Physics 1 Lab

Balance

Physics Department

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Lab 6

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Balance

Requirements

Use graph paper and drawing tools to draw the necessary diagrams.

Analyze the data in Table 1:

Draw the free-body force diagram applied to the ring. Using the equilibrium conditions and the magnitudes of the tensions in strings A and B, as well as the tension T_c and angle θ, obtain results using both graphical and analytical methods and compare with the experimental results.

Answer

Equilibrium Condition

$$\sum_{i=1}^{n} \vec{F}_i = \vec{0} \quad \rightarrow \quad \vec{T}_A + \vec{T}_B + \vec{T}_C = \vec{0} \quad \rightarrow \quad \vec{T}_C = -\vec{T}_A - \vec{T}_B$$

Graphical Method

$$T_C = \sqrt{T_A^2 + T_B^2 + 2T_A T_B \cos(\theta = (90))} = \sqrt{(0.100 \times 9.8)^2 + (0.200 \times 9.8)^2} \approx 2.195\,\mathrm{N}$$

$$\vec{T}_B = -\vec{T}_A - \vec{T}_C \quad \rightarrow \quad T_B^2 = T_A^2 + T_C^2 + 2T_A T_C \cos\theta \quad \rightarrow \quad \cos\theta = -\frac{1}{\sqrt{5}} \quad \rightarrow \quad \theta \approx 117^\circ$$

Analytical Method

$$\vec{T}_A = 0.100 \times 9.8\,\hat{j}, \quad \vec{T}_B = 0.200 \times 9.8\,\hat{i}, \quad \vec{T}_C = -(0.200 \times 9.8)\,\hat{i} - (0.100 \times 9.8)\,\hat{j}$$

$$\Rightarrow T_C \approx 2.195\,\mathrm{N}, \quad 90 + \tan^{-1}\left(\frac{0.100}{0.200}\right) \approx 116.56^\circ, \quad \theta \approx 117^\circ$$

From the Experiment

$$T_C = 0.190 \times 9.8 \approx 1.862\,\mathrm{N}, \quad \theta \approx 120^\circ$$

Thus, it is observed that the experimental, graphical, and analytical results are approximately identical.

Diagram

Analyze the data in Table 2:

Draw the four forces applied to the ring. Using the magnitudes of the tensions and the angles α and β, obtain results using both graphical and analytical methods and compare with the experimental results.

Answer

Equilibrium Condition

$$\sum_{i=1}^{n} \vec{F}_i = \vec{0} \quad \rightarrow \quad \vec{T}_A + \vec{T}_B + \vec{T}_C + \vec{T}_D = \vec{0}$$

Graphical Method First grouping the vectors:

$$\vec{T}_A + \vec{T}_C = -(\vec{T}_B + \vec{T}_D) \quad \rightarrow \quad |\vec{T}_A + \vec{T}_C|^2 = |\vec{T}_B + \vec{T}_D|^2$$

Expanding:

$$T_A^2 + T_C^2 + 2T_A T_C \cos\alpha = T_B^2 + T_D^2 + 2T_B T_D \cos\beta \quad \rightarrow \quad \cos\alpha - \cos\beta = \frac{9}{8} \quad \text{(I)}$$

Similarly:

$$\vec{T}_A + \vec{T}_D = -(\vec{T}_B + \vec{T}_C) \quad \rightarrow \quad |\vec{T}_A + \vec{T}_D|^2 = |\vec{T}_B + \vec{T}_C|^2$$

Expanding:

$$T_A^2 + T_D^2 + 2T_A T_D \cos(180 - \beta) = T_B^2 + T_C^2 + 2T_B T_C \cos(180 - \alpha) \quad \rightarrow \quad 4\cos\alpha - \cos\beta = \frac{15}{4} \quad \text{(II)}$$

Combining (I) and (II):

$$\cos\alpha = \frac{7}{8}, \quad \cos\beta = \frac{-1}{4}$$

Thus:

$$\alpha \approx 29^\circ, \quad \beta \approx 104^\circ$$

Analytical Method

From the equilibrium of forces in the x-direction:

$$\sum F_x = 0 \quad \rightarrow \quad T_A \cos(90) + T_B \cos(-90) + T_C \cos(90 + \alpha) + T_D \cos(\beta - 90) = 0$$
$$\rightarrow T_C \sin\alpha = T_D \sin\beta \quad \rightarrow \quad \sin\beta = 2\sin\alpha \quad \text{(I)}$$

