ganjefar et al. [11] used an adaptive sliding mode controller. Bouteraa et al. [12] applied a robust adaptive fuzzy control method, and transmitted environmental parameters rather than torque signals. Sarajchi et al. [13] proposed the Lambert W function to obtain feasible eigenvalues. Teleoperation systems inherently involve the remote manipulation of a slave robot by a human operator through a master robot. The establishment of communication channels, often facilitated by the Internet, introduces a myriad of challenges. Communication instabilities, primarily stemming from delays. Ganjefar et al. [14] proposed the utilization of wave-variables and the Smith predictor as strategies to minimize the impact of delays.

Cooperative teleoperation systems, extending beyond individual robots to encompass multiple entities on either the master or slave side, have become instrumental in undertaking complex tasks such as transporting heavy loads. The control landscape for these systems is vast and varied, with adaptive controllers [15], impedance-based control [16], nonlinear proportional plus damping strategies [17], Wave-Variables approaches [18, 19], passive decomposition techniques [20, 21], neural networks [22],  $\mu$ -synthesis-based [23], adaptive fuzzy [24, 25], adaptive neural networks [26] and deep learning [27]. In recent times, the application of deep learning methodologies has extended beyond teleoperation to tasks such as image classification [28, 29]. Shahbazi et al. [16] illuminated the nuanced challenges that emerge when employing more than one master and one slave robot, highlighting the non-passive nature of the system under certain conditions. Zhai et al. [25] contributed an adaptive fuzzy control approach for a four-master, four-slave asymmetric teleoperation system, integrating a switched error filtering method to fortify control stability. Zakerimanesh et al. [17] used nonlinear proportional plus damping controller, and applied nonlinear functions in the proportional section. Z. Li et al. [24] presented an adaptive fuzzy control demonstrating stochastic stability. Yasrebi et al. [19] considered one slave robot and two master robots controlled by a wave-based method, assuming constant delay to reduce complexity. Farahmandrad et al. [21] introduced passive decomposition, where they divided slave systems into locked and shape systems, and controlled them using higher-order sliding mode controllers. They evaluated their method using a surgical robot at the Realtime Systems Lab at Amirkabir University of Technology.

Chen et al. [30] employed adaptive robust control to attain stability and transparency in the presence of arbitrary time delays. Yang et al. [1] employed a master robot and two slave robots, implementing an adaptive synchronization controller to address delays and modeling uncertainties. Additionally, Yang et al. [31] took into account intermittent communications, utilizing adaptive formation control for cooperative teleoperation systems.

In this paper, we introduce an innovative adaptive synchronization controller to enhance transparency and stability in cooperative teleoperation systems. The stability of this controller will be assessed using Lyapunov functional and validated through simulation involving one master and three slave robots.

This paper has the following structure. The second section delves into elucidating the dynamics of the cooperative teleoperation system, followed by the introduction of a novel adaptive synchronization controller in the third section. The stability of this control method will be scrutinized in the fourth section. The fifth section will showcase numerical simulations. Finally, the sixth section provides the conclusion for this article.

## II. COOPERATIVE TELEOPERATION MODEL

The master robot commands and coordinates three slave robots, establishing a synchronized system for efficient task execution and complex operations. To control a cooperative teleoperation system, we consider the communication topology of a cooperative teleoperation system according to Fig.1. Data exchange between three slaves is denoted by a graph  $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ , where  $\mathbf{V} = \{v_1, v_2, v_3\}$  is the set of vertices representing three slaves and,  $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$  is the set of edges. The adjacency matrix  $\mathbf{A} = [a_{ij}]$  is described as  $a_{ij} = 0$  if slave  $i$  does not exchange data between slave  $j$ . Otherwise,  $a_{ij} = 1$ . The adjacency matrix  $\mathbf{B} = \text{diag}[b_1, b_2, b_3]$  is defined as  $b_i = 0$  if slave  $i$  does not exchange information between master. Otherwise  $b_i = 1$ , and  $\mathbf{B}$  is a non-zero matrix [32]. Then we expand a nonlinear model of one master and one slave from the [33] to the one master and three slaves model:

$$\begin{cases} M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) = -\tau_m + \tau_h, \\ M_1(q_1)\ddot{q}_1 + C_1(q_1, \dot{q}_1)\dot{q}_1 + g_1(q_1) = \tau_1 - \tau_{e1}, \\ M_2(q_2)\ddot{q}_2 + C_2(q_2, \dot{q}_2)\dot{q}_2 + g_2(q_2) = \tau_2 - \tau_{e2}, \\ M_3(q_3)\ddot{q}_3 + C_3(q_3, \dot{q}_3)\dot{q}_3 + g_3(q_3) = \tau_3 - \tau_{e3}, \end{cases} \quad (1)$$

