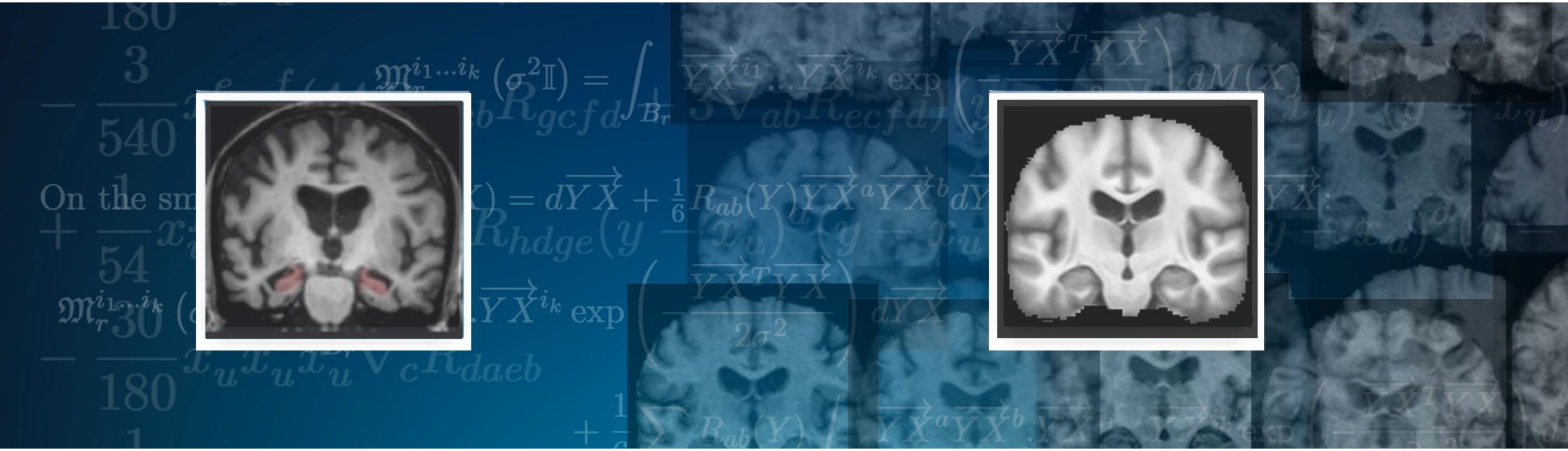
UC **SANTA BARBARA**

Introduction to ECE 594n

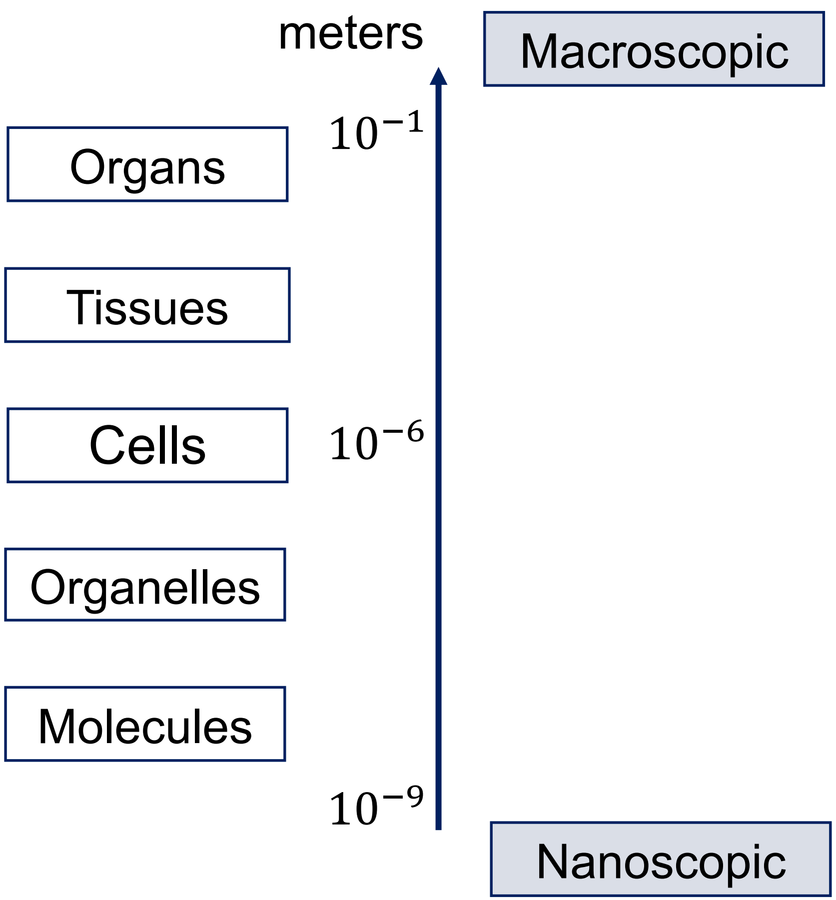
Geometric Machine Learning for Biomedical Imaging and Shape Analysis

Nina Miolane, Assistant Professor



Bioimaging at Different Scales

- Biomedical research: understand mechanisms of life.



Bioimaging at Different Scales

- Biomedical research: understand mechanisms of life.

meters

Macroscopic

Organs

Tissues

Cells

Organelles

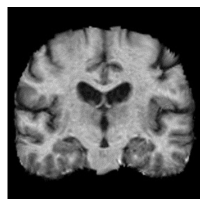
Molecules

10^{-1}

10^{-6}

10^{-9}

25cm



Human brain

Magnetic resonance imaging (Nobel Prize 2003)

Nanoscopic

Bioimaging at Different Scales

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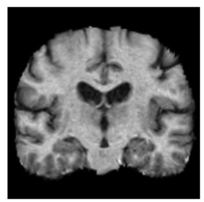


Human brain

Magnetic resonance imaging (Nobel Prize 2003)

Tissues

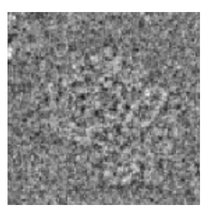
X-ray computed tomography
Positron emission tomography
Photoacoustic microscopy
Two-photon microscopy
Optical coherence tomography
Confocal microscopy
Etc.

Cells

10^{-6}

Organelles

40nm



Human ribosome

Cryo-electron microscopy (Nobel Prize 2017)

