ARQ for Interference Packet Networks

Alireza Vahid Robert Calderbank

***Abstract*—In multi-user wireless packet networks interference is the throughput bottleneck. Users become aware of the interference pattern via feedback and use this information for contention resolution and for packet retransmission. We consider networks with spatially correlated wireless links, and we develop an opportunistic automatic repeat request function for these networks. We prove the optimality of our protocol using an extremal rank-ratio inequality for spatially correlated channels.**

***Index Terms*—Wireless packet networks, interference management, spatial correlation, network throughput, ARQ.**

I. Introduction

Interference is the main bottleneck in modern communication networks. In the context of wireless packet networks, information about the interference pattern arrives only after packets are communicated and is therefore delayed. There is a large body of work on wireless networks with delayed interference and channel knowledge (e.g., [1]–[6]). In most cases wireless links are assumed to be independently and identically distributed across time and space. However, such assumptions are not realistic. Temporal correlation allows transmitters to estimate what will happen next, and to adjust their transmission strategies accordingly [7]–[9]. There are also some results that develop transmission strategies for wireless networks with spatially correlated links [10]–[14]. However, these results do not provide any capacity results for spatially correlated networks. Recently in [15], we presented the capacity region of a class of two-user interference channels with spatial correlation at the transmitters.

In this work we consider a wireless packet network with two transmitter-receiver pairs and with spatially correlated links. We capture spatial correlation by introducing a correlation coefficient between the channels connected to *each* user. We develop an Automatic Repeat reQuest (ARQ) function for this problem and prove its optimality. In essence, this work extends the results of [15] to include correlation at *all* wireless nodes.

We adopt the physical layer model for wireless packet networks introduced in [16] and depicted in Fig. 1. In this model, depending on the aggregate interference from other users, there will be four channel states at each receiver: (1) the desired signal is received at a power level such that it can be decoded; (2) the interfering signal is received at a power level such that it can be decoded; (3) the desired signal and the interfering signal are received at similar power levels and the receiver obtains a linear combination of them; (4) no useful signal is obtained. These cases are modeled using a shadowing coefficient for each wireless link, and the transmitters learn these shadowing coefficients with some delay. We assume that there is a certain spatial correlation pattern between the shadowing coefficients and that this knowledge is available to wireless nodes as side information. We show that spatial correlation can greatly affect the throughput region of wireless packet networks: spatial correlation on the one hand can take away any potential gain of delayed interference pattern knowledge and on the other hand, it can help us perform as well as having instantaneous knowledge of interference pattern. We discuss this dichotomy in Section III.

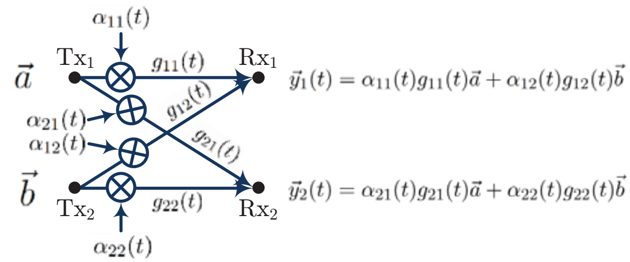

Fig. 1. At time t, Tx_1 and Tx_2 communicate data packets $\vec{a}$ and $\vec{b}$ respectively and $\alpha_{ji}(t)$'s are the shadowing coefficients.

To derive the outer-bounds that capture spatial correlation, we develop an extremal rank-ratio inequality in Section V. We then use a genie-aided argument and apply our extremal rank-ratio inequality to obtain the outer-bounds. We observe that spatial correlation at the transmitter side defines the shape of the capacity region while spatial correlation at the receiver side determines its size.

To achieve the outer-bounds, we develop an ARQ function in Section IV that updates the status of each transmitted packet into three queues: 1) delivered packets, 2) packets that arrived (and potentially interfered) at both receivers, and 3) packets that arrived (and potentially interfered) at the unintended receivers. The ARQ function then combines and retransmits the packets in the latter two queues taking into account the spatial correlation structure of the network.

II. Problem Formulation

We quantify the impact of spatial correlation on the throughput region of wireless packet networks. To do so, we consider a wireless packet network in which multiple transmitter-receiver pairs wish to communicate with each other and within this setup, we focus on two nearby transmitter-receiver pairs, namely Tx_1-Rx_1 and Tx_2-Rx_2.

