Embedding Information in Radiation Pattern Fluctuations

Milad Johnny Alireza Vahid

***Abstract*—The radiation pattern of transmit antennas varies and fluctuates as receivers change their location, other objects move around, and due to the antenna design itself. In this paper, we demonstrate how this observation can be exploited to align most of the interference signal power and significantly increase the average achievable communication rates. More precisely, in the context of K-user interference channels, we propose a blind interference alignment scheme that combines multi-layer coding at the transmitters and a post-processing methodology at the receivers to align a significant portion of the interference signal power. Our scheme does not rely on any channel state information (CSI), hence the term blind, and only relies on the statistics of the radiation pattern fluctuations. Our proposed communication methodology overcomes some of the barriers in practical implementation of the interference alignment concept. Due to the complexity of the expressions, in this work, we numerically evaluate the achievable rates in different scenarios, demonstrate the gains of our proposed strategy, and compare our results to the prior works with perfect CSI.**

***Index Terms*—Radiation pattern fluctuations, coherence time variations, blind interference alignment, multi-layer coding.**

I. Introduction

In recent years, alleviating the negative impact of interference in wireless systems has been at the center of attention. From a physical-layer point of view, there are well-known methods such as TDMA and FDMA schemes that reduce the impact of interference signals at the receivers at the cost of spectrum efficiency. Considering the ever-growing demand for higher communication rates and the growth of data network traffic, we need to replace these suboptimal methods with more bandwidth-efficient schemes. Interference alignment (IA) is an attractive concept to manage interference in wireless networks which creates a multiplexing gain proportional to the number of users in the network [1]. However, the requirement of perfect CSI, and in some cases global perfect CSI, the fast fading assumption, and long precoder lengths at the transmitters are some of the main barriers in implementing IA [2]–[4].

Broadly speaking, in terms of reliance on CSI, we have four different types of interference alignment schemes: The first one is based on perfect channel state information (CSI) [1], [5]; the second one is instantaneous but imperfect CSI [1], [6], [7]; the third one is delayed CSI [8]–[11]; the last one is known as blind IA (BIA) [12], [13]. For the first three cases of (perfect, imperfect, and delayed) CSI, one of the main practical challenges is attaining high-resolution CSI cannot be realized (with acceptable overhead) in real-world networks [3]. The last one, the BIA, does not rely on CSI at the transmitters but typically assumes particular channel variation structures that may not be feasible either.

In this paper, we propose a new IA technique based on the observation that the radiation pattern of transmit antennas varies and fluctuates as receivers change their location, other objects move around, and due to the antenna design itself. In fact, in [14], we propose a transmit antenna hardware design that creates such an environment even when wireless nodes are stationary. Thus, more specifically, our main contribution in this paper is the introduction of a new BIA scheme based on antenna radiation pattern fluctuations which result in different coherence times for different wireless links. Using the statistics of channel coherence times, we introduce a multi-layer encoding and a successive decoding strategy that maximizes the average transmission rate. We show that without accessing channel state information and even channel variations pattern, we can improve the average achievable sum-rate drastically similar to prior IA schemes (even those with perfect CSI).

In the next section, we introduce our system and channel models, and explain how different receivers may experience different channel coherence times from different transmitters. In Section III, using differences in channel coherence times, without accessing channel state information at the transmitters, we devise our BIA scheme. Finally, through numerical analysis, we show our scheme increases the average transmission rate, and we compare our results to known outer-bound.

II. Problem Formulation

In this paper, we consider the K-user interference channel (IC) in which K single-antenna transmitters $\mathrm{TX}_k, k \in \mathcal{K} = \{1, \ldots, K\}$, send their messages to K single-antenna receivers $\mathrm{RX}_k, k \in \mathcal{K}$. The received signal at RX_i is given by

$$\bar{\mathbf{Y}}^{[i]} = \sum_{j=1}^{K} \bar{\mathbf{H}}^{[ij]} \bar{\mathbf{X}}^{[j]} + \bar{\mathbf{Z}}^{[i]}, \qquad i, j \in \mathcal{K}, \tag{1}$$

where the $n \times 1$ column vectors $\bar{\mathbf{Y}}^{[i]}$ and $\bar{\mathbf{X}}^{[j]}$ represent the received and the transmitted signals of RX_i and TX_j, respectively. Here, n is the communication block length. The $n \times 1$ column vector $\bar{\mathbf{Z}}^{[i]}$ denotes the additive white Gaussian noise at RX_i distributed according to $\mathcal{CN}(0,1)$, and $\bar{\mathbf{H}}^{[ij]} = \mathrm{diag}\left(\left[h_1^{[ij]}, \ldots, h_n^{[ij]}\right]\right), i, j \in \mathcal{K}$, is the $n \times n$ diagonal matrix of channel coefficients from TX_j to RX_i. As motivated later in this section, and due to transmitter antenna radiation pattern fluctuations, we assume a block fading model

