به نام یگانه معبود بخشنده مهربان

مبانی یادگیری ماشین

Machine Learning Foundations

گروه هوش مصنوعی، دانشکده مهندسی کامپیوتر، دانشگاه اصفهان

ترم اول سال تحصیلی ۰۲-۰۳

ارائه دهنده : پیمان ادیبی

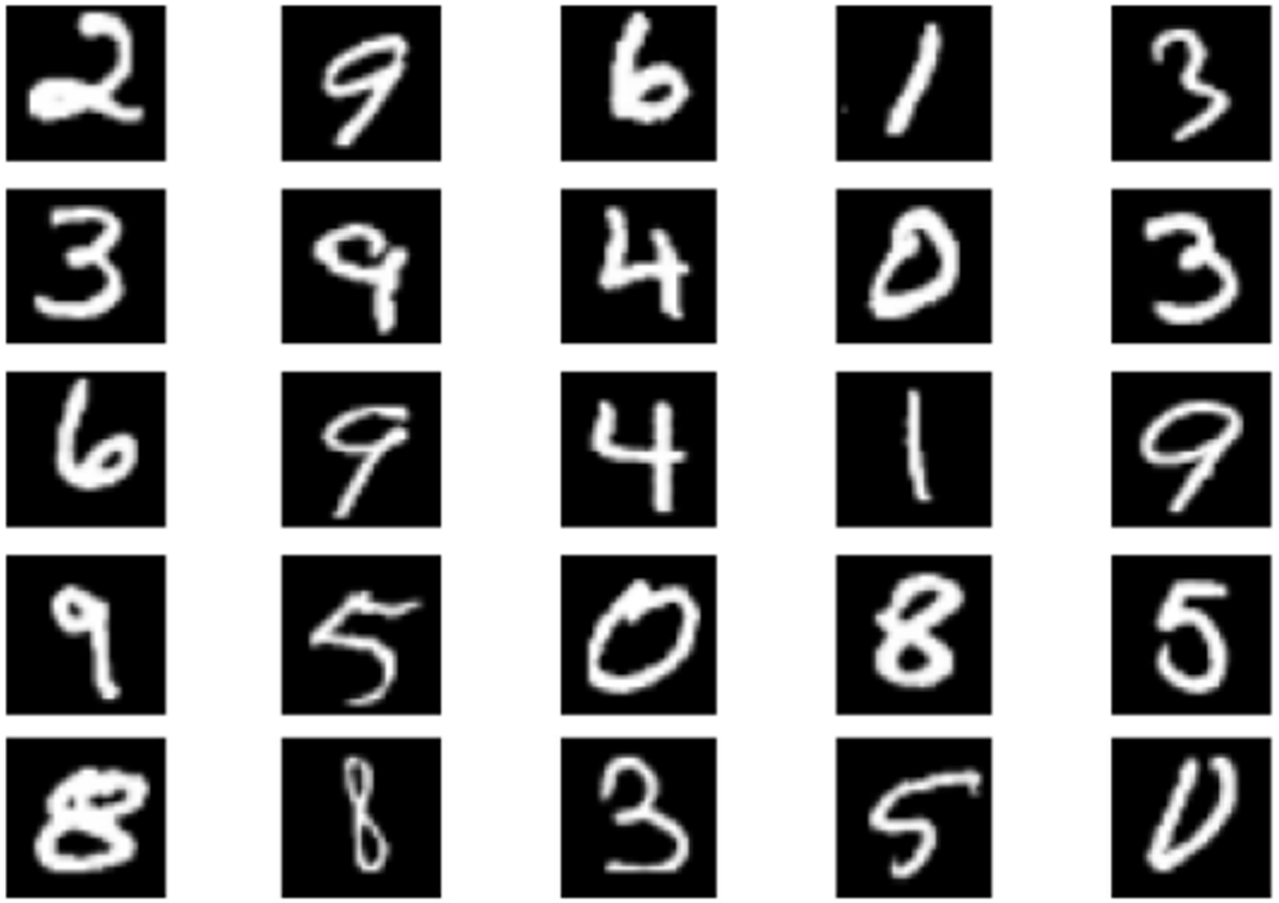
دسته بندی خطی

Linear Classification

مثال های کاربردی: تشخیص پزشکی

Do I have diabetes?

مثال های کاربردی: شناسایی ارقام



What digit is this?
How can I predict this? What are my input features?

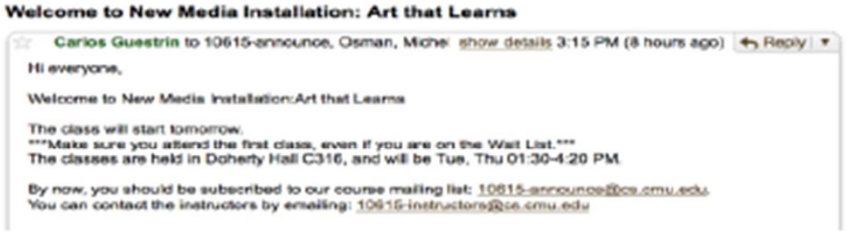
مثال های کاربردی: فیلتر کردن اسپم


Osman Khan to Carlos show details Jan 7 (6 days ago) Reply

sounds good
+ok

Carlos Guestrin wrote:
Let's try to chat on Friday a little to coordinate and more on Sunday in person?

Carlos


Welcome to New Media Installation: Art that Learns

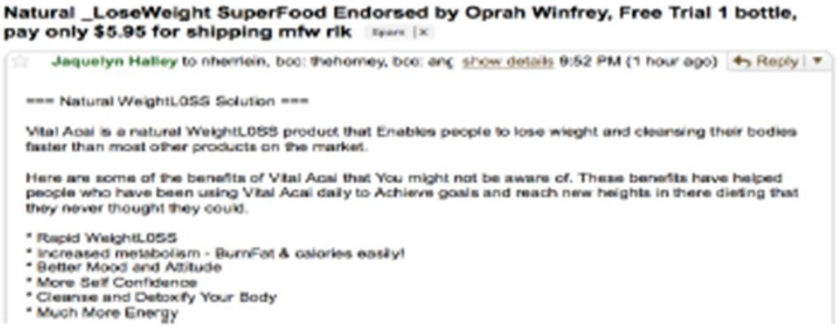
Carlos Guestrin to 10615-announce, Osman, Michel show details 3:15 PM (8 hours ago) Reply

Hi everyone,

Welcome to New Media Installation:Art that Learns

The class will start tomorrow.
Make sure you attend the first class, even if you are on the Wait List.
The classes are held in Doherty Hall C316, and will be Tue, Thu 01:30-4:20 PM.