Molecules

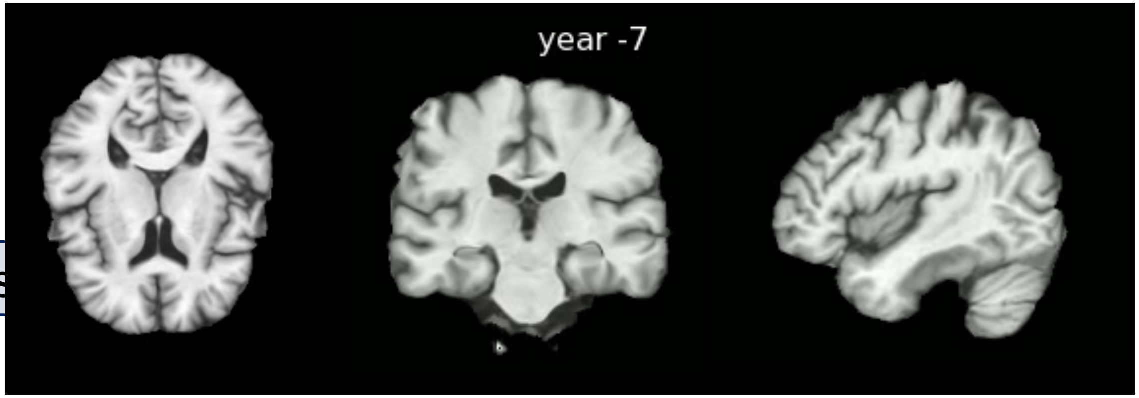
10^{-9}

Nanoscopic

The Shapes Of You

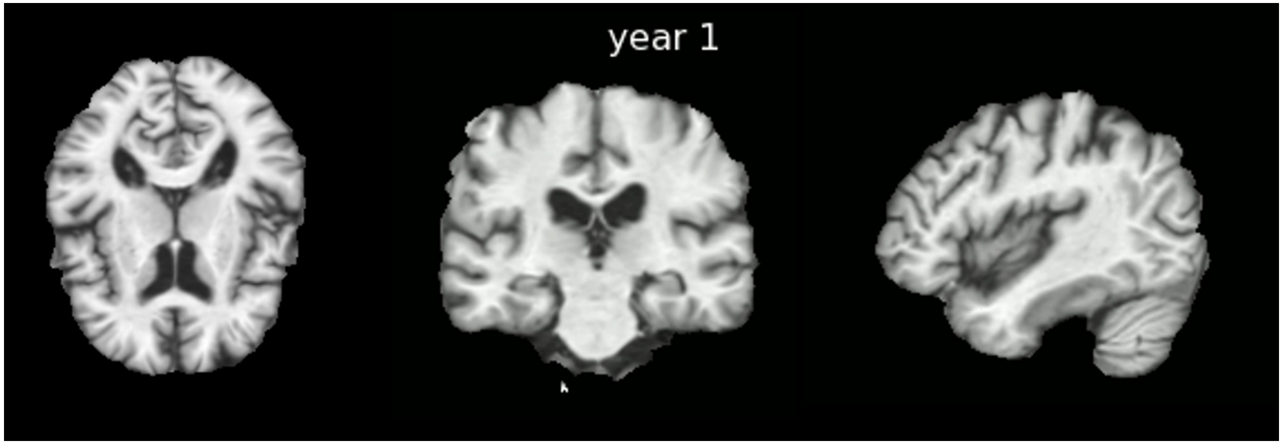
Macros

Nanos



Simulation credits: Lorenzi, Ayache, Frisoni, Pennec (Inria). Video credits: US National Institute on Aging (NIH).

Shapes are linked to Functions



Function
Healthy/pathological state → Geometry

Credits: Lorenzi, Ayache, Frisoni, Pennec (Inria), US National Institute on Aging (NIH), Ashok Prasad (Colorado University)

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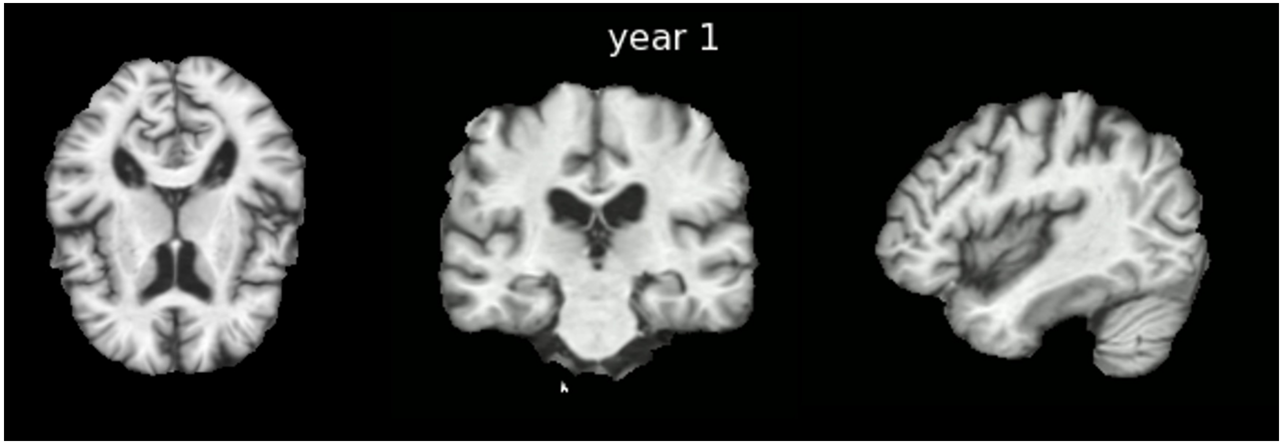

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Healthy/pathological state $\longrightarrow$ Geometry

Inverse model?

Biomedical discoveries $\longleftarrow$ Geometry

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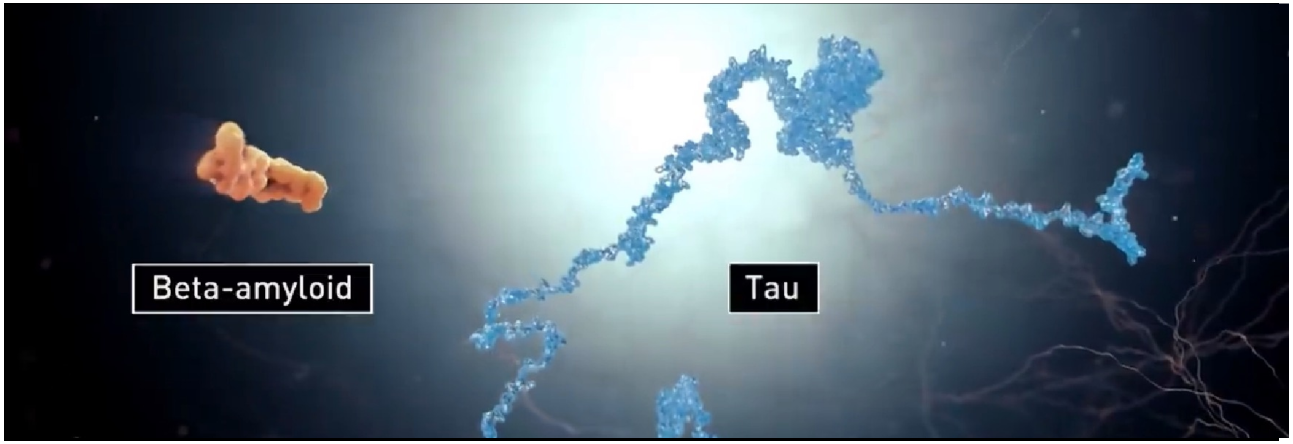

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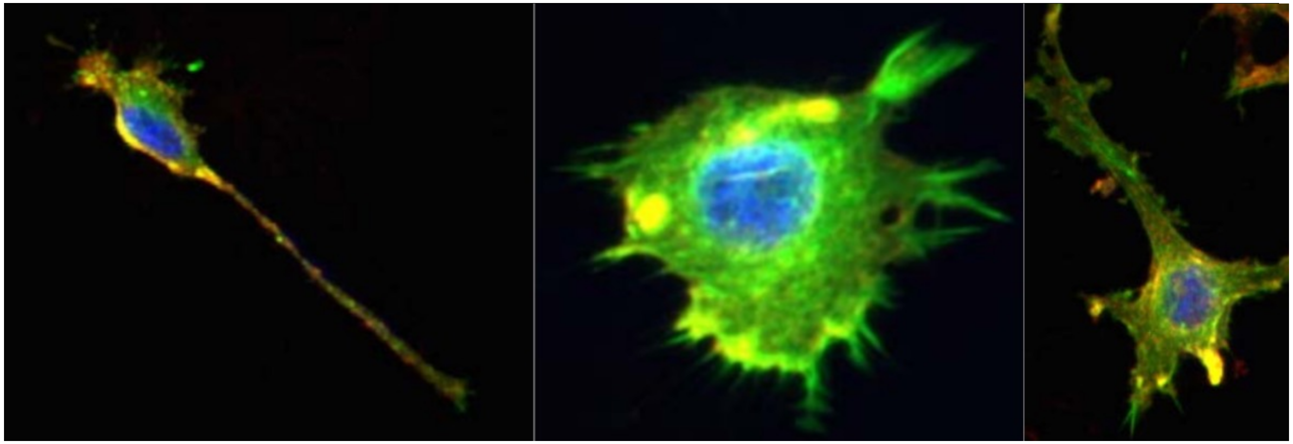

