Finite Field X-Channels with Delayed CSIT and Common Messages

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***Abstract*—We establish the capacity region of the two-user finite field X-Channels with delayed channel state information at the transmitters. We consider the most general case in which each transmitter has a common message for both receivers and a private message for each one of them. We derive a new set of outer-bounds for this problem that rely on an extremal entropy inequality. This inequality quantifies the ability of each transmitter in favoring one receiver over the other in terms of delivered entropy when both receivers must obtain some baseline entropy. We also propose a transmission strategy that harvests the delayed channel state information to combine and to repackage previously communicated signals in order to deliver them efficiently. We show that this transmission strategy matches the outer-bounds.**

***Index Terms*—X-Channel, finite-field model, capacity region, interference channel, delayed CSIT.**

I. Introduction

Alongside the two-user interference channel [1]–[5], the two-user X-Channel is a canonical example to study the impact of interference in communication networks. In the X-Channel, each transmitter has a *common message* intended for both receivers as well as a *private message* intended for each receiver. This problem has been widely studied in the literature and several interference management techniques have been proposed [6]–[8]. For instance under instantaneous channel state information (CSI) model, it was shown in [7] that interference alignment can provide a gain over baseline techniques (*e.g.*, orthogonalization). This gain is expressed in terms of degrees-of-freedom (DoF) which captures the asymptotic behavior of the network normalized by the capacity of the point-to-point channel when power tends to infinity.

Attaining instantaneous channel state information at the transmitters (CSIT) in many real-world scenarios may not be feasible. In such cases, a more realistic model is the delayed CSIT in which by the time the CSI arrives at the transmitters, the channel has already changed to a new state. Under the delayed CSIT model, authors in [9] developed a scheme that achieves $6/5$ DoF. Later, it was shown that if we limit ourselves to *linear* encoding functions, then $6/5$ is indeed the optimal DoF [10]. These results provide valuable insight into the behavior of X-Channels. However, in information and communication theory, the ultimate goal is to understand the behavior of wireless networks for any signal-to-noise ratio (SNR). In other words, we are interested in capacity results rather than DoF-type results. Moreover, while one might argue that most practical communication protocols are linear, limiting the encoding functions to be linear removes the majority of potential encoding functions and from an information-theoretic perspective, this is not desirable. Finally, authors in [9] and [10] study a subset of X-Channels in which transmitters only have private messages for the receivers and the issue of common messages in X-Channels is not addressed.

In this work we address these issues by deriving the capacity region of X-Channels with delayed CSIT and common messages under a finite field fading model introduced in [11], [12]. To derive the outer-bounds, we rely on an extremal entropy inequality to capture both the impact of delayed CSIT and the issue of delivering the common messages. This inequality quantifies the ability of a transmitter to favor one receiver over the other in terms of provided entropy when: (1) both receivers need to obtain some common entropy, and (2) the transmitter has access to the delayed channel state information. The extremal entropy inequality in this work extends a similar inequality in [11] to capture the impact of common messages on the capacity region of X-Channels.

To achieve the outer-bounds, we treat the X-Channel as a combination of a number of well-known problems for which the capacity region is known. By adjusting different rates for the X-Channel, we can recover several other problems such as the interference channel, the multicast channel, the broadcast channel, and the multiple-access channel. We demonstrate how to utilize the capacity-achieving strategies of such problems in a systematic way in order to achieve the capacity region of the X-Channel. We show that, however, if we treat the X-Channel as a number of disjoint sub-problems, we will not achieve the capacity, and in some regimes we need to interleave the capacity-achieving strategies of different sub-problems and execute them simultaneously.

The rest of the paper is organized as follows. In Section II we formulate the problem. In Section III we present our main results and provide some insights. Sections IV and V are dedicated to the proof of the main results. Section VI concludes the paper.

II. Problem Formulation

We consider the two-user Binary Fading X-Channel as illustrated in Fig. 1 with two transmitters and two receivers. In the binary fading model, the channel gain from transmitter Tx_j to receiver Rx_i at time t is denoted by $G_{ij}[t]$, $i, j \in \{1, 2\}$.