By now, you should be subscribed to our course mailing list: 10615-announce@cs.cmu.edu.
You can contact the instructors by emailing: 10615-instructors@cs.cmu.edu


Natural _LoseWeight SuperFood Endorsed by Oprah Winfrey, Free Trial 1 bottle, pay only $5.95 for shipping mfw rlk Spam x

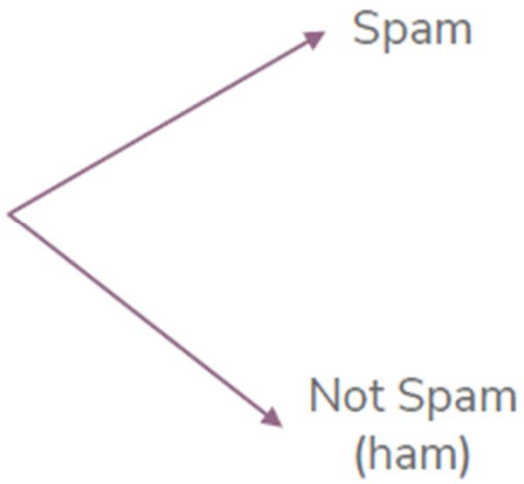
Jaquelyn Halley to nherrlein, bcc: thehorney, bcc: anc show details 9:52 PM (1 hour ago) Reply

=== Natural WeightLOSS Solution ===

Vital Acai is a natural WeightLOSS product that Enables people to lose wieght and cleansing their bodies faster than most other products on the market.

Here are some of the benefits of Vital Acai that You might not be aware of. These benefits have helped people who have been using Vital Acai daily to Achieve goals and reach new heights in there dieting that they never thought they could.

* Rapid WeightLOSS
* Increased metabolism - BurnFat & calories easily!
* Better Mood and Attitude
* More Self Confidence
* Cleanse and Detoxify Your Body
* Much More Energy



Input: x

Text of email
Sender
Subject
...

Output: y

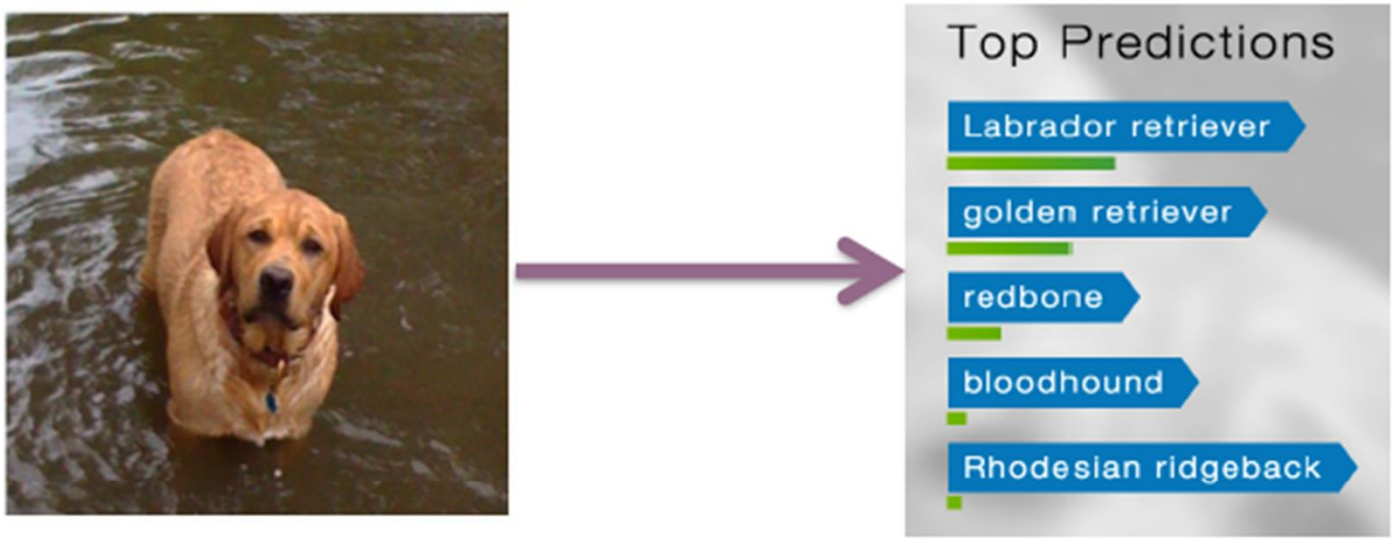
مثال های کاربردی: شناسایی اشیاء

Is this a dog?

مثال های کاربردی: شناسایی اشیاء

what about this one?

مثال های کاربردی: شناسایی اشیاء

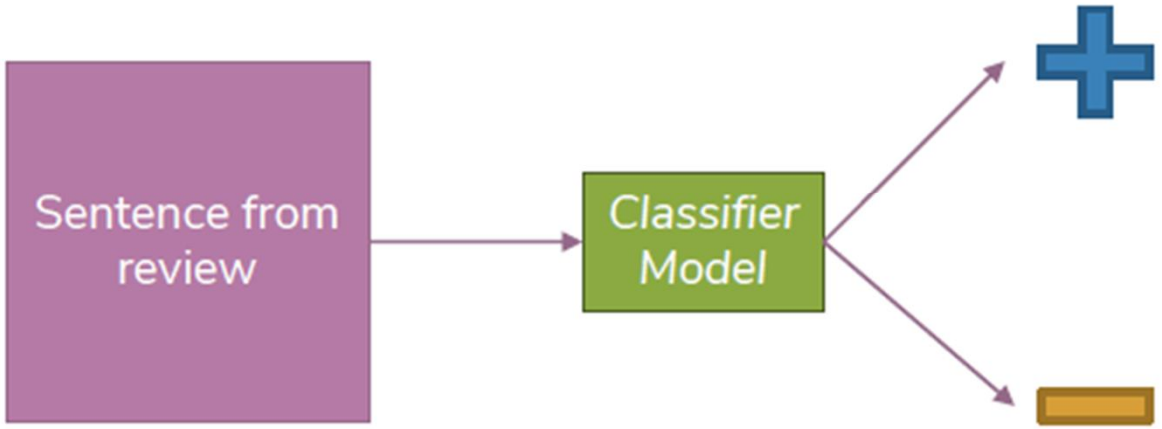


Input: x
Pixels

Output: y
Class
(+ Probability)

مثال های کاربردی: دسته بندی احساسات

In our example, we want to classify a restaurant review as positive or negative.



