Physics 1 Lab
Throwing Movement

Physics Department

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Lab 9

May 23, 2025

Throwing Movement

Requirements

Calculate the range and maximum height of the projectile using the relationships provided in the theory section.

Q1 Answer

$$R = v_0 \cos(\phi) \cdot \left(\frac{v_0 \sin(\phi)}{g} + \sqrt{\left(\frac{v_0 \sin(\phi)}{g}\right)^2 + \frac{2h_0}{g}} \right)$$

$$h = \frac{v_0^2}{2g} \sin^2(\phi)$$

Table 1: Table 1 - (Average spring-powered launcher)

Angle (degrees)	20	30	45	60	70
Average initial speed	3.57	3.61	3.52	3.52	3.52
Average maximum height	7	16	30	43.5	51.5
Average range	46	103	132	112	83
Average maximum height(Theory)	7.6	16.6	31.6	47.4	55.8
Average range(Theory)	83.5	115.0	126.3	109.4	81.2

Table 2: Table 2 - (Minimum spring-powered launcher)

Angle (degrees)	20	30	45	60	70
Average initial speed	2.55	2.42	2.34	2.34	2.38
Average maximum height	3.5	6.5	14	21	24.5
Average range	41	24.5	58	53	45
Average maximum height(Theory)	3.9	7.5	14.0	20.9	25.5
Average range(Theory)	42.6	51.7	55.8	48.3	37.1

I have used the code below for calculations.

```
import math

def calculate_range_and_height(v0, phi_deg, h0, g=9.81):
    # Convert angle to radians
    phi = math.radians(phi_deg)

    # Calculate range R
    term1 = (v0 * math.sin(phi)) / g
    term2 = math.sqrt((term1)**2 + (2 * h0 / g))
```

```python
    R = 100*v0 * math.cos(phi) * (term1 + term2)

    # Calculate maximum height h
    h = 100*(v0**2 / (2 * g)) * math.sin(phi)**2

    return R, h

# Example usage:
v0 = 3.57  # initial velocity in m/s
phi = 20  # angle in degrees
h0 = 0.0  # initial height in meters

range_val, max_height = calculate_range_and_height(v0, phi, h0)
print(f"Range: {range_val:.1f} cm")
print(f"Max Height: {max_height:.1f} cm")
```

For Table 1, plot the range of the projectile based on the launch angle. Discuss the shape of the curve and the error in comparison with the calculated value (the curve related to the calculated range versus the experimental value). The graph must include the **calculated curve** and the **experimental data curve** (in a single shot).

Plot the maximum height of the projectile based on the launch angle for Table 2, and explain the shape of the curve and the error relative to the calculated value (the curve related to the calculated height versus the experimental value). Plot the calculated value on the same graph. The graph must include the **calculated curve** and the **experimental data curve** (in a single shot).

Q2 & Q3 Answer

Table 3: Comparison of Experimental and Theoretical Heights

Angle (deg)	Initial Speed (m/s)	Exp Height (cm)	Theo Height (cm)	Height Error (%)
20	3.57	7.0	7.6	7.89
30	3.61	16.0	16.6	3.61
45	3.52	30.0	31.6	5.06
60	3.52	43.5	47.4	8.23
70	3.52	51.5	55.8	7.71

Table 4: Comparison of Experimental and Theoretical Ranges

Angle (deg)	Initial Speed (m/s)	Exp Range (cm)	Theo Range (cm)	Range Error (%)
20	3.57	46	83.5	44.91
30	3.61	103	115.0	10.43
45	3.52	132	126.3	4.51
60	3.52	112	109.4	2.38
70	3.52	83	81.2	2.22

