BIA with Embedding Information in Radiation Pattern Fluctuations

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Abstract

In this paper, we propose blind interference alignment (BIA) schemes using channel variations to use BIA scheme and achieve high diversity gain for the $K-$user interference channels (IC). At first using channel variation pattern, in an opportunistic manner the transmitters wait for proper channel conditions to transmit their data. The more direct channel variations compared with cross channels give us more chances to align most of the interference signal power. The core of this idea is based on dividing the transmission time into some distinct blocks and selecting a subset of transmitters and their corresponding receivers that have proper conditions to align interference signals. To increase the chance of alignment conditions for our scheme, we propose a new transmitter antenna structure where its pattern has a lot of fluctuations. This antenna pattern structure leads to increase the chance of channel variations for the direct links (desired links) where generally have lower distances in compare with cross channels (interference links). Finally, for a causal and more practical case where there exists no feedback channel to inform transmitter regarding the channel variations, we propose a multi-layer encoding strategy to maximize the average transmission rate. We show that using a simple antenna structure at transmitters and without additional hardware complexity our method can improve the average achievable transmission rate.

Index Terms

Blind interference alignment (BIA), average achievable sum rate, channel state information (CSI), channel changing pattern, multi-layer encoding.

I. Introduction

The interference channel (IC) is a communication network with many pairs of transceivers. Many wireless networks can be categorized as a type of IC networks, e.g., cellular communication networks and wireless local area networks. Today, due to the increasing number of users and their request for higher quality services such as HD voice and video connection, there is an increasing demand for higher data rate, especially in the age of 5G which introduces boundless connectivity, intelligent automation and internet of things (IoT). Therefore, there is a lot of competition among many service provider companies to attract users with a higher data rate and the lower price. One of the great barriers for this vision is the interference which motivates researchers to find solutions for this problem. Accordingly, using any method to reduce interference effects on the communication rate and improving bandwidth assignment for the users is an essential field of research in the wireless networks.

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One of the approaches which mitigate the effect of interference is time and frequency division medium access scheme, also known as orthogonal access scheme, which divides the entire transmission signal among orthogonal dimensions. Therefore, each user can use a part of signaling dimension which has no overlap with other user signaling space. Another approach to improve channel spectral efficiency and achieve higher data rate is to provide full cooperation either among the transmitters or the receivers [1], [2]. As an example, employing full cooperation among the transmitters can lead to reducing the system to a single MIMO broadcast channel with higher channel capacity. But using the full cooperation needs joint processing and data sharing over distinct nodes. It seems to be infeasible in practical scenarios to provide full cooperation among transmitters and receivers. In recent years there has been a lot of interest in the main idea of interference alignment (IA) and its potential benefits [3], [4]. Degrees of freedom (DoF) or capacity pre-log can characterize the capacity behavior of IC networks at high SNR regime.

Due to the simple intuitive concepts of the term DoF, many recent research activities aimed at characterizing the DoF region of wireless networks. As an outstanding work [3], for the $K-$user fast fading IC by the aid of IA, authors have shown that one can achieve the sum DoF of $\frac{K}{2}$. This achievement theoretically shows a major improvement compared to the trivial time or frequency division multiple access schemes where totally achieve the sum DoF of one. This has drawn the attention of researchers interest to the novel insights, especially those which are related to perfect IA that changes the effects of many interference signals to the one interference signal. Although IA has a lot of benefits, unfortunately there exist some problems and barriers to convert this method into a feasible one. The assumption of perfect CSI, in some cases global CSI for the $K-$user IC, the need of fast fading channel and long precoder lengths at transmitters are just a bunch of problems that we are engaged with [5]- [7]. The depth of such problems is so high that practically we cannot use and implement the benefits of the IA method.

In fact, in the case of blind channel knowledge, it is well known that the sum DoF of many networks collapse entirely to what is achievable by time or frequency sharing methods [8], [9]. In [10], the authors show that if the channel coherence time complies with the specific structure one can use the benefits of the IA method without accessing CSI at transmitters.

Regarding blind CSI, one basic idea to control channel coherence time and utilize partial IA is to use multi-mode switching antenna at receivers [13]- [14]. In this case, every receiver is equipped with an antenna that can switch among different reception modes. The framework in this case is to design proper precoders and switching patterns at transmitters and receivers, respectively. In [15], the author analyzes the network for the 3-user IC and shows that in the case of blind CSI using a re-configurable antenna at receivers the sum DoF is $\frac{6}{5}$.