Input: x

Output: y
Predicted class

دسته بندی

- What do all these problems have in common?
- Categorical outputs, called labels
  (eg, yes/no, dog/cat/person/other)
- Assigning each input vector to one of a finite number of labels is called classification
- **Binary classification**: two possible labels (eg, yes/no, 0/1, cat/dog)
- **Multi-class classification**: multiple possible labels
- We will first look at binary problems, and discuss multi-class problems later in class

دسته بندی در برابر رگرسیون

- Linear Classification (binary)
- Key Concepts:
  - Classification as regression
  - Decision boundary
  - Loss functions
  - Metrics to evaluate classification

- We are interested in mapping the input $\mathbf{x} \in \mathcal{X}$ to a *label* $t \in \mathcal{Y}$
- In regression typically $\mathcal{Y} = \Re$
- Now $\mathcal{Y}$ is categorical

دسته بندی به عنوان رگرسیون

- Can we do this task using what we have learned in previous lectures?
- Simple hack: Ignore that the output is categorical!
- Suppose we have a binary problem, $t \in \{-1, 1\}$
- Assuming the standard model used for (linear) regression

$$y(\mathbf{x}) = f(\mathbf{x}, \mathbf{w}) = \mathbf{w}^T \mathbf{x}$$

- How can we obtain $\mathbf{w}$?
- Use least squares, $\mathbf{w} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{t}$. How is $\mathbf{X}$ computed? and $\mathbf{t}$?
- Which loss are we minimizing? Does it make sense?

$$\ell_{square}(\mathbf{w}, t) = \frac{1}{N} \sum_{n=1}^{N} (t^{(n)} - \mathbf{w}^T \mathbf{x}^{(n)})^2$$

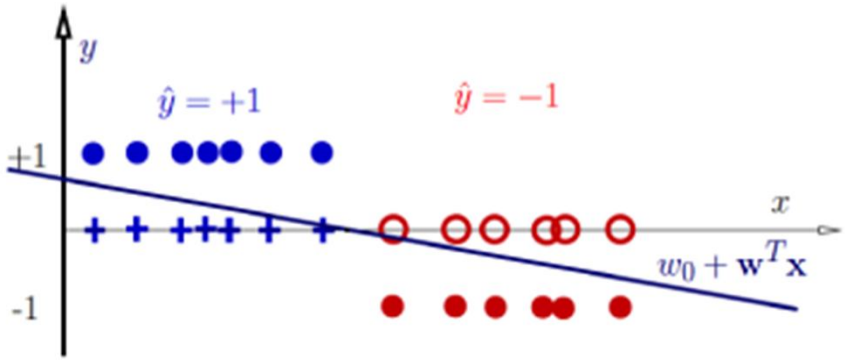
- How do I compute a label for a new example? Let's see an example

دسته بندی به عنوان رگرسیون

- One dimensional example (input x is 1-dim)

- The colors indicate labels (a blue plus denotes that $t^{(i)}$ is from the first class, red circle that $t^{(i)}$ is from the second class)

دسته بندی به عنوان رگرسیون: قانون تصمیم



- Our classifier has the form

$$f(\mathbf{x}, \mathbf{w}) = w_o + \mathbf{w}^T\mathbf{x}$$

- A reasonable decision rule is

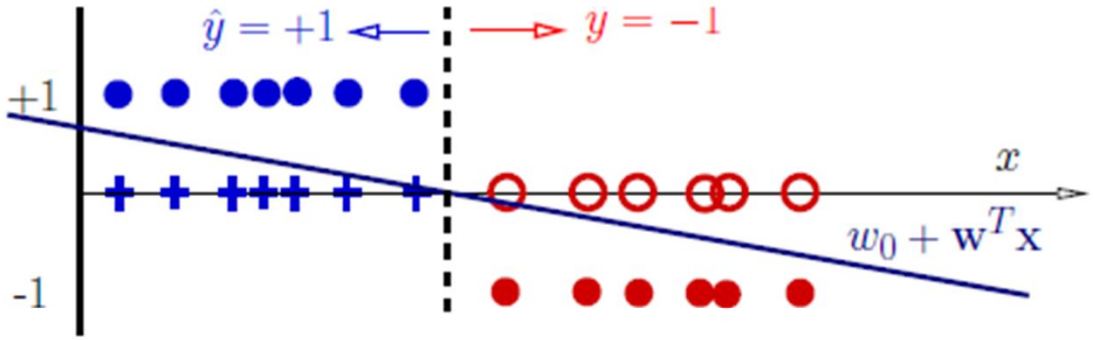
$$y = \begin{cases} 1 & \text{if } f(\mathbf{x}, \mathbf{w}) \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

- How can I mathematically write this rule?

$$y(\mathbf{x}) = \text{sign}(w_0 + \mathbf{w}^T\mathbf{x})$$

- What does this function look like?

دسته بندی به عنوان رگرسیون: قانون تصمیم



- How can I mathematically write this rule?

$$y(\mathbf{x}) = \text{sign}(w_0 + \mathbf{w}^T\mathbf{x})$$

- This specifies a linear classifier: it has a linear boundary (hyperplane)

$$w_0 + \mathbf{w}^T\mathbf{x} = 0$$

which separates the space into two "half-spaces"

مثال: دسته بندی احساسات

Input x: Sentence from review

- Count the number of positive and negative words, in x
- Example: "Sushi was **great**, the food was **awesome**, but the service was **terrible**"

Idea: Use labelled training data to learn a weight for each word.
Use weights to score a sentence.

Linear Classifier

Input x: Sentence from review

- Compute $Score(x)$
- If $Score(x) \geq 0$:
  - $\hat{y} = +1$
- Else:
  - $\hat{y} = -1$

Model: $\hat{y}_i = sign(Score(x_i))$

$$Score(x_i) = w_0 h_0(x_i) + \ldots + w_D h_D(x_i)$$

$$= \sum_{j=0}^{D} w_j h_j(x_i)$$

$$= w^T h(x_i)$$

We will also use the notation

$$\hat{s}_i = Score(x_i) = w^T h(x_i)$$

$$\hat{y}_i = sign(\hat{s}_i)$$

مثال: دسته بندی احساسات – وزن ویژگیها

- یادگرفته می شوند:

Word	Weight
good	1.0
great	1.5
awesome	2.7
bad	-1.0
terrible	-2.1
awful	-3.3
~~restaurant, the, we, where, ...~~	0.0
...	...