Where  $q_r, \dot{q}_r, \ddot{q}_r \in \mathbb{R}^n$  denote the joint positions, velocities, and accelerations.  $M_r \in \mathbb{R}^{n \times n}$  are the inertia matrices.  $C_r \in \mathbb{R}^{n \times n}$  represent the Coriolis and centrifugal effects,  $g_r \in \mathbb{R}^n$  are the gravitational vector,  $\tau_r \in \mathbb{R}^n$  are the input control torque, and  $\tau_h \in \mathbb{R}^n$ ,  $\tau_{e1}, \tau_{e2}, \tau_{e3} \in \mathbb{R}^n$  are the operator and environment torque, where  $r = m, 1, 2, 3$  and  $r = m$  denotes the master robot, and  $r = 1, 2, 3$  indicate three slave robots. This model has the following properties [32].

**Property 1.** The inertia matrices  $M_r(q_r)$  are positive definite and bounded.

**Property 2.** The matrix  $\dot{M}_r(q_r) - 2C_r(q_r, \dot{q}_r)$  are skew-symmetric,  $\dot{M}_r(q_r) = C_r(q_r, \dot{q}_r) + C_r^T(q_r, \dot{q}_r)$ .

**Property 3.** The dynamics can be linearly parameterized. Therefore,  $M_r(q_r)\ddot{q}_r + C_r(q_r, \dot{q}_r)\dot{q}_r + g_r(q_r) = Y_r\theta_r$  where  $Y_r \in \mathbb{R}^{n \times p}$  are matrices of known functions and  $\theta_r \in \mathbb{R}^p$  represent constant vectors of the manipulator physical parameters such as link masses and moments of inertia [33].

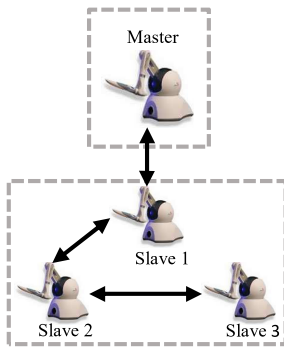


Fig. 1. Topology relationship between one master and three slaves.

In the subsequent section, we aim to attain specific control objectives as delineated in Equation (2):

$$\lim_{t \rightarrow \infty} \|q_m(t-d) - q_i(t)\| = \lim_{t \rightarrow \infty} \|\dot{q}_m(t-d) - \dot{q}_i(t)\| = 0 \quad (2)$$

Where  $\|\cdot\|$  represents the Euclidean norm of  $\cdot$ , position and velocity errors converge towards zero [33].

## III. ADAPTIVE SYNCHRONIZATION CONTROLLER

The adaptive controller enhances slave robots' position tracking, aligning with the master's instructions in cooperative teleoperation fostering improved precision and coordination. This method is shown in Fig.2. In order to aim to control goal (2) by inspiring [33], a novel adaptive controller is designed for cooperative teleoperation system as:

$$\tau_m = -\hat{M}_m(q_m)\lambda(\ddot{e}_m + \dot{e}_m) - \hat{C}_m(q_m, \dot{q}_m)\lambda(\dot{e}_m + e_m) - \hat{g}_m(q_m) + \bar{\tau}_m \quad (3)$$

$$\tau_i = \hat{M}_i(q_i)\lambda(\ddot{e}_i + \dot{e}_i) + \hat{C}_i(q_i, \dot{q}_i)\lambda(\dot{e}_i + e_i) + \hat{g}_i(q_i) - \bar{\tau}_i \quad (4)$$

Where  $i=1,2,3$  denote three slaves. So equation (3) and (4) can be written as  $\tau_m = Y_m\hat{\theta}_m + \bar{\tau}_m$  and  $\tau_i = Y_i\hat{\theta}_i - \bar{\tau}_i$ . Then by inspiring [32] we present tracking errors for cooperative teleoperation systems as:

$$\begin{aligned} e_m &= \sum_{i=1}^N b_i(q_i(t-d_{im}) - q_m) + \delta_i(t-d_{im}) \\ e_i &= \sum_{j=1}^N a_{ij}(q_j(t-d_{ji}) - q_i - \delta_i + \delta_j(t-d_{ji})) \\ &\quad + b_i(q_m(t-d_{mi}) - q_i). \end{aligned} \quad (5)$$

Where  $d$  represents time-varying delay and  $\delta$  is a time-varying formation.