We adopt the abstraction of wireless packet networks introduced in [16]. In this model, transmitter Tx_1 has m_1 packets (data frames) with corresponding physical layer codewords of

Alireza Vahid is with the Electrical Engineering Department, University of Colorado Denver, USA. Email: alireza.vahid@ucdenver.edu.

Robert Calderbank is with the Electrical and Computer Engineering Department, Duke University, USA. Email: robert.calderbank@duke.edu.

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length τ denoted by $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_{m_1}$, and wishes to communicate them to receiver Rx_1. Similarly, transmitter Tx_2 has m_2 packets denoted by codewords $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_{m_2}$ for receiver Rx_2. It is assumed that the mapping from the packets to their corresponding physical layer codewords is fixed (*e.g.*, LDPC codes, Reed-Solomon codes, etc) and if a codeword is received with a signal-to-interference-plus-noise ratio of above some threshold, γ, the receiver is able to decode its packet.

Suppose Tx_1 and Tx_2 communicate data packets $\vec{a}$ and $\vec{b}$ respectively at time t. Then, Rx_1 obtains:

$$\vec{y}_1(t) = g_{11}(t)\vec{a} + g_{12}(t)\vec{b} + \vec{z}_1(t), \tag{1}$$

where $g_{11}(t)$ and $g_{12}(t)$ are real-valued channel gains and $\vec{z}_1(t)$ is the ambient interference plus noise. The signal-to-interference-plus-noise ratio of link ji at Rx_j, $i,j \in \{1,2\}$, is defined as:

$$\mathrm{SINR}_{ji} \triangleq 10\log_{10}\left(\frac{P|g_{ji}|^2}{\mathbb{E}\left[\vec{z}_j^\top(t)\vec{z}_j(t)\right] + P|g_{j\bar{i}}|^2}\right), \tag{2}$$

where $\bar{i} \triangleq 3-i$ and P is the average transmit power constraint. Furthermore, we define the signal-to-noise ratio (SNR) of link ji at Rx_j, $i,j \in \{1,2\}$, as:

$$\mathrm{SNR}_{ji} \triangleq 10\log_{10}\left(\frac{P|g_{ji}|^2}{\mathbb{E}\left[\vec{z}_j^\top(t)\vec{z}_j(t)\right]}\right). \tag{3}$$

Based on the SINR and the SNR values of different links at each time t, we have one of the following states at any of the receivers, say Rx_1:

- State 1 ($\mathrm{SINR}_{11} \geq \gamma$): In this state the SINR of the desired packet (*i.e.* $\vec{a}$) at Rx_1 is above the threshold and Rx_1 can decode the corresponding packet.
- State 2 ($\mathrm{SINR}_{12} \geq \gamma$): Similar to State 1, but in this case the SINR of the interfering packet (*i.e.* $\vec{b}$) at Rx_1 is above the threshold and Rx_1 (the unintended receiver in this case) can decode the packet.
- State 3 ($\mathrm{SINR}_{1i} < \gamma$ but $\mathrm{SNR}_{1i} \geq \gamma$ for $i = 1,2$): This state corresponds to the scenario in which the SINRs of both packets are below the threshold at Rx_1 but the individual links are strong. Thus, receiver Rx_1 obtains a linear combination of the packets.
- State 4: In any other scenario, Rx_1 discards its signal.

The following abstraction of the physical layer at Rx_1 captures these four different states:

$$\vec{y}_1(t) = \alpha_{11}(t)g_{11}(t)\vec{a} + \alpha_{12}(t)g_{12}(t)\vec{b}, \tag{4}$$

where the shadowing coefficients $\alpha_{11}(t)$ and $\alpha_{12}(t)$ are in the binary field. Similarly, $\alpha_{21}(t)$ and $\alpha_{22}(t)$ can be used for Rx_2. The channel state at time t is represented by

$$\alpha(t) = \{\alpha_{11}(t), \alpha_{12}(t), \alpha_{21}(t), \alpha_{22}(t)\}. \tag{5}$$

We assume that $\alpha_{ji}(t)$'s follow a Bernoulli distribution $\mathcal{B}(p)$ and that at time t, each transmitter knows $\alpha^{t-1} = (\alpha(\ell))_{\ell=1}^{t-1}$ and each receiver has access to $\alpha^t = (\alpha(\ell))_{\ell=1}^{t}$. We note that transmitters do *not* learn the real-valued channel gains.