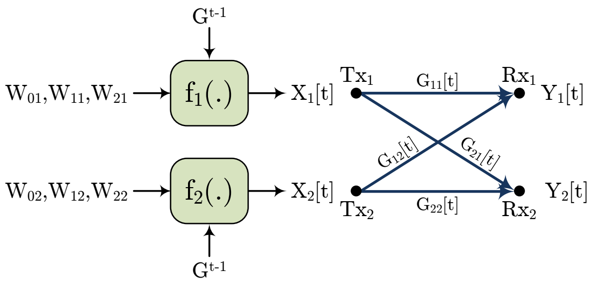

Fig. 1. Two-user Binary Fading X-Channel. All signals and the channel gains are in the binary field.

The channel gains are either 0 or 1 (*i.e.* $G_{ij}[t] \in \{0,1\}$), and they are distributed as independent Bernoulli random variables (independent across time and space). We consider the homogeneous setting in which

$$G_{ij}[t] \overset{d}{\sim} \mathcal{B}(p), \qquad i,j = 1,2, \tag{1}$$

for $0 \leq p \leq 1$, and we define $q \triangleq 1-p$.

At each time t, the transmit signal of Tx_j is denoted by $X_j[t] \in \{0,1\}$, and the received signal at Rx_i is given by

$$Y_i[t] = G_{ii}[t]X_i[t] \oplus G_{i\bar{i}}[t]X_{\bar{i}}[t], \quad i=1,2, \tag{2}$$

where the summation is in $\mathbb{F}_2$, and $\bar{i} = 3-i$.

We define the channel state information (CSI) at time t to be the quadruple

$$G[t] \triangleq \{G_{11}[t], G_{12}[t], G_{21}[t], G_{22}[t]\}. \tag{3}$$

In this work, we consider the delayed CSIT model in which at time t each transmitter has the knowledge of the channel state information up to the previous time instant (*i.e.* G^{t-1}) and the distribution from which the channel gains are drawn (*i.e.* $\mathcal{B}(p)$), $t = 1,2,\ldots,n$. Since receivers only decode the messages at the end of the communication block, without loss of generality, we assume that the receivers have instantaneous knowledge of the CSI. We consider the scenario in which Tx_j, $j=1,2$, wishes to reliably communicate

1) message $W_{0j} \in \{1,2,\ldots,2^{nR_{0j}}\}$ to both receivers,
2) message $W_{1j} \in \{1,2,\ldots,2^{nR_{1j}}\}$ to Rx_1,
3) and message $W_{2j} \in \{1,2,\ldots,2^{nR_{2j}}\}$ to Rx_2,

during n uses of the channel. We assume that the messages and the channel gains are *mutually* independent and the messages are chosen uniformly at random.

For transmitter Tx_j, let messages W_{0j}, W_{1j}, and W_{2j} be encoded as X_j^n using the encoding function $f_j(.)$, which depends on the available CSI at Tx_j, see Fig. 1. Receiver Rx_i is interested in decoding W_0 and W_i given by

$$W_0 \triangleq (W_{01}, W_{02}), \ W_i \triangleq (W_{i1}, W_{i2}), \tag{4}$$

and it will decode the messages using the decoding function $(\widehat{W}_0, \widehat{W}_i) \triangleq g_i(Y_i^n, G^n)$. An error occurs when $\left(\widehat{W}_0, \widehat{W}_i\right) \neq (W_0, W_i)$. The average probability of decoding error is given by

$$\lambda_{i,n} \triangleq \mathbb{E}[P[\left(\widehat{\mathrm{W}}_0, \widehat{\mathrm{W}}_i\right) \neq (\mathrm{W}_0, \mathrm{W}_i)]], \qquad i=1,2, \tag{5}$$

where the expectation is taken with respect to the random choice of messages.

We define

$$R_0 \triangleq R_{01} + R_{02}, \ R_1 \triangleq R_{11} + R_{12}, \ R_2 \triangleq R_{21} + R_{22}. \tag{6}$$

A rate tuple (R_0, R_1, R_2) is said to be achievable, if there exists encoding and decoding functions at the transmitters and the receivers respectively, such that the decoding error probabilities $\lambda_{1,n}, \lambda_{2,n}$ go to zero as n goes to infinity. The capacity region is the closure of all achievable rate tuples.

III. Main Results

In this section, we present the capacity region of the two-user Binary Fading X-Channel under the delayed CSIT assumption, and we provide some technical insights and interpretations of the main results.