Alireza Vahid is with the Department of Electrical Engineering at the University of Colorado Denver, Denver, USA. Email: alireza.vahid@ucdenver.edu

in time, where channels remain constant for some duration, and these durations may vary for different channels. Therefore, for the channel matrix of $\bar{\mathbf{H}}^{[ij]}$, we have:

$$h^{[ij]}_{c^{[ij]}_l} = \ldots = h^{[ij]}_{c^{[ij]}_{l+1}-1} \tag{2}$$

where $l \in \{1, \ldots, \eta(i,j)\}$ and $\eta(i,j)$ is the number of channel "altering points" in time between TX_j and RX_i. The value of $c^{[ij]}_l$, $l \in \{1, \ldots, \eta(i,j)\}$ represents the l^{th} point of altering state of the channel between TX_j and RX_i and as n goes to infinity $\frac{\eta(i,j)}{n}$ represents channel variation rate. It is assumed that all $h^{[ij]}_l$ are $i.i.d$ random variables with a specific distribution with *magnitude* bounded between a nonzero value and a finite maximum value. Since the channel coherence time is a random variable, the channel at each altering point in time may remain in its previous state with probability p_{ij}, and may transition to a new state with probability $(1 - p_{ij})$. For the channel described above, we have the following definition.

Definition 1: We define the function $\mathcal{F}\left(\bar{\mathbf{H}}^{[ij]}\right)$ to be the maximum number of the time snapshots in which the channel matrix between TX_j and RX_i is constant. We refer the reader to [14] for more details and intuitions on this definition.

In this paper, we use a multi-layer encoding and decoding strategy in which transmitter TX_i wishes to send uniformly distributed message $W^{[i]} \in \mathcal{W}^{[i]} = \mathcal{W}^{[i]}_1 \times \cdots \times \mathcal{W}^{[i]}_M$ to RX_i, $i \in \mathcal{K}$, over n uses of the channel. Each set $\mathcal{W}^{[i]}_j$, $1 \leq j \leq M$, represents the message set for the j^{th} transmission layer. We further assume that the messages are independent from each other and the channel gains. Each transmitter is subject to a total average transmission power constraint of P_t. Transmitter TX_i encodes its message $W^{[i]}$ using the encoding function $\bar{\mathbf{X}}^{[i]} = e_i\left(W^{[i]}, \mathrm{SI}_{\mathsf{TX}_i}\right)$ where $i \in \mathcal{K}$ and $\mathrm{SI}_{\mathsf{TX}_i}$ is the available side information at TX_i which is a cumulative distribution function discussed in Section III-C. The value of $R^{[i]}_j = \log_2 |\mathcal{W}^{[i]}_j|/n$ is the transmission rate of of j^{th} layer at TX_i. Therefore, the total transmission rate at TX_i is $R^{[i]} = \sum_{j=1}^{M} R^{[i]}_j$. We assume that each receiver RX_i is aware of its channel state information and decodes its intended message $W^{[i]}_j \in \mathcal{W}^{[i]}_j$ using the decoding function $\hat{W}^{[i]}_j = \phi_{ij}\left(\bar{\mathbf{Y}}^{[i]}, \mathrm{SI}_{\mathsf{RX}_i}\right)$, $1 \leq j \leq M$, where $\mathrm{SI}_{\mathsf{RX}_i}$ is the side information available to the receiver (in this case channel state information). Then, the decoding error probability at receiver RX_i for the j^{th} layer is given by

$$\lambda^{[i]}_j(n) = \mathbb{E}\left[\Pr\left(\hat{W}^{[i]}_j \neq W^{[i]}_j\right)\right], \tag{3}$$

