On Duality of Stability and Capacity Regions in Interference Networks

Sajjad Nassirpour and Alireza Vahid

Abstract

Recent information-theoretic studies of wireless networks have resulted in several interference management (IM) techniques that promise significant gains over orthogonalization techniques (e.g., TDMA and FDMA). However, some underlying assumptions raise concerns about the practicality of these results. It is typically assumed that all messages are available to the transmitters at the beginning of communications, resulting in attempts to characterize the (asymptotic) capacity region. In this work, we investigate the stable throughput region of a canonical wireless network with distributed transmitters as opposed to the capacity region. In this context, we translate physical-layer IM protocols to accommodate stochastic message arrivals. We show the equivalence of the stable throughput region and the capacity region via simulations. We observe that in order to achieve the optimal stable throughput region, significant changes are needed in prior techniques. We quantify the trade-off between encoding/decoding complexity of the proposed scheme (in terms of number of required algebraic operations), and the achievable rates. Finally, we study the lifetime of each packet (i.e. the duration from arrival to successful delivery) vis-a-vis the total communication time, and we observe that the average lifetime scales as the square root of the total communication time.

Index Terms

Stable throughput, interference management, distributed transmitters, backlogged traffic, delayed feedback.

I. Introduction

As wireless networks grow in size, number of simultaneous transmissions increases resulting in higher chance of signal interference. In fact, interference has become the main bottleneck

The authors are with the Department of Electrical Engineering at the University of Colorado, Denver, USA. Email: sajjad.nassirpour@ucdenver.edu and alireza.vahid@ucdenver.edu.

for network throughput improvement in modern wireless systems. Therefore, Interference Management (IM) techniques play a central role in enhancing throughput rates, and in recent years, we have seen a variety of IM techniques in Information Theory literature. One of the most prominent proposed solutions is Interference Alignment (IA). This technique was first implicitly introduced in [1] to reduce the subspace occupied by interference at each receiver in the context of single-input single-output X Channels. In [2–5], IA is utilized in K-user interference channels to reveal the significant gain over simple orthogonalization techniques. Retrospective IA of [6] manages interfering signals in dynamic, distributed networks. In the context of multi-antenna users [7–13], explain IA approaches with feedback.

There are also several results investigating an interesting aspect of wireless networks, namely stable throughput region. This study corresponds to the scenario in which messages arrive stochastically at the transmitters, and thus, introducing new challenges in interference management. Stable throughput region of multi packet reception channel model under ALOHA mechanism is described in [14, 15]. Authors in [16] characterize the stable throughput region of broadcast channel (BC) and multiple-access channel (MAC) with backlogged traffic. The stable throughput region of the two-user BC with feedback is characterized in [17, 18]. Authors in [19] show that the complexity of analyzing the information-theoretic capacity region of K-user ($K \geq 4$) BCs grows rapidly with the number of users, and tracking all interfering signals becomes a daunting task, and in turn, for the stable throughput region as well. These results, for the most part, consider a central transmitter which has access to all messages in the network.

In this work, we consider a distributed interference network with two transmitter-receiver pairs. In this setting, transmitters do not exchange any messages and thus, we ought to make decentralized and distributed decisions for interference management. For this problem, we study the stable throughput region. We show, through simulations, that the stable throughput region matches the capacity region. We use simulations, as the complexity of obtaining analytical expression for the stable throughput region in distributed networks is prohibitive. In our work, we take into account several practical challenges in implementing IA techniques: (1) decreasing heavy reliance on accurate channel state information; (2) impact of encoder block length on throughput region; (3) stochastic packet arrival.

Motivated by recent IM techniques for networks with delayed channel state information at the transmitters (CSIT) which is a more suitable model for dynamic networks (*e.g.* [20–22]), we consider wireless packet network with low feedback bandwidth where only a couple of feedback bits are needed per packet. We note that each data packet in the forward channel may contain thousands of bits and thus, feedback overhead is negligible. In the context of packet networks, transmitters send out data packets to desired destinations and receive delayed ACK/NACK messages (*i.e.* indicating successful delivery or failure).

There are several communication protocols that achieve the information-theoretic bounds on throughput rates of networks with ACK/NACK feedback. However, these results are again mostly limited to centralized transmitters. XOR combination technique which incorporates with feedback from receivers is introduced in [23] to reduce the retransmission attempts and improve throughput rate under repetitive Automatic Repeat reQuest (ARQ) strategy in multi-users BC model. Authors in [24] show that by employing the XOR operation and random coding mechanism, the upper bound of the capacity region in BC model is achievable. A threshold-based scheduling procedure is defined in [25] which describes the policies of threshold value determination to handle the packets and maximize the throughput rate in K-user interference wireless network.

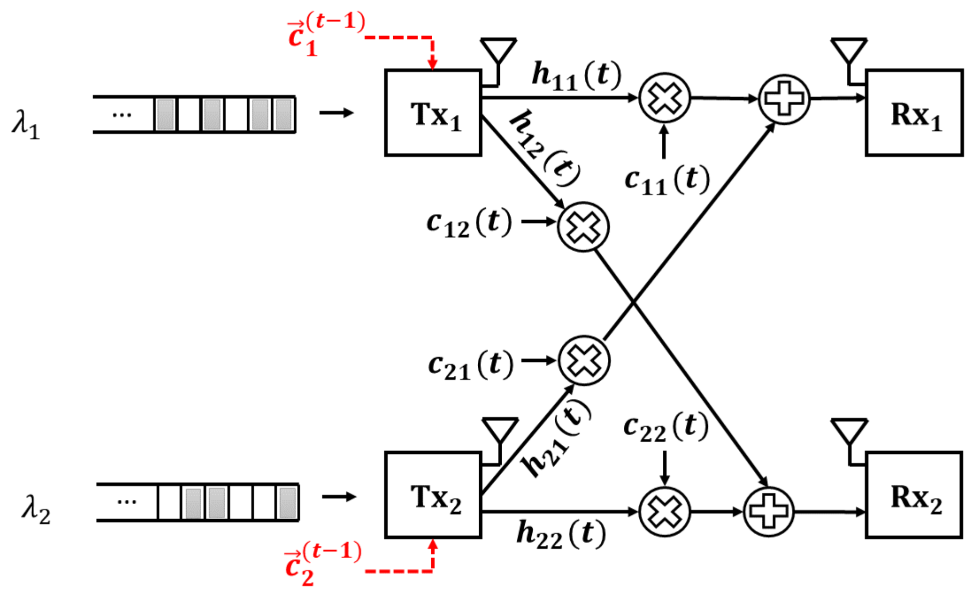

Fig. 1. *An interference network with two transmitter-receiver pairs where packets are arriving through stochastic processes and binary shadowing coefficients are available at transmitters with unit delay.*

More recently, authors in [26] and [27] propose a new abstraction for wireless networks with distributed transmitters and characterize the optimal throughput rates for a canonical network with two transmitter-receiver pairs. In this work, we take the model introduced in [26] and [27] which is depicted in Fig. 1. As we discuss later in Section II, this model consists of two transmitter-receiver pairs and channel connectivity is governed by binary shadowing coefficients $c_{ij}(t)$'s which capture the relative signal-to-noise ratios (SNRs) and signal-to-interference-plus-noise ratios (SINRs) of different links. We assume all packets arrive through stochastic flows and may collide (or interfere) at receivers. For this model, we propose a new queue-based protocol that is well-suited for stochastic packet arrival. Our queue analysis is similar to [28–30] which propose a protocol to improve stable rate through multiple stochastic unicast transmissions.

It is worth noting that authors in [31] study a similar problem, and based on the coding ideas of [26], they propose a modified version to accommodate stochastic packet arrivals. However, there are a number of errors in that paper which we will discuss in Remark 1 of Section VI-B.

We provide an overview of our main contributions in Section IV which are summarized below:

C1: A new communication protocol for interference wireless networks with two transmitter-receiver pairs and stochastic packet arrival; showing the equivalence of the stable throughput region and the capacity region in this context;

C2: Revealing the impact of limited computing power (in terms of number of algebraic operations) on stable rates;

C3: Simulating the lifetime (or delay in delivery) of packets in the proposed protocol.

The rest of the paper is organized as follows. Section II describes the channel model we use in this paper. In section III, we compare the capacity regions in different schemes. Section V and IV contain overview of the results and comparison to non-stochastic arrival, respectively. We present our transmission protocol (contribution **C1** above) in section VI and show that the stable throughput region matches the capacity region. Section VII includes our simulation results (**C2** and **C3**). In section VIII, we conclude the discussion about queue-based algorithm.

II. Problem Setting

We consider the channel model for wireless networks introduced in [27] which includes two transmitter-receiver pairs as in Fig. 1. In this network, transmitter Tx_i only wishes to communicate with receiver Rx_i, $i = 1, 2$. The received signal at Rx_i is given by:

$$\vec{r}_i(t) = c_{ii}(t)h_{ii}(t)x_i(t) + c_{ji}(t)h_{ji}(t)x_j(t), \qquad i = 1, 2 \text{ and } j = 3 - i. \tag{1}$$

Here, $h_{ji}(t)$ captures the continuous channel gain from transmitter Tx_j to receiver Rx_i at time instant t and follows some distribution such as Rayleigh fading (the actual distribution is irrelevant in our work as discussed in Table I), $x_j(t)$ is the transmit signal of Tx_j at time instant

t, and $c_{ji}(t)$ is the binary shadowing coefficient of the link from transmitter Tx_j to receiver Rx_i at time instant t. The shadowing coefficients, as described in Table I, capture the relative signal-to-noise ratio (SNR) and signal-to-interference-plus-noise ratio (SINR) of the corresponding wireless links defined as:

$$SNR_{ji} \triangleq 10\log_{10}(\frac{P|h_{ji}|^2}{P_I}), \qquad SINR_{ji} \triangleq 10\log_{10}(\frac{P|h_{ji}|^2}{P_I + P|h_{ji}|^2}). \tag{2}$$

where P is the average power budget and P_I represents summation of noise and ambient interference power.

TABLE I

DIFFERENT STATES AT Rx_1. HERE, γ IS THE THRESHOLD BEYOND WHICH A PACKET CAN BE DECODED.

State	Power Ratio	Received Packet	$c_{11}(t)$	$c_{21}(t)$
1	$SINR_{11} \geq \gamma$	Tx_1	1	0
2	$SINR_{21} \geq \gamma$	Tx_2	0	1
3	$SINR_{j1} \leq \gamma$ and $SNR_{j1} \geq \gamma$ for $j \in 1, 2$	Linear Combination	1	1
4	$SNR_{j1} \leq \gamma$ for $j \in 1, 2$	Dropped	0	0

We assume packets arrive at Tx_i according to a Poisson(λ_i) distribution, $i = 1, 2$, $\lambda_i > 0$, and that the two processes are independent from each other. The mapping of data packets to codewords and then the transmit signal is not the focus of this work. We assume a standard coding scheme such as Reed-Solomon or Low-Density-Parity-Check (LDPC) codes for encoding data bits. All we need is the existence of some signal-to-noise ratio threshold, γ, beyond which data bits can be decoded from the received signal at each receiver. In multi-terminal communications, interference is added to the received signals, and we devise a communication protocol that will guarantee decodability of data at the receivers. Table I describes channel connectivity by assigning binary values to $c_{ji}(t)$'s based on corresponding SNR and SINR values.