In general, $\alpha_{ji}(t)$'s are correlated across time and space. Since we study the impact of spatial correlation, we assume that $\alpha_{ji}(t)$'s are distributed independently across time and are drawn from the same joint distribution at each time. To capture spatial correlation, we assume a symmetric setting in which the shadowing coefficients corresponding to the links connected to transmitter Tx_i have a correlation coefficient ρ_{Tx}, *i.e.*

$$\rho_{\mathsf{Tx}} = \frac{\mathrm{cov}\left(\alpha_{1i}(t), \alpha_{2i}(t)\right)}{\sigma_{\alpha_{1i}(t)}\sigma_{\alpha_{2i}(t)}}, \qquad i = 1,2. \tag{6}$$

and similarly, the links connected to receiver Rx_j have a correlation coefficient ρ_{Rx}.

We note that fixing $-1 \leq \rho_{\mathsf{Tx}}, \rho_{\mathsf{Rx}} \leq 1$ imposes a feasible set on p. More precisely, we have $p \in \mathcal{S}_{\rho_{\mathsf{Tx}}} \cap \mathcal{S}_{\rho_{\mathsf{Rx}}}$ where $\mathcal{S}_{\rho_{\mathsf{Tx}}}$ (and similarly $\mathcal{S}_{\rho_{\mathsf{Rx}}}$) is defined below.

$$\mathcal{S}_{\rho_{\mathsf{Tx}}} \triangleq \left[\max\left\{0, \frac{-\rho_{\mathsf{Tx}}}{1-\rho_{\mathsf{Tx}}}\right\}, \min\left\{1, \frac{1}{1-\rho_{\mathsf{Tx}}}\right\}\right], \tag{7}$$

and we set $\mathcal{S}_{\rho_{\mathsf{Tx}}=1}, \mathcal{S}_{\rho_{\mathsf{Rx}}=1} \triangleq [0,1]$. The derivation of $\mathcal{S}_{\rho_{\mathsf{Tx}}}$ and $\mathcal{S}_{\rho_{\mathsf{Rx}}}$ is provided in [17]. We also define:

$$\begin{aligned} p_{k\ell}^{\mathsf{Tx}} &\triangleq \Pr\left(\alpha_{11}(t) = k, \alpha_{21}(t) = \ell\right), \\ p_{k\ell}^{\mathsf{Rx}} &\triangleq \Pr\left(\alpha_{11}(t) = k, \alpha_{12}(t) = \ell\right), \qquad k,\ell \in \{0,1\}, \end{aligned} \tag{8}$$

As mentioned above, Tx_i wishes to reliably communicate m_i packets to Rx_i during n uses of the channel, $i = 1,2$. We assume that the packets and the channel gains are mutually independent. Receiver Rx_i is only interested in packets from Tx_i, and it will recover (decode) them using the received signal $\vec{y}_i^n$, the knowledge of the channel state information, and the knowledge of ρ_{Tx} and ρ_{Rx}.

At each time instant, transmitter i creates a linear combination of the m_i packets it has for receiver i by choosing a precoding vector $\vec{v}_i(t) \in \mathbb{R}^{1\times m_i}$, $i = 1,2$. Transmit signals at time t at Tx_1 and Tx_2 are given by $\vec{v}_1(t)\mathbf{A}$ and $\vec{v}_2(t)\mathbf{B}$ respectively, where $\mathbf{A} = \left[\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_{m_1}\right]^\top$, and $\mathbf{B} = \left[\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_{m_2}\right]^\top$. We impose $||\vec{v}_1(t)||, ||\vec{v}_2(t)|| \leq 1$ to satisfy the power constraint at the transmitters where $||.||$ represents the Euclidean norm. Since transmitters learn $\alpha(t)$ with unit delay, $\vec{v}_i(t)$ is only a function of α^{t-1} and the correlation coefficients ρ_{Tx} and ρ_{Rx}. The received signal of receiver i at time t, can be represented by

$$\vec{y}_i(t) = \alpha_{i1}(t)g_{i1}(t)\vec{v}_1(t)\mathbf{A} + \alpha_{i2}(t)g_{i2}(t)\vec{v}_2(t)\mathbf{B}. \tag{9}$$