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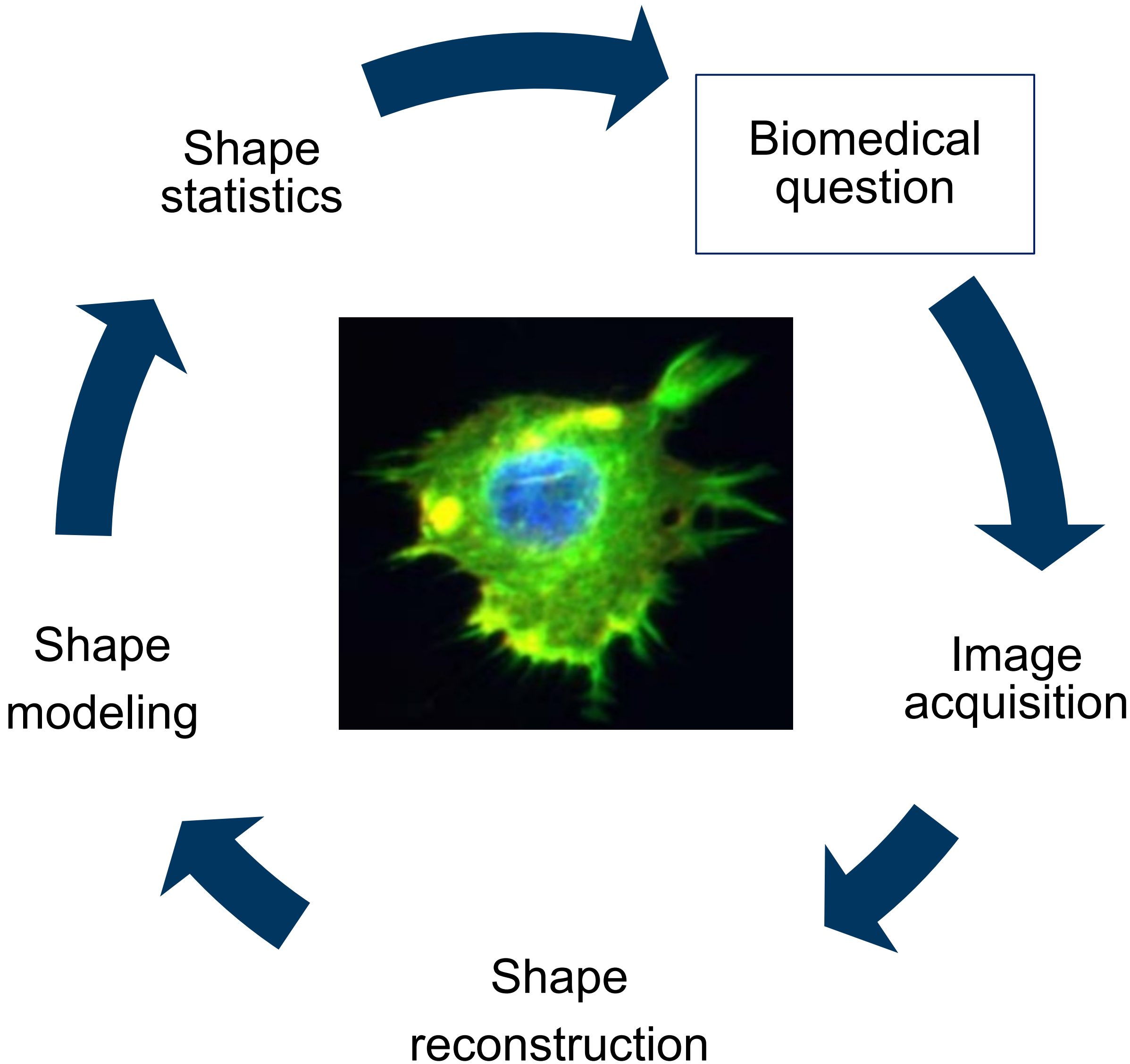
Shape Analysis from Biomedical Imaging