A packet is said to be delivered successfully if it is received interference-free at its intended receiver, or within a finite time horizon, the intended receiver obtains sufficient number of equations to recover (at least) this packet. The achievable rate at receiver Rx_i is the number of successfully delivered packets to that receiver divided by the communication time, and is denoted by $\hat{\lambda}_i$. We say a communication protocol accommodates stable rates λ_1 and λ_2 if

$$\lambda_i(1-\epsilon) \leq \hat{\lambda}_i \leq \lambda_i(1+\epsilon), \quad \text{and} \quad \epsilon \ll 1, \tag{3}$$

is simultaneously satisfied for $i = 1, 2$, where ϵ represents the acceptable error rate at the receivers. We note that the non-stochastic result, *i.e.* the capacity region, serves as an outer-bound for our formulation. Meaning that λ_i cannot exceed the maximum throughput rates defined by the boundary of the non-stochastic capacity region.

In our model, we assume shadowing coefficients $c_{ij}(t)$'s are independently and identically distributed (i.i.d.) Bernoulli random variables, ($\mathcal{B}(1-\delta)$, for $0 \le \delta \le 1$) where δ is the erasure probability. In other words, we focus on the homogeneous setting in which all links have the same probability of erasure. Receiver Rx_i broadcasts $c_{1i}(t)$ and $c_{2i}(t)$ at the end of each time instant t to all other nodes and transmitters do not learn the continuous channel values $h_{ij}(t)$ during the transmission process. To exploit the benefits of saving the feedback signals from the receivers, transmitters employ the precoding vector $\vec{v}_i(t) \in \mathbb{R}^{1 \times N_i}, i = 1, 2$ where N_i is the number of packets which are sent to Rx_i to construct a linear combination of their data $L(\vec{x}_1, \vec{x}_2)$ at each time and apply it to original data. We describe how Rx_1 can decode N_1 packets as an example and similar statement holds for Rx_2. By using the precoding vector, the original data of Tx_1 converts to a transmit signal $\vec{v}_1(t)\mathbf{H}_1$ where $\mathbf{H}_1 = [\vec{x}_{11}\vec{x}_{12}\ldots\vec{x}_{1N_1}]^T$. In order not to change the total power of transmit signal, we confine $\|\vec{v}_i(t)\| \le 1, i = 1, 2$ for precoding vectors where $\|.\|$ represents the Euclidean norm. As a result, (1) can be formulated as below:

$$\vec{r}_i(t) = c_{1i}h_{1i}(t)\vec{v}_1(t)\mathbf{H}_1 + c_{2i}h_{2i}(t)\vec{v}_2(t)\mathbf{H}_2, \quad i = 1, 2. \tag{4}$$

We assign $\vec{v}_i(t)$ as a row of a precoding matrix $\mathbf{V}_i^m \in \mathbb{R}^{m \times N_i}$ and $c_{ij}(t)h_{ij}(t), i, j = 1, 2$ as the t^{th} element of a diagonal matrix $\mathbf{A}_{ij}^m(m \times m)$ to represent (4) algebraically as below:

$$\vec{r}_i^m = \mathbf{A}_{1i}^m\mathbf{V}_1^m\mathbf{H}_1 + \mathbf{A}_{2i}^m\mathbf{V}_2^m\mathbf{H}_2, \quad i = 1, 2. \tag{5}$$

Furthermore, interference elements at Rx_1 can be denoted by I_1 as below:

$$I_1 = \text{Rowspan}(\mathbf{A}_{21}\mathbf{V}_2), \tag{6}$$

where Rowspan is subspace of a matrix which is spanned by row space of the matrix. We identify a subspace I_1^c which is orthogonal to I_1 and I_1^{proj} as the projection matrix of $\mathbf{A}_{11}^m\mathbf{V}_1^m$ on I_1^c. According to linear algebra, if the rank of $\mathbf{V}_1^m$ is equal to rank of I_1^{proj}, Rx_1 will be able to decode N_1 packets from N_1 independent equations. It means,

$$\text{rank}(I_1^{proj}) = \text{rank}(\mathbf{V}_1^m) = N_1. \tag{7}$$

We say rate-tuple $(\lambda_1, \lambda_2) = (N_1/m, N_2/m)$ is achievable if there exist precoding vectors satisfying both (3) and (7). Then, the stable throughput region is defined as closure of all stable rates.

III. Benchmarks

In this section, we compare the capacity region of three different benchmarks:

Time Division Multiplexing (TDM): The wireless link between each transmitter and receiver pair is a binary erasure channel with capacity $(1-\delta)$. In TDM, the communication time is divided into segments where in any given segment only one transmitter-receiver pair communicates, and thus, interference is avoided. The maximum achievable sum-rate in this case is then $(1-\delta)$.

No-CSIT: In this case, there is not any feedback information from receivers to transmitters and the distribution of channel gain is the main information source for the transmitters. Therefore, the capacity region is same as the Multiple Access Channel which is given below:

$$\begin{cases} 0 \leq \hat{\lambda}_i \leq (1-\delta), \quad i = 1, 2, \\ \hat{\lambda}_1 + \hat{\lambda}_2 \leq 1 - \delta^2. \end{cases} \tag{8}$$

Delayed-CSIT: It means, $c_{ij}(t-1)$ are available at transmitters at the end of time instant t. Capacity region of distributed interference channel with delayed-CSIT which is derived in [26] can be shown as below:

$$\begin{cases} 0 \leq \hat{\lambda}_i \leq (1-\delta), & i = 1, 2, \\ \hat{\lambda}_i + (1+\delta)\hat{\lambda}_j \leq (1-\delta)(1+\delta)^2, & i = 1, 2, j = 3 - i. \end{cases} \tag{9}$$

Fig. 2 depicts capacity region for these three different schemes when $\delta = 0.4$ and shows delayed-CSIT provides better capacity region in comparison to no-CSIT and TDM. As a result, we use the ACK/NACK packets as delayed-CSIT to communicate between network's and show later in section VII that the stable throughput region of our proposed network is identical with capacity region of delayed-CSIT in Fig. 2.

IV. Overview of the Results

In this paper, we assume packets arrive at each transmitter according to independent Poisson processes. We use a queue-based network to transmit packets and manage interference. At the beginning of each time instant t, we face two broad categories of packets:

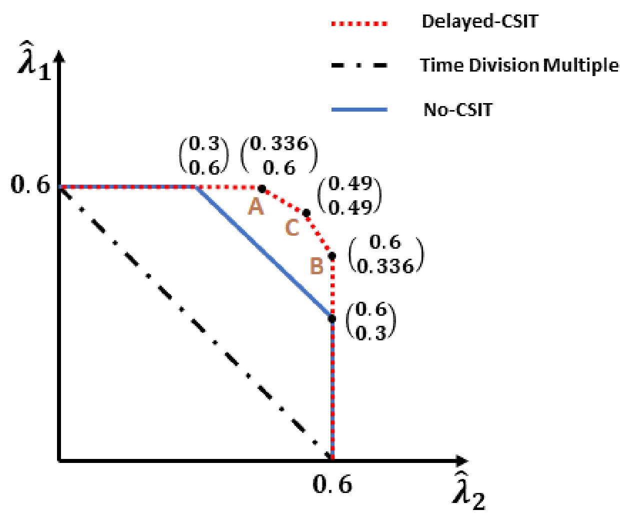

Fig. 2. *Comparison of information-theoretic capacity regions for three different types of CSIT with* $\delta = 0.4$.

- Newly arrived packets which join "initial" queues;
- Previously transmitted packets saved whose delivery requires further re-transmission.

We denote the status of these packets at the beginning and the ending of time instant t by origin and destination queues, respectively. According to the nature of stochastic arrival, it is impossible to estimate which origin queue has a packet to send at each time instant. Therefore, we assign a priority policy to determine which queue should send its packets at time instant t. We identify a control table to describe packet movements between origin and destination queues based on channel state information. At the end of each time instant, we look for coding opportunities, and if any is available, packets are removed from origin queues and a combination of them joins a queue containing packets that are simultaneously beneficial to both receivers.

Moreover, an important concern in deploying existing interference techniques is the delay they introduce in delivering the messages. Delivery delay (i.e. lifetime of packet) in non-stochastic procedures equals the communication time. However, we show that our dynamic protocol significantly decreases the delivery delay to the square root of communication time.

V. Comparison to related work in information theory

As we mentioned earlier, we build upon the results of [26], and in this section, we highlight the differences between the two manuscripts.

In [26], authors assume a non-stochastic model in which all messages are available to the transmitters at the beginning of the communication block. As a result, it is possible to develop communication protocols that are carried on over several phases and in each phase, signals are communicated and at the end of the phase, using delayed feedback, the status of the transmitted signals is updated. Then, new combinations are created for future phases. However, in this paper due to the stochastic nature of packet arrivals, we need to make real-time decisions, and as we mention in Section VI, new cases arise that do not fall into any of the categories considered in [26]. Moreover, the non-stochastic model of [26], allows for the transmitters to wait and create combinations of large number of packets using random matrices. In this work, on the other hand, we take a close look at the encoder and the decoder and the number of operations they need to perform to successfully transmit/decode each packet. We also investigate the delivery delay, i.e. lifetime of the packets, which tends to is linearly related to the communication block length for the scheme of [26]. We show, through simulations, that for our scheme that this delay scales as square root of communication block length.

VI. Transmission Protocol

The transmission protocol in general requires careful consideration of various queues and coding opportunities. To simplify the description of the transmission protocol, in this section, we focus on $\frac{(3-\sqrt{5})}{2} \leq \delta < 0.5$ which results in a more compact formulation of the protocol. We defer the detailed description of our protocol for other regimes to appendices. Furthermore, we demonstrate the protocol as ordinary procedure for the maximum sum-rate corner point "C" in Fig. 2 through the followings: (1) Identifying a priority policy to distinguish which queue should send its packet before others; (2) Defining control table to determine packet movement rules between queues; (3) Considering a pseudo-code to explain how we can implement the protocol. The details of the modified procedure for other corner points "A" and "B" in Fig. 2 is presented in Appendix C.

