CS 500 Problem Set 5 18 Nov 2013
W. Evans Due: 25 Nov 2013

Optional background reading from Algorithms by Dasgupta, Papadimitriou, Vazirani (DPV).
Linear Programming pages 188–213

You may discuss problems with other people in the class, but you must write up your own solutions. Please acknowledge the people you work with and any sources you use (web pages, books, etc.).

1. (from DPV Problem 7.28) *A linear program for shortest path.* Suppose we want to compute the shortest path from node s to node t in a directed graph with edge lengths $\ell_e > 0$.
   (a) Show that this is equivalent to finding an s—t flow f that minimizes $\sum_e \ell_e f_e$ subject to $\text{size}(f) = 1$. There are no capacity contraints. (This requires a proof.)
   (b) Write the shortest path problem as a linear program.
   (c) Show that the dual LP can be written as

$$\max x_s - x_t$$
$$x_u - x_v \leq \ell_{uv} \text{ for all } (u, v) \in E$$

2. Let G be a bipartite graph. We want to find a minimum covering of vertices by edges, i.e., a minimum subset A of edges such that each vertex is incident to at least one edge in A. Show how to solve this problem using a maximum flow algorithm.

   Hint: Create a flow network so the maximum flow tells you which edges are *not* in A.

3. Each of two players hides a nickel (5 cents) or a dime (10 cents). If the two coins match then A gets both; if they don't match then B gets both. Show the payoff matrix. (The payoff is the *additional* money a player gets.) Describe the optimal strategy for A using a linear program. What is the optimal strategy? (If you need to, you may use any simplex implementation you like. There are several on the Web.) What is the optimal strategy if instead of a nickel or a dime the players hide a c-cent coin or a d-cent coin?