Shape Analysis from Biomedical Imaging

Shape statistics

Biomedical question

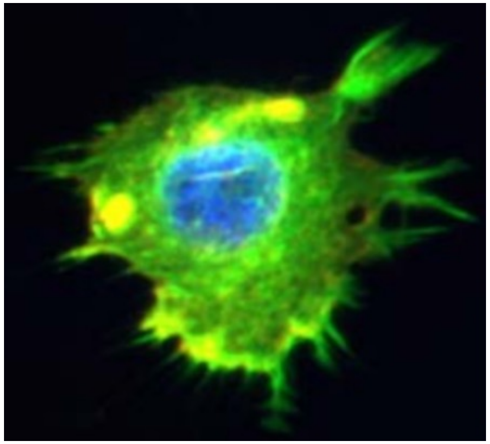


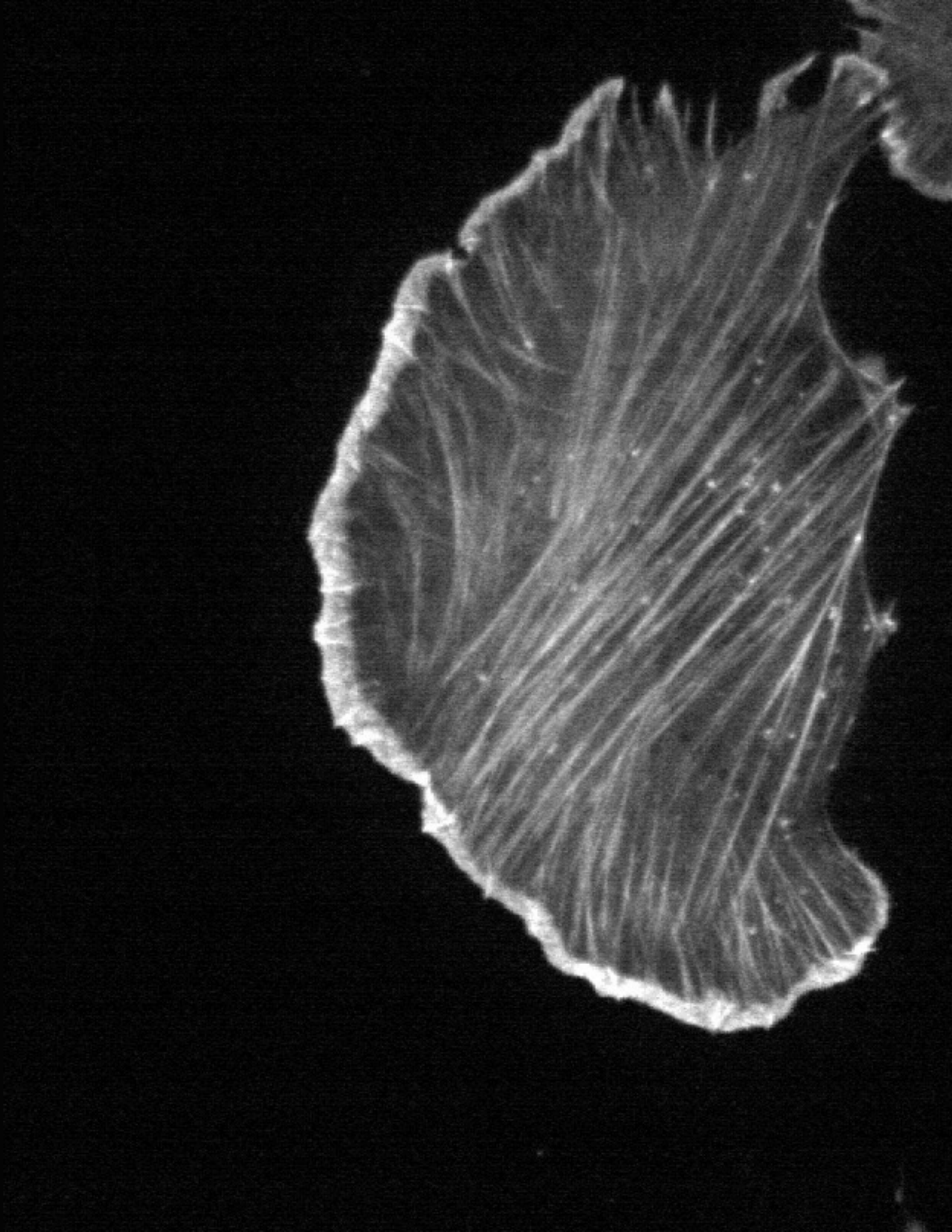
Image acquisition

Shape reconstruction

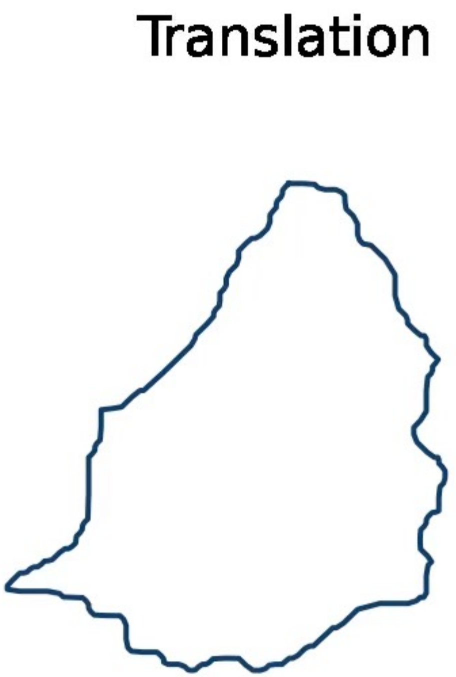
Shape modeling

- Mathematical…
- Computational…
- Statistical…

…shape models



Maths of Shapes & Shape Transformations



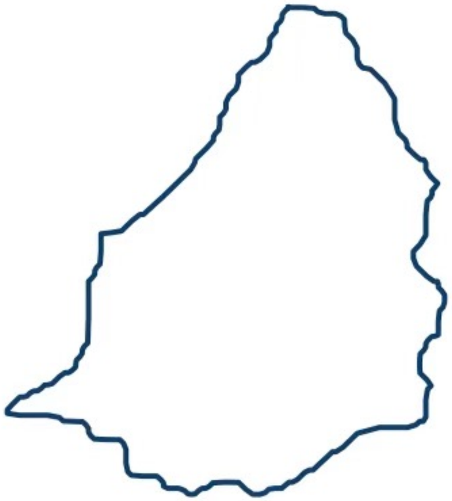
Shapes ↔ Equivalence classes

Maths of Shapes & Shape Transformations

$\boxed{G_0}$

Translation

$g_0 * x$



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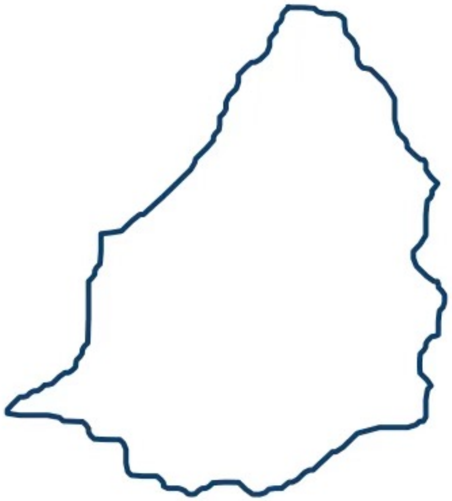

Shapes ↔ Equivalence classes
= Elements of “Quotient space” Q

$$Q = \{[x] | x \in M)$$

where $[x] = \{y \in M \mid \exists g_0 \in G_0 \ s.t. \ y = g_0 * x\}$

Maths of Shapes & Shape Transformations

G_0

$g_0 * x$

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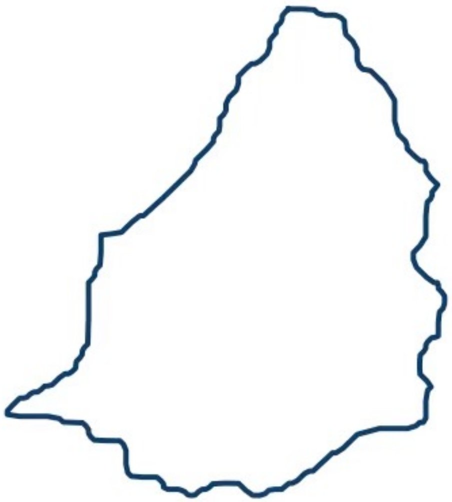
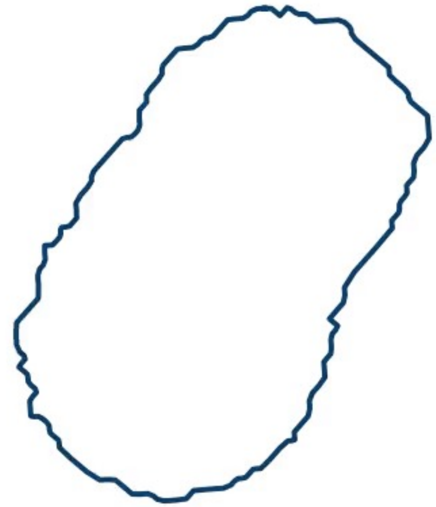


Smooth deformation



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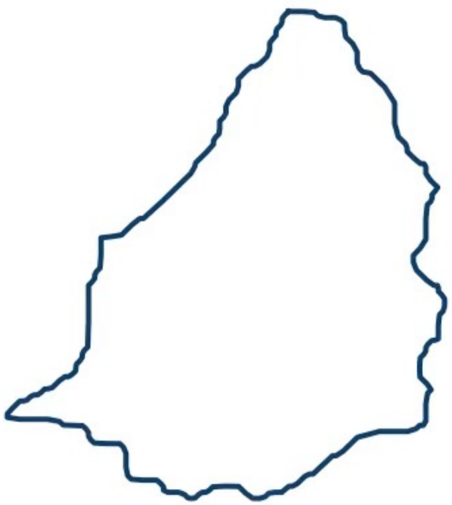
Maths of Shapes & Shape Transformations

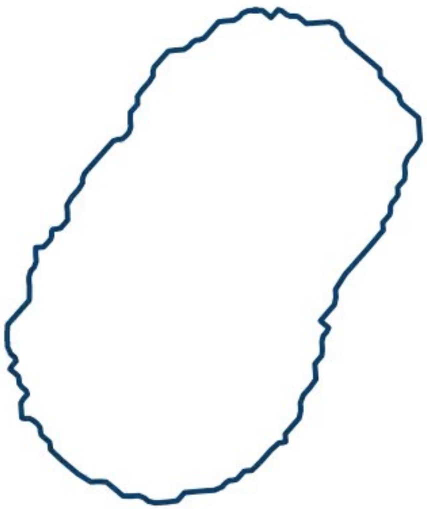
G_0 Translation

$g_0 * x$



G Smooth deformation

$g * x$



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Maths of Shapes & Shape Transformations

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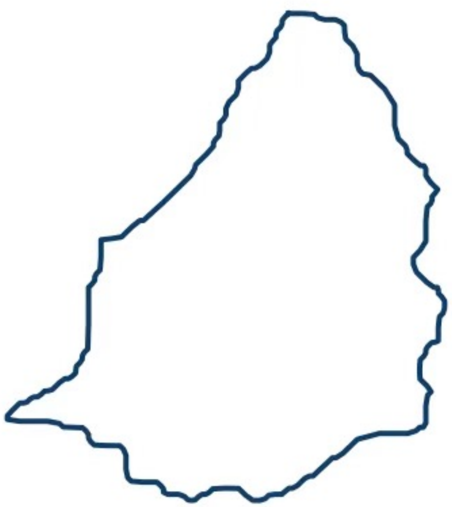

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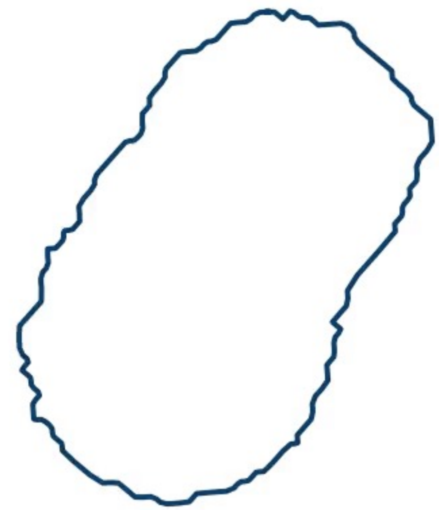

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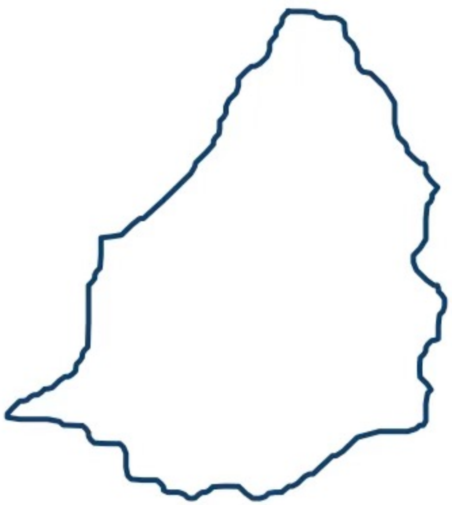
Shapes ↔ Deformations
= Elements of “Lie group” G

Maths of Shapes & Shape Transformations

Translation



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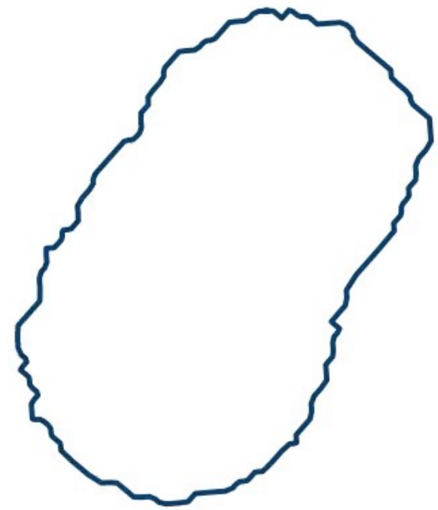