We denote the overall precoding matrix of transmitter i by $\mathbf{V}_i^n \in \mathbb{R}^{n\times m_i}$, where the t^{th} row of $\mathbf{V}_i^n$ is $\vec{v}_i(t)$. Furthermore, let $\mathbf{G}_{ij}^n$ be an $n \times n$ diagonal matrix where the t^{th} diagonal element is $\alpha_{ij}(t)g_{ij}(t)$, $i,j = 1,2$. Thus, we can write the output at receiver i as

$$\vec{y}_i^n = \mathbf{G}_{i1}^n\mathbf{V}_1^n\mathbf{A} + \mathbf{G}_{i2}^n\mathbf{V}_2^n\mathbf{B}, \quad i = 1,2. \tag{10}$$

We denote the interference subspace at receiver i by $\mathcal{I}_i$ and is given by

$$\mathcal{I}_i = \mathrm{colspan}\left(\mathbf{G}_{i\bar{i}}\mathbf{V}_{\bar{i}}\right), \quad i = 1,2, \tag{11}$$

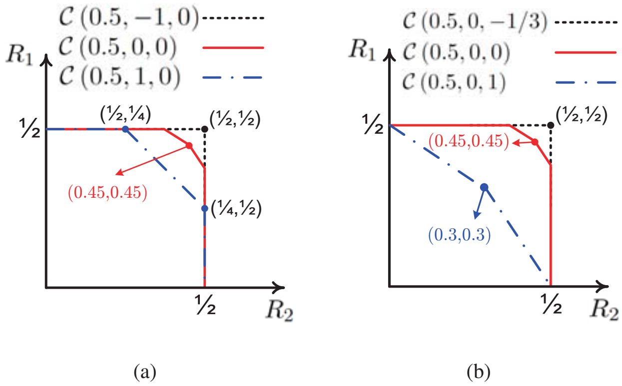


Fig. 2. Throughput region for $p = 0.5$: (a) $\rho_{\mathsf{Rx}} = 0$, and $\rho_{\mathsf{Tx}} \in \{-1, 0, 1\}$, and (b) $\rho_{\mathsf{Tx}} = 0$, and $\rho_{\mathsf{Rx}} \in \{-1/3, 0, 1\}$.

where $\mathrm{colspan}(.)$ of a matrix represents the sub-space spanned by its column vectors, and let $\mathcal{I}_i^c$ denote the subspace orthogonal to $\mathcal{I}_i$. Then, in order for decoding to be successful at receiver i, it should be able to create m_i linearly independent equations that are solely in terms of its intended packets. Mathematically speaking, this means that the image of $\mathrm{colspan}\left(\mathbf{G}_{ii}^n \mathbf{V}_i^n\right)$ on $\mathcal{I}_i^c$ should have the same dimension as $\mathrm{colspan}\left(\mathbf{V}_i^n\right)$ itself. In other words, we require

$$\begin{aligned} &\dim\left(\mathrm{Proj}_{\mathcal{I}_i^c}\ \mathrm{colspan}\left(\mathbf{G}_{ii}^n \mathbf{V}_i^n\right)\right) \\ &\qquad = \dim\left(\mathrm{colspan}\left(\mathbf{V}_i^n\right)\right) = m_i, \quad i = 1, 2. \end{aligned} \tag{12}$$

We say that a throughput tuple of $(R_1, R_2) = (m_1/n, m_2/n)$ is achievable, if there exists a choice of $\mathbf{V}_1^n$ and $\mathbf{V}_2^n$, such that (12) is satisfied for $i = 1, 2$ with probability 1. The throughput region, $\mathcal{T}(p, \rho_{\mathsf{Tx}}, \rho_{\mathsf{Rx}})$, is the closure of all achievable throughput tuples (R_1, R_2).

III. Statement of the Main Results

The following theorem establishes the throughput region.

Theorem 1. *For the spatially correlated wireless packet networks with two transmitter-receiver pairs as described in Section II and for $p \in \mathcal{S}_{\rho_{\mathsf{Tx}}} \cap \mathcal{S}_{\rho_{\mathsf{Rx}}}$, we have*

$$\mathcal{T}(p, \rho_{\mathsf{Tx}}, \rho_{\mathsf{Rx}}) = \tag{13}$$
$$\left\{ \begin{array}{ll} 0 \leq R_i \leq p, & i = 1, 2, \\ R_i + \beta(p, \rho_{\mathsf{Tx}}) R_{\bar{i}} \leq \beta(p, \rho_{\mathsf{Tx}}) \left(1 - p_{00}^{\mathsf{Rx}}\right), & i = 1, 2. \end{array} \right\}$$

where

$$\begin{aligned} \beta(p, \rho_{\mathsf{Tx}}) &= 2 - \rho_{\mathsf{Tx}} - p(1 - \rho_{\mathsf{Tx}}), \\ p_{00}^{\mathsf{Rx}} &= 1 + p^2 + p(1-p)\rho_{\mathsf{Rx}} - 2p. \end{aligned} \tag{14}$$

