Physics 1 Lab
Rotational Inertia

Physics Department

Parsa Hatami	400100962
Alireza Haghshenas	400101067

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Rotational Inertia

Requests

For each of the data entries in Tables 1 to 5, calculate the moment of inertia using the definition

$$I = \int r^2 dm = \sum m_i r_i^2$$

and compare the result with the value obtained from the definition of rotational moment of inertia (report the relative error in percent). Explain the cause of the error in each case separately.

Answer

Table 1

$$I = mr^2 \left(\frac{9.78}{a} - 1 \right)$$

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{5.406^2}{2 \times 0.50} \right) - 1 \right) = 0.003\, kg\, m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{4.303^2}{2 \times 0.50} \right) - 1 \right) = 0.003\, kg\, m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.280^2}{2 \times 0.50} \right) - 1 \right) = 0.003\, kg\, m^2$$

According to the definition of rotational inertia, the moment of inertia of a slender rod is given by the formula:

$$I = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$$

We have:

$$I = \frac{1}{4} \times (0.313) \left[0.0010^2 + \frac{1}{3} \times 0.508^2 \right] = 0.007\, kg\, m^2$$

Errors:

- 1: 57.1 %
- 2: 57.1 %
- 3: 57.1 %

Table 2

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{4.456^2}{2 \times 0.50} \right) - 1 \right) = 0.0017\, kg\, m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.208^2}{2 \times 0.50}\right) - 1\right) = 0.0018\,kg\,m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{2.339^2}{2 \times 0.50}\right) - 1\right) = 0.0014\,kg\,m^2$$

According to the definition of rotational inertia, the moment of inertia of a spherical shell is given by the formula:

$$I = \frac{2}{3}mr^2$$

We obtain:

$$I = \frac{2}{3} \times (0.638) \times (0.098)^2 = 0.0041\,kg\,m^2$$

Errors:

- 1: 58.5 %
- 2: 56.1 %
- 3: 65.9 %

Table 3

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{5.025^2}{2 \times 0.50}\right) - 1\right) = 0.0023\,kg\,m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.280^2}{2 \times 0.50}\right) - 1\right) = 0.0019\,kg\,m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{1.671^2}{2 \times 0.50}\right) - 1\right) = 0.0007\,kg\,m^2$$

According to the definition of rotational inertia, the moment of inertia of a solid sphere is given by the formula:

$$I = \frac{2}{5}mr^2$$

We obtain:

$$I = \frac{2}{5} \times (1.7216) \times (0.074)^2 = 0.0038\,kg\,m^2$$

Errors:

- 1: 39.4 %
- 2: 50.0 %
- 3: 81.6 %

Table 4

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.177^2}{2 \times 0.50}\right) - 1\right) = 0.0009\, kg\, m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{2.428^2}{2 \times 0.50}\right) - 1\right) = 0.0010\, kg\, m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{2.189^2}{2 \times 0.50}\right) - 1\right) = 0.0013\, kg\, m^2$$

Also, according to the definition of rotational inertia, the moment of inertia of a thin cylindrical shell is given by:

$$I = \frac{1}{2} m (R_{in}^2 + R_{out}^2)$$

But since we only have access to the average radius, we use the approximation:

$$I = m(R_{avg}^2)$$

We obtain:

$$I = (0.5511) \times (0.116)^2 = 0.0074\, kg\, m^2$$

Errors:

- 1: 87.8 %
- 2: 86.4 %
- 3: 82.4 %

Table 5

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{4.858^2}{2 \times 0.50}\right) - 1\right) = 0.0021\, kg\, m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.433^2}{2 \times 0.50}\right) - 1\right) = 0.0021\, kg\, m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{2.655^2}{2 \times 0.50}\right) - 1\right) = 0.0019\, kg\, m^2$$

Also, according to the definition of rotational inertia, the moment of inertia of a solid cylinder is given by:

$$I = \frac{1}{2} m R^2$$

We obtain:

$$I = \frac{1}{2} \times (1.5139) \times (0.065)^2 = 0.0032\, kg\, m^2$$

Errors:

- 1: 34.3 %
- 2: 34.3 %
- 3: 40.6 %

There are many sources of error that can significantly influence the results. Some of the most important ones include:

- Inaccurate measurement of quantities such as mass, time, and length — whether due to the experimenter or the measurement tools
- The presence of friction on the pulley, and the resulting torque
- Slipping of the thread on the pulley
- The thread and pulley having non-negligible mass
- Non-zero initial velocity when passing the first sensor
- Lack of symmetry in the test objects
- Imperfections in rotational components such as bearings

[redacted]

Calculate the rotational moment of inertia of a flat disk in different rotational states using the data from Tables 6 to 9, and compare the results with the values obtained from the definition and the parallel axis theorem (report the relative error in percent). Does the relative error depend on the distance from the rotation axis to the center of the disk? Explain.