where the expectation is over the random choice of messages. For $\mathcal{J}_i \subseteq \{1, \ldots, M\}$, we define $R^{[i]}_{\mathcal{J}_i} = \sum_{j \in \mathcal{J}_i} R^{[i]}_j$. Then, rate-tuple $\left(R^{[1]}_{\mathcal{J}_1}, R^{[2]}_{\mathcal{J}_2}, \ldots, R^{[K]}_{\mathcal{J}_K}\right)$, is achievable if there exist encoders and decoders such that $\lambda^{[i]}_j(n) \to 0$, when $n \to \infty$ for all $i \in \mathcal{K}$, and $j \in \mathcal{J}_i$. Based on this multi-layer definition for encoding and decoding, we are able to present and evaluate the average achievable rates in Section III-D.

We further limit our study to the scenario in which cross channels vary at a much slower pace compared to direct channels, and a more general setting is considered in the extended version of this work [14]. Our channel model is motivated by the physics of wireless networks. Consider an interference channel in which receivers have random and time-varying physical locations, and each transmitter is equipped with an antenna with high fluctuation pattern gain. In Fig. 1, a receiver is in the vicinity of two transmitters (the intended and the interfering transmitters). The intended transmitter is typically closer to its receiver, and thus, the receiver observes a higher fluctuation rate for the intended signal.

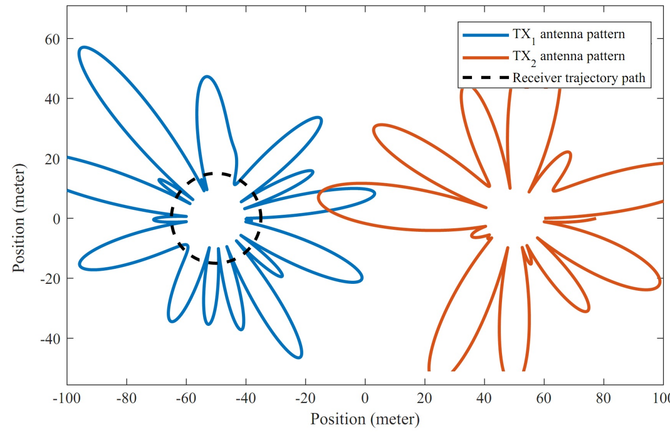

Fig. 1. A receiver moves around its corresponding transmitter and a second nearby transmitter creates interference at this receiver. The receiver experiences higher channel gain fluctuations from the corresponding transmitter.

III. Opportunistic Blind Interference Alignment

We now present our transmission strategy that opportunistically utilizes the channel variation pattern in order to align a part of interference signal power at the receivers. We note that the transmitters do not have access to the channel state information or the channel variation knowledge, and are only aware of the statistics of the channel parameters. We will deploy multi-layer superposition coding at the transmitters to maximize the average achievable sum-rate. More precisely, transmitters $\mathrm{TX}_k, 1 \leq k \leq K$, use a multi-layer encoding scheme with M distinct layers $l^{[k]}_i, 1 \leq i \leq M$. In this coding procedure, the transmit signal in each layer $l^{[k]}_i, 1 \leq i \leq M$ is represented by a Gaussian random variable with the length of n. As discussed later, these transmission layers are designed such that at each receiver, the decoder (based on the fluctuation pattern) is able to recover some parts of the transmitted data.

To present our Interference Alignment technique, we need the following lemma whose proof is presented in [14].

Lemma 1: Consider $\mathbf{V}$ is a nonzero elements random matrix with the size of $n \times d_v$ and the rank of d_v. Let, $\bar{\mathbf{H}}$ be a random diagonal matrix with the size of $n \times n$, we can conclude that:

$$\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) = \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right),$$

where $\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]$ is obtained by concatenating $\mathbf{V}$ and $\bar{\mathbf{H}}\mathbf{V}$.

A. Encoding

The encoding strategy has the following steps:

(1) Let $X_i^{[k]}$, $1 \leq i \leq M$, be M Gaussian distributed continuous random variables with limited variance for the i^{th} transmission layer at TX_k.

