

# Statistical Machine Learning

## Reading Assignment 1 Report

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### 1 A Taste of Real Analysis

Real Analysis is the field of mathematics on which probability theory is founded, it is therefore convenient to first introduce some basic concepts from real analysis. We begin this chapter with the definition of metric space and use the notion of metric to define two core concepts: convergence and continuity. In 1.1.2 we review the topology defined on a metric space based on the metric function and then in 1.2 we provide a more general view of topological spaces.

#### 1.1 Metric Space

A metric space is a set  $M$  together with a metric  $d : M \times M \rightarrow \mathcal{R}$  satisfying following four properties

- $d$  is non-negative
- $d(x, y) = 0$  iff  $x = y$
- Symmetry:  $d(x, y) = d(y, x)$
- Triangle Inequality:  $d(x, z) \leq d(x, y) + d(y, z)$

Strictly speaking, the pair  $(M, d)$  is the metric space as different metric functions can be defined on same  $M$ , consider for example  $\mathcal{R}^n$ ; a well-known class of metrics defined on  $\mathcal{R}^n$  is **Minkowski Norm**:

$$d_p(x, y) = \left( \sum_{i=1}^n (x_i - y_i)^p \right)^{1/p}$$

which for all values of  $p \geq 1$  is a valid metric function.

### 1.1.1 Convergence & Continuity

There are different ways for defining convergence, we follow [?] and use sequence/subsequence approach. A sequence  $(p_n)$  is a list of points  $p_1, p_2, \dots$  in  $M$ . Formally, a sequence is a function  $f : \mathbb{N} \rightarrow M$  in which  $f(n) = p_n$ . The sequence  $(p_n)$  **converges to the limit**  $p$  in  $M$  if:

$$\begin{aligned} \forall \epsilon > 0 \quad \exists N \in \mathbb{N} \quad \text{such that} \\ n \geq N \Rightarrow d(p_n, p) < \epsilon \end{aligned}$$

### 1.1.2 Topology of Metric Space

## 1.2 Topology: a More General Perspective

# 2 Theory of Probability

## 2.1 Measure Theory

## 2.2 Measurable Functions

## 2.3 Absolute Continuity

## 2.4 Mathematical Probability

## 2.5 Random Variable

## 2.6 Conditioning

# 3 Stochastic Processes

## 3.1 Definition

## 3.2 Random Functions