In [14] and with much more practical insights in [16] the authors propose a method to implement interference alignment in a multipath fading environment. They suggest a new antenna structure that can find proper channel conditions to align most of interference signal power. But they need to add specific antenna structure at the receivers which cannot be used for today provided communication platform unless we change the receivers structure. So, it will become useful to find a solution at the transmitters to use the benefits of IA without no much changes.

In this paper, we show that by dividing transmission time into distinct blocks and searching for some transceivers with proper channel state, with an opportunistic manner one can find conditions to align most of interference signals while the desired signal is linearly independent. Since finding these channel conditions is very hard to be materialized and the transmitters need to access channel changing pattern structure before transmission (non-causal transmission strategy). Therefore, we propose an antenna structure which has a lot of fluctuations in its transmission pattern gain. This antenna pattern structure help us to generate proper channel conditions with higher probability. So, the average achievable sum rate can be increase and in this case the transmitters don't need to access channel changing patterns. In fact the transmitters use a multi-layer encoding strategy with different transmission rate. Better conditions for data transmission lead to higher transmission rate while worse channel conditions lead lower data transmission.

In the next section, we discuss system model, channel setup, our propose antenna structure and preliminaries. In section III, we express a lemma and its proof which continues with an example and a searching algorithm. In section IV, considering our proposed technique we will analyze the probability distribution of achievable DoF for each and the average achievable sum DoF. In section V, we generalize our discussion for a blind interference alignment scheme in which the transmitters not only have not access to CSI but also, not access to the channel variation pattern. In this case, with the aid of cumulative distribution free interference dimensions at each receiver, we design a multi-layer encoding strategy to maximize average transmission rate. With many practical assumption, the numerical results show that we can trace the sum rate of $K-$user SISO interference channel within a small value. Finally, in section VI, we conclude the paper.

II. System Model and Definitions

Consider the $K-$user IC which consists of K transmitters $\mathrm{TX}_k, k \in \mathcal{K} = \{1, \dots, K\}$ and K receivers $\mathrm{RX}_k, k \in \mathcal{K}$ which are equipped with a single antenna which we discuss the structure of it later. The received signal at RX_i can be modeled as follows [7]:

$$\bar{\mathbf{Y}}^{[i]} = \sum_{j=1}^{K} \bar{\mathbf{H}}^{[ij]} \bar{\mathbf{X}}^{[j]} + \bar{\mathbf{Z}}^{[i]}, i \in \mathcal{K}, \tag{1}$$

where the $n \times 1$ column vectors of $\bar{\mathbf{Y}}^{[i]}$ and $\bar{\mathbf{X}}^{[j]}$ indicate the received and transmitted signals at RX_i and TX_j, respectively. The $n \times 1$ column vectors of $\bar{\mathbf{Z}}^{[i]}$ indicates the additive white Gaussian noise which is distributed according to $\mathcal{CN}(0,1)$. The matrix $\bar{\mathbf{H}}^{[ij]}$ is an $n \times n$ diagonal matrix which represents the channel matrix coefficients from TX_j to RX_i. At the transmission time, we can assume all the transmitters $\mathrm{TX}_j,\ j \in \mathcal{K}$ encode their messages $W^{[j]} \in \{1, \dots, 2^{nR^{[j]}}\}$ using encoding function $\bar{\mathbf{X}}^{[j]} = e_j\left(W^{[j]}\right),\ p, q \in \mathcal{K}$ where $R[j]$ represents the transmission rate of TX_j. The output of the encoding function $\bar{\mathbf{X}}^{[j]}$, is an $n \times 1$ column vector with the following relation:

$$\bar{\mathbf{X}}^{[j]} = \sum_{d=1}^{d_j} x_d^{[j]} \mathbf{v}_d^{[j]} \tag{2}$$