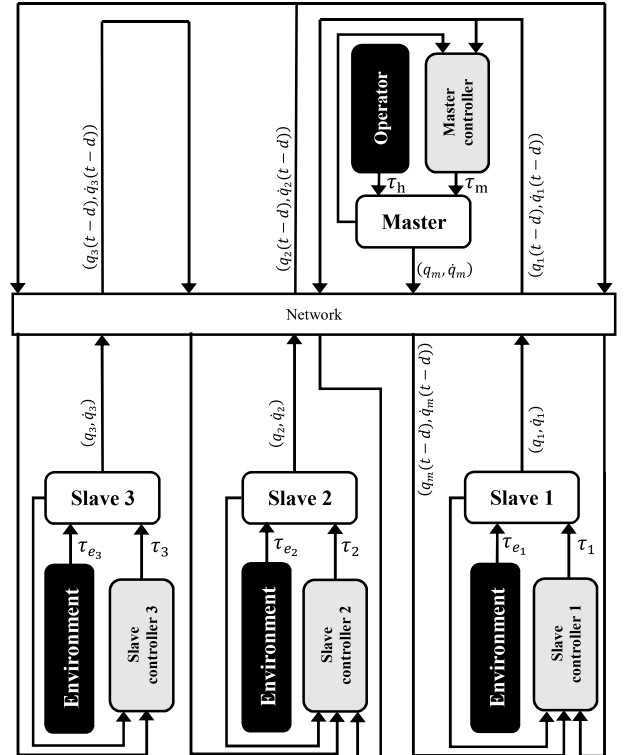


Fig. 2. The control structure of the cooperative teleoperation system.

Derivative of tracking error yields as:

$$\begin{aligned}\dot{e}_m &= \sum_{i=1}^N \left( (1 - \dot{d}_{im})\dot{q}_i(t - d_{im}) - \dot{q}_m \right) \\ &\quad + (1 - \dot{d}_{im})\dot{\delta}_i(t - d_{im}) \\ \dot{e}_i &= \sum_{j \neq i}^N \left( (1 - \dot{d}_{ji})\dot{q}_j(t - d_{ji}) - \dot{q}_i \right) - \dot{\delta}_i \\ &\quad + (1 - \dot{d}_{ji})\dot{\delta}_j(t - d_{ji}) \\ &\quad + \left( (1 - \dot{d}_{mi})\dot{q}_m(t - d_{mi}) - \dot{q}_i \right) \quad (6)\end{aligned}$$

The synchronizing signal in reference [33] is defined as:

$$\epsilon_r = \dot{q}_r - \lambda e_r \quad (7)$$

Where  $\lambda$  is a constant diagonal positive definite matrix.

In this paper we improved signal (7) and achieved new synchronizing signal (8), which will be discussed in the numerical simulation.

$$\epsilon_r = \dot{q}_r - \lambda(e_r + \dot{e}_r) \quad (8)$$

Then applying control signal (3), (4), synchronizing signal (8) and considering **Property 3** to the cooperative teleoperation system in (1), yields:

$$\begin{aligned}M_m(q_m)\dot{e}_m + C_m(q_m, \dot{q}_m)\epsilon_m &= Y_m\Delta\theta_m - \bar{\tau}_m + \tau_h \\ M_i(q_i)\dot{e}_i + C_i(q_i, \dot{q}_i)\epsilon_i &= Y_i\Delta\theta_i - \bar{\tau}_i - \tau_e.\end{aligned} \quad (9)$$

Adaptive law can be described as [33]:

$$\dot{\hat{\theta}}_r = \Gamma_r Y_r^T \epsilon_r \quad (10)$$

The torques  $\bar{\tau}_r$  in reference [33] is defined as  $\bar{\tau}_r = K_r \epsilon_r - B \dot{e}_r$  and we consider as [34]:

$$\bar{\tau}_r = K_r \epsilon_r \quad (11)$$

Where  $K_r$  are positive definite matrices. As a final step, we define controller as [33]:

$$\tau_r(t) = \bar{\tau}_r(t) - Y_r(q_r, \dot{q}_r, e_r, \dot{e}_r)\hat{\theta}_r \quad (12)$$

Where  $Y_r(q_r, \dot{q}_r, e_r, \dot{e}_r)$  are matrices of known functions, and  $\hat{\theta}_r$  are the estimated parameters,  $\Delta\theta_r = \theta_r - \hat{\theta}_r$  are the estimation errors.