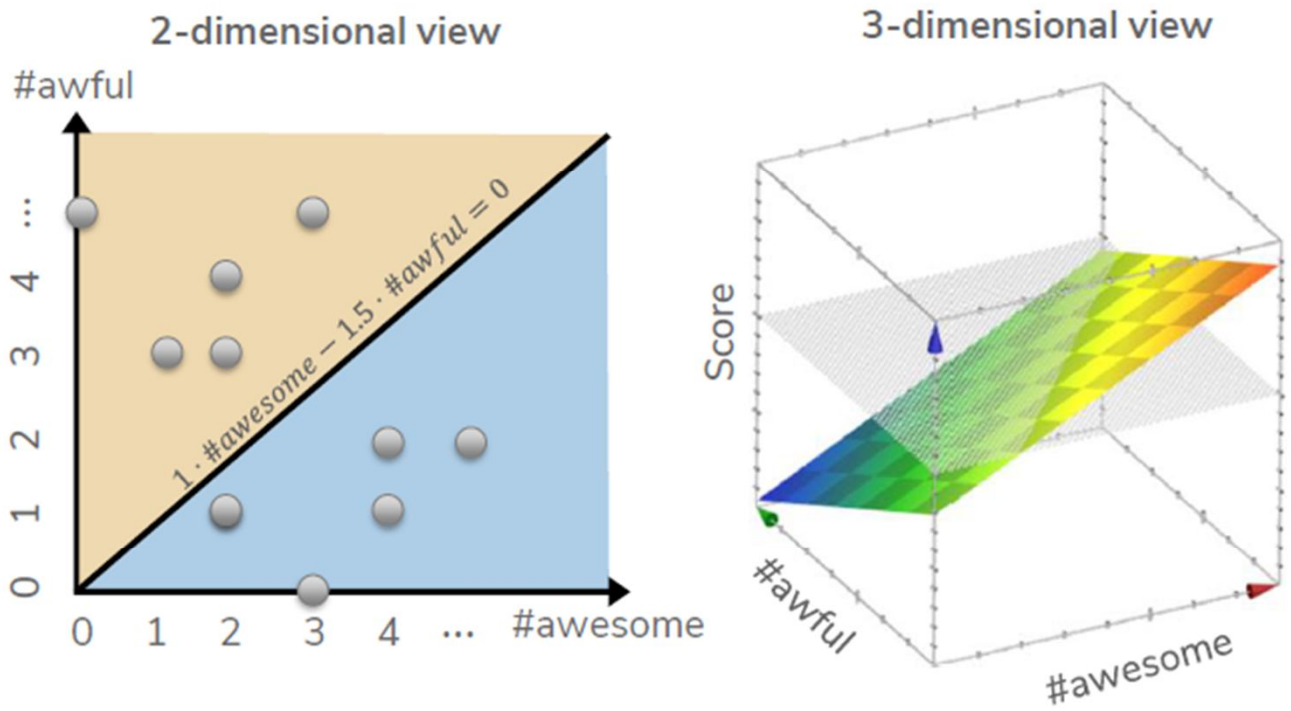
ویژگی نیستند

- فرضاً تنها دو ویژگی وزن غیر صفر پیدا کرده اند:

Word	Coefficient	Weight
	w_0	0.0
awesome	w_1	1.0
awful	w_2	-1.5

مثال: دسته بندی احساسات – مرز تصمیم

$$Score(x) = 1 \cdot \#awesome - 1.5 \cdot \#awful$$



Generally, with classification we don't us a plot like the 3d view since it's hard to visualize, instead use 2d plot with decision boundary

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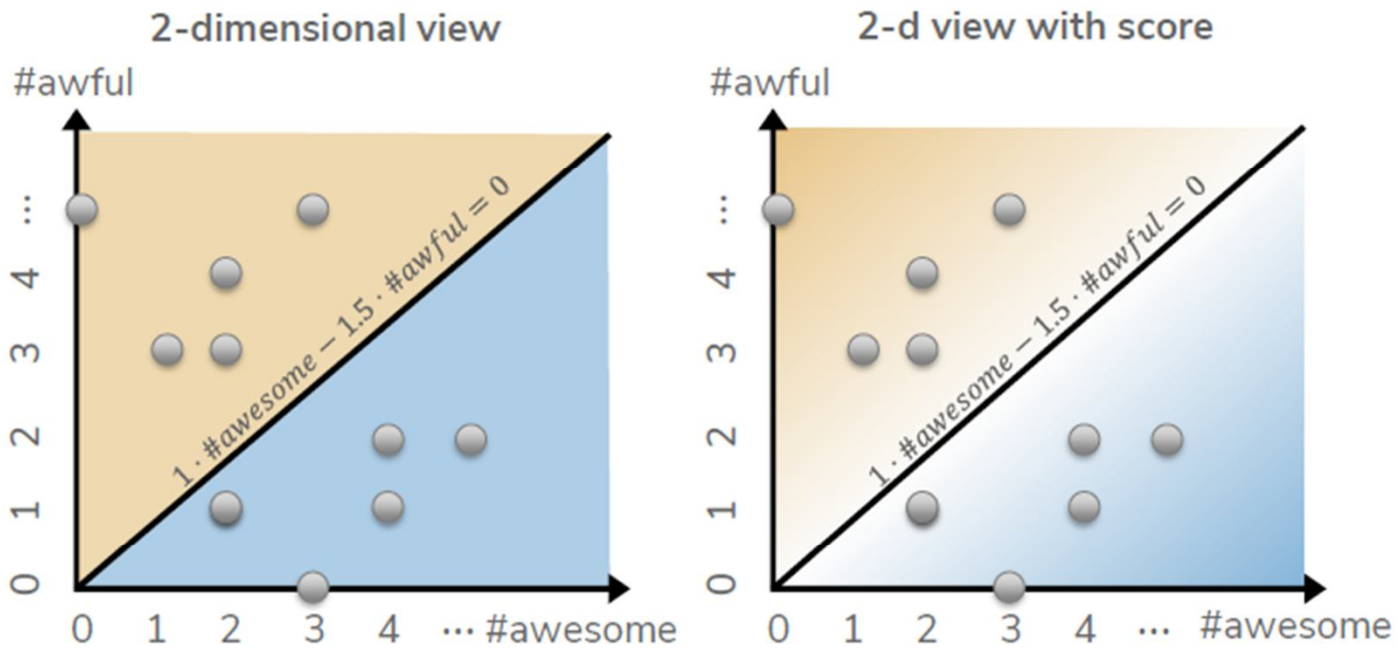
مثال: دسته بندی احساسات – مرز تصمیم