$$= \bar{\mathbf{V}}^{[j]} \mathbf{x}^{[j]}, \tag{3}$$

where d_j is the number of symbols transmitted by TX_j over n channel uses, $x_d^{[j]}$ is the d^{th} transmitted symbol and $\mathbf{v}_d{}^{[j]}$ is an $n \times 1$ transmit beamforming vector for the d^{th} symbol. Also, $\mathbf{x}^{[j]} = \left[x_1^{[j]}, \dots, x_{d_j}^{[j]}\right]^T$ is an $d_j \times 1$ column vector that represents transmitted data symbols and $\bar{\mathbf{V}}^{[j]} = [\mathbf{v}_1^{[j]}\ \mathbf{v}_2^{[j]}\ \dots\ \mathbf{v}_{d_j}^{[j]}]$ is an $n \times d_j$ matrix that represents precoder matrix. For the precoder matrix $\bar{\mathbf{V}}^{[j]}$, the vector $\mathbf{v}_d^{[j]}$ indicates one of the basis vectors of the designed precoder at TX_j.

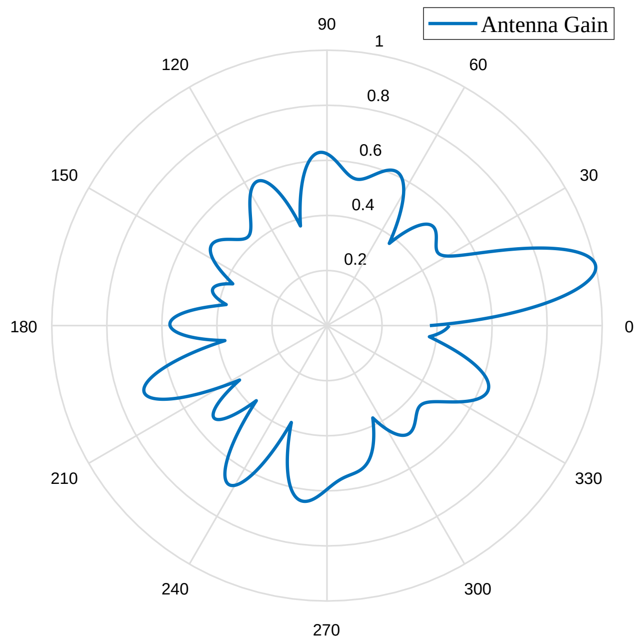

Fig. 1. Implemented antenna pattern structure at the transmitter. This figure indicates that the antenna pattern has a lot of fluctuation rate.

In the K-user interference channel the total power across all transmitters is ρ. The capacity region $C(\rho)$ of the K-user interference channel is set to be $R(\rho) = \left(R^{[1]}(\rho), \ldots, R^{[K]}(\rho)\right)$.

A. DoF Region and the Sum DoF for the K-user Interference Channel

If $(d_1, \ldots, d_K)$ be the DoF region of the $K-$user IC [3], the sum DoF can be defined as follows:

$$d_{\text{sum}} = \max_{d_1, \ldots, d_K} \left(\sum_{i=1}^{K} d_i \right). \tag{4}$$

B. Motivation, Definitions and Channel Setup

Consider K-user IC, where all the receivers have a random and time-varying physical location. In this scenario we assume that every transmitter is equipped with an antenna with high fluctuation pattern gain which is depicted in Fig. 1. Fig. 4 shows a receiver that is affected by two different transmitters by the antenna structure of Fig. 1 (corresponding and interference transmitters). Since in the most of cases the corresponding transmitter has more closer distance to the receiver than interference transmitters, the receiver see the the desired transmitter antenna pattern with more fluctuation rate.

In this problem, we assume that all the channel matrices are diagonal which are represented by $\bar{\mathbf{H}}^{[ij]} = \operatorname{diag}\left(\left[h_1^{[ij]}, \ldots, h_n^{[ij]}\right]\right), i, j \in \mathcal{K}$. We assume a block fading model in time, where channel states are constant