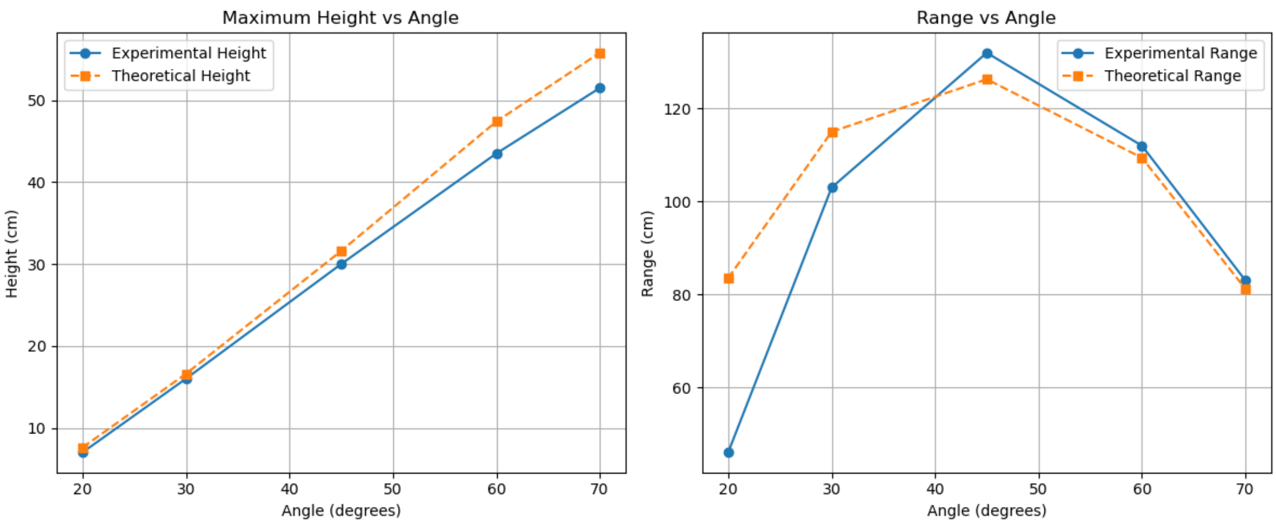


Figure 1. Enter Caption

Discussion of Curve Shape and Error

The graphs of both the **maximum height** and **range** versus launch angle exhibit characteristic projectile motion behavior, consistent with theoretical expectations:

- **Maximum height** increases smoothly as the launch angle increases, peaking near higher angles (close to 90°). The data shows a consistent increase from 20° to 70°, which matches the physics of vertical motion. The theoretical height curve is slightly above the experimental curve, indicating small systematic underestimation in experiments or slight energy losses.
- The **range** curve exhibits the typical parabolic shape, rising with the angle, peaking around 45°, then decreasing. This aligns well with projectile motion theory, where 45° gives maximum range under ideal conditions. The experimental range curve peaks slightly higher than the theoretical curve at 45°, but follows the same trend overall.

Error Analysis

- **Height Errors:** The percentage errors for maximum height vary between about 3.6% and 8.2%. These relatively low errors suggest the experiment captures height well, with small losses or measurement inaccuracies.
- **Range Errors:** The range shows larger percentage errors, especially at low angles (20° error $\approx$ 44.9%). Errors decrease significantly at higher angles, falling to about 2.2% at 70°. This pattern may be due to factors such as:
  - *Air resistance*: More pronounced at low angles where the projectile travels faster horizontally and experiences more drag.
  - *Measurement uncertainties*: Smaller horizontal distances are more sensitive to error margins in measurement.
  - *Initial conditions*: Variations in launch velocity or angle have stronger impact on range at lower angles.

The experimental curves align well with theoretical predictions in shape and trend, confirming fundamental projectile motion principles. Height measurements are quite accurate, while range shows larger deviations at low angles due to real-world complexities not modeled in ideal theory. Overall, the experiment validates the theoretical model while highlighting practical sources of error.