A. Constructing virtual Queues at the transmitters

In queue-based transmissions, queues play the main role to handle the packets. Therefore, we introduce some queues for packet controlling as follow:

$Q_i^{i|\phi}$: It is the first queue where each packet of Tx_i goes in. It means the new arrived packets at each time instant must save in $Q_i^{i|\phi}$;

$Q_i^{1,2|\phi}$: It contains packets which belong to Tx_i and are helpful to both Rx_1 and Rx_2 (named as common interest packets);

$Q_i^{i|\bar{i}}$: It contains the packets which are needed by Rx_i but are attained by $\mathsf{Rx}_{\bar{i}}$ without interference. In other words, $\mathsf{Rx}_{\bar{i}}$ can use these packets as side information in future process;

$Q_i^{\bar{i}|i}$: It contains the packets which are needed by $\mathsf{Rx}_{\bar{i}}$ but Rx_i gets them without any trouble. Hence, Rx_i will be able to employ them as side information;

$Q_i^{c_1}$: It contains the packets which are received by Rx_i from both Tx_1 and Tx_2 (named as communication packets) and we will describe later in Section VI-E that this queue provides combining models for our transmission protocol;

Q_i^F: Packets that are either delivered or upon successful delivery of other packets can be decoded. The packets of Q_i^F will not be retransmitted.

B. Delivering Options

In this part, we identify two delivering opportunities based on prior values of channel state $c_{ij}(i, j = 1, 2)$. Later, we will use these opportunities to deliver packets to the intended destinations by exploiting the side information created due to channel statistics.

Creating common interest packets: In this category, Tx_i exploits the saved analog signals from receivers to deliver its packets to Rx_i. We use Fig. 3 as an example to clarify this point. In Fig. 3, Tx_1 and Tx_2 send out p_1 and p_2, respectively, at the same time to intended receivers. We assume channel state 3 (which discussed in Table I) happens and both receivers obtain linear combinations of p_1 and p_2. We observe that by delivering p_1 to both receivers, Rx_1 receives its desired packet p_1 and Rx_2 removes p_1 from $L_2(p_1, p_2)$ and decodes p_2 easily. As a result, p_1 is considered as a common interest packet and joins $Q_1^{1,2|\phi}$ and it is not needed to retransmit p_2 and p_2 joins Q_2^F. In this case, p_2 is considered as a virtually delivered packet which means p_2 can be decoded by Rx_2 if p_1 is delivered to Rx_2.

Exploiting available side information: In this category, we describe how Tx_i delivers the packet which is received with interference by Rx_i. Assume an example as Fig. 4 to explain its concept. Suppose $\{p_1, p_3, p_5\}$ and $\{p_2, p_4, p_6\}$ are transmitted by Tx_1 and Tx_2 at three time instants which are shown in Fig.4 as case (a), (b) and (c), respectively. Fig. 4(a) describes the case that Rx_1 receives p_1 and Rx_2 obtains $L_1(p_1, p_2)$ and then Tx_2 should retransmit p_2. In Fig. 4(b), p_3 and p_4 are obtained by unintended receivers which means that p_3 and p_4 should be retransmitted. By changing the labels of transmitters and receivers, (c) is similar to case (a) and it is easy to find that Tx_1 needs to retransmit p_5. While these retransmission attempts are obvious to understand, we introduce an efficient delivery option which uses less number of

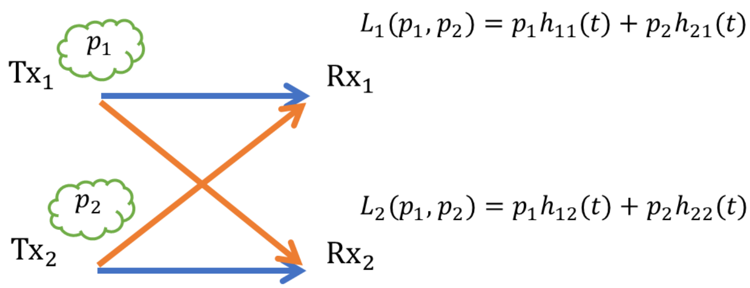

Fig. 3. Rx_1 *and* Rx_2 *receive linear combinations of* p_1 *and* p_2 *at time* t *and by delivering* p_1 *to both receivers,* Rx_1 *receives* p_1 *and* Rx_2 *removes* p_1 *from* $L_2(p_1, p_2)$ *and decodes* p_2 *easily. As a result,* p_1 *is considered as a common interest packet and saves in* $Q_1^{1,2|\phi}$ *and* p_2 *is not needed to retransmit and joins* Q_2^F*.*

retransmission. More precisely, if linear combinations $L_3(p_1, p_3)$ and $L_4(p_4, p_6)$ are delivered to both receivers, all desired packets will be decoded by intended receivers. Suppose $L_3(p_1, p_3)$ and $L_4(p_4, p_6)$ are delivered to Rx_2. Therefore, Rx_2 recovers p_4 by removing p_6 from $L_4(p_4, p_6)$ and then uses p_3 to obtain p_1 from $L_3(p_1, p_3)$ and then plugs p_1 into $L_1(p_1, p_2)$ to decode p_2. Similarly, if $L_3(p_1, p_3)$ and $L_4(p_4, p_6)$ are delivered to Rx_1, it will be easy for Rx_1 to recover p_3 by removing p_1 from $L_3(p_1, p_3)$. Furthermore, Rx_1 uses $L_4(p_4, p_6)$ and p_4 to decode p_6 and then uses p_6 to obtain p_5 from $L_2(p_5, p_6)$.

Although channel coefficients $h_{ij}(t)$ have basic role in reality, in this paper we focus on the impact of interference which described by binary coefficients $c_{ij}(t)$ on our proposed protocol. Therefore, we consider a binary system with binary transmitted packets and all operations will be in the binary field, $\mathbb{F}_2$. Including the real value channel gains includes normalization at the transmitters to guarantee the power constraint [32]. As a result, from now on we exploit XOR combination to describe the linear combination between packets.

Remark 1. *To illustrate the main error in the communication protocol of [31], we consider the example illustrated in Fig. 5 which happens during four time instants* t_1*,* t_2*,* t_3 *and* t_4*. Suppose* Tx_1 *and* Tx_2 *send out* p_1 *and* p_2*, respectively, and only* $c_{11}(t)$ *and* $c_{21}(t)$ *are on at time instant* t_1 *(i.e.* Rx_1 *receives* $p_1 \oplus p_2$*). Similarly, assume* Tx_1 *and* Tx_2 *send out* p_3 *and* p_4*, respectively, and* $c_{22}(t)$ *and* $c_{12}(t)$ *are on at time instant* t_2*. At time instant* t_3*, consider* Tx_1 *and* Tx_2 *transmit* p_5 *and* p_6*, respectively, and* $c_{11}(t)$*,* $c_{12}(t)$ *and* $c_{22}(t)$ *are on. Finally, assume* Tx_1 *and* Tx_2 *send out* p_7 *and* p_8*, respectively, and* $c_{11}(t)$*,* $c_{21}(t)$ *and* $c_{22}(t)$ *are on at time instant* t_4*. According to [31]'s approach,* $p_1 \rightarrow Q_1^F$*,* $p_2 \rightarrow Q_2^{2|1}$*,* $p_3 \rightarrow Q_1^{1|2}$*,* $p_4 \rightarrow Q_2^F$*,* $p_5 \rightarrow Q_1^{2|1}$*,* $p_6 \rightarrow Q_2^F$*,* $p_7 \rightarrow Q_1^F$

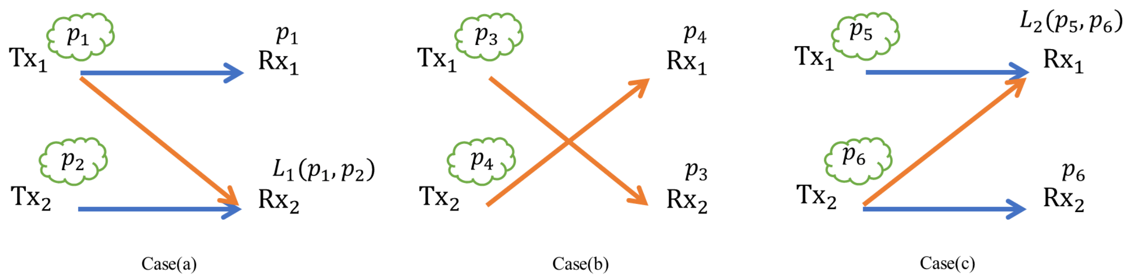

Fig. 4. *Consider* $\{p_1, p_3, p_5\}$ *and* $\{p_2, p_4, p_6\}$ *are available at* Tx_1 *and* Tx_2*, respectively and case(a), (b) and (c) happen at three different time instants. By delivering* $L_3(p_1, p_3)$ *and* $L_4(p_4, p_6)$ *to both receivers,* Tx_1 *and* Tx_2 *are able to decode their desired packets. Therefore,* $\{p_1, p_3\}$ *join* $Q_1^{1,2|\phi}$ *and* $\{p_4, p_6\}$ *save in* $Q_2^{1,2|\phi}$.

and $p_8 \to Q_2^{1|2}$ *and it is sufficient to deliver* $p_2 \oplus p_8$ *and* $p_3 \oplus p_5$ *to both receivers to recover* $\{p_1, p_3, p_5, p_7\}$ *and* $\{p_2, p_4, p_6, p_8\}$ *by* Rx_1 *and* Rx_2*, respectively. However, we observe proposed decoding method does not work properly. For example, if* Rx_1 *receives* $p_2 \oplus p_8$ *and* $p_3 \oplus p_5$*, it will decode* p_3 *from* $p_3 \oplus p_5$ *but there is no way to decode* p_1 *and* p_7 *for* Rx_1*. A similar statement holds for* Rx_2 *which cannot decode* p_4 *and* p_6*. According to our strategy which we will discuss shortly, transmitted packets should join the following queues:* $p_1 \to Q_1^F$, $p_2 \to Q_2^{1,2|\phi}$, $p_3 \to Q_1^{1,2|\phi}$, $p_4 \to Q_2^F$, $p_5 \to Q_1^{2|1}$, $p_6 \to Q_2^F$, $p_7 \to Q_1^F$ *and* $p_8 \to Q_2^{1|2}$*. In this case, it is enough to deliver* p_2 *and* p_3 *to both receivers,* p_5 *to* Rx_2*, and* p_8 *to* Rx_1.