(2) At TX_k, we partition our message $W^{[k]}$ into M sub-messages of $\left(W_1^{[k]}, \cdots, W_M^{[k]}\right)$, and we send each sub-message via a single transmission layer. For each layer $l_i^{[k]}, 1 \leq i \leq M$, consisting of $\frac{n}{2}$ time snapshots, we consider a $\left(2^{nR_i^{[k]}}, \frac{n}{2}\right)$ code for the Gaussian channel where $W_i^{[k]} \in \{1, \ldots, 2^{nR_i^{[k]}}\}$, and we generate codeword of $x_{i1}^{[k]\frac{n}{2}}(W_i^{[k]})$ as follows:

$$x_{i1}^{[k]\frac{n}{2}}\left(W_i^{[k]}\right) = \left(x_{i1}^{[k]}(W_i^{[k]}), \ldots, x_{i\frac{n}{2}}^{[k]}\left(W_i^{[k]}\right)\right)^{\mathsf{T}}. \tag{4}$$

(3) TX_k computes $X_j^{[k]} = \sum_{i=1}^{M} x_{ij}^{[k]}(W_i^{[k]})$ of M independent layers to generate the following matrix:

$$\bar{\mathbf{x}}^{[k]} = \left[X_1^{[k]}, \ldots, X_{\frac{n}{2}}^{[k]}\right]^{\mathsf{T}}. \tag{5}$$

(4) Transmitters use a random full rank matrix of $\bar{\mathbf{V}}$ with the size of $n \times \frac{n}{2}$ as their precoder matrices. We can design $\bar{\mathbf{V}}$ such that $\lim_{n\to\infty} \frac{1}{n}\mathrm{tr}\left(\bar{\mathbf{V}}\bar{\mathbf{x}}^{[k]}(\bar{\mathbf{V}}\bar{\mathbf{x}}^{[k]})^H\right) = P_t$, where $\mathrm{tr}\left(\mathbf{A}\right)$ computes the trace of the square matrix $\mathbf{A}$.

(5) The output of the encoding function at TX_k, $\bar{\mathbf{X}}^{[k]}$, is an $n \times 1$ column vector with the following relation:

$$\bar{\mathbf{X}}^{[k]} = \bar{\mathbf{V}}\bar{\mathbf{x}}^{[k]}. \tag{6}$$

and for i^{th} layer we have the following constraint:

$$\lim_{n\to\infty} \frac{1}{n}\mathrm{tr}\left(\bar{\mathbf{V}}x_{i1}^{[k]\frac{n}{2}}\left(W_i^{[k]}\right)\left(\bar{\mathbf{V}}x_{i1}^{[k]\frac{n}{2}}\left(W_i^{[k]}\right)\right)^H\right) = P_i^{[k]}, \tag{7}$$

where $P_i^{[k]}$ indicates the total transmission power for i^{th} layer at TX_k and $\sum_{i=1}^{M} P_i^{[k]} = P_t$, which shows that the summation of transmission power for all of the layers is equal to P_t.

B. Analyzing the received signal

Fig. 2 shows a representation of the reception space at a receiver in a 3-user IC. In this case, we assume $p_{ij} >> p_{ii}, i \neq j$, meaning that the probability of changing channel value for the cross channels is much lower than the direct channels. The goal is to align the interference signals at each receiver, and depending on the value of $\mathcal{F}\left(\bar{\mathbf{H}}\right)$, the direct channel signal space has some free interference dimensions which are linearly independent from the interference signals. The decoder uses the following zero-forcing decoding matrix:

$$\bar{\mathbf{D}} = \left(\bar{\mathbf{I}} - \bar{\mathbf{V}}(\bar{\mathbf{V}}^H\bar{\mathbf{V}})^{-1}\bar{\mathbf{V}}^H\right). \tag{8}$$

If we assume during n transmission time snapshots, all cross links have a constant diagonal value, we have $\bar{\mathbf{D}}\bar{\mathbf{H}}^{[ij]}\mathbf{V}\bar{\mathbf{x}}^{[j]} = \bar{\mathbf{H}}^{[ij]}\bar{\mathbf{D}}\mathbf{V}\bar{\mathbf{x}}^{[j]} = \mathbf{0}$. Since all transmitters use the same precoder of $\bar{\mathbf{V}}$, for the encoding scheme of the previous subsection