Theorem 1. *The capacity region, $\mathcal{C}$, of the two-user Binary Fading X-Channel with private and common messages under delayed CSIT assumption as described in Section II is the set of all rates satisfying:*

$$\begin{cases} \text{BC Bounds}: & 0 \leq R_{ij} + \beta\left(R_{\bar{i}j} + R_{0j}\right) \leq \beta p, \\ \text{XC Bounds}: & R_i + \beta\left(R_{\bar{i}} + R_0\right) \leq \beta\left(1-q^2\right), \end{cases} \tag{7}$$

for $i,j \in \{1,2\}$, R_0, R_1, R_2 defined in (6), and

$$\beta = 2-p. \tag{8}$$

The capacity region is described by two sets of outer-bounds. The first set is referred to as the Broadcast Channel (BC) bounds. These bounds describe the capacity region of the broadcast channel formed by one of the transmitters and the two receivers assuming the other transmitter is eliminated. These bounds can be thought of as the generalization of the results in [13]–[15] for the two-user case to include common messages. The second set is referred to as the X-Channel (XC) bounds which cannot be obtained from the BC bounds.

The derivation of the outer-bounds relies on an extremal entropy inequality that quantifies the ability of each transmitter in favoring one receiver over the other in terms of the available entropy subject to two constraints: (1) both receivers need to obtain a baseline entropy (to capture the common messages), and (2) transmitters have access to the delayed CSI. This inequality characterizes the limit to which the unwanted subspace at one receiver can be scaled down while the desired subspace at the other receiver is maximized. We use this inequality and a genie-aided argument to derive the new outer-bounds.

The two-user X-Channel can be thought of as a generalization and a combination of several well-known problems. For instance, if R_{11} and R_{12} are the only non-zero rates, then the problem is equivalent to the multiple-access channel formed at Rx_1, and if R_{12} and R_{22} are the only non-zero rates, then the problem is equivalent to the broadcast channel formed by Tx_2. We demonstrate how to utilize the capacity-achieving strategies of other problems, such as the Interference Channel and the Multicast Channel, in a systematic way in order to achieve the capacity region of the X-Channel. We show that, however, if we treat the X-Channel as a number of disjoint sub-problems, we will not achieve the capacity in some regimes. In fact, in such regimes, we need to interleave the capacity-achieving strategies of different sub-problems and execute them simultaneously.

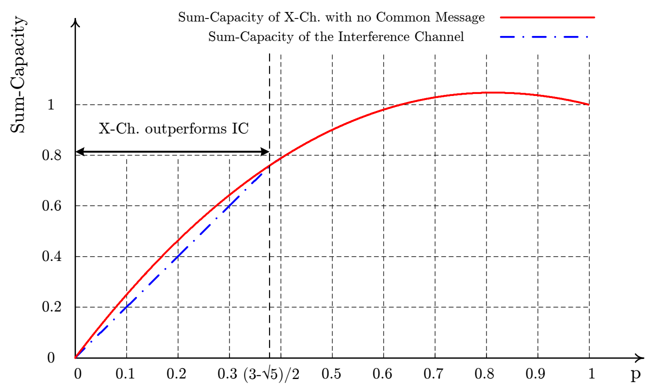

Fig. 2. *The sum-capacity of the X-Channel vs. that of the Interference Channel. To have a fair comparison, in the X-Channel we set* $R_0 = 0$.

An important difference between the X-Channel and the Interference Channel is the fact that in the latter scenario, the individual rates are limited by the capacity of a point-to-point channel, *i,e,* p. As a result, for the Interference Channel we have [11]:

$$\sup\left(R_1 + R_2\right) = \min\left\{2p, \frac{2\beta\left(1-q^2\right)}{1+\beta}\right\}. \tag{9}$$

However, in the X-Channel no such limitation exists, and we have

$$\sup\left(R_1 + R_2\right) = \frac{2\beta\left\{\left(1-q^2\right) - R_0\right\}}{1+\beta}. \tag{10}$$

The difference is depicted in Fig. 2 for $R_0 = 0$. This means that if we naively try to use the capacity-achieving strategies of the sub-problems, we cannot achieve the capacity region of the X-Channel. The key idea to overcome this challenge is to interleave different strategies and execute them simultaneously. For more details, we refer the reader to [16].

IV. Converse Proof of Theorem 1

In this section we provide the converse proof of Theorem 1. The proof of the BC bounds has some similarities to that of the XC bounds and are presented in [16].