$$Score(x) = 1 \cdot \#awesome - 1.5 \cdot \#awful$$



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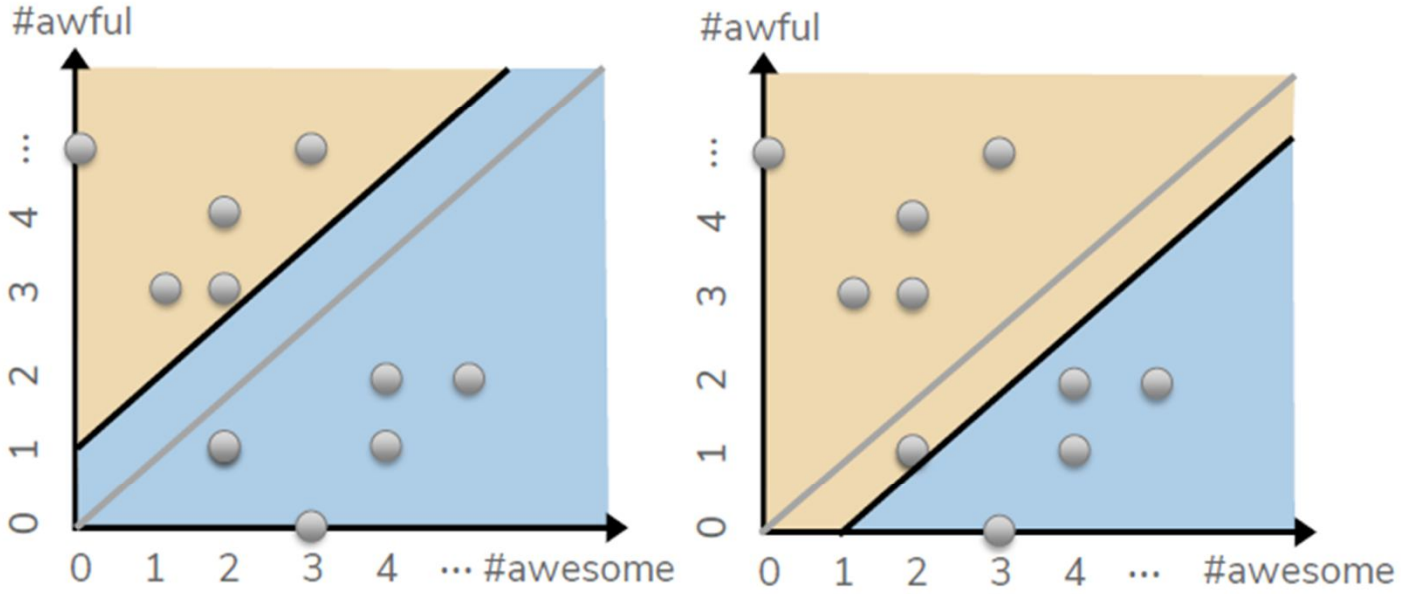
مثال: دسته بندی احساسات – مرز تصمیم

What happens to the decision boundary if we add an intercept?

$$Score(x) = 1.0 + 1 \cdot \#awesome - 1.5 \cdot \#awful$$



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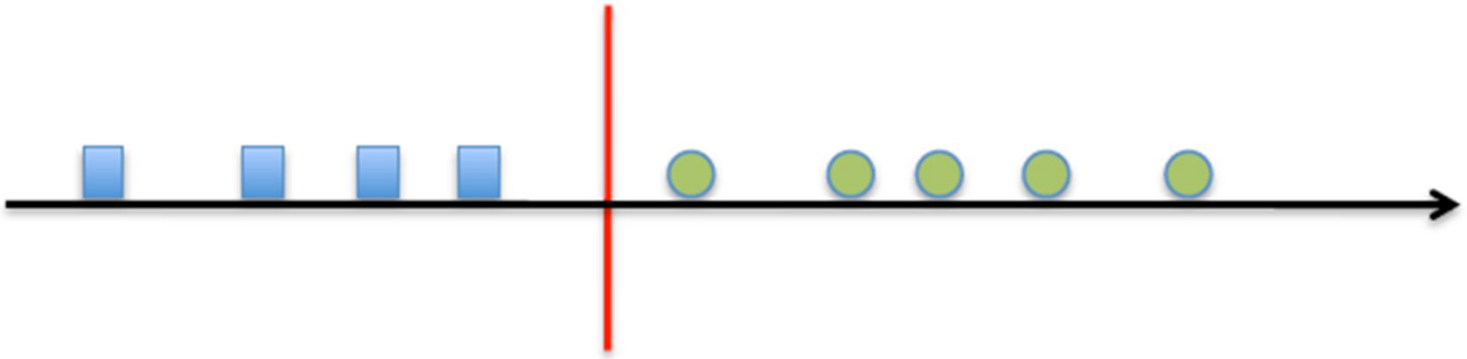
مرز تصمیم خطی: مثال یک بعدی (1D)