The converse proof of Theorem 1 relies on an extremal rank-ratio inequality for correlated channels that we present in Section V. The slope of the outer-bounds (*i.e.* $\beta(p, \rho_{\mathsf{Tx}})$) is determined by this inequality and depends on spatial correlation at the transmitters. Using the extremal inequality and genie-aided arguments, we obtain the outer-bound.

The communication protocol that achieves the capacity has multiple phases of communications and after each phase, transmitters use the delayed interference pattern knowledge to update the status of the previously communicated packets. The goal is to retransmit linear combination of packets in a way to help receivers decode their corresponding packets faster than the scenario in which individual packets are retransmitted.

Before presenting the proofs, we provide further interpretation of Theorem 1. As shown in Fig. 2(a), for $p = 0.5$ and fully correlated links at each transmitter, $\mathcal{C}(0.5, 1, 0)$ coincides with the one where transmitters do not have any access to interference pattern [16]. On the other hand, $\mathcal{C}(0.5, -1, 0)$ includes $(R_1, R_2) = (0.5, 0.5)$ which implies that the throughput region coincides with the throughput region of a network in which a genie informs wireless nodes of the interference pattern before it even happens. Intuitively this is due to the fact that with fully correlated channels, each transmitter cannot distinguish between the two receivers and as a result, it is not able to perform interference cancellation or interference alignment. However, with negative correlation a transmitter's power to favor one receiver over the other improves (in terms of the received number of new equations). This in turn enables the transmitters to perform interference alignment and interference cancellation more efficiently.

To study the impact of spatial correlation at the receivers on the throughput region, we consider $\rho_{\mathsf{Tx}} = 0$ (*i.e.* independent links at the transmitters), $\rho_{\mathsf{Rx}} \in \{-1/3, 0, 1\}$, and $p = 0.5$. As ρ_{Rx} moves from $+1$ to -1, the maximum achievable sum-rate improves as in Fig. 2(b). For $\rho_{\mathsf{Rx}} \in [-1, -1/3]$, $\mathcal{C}(0.5, 0, \rho_{\mathsf{Rx}})$ includes $(R_1, R_2) = (0.5, 0.5)$ which implies that the capacity region coincides with that of instantaneous knowledge. Intuitively, negative spatial correlation at receivers separates the signal subspace from the interference subspace which results in higher network throughput, see Section IV for more details.

IV. ARQ for Correlated Packet Networks

In this section we present the communication protocol of Theorem 1 for a particular example ($p = \rho_{\mathsf{Tx}} = \rho_{\mathsf{Rx}} = 0.5$). The complete proof can be found in [17]. We present the ARQ function for the maximum symmetric sum-rate point as given by

$$\left.\frac{2\beta(p, \rho_{\mathsf{Tx}})\left(1 - p_{00}^{\mathsf{Rx}}\right)}{1 + \beta(p, \rho_{\mathsf{Tx}})}\right|_{p=\rho_{\mathsf{Tx}}=\rho_{\mathsf{Rx}}=0.5} = \frac{25}{36}. \tag{15}$$

Suppose each transmitter wishes to communicate m packets to its intended receiver. It suffices to show that this task can be accomplished (with vanishing error probability as $m \to \infty$) in $72/25m + \mathcal{O}\left(m^{\frac{2}{3}}\right)$ time instants. The ARQ function is divided into two phases described below.

Phase 1: At the beginning of the communication block, we assume that the m packets at Tx_i are in an initial queue[1] denoted by $Q_{i\to i}$, $i = 1, 2$. At each time instant t, Tx_i transmits a packets from $Q_{i\to i}$ and this packet will either stay in this initial queue or will transition to one of the queues listed

[1] We assume that the queues are column vectors and packets are placed in each queue according to the order they join in.