XC Bounds: To derive these bounds, we have

$$\begin{aligned}
&n\left(R_1 + \beta\left(R_2 + R_0\right)\right) = H\left(W_1\right) + \beta\left\{H\left(W_2\right) + H\left(W_0\right)\right\} \\
&\stackrel{(a)}{=} H\left(W_1|W_0, W_2\right) + \beta\left\{H\left(W_2\right) + H\left(W_0|W_2\right)\right\} \\
&\stackrel{(b)}{\leq} I\left(W_1; Y_1^n|W_0, W_2, G^n\right) + \beta\left\{I\left(W_2; Y_2^n|G^n\right)\right. \\
&\left.+ I\left(W_0; Y_2^n|W_2, G^n\right)\right\} + n\epsilon_n \\
&= H\left(Y_1^n|W_0, W_2, G^n\right) - \underbrace{H\left(Y_1^n|W_0, W_1, W_2, G^n\right)}_{=0} \\
&+ \beta H\left(Y_2^n|G^n\right) - \beta\left\{H\left(Y_2^n|W_2, G^n\right)\right. \\
&\left.- I\left(W_0; Y_2^n|W_2, G^n\right)\right\} + n\epsilon_n \\
&\stackrel{(d)}{\leq} \beta H\left(Y_2^n|G^n\right) + n\epsilon_n \stackrel{(e)}{\leq} n\beta(1-q^2) + n\epsilon_n.
\end{aligned} \tag{11}$$

where $\epsilon_n \to 0$ as $n \to \infty$; (a) follows from the independence of messages; (b) follows from Fano's inequality and the fact that messages are independent of channel realizations; (d) follows from Claim 1 below; (e) holds since

$$H\left(Y_2^n|G^n\right) \leq \sum_{t=1}^{n} H\left(Y_2[t]|G^n\right) \leq n(1-q^2). \tag{12}$$

Dividing both sides by n and letting $n \to \infty$, we get

$$R_1 + \beta\left(R_2 + R_0\right) \leq \beta(1-q^2). \tag{13}$$

Similarly, we can obtain the other XC bound.

Claim 1. *For the two-user Binary Fading X-Channel with private and common messages under delayed CSIT assumption as described in Section II, we have*

$$\begin{aligned}
&H\left(Y_1^n|W_0, W_2, G^n\right) \\
&- \beta\left\{H\left(Y_2^n|W_2, G^n\right) - I\left(W_0; Y_2^n|W_2, G^n\right)\right\} \leq 0.
\end{aligned} \tag{14}$$

Proof. We first note that

$$\begin{aligned}
&H\left(Y_2^n|W_0, W_2, G^n\right) \\
&= H\left(Y_2^n|W_2, G^n\right) - I\left(W_0; Y_2^n|W_2, G^n\right).
\end{aligned} \tag{15}$$

Thus, proving (14) is equivalent to proving

$$H\left(Y_1^n|W_0, W_2, G^n\right) - \beta H\left(Y_2^n|W_0, W_2, G^n\right) \leq 0. \tag{16}$$

We have

$$\begin{aligned}
&H\left(Y_2^n|W_0, W_2, G^n\right) \\
&\stackrel{(a)}{=} \sum_{t=1}^{n} H\left(Y_2[t]|Y_2^{t-1}, W_0, W_2, G^t\right) \\
&\stackrel{(b)}{=} \sum_{t=1}^{n} pH\left(X_1[t]|Y_2^{t-1}, W_0, W_2, G_2[t]=1, G_1[t], G^{t-1}\right) \\
&\stackrel{(c)}{=} \sum_{t=1}^{n} pH\left(X_1[t]|Y_2^{t-1}, W_0, W_2, G^t\right) \\
&\stackrel{(d)}{\geq} \sum_{t=1}^{n} pH\left(X_1[t]|Y_1^{t-1}, Y_2^{t-1}, W_0, W_2, G^t\right) \\
&\stackrel{(e)}{=} \sum_{t=1}^{n} \frac{1}{\beta} H\left(Y_1[t], Y_2[t]|Y_1^{t-1}, Y_2^{t-1}, W_0, W_2, G^t\right) \\
&\stackrel{(f)}{=} \sum_{t=1}^{n} \frac{1}{\beta} H\left(Y_1[t], Y_2[t]|Y_1^{t-1}, Y_2^{t-1}, W_0, W_2, G^n\right) \\
&= \frac{1}{\beta} H\left(Y_1^n, Y_2^n|W_0, W_2, G^n\right) \stackrel{(g)}{\geq} \frac{1}{\beta} H\left(Y_1^n|W_0, W_2, G^n\right),
\end{aligned} \tag{17}$$

where (a) follows from the fact that all signals at time t are independent of future channel realizations; (b) holds since $\Pr(G_2[t]=1)=p$; (c) is true since transmit signal $X_1[t]$ is independent of the channel realization at time t; (d) holds since conditioning reduces entropy; (e) holds since $\Pr(G_1[t]=G_2[t]=0)=q^2$ and the definition of β; (f) is true since all signals at time t are independent of future channel realizations; (g) follows from the chain rule and the non-negativity of the entropy function for discrete random variables. □

This completes the converse proof of Theorem 1, and in the following section we present the achievability proof.