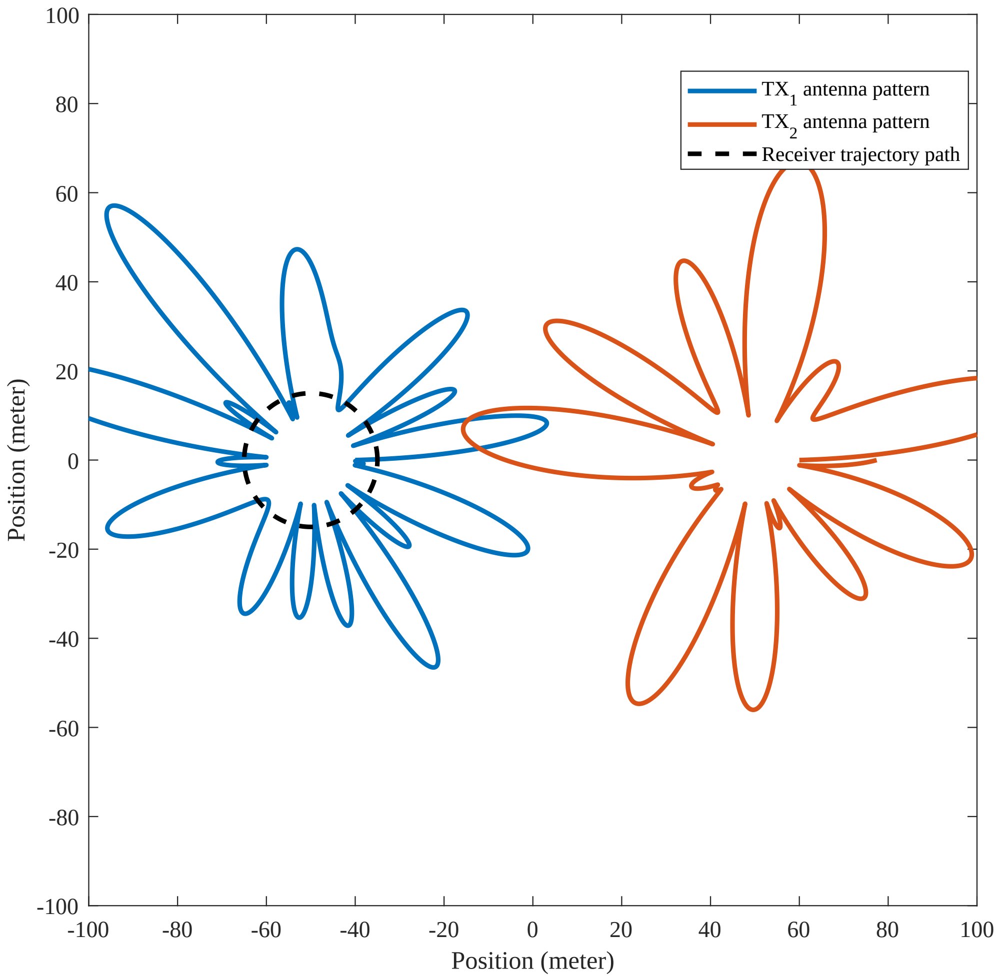

Fig. 2. This figure shows two transmitters and a receiver that moves through a circular trajectory path. Both of the transmitter antennas have the same fluctuation pattern rate. As it indicated in this figure the receiver through the circular path experiences many channel variations from the desired transmitter but the interference link generates lower variation rate.

for a specific time duration. Therefore for the channel matrix of $\bar{\mathbf{H}}^{[ij]}$, we have:

$$\begin{aligned} h_1^{[ij]} &= \cdots = h_{c_1^{[ij]}-1}^{[ij]} \\ h_{c_1^{[ij]}}^{[ij]} &= \cdots = h_{c_2^{[ij]}-1} \\ \vdots \quad & \vdots \qquad\quad \vdots \\ h_{c_{\eta(i,j)}^{[ij]}} &= \cdots = h_n^{[ij]}, \end{aligned} \tag{5}$$

where, $\eta\left(i,j\right)$ indicates the number of altering points between TX_j and RX_i. The value of $c_l^{[ij]},\ l \in \{1,\ldots,\eta\left(i,j\right)\}$ represents the l^{th} point of altering state of the channel between TX_j and RX_i. It is assumed that all $h_l^{[ij]}$ are $i.i.d$ random variables with a specific distribution whose magnitude are bounded between a nonzero and a finite maximum value. Since the channel coherence time is a random variable, we can model each signaling channel similar to the state diagram which is shown in Fig. 3. In this figure during n time snapshots the value of $\eta\left(i,j\right)$ indicates the

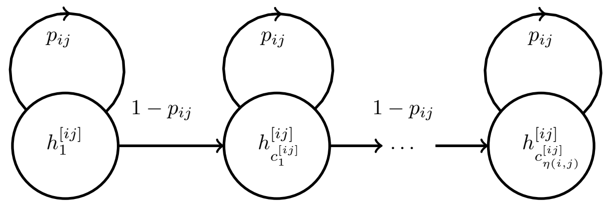

Fig. 3. The state diagram of the channel between TX_j and RX_i. This diagram is consisted of $\eta(i,j)$ states where each state has been shown by a circle.

number of channel states, in each time snapshot the state of the channel remains in its previous state with the probability of p_{ij} and changes with the probability of $1-p_{ij}$.