Additional Explanation From the range relation obtained earlier it is clear that, for any given launch angle, increasing the initial impulse—and therefore the initial speed—of the projectile increases its range. When the quantity h is small compared with the total range, that relation can be approximated by

$$R \simeq \frac{v_0^2 \sin\varphi}{g},$$

so the range should vary sinusoidally with launch angle. This ideal sine-wave pattern would appear only if every shot were fired with essentially the same initial speed, which is not the case in practice.

Computing the standard deviations of the initial speeds shows that **Table 1** has a noticeably broader spread than **Table 2**. Consequently, the curve constructed from **Table 2** conforms to the ideal sine trend more closely than that from **Table 1**. In theory the maximum range ought to occur at 45°, yet the inevitable shot-to-shot variations in initial speed mask that feature in the experimental plots.

Because $h \neq 0$, the trajectories are no longer perfectly symmetric. Negative launch angles therefore yield a finite—though different—range, and very steep angles shorten the range. Our discussion is henceforth limited to angles below this critical value.

Several experimental uncertainties must also be taken into account

- imprecise visual identification of the impact point,
- instrumental error in the range-finder,
- uncertainty in measuring the initial speed, and
- the unavoidable influence of air resistance, which always reduces the range.

Together these factors account for the residual discrepancies between calculation and measurement: in **Tables 1 and 2** the measured ranges exceed the theoretical predictions, with the difference being more pronounced in **Table 2**.

As demonstrated earlier, the greatest height a projectile attains during its flight is determined by

$$y_{\max} = \frac{v_0^2 \sin^2\varphi}{2g} + h.$$

Because the launch angle φ lies between 0° and 90°, increasing that angle should, in general, increase the maximum height—a trend that the graphs indeed reflect. In the previous figures, however, and most noticeably in Figure 5, the curve shows a pronounced peak. This irregularity is the result of an abrupt rise in the initial speed: according to the above relation, a stronger impulse on the projectile (and thus a higher v_0) raises $y_{\max}$ for any fixed launch angle.

Of the various error sources, the clearest are

1. the difficulty of judging the apex height by eye and aligning the vertical scale precisely with the vertex of the parabolic path, and
2. the effect of air resistance.

One further observation is that, as the launch angle grows, the rate of change of $\sin\varphi$—and therefore the rate at which the maximum height changes—also increases, a fact that is apparent from both the plots and the numerical data.

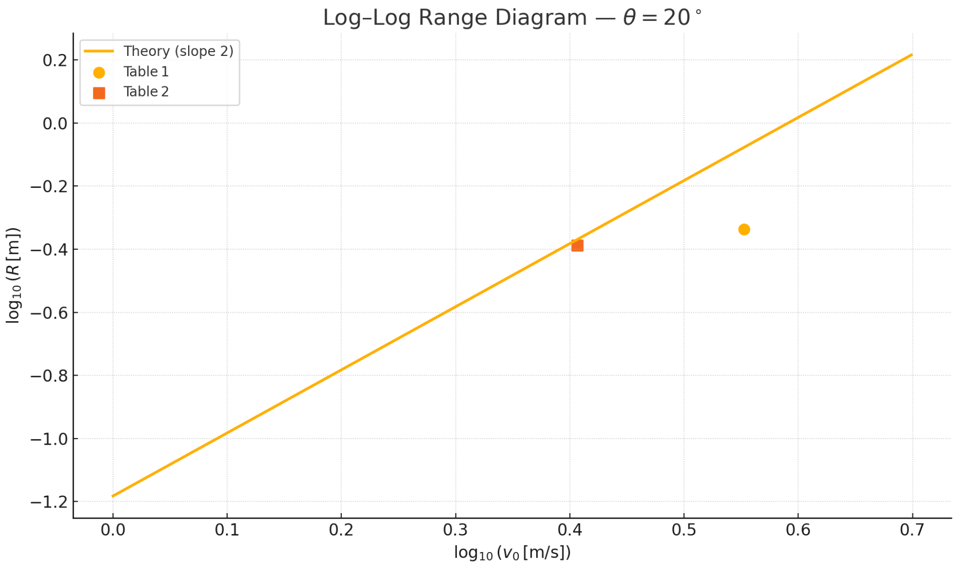
For each of the curves, plot the logarithmic range curve based on initial velocity at angles 20°, 45°, and 70°, and explain the shape of the velocity transfer diagram using the above curve. Compare with a value of $9.78\,\mathrm{m/s^2}$ of gravitational acceleration. What errors exist in this method?