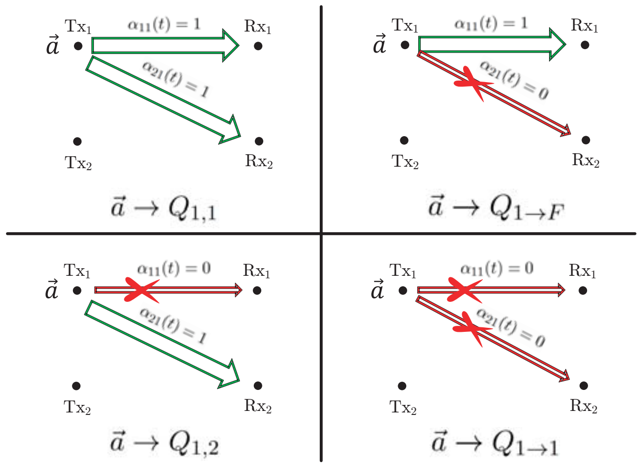


Fig. 3. Based on the shadowing coefficients at the time of communication, the status of packet $\vec{a}$ is updated. We note that the shadowing coefficients are learned with unit delay.

in Fig. 3. If at time instant t, $Q_{i \rightarrow i}$ is empty, then Tx_i, $i = 1, 2$, remains silent until the end of Phase 1. $Q_{i \rightarrow F}$ includes the packets for which no retransmission is required and potential interference from $\mathsf{Tx}_{\bar{i}}$ will be resolved by $\mathsf{Tx}_{\bar{i}}$.

Phase 1 continues for

$$\frac{1}{1 - p_{00}^{\mathsf{Tx}}} m + m^{\frac{2}{3}} = \frac{8}{5} m + m^{\frac{2}{3}} \tag{16}$$

time instants, and if at the end of this phase, either of the queues $Q_{i \rightarrow i}$ is not empty, we declare error type-I and halt the transmission (we assume m is chosen such that $m^{\frac{2}{3}} \in \mathbb{Z}$).

Assuming that the transmission is not halted, let $N_{i,1}$ and $N_{i,2}$ denote the number of packets in queues $Q_{i,1}$ and $Q_{i,2}$ respectively at the end of the first phase, $i = 1, 2$. The transmission strategy will be halted and error type-II occurs, if any of the following events happens.

$$\begin{aligned} N_{i,1} &> \mathbb{E}[N_{i,1}] + 2m^{\frac{2}{3}} \stackrel{\triangle}{=} n_{i,1}, \quad i = 1, 2; \\ N_{i,2} &> \mathbb{E}[N_{i,2}] + 2m^{\frac{2}{3}} \stackrel{\triangle}{=} n_{i,2}, \quad i = 1, 2. \end{aligned} \tag{17}$$

From basic probability, we have

$$\mathbb{E}[N_{i,1}] = \frac{p_{11}^{\mathsf{Tx}} m}{1 - p_{00}^{\mathsf{Tx}}} = \frac{3}{5} m, \ \mathbb{E}[N_{i,2}] = \frac{p_{01}^{\mathsf{Tx}} m}{1 - p_{00}^{\mathsf{Tx}}} = \frac{1}{5} m. \tag{18}$$

At the end of Phase 1, we add deterministic packets (if necessary) in order to make queues $Q_{i,1}$ and $Q_{i,2}$ of size equal to $n_{i,1}$ and $n_{i,2}$ respectively as given above, $i = 1, 2$. Statistically a fraction $p_{01}^{\mathsf{Rx}}/p = 1/4$ of the packets in $Q_{i,1}$ and the same fraction of the bits in $Q_{i,2}$ are known to $\mathsf{Rx}_{\bar{i}}$, $i = 1, 2$. Denote the number of bits in $Q_{i,j}$ known to $\mathsf{Rx}_{\bar{i}}$ by $N_{i,j|\mathsf{Rx}_{\bar{i}}}$. At the end of communication, if we have

$$N_{i,j|\mathsf{Rx}_{\bar{i}}} < \frac{p_{01}^{\mathsf{Rx}}}{p} n_{i,j} - 2m^{\frac{2}{3}} = \frac{1}{4} n_{i,j} - 2m^{\frac{2}{3}}, \quad i, j \in \{1, 2\},$$

we declare error type-III.

Furthermore using the Bernstein inequality, we can show that the probability of errors of types I, II, and III decreases and approaches exponentially to zero as $m \rightarrow \infty$ (see [17]). For the rest of this section, we assume that Phase 1 is completed with no errors.