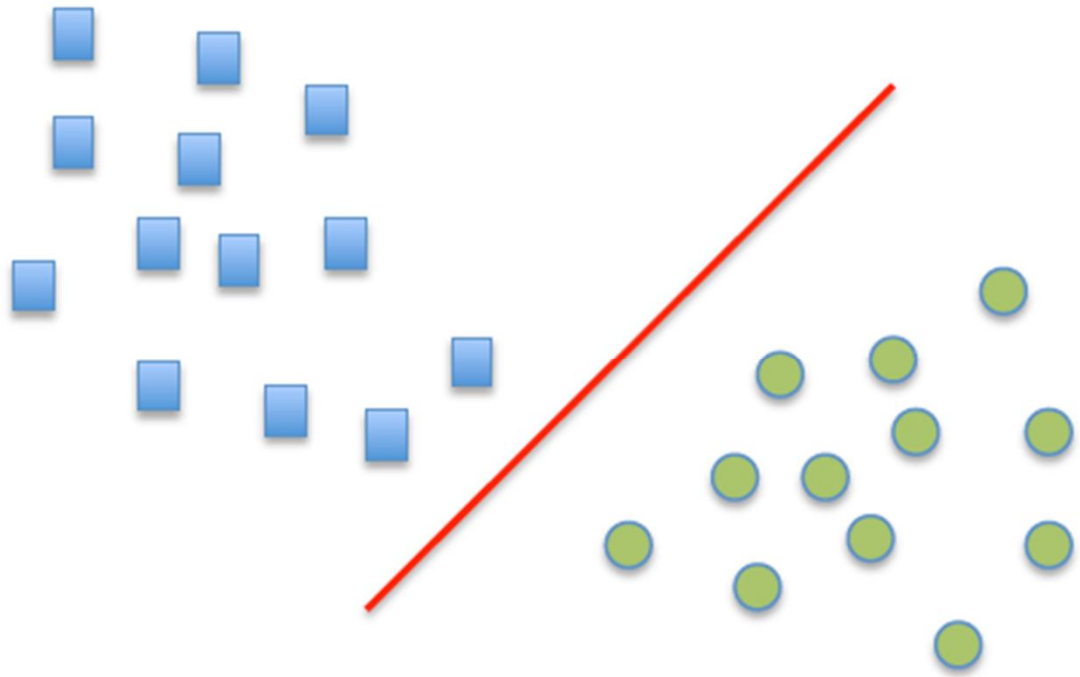
- The linear classifier has a linear boundary (hyperplane)

$$w_0 + \mathbf{w}^T \mathbf{x} = 0$$

which separates the space into two "half-spaces"

- In 1D this is simply a threshold

مرز تصمیم خطی: مثال دو بعدی (2D)



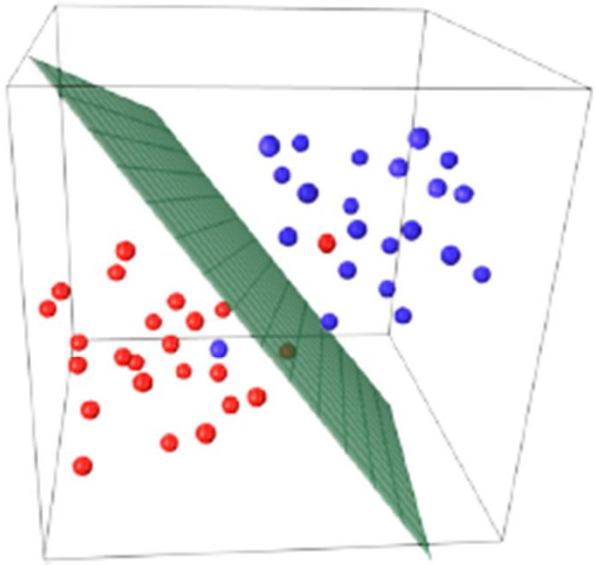
- The linear classifier has a linear boundary (hyperplane)

$$w_0 + \mathbf{w}^T \mathbf{x} = 0$$

which separates the space into two "half-spaces"

- In 2D this is a line

مرز تصمیم خطی: مثال سه بعدی (3D)



- The linear classifier has a linear boundary (hyperplane)

$$w_0 + \mathbf{w}^T \mathbf{x} = 0$$

  which separates the space into two "half-spaces"
- In 3D this is a plane
- What about higher-dimensional spaces?

مرز تصمیم خطی: هندسه

$\mathbf{w}^T\mathbf{x} = 0$ a line passing though the origin and orthogonal to $\mathbf{w}$
$\mathbf{w}^T\mathbf{x} + w_0 = 0$ shifts it by w_0

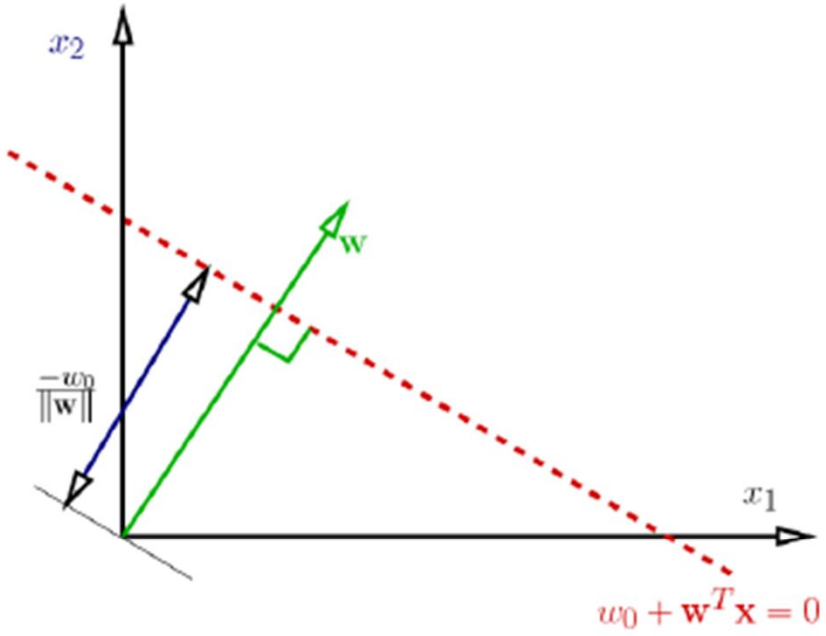
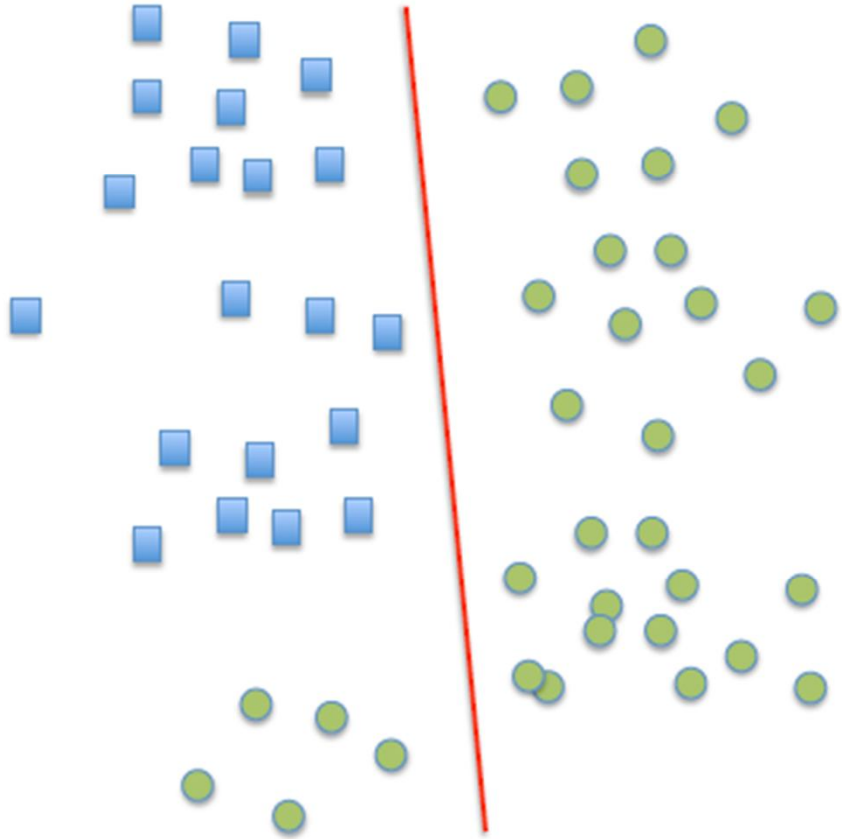