C. Priority Policy

In non-stochastic queue-based transmissions, all packets are available in $Q_i^{i|\phi}$ at time instant $t = 0$ and transmission strategy is divided into two phases [26]. During the first phase, transmitters send their packets and find some combining opportunities between packets in the second phase. However, in stochastic packet arrival, the packets are arriving during communication time and always there are some packets which leave their origin queues and join destination queues. Therefore, It is important to determine which packets have higher priority to move between queues at each time instant. Hence, we assign a priority policy (see Table II) to determine which packets should send before others. Since common interest packets include more information rather than other packets (i.e. they help both receivers to decode their packets) $Q_i^{1,2|\phi}$ has higher priority. $Q_i^{i|\phi}$ contains new arrived packets at Tx_i and occupies second place. Usually, Tx_i should deliver desired packets to Rx_i and some of the times, it is beneficial to deliver the interfering

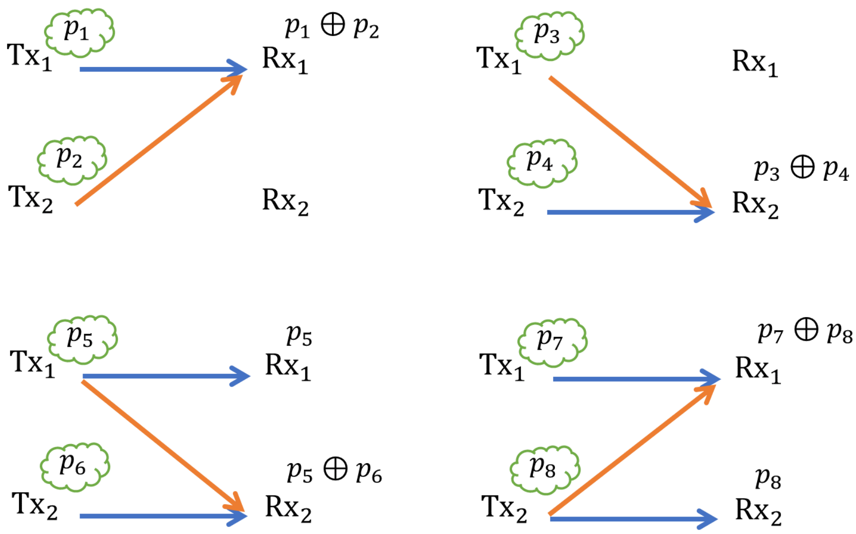

Fig. 5. *An example of decoding problem in [31] when packets are transmitted through four time instants. Although [31] explains that delivering $p_2 \oplus p_8$ and $p_3 \oplus p_5$ to both receivers is sufficient to decode $\{p_1, p_3, p_5, p_7\}$ and $\{p_2, p_4, p_6, p_8\}$ by* Rx_1 *and* Rx_2*, respectively, it does not work properly and (as an example) there is no way to decode p_1 and p_7 for* Rx_1.

packet to $\mathsf{Rx}_{\bar{i}}$. Therefore, we assign third and fourth priority to $Q_i^{i|\bar{i}}$ and $Q_i^{\bar{i}|i}$, respectively. Next place is occupied by $Q_i^{c_1}$ which contains the packets which provide XOR options for transmitters. The main role of $Q_i^{c_1}$ is providing XOR opportunities to create linear combinations of packets that simultaneously deliver packets to both receivers.

TABLE II

PRIORITY POLICY INDICATES WHICH PACKETS SHOULD LEAVE THEIR ORIGIN QUEUES AT TIME INSTANT t

Priority	Queue	Policy
1	$Q_i^{1,2\|\phi}$	If there is a packet in $Q_i^{1,2\|\phi}$
2	$Q_i^{i\|\phi}$	If $Q_i^{1,2\|\phi}$ is empty and there is a packet in $Q_i^{i\|\phi}$
3	$Q_i^{i\|\bar{i}}$	If $Q_i^{1,2\|\phi}$ and $Q_i^{i\|\phi}$ are empty and there is a packet in $Q_i^{i\|\bar{i}}$
4	$Q_i^{\bar{i}\|i}$	If $Q_i^{1,2\|\phi}$, $Q_i^{i\|\phi}$ and $Q_i^{i\|\bar{i}}$ are empty and there is a packet in $Q_i^{\bar{i}\|i}$
5	$Q_i^{c_1}$	If all other queues are empty and only there is a packet in $Q_i^{c_1}$

D. Control Table

Stochastic packet arrival introduces new challenges when compared to the non-stochastic model as we discuss shortly. In this paper, we propose a procedure to determine how packets move between queues in different conditions. In this part, we identify a four-bit sequence as ($\vec{c} = (c_{11}, c_{21}, c_{22}, c_{12})$) to explain the channel state information and name $\vec{c}$ as Situation Number (SN) to make its description easily. Let's give an example to illustrate packet movement rules. Assume, p_1 and $\{p_2, p_3\}$ arrive at Tx_1 and Tx_2, respectively, at time instant $t = 0$ ($p_1 \rightarrow Q_1^{1|\phi}$ and $\{p_2, p_3\} \rightarrow Q_2^{2|\phi}$). Suppose SN-11 happens at time instant $t = 1$ (as Fig. 6), Rx_2 receives a linear combination of p_1 and p_2 as $L(p_1, p_2)$ and Rx_1 does not receive anything. In this case, by delivering p_1 to both receivers, Rx_2 removes p_1 from $L(p_1, p_2)$ and obtains p_2 and Rx_1 receives p_1 and then, $p_1 \rightarrow Q_1^{1,2|\phi}$ and $p_2 \rightarrow Q_2^F$ at the end of time instant $t = 1$. According to Table II, $Q_1^{1,2|\phi}$ and $Q_2^{2|\phi}$ which contain p_1 and p_3, respectively, are considered as the origin queues at $t = 2$ and face with 16 different situations for packet movement (we explain packet movement rules for another case when $Q_1^{1|\phi}, Q_2^{c_1}$ are origin queues in [33]). Table III summarizes the control table for $Q_1^{1,2|\phi}$ and $Q_2^{2|\phi}$ as the origin queues at $t = 2$ and it can be illustrated as follows:

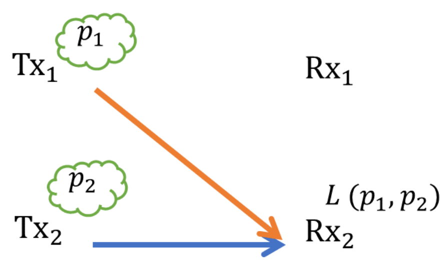

Fig. 6. Tx_1 *and* Tx_2 *transmit their desired packets to intented receivers and SN-11 happens at* $t = 1$*. Therefore,* Rx_2 *receives a linear combination of* p_1 *and* p_2 *and* Rx_1 *does not receive anything and it leads to* $p_1 \rightarrow Q_1^{1,2|\phi}$ *and* $p_2 \rightarrow Q_2^F$.

$\diamond$ SN-1: At this time, both receivers get linear combinations of p_1 and p_3 from both transmitters. According to existing packets (i.e. $L_1(p_1, p_3)$ and $L_2(p_1, p_3)$), it is enough to deliver one of p_1 or p_3 to Rx_1 and Rx_2. Therefore, they can use the packets as communication packets and $p_1 \rightarrow Q_1^{c_1}$ and $p_3 \rightarrow Q_2^{c_1}$. It is important to note that $Q_i^{c_1}, i \in \{1, 2\}$ are used to make the XOR combinations which will be described later.

$\diamond$ SN-2: In this SN, Rx_1 and Rx_2 receive p_1 and $L_3(p_1, p_3)$), respectively. Therefore, Rx_1 can decode p_1 easily and Rx_2 needs p_1 to decode both p_1 and p_3. Hence, $Q_1^{2|1}$ and Q_2^F are the destination queues for p_1 and p_3, respectively.

$\diamond$ SN-3: In this case, Rx_2 receives p_3 and decodes it easily but both receivers still need to obtain p_1. As a result, $p_1 \rightarrow Q_1^{1,2|\phi}$ and $p_3 \rightarrow Q_2^F$.

TABLE III

DETERMINE PACKET MOVEMENT RULES BETWEEN QUEUES WHEN PACKETS p_1 AND p_3 ARE AVAILABLE IN $Q_1^{1,2|\phi}, Q_2^{2|\phi}$ FOR $\frac{(3-\sqrt{5})}{2} \leq \delta < 0.5$

SN	$\vec{c}$	Existing Packets	Destination Queues	SN	$\vec{c}$	Existing Packets	Destination Queues
1	(1,1,1,1)	$\begin{cases} L_1(p_1,p_3) \\ L_2(p_1,p_3) \end{cases}$	$Q_1^{c_1}, Q_2^{c_1}$	9	(0,0,1,0)	$\begin{cases} \phi \\ p_3 \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^F$
2	(1,0,1,1)	$\begin{cases} p_1 \\ L_3(p_1,p_3) \end{cases}$	$Q_1^{2\|1}, Q_2^F$	10	(0,1,1,0)	$\begin{cases} p_3 \\ p_3 \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^F$
3	(1,1,1,0)	$\begin{cases} L_4(p_1,p_3) \\ p_3 \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^F$	11	(0,0,1,1)	$\begin{cases} \phi \\ L_7(p_1,p_3) \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^F$
4	(1,0,1,0)	$\begin{cases} p_1 \\ p_3 \end{cases}$	$Q_1^{2\|1}, Q_2^F$	12	(0,1,1,1)	$\begin{cases} p_3 \\ L_8(p_1,p_3) \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^F$
5	(1,0,0,0)	$\begin{cases} p_1 \\ \phi \end{cases}$	$Q_1^{2\|1}, Q_2^{2\|\phi}$	13	(0,1,0,0)	$\begin{cases} p_3 \\ \phi \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^{2\|1}$
6	(1,0,0,1)	$\begin{cases} p_1 \\ p_1 \end{cases}$	$Q_1^F, Q_2^{2\|\phi}$	14	(0,0,0,1)	$\begin{cases} \phi \\ p_1 \end{cases}$	$Q_1^{1\|2}, Q_2^{2\|\phi}$
7	(1,1,0,0)	$\begin{cases} L_5(p_1,p_3) \\ \phi \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^{2\|1}$	15	(0,1,0,1)	$\begin{cases} p_3 \\ p_1 \end{cases}$	$Q_1^{1\|2}, Q_2^{2\|1}$
8	(1,1,0,1)	$\begin{cases} L_6(p_1,p_3) \\ p_1 \end{cases}$	$Q_1^F, Q_2^{1,2\|\phi}$	16	(0,0,0,0)	$\begin{cases} \phi \\ \phi \end{cases}$	$Q_1^{1,2\|\phi}, Q_2^{2\|\phi}$

$\diamond$ SN-4: Both receivers obtain packets from direct channels and p_1 and p_3 are the existing packets at Rx_1 and Rx_2, respectively. As a result, Rx_1 does not need to receive any other packets but Rx_2 still needs p_1 and $Q_1^{2|1}$ and Q_2^F are the destination queues for p_1 and p_3, respectively.

$\diamond$ SN-5 and SN-6: In these SNs, the existing packets consist of only Tx_1'packet. Therefore, p_3 stays at its origin queue and then $p_3 \rightarrow Q_2^{2|\phi}$. In SN-5, Rx_2 still needs p_1 and $p_1 \rightarrow Q_1^{2|1}$ and in SN-6 it is not needed to retransmit p_1 and $p_1 \rightarrow Q_1^F$.

$\diamond$ SN-7: At this time, Rx_1 receives $L_5(p_1, p_3)$ and Rx_2 does not receive anything. Thus, by delivering p_1 to both receivers and p_3 to Rx_2, both receivers can recover their desired packets easily then $Q_1^{1,2|\phi}$ and $Q_2^{2|1}$ are the destination queues for p_1 and p_3, respectively.