Note: For the case of using the proposed antenna structure the value of p_{11} *is much lower than the value of* p_{12}.

Definition 1: The set $\mathcal{C}^{[ij]} = \left\{c_1^{[ij]}, \dots, c_{\eta(i,j)}^{[ij]}\right\}$ *is called changing pattern of the channel between* TX_j *and* RX_i.

Definition 2: We represent all the collection of the sets $\mathcal{C}^{[ii]},\ i \in \mathcal{K}_r \subseteq \mathcal{K}$ *by the set* $\mathcal{C}_{\mathcal{K}_r}$ *as follows:*

$$\mathcal{C}_{\mathcal{K}_r} = \bigcup_i \mathcal{C}^{[ii]},\ i \in \mathcal{K}_r \subseteq \mathcal{K}. \tag{6}$$

The extended form of the set $\mathcal{C}_{\mathcal{K}_r}$ can be written as follows:

$$\mathcal{C}_{\mathcal{K}_r} = \left\{c_1^{\mathcal{K}_r}, c_2^{\mathcal{K}_r}, \dots, c_{\sigma_{\mathcal{K}_r}}^{\mathcal{K}_r}\right\},\ |\mathcal{C}_{\mathcal{K}_r}| = \sigma_{\mathcal{K}_r}, \tag{7}$$

where $|\mathcal{A}|$ indicates the cardinality of the set $\mathcal{A}$. During the transmission time, the value of $\sigma_{\mathcal{K}_r}$ indicates the total number of altering points of the direct channels in the set of $\mathcal{K}_r$.

Definition 3: We define subset of $\mathcal{C}'_{\mathcal{K}_r}$ *as follows:*

$$\mathcal{C}'_{\mathcal{K}_r} = \bigcup_{i,j} \mathcal{C}^{[ij]},\ i \neq j \in \mathcal{K}_r \subseteq \mathcal{K}, \tag{8}$$

where $\mathcal{C}'_{\mathcal{K}_r}$ indicates the collection of changing pattern sets of all the cross links with the indices in the set of $\mathcal{K}_r$. Similar to (7), we can write the extended form of the set $\mathcal{C}'_{\mathcal{K}_r}$ as follows:

$$\mathcal{C}'_{\mathcal{K}_r} = \left\{c_1'^{\mathcal{K}_r}, c_2'^{\mathcal{K}_r}, \dots, c_{\sigma'_{\mathcal{K}_r}}'^{\mathcal{K}_r}\right\},\ |\mathcal{C}'_{\mathcal{K}_r}| = \sigma'_{\mathcal{K}_r} \tag{9}$$

During the transmission time, the value of $\sigma'_{\mathcal{K}_r}$ indicates the total number of altering points of the cross channels in the set of $\mathcal{K}_r$.

Definition 4: We define the set $\mathcal{B}_t^{\mathcal{K}_r}, t \in \mathbb{N}$ *as follows:*

$$\mathcal{B}_t^{\mathcal{K}_r} = \left\{i | i \in [c_{t-1}'^{\mathcal{K}_r} : c_t'^{\mathcal{K}_r} - 1], i \in \mathbb{N}\right\}, \tag{10}$$

which indicates t^{th} *transmission block time snapshots where all the cross links among the transceivers with the indexes in the set of* $\mathcal{K}_r$ *have constant value without altering state.*

Definition 5: We define the set $\mathcal{C}_{\mathcal{B}_t^{\mathcal{K}_r}}^{[ij]}, t \in \mathbb{N}$ *as follows:*

$$\mathcal{C}_{\mathcal{B}_t^{\mathcal{K}_r}}^{[ij]} = \mathcal{C}^{[ij]} \cap \mathcal{B}_t^{\mathcal{K}_r}. \tag{11}$$