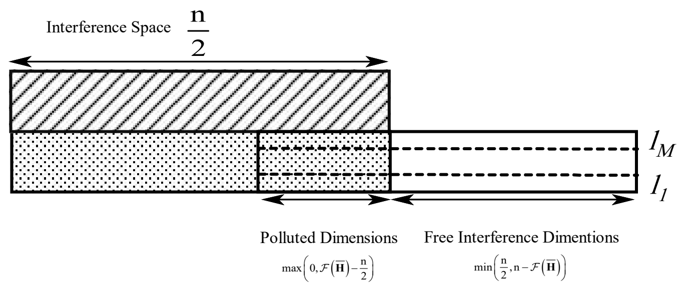

Fig. 2. The reception space at each receiver, prior to applying the zero-forcing decoding matrix, consists of three different subspaces: the desired interference-free subspace of dimension $\min\left(\frac{n}{2}, n - \mathcal{F}(\bar{\mathbf{H}})\right)$; the desired signal sub-space corrupted by interference signals of dimension $\max\left(0, \mathcal{F}(\bar{\mathbf{H}}) - \frac{n}{2}\right)$; the remaining interference sub-space. Note that we assume that all cross links have constant values.

where all transmitted signals have the same number of dimensions $\frac{n}{2}$, the number of free interference subspace dimensions at each receiver can be calculated from Lemma 1 as:

$$\begin{aligned}&\mathrm{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) - \mathrm{rank}\left(\left[\mathbf{V}\right]\right)\\ &= \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right) - d_v,\end{aligned} \tag{9}$$

substituting $d_v = \frac{n}{2}$ in the above equation, we have:

$$\mathrm{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) - \mathrm{rank}\left(\left[\mathbf{V}\right]\right)$$

$$= \min\left(\frac{n}{2} + \min\left(\frac{n}{2}, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right) - \frac{n}{2} \tag{10}$$

$$= \min\left(\min\left(\frac{n}{2}, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), \frac{n}{2}\right) \tag{11}$$

$$= \min\left(\frac{n}{2}, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), \tag{12}$$

where in the above relations $\bar{\mathbf{H}}$ represents the direct channel matrix.

C. Decoding

Without loss of generality, we consider the decoding operation at the first receiver. As mentioned earlier, we assume all channel matrices are known to their corresponding receivers. Furthermore, for simplicity and in order to provide a comparison to prior results that focus on DoF, we assume $\det\left(\bar{\mathbf{H}}\bar{\mathbf{H}}^H\right) = 1$. In other words, in this work, we ignore the potential gain of the exact knowledge of channel gain values and assume channel matrices give no power gain to the received signals during transmission block. With slight abuse of notation, we denote the set of jointly ϵ-typical sequences of length n by $\mathcal{T}_\epsilon^n$.

After applying zero forcing matrix, $\bar{\mathbf{D}}\mathbf{Y}_1^{[1]n}$ is available at the first receiver. Based on the following successive decoding strategy, we can decode part of data:

(1) First layer decoding: Based on $\bar{\mathbf{H}}^{[ij]}$ and the decoder matrix of $\bar{\mathbf{D}}$, the decoder finds that $\widehat{w}_1$ is sent if there exists a unique message such that 2-tuple $n-$sequence of $\left(\bar{\mathbf{D}}\bar{\mathbf{H}}^{[11]}\bar{\mathbf{V}}x_{11}^{[1]\frac{n}{2}}(\widehat{w}_1), \bar{\mathbf{D}}\mathbf{Y}_1^{[1]n}\right) \in \mathcal{T}_\epsilon^n$, otherwise it declares error in the first decoding step.

(2) Second layer decoding: After decoding the first layer and if such $\widehat{w}_1$ is found, the decoder tries

to find $\widehat{w}_2$ such that the 3-tuple $n-$sequence of $\left(\bar{\mathbf{D}}\bar{\mathbf{H}}^{[11]}\bar{\mathbf{V}}x_{11}^{[1]\frac{n}{2}}(\widehat{w}_1), \bar{\mathbf{D}}\bar{\mathbf{H}}^{[11]}\bar{\mathbf{V}}x_{21}^{[1]\frac{n}{2}}(\widehat{w}_2), \bar{\mathbf{D}}\mathbf{Y}_1^{[1]n}\right) \in \mathcal{T}_\epsilon^n$, otherwise it declares error in the second decoding stage.