$\diamond$ SN-8: In this SN, Rx_1 and Rx_2 receive $L_6(p_1, p_3)$ and p_1, respectively. Hence, it is required to retransmit p_3 to both receivers and $p_1 \rightarrow Q_1^F$ and $p_3 \rightarrow Q_2^{1,2|\phi}$.

$\diamond$ SN-9, SN-10, SN-11 and SN-12: In these situations, as p_1 is not available in existing packets list which are belong to Rx_1, p_1 should be delivered to Rx_1. Moreover, Rx_2 receives p_3 in SN-

9 and SN-10 and $L_7(p_1, p_3)$ and $L_8(p_1, p_3)$ in SN-11 and SN-12, respectively. Therefore, it is needed to retransmit p_1 to Rx$_2$ and $p_1 \to Q_1^{1,2|\phi}$ and $p_3 \to Q_2^F$.

⋄ SN-13: In this SN, only Rx$_2$ receives p_3 and then $p_3 \to Q_2^{2|1}$ while p_1 stays at its origin queue.

⋄ SN-14 and SN-15: In these SNs, Rx$_2$ receives p_1 and $p_1 \to Q_1^{1|2}$. Furthermore, p_3 is not available in existing packets in SN-14 and $p_3 \to Q_2^{2|\phi}$. However, Rx$_1$ receives p_3 in SN-15 and $p_3 \to Q_2^{2|1}$.

⋄ SN-16: At this time, both receivers do not obtain anything and p_1 and p_3 stay at their origin queues and $p_1 \to Q_1^{1,2|\phi}$ and $p_3 \to Q_2^{2|\phi}$.

E. XOR Combining models

In this part, we discuss delivering options to construct common interest packets based on the constraints explained in section VI-B. We show that three models of XOR combination exist in $\frac{(3-\sqrt{5})}{2} \leq \delta < 0.5$ as below:

• **model-1 (XOR between $\mathbf{Q_i^{c_1}}$ and $\mathbf{Q_i^{i|\bar{i}}}$)**:

Suppose SN-1 and SN-15 happen in two time instants (as in Fig. 7). According to packet movement rules in section VI-D, it is easy to find that $p_1 \to Q_1^{c_1}$, $p_2 \to Q_2^{c_1}$, $p_3 \to Q_1^{1|2}$ and $p_4 \to Q_2^{2|1}$. In this part, we show that delivering $p_1 \oplus p_3$ and $p_2 \oplus p_4$ to both receivers leads to deliver the desired packets to corresponding receivers. We describe the details of decoding procedure of p_2 and p_4 by Rx$_2$ as an example. If $p_1 \oplus p_3$ and $p_2 \oplus p_4$ are delivered to Rx$_2$, it can use p_3 to decode p_1 from $p_1 \oplus p_3$ easily and then employs p_1 to obtain p_2 from $p_1 \oplus p_2$ and use p_2 to decode p_4 from $p_2 \oplus p_4$. Based on the symmetry, it is easy to find the decoding approach for Rx$_1$. Hence, combining $Q_i^{c_1}$ and $Q_i^{i|\bar{i}}$ is a XOR opportunity in our network. At this time, we can omit the packets from $Q_i^{c_1}$ and $Q_i^{i|\bar{i}}$ and put XOR values of them in $Q_i^{1,2|\phi}, i \in \{1, 2\}$.

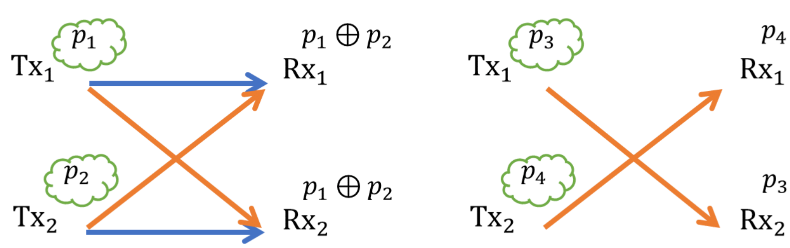

Fig. 7. *Consider SN-1 and SN-15 happen at two different time instants. It is enough to deliver $p_1 \oplus p_3$ and $p_2 \oplus p_4$ to both receivers. As a result, combining $Q_i^{c_1}$ and $Q_i^{i|\bar{i}}$ is a XOR opportunity in our network as XOR model-1.*

• **model-2 (XOR between $\mathbf{Q_i^{c_1}}$ and $\mathbf{Q_i^{\bar{i}|i}}$)**:

Assume SN-1, SN-2 and SN-3 occur at three time instants and $p_1 \rightarrow Q_1^{c_1}$, $p_2 \rightarrow Q_2^{c_1}$, $p_3 \rightarrow Q_1^{2|1}$, $p_4 \rightarrow Q_2^F$ $p_5 \rightarrow Q_1^F$, $p_6 \rightarrow Q_2^{1|2}$ based on packet movement rules between queues. We indicate that, it is enough to deliver $p_1 \oplus p_3$ and $p_2 \oplus p_6$ to both receivers. At this time, we describe decoding procedure for Rx_2 as an instance and it is similar for Rx_1. According to Fig. 8, Rx_2 recovers p_2 by removing p_6 from $p_2 \oplus p_6$ and then uses p_2 to decode p_1 from linear combination in SN-1. Furthermore, Rx_2 obtains p_3 from $p_1 \oplus p_3$ and uses p_3 to decode p_4 from linear combination in SN-2. As a result, $Q_i^{c_1}$ and $Q_i^{\bar{i}|i}$ is a model of XOR combination and packets leave $Q_i^{c_1}$ and $Q_i^{\bar{i}|i}$ and XOR values of them move to $Q_i^{1,2|\phi}, i \in \{1, 2\}$.

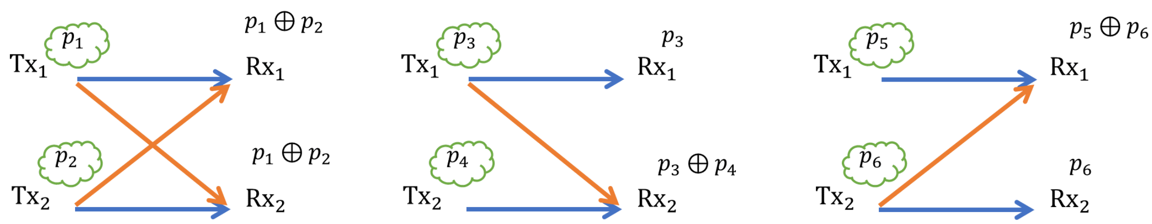

Fig. 8. *Suppose SN-1, SN-2 and SN-3 occur at three different time instants. We show that by delivering $p_1 \oplus p_3$ and $p_2 \oplus p_6$ to both receivers, $\mathsf{Rx}_i, i \in \{1, 2\}$ can decode its desired packets. Therefore, XOR combination between $Q_i^{c_1}$ and $Q_i^{\bar{i}|i}$ creates a XOR opportunity in our proposed scheme as XOR model-2.*

• **model-3 (XOR between $\mathbf{Q_i^{i|\bar{i}}}$ and $\mathbf{Q_i^{\bar{i}|i}}$)**:

We describe this XOR model through an example of decoding procedure for Rx_1. Imagine SN-2 and SN-14 happen at two time instants (as in Fig. 9). According to packet movement rules, we find that $p_1 \rightarrow Q_1^{2|1}$, $p_2 \rightarrow Q_2^F$, $p_3 \rightarrow Q_1^{1|2}$ and $p_4 \rightarrow Q_2^{2|\phi}$. We show that by delivering $p_1 \oplus p_3$ to both receivers, Rx_1 obtains p_3 by removing p_1 from $p_1 \oplus p_3$ and Rx_2 obtains p_1 from $p_1 \oplus p_3$ and uses p_1 to decode p_2 from $p_1 \oplus p_2$. Thus, $Q_i^{i|\bar{i}}$ and $Q_i^{\bar{i}|i}$ can be considered as a model of XOR combination and p_1 and p_3 leave $Q_1^{1|2}$ and $Q_1^{2|1}$ and XOR value of them join $Q_1^{1,2|\phi}$. Similar statement holds for Rx_2 when SN-3 and SN-13 happen at two time instants.

F. Pseudo code of protocol

Until now, we described the details of our proposed protocol through definition of a priority policy to determine which packets have higher priority than others to send and packet movement rules between queues in Table III. Now it is essential to describe how we can implement our protocol. Thus, we use a pseudo code in Table IV to illustrate our proposed protocol.

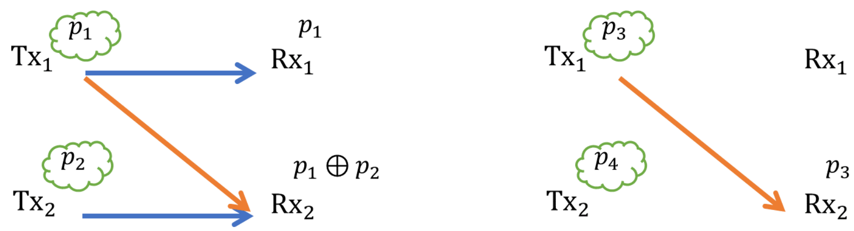

Fig. 9. *Assume SN-2 and SN-14 happen at two time instants. In this case, by delivering $p_1 \oplus p_3$ to both receivers,* Rx_1 *obtains p_1 and p_3 and it makes a type of XOR combination as XOR model-3 between $Q_i^{i|\bar{i}}$ and $Q_i^{\bar{i}|i}$.*

TABLE IV

TRANSMISSION PROTOCOL PSEUDO CODE FOR $\frac{(3-\sqrt{5})}{2} \leq \delta < 0.5$

Step	Code
1	Save new arrived packets in $Q_i^{i\|\phi}$.
2	Check which queues have higher priority based on Table II.
3	According to $c_{ij}(t-1)$, transfer packets from highest prior queues based on Table III.
4	Find XOR opportunities from model-1, 2 and 3.
5	Remove packets (which participate in XOR combination) from their origin queues and join to $Q_i^{1,2\|\phi}$

VII. SIMULATION

A. Throughput Region vs. Capacity Region

In this section, we compare stable throughput region of our proposed scheme for stochastic arrivals with non-stochastic capacity region of [26]. We assume packets arrive at Tx_i according to a Poisson (λ_i) distribution, $i = 1, 2$, each transmitter sends one packet per time instant and all links are distributed as i.i.d. Bernoulli random variables across time and users. On the other hand, $c_{ij}(t) \sim \mathcal{B}(1-\delta)$ where $0 \leq \delta \leq 1$ is the erasure probability of the channel between Tx_i and Rx_j. Furthermore, we calculate the stable throughput region during a final window which equals a fraction of the communication time and set acceptable error ϵ equal to 1 percent.