Shapes $\leftrightarrow$ Equivalence classes
= Elements of "Quotient space" Q

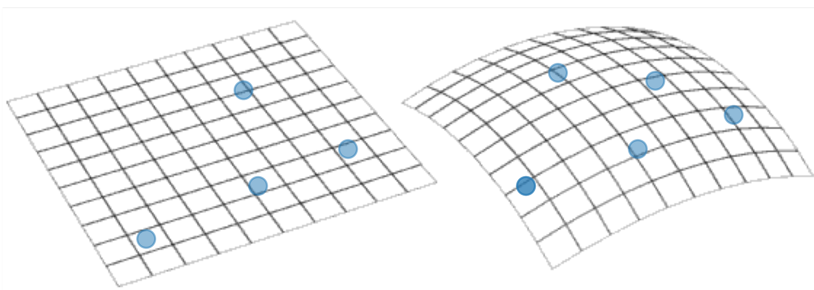
Shapes $\leftrightarrow$ Deformations
= Elements of "Lie group" G

= Manifolds with additional geometric structures

Quotients, Lie Groups = Manifolds

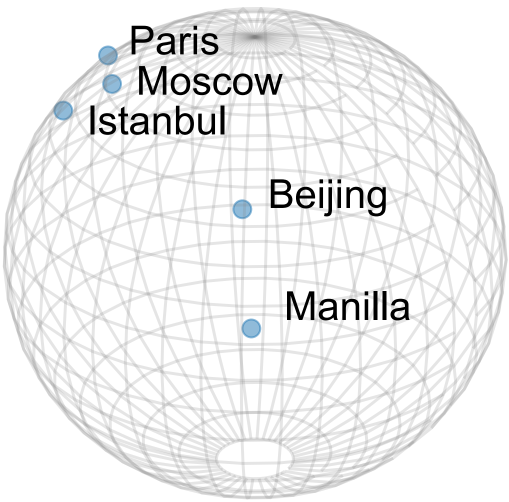
Generalize computing, statistics & (deep) learning to data on manifolds

- Geographic data, e.g. coordinates of cities or earthquakes.



Data on a vector space   Data on a manifold

Example: Data on the sphere



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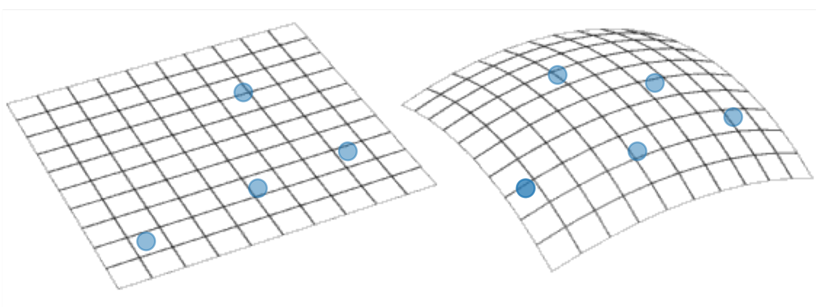

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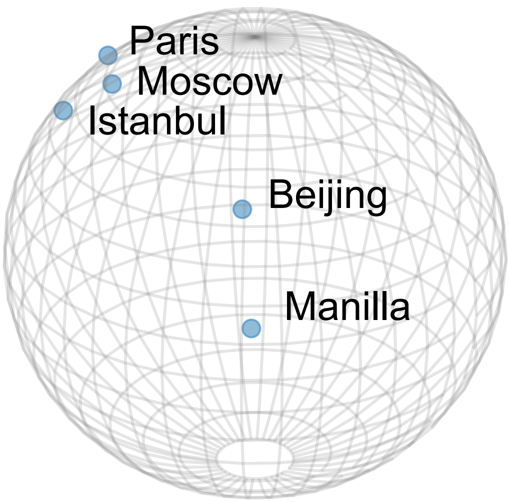


Geomstats: Open-source Python package for Geometric Statistics

Why Geometric Statistics

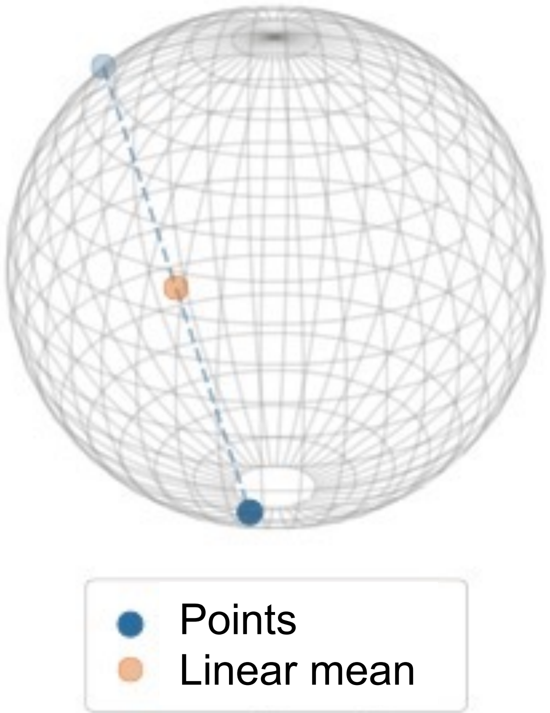
- Mean: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \rightarrow$ linear
- Manifold $\rightarrow$ non-linear

$\rightarrow$ Mean may not belong to the manifold

```python
from geomstats.geometry.hypersphere \
    import Hypersphere

sphere = Hypersphere(dim=2)
points = sphere.random_uniform(
    n_samples=2)

linear_mean = gs.sum(
    points, axis=0) / n_samples
```



Why Geometric Statistics

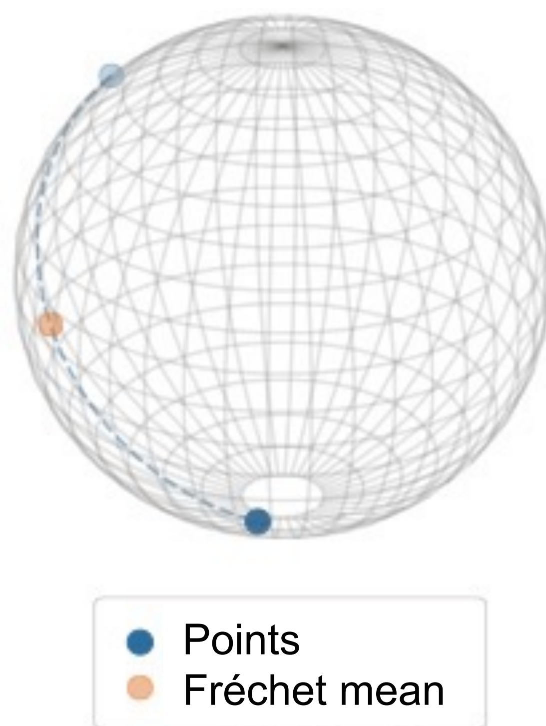
- Fréchet mean $\bar{x} = \mathrm{argmin}_{x \in M} \sum_{i=1}^{n} \mathrm{dist}^2_{M(x,x_i)}$

→ Mean belongs to the manifold

```
from geomstats.learning.frechet_mean import \
    FrechetMean

estimator = FrechetMean(metric=sphere.metric)
estimator.fit(points)

frechet_mean = estimator.estimate_
```



Why Geometric Statistics

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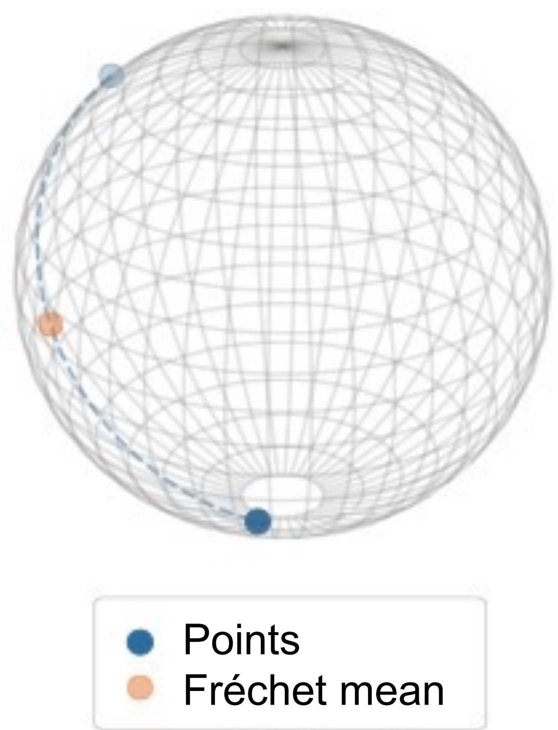