(3) i-th layer decoding: After finding unique messages $\widehat{w}_1, \cdots, \widehat{w}_{i-1}$ in the previous steps, the decoder finds $\widehat{w}_i$ such that $(i+1)-$tuple of $n-$sequences $\left(\bar{\mathbf{D}}\bar{\mathbf{H}}^{[11]}\bar{\mathbf{V}}x_{11}^{[1]\frac{n}{2}}(\widehat{w}_1), \ldots, \bar{\mathbf{D}}\bar{\mathbf{H}}^{[11]}\bar{\mathbf{V}}x_{i1}^{[1]\frac{n}{2}}(\widehat{w}_i), \bar{\mathbf{D}}\mathbf{Y}_1^{[1]n}\right) \in \mathcal{T}_\epsilon^n$, otherwise it declares error in the i-th decoding step.

The error analysis of this decoding approach is presented in [14] where we show we can set the following transmission rate for i-th transmission layer:

$$nR_i^{[1]} \leq \frac{i}{2}\log\left(1+\frac{P_i^{[1]}}{N+\sum_{j=i+1}^{M}P_j^{[1]}}\right),\ M=\frac{n}{2} \tag{13}$$

where N is the noise power, and we treat interference from all layers that are not yet decoded as noise, also the value of i indicates the number of free interference dimension at the receiver which is appeared in multiplication form in the above relation. Each layer which is decoded at receiver should be subtract from the received signal to decode the next layer.

Note: We assume that the channel matrices and the zero-forcing operation add no gain to decoding or have the same impact on signals and noise. Therefore, for the sake of simplicity, we omit their impact in our analysis.

Depending on channel realizations, each receiver will be able to decode part of its intended data. More precisely, if the number of interference-free dimensions at the receiver is l_s, we except the receiver to be able to decode the messages of layers 1 to l_s. Based on (12), in the next subsection, we will divide the total transmission power among $M=n/2$ layers in order to maximize the average transmission rate.

D. Maximizing the average transmission rate

Based on the argument presented in the previous subsection, the average transmission rate is given by:

$$\bar{R}_{\text{avg}} = \frac{1}{n}\sum_{i=1}^{\frac{n}{2}}\frac{i}{2}\log\left(1+\frac{P_i^{[1]}}{N+\sum_{j=i+1}^{M}P_j^{[1]}}\right)F_I\left(\frac{n}{2}-i\right) \tag{14}$$

where

$$F_I\left(\frac{n}{2}-i\right) = p\left(\min\left(\frac{n}{2}, n-\mathcal{F}\left(\bar{\mathbf{H}}\right)\right)\geq i\right). \tag{15}$$

For large n, $\frac{i}{n}, 1\leq i\leq\frac{n}{2}$ and $P_i^{[1]}$ can be approximated by continues parameters of $0\leq z\leq\frac{1}{2}$ and $\rho(z)dz$, respectively.

We set $P(1/2-z) = \int_z^{\frac{1}{2}}\rho(x)dx$, and thus, $P(0) = \int_{\frac{1}{2}}^{\frac{1}{2}}\rho(x)dx = 0$ and $P(1/2) = \int_0^{\frac{1}{2}}\rho(x)dx = P_t$. Therefore, for the large values of n, (14) can be approximated as:

$$\bar{R}_{\text{avg}} = \frac{1}{2\ln 2}\int_0^{\frac{1}{2}}\frac{z\rho(1/2-z)F_Z(1/2-z)}{N+P(1/2-z)}dz \tag{16}$$

where $F_Z(1/2-z) = F_I(n/2-\lfloor nz\rfloor),\ 0\leq z\leq 1/2$ and $\rho(z) = dP(z)/dz$. We note that as the number of layers $M = n/2$ increases, our approximation becomes more accurate.

The following theorem helps us maximize the above average transmission rate. For simplifying our notation, we substitute $\frac{1}{2}-z$ with u to get:

$$\bar{R}_{\text{avg}} = \frac{1}{2\ln 2}\int_{\frac{1}{2}}^{0}\frac{-(1/2-u)\rho(u)F_U(u)}{N+P(u)}du \tag{17}$$

$$= \frac{1}{2\ln 2}\int_0^{\frac{1}{2}}\frac{(1/2-u)\rho(u)F_U(u)}{N+P(u)}du. \tag{18}$$

Theorem 3.1: A necessary condition for function $y(u)$ to be an extremum of:

$$\int_{z_1}^{z_2}D\left(y,y',u\right)du, \tag{19}$$

is that y satisfies the following Euler differential equation:

$$D_y - \frac{dD_{y'}}{du} = 0,\ z_1\leq u\leq z_2. \tag{20}$$

where the subscripts denote the partial derivatives with respect to corresponding arguments [15], [16].