Our simulations show that our proposed transmission protocol achieves the information-theoretic outer-bounds for this problem as presented in [26]. Fig. 10 depicts the stable throughput regions of ordinary procedure and modified+ordinary procedure when $\delta = 0.4$, $n = 20k$, window time $= 3k$ and $\epsilon = 1\%$ and compares them with information-theoretic boundary and shows the maximum sum-rate corner point "C" is achievable through the proposed scheme. In Appendix C, we show the other two corner points "A" and "B" in Fig. 10 are also achievable by modifying our scheme to accommodate $\lambda_1 \neq \lambda_2$. Moreover, Fig. 11 shows the stable throughput region for $\epsilon = 0.1\%$ when other parameters are identical to those in Fig. 10. As we observe, the overall shape of the stable throughput region remains similar.

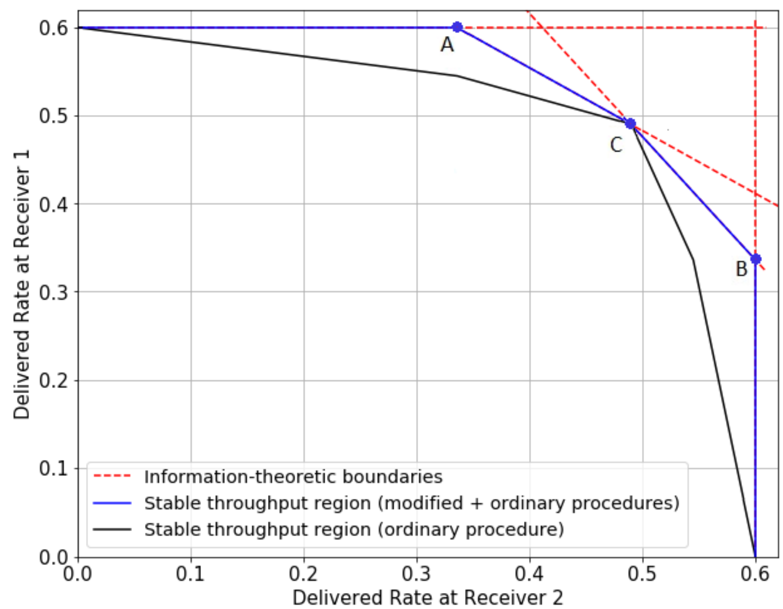

Fig. 10. *Comparing the stable throughput regions of ordinary procedure, modified+ordinary procedures and information-theoretic capacity region when $\delta = 0.4$, $n = 20k$, window time $= 3k$ and $\epsilon = 1\%$.*

B. Coding Complexity

The power of wireless is in multi-casting, and thus, the key idea behind optimal feedback-based transmission strategies for BC and distributed interference channels is to create as many linear combinations as possible in order to simultaneously satisfy multiple users. However, increasing the number of XOR operations raises encoding and decoding complexity. Therefore, we analyze the number of XOR operations in our proposed protocol to gain a deeper understanding of this tradeoff. Fig. 12 describes a histogram of the number of packets which are delivered through XOR operations for the maximum sum-rate corner point "C" in Fig. 10 when $\delta = 0.4$, $n = 10k$ and window time $= 1.5k$. We observe that about 42 percent of total packets are delivered through the XOR combinations. If we use an encoder-decoder mechanism which does not allow linear combinations with more than 3 participating packets, about 27 percent of total packets will be affected based on Fig. 12. It is thus an interesting future direction to characterize the stable throughput region of wireless networks with limitations on encoding and decoding complexity.

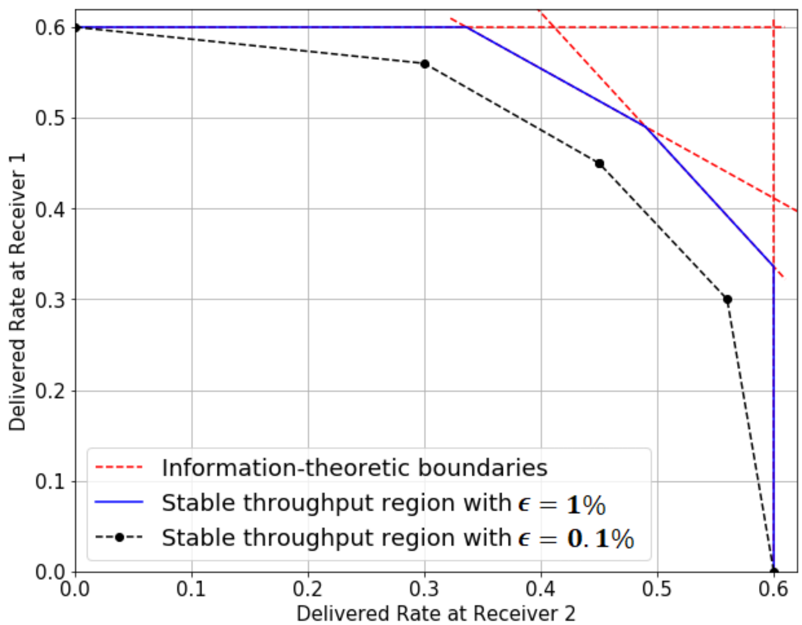

Fig. 11. *Stable throughput regions of proposed procedures for different values of ϵ (1% and 0.1%) versus information-theoretic capacity region when $\delta = 0.4$, $n = 20k$ and window time $= 3k$.*

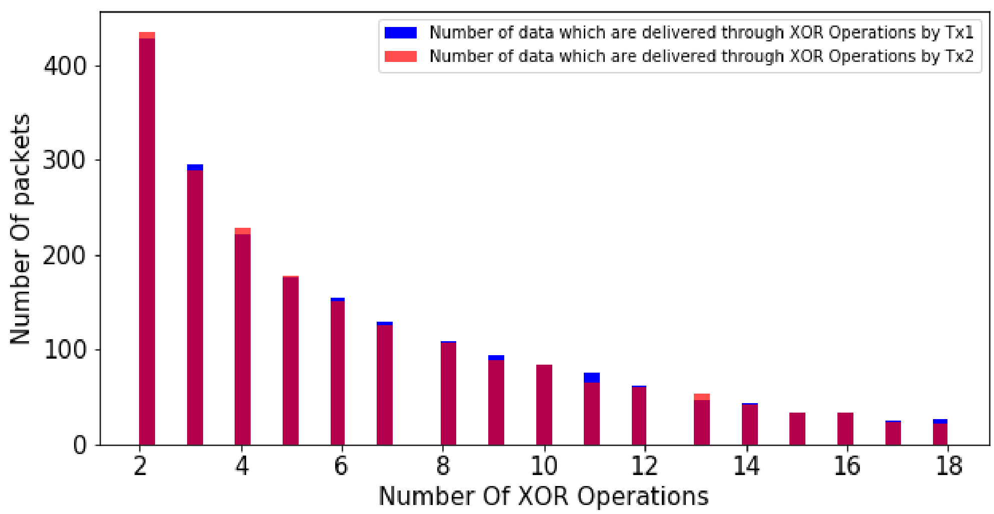

Fig. 12. *Number of packets versus number of XOR operations when $\delta = 0.4$, $n = 10k$, window time $= 1.5k$ and $\epsilon = 1\%$.*

C. *Packet Lifetime (delivery delay)*

In general, some packets may have delivery deadline that transmission protocols must take into account. To evaluate this delay for our protocol, we define the lifetime of a packet as the time interval between its arrival to the system (joins $Q_i^{i|\phi}$) and its successful delivery time (joins

Q_i^F). Fig. 13 depicts the mean and the max values of packet lifetime for maximum sum-rate corner point "C" of Fig. 10 when $\delta = 0.4$ and $n \in [100, 50k]$ and shows the mean and the max values of packet lifetime behaves as the square root of the total communication time and fit with $0.84\sqrt{0.8n} + 4.31$ and $3.32\sqrt{0.38n} + 43.75$, respectively.

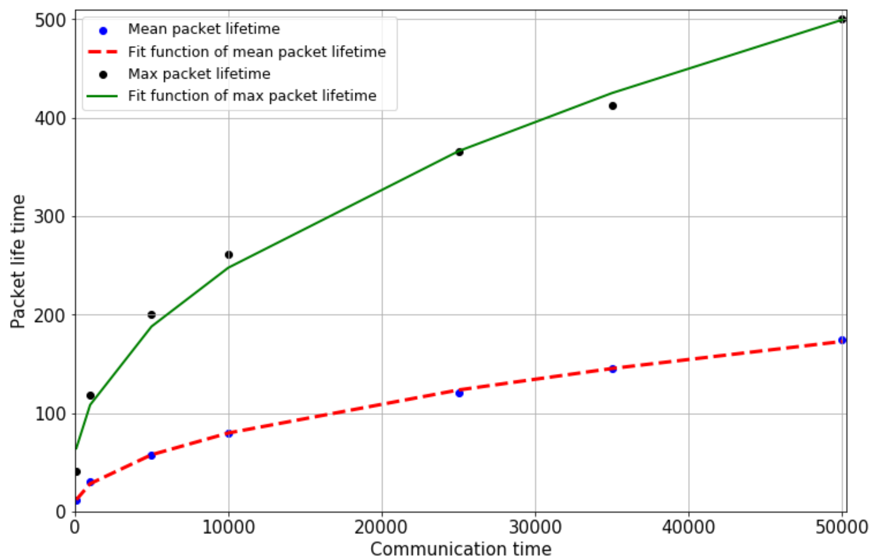

Fig. 13. *Mean and max values of* Tx_1*'s packet lifetime and* $0.84\sqrt{0.8n}+4.31$ *and* $3.32\sqrt{0.38n}+43.75$ *as their fitting curves, respectively, versus communication time for maximum sum-rate corner point "C" in Fig. 10 when* $\delta = 0.4$ *and* $n \in [100, 50k]$.

VIII. Conclusion

We described stable throughput region of distributed interference channel with two pairs of transmitters and receivers and show that it is identical with information-theoretic capacity region. We presented different transmission protocols which make all corner points of stable throughput region achievable. We defined priority and control tables to determine which packet should send at each time and manage the packets between queues in stochastic model, respectively. Moreover, we show life time of packets follow square root pattern of communication time and investigated how many of XOR operations are needed to deliver all packets to the receivers. However, if we employ a limited computing system which can not solve all XOR operations, it will not clear what happen on XORed packets and it will be an interesting topic for future works to pay attention what should have done to recover the XORed packets in limited computing systems.

Appendix A
Ordinary procedure for $0 \leq \delta < \frac{(3-\sqrt{5})}{2}$

In this appendix, we investigate ordinary procedure for $0 \leq \delta < \frac{(3-\sqrt{5})}{2}$. In this condition, types of the queues, priority policy between the queues, XOR combination models and all SNs in control table except SN-1 are same as $\frac{(3-\sqrt{5})}{2} \leq \delta < 0.5$. More precisely, SN-1 where both receivers obtain packets from both transmitters occurs more than other situations (i.e. SN-1 in Table III has higher chance to happen than others). However, if we employ the packet movement rules in Table III, the number of remained packets in $Q_i^{c_1}, i \in \{1,2\}$ at the end of communication time will grow up. To solve this problem, we separate the packets of SN-1 to two categories as follow: At each time instant t, one of the received packets is considered as common interest packet and goes to $Q_i^{1,2|\phi}$ and one of them is considered as virtual dilivered packet and joins $Q_i^F, i \in \{1,2\}$. We execute a random function with probability = 0.5 to divide the packets.