#### IV. STABILITY

In references [34-36], the stability of teleoperation systems was analyzed in two methods. First, free motion. After that, it will be done in the presence of human and environment torques or forces. Similarly, this paper first discusses free motion stability, in which interaction torques are zero. Then we consider these interaction torques as a passive spring-damped model.

In free motion ( $\tau_k = 0$ ), where  $k = h, e_1, e_2, e_3$  indicate operator and environment torques respectively. The following Lyapunov functional is proposed to verify the stability

$$V = V_1 + V_2 \quad (13)$$

$$V_1 = \frac{1}{2} \epsilon_r^T M_r \epsilon_r \quad (14)$$

$$V_2 = \frac{1}{2} \Delta\theta_r^T \Gamma_r^{-1} \Delta\theta_r \quad (15)$$

The function (13) is positive definite. Considering **Property 2**, (9) and (11) derivative of Lyapunov functional is simplified to:

$$\dot{V}_1 = \epsilon_r^T (Y_r \Delta\theta_r) - \epsilon_r^T K_r \epsilon_r \quad (16)$$

$$\dot{V}_2 = -\Delta\theta_r^T \Gamma_r^{-1} \Delta\theta_r \quad (17)$$

Then,  $\dot{V} = -[\epsilon_r^T K_r \epsilon_r]$ . Since  $V \geq 0$ ,  $\dot{V} \leq 0$ ,  $\epsilon_r \in \mathcal{L}_2$  and  $\epsilon_r, \Delta\theta_r, e_r \in \mathcal{L}_\infty$ . Hence,  $\epsilon_r \in \mathcal{L}_\infty \cap \mathcal{L}_2$ ,  $\dot{e}_r \in \mathcal{L}_\infty$  support that  $|\epsilon_r| \rightarrow 0$  and,  $\lim_{t \rightarrow \infty} |\epsilon_r| = 0$ , imply that when  $t \rightarrow \infty$  ensure that  $q_m(t - d) - q_i \rightarrow 0$ . It can be concluded that the proposed Lyapunov functional is positive, and the derivative of the Lyapunov functional is negative. This shows that position tracking errors converge to zero, thereby proving the stability of this cooperative teleoperation system.

In general, interaction torques are not zero. In this section we describe the interaction torques between the environment and slaves and between operator and master as a passive spring-damped model as follows [32, 35]:

$$\tau_k = S_r q_r + D_r \dot{q}_r, \quad k = h, e_1, e_2, e_3 \quad (18)$$

Where  $r = m, 1, 2, 3$  and  $r = m$  denotes the master robot, and  $r = 1, 2, 3$  indicate three slave robots.  $S_r \in \mathbb{R}^{n \times n}$  and  $D_r \in \mathbb{R}^{n \times n}$  are diagonal and positive definite matrices of the spring and damping. The following positive-definite Lyapunov functional is used when human and environment torques are passive.

$$W = \frac{1}{2} \epsilon_r^T M_r \epsilon_r + \frac{1}{2} \Delta\theta_r^T \Gamma_r^{-1} \Delta\theta_r + V_{pass} \quad (19)$$

According to (18) interaction torques are passive so  $V_{pass} \geq 0$  and we get

$$-\int_0^t (\epsilon_m^T \tau_h) d\sigma \geq -c_b, \quad \int_0^t (\epsilon_i^T \tau_{e_i}) d\sigma \geq -c_e \quad (20)$$

Where  $c_e, c_b$  as the constant depending on the initial condition and the storage function of the passive interaction torques is given as [37].

$$V_{pass} = -\int_0^t (\epsilon_m^T \tau_h) d\sigma + \int_0^t (\epsilon_i^T \tau_{e_i}) d\sigma + c_e + c_b \quad (21)$$

Using **Property 2**, (9), (11) and passive interaction torques, the time derivatives of  $W$  yields as:

$$\dot{W} = -[\epsilon_r^T K_r \epsilon_r] - \epsilon_i^T \tau_{e_i} + \epsilon_m^T \tau_h + \dot{V}_{pass} \quad (22)$$

The operator and environment torques are bounded and passive, the storage function  $V_{pass}$  is also bounded. The equation of (22) gives that  $\dot{W} = -[\epsilon_r^T K_r \epsilon_r]$ . Since  $W \geq 0$ ,  $\dot{W} \leq 0$  the position errors asymptotically converge to zero when the interaction torques are bounded and passive.