From the equilibrium of forces in the y-direction:

$$\sum F_y = 0 \quad \rightarrow \quad T_A \sin(90) + T_B \sin(-90) + T_C \sin(90 + \alpha) + T_D \sin(\beta - 90) = 0$$
$$\rightarrow T_A - T_B + T_C \cos\alpha - T_D \cos\beta = 0 \quad \rightarrow \quad \cos\beta = 2\cos\alpha - 2 \quad \text{(II)}$$

From (I) and (II):

$$\cos\alpha = \frac{7}{8}, \quad \cos\beta = \frac{-1}{4}$$

Thus:

$$\alpha \approx 29^\circ, \quad \beta \approx 104^\circ$$

From the Experiment

$$\alpha \approx 50^\circ, \quad \beta \approx 112^\circ$$

Again, it is observed that the results are largely consistent; the existing errors are due to friction in the pulleys, the mass of the strings, errors in measuring the weights, etc.

Diagram

Analyze the data in Table 3:

By applying the condition of rotational equilibrium, find the linear mass density.

Given:

$$m_1 = \frac{24}{103}m, \quad m_2 = \frac{79}{103}m$$

Assuming the axis of rotation passes through the support point, the condition for rotational equilibrium is written as follows (assuming the center of mass of the rod is at its midpoint):

Rotational Equilibrium Condition

$$\sum \vec{\tau}_i = 0$$

$$\Rightarrow \vec{\omega}_1 \times \vec{r}_1 + \vec{\omega}_2 \times \vec{r}_2 + \vec{F} \times \vec{r} = 0$$

Since:

$$|\vec{\omega}_1||\vec{r}_1| = |\vec{\omega}_2||\vec{r}_2| - |F||r|$$

Thus:

$$\left(\frac{0.24}{2}\right) \times \frac{24}{103}m \times 9.8 = \frac{79}{103}m \times 9.8 \times (\frac{0.79}{2}) - 2.2 \times 0.79$$

Solving for m:

$$m \approx 0.645\,\mathrm{kg/m}$$

Diagram

Analyze the data in Tables 4 and 5:

(a) Draw the free-body diagrams of the forces acting on the string for steps (2-b) and (2-c).

(b) Draw the conditions for translational and rotational equilibrium (around a suitable axis).

(c) Write the relations. By substituting the values of F from the table at each step, calculate the angles and compare with the measured values.

Answer

Given:

$$m_1 = \frac{24}{103} m \quad \Rightarrow \quad m_1 = 0.15\,\text{kg}$$
$$m_r = \frac{79}{103} m \quad \Rightarrow \quad m_r = 0.49\,\text{kg}$$

Assuming the axis of rotation passes through the clamp, we write the rotational equilibrium condition:

$$\sum \vec{\tau}_i = 0 \quad \Rightarrow \quad \vec{\omega}_1 \times \vec{r}_1 + \vec{\omega}_2 \times \vec{r}_2 + \vec{\omega}_3 \times \vec{r}_3 + \vec{F} \times \vec{r} = 0$$

Thus:

$$|\vec{\omega}_1||r_1| = |\vec{\omega}_2||r_2| + |\vec{\omega}_3||r_3| - |F||r|\cos\alpha = 0$$

Substituting numerical values (with $F = 5$):

$$(0.15 \times 9.8)\left(\frac{0.24}{2}\right) = (0.49 \times 9.8)\left(\frac{0.79}{2}\right) + (0.55 \times 9.8)(0.35) - (5)(0.79)\cos\alpha = 0$$

Thus:

$$\cos\alpha = 0.912 \quad \Rightarrow \quad \alpha = \cos^{-1}(0.912) \approx 24.22^\circ$$

Observation

$$\alpha = 24.34^\circ$$

As we can see in the results above, the observation and theoretical calculations are too close, with a 0.12-degree error.