TABLE I

EXAMPLE OF CHANNEL STRUCTURE FOR 3-USER IC

Channel matrices $\bar{\mathbf{H}}^{[ij]},\ i,j \in \mathcal{K}$	$\mathcal{C}^{[ij]}$	$\mathcal{C}'_{\mathcal{K}}$	$\mathcal{B}_t^{\mathcal{K}}, 1 \le t \le 5$	$\mathcal{C}^{[33]}_{\mathcal{B}_t^{\mathcal{K}}}$
$\bar{\mathbf{H}}^{[11]} = \mathrm{diag}\left([1.5, 1.1, 2.3, 2.3, 0.6, 0.6, 3.0, 3.0, 1.0, 1.0]\right)$	$\mathcal{C}^{[11]} = \{2,3,5,7,9\}$			
$\bar{\mathbf{H}}^{[12]} = \mathrm{diag}\left([1.2, 1.2, 1.2, 1.2, 1.5, 1.5, 1.5, 1.0, 1.0, 1.0]\right)$	$\mathcal{C}^{[12]} = \{5,8\}$		$\mathcal{B}_1^{\mathcal{K}} = \{1,2,3,4\}$	$\mathcal{C}^{[33]}_{\mathcal{B}_1^{\mathcal{K}}} = \{2,3\}$
$\bar{\mathbf{H}}^{[13]} = \mathrm{diag}\left([1.0, 1.0, 1.0, 1.0, 0.8, 0.8, 1.5, 3.0, 3.0, 3.0]\right)$	$\mathcal{C}^{[13]} = \{5,7,8\}$		$\mathcal{B}_2^{\mathcal{K}} = \{5,6\}$	$\mathcal{C}^{[33]}_{\mathcal{B}_2^{\mathcal{K}}} = \{6\}$
$\bar{\mathbf{H}}^{[21]} = \mathrm{diag}\left([2.0, 2.0, 2.0, 2.0, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3]\right)$	$\mathcal{C}^{[21]} = \{5\}$		$\mathcal{B}_3^{\mathcal{K}} = \{7\}$	$\mathcal{C}^{[33]}_{\mathcal{B}_3^{\mathcal{K}}} = \varnothing$
$\bar{\mathbf{H}}^{[22]} = \mathrm{diag}\left([2.5, 2.1, 2.1, 2.1, 1.1, 1.1, 3.1, 1.1, 1.1, 1.1]\right)$	$\mathcal{C}^{[22]} = \{2,5,7,8\}$	$\{5,7,8,10\}$	$\mathcal{B}_4^{\mathcal{K}} = \{8,9\}$	$\mathcal{C}^{[33]}_{\mathcal{B}_4^{\mathcal{K}}} = \{9\}$
$\bar{\mathbf{H}}^{[23]} = \mathrm{diag}\left([0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 3.0, 3.0, 3.0, 3.0]\right)$	$\mathcal{C}^{[23]} = \{7\}$		$\mathcal{B}_5^{\mathcal{K}} = \{10\}$	$\mathcal{C}^{[33]}_{\mathcal{B}_5^{\mathcal{K}}} = \varnothing$
$\bar{\mathbf{H}}^{[31]} = \mathrm{diag}\left([0.8, 0.8, 0.8, 0.8, 0.4, 0.4, 0.4, 0.4, 0.4, 0.5]\right)$	$\mathcal{C}^{[31]} = \{5,10\}$		$\mathcal{B}_6^{\mathcal{K}} = \varnothing$	$\mathcal{C}^{[33]}_{\mathcal{B}_6^{\mathcal{K}}} = \varnothing$
$\bar{\mathbf{H}}^{[32]} = \mathrm{diag}\left([0.2, 0.2, 0.2, 0.2, 0.3, 0.3, 1.2, 1.2, 1.2, 1.2]\right)$	$\mathcal{C}^{[32]} = \{5,7\}$			
$\bar{\mathbf{H}}^{[33]} = \mathrm{diag}\left([0.1, 0.2, 0.2, 0.3, 0.3, 1.7, 1.7, 1.7, 1.3, 1.3]\right)$	$\mathcal{C}^{[33]} = \{2,4,6,9\}$			

Definition 6: We define the function $\mathcal{F}\left(\bar{\mathbf{H}}^{[ij]}\right)$ as the maximum number of the time snapshots in which the channel matrix between TX_j and RX_i is constant.