Figure from G. Shakhnarovich

مرز تصمیم خطی: همواره جداساز دسته هاست؟

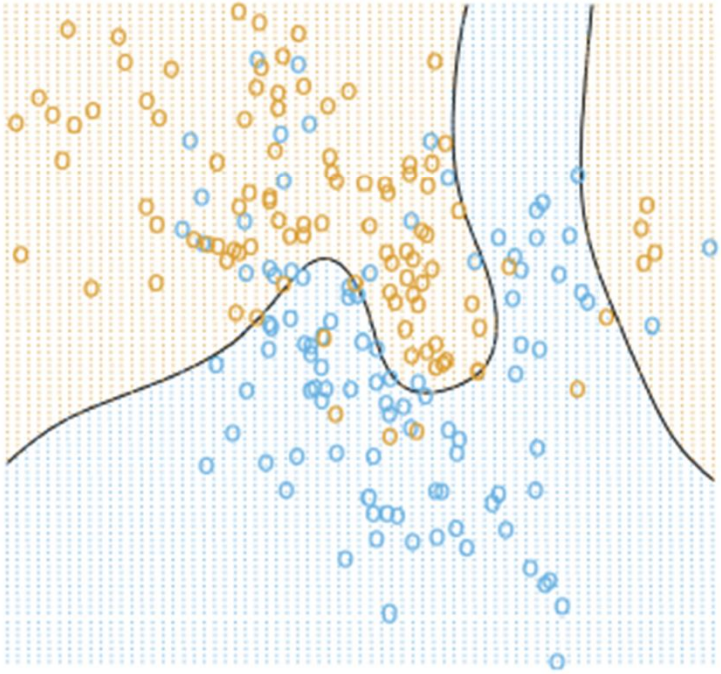
- If we can separate the classes, the problem is linearly separable



مرز تصمیم غیرخطی: برای مسایل پیچیده

What if we want to use a more complex decision boundary?

- Need more complex model/features!
- Covered next lecture!



ارزیابی دسته بندی

For binary classification:

- Should at least beat random guessing...
- Accuracy should be at least 0.5

For multi-class classification (k classes):

- Should still beat random guessing
- Accuracy should be at least $^1/_k$
  - 3-class: 0.33
  - 4-class: 0.25
  - ...

Besides that, higher accuracy means better, right?

ارزیابی دسته بندی

Imagine I made a "Dummy Classifier" for detecting spam

- The classifier ignores the input, and always predicts spam.
- This actually results in 90% accuracy! Why?
  - Most emails are spam...

This is called the **majority class classifier.**

A classifier as simple as the majority class classifier can have a high accuracy if there is a **class imbalance**.

- A class imbalance is when one class appears much more frequently than another in the dataset

This might suggest that accuracy isn't enough to tell us if a model is a good model.

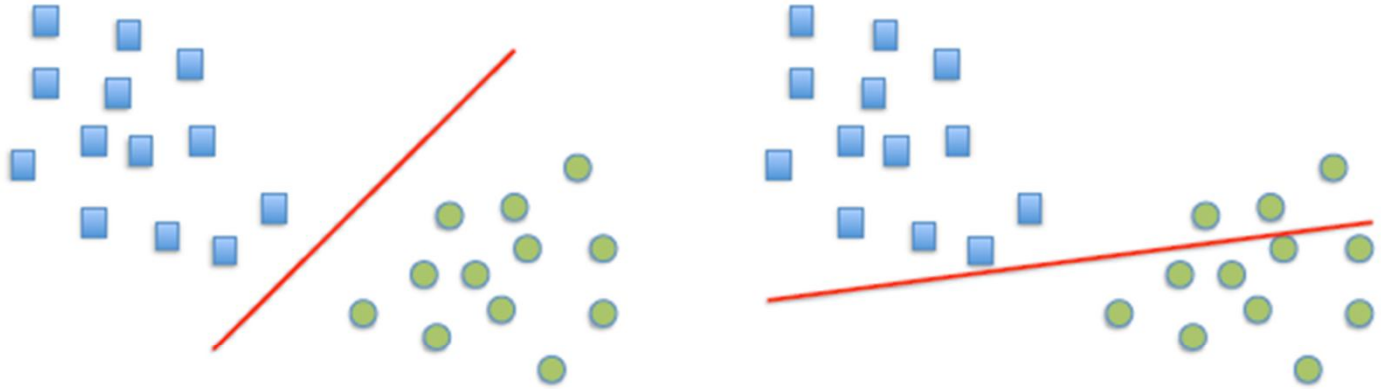
ارزیابی دسته بندی

Always digging in and ask critical questions of your accuracy.

- Is there a **class imbalance**?
- How does it compare to a baseline approach?
  - Random guessing
  - Majority class
  - ...
- Most important: **What does my application need?**
  - What's good enough for user experience?
  - What is the impact of a mistake we make?

ارزیابی دسته بندی

- Learning consists in estimating a "good" decision boundary
- We need to find $\mathbf{w}$ (direction) and w_0 (location) of the boundary
- What does "good" mean?
- Is this boundary good?