#### V. NUMERICAL SIMULATION

In the analytical simulations, the master and three slaves manipulators are modeled as two-degree-of-freedom (2-DOF) as follows [32]:

$$M_r(q_r) = \begin{bmatrix} M_{11r} & M_{12r} \\ M_{21r} & M_{22r} \end{bmatrix}, \quad C_r(q_r, \dot{q}_r) = \begin{bmatrix} C_{11r} & C_{12r} \\ C_{21r} & C_{22r} \end{bmatrix},$$

$$G_r(q_r) = \begin{bmatrix} g_{1r} \\ g_{2r} \end{bmatrix},$$

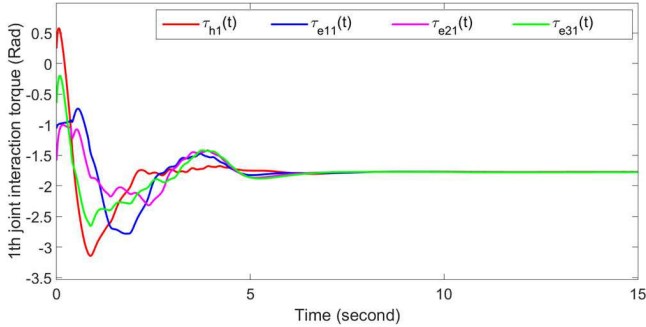
$$M_{11r} = l_{r2}^2 m_{r2} + l_{r1}^2 (m_{r1} + m_{r2}) + 2l_{r1} l_{r2} m_{r2} \cos(q_{r2}),$$

$$\begin{aligned}
M_{12r} &= M_{21r} = l_{r2}m_{r2}(l_{r2} + l_{r1}\cos(q_{r2})), \\
M_{22r} &= l_{r2}^2m_{r2}, \\
C_{11r} &= -2l_{r1}l_{r2}m_{r2}\sin(q_{r2})\dot{q}_{r2}, \\
C_{12r} &= -l_{r1}l_{r2}m_{r2}\sin(q_{r2})(\dot{q}_{r1} + \dot{q}_{r2}), \\
C_{21r} &= l_{r1}l_{r2}m_{r2}\sin(q_{r2})\dot{q}_{r1}, \quad C_{22r} = 0, \\
g_{1r} &= gl_{r2}m_{r2}\cos(q_{r1} + q_{r2}) + l_{r1}(m_{r1} + m_{r2})\cos(q_{r1}), \\
g_{2r} &= gl_{r2}m_{r2}\cos(q_{r1} + q_{r2}).
\end{aligned}$$

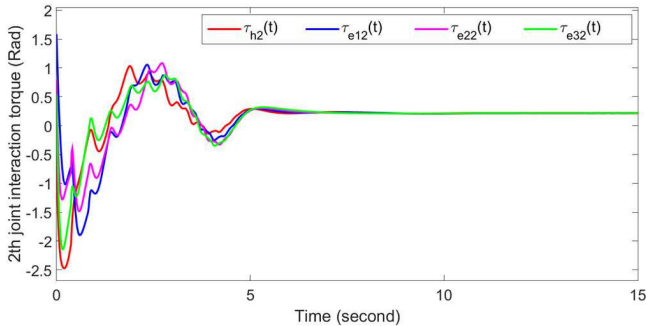
$$\begin{aligned}
Y_r(q_r, \dot{q}_r, e_r, \dot{e}_r) &= [Y_{r1}^T, Y_{r2}^T]^T, \\
Y_{r1} &= [\lambda_r(\dot{e}_{r1} + \dot{e}_{r2}), \lambda_r\dot{e}_{r1}, \lambda_r\cos(q_{r2})(2\dot{e}_{r1} + \dot{e}_{r2}) \\
&\quad - \lambda_r\sin(q_{r2})(2\dot{q}_{r2}e_{r1} + (\dot{q}_{r1} + \dot{q}_{r2})e_{r2}), \cos(q_{r1} \\
&\quad + q_{r2}), \cos(q_{r1})] \\
Y_{r2} &= [\lambda_r(\dot{e}_{r1} + \dot{e}_{r2}), 0, \lambda_r(\cos(q_{r2})\dot{e}_{r1} + \\
&\quad \sin(q_{r2})\dot{q}_{r1}e_{r1}), \cos(q_{r1} + q_{r2}), 0],
\end{aligned}$$

$$\theta_r = [l_{r2}^2m_{r2}, l_{r1}^2(m_{r1} + m_{r2}), l_{r1}l_{r2}m_{r2}, gl_{r2}m_{r2}, l_{r1}(m_{r1} + m_{r2})]^T,$$