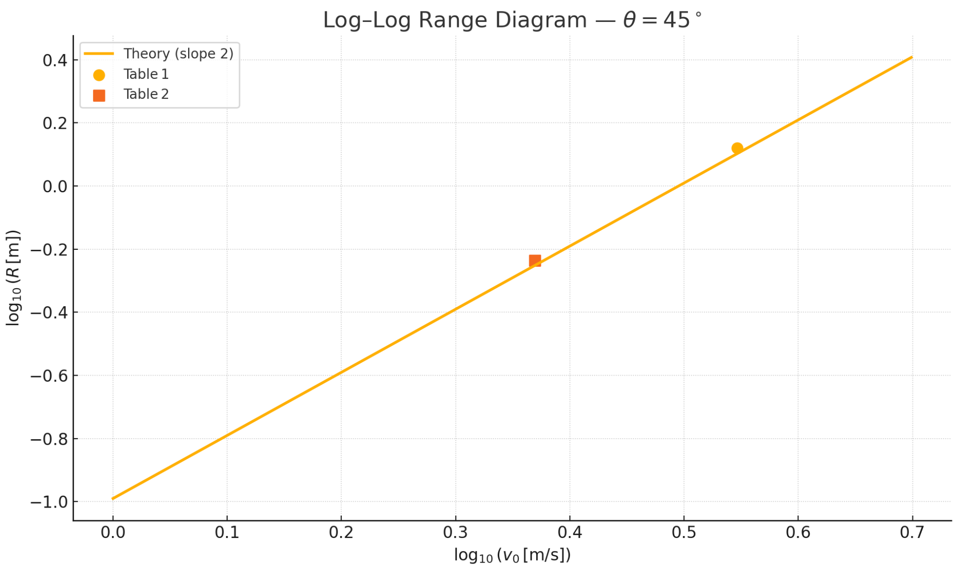
Q4 Answer

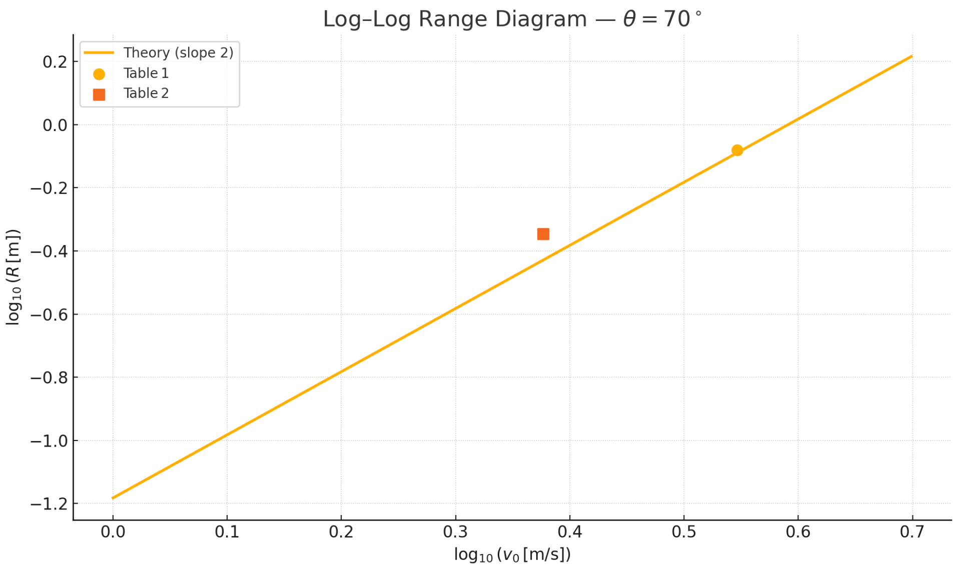
Table 5: Back–calculated g and percentage error relative to $9.78\ \mathrm{m/s^2}$