V. Achievability Proof of Theorem 1

In the previous section we developed a set of new outer-bounds for this problem. In this section we show that a careful combination of the capacity-achieving strategies for other known known problems will achieve the capacity region of the X-Channel. However, we show that if we simply treat the X-Channel as a number of disjoint sub-problems, we will not achieve the capacity, and in some regimes we need to interleave the capacity-achieving strategies of different sub-problems and execute them simultaneously. This issue is discussed later in this section. For more details, we refer the reader to [16].

To describe the key idea in our transmission strategy, we present two examples. The general scheme is presented in [16].

Example 1: Symmetric Sum-Rate: Suppose for $p=0.5$, we wish to achieve

$$\begin{aligned} &R_{01}=R_{02}=1/8, \\ &R_{11}=R_{12}=R_{21}=R_{22}=0.15. \end{aligned} \tag{18}$$

To achieve the capacity region, we treat the X-Channel as three separate problems at different times:

- For the first third of the communication block, we treat the X-Channel as a two-user multicast channel as depicted in Fig. 3(a) in which each transmitter has a message for *both* receivers. For the two-user multicast channel with fading parameter $1/2$, the capacity region matches that of the multiple-access channel formed at each receiver [11] and depicted in Fig. 3(a) as well.
- For the second third of the communication block, we treat the X-Channel as a two-user interference channel in which Tx_j wishes to communicated with Rx_j, see Fig. 3(b). The capacity region of this problem is given in [11] and depicted in Fig. 3(b).
- During the final third of the communication block, we treat the X-Channel as a two-user interference channel with swapped IDs in which Tx_j wishes to communicated with $\mathsf{Rx}_{\bar{j}}$, see Fig. 3(c). In the homogeneous setting of this work, the capacity region of this interference channel with swapped IDs matches that of the previous case and is depicted in Fig. 3(c).

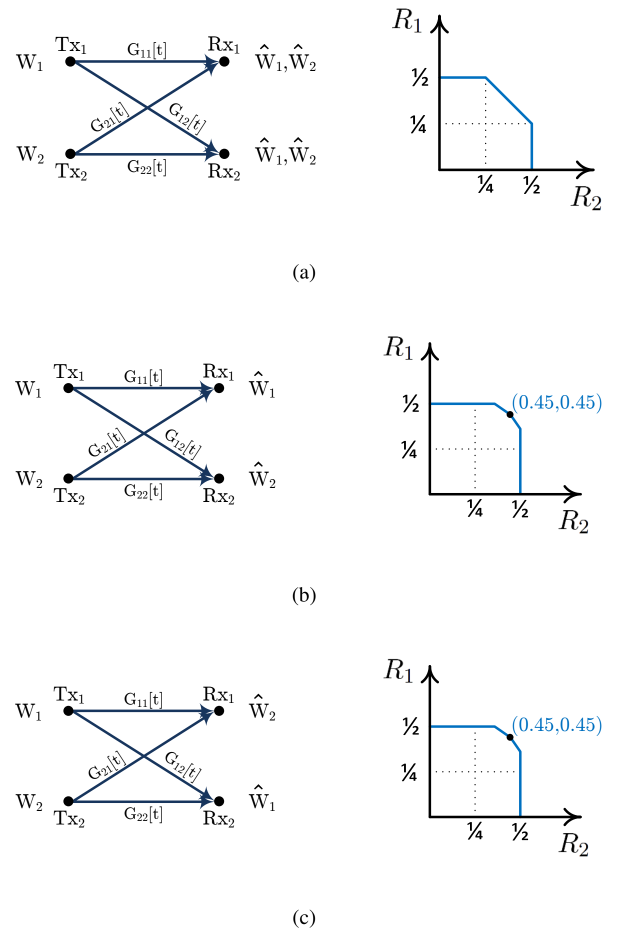

Fig. 3. *(a) The two-user multicast channel and its capacity region, the capacity can be achieved without using the delayed CSI; (b) The two-user binary fading interference channel and its capacity region; (c) The two-user binary fading interference channel with swapped IDs and its capacity region.*