The parameters of master and slaves model for simulations are chosen as follows:  $m_{r1} = 10$  kg,  $m_{r2} = 5$  kg,  $l_{r1} = 0.7$  m,  $l_{r2} = 0.5$  m,  $g = 9.8$  m/s<sup>2</sup>,  $\lambda_r = 1$ ,  $\Gamma_m = 0.25$  I and  $\Gamma_s = I$ , where  $r = m, 1, 2, 3$ . The initial values selected,  $q_m = [\frac{\pi}{12}, -\frac{\pi}{12}]^T$ ,  $q_1 = [-\frac{\pi}{3}, \frac{\pi}{2}]^T$ ,  $q_2 = [-\frac{\pi}{2}, \frac{\pi}{4}]^T$ ,  $q_3 = [-\frac{\pi}{5}, \frac{\pi}{6}]^T$ ,  $\dot{q}_m = [0, 0]^T$ ,  $\dot{q}_1 = [0, 0]^T$ ,  $\dot{q}_2 = [0, 0]^T$ ,  $\dot{q}_3 = [0, 0]^T$ ,  $K_r = 3I$  and  $B = I$ . Interaction torques between operator and master and between environment and slaves are defined as  $S_r = 1$ ,  $D_r = 1$ . The time-varying delays are given as  $d_l(t) = 0.3 + 0.2\sin(t)$ ,  $l = mi, im, ji$ . The time-varying formation are  $\delta_i = [0.1\sin(0.06t + (i-1)/3), 0.1\sin(0.06t + (i-1)/3)]^T$ ,  $i = 1, 2, 3$ , where  $l_{r1}$ ,  $l_{r2}$  are link lengths and  $m_{r1}$ ,  $m_{r2}$  are the masses of first and second links [32].



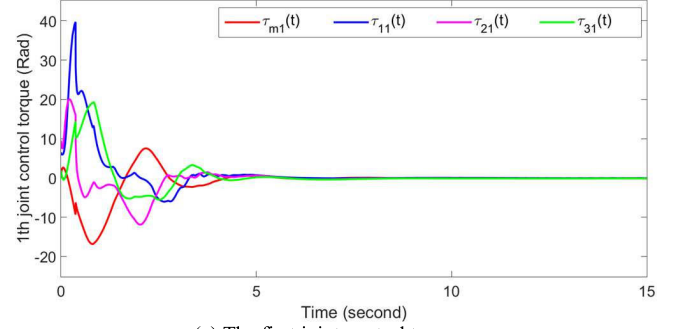
(a) The first joint interaction torques



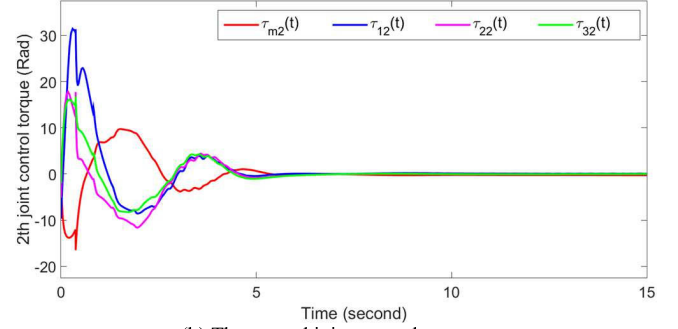
(b) The second joint interaction torques

Fig. 3. The interactional torques for cooperative teleoperation system.

The simulation results for interaction torques by the operator to the master  $\tau_{h1}$  and environment to the slaves  $\tau_{ei1}$  are shown in Fig.3a, and for the second joint  $\tau_{h2}$ ,  $\tau_{ei2}$  are demonstrated in Fig.3b. The control torques of the first joint  $\tau_{r1}$  are shown in Fig.4a. The control torques for the second joint  $\tau_{r2}$  are shown in Fig.4b. Despite the presence of time-varying formation and time delay and the aforementioned operator and environmental torques, all three slave robots were able to track the master robot after five seconds in Fig.5.