Phase 2: During this phase, we deliver sufficient number of linearly independent combinations of packets to each receiver so that they can decode their corresponding packets. We create these linear combinations in such a way that they are of interest to both receivers. Denote by $Q_{1,1}^{c}$ and $Q_{1,2}^{c}$ the fraction of the packets in $Q_{1,1}$ and $Q_{1,2}$ respectively for which at the time of transmission $\alpha_{22}(t) = 1$, and by $Q_{1,1}^{nc}$ and $Q_{1,2}^{nc}$ the fraction of the packets in $Q_{1,1}$ and $Q_{1,2}$ respectively for which at the time of transmission $\alpha_{22}(t) = 0$. Similar definitions apply to $Q_{2,1}^{c}, Q_{2,2}^{c}, Q_{2,1}^{nc}$ and $Q_{2,2}^{nc}$. We have

$$\mathbb{E}[N_{i,1}^{c}] = \frac{9}{20} m, \ \mathbb{E}[N_{i,2}^{c}] = \frac{3}{20} m, \quad i = 1, 2. \tag{19}$$

Packets in $Q_{i,2}^{nc}$ can be combined with packets in $Q_{i,1}^{c}$ to create packets of common interest as depicted in Fig. 4(a) and Fig. 4(b). Packets in $Q_{i,2}^{c}$ are needed at both receivers (no combination in this case) as depicted in Fig. 4(c). Finally, packets in $Q_{i,1}^{c}$ minus the ones combined with packets in $Q_{i,2}^{nc}$ are needed at both receivers, e.g., packet $\vec{a}$ in Fig. 4(a) is useful for both receivers. However for packets in $Q_{i,1}^{c}$ and $Q_{\bar{i},1}^{c}$ only half of them need to be provided to both receivers. As a result, the expected number of total linearly independent combinations needed at both receivers is given by:

$$2 \times \underbrace{\frac{3}{20} m}_{Q_{i,2}^{c}} + 2 \times \underbrace{\frac{1}{20} m}_{Q_{i,2}^{nc} \text{ + w/ } Q_{i,1}^{c}} + \underbrace{\frac{8}{20} m}_{\text{remaining in } Q_{i,1}^{c}} = \frac{4}{5} m. \tag{20}$$

We conclude that Tx_i only needs to create $2/5m + \mathcal{O}\left(m^{\frac{2}{3}}\right)$ linearly independent equations of the packets in $Q_{i,1}$ and $Q_{i,2}$, $i = 1, 2$, and deliver them to both receivers.

The problem of delivering packets to both receivers has the same capacity as the multiple-access channel formed at each receiver for which a rate-tuple of $(R_1, R_2) = (5/16, 5/16)$ is achievable. Using the communication protocol of the two-multicast problem, Phase 2 lasts for

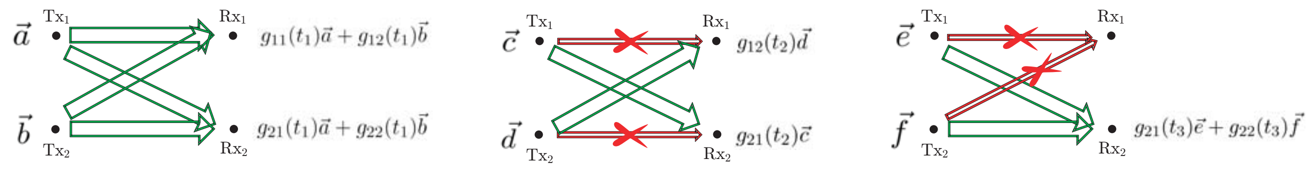
$$\underbrace{\frac{8}{5}}_{\text{multicast}} \times \underbrace{\frac{4}{5} m}_{\text{\# of eqs needed}} + \mathcal{O}\left(m^{\frac{2}{3}}\right) = \frac{32}{25} m + \mathcal{O}\left(m^{\frac{2}{3}}\right) \tag{21}$$

time instants. If any error takes place, we declare error and terminate the communication. Thus to continue, we assume that the transmission is successful and no error has occurred.

Decoding: The idea is that upon completion of the communication protocol, each receiver has sufficient number of linearly independent equations of the desired packets to decode them. A detailed discussion can be found in [17].