Achievable Rates: We note that as the communication block length, n, goes to infinity, so do the communication block lengths for each sub-problem. Thus, during the first third of the communication block, we can achieve symmetric common rates arbitrary close to $(3/8, 3/8)$. Normalizing to the total communication block, we achieve $(R_{01}, R_{02}) = (1/8, 1/8)$ which matches the requirements of (18). From [11] we know that for the two-user binary fading interference channel with delayed CSIT and $p=1/2$, we can achieve symmetric rates of $(0.45, 0.45)$. Normalizing to the total communication block, we achieve $(R_{11}, R_{22}) = (0.15, 0.15)$ which matches the requirements of (18). Finally, during the final third of the communication block we treat the problem as a two-user interference channel with swapped IDs in which we can achieve symmetric rates of $(0.45, 0.45)$. Normalizing to the total communication block, we achieve $(R_{21}, R_{12}) = (0.15, 0.15)$ which again matches the requirements of (18). Thus, with splitting up the X-Channel into a combination of three known sub-problems, we can achieve the capacity region described in Theorem 1.

Example 2: Unequal Rates: In the previous subsection we focused on a symmetric setting. Here, we discuss a scenario

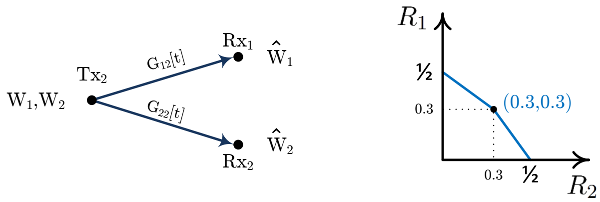

Fig. 4. *The two-user broadcast channel with delayed CSIT and private messages, and its capacity region.*

in which transmitters have different types of messages with different rates for each receiver. More precisely, we consider $p = 0.5$, and

$$R_{01} = 1/2, \qquad R_{02} = 0,$$
$$R_{11} = R_{21} = 0, \qquad R_{12} = R_{22} = 0.15, \tag{19}$$

In this case, we can think of the X-Channel in this case as two sub-problems that *coexist* at the same time as described below.

- The Binary Fading Broadcast Channel from Tx_1 in which a single message is intended for both receivers. For this problem, the capacity can be achieved using a point-to-point erasure code of rate $1/2$.
- The Binary Fading Broadcast Channel from Tx_2 as in Fig. 4 with delayed CSIT in which the transmitter has a private message for each receiver. For this problem, the capacity region is given in [13], [14] and depicted Fig. 4. As described below, in order to be able to decode the messages in the presence of the Broadcast Channel from Tx_1, we first encode W_{12} and W_{22} using point-to-point erasure codes of rate $1/2$, and treat the resulting codes as the input messages to the Broadcast Channel of Fig. 4.

Achievable Rates: At each receiver the received signal from Tx_2 is corrupted (erased) half of times by the signal from Tx_1. As a result, when we implement the capacity-achieving strategy of [13], [14], we only deliver half of the bits intended for each receiver. However, since we first encode W_{12} and W_{22} using point-to-point erasure codes of rate $1/2$, obtaining half of the bits is sufficient for reliable decoding of W_{12} and W_{22}. Thus, we achieve

$$(R_{12}, R_{22}) = \frac{1}{2}(0.3, 0.3) = (0.15, 0.15), \tag{20}$$

which again matches the requirements of (19). At the end of the communication block, receivers decode W_{12} and W_{22}, and remove the contribution of Tx_2 from their received signals. After removing the contribution of X_2^n, the problem is identical to the Broadcast Channel from Tx_1 as in Fig. 4(a) for which we can achieve a common rate R_{02} of $1/2$.

VI. Conclusion

We established the capacity region of finite-field fading X-Channels with common messages and delayed CSIT. We presented a new set of outer-bounds for this problem that relied on an extremal entropy inequality developed specifically for this problem. We then showed how the outer-bounds can be achieved by treating the X-Channel as a combination of a number of well-known problems such the interference channel and the multicast channel.

An important future work is to study Gaussian X-Channels with delayed CSIT. One approach could be to extend our results to the multi-layer finite-field fading setting similar to [17] and then, derive the capacity region of the Gaussian X-Channels to within a constant number of bits. One could also follow the steps taken in [18] to generalize the results to larger finite fields.

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