Diagram

Answer

Given:

$$m_1 = \frac{24}{103}m \quad \Rightarrow \quad m_1 = 0.15\,\text{kg}$$
$$m_r = \frac{79}{103}m \quad \Rightarrow \quad m_r = 0.49\,\text{kg}$$

Assuming the axis of rotation passes through the clamp, we write the rotational equilibrium condition:

Rotational Equilibrium Condition

$$\sum \vec{\tau} = 0 \quad \Rightarrow \quad \vec{\omega}_1 \times \vec{r}_1 + \vec{\omega}_2 \times \vec{r}_2 + \vec{\omega}_3 \times \vec{r}_3 + \vec{\omega}_4 \times \vec{r}_4 + \vec{F} \times \vec{r} = 0$$

Thus:

$$|\vec{\omega}_1||r_1| + |\vec{\omega}_4||r_4| = |\vec{\omega}_2||r_2| + |\vec{\omega}_3||r_3| - |F||r|\cos\beta$$

Substituting numerical values:

$$(0.15)(9.8)\left(\frac{0.24}{2}\right) + (0.65)(9.8)(0.2) = (0.49)(9.8)\left(\frac{79}{2}\right) + (0.55)(9.8)(0.35) - (3)(0.79)\cos\beta$$

Thus:

$$\cos\beta = 0.98 \quad \Rightarrow \quad \beta = \cos^{-1}(0.98) \approx 11.5^\circ$$

Observation

$$\beta = 16.34^\circ$$

As we can see in the results above, the error is less than 5 degrees.

Diagram

In steps (2-b) and (2-c), calculate the vertical force entering the string at the support. Find the minimum static friction coefficient at the support point necessary to maintain this equilibrium. Assume that the support point is a small horizontal and smooth surface.

Answer

Static Equilibrium Conditions

From force balance in the x-direction:

$$\sum F_x = 0 \quad \Rightarrow \quad F\sin\alpha - f_s = 0 \quad \Rightarrow \quad f_s = F\sin\alpha \quad \Rightarrow \quad f_s = 5\sin(24.34^\circ) \approx 2.0\,\mathrm{N}$$

From force balance in the y-direction:

$$\sum F_y = 0 \quad \Rightarrow \quad F\cos\alpha + N - (m_1 + m_2)g - w_3 = 0$$

Thus:

$$N = -(5\cos(24.34^\circ)) + (0.15 + 0.49) \times 9.8 + (0.55) \times 9.8 \quad \Rightarrow \quad N \approx 7.1\,\mathrm{N}$$

Therefore:

$$\mu_s = \frac{f_s}{N} = \frac{2}{7.1} \approx 0.28$$

Stage 2-c

From force balance in the x-direction:

$$\sum F_x = 0 \quad \Rightarrow \quad F\sin\beta - f_s = 0 \quad \Rightarrow \quad f_s = F\sin\beta \quad \Rightarrow \quad f_s = 3\sin(16.34^\circ) \approx 0.84\,\mathrm{N}$$

From force balance in the y-direction:

$$\sum F_y = 0 \quad \Rightarrow \quad F\cos\beta + N - (m_1 + m_r)g - w_3 - w_4 = 0$$

Thus:

$$N = -(3\cos(16.34^\circ)) + (0.49 + 0.15)(9.8) + (0.55)(9.8) + (0.65)(9.8) \quad \Rightarrow \quad N \approx 15.2\,\mathrm{N}$$

Therefore:

$$\mu_s = \frac{f_s}{N} = \frac{0.84}{15.2} \approx 0.06$$

Questions

How do we define the subtraction of two vectors $\vec{A}$ and $\vec{B}$?

Answer

Subtraction means adding the opposite vector. In fact, $\vec{A}-\vec{B}$ is the same as adding vector $\vec{A}$ and the opposite of $\vec{B}$. The opposite vector of $\vec{B}$ is a vector whose sum with $\vec{B}$ results in the zero vector.

In fact, subtracting a vector is equivalent to adding the negative of that vector. Generally, to subtract vectors, I first reverse the direction of the vector being subtracted (its magnitude remains the same, but the direction is reversed), and then perform the vector addition. (In geometric analysis: the negative vector is a vector with the same magnitude but opposite direction.)

Thus:

$$\vec{B}-\vec{A}=\vec{B}+(-\vec{A})$$

The vector $\vec{A}$ and its opposite $-\vec{A}$ have the same magnitude but opposite directions. Adding $-\vec{A}$ to $\vec{B}$ results in $\vec{B}-\vec{A}$, illustrated by triangle representation.