Appendix B
Ordinary procedure for $0.5 \leq \delta \leq 1$

In this appendix, we analyze ordinary procedure for $0.5 \leq \delta \leq 1$. In this condition, we identify a new queue as $Q_i^{INT}, i \in \{1,2\}$ to make a retransmission chance for the SNs which are not participated in XOR combination models in section VI-D. Therefore, we replace the $Q_i^{1,2|\phi}$ with $Q_i^{INT}, i \in \{1,2\}$. In this case, since the packets in Q_i^{INT} have to be delivered to both receivers, we consider Q_i^{INT} same as $Q_i^{1,2|\phi}$ and assign the second level of priority to Q_i^{INT} (see Table V). It means, if there is not any packet in $Q_i^{1,2|\phi}$, transmitter i will send the packets in Q_i^{INT}.

TABLE V

PRIORITY POLICY INDICATES WHICH PACKETS SHOULD LEAVE THEIR ORIGIN QUEUES AT TIME INSTANT t FORE THE CASE $0.5 \leq \delta \leq 1$

Priority	Queue	Policy
1	$Q_i^{1,2\|\phi}$	If there is a packet in $Q_i^{1,2\|\phi}$
2	Q_i^{INT}	If $Q_i^{1,2\|\phi}$ is empty and there is a packet in Q_i^{INT}
3	$Q_i^{i\|\phi}$	If $Q_i^{1,2\|\phi}$ and Q_i^{INT} are empty and there is a packet in $Q_i^{i\|\phi}$
4	$Q_i^{i\|\bar{i}}$	If $Q_i^{1,2\|\phi}$, Q_i^{INT} and $Q_i^{i\|\phi}$ are empty and there is a packet in $Q_i^{i\|\bar{i}}$
5	$Q_i^{\bar{i}\|i}$	If $Q_i^{1,2\|\phi}$, Q_i^{INT}, $Q_i^{i\|\phi}$ and $Q_i^{i\|\bar{i}}$ are empty and there is a packet in $Q_i^{\bar{i}\|i}$
6	$Q_i^{c_1}$	If all other queues are empty and only there is a packet in $Q_i^{c_1}$

In this part, we demonstrate the control table of packet movement for $0.5 \leq \delta \leq 1$ through an example. Suppose p_1 and $\{p_2, p_3\}$ arrive at Tx_1 and Tx_2, respectively ($p_1 \to Q_1^{1|\phi}$ and $\{p_2, p_3\} \to Q_2^{2|\phi}$), and SN-11 happens at $t = 1$. According to packet movement rules in Section VI-D, $p_1 \to Q_1^{INT}$ and $p_2 \to Q_2^{F}$ at the end of time instant $t = 1$. As a result, Q_1^{INT} and $Q_2^{2|\phi}$ are considered as the origin queues at the beginning of time instant $t = 2$ and contain p_1 and p_3, respectively. Table VI describes packet movement rules for p_1 and p_3 in 16 different SNs at $t = 2$ which can be explained as follows:

TABLE VI

DETERMINE PACKET MOVEMENT RULES BETWEEN QUEUES WHEN PACKETS p_1 AND p_3 ARE AVAILABLE IN $Q_1^{INT}, Q_2^{2|\phi}$ FOR $0.5 \leq \delta \leq 1$

SN	$\vec{c}$	Existing Packets	Destination Queues	SN	$\vec{c}$	Existing Packets	Destination Queues
1	(1,1,1,1)	$\begin{cases} L_9(p_1,p_3) \\ L_{10}(p_1,p_3) \end{cases}$	$Q_1^{c_1}, Q_2^{c_1}$	9	(0,0,1,0)	$\begin{cases} \phi \\ p_3 \end{cases}$	Q_1^{INT}, Q_2^{F}
2	(1,0,1,1)	$\begin{cases} p_1 \\ L_{11}(p_1,p_3) \end{cases}$	$Q_1^{2\|1}, Q_2^{F}$	10	(0,1,1,0)	$\begin{cases} p_3 \\ p_3 \end{cases}$	Q_1^{INT}, Q_2^{F}
3	(1,1,1,0)	$\begin{cases} L_{12}(p_1,p_3) \\ p_3 \end{cases}$	Q_1^{INT}, Q_2^{F}	11	(0,0,1,1)	$\begin{cases} \phi \\ L_{15}(p_1,p_3) \end{cases}$	Q_1^{INT}, Q_2^{F}
4	(1,0,1,0)	$\begin{cases} p_1 \\ p_3 \end{cases}$	$Q_1^{2\|1}, Q_2^{F}$	12	(0,1,1,1)	$\begin{cases} p_3 \\ L_{16}(p_1,p_3) \end{cases}$	Q_1^{INT}, Q_2^{F}
5	(1,0,0,0)	$\begin{cases} p_1 \\ \phi \end{cases}$	$Q_1^{2\|1}, Q_2^{2\|\phi}$	13	(0,1,0,0)	$\begin{cases} p_3 \\ \phi \end{cases}$	$Q_1^{INT}, Q_2^{2\|1}$
6	(1,0,0,1)	$\begin{cases} p_1 \\ p_1 \end{cases}$	$Q_1^{F}, Q_2^{2\|\phi}$	14	(0,0,0,1)	$\begin{cases} \phi \\ p_1 \end{cases}$	$Q_1^{1\|2}, Q_2^{2\|\phi}$
7	(1,1,0,0)	$\begin{cases} L_{13}(p_1,p_3) \\ \phi \end{cases}$	$Q_1^{c_1}, Q_2^{INT}$	15	(0,1,0,1)	$\begin{cases} p_3 \\ p_1 \end{cases}$	$Q_1^{1\|2}, Q_2^{2\|1}$
8	(1,1,0,1)	$\begin{cases} L_{14}(p_1,p_3) \\ p_1 \end{cases}$	Q_1^{F}, Q_2^{INT}	16	(0,0,0,0)	$\begin{cases} \phi \\ \phi \end{cases}$	$Q_1^{INT}, Q_2^{2\|\phi}$

$\diamond$ SN-1, 2, 3, 4, 5 and 6: The packet movement rules of these SNs are similar to Table III.

$\diamond$ SN-7: At this time, the existing packets consist of $L_{13}(p_1, p_3)$ for Rx_1 and ϕ for Rx_2. In this SN, Rx_1 needs one of p_1 or p_3 to recover p_1 form $L_{13}(p_1, p_3)$ and we consider a type of communication packet for receiving packet with interference at Rx_1. It means, Tx_1 retransmits p_1 to both receivers while p_1 has an opportunity to combine with packets in $Q_1^{1|2}$ or $Q_1^{2|1}$. Therefore, $Q_1^{c_1}$ is destination queue for p_1 to provide XOR option at Rx_1 and $p_3 \to Q_2^{INT}$.

$\diamond$ SN-8: In this SN, Rx_1 receives $L_{14}(p_1, p_3)$ and Rx_2 obtains unintended packet. Thus, Rx_1 needs p_3 to recover p_1 form $L_{14}(p_1, p_3)$ and then $p_1 \to Q_1^F$ and $p_3 \to Q_2^{INT}$.
$\diamond$ SN-9 and SN-10: In these situations, only Tx_2's packet is delivered and Rx_1 does not obtain any packet of Tx_1. As a result, $p_3 \to Q_2^F$ and p_1 stays at the its origin queue.
$\diamond$ SN-11 and SN-12: In these SNs, Rx_2 receives a linear combination of p_1 and p_3. As a result, Rx_2 can decode p_3 by using either p_1 or p_3 and then $p_1 \to Q_1^{INT}$ and $p_3 \to Q_2^F$
$\diamond$ SN-13, SN-14 and SN-15: In these SNs, $\mathsf{Rx}_{\bar{i}}$ receives packet from $\mathsf{Tx}_i (i, \bar{i} = 1, 2)$ and if Rx_1 receives a packet then $p_1 \to Q_1^{1|2}$ otherwise p_1 stays at its origin queue and it is similar for Rx_2.
$\diamond$ SN-16: In this SN, both receivers do not obtain anything. Therefore, p_1 and p_3 stay at their origin queues.

APPENDIX C
MODIFIED PROCEDURE FOR BOUNDARY POINTS

In this appendix, we explain modified procedure for boundary points between "C"and "B" in Fig. 10 where $\lambda_2 > \lambda_1$ and due to the symmetric feature, the procedures for boundary points between "A" and "C" are similar. We identify a new queue as $Q_i^{OP}, i \in \{1, 2\}$ to describe the packets which are received with interference at unintended receivers. Our proposed scheme gives a chance of retransmission through Q_i^{OP} to solve this type of interference. In modified procedure, $\lambda_2 > \lambda_1$ (*i.e.* Tx_2 should deliver more packets than Tx_1) and we modify the priority policy between queues in Table V by replacing $Q_i^{\bar{i}|i}$ with Q_i^{OP} at level fifth. Control table in modified procedure is similar to the control table for $0.5 \leq \delta \leq 1$ except SN-1, 3, 7 and SN-8. To describe packet movement rules, we divide boundary points to two categories as follows:

Corner point "B": It is the extreme boundary point where value of $(\lambda_2 - \lambda_1)$ is maximum. We change the packet movement rules in SN-1, 3 and SN-7 in order to make more XOR options regarding to deliver more packets from Tx_2 than Tx_1. The control table for corner point "B" of Fig. 10 is shown in Table VII.