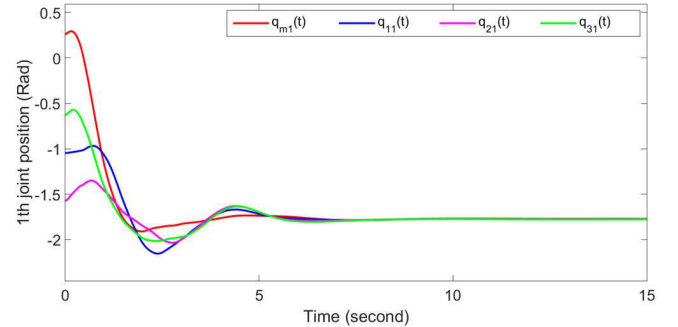


(a) The first joint control torques

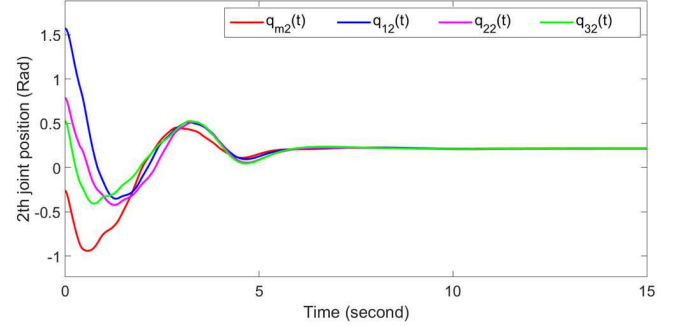


(b) The second joint control torques

Fig. 4. The control torques for cooperative teleoperation system.



(a) The first joint positions



(b) The second joint positions

Fig. 5. The position tracking of cooperative teleoperation system.



The position of the cooperative teleoperation system for the first joint is shown in Fig.5a, and Fig.5b confirms all slaves can pursue the master robot. The final values of the first and second joint positions depend on the gravitational vectors in (1) a nonlinear model of a cooperative system. The error signals for the first joint are shown in Fig.6a, and the error signals of the second joint in Fig.6b dwindle to zero after five seconds by utilizing this adaptive controller.

Then we compared our cooperative teleoperation system with the synchronizing controller proposed in [33]. In [33], a synchronization signal was introduced to govern single master and single slave teleoperation systems. In our study, we adapted this method for cooperative teleoperation systems characterized by one master and three slave manipulators, modeled as two-degree-of-freedom (2-DOF) systems.

Fig.7a illustrates a comparison of control torques for the first joint. The adaptive synchronization controller introduced in our paper resulted in control torques of less than 40 radians, whereas the controller [33] produced control torques exceeding 50 radians for the same joint. Moving to Fig.7b, which focuses on the control torques of the second joint, our synchronization method yielded torques surpassing 30 radians. Conversely, the controller [33] produced interaction torques of around 40 radians for the second joint.

Figures 8a and 8b showcase the positions of the first and second joints within the cooperative teleoperation system. These figures highlight the capability of all slaves to follow the master robot using two distinct methods. Notably, our proposed synchronization signal, as depicted in this paper, exhibits greater efficiency in reaching the final position in less time and achieving more accurate tracking when compared to the approach outlined in reference [33]. These findings underscore the enhanced performance potential of the teleoperation system through the utilization of the synchronization signal employed in our study.