Remarks:

- Whitney embedding theorem $M_m \subset \mathbb{R}^{2m}$
- Mean respecting additional geometries

Geomstats

Geomstats: Computing, statistics & (deep) learning for data on manifolds

- Backends : NumPy, Autograd, TensorFlow and PyTorch

1. Instantiate manifold of interest

```
sphere = Hypersphere(dim=2)
```

2. Apply estimation or learning method

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Miolane et al. *Geomstats: a Python package for Riemannian geometry in machine learning.* Journal of Machine Learning Research (2020).
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Geomstats Objectives :

- Teach "hands-on" Geometric Statistics and Learning
- Democratize the use of Geometric Statistics and Learning
- Support research in Geometric Statistics and Learning

→ Compute with shape data

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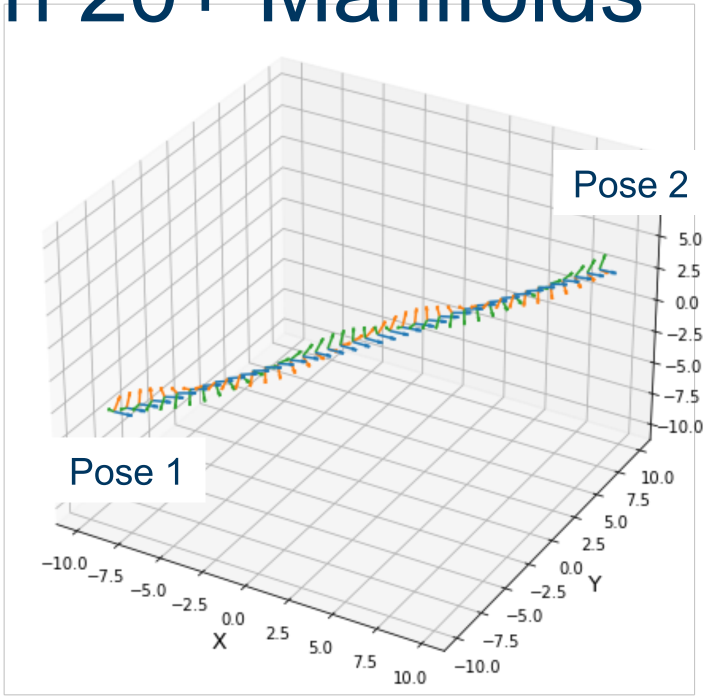
Basic Operations Coded on 20+ Manifolds

```
from geomstats.geometry.special_euclidean \
    import SpecialEuclidean

se3 = SpecialEuclidean(n=3, point_type='vector')
metric = se3.left_canonical_metric

initial_point = se3.identity
initial_tangent_vec = gs.array(
    [1.8, 0.2, 0.3, 3., 3., 1.])
geodesic = metric.geodesic(
    initial_point=initial_point,
    initial_tangent_vec=initial_tangent_vec)
```



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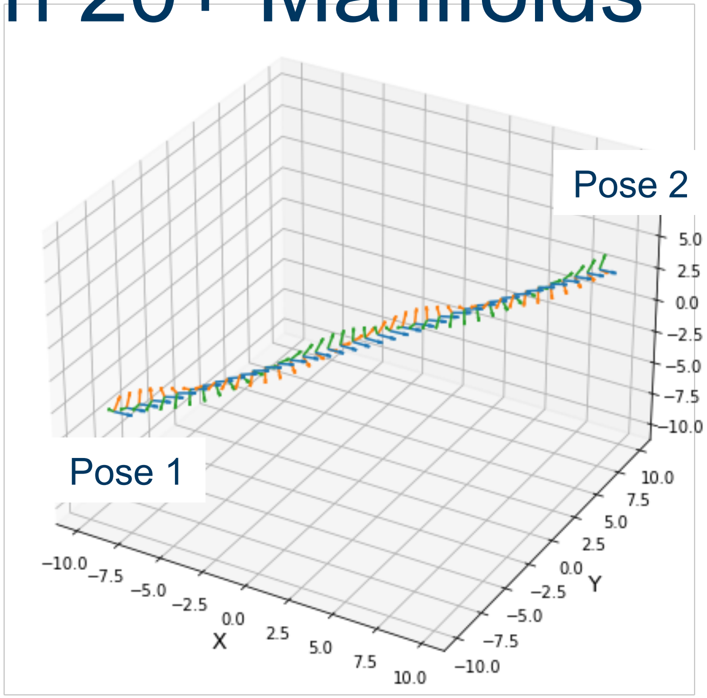
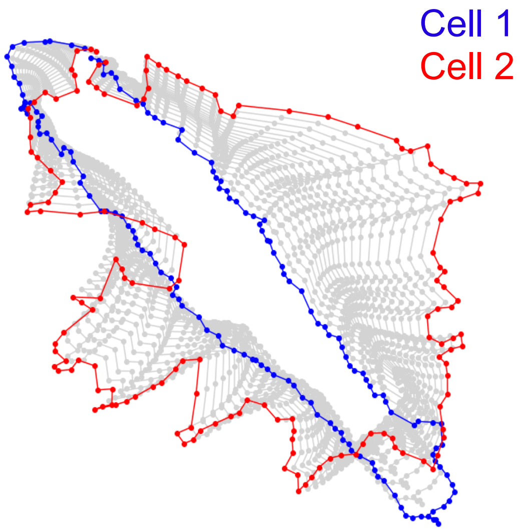



```
from geomstats.geometry.discrete_curves \
    import R2, DiscreteCurves

curves = DiscreteCurves(R2)
metric = curves.square_root_velocity_metric

geodesic = metric.geodesic(
    initial_curve=cells_shape[i],
    end_curve=cells_shape[j])
```



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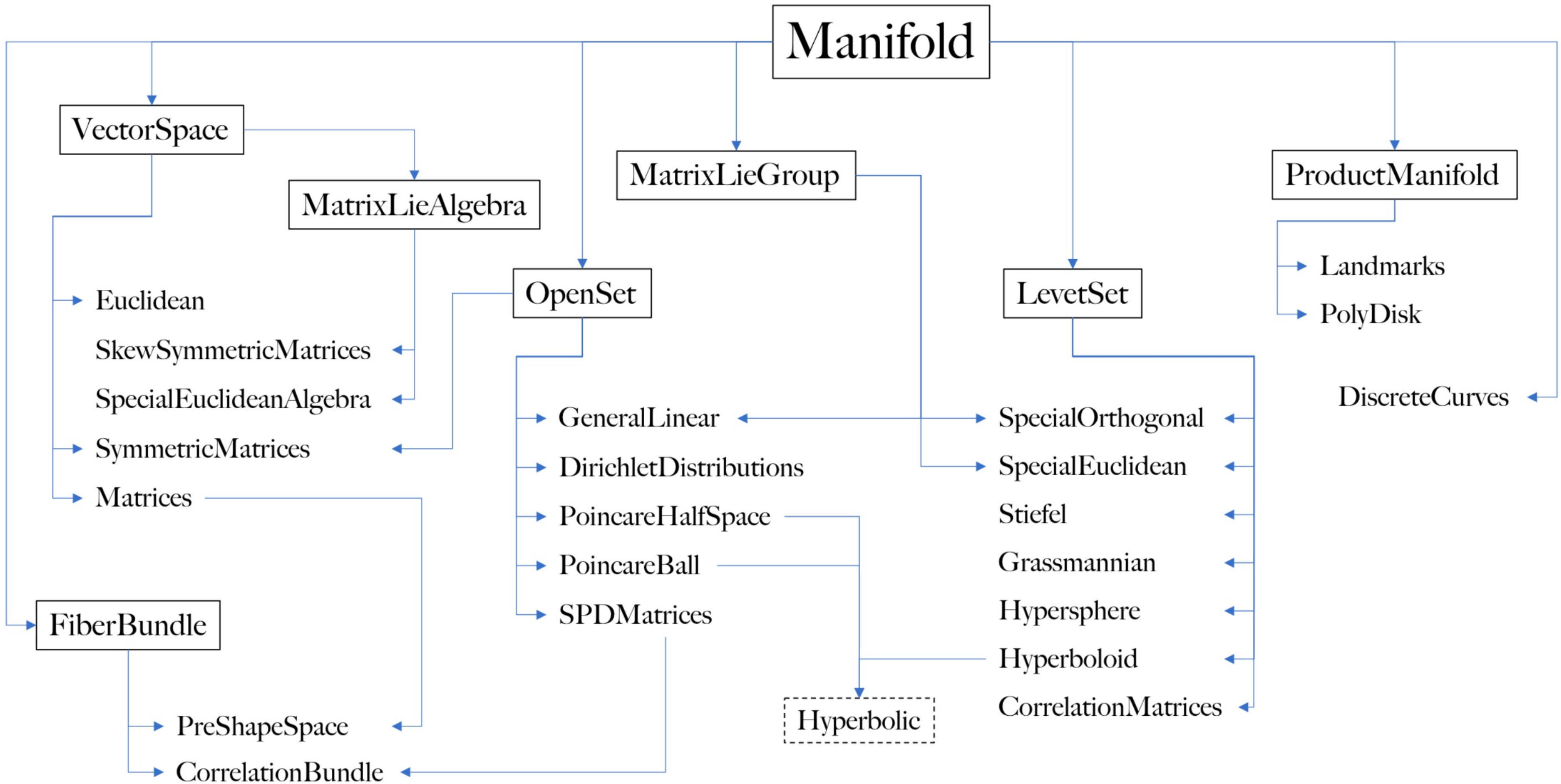