- We need a criteria that tell us how to select the parameters
- Do you know any?

ارزیابی دسته بندی – توابع اتلاف

- Classifying using a linear decision boundary reduces the data dimension to 1

$$y(\mathbf{x}) = \text{sign}(w_0 + \mathbf{w}^T \mathbf{x})$$

- What is the cost of being wrong?
- Loss function: $L(y, t)$ is the loss incurred for predicting y when correct answer is t
- For medical diagnosis: For a diabetes screening test is it better to have false positives or false negatives?
- For movie ratings: The "truth" is that Alice thinks E.T. is worthy of a 4. How bad is it to predict a 5? How about a 2?

ارزیابی دسته بندی – توابع اتلاف

- Zero/one loss for a classifier

$$L_{0-1}(y(\mathbf{x}), t) = \begin{cases} 0 & \text{if } y(\mathbf{x}) = t \\ 1 & \text{if } y(\mathbf{x}) \neq t \end{cases}$$

- Asymmetric Binary Loss

$$L_{ABL}(y(\mathbf{x}), t) = \begin{cases} \alpha & \text{if } y(\mathbf{x}) = 1 \wedge t = 0 \\ \beta & \text{if } y(\mathbf{x}) = 0 \wedge t = 1 \\ 0 & \text{if } y(\mathbf{x}) = t \end{cases}$$

- Squared (quadratic) loss

$$L_{squared}(y(\mathbf{x}), t) = (t - y(\mathbf{x}))^2$$

- Absolute Error

$$L_{absolute}(y(\mathbf{x}), t) = |t - y(\mathbf{x})|$$

ارزیابی دسته بندی – معیارها

How to evaluate how good my classifier is? How is it doing on dog vs no-dog?

TP (True Positive) FP (False Positive) FN (False Negative)

ارزیابی دسته بندی – معیارها

How to evaluate how good my classifier is?

- Recall: is the fraction of relevant instances that are retrieved

$$R = \frac{TP}{TP + FN} = \frac{TP}{\text{all groundtruth instances}}$$

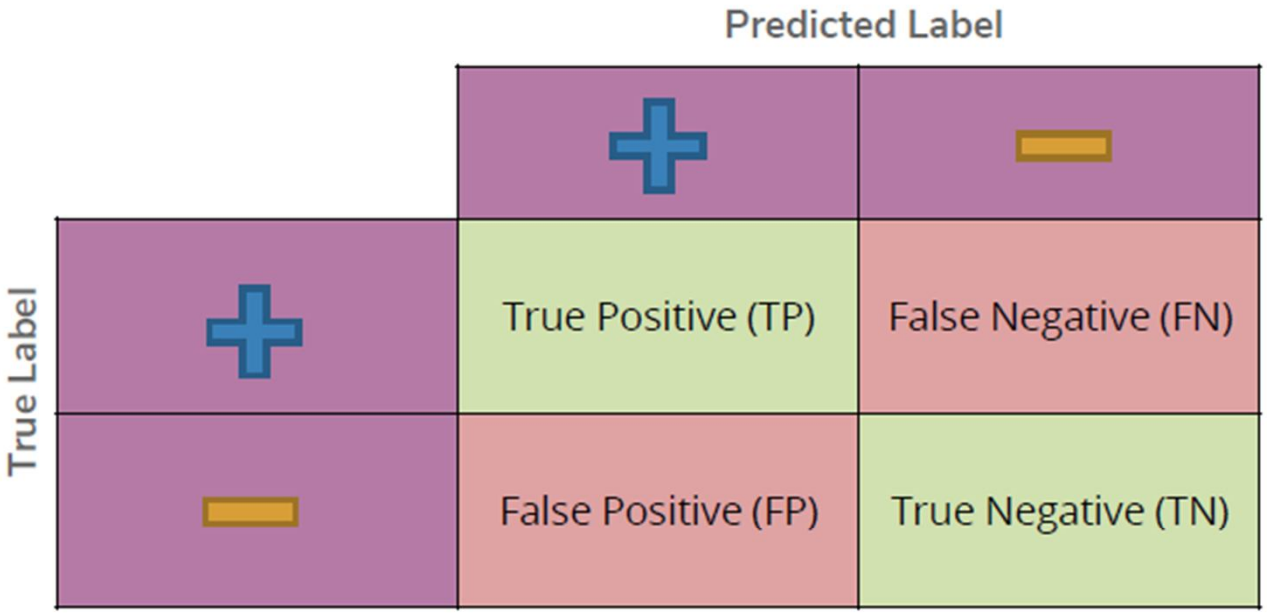
- Precision: is the fraction of retrieved instances that are relevant

$$P = \frac{TP}{TP + FP} = \frac{TP}{\text{all predicted}}$$

- F1 score: harmonic mean of precision and recall

$$F1 = 2\frac{P \cdot R}{P + R}$$

ارزیابی دسته بندی – معیارها



What's worse, a false negative or a false positive?

- It entirely depends on your application!

Detecting Spam	**Medical Diagnosis**
False Negative: Annoying	False Negative: Disease not treated
False Positive: Email lost	False Positive: Wasteful treatment

In almost every case, how treat errors depends on your context.

ارزیابی دسته بندی – معیارها

Notation

- $C_{TP} = \#\text{TP}, \quad C_{FP} = \#\text{FP}, \quad C_{TN} = \#\text{TN}, \quad C_{FN} = \#\text{FN}$
- $N = C_{TP} + C_{FP} + C_{TN} + C_{FN}$
- $N_P = C_{TP} + C_{FN}, \quad N_N = C_{FP} + C_{TN}$

Error Rate

$$\frac{C_{FP} + C_{FN}}{N}$$

Accuracy Rate

$$\frac{C_{TP} + C_{TN}}{N}$$

False Positive rate (FPR)

$$\frac{C_{FP}}{N_N}$$

False Negative Rate (FNR)

$$\frac{C_{FN}}{N_P}$$

True Positive Rate or Recall

$$\frac{T_P}{N_P}$$

Precision

$$\frac{T_P}{C_{TP} + C_{FP}}$$

F1-Score

$$2\frac{Precision \cdot Recall}{Precison + Recall}$$

See more!

ارزیابی دسته بندی – معیارها

Consider predicting (*Healthy*, *Cold*, *Flu*)

True Label \ Predicted Label	Healthy	Cold	Flu
Healthy	60	8	2
Cold	4	12	4
Flu	0	2	8