1 Question 3(a)

If we define X_i as the indicator for wrong labeling of i_{th} sample, we then want to prove:

$$Pr[|\hat{R}_{S'}(h) - R(h)| \leq \epsilon/2] \geq 1/2$$
$$\Rightarrow 1 - Pr[|\hat{R}_{S'}(h) - R(h)| \leq \epsilon/2] \leq 1/2$$
$$\Rightarrow Pr[|\hat{R}_{S'}(h) - R(h)| > \epsilon/2] \leq 1/2$$

Now we apply Hoeffding with $X_i \in [0, 1], \epsilon := m\epsilon/2$:

$$\Rightarrow Pr[|Z/m - \mathbb{E}[Z]/m| \geq \epsilon/2] \leq 2.e^{-\frac{2}{4}\epsilon^2 m^2 \frac{1}{\Sigma_{i=1}^m (b_i - a_i)^2}}$$
$$\Rightarrow Pr[|\hat{R}_{S'}(h) - R(h)| \geq \epsilon/2] \leq 2.e^{-1/2\epsilon^2 m^2 \frac{1}{m}}$$
$$\Rightarrow Pr[|\hat{R}_{S'}(h) - R(h)| \geq \epsilon/2] \leq 2.e^{-1/2\epsilon^2 m}$$

Replacing m with $\frac{K}{\epsilon^2}(log(\Pi_H(m)) + log(1/\delta))$:

$$\Rightarrow Pr[|\hat{R}_{S'}(h) - R(h)| \geq \epsilon/2] \leq 2.e^{-K/2(log(\Pi_H(m))+log(1/\delta))}$$
$$\Rightarrow Pr[|\hat{R}_{S'}(h) - R(h)| \geq \epsilon/2] \leq 2.(\frac{\delta}{\Pi_H(m)})^{K/2}$$

Now it suffices to show:

$$2.(\frac{\delta}{\Pi_H(m)})^{K/2} \leq \frac{1}{2}$$
$$\delta \leq \frac{1}{2} \text{ and } \Pi_H(m) \geq 1$$
$$\Rightarrow 2.(\frac{\delta}{\Pi_H(m)})^{K/2} \leq 2.(\frac{1}{2})^{K/2} \leq \frac{1}{2}$$

And that's always true if we assume $K \geq 4$.

2 Question 3(b)

Assuming instead of $\frac{1}{2.\Pi_H(2m)}$ you meant $\frac{\delta}{2.\Pi_H(2m)}$ so that at the end we'd have $Pr[B] \leq \delta/2$. We proceed to applying Hoeffding right away. $Z_i \in [-1, +1], \epsilon := m\epsilon$.

$$Pr\left[|\Sigma_{i=1}^{d} Z_i - \mathbb{E}[\Sigma_{i=1}^{d} Z_i]| \geq m\epsilon\right] \leq 2.e^{-2\epsilon^2 m^2 \frac{1}{\Sigma_{i=1}^{m}(b_i - a_i)^2}}$$
$$Pr\left[|\Sigma_{i=1}^{d} Z_i| \geq m\epsilon\right] \leq 2.e^{-2\epsilon^2 m^2 \frac{1}{4m}}$$
$$\Rightarrow Pr\left[|\Sigma_{i=1}^{d} Z_i| \geq m\epsilon\right] \leq 2.e^{\frac{-1}{2}\epsilon^2 m}$$
$$\Rightarrow Pr\left[|\Sigma_{i=1}^{d} Z_i| \geq m\epsilon\right] \leq 2.e^{\frac{-K}{2}(log(\Pi_H(2.m)) + log(1/\delta))}$$
$$\Rightarrow Pr\left[|\Sigma_{i=1}^{d} Z_i| \geq m\epsilon\right] \leq 2.(\frac{\delta}{\Pi_H(2m)})^{\frac{K}{2}}$$

Now we only need to show:

$$2.(\frac{\delta}{\Pi_H(2m)})^{\frac{K}{2}} \leq \frac{\delta}{2.\Pi_H(2m)}$$
$$\text{If we further assume } K \geq 6,\ \delta \leq \frac{1}{2},\ \Pi_H(2m) > 1$$
$$2.(\frac{\delta}{\Pi_H(2m)})^3 = 2.(\frac{\delta}{\Pi_H(2m)})^2.(\frac{\delta}{\Pi_H(2m)})^1 \leq \frac{\delta}{2.\Pi_H(2m)}$$
$$\Rightarrow 2.(\frac{\delta}{\Pi_H(2m)})^3 \leq 2.\frac{1}{4}.\frac{\delta}{\Pi_H(2m)} \leq \frac{\delta}{2.\Pi_H(2m)}$$

Thus, if $K \geq 6$ both cases will always be true. Now that $Pr[B] \leq \delta/2$ and $Pr[A] \leq 2.Pr[B]$, then $Pr[A] \leq \delta$.

The catch. If we take samples from Z_i uniformly with replacement, we will have a new random variable which takes values independently. If X_i is the result of the i_{th} sample, we have:

$$\frac{1}{d}\sum_{i=1}^{d} Z_i = \mathbb{E}[\frac{1}{n}\sum_{i=1}^{n} X_i].$$

We can probably use this as a proxy for Z_i, but I couldn't figure it out.