Achievable rates: Assuming that no error occurs, the total communication time is:

$$\underbrace{\frac{8}{5} m}_{\text{Phase 1}} + \underbrace{\frac{32}{25} m}_{\text{Phase 2}} + \mathcal{O}\left(m^{\frac{2}{3}}\right) = \frac{72}{25} m + \mathcal{O}\left(m^{\frac{2}{3}}\right) \tag{22}$$



(a) (b) (c)

Fig. 4. (a) and (b): Packets in $Q_{i,2}^{nc}$ can be combined with packets in $Q_{i,1}^{c}$ to create packets of common interest: $\vec{a}+\vec{c}$ and $\vec{b}+\vec{d}$ are of common interest to both receivers; (c) Packets in $Q_{i,2}^{c}$ are needed at both receivers.

time instants which is what we expected.

V. Converse Proof of Theorem 1

The derivation of the bounds on individual rates is straightforward and thus omitted. To derive the other bounds, we first present an extremal rank-ratio lemma tailored to correlated channels. The proof of this lemma can be found in [17].

Lemma 1 (Extremal Rank-Ratio Inequality for Spatially Correlated Channels)**.** *For the two-user spatially correlated wireless packet network as described in Section II and for* $p \in \mathcal{S}_{\rho_{\mathsf{Tx}}} \cap \mathcal{S}_{\rho_{\mathsf{Rx}}}$, $p \neq 0$, *and* β *given in (14), we have*

$$\mathbb{E}\left[\mathrm{rank}\left[\mathbf{G}_{21}^{n}\mathbf{V}_{1}^{n}\right]\right] \geq \frac{1}{\beta\left(p,\rho_{\mathsf{Tx}}\right)}\mathbb{E}\left[\mathrm{rank}\left[\mathbf{G}_{11}^{n}\mathbf{V}_{1}^{n}\right]\right]. \tag{23}$$

Using Lemma 1, we have

$$\begin{aligned}
& n\left(R_1 + \beta\left(p,\rho_{\mathsf{Tx}}\right)R_2\right) \\
& \overset{a.s.}{=} \mathbb{E}\left[\dim\left(\mathrm{Proj}_{\mathcal{I}_1^c}\ \mathrm{colspan}\left(\mathbf{G}_{11}^{n}\mathbf{V}_{1}^{n}\right)\right)\right] \\
& + \beta\left(p,\rho_{\mathsf{Tx}}\right)\mathbb{E}\left[\dim\left(\mathrm{Proj}_{\mathcal{I}_2^c}\ \mathrm{colspan}\left(\mathbf{G}_{22}^{n}\mathbf{V}_{2}^{n}\right)\right)\right] \\
& \overset{(a)}{\leq} \mathbb{E}\left[\mathrm{rank}\left[\mathbf{G}_{11}^{n}\mathbf{V}_{1}^{n}\right]\right] - \beta\left(p,\rho_{\mathsf{Tx}}\right)\mathbb{E}\left[\mathrm{rank}\left[\mathbf{G}_{21}^{n}\mathbf{V}_{1}^{n}\right]\right] \\
& + \beta\left(p,\rho_{\mathsf{Tx}}\right)\mathbb{E}\left[\mathrm{rank}\left[\mathbf{G}_{21}^{n}\mathbf{V}_{1}^{n} + \mathbf{G}_{22}^{n}\mathbf{V}_{2}^{n}\right]\right] \\
& \overset{\text{Lemma 1}}{\leq} \beta\left(p,\rho_{\mathsf{Tx}}\right)\mathbb{E}\left[\mathrm{rank}\left[\mathbf{G}_{21}^{n}\mathbf{V}_{1}^{n} + \mathbf{G}_{22}^{n}\mathbf{V}_{2}^{n}\right]\right] \\
& \overset{(b)}{\leq} n\beta\left(p,\rho_{\mathsf{Tx}}\right)\left(1 - p_{00}^{\mathsf{Rx}}\right),
\end{aligned} \tag{24}$$

where the first equality is needed to guarantee that (12) holds; (a) follows since we ignored interference at Rx_1; (b) holds since Rx_2 does not receive any useful information p_{00}^{Rx} fraction of the time. Dividing both sides of (24) by n and let $n \to \infty$, we get $R_1 + \beta\left(p,\rho_{\mathsf{Tx}}\right)R_2 \leq \beta\left(p,\rho_{\mathsf{Tx}}\right)\left(1 - p_{00}^{\mathsf{Rx}}\right)$. Similarly, we can obtain the other bound.

VI. Conclusion

We characterized the throughput region of spatially correlated interference packet networks. We learned that spatial correlation at the transmitters defines the shape of the throughput region while spatial correlation at the receivers defines its size. An interesting future direction is to consider stochastic packet arrival with delivery deadline.

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