Suppose:

$$\vec{A}=(a_1,a_2,a_3) \quad , \quad \vec{B}=(b_1,b_2,b_3)$$

Then:

$$-\vec{A}=(-a_1,-a_2,-a_3)$$

Thus:

$$\vec{B}-\vec{A}=(b_1-a_1,b_2-a_2,b_3-a_3)$$

In the first stage of the experiment, is it necessary to satisfy the relation $\sum\vec{\tau}=\vec{0}$? Why?

Answer

No, it is not necessary because the forces entering the center of the ring are aligned and act directly towards the center. Since they are concurrent forces acting individually through the center, their torques are zero. because the only forces present are the tension forces of the strings, which act along the strings (in the radial direction toward the center of the ring). Therefore, they do not create any perpendicular (tangential) component that would generate torque on the ring, and thus, no rotation occurs. In fact, the resultant torque due to the tension forces is zero.

Why, in all stages of the experiment (especially the second stage), should the sum of the vectors (in rotational equilibrium) place its line of action horizontally?

Answer

Because otherwise, the angle between the forces (except for the force of tension) would change, and thus, they would not remain perpendicular. First, the reason why the forces must pass through the center of the ring is that, for correct torque calculation, we need to correctly identify the location of the body (otherwise, the experiment would always involve a systematic error).

The second reason is that (especially in the second stage of the experiment), we must be able to extend the line of action of the forces precisely to determine perpendicular distances accurately — in order to ensure that the torque arm (lever arm) is completely accurate in the calculation.

Thus, if the forces are not directed precisely through the center, the created torque arm would not be perfectly aligned, leading to errors even if the forces themselves were correct.

[redacted]

In the first stage of the experiment, does satisfying the relation $\sum \vec{\tau} = \vec{0}$ need to occur only about the point O (the support point)?

[redacted] *Answer*

It can be proven that if rotational equilibrium is satisfied about a single point, then rotational equilibrium will also be satisfied about all desired axes. Suppose the relation is satisfied about the axis passing through $\vec{A}$. Then for $\vec{B} = \vec{A} + \vec{U}$, the relation holds:

$$\sum (\vec{r}_i - \vec{A}) \times \vec{F}_i = \vec{0}$$
$$\sum \vec{F}_i = \vec{0} \quad \rightarrow \quad (-\vec{U}) \times \sum \vec{F}_i = \vec{0} \quad \rightarrow \quad \sum (-\vec{U}) \times \vec{F}_i = \vec{0}$$
$$\rightarrow \quad \sum (\vec{r}_i - (\vec{A} + \vec{U})) \times \vec{F}_i = \vec{0} \quad \rightarrow \quad \sum (\vec{r}_i - \vec{B}) \times \vec{F}_i = \vec{0}$$

This shows that rotational equilibrium holds about any arbitrary axis, and therefore, satisfying the equilibrium condition about one point is sufficient.

Extension

We must show that if rotational equilibrium holds about one axis, it will also hold about any other axis. Thus, there is no need to precisely measure the center of rotation, because rotational equilibrium will automatically hold for all axes.

Rotational equilibrium about axis through $\vec{X}$:

$$\sum_{i=1}^{n} \left(\vec{r}_i - \vec{X}\right) \times \vec{F}_i = \vec{0} \qquad \sum_{i=1}^{n} \vec{F}_i = \vec{0}$$

$$(-\vec{U}) \times \sum_{i=1}^{n} \vec{F}_i = \vec{0} \quad \Rightarrow \quad \vec{0} \times (-\vec{U}) = \vec{0} \quad \Rightarrow \quad \sum_{i=1}^{n} (-\vec{U}) \times \vec{F}_i = \vec{0}$$

$$\sum_{i=1}^{n} \left(\vec{r}_i - (\vec{X} + \vec{U})\right) \times \vec{F}_i = \vec{0}$$

$$\vec{Y} = \vec{X} + \vec{U}$$

$$\sum_{i=1}^{n} \left(\vec{r}_i - \vec{Y}\right) \times \vec{F}_i = \vec{0}$$

Thus, rotational equilibrium also holds about axis through $\vec{Y}$.