[redacted] *Answer*

[redacted] *Table 6*

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{5.047^2}{2 \times 0.50}\right) - 1\right) = 0.0023\, kg\, m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.331^2}{2 \times 0.50}\right) - 1\right) = 0.0020\, kg\, m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{1.975^2}{2 \times 0.50}\right) - 1\right) = 0.0011\, kg\, m^2$$

Also, according to the definition of rotational inertia, the moment of inertia of a disk with distance from the rotation axis is given by:

$$I' = \frac{1}{2} m R^2$$

We have:

$$I = \frac{1}{2} \times (0.5540) \times (0.12)^2 = 0.0039\, kg\, m^2$$

Errors:

- 1: 41.0 %
- 2: 48.7 %
- 3: 71.8 %

Table 7

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{5.053^2}{2 \times 0.50}\right) - 1\right) = 0.0023\, kg\, m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{4.251^2}{2 \times 0.50}\right) - 1\right) = 0.0032\, kg\, m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.319^2}{2 \times 0.50}\right) - 1\right) = 0.0030\, kg\, m^2$$

Also, according to the definition of rotational inertia, the moment of inertia of a disk at a distance h from the axis of rotation is given by the relation:

$$I' = \frac{1}{2} m R^2 + m h^2$$

We have:

$$I = 0.0039 + (0.5540) \times 0.03^2 = 0.0044\, kg\, m^2$$

Errors:

- 1: 47.7 %
- 2: 27.2 %
- 3: 31.8 %

Table 8

$$I_1 = 0.050 \times 0.01356^2 \times \left(9.78 \times \left(\frac{6.683^2}{2 \times 0.50}\right) - 1\right) = 0.0040\, kg\, m^2$$

$$I_2 = 0.100 \times 0.01356^2 \times \left(9.78 \times \left(\frac{5.936^2}{2 \times 0.50}\right) - 1\right) = 0.0063\, kg\, m^2$$

$$I_3 = 0.150 \times 0.01356^2 \times \left(9.78 \times \left(\frac{3.481^2}{2 \times 0.50}\right) - 1\right) = 0.0032\, kg\, m^2$$

Also, according to the definition of rotational inertia, the moment of inertia of a disk at a distance h from the axis of rotation is given by the relation:

$$I' = \frac{1}{2} m R^2 + m h^2$$

We have:

$$I = 0.0039 + (0.5540) \times 0.06^2 = 0.0059\,kg\,m^2$$

Errors:

- 1: 32.2 %
- 2: 6.7 %
- 3: 45.7 %

Since moving away from the main axis increases friction, one would expect the error to increase. However, due to various other sources of error and limitations in measurement accuracy, this trend is not clearly observable in the results.

For each of the above cases, calculate the torque due to friction. (For this purpose, assume that equation 5 regarding the existence of friction torque is valid and write out the relations explicitly.)

Answer

I used the code below to calculate the measures for this part.

```python

def calculate_frictional_torque(r, m, g, l, t, I):
    term1 = r * m * (g - (2 * l) / (t**2))
    term2 = I * (2 * l) / (r * t**2)
    tau_f = term1 - term2
    return tau_f

# Example usage:
r = 0.01356  # radius in meters
m = 0.15     # mass in kg
g = 9.78     # gravitational acceleration in m/s^2
l = 0.5      # length in meters
t = 3.481      # time in seconds
I = 0.0059     # moment of inertia in kg m2

tau_f = calculate_frictional_torque(r, m, g, l, t, I)
print(f"Frictional torque T_f = {tau_f:.6f} N*m")
```

Table 1

- 1: 0.0111
- 2: 0.0147
- 3: 0.0283

Table 2

- 1: 0.0086
- 2: 0.0163

- 3: 0.0357

Table 3

- 1: 0.0045
- 2: 0.0129
- 3: 0.0812

Table 4

- 1: 0.0475
- 2: 0.0795
- 3: 0.0944

Table 5

- 1: 0.0034
- 2: 0.0069
- 3: 0.0139

Table 6

- 1: 0.0047
- 2: 0.0128
- 3: 0.0544

Table 7

- 1: 0.0061
- 2: 0.0048
- 3: 0.0097

Table 8

- 1: 0.0031
- 2: 0.0009
- 3: 0.0162

In what cases is the friction torque greater? Discuss.

Answer

By analyzing the results obtained from the tables, it is observed that within each table, frictional torque generally increases as the mass of the hanging weights increases—that is, as we move from the top rows to the bottom ones. This indicates a direct relationship between the increase in mass and the increase in frictional torque.

However, when examining the effect of the rotational inertia of the test object on frictional torque, the experimental results do not demonstrate a clear or consistent pattern. In most cases, an inverse relationship between rotational inertia and frictional torque can be observed, but due to variations across the tables, a definitive conclusion cannot be drawn.

Do you have another method for obtaining the friction torque using the tools available in this experiment? What do you propose?

Answer

An alternative method to calculate the frictional torque τ_f using the available tools in the experiment is to measure the time required for the rotating disk to come to a complete stop after detaching the hanging mass. Once the mass passes the first sensor, it is quickly removed to eliminate its effect. From that point on, the only torque acting on the system is due to friction.

Assuming constant angular deceleration, the torque can be computed based on the angular velocity at the moment of mass release and the time it takes to come to rest.

Parameters

- Δt_{fall}: Time taken for the weight to pass between the two sensors
- t_{halt}: Time until the disk fully stops after weight removal
- I_d: Rotational inertia of the disk
- R_b: Radius of the pulley or bearing
- τ_f: Frictional torque

Derivation

Velocity of the falling weight before release:

$$v_{\text{rel}} = g\Delta t_{\text{fall}}$$

Angular velocity of the rim before release:

$$R_b\omega_0 = v_{\text{rel}} \Rightarrow \omega_0 = \frac{g\Delta t_{\text{fall}}}{R_b}$$

Using constant angular deceleration:

$$\omega(t_{\text{halt}}) = 0 = \omega_0 + \alpha t_{\text{halt}} \Rightarrow \alpha = -\frac{\omega_0}{t_{\text{halt}}} = -\frac{g\Delta t_{\text{fall}}}{R_b t_{\text{halt}}}$$

Finally, the frictional torque is:

$$\tau_f = I_d|\alpha| = \frac{I_d g\Delta t_{\text{fall}}}{R_b t_{\text{halt}}}$$