Let's give an example to illustrate packet movement rules in control table for Corner point "B". Suppose same scenario in Appendix B happens and p_1 and p_3 are available in Q_1^{INT} and $Q_2^{2|\phi}$ at $t = 2$. According to shadowing coefficients, the origin queues will deal with 16 SNs which are described in Table VII as follows:

TABLE VII

DETERMINE PACKET MOVEMENT RULES BETWEEN QUEUES WHEN PACKETS p_1 AND p_3 ARE AVAILABLE IN $Q_1^{INT}, Q_2^{2|\phi}$ FOR CORNER POINT "B" IN FIG. 10

SN	$\vec{c}$	Existing Packets	Destination Queues	SN	$\vec{c}$	Existing Packets	Destination Queues
1	(1,1,1,1)	$\begin{cases} L_{17}(p_1,p_3) \\ L_{18}(p_1,p_3) \end{cases}$	Q_1^{INT}, Q_2^F	9	(0,0,1,0)	$\begin{cases} \phi \\ p_3 \end{cases}$	Q_1^{INT}, Q_2^F
2	(1,0,1,1)	$\begin{cases} p_1 \\ L_{19}(p_1,p_3) \end{cases}$	Q_1^{OP}, Q_2^F	10	(0,1,1,0)	$\begin{cases} p_3 \\ p_3 \end{cases}$	Q_1^{INT}, Q_2^F
3	(1,1,1,0)	$\begin{cases} L_{20}(p_1,p_3) \\ p_3 \end{cases}$	Q_1^{OP}, Q_2^{OP}	11	(0,0,1,1)	$\begin{cases} \phi \\ L_{23}(p_1,p_3) \end{cases}$	Q_1^{INT}, Q_2^F
4	(1,0,1,0)	$\begin{cases} p_1 \\ p_3 \end{cases}$	Q_1^{OP}, Q_2^F	12	(0,1,1,1)	$\begin{cases} p_3 \\ L_{24}(p_1,p_3) \end{cases}$	Q_1^{INT}, Q_2^F
5	(1,0,0,0)	$\begin{cases} p_1 \\ \phi \end{cases}$	$Q_1^{OP}, Q_2^{2\|\phi}$	13	(0,1,0,0)	$\begin{cases} p_3 \\ \phi \end{cases}$	$Q_1^{INT}, Q_2^{2\|1}$
6	(1,0,0,1)	$\begin{cases} p_1 \\ p_1 \end{cases}$	$Q_1^F, Q_2^{2\|\phi}$	14	(0,0,0,1)	$\begin{cases} \phi \\ p_1 \end{cases}$	$Q_1^{1\|2}, Q_2^{2\|\phi}$
7	(1,1,0,0)	$\begin{cases} L_{21}(p_1,p_3) \\ \phi \end{cases}$	Q_1^{OP}, Q_2^{INT}	15	(0,1,0,1)	$\begin{cases} p_3 \\ p_1 \end{cases}$	$Q_1^{1\|2}, Q_2^{2\|1}$
8	(1,1,0,1)	$\begin{cases} L_{22}(p_1,p_3) \\ p_1 \end{cases}$	Q_1^F, Q_2^{INT}	16	(0,0,0,0)	$\begin{cases} \phi \\ \phi \end{cases}$	$Q_1^{INT}, Q_2^{2\|\phi}$

◇ SN-1: At this time, both receivers obtain linear combinations from Tx_1 and Tx_2. Since $\lambda_2 > \lambda_1$, both receivers can decode their packets by using p_1. Therefore, Q_1^{INT} and Q_2^F are destination queues for p_1 and p_3, respectively.

◇ SN-3: In this SN, Rx_1 and Rx_2 receive linear combination of transmitted packets and p_3, respectively, and Rx_1 can decode p_1 by using either p_1 or p_3. To make more XOR options p_3 is selected to deliver to Rx_1. Therefore, $p_1 \rightarrow Q_1^{OP}$ and $p_3 \rightarrow Q_2^{OP}$.

◇ SN-2, 4, 5 , 6, 8, 9, 10, 11, 12, 13, 14, 15 and SN-16: According to Appendix B, it is easy to find the destination queues in these SNs.

Boundary points between "B" and "C": In these points, the difference between λ_2 and λ_1 is less than what is in corner point "B". Therefore, we change our goal to create as many XOR combinations as possible. As a result, we modify SN-1 and SN-8 in Table VII to make more XOR opportunities instead of delivering packet to Rx_1 and other SNs are the same. Packet movement rules between queues for boundary points between "B" and "C" in SN-1 and SN-8 can be described as follows:

◇ SN-1: This SN is similar to SN-1 in Table VI.

◇ SN-8: In this SN, since Rx_1 receives a linear combination of transmitted packets ($L_{22}(p_1, p_3)$), both of p_1 and p_3 can be used by Rx_1 to recover p_1. At this time, p_1 is chosen to deliver to Rx_1 in order to make more XOR options and $p_1 \to Q_1^{1|2}$. Furthermore, Rx_2 receives p_1 and needs to receive p_3. As a result, $p_3 \to Q_2^{2|1}$.

References

[1] M. A. Maddah-Ali, A. K. Khandani, and A. S. Motahari, *Communication over X channel: Signalling and multiplexing gain*. Citeseer, 2006.

[2] V. R. Cadambe and S. A. Jafar, "Interference alignment and spatial degrees of freedom for the k user interference channel," in *2008 IEEE International Conference on Communications*. IEEE, 2008, pp. 971–975.

[3] B. Nazer, M. Gastpar, S. A. Jafar, and S. Vishwanath, "Ergodic interference alignment," *IEEE Transactions on Information Theory*, vol. 58, no. 10, pp. 6355–6371, 2012.

[4] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *IEEE GLOBECOM 2008-2008 IEEE Global Telecommunications Conference*. IEEE, 2008, pp. 1–6.

[5] R. Tresch, M. Guillaud, and E. Riegler, "On the achievability of interference alignment in the k-user constant MIMO interference channel," in *2009 IEEE/SP 15th Workshop on Statistical Signal Processing*. IEEE, 2009, pp. 277–280.

[6] H. Maleki, S. A. Jafar, and S. Shamai, "Retrospective interference alignment over interference networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 228–240, 2011.

[7] G. Bresler, D. Cartwright, and D. Tse, "Feasibility of interference alignment for the MIMO interference channel," *IEEE Transactions on Information Theory*, vol. 60, no. 9, pp. 5573–5586, 2014.

[8] R. T. Krishnamachari and M. K. Varanasi, "Interference alignment under limited feedback for MIMO interference channels," *IEEE Transactions on Signal Processing*, vol. 61, no. 15, pp. 3908–3917, 2013.

[9] O. El Ayach and R. W. Heath, "Interference alignment with analog channel state feedback," *IEEE Transactions on Wireless Communications*, vol. 11, no. 2, pp. 626–636, 2011.

[10] J.-S. Kim, S.-H. Moon, S.-R. Lee, and I. Lee, "A new channel quantization strategy for MIMO interference alignment with limited feedback," *IEEE Transactions on Wireless Communications*, vol. 11, no. 1, pp. 358–366, 2011.

[11] X. Chen and C. Yuen, "Performance analysis and optimization for interference alignment over MIMO interference channels with limited feedback," *IEEE Transactions on Signal Processing*, vol. 62, no. 7, pp. 1785–1795, 2014.

[12] X. Rao, L. Ruan, and V. K. Lau, “Limited feedback design for interference alignment on MIMO interference networks with heterogeneous path loss and spatial correlations,” *IEEE Transactions on Signal Processing*, vol. 61, no. 10, pp. 2598–2607, 2013.

[13] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, “On the degrees of freedom of k-user siso interference and x channels with delayed CSIT,” *IEEE transactions on Information Theory*, vol. 59, no. 10, pp. 6542–6561, 2013.

[14] G. Mergen and L. Tong, “Stability and capacity of regular wireless networks,” *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 1938–1953, 2005.

[15] J. Luo and A. Ephremides, “On the throughput, capacity, and stability regions of random multiple access,” *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2593–2607, 2006.

[16] V. R. Cadambe and S. A. Jafar, “Duality and stability regions of multi-rate broadcast and multiple access networks,” in *2008 IEEE International Symposium on Information Theory*. IEEE, 2008, pp. 762–766.

[17] Y. E. Sagduyu, L. Georgiadis, L. Tassiulas, and A. Ephremides, “Capacity and stable throughput regions for the broadcast erasure channel with feedback: An unusual union,” *IEEE Transactions on Information Theory*, vol. 59, no. 5, pp. 2841–2862, 2012.

[18] S. Li, H. Seferoglu, D. Tuninetti, and N. Devroye, “On the stability region of the layered packet erasure broadcast channel with output feedback,” *Available online: https://devroye.lab.uic.edu/wp-content/uploads/sites/570/2019/09/Li-ITW2019.pdf*, 2019.

[19] C.-C. Wang, “On the capacity of 1-to-k broadcast packet erasure channels with channel output feedback,” *IEEE Transactions on Information Theory*, vol. 58, no. 2, pp. 931–956, 2012.

[20] M. A. Maddah-Ali and D. Tse, “Completely stale transmitter channel state information is still very useful,” *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4418–4431, 2012.

[21] H. Viswanathan, “Capacity of markov channels with receiver CSI and delayed feedback,” *IEEE Transactions on Information Theory*, vol. 45, no. 2, pp. 761–771, 1999.

[22] C. Vaze and M. Varanasi, “The degrees of freedom regions of two-user and certain three-user MIMO broadcast channels with delayed CSIT [arxiv: 1101.0306 v2]. submitted to ieee trans. inf,” *Theory, Dec*, 2011.

[23] M. Jolfaei, S. Martin, and J. Mattfeldt, “A new efficient selective repeat protocol for point-to-multipoint communication,” in *Proceedings of ICC’93-IEEE International Conference on Communications*, vol. 2. IEEE, 1993, pp. 1113–1117.

[24] L. Georgiadis and L. Tassiulas, “Broadcast erasure channel with feedback-capacity and algorithms,” in *2009 Workshop on Network Coding, Theory, and Applications*. IEEE, 2009, pp. 54–61.

[25] A. A. Reddy, S. Banerjee, A. Gopalan, S. Shakkottai, and L. Ying, “On distributed scheduling with heterogeneously delayed network-state information,” *Queueing Systems*, vol. 72, no. 3-4, pp. 193–218, 2012.

[26] A. Vahid, M. A. Maddah-Ali, and A. S. Avestimehr, “Capacity results for binary fading interference channels with delayed CSIT,” *IEEE Transactions on Information Theory*, vol. 60, no. 10, pp. 6093–6130, 2014.

[27] ——, “Communication through collisions: Opportunistic utilization of past receptions,” in *IEEE INFOCOM*

2014-IEEE Conference on Computer Communications. IEEE, 2014, pp. 2553–2561.

[28] G. S. Paschos, L. Georgiadis, and L. Tassiulas, "Scheduling with pairwise XORing of packets under statistical overhearing information and feedback," *Queueing Systems*, vol. 72, no. 3-4, pp. 361–395, 2012.

[29] S. Athanasiadou, M. Gatzianas, L. Georgiadis, and L. Tassiulas, "Stable XOR-based policies for the broadcast erasure channel with feedback," *IEEE/ACM Transactions on Networking (TON)*, vol. 24, no. 1, pp. 476–491, 2016.

[30] S. Athanasiadou, "Low overhead xor algorithms for the broadcast erasure channel with feedback," in *IEEE EUROCON 2019-18th International Conference on Smart Technologies*. IEEE, 2019, pp. 1–7.

[31] H. Pan, J. Li, and K. Wang, "On stability region of two-user interference network with permission to utilize past receptions," *IEEE Access*, vol. 5, pp. 4913–4926, 2016.

[32] A. Vahid, M. A. Maddah-Ali, and A. S. Avestimehr, "Approximate capacity region of the MISO broadcast channels with delayed CSIT," *IEEE Transactions on Communications*, vol. 64, no. 7, pp. 2913–2924, 2016.

[33] S. Nassirpour and A. Vahid, "Throughput, delay, and complexity tradeoffs in interference channels," *under review*, 2019.