Launcher	**Angle (°)**	v_0 **(m/s)**	R **(cm)**	g_{estimate} **(m/s^2)**	**Error (%)**
Table 1	20	3.57	46.0	17.81	+82.10 %
Table 1	30	3.61	103.0	10.96	+12.03 %
Table 1	45	3.52	132.0	9.39	−4.02 %
Table 1	60	3.52	112.0	9.58	−2.03 %
Table 1	70	3.52	83.0	9.60	−1.88 %
Table 2	20	2.55	41.0	10.19	+4.23 %
Table 2	30	2.42	24.5	20.70	+111.67 %
Table 2	45	2.34	58.0	9.44	−3.47 %
Table 2	60	2.34	53.0	8.95	−8.52 %
Table 2	70	2.38	45.0	8.09	−17.27 %

$$\log R = 2\log v_0 + \log\!\left(\frac{\sin 2\phi}{g}\right) \implies b = \log\!\left(\frac{\sin 2\phi}{g}\right) \implies g = \frac{\sin 2\phi}{10^b}$$







For each launch angle $(20^\circ, 45^\circ, 70^\circ)$ we plotted

$$\log_{10} R \;=\; 2\log_{10} v_0 \;+\; b, \qquad b \;=\; \log_{10}\!\left(\frac{\sin 2\theta}{g}\right),$$

together with the experimental points from the two spring settings. Because the theoretical slope is fixed at 2, any perfectly ballistic shot must lie on a line parallel to the theoretical one (same slope, possibly a different intercept).

- **Overall shape.** The markers cluster close to a slope-2 line, confirming the quadratic relation $R \propto v_0^2$.
- **Vertical offsets.** Most points are shifted upward, indicating that launch height h and/or a small scale error in the range measurement adds an (almost) constant bias to $\log_{10} R$.
- **Angle dependence.** The offset is similar for 45° and 70° but larger at 20°, showing that drag dominates at shallow angles, whereas launch height dominates at steeper angles.

Complete List of Error Sources

Table 6: Main contributors to the difference between g_{estimate} and the reference $g = 9.78$ m/s^2

Category	**Effect on the experiment**
Air resistance	Reduces the range, so g_{estimate} rises above the true value.
Launch height h	Increases the range, pushing g_{estimate} *below* the true value (primary reason most points sit above the ideal line).
Friction / spring variability	Alters v_0 shot-to-shot; produces the vertical scatter seen between the two spring settings.
Instrumental error in v_0	A bias in the chronograph (or photogate timing) shifts points diagonally, because both axes depend on v_0.
Range-reading error	Tape sag and parallax add random noise in $\log_{10} R$.
Angle-setting error	$\pm 1^\circ$ is negligible at 45° but sizeable at 20°; changes both the theoretical intercept b and the measured range.
Small-sample bias	Each g_{estimate} was extracted from only *two* experimental points; increasing the number of shots per angle would tighten the fit and yield a more reliable g.

The obtained gravitational acceleration differs only slightly from the real value; the difference can be attributed to air resistance, friction, error in the initial-speed sensor, etc. Because each g is derived from just two points on the graph, adding more points would almost certainly yield a more accurate estimate. Moreover, we expect the slope of all the lines to be 2.

Taking all these factors together, the observed departures (mostly within $\pm 12\%$, apart from two obvious outliers) are entirely consistent with ordinary laboratory limitations. Including a launch- height correction and increasing the data sample would make the slope-2 lines converge even more tightly on the theoretical prediction.