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…Statistics and Learning

Geometric…

	Point estimation	Dimension Reduction	Stochastic processes	…
Riemannian		(2019)		
Finsler				
Affine				
Stratified spaces	(2017-18)	(2020)		
Lie groups	(2015)			
Quotient spaces	(2017-21)			
Subriemannian			(2015)	
…				

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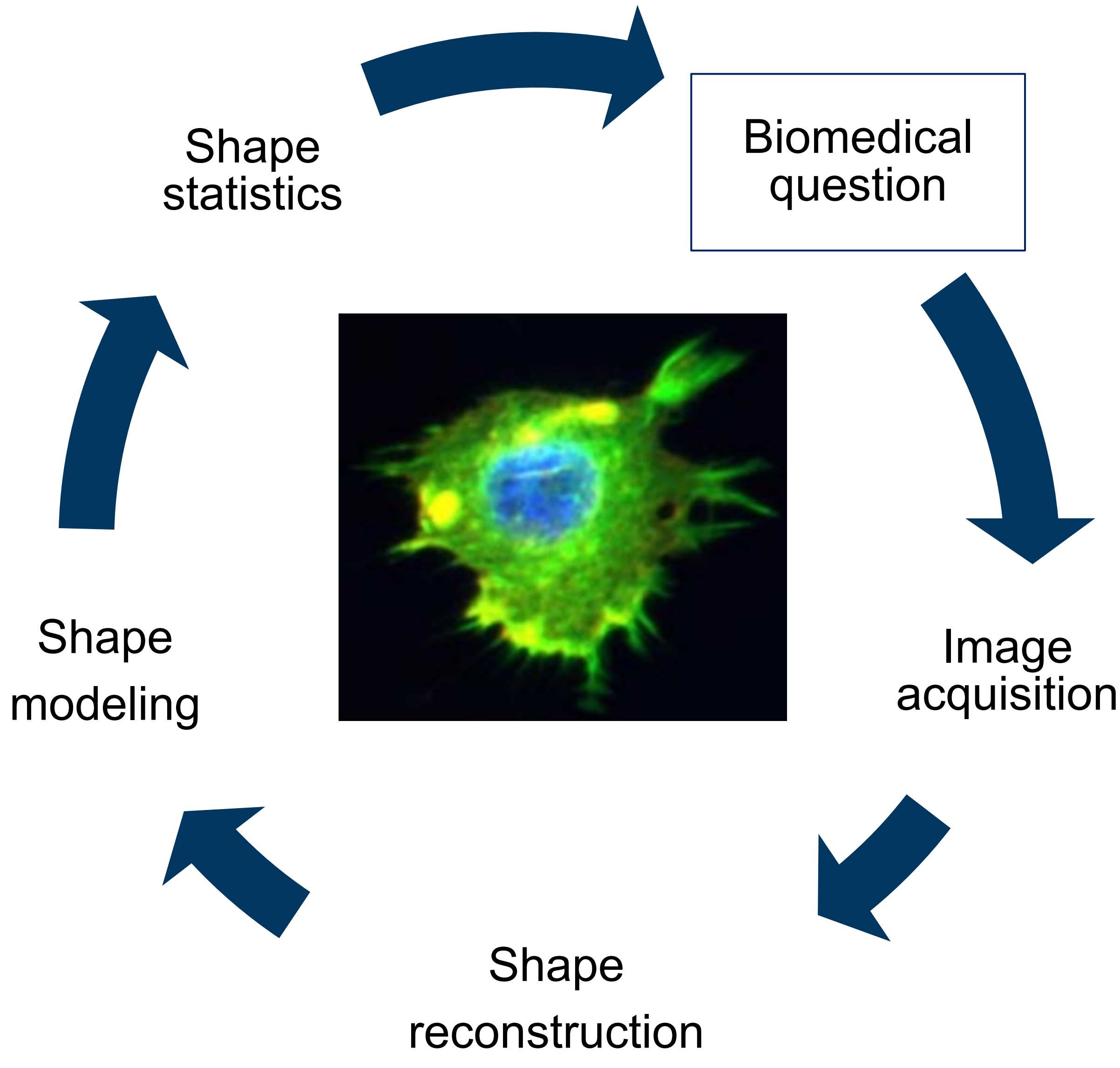
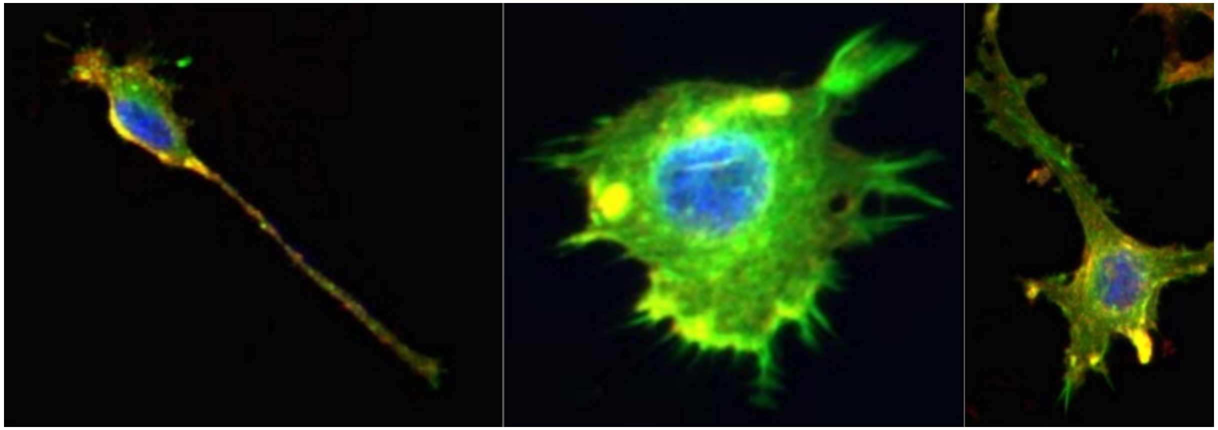


Goal: Impact of Drug Treatment on Cancer Cell Shapes

- Tumor grading → low accuracy and reproducibility [1]
- Cell morphometrics: cell state → cell shape
  - e.g. actin activity → irregular perimeter



- Questions:
  - How can we quantify differences in cell shapes?
  - How do cancer treatments affect cell?