In our problem, we have:

$$D(y,y',u) = \frac{1}{2\ln 2}\frac{(1/2-u)\rho(u)F_U(u)}{N+P(u)} \tag{21}$$

where $y(u) = P(u)$ and $y'(u) = \rho(u)$. We have:

$$D_y = \frac{1}{2\ln 2}\frac{-(1/2-u)\rho(u)F_U(u)}{(N+P(u))^2} \tag{22}$$

$$D_{y'} = \frac{1}{2\ln 2}\frac{(1/2-u)F_U(u)}{N+P(u)}. \tag{23}$$

Thus, we get

$$\frac{dD_{y'}}{du} = \frac{(-F_U(u)+(1/2-u)f_U(u))(N+P(u))}{2\ln 2(N+P(u))^2}, \tag{24}$$

where $f_U(u) = \frac{dF_U(u)}{du}$, and from the first theorem we have:

$$\frac{dP(u)}{N+P(u)} = -\frac{(-F_U(u)+(1/2-u)f_U(u))du}{(1/2-u)F_U(u)}, \tag{25}$$

and finally:

$$\ln(N+P(u)) = -\ln((1/2-u)F_U(u)) + C. \tag{26}$$

Therefore:

$$P(1/2-z) = \frac{C}{zF_Z(1/2-z)} - N, \tag{27}$$

where $P(1/2-z=0) = 0$. Setting $z=1$, we get $C = N\ F_Z(0)/2$, and function $P(z)$ can be calculated as:

$$P(z) = \frac{N}{2(1/2-z)F_Z(z)}F_Z(0) - N. \tag{28}$$

We note that when $P(z_0)\geq P_t$ then $P(z>z_0) = P_t$. Fig. 3 depicts numerical evaluation of $F_Z(z)$ for precoder length of $n=20$, and the altering probability of $p_{ii}=0.9$. Given $F_Z(z)$, we can calculate $P(z)$ (as in Fig. 4), and then, the transmission power for each transmission layer can be calculated as:

$$P_j^{[1]} = P_Z\left(\frac{j-1}{n}\right) - P_Z\left(\frac{j}{n}\right). \tag{29}$$

For $P_t = 100$, $n=20$ and $N=1$, the total transmission power for each transmission layer is depicted in Fig. 5.

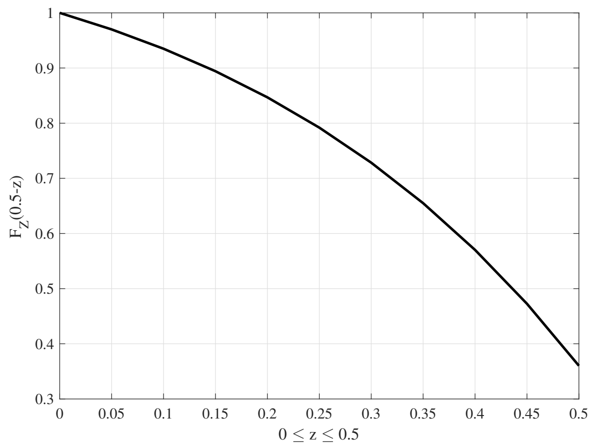

Fig. 3. Numerical evaluation of the cumulative distribution function $F_Z(z)$ from $F_I(i)$. See [14] for more details.