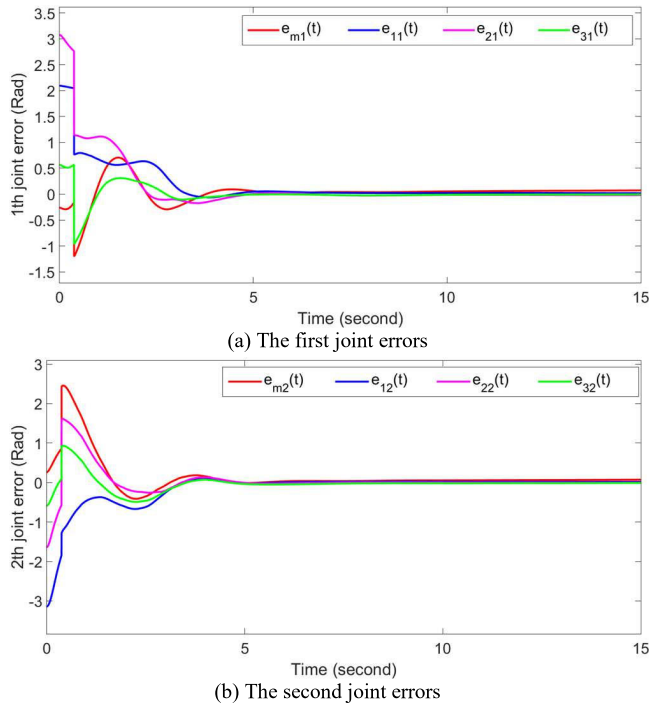


Fig. 6. The errors of cooperative teleoperation system

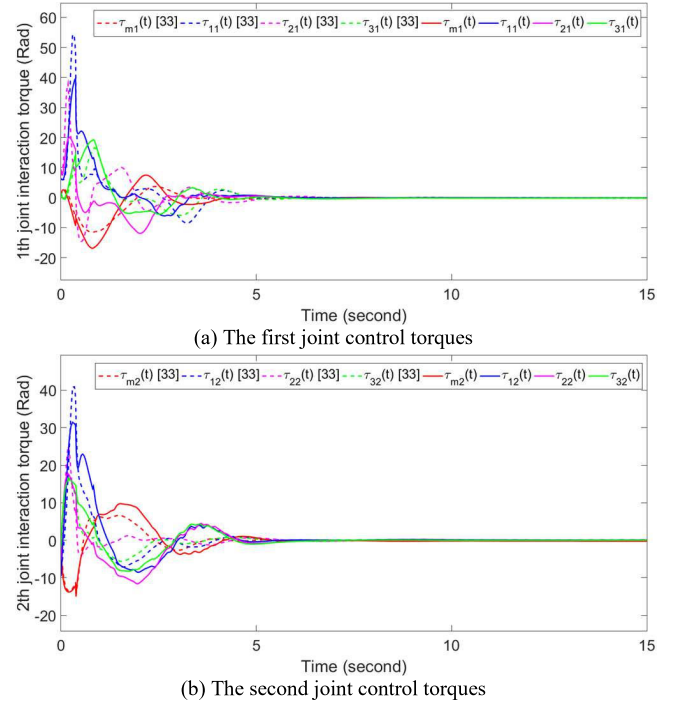


Fig. 7. Comparison of the interactional torques with ref [33].

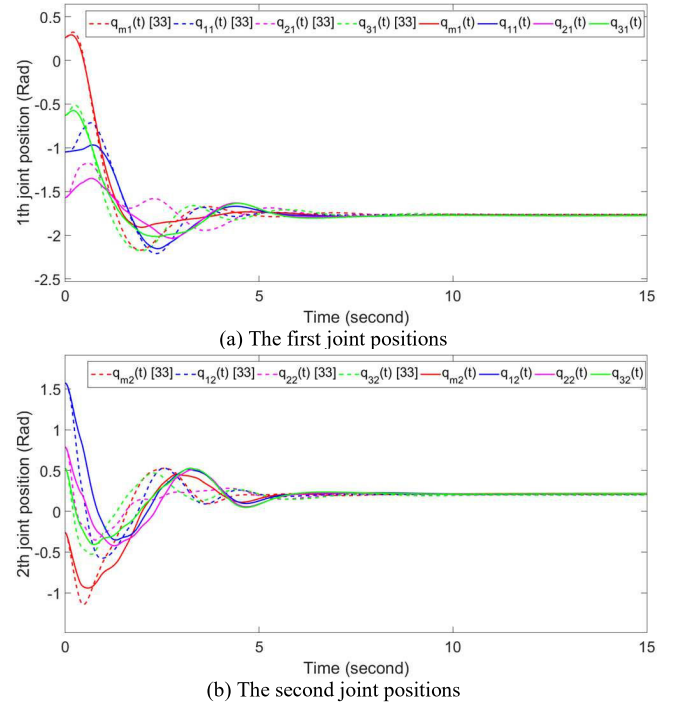


Fig. 8. Comparison of the position tracking with ref [33].

## VI. CONCLUSION

In this paper, a novel adaptive synchronization controller is presented that undergoes a comparative analysis with previous methods. The stability of the proposed controller is evaluated using the Lyapunov function. The application of this controller is demonstrated in a teleoperation system featuring a single master and three slaves. Notably, the controller showcases stability, precise position tracking across all joints, and effective reduction of error signals to zero. These accomplishments are achieved even in the face of challenges such as time-varying formations, modeling uncertainties, environment torques, and time-varying delays.

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