- Collaborators: A. Prasad, K. Dao Duc, F. Michel, A. Le Brigant

[1] Alizadeh E. et al. TISMorph: A tool to quantify texture, irregularity and spreading of single cells. (2019)

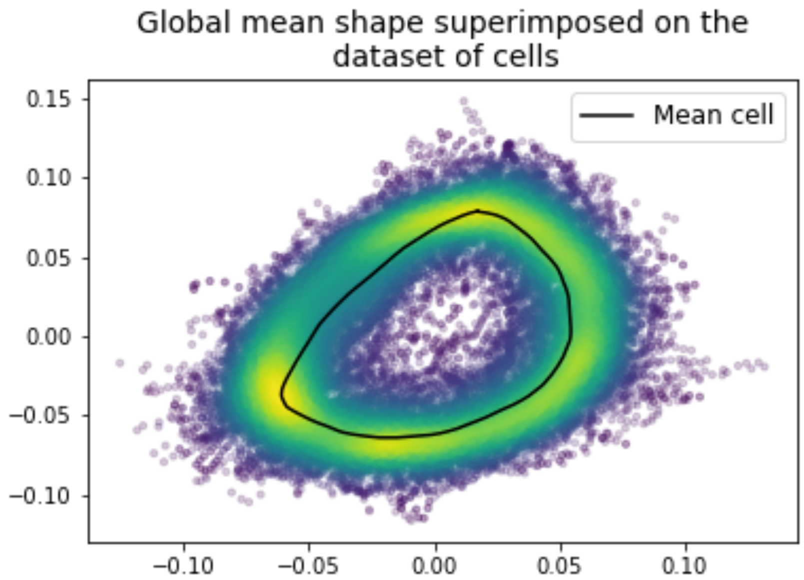
Goal: Impact of Drug Treatment on Cancer Cell Shapes

```
curves, treatments, lines = geomstats.datasets.utils.load_cells()
```

Computing the Mean Shape

```
curves = DiscreteCurves(R2)
mean =FrechetMean(metric=curves.square_root_velocity_metric)
mean.fit(cell_shapes)

mean_estimate = mean.estimate_
```



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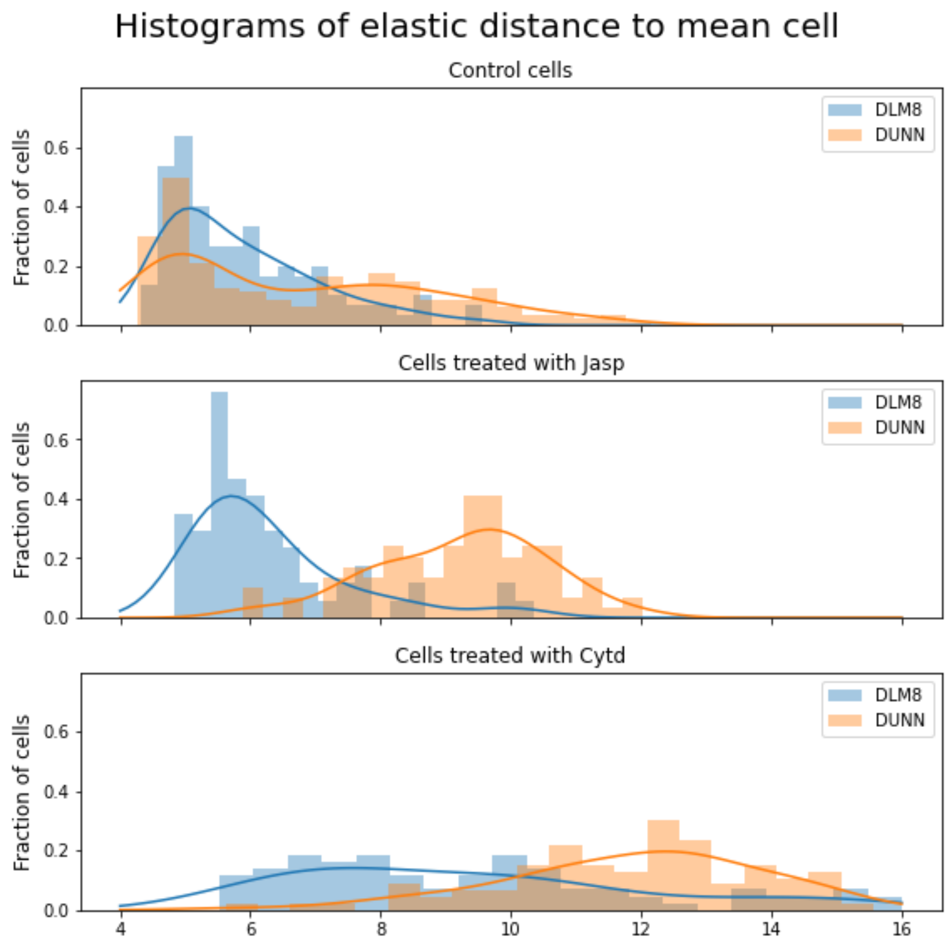
Hypothesis Testing on Shape Transformations

- Dunn :
  - higher variability in shapes
  - further from mean shape
- Jasp treatment:
  - Effective on Dunn
  - Less effective on DLM8
- Cytd treatment:
  - Very effective on:
    - Dunn
    - DLM8



Statistically significant ($p < 0.01$)

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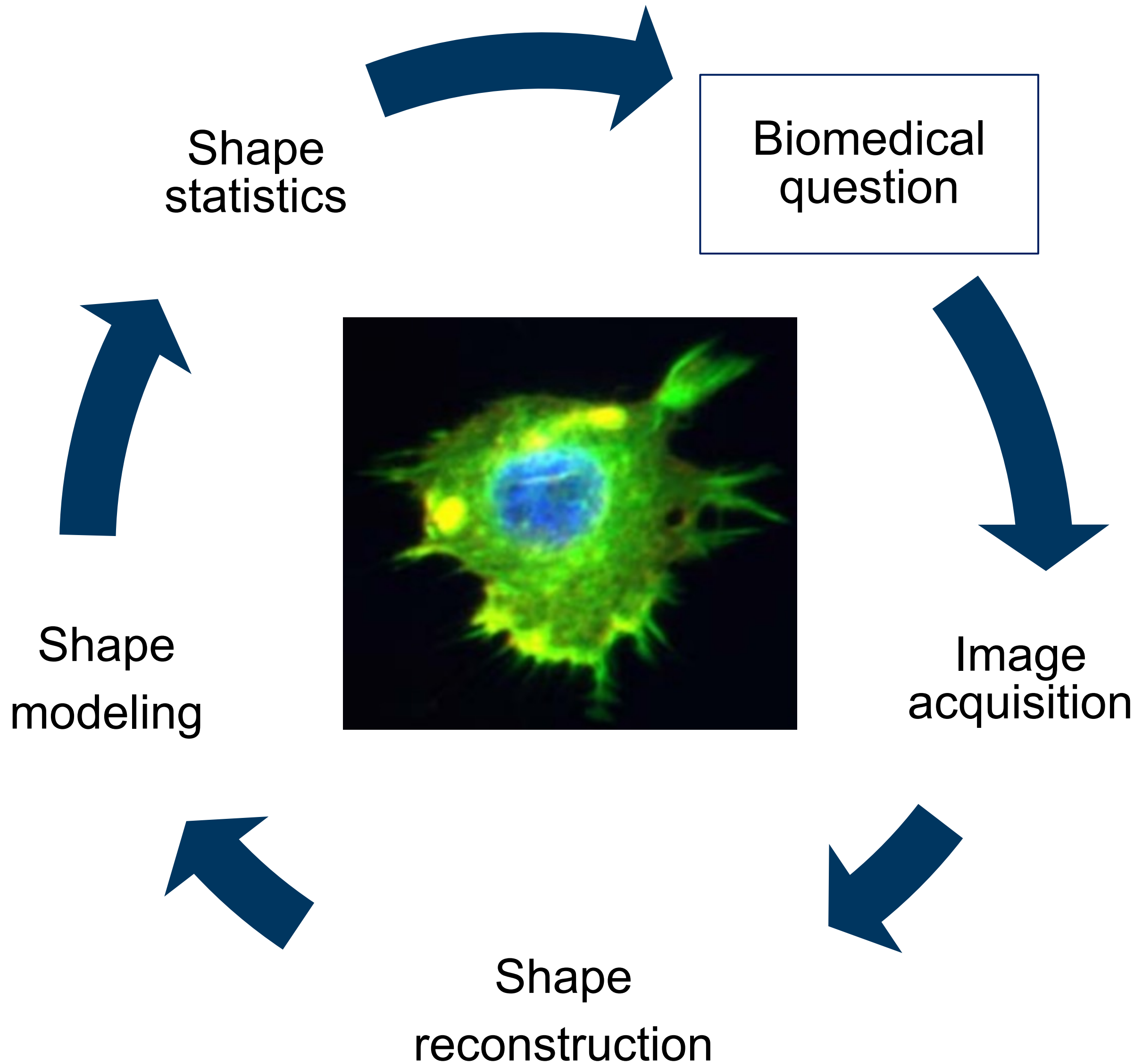
Shape Analysis from Biomedical Imaging



…Statistics and Learning

Geometric…:

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Lie groups				
Quotient spaces				
Subriemannian				

Geometric Learning beyond shape modeling: equivariance, invariance properties

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Exploring the Geometries of Life

Thank you for your attention.