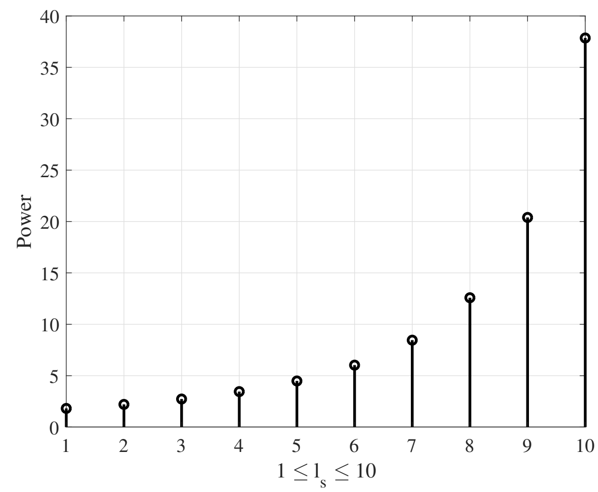

Fig. 5. Dedicated power to each transmission layer for $n = 20$ and $p_{11} = 0.9$.

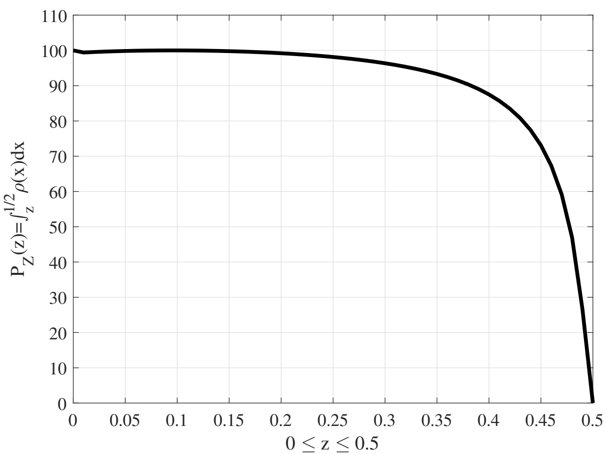

Fig. 4. $P_Z(z)$ as a function of $0 \leq z \leq \frac{1}{2}$ using $F_Z(z)$ in Fig. 3 for in which $p_{11} = 0.9$ and $P_t^{[1]} = 100$.

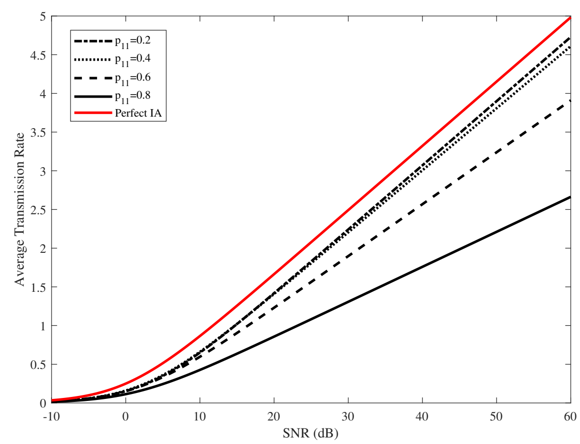

Fig. 6. Average achievable rate for each user in the $K-$user IC for $p_{ii} = \{0.2, 0.4, 0.6, 0.8\}$, constant-value cross links, and $n = 20$. The red solid line indicates the achievable rates for IA with perfect CSI.

E. Numerical Results

In this subsection, to demonstrate the gain of our proposed strategy, we consider a relatively short transmission block-length, $n = 20$, and numerically evaluate the results. Fig. 6 shows the average achievable rates for different values of p_{ii} using our strategy. This figure indicates that when p_{ii} decreases and direct channels have a higher variation rate, the chance of finding proper channel states with more interference-free dimensions increases. In other words, when the direct channels have higher variation probability, we can achieve a higher average transmission rate. This figure also includes the achievable rates of IA with perfect CSI. Although, IA with perfect CSI is meant for asymptotic analysis, *i.e.* degrees-of-freedom, as discussed in Section III-C, we eliminated the channel gains from our analysis, therefore, allowing a comparison to this perfect CSI case.

IV. Conclusion

We proposed a new blind interference alignment strategy for the K-user ICs that exploits the radiation pattern fluctuations of transmit antennas. Our strategy includes a multi-layer encoding strategy at the transmitters, an interference alignment phase and a successive decoding phase at each receiver. Our strategy does not rely on any channel state information beyond the CDF of channel variations. We evaluated the achievable rates numerically and compared it to the performance of IA techniques with perfect instantaneous CSI at the transmitters. We divided the total transmission power among many layers to achieve the highest average achievable sum rate. The next steps include obtaining a closed-form expression for the achievable rates, and extending the results to the scenario in which cross links vary at similar rates when compared to direct links.

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