Note: The set $\mathcal{K}_r$ shows a subset of the set $\mathcal{K}$ except $\varnothing$. Therefore, the value of r is between one and $2^K - 1$.

Table 1 shows an example for 3-user IC, in this table one can realize how we can extract the channel sets of $\mathcal{C}^{[ij]}$, $\mathcal{C}'$, $\mathcal{B}_m^{\mathcal{K}}$ and $\mathcal{C}^{[33]}_{\mathcal{B}_m^{\mathcal{K}}}$ from the channel matrices of $\bar{\mathbf{H}}^{[ij]},\ i,j \in \mathcal{K}$.

III. NON-CAUSAL TRANSMISSION STRATEGY

In this section using channel changing pattern, we analyze a method to transmit data. Although we cannot use this method without accessing channel changing pattern before transmitting data but it give us enough intuition to use channel changing pattern to align interference signals. In the next section we generalize this scheme to a more practical method which is not access to any information of the channel. Before we pin point our proposed method we discuss following lemma which plays an important role in our study.

Lemma 1: Consider $\mathbf{V}$ is a nonzero elements random matrix with the size of $n \times d_v$ and the rank of d_v. Let, $\bar{\mathbf{H}}$ be a random diagonal matrix with the size of $n \times n$, we can conclude that:

$$\mathrm{rank}\left(\left[\mathbf{V}^{\top}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) = \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right).$$

Proof: Consider $t_1,\ t_2 \in \mathbb{N}$ $(t_1, t_2 \le n)$, are two time snapshots in which the random matrix $\bar{\mathbf{H}}$ from t_1-th time snapshot to t_2-th time snapshot has the constant value of α and $\mathcal{F}\left(\bar{\mathbf{H}}\right) = t_2 - t_1 + 1$. For the fast fading channel model of [3], $t_1 = t_2$ and the value of $\mathcal{F}\left(\bar{\mathbf{H}}\right)$ is one. In this case the rank of the matrix $\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]$ can be upper bounded as follows:

$$\text{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) = \text{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V} - \alpha\bar{\mathbf{I}}\mathbf{V}\right]\right) \tag{12}$$

$$\leq \underbrace{\text{rank}\left(\mathbf{V}\right)}_{=d_v} + \text{rank}\left(\bar{\mathbf{H}}\mathbf{V} - \alpha\bar{\mathbf{I}}\mathbf{V}\right), \tag{13}$$

where $\alpha\bar{\mathbf{I}}$ is a square diagonal matrix which all its main diagonal entries are equal to α. Since the matrix $\bar{\mathbf{H}}\mathbf{V} - \alpha\bar{\mathbf{I}}\mathbf{V}$ has $n - \mathcal{F}\left(\bar{\mathbf{H}}\right)$ nonzero rows, we can upper bound the rank of matrix $\bar{\mathbf{H}}\mathbf{V} - \alpha\bar{\mathbf{I}}\mathbf{V}$ by $\min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right)$. The rank of $\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)$ can not be greater than n, therefore from (13) we get:

$$\text{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) \leq \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right). \tag{14}$$

For the case where the diagonal matrix $\bar{\mathbf{H}}$ has one state, $\mathcal{F}\left(\bar{\mathbf{H}}\right) = n$ and the value of $n - \mathcal{F}\left(\bar{\mathbf{H}}\right)$ is zero and consequently the rank of $\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]$ is equivalent to the rank of the matrix $\mathbf{V}$. Subsequently, as another conclusion from (14), we have:

$$\text{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) = d_v \tag{15}$$

$$= \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right). \tag{16}$$

So, in the case where the matrix $\bar{\mathbf{H}}$ has one state the proof has been completed.

For the case where the matrix $\bar{\mathbf{H}}$ has 2 states of α_1 and α_2, rank of the matrix $\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]$ can be simplified as follows:

$$\text{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)$$

$$= \text{rank}\left(\begin{bmatrix}\mathbf{V}_1 & \alpha_1\mathbf{V}_1 \\ \mathbf{V}_2 & \alpha_2\mathbf{V}_2\end{bmatrix}\right) \tag{17}$$

$$\overset{(a)}{=} \text{rank}\left(\begin{bmatrix}\mathbf{V}_1 & \mathbf{0} \\ \mathbf{V}_2 & (\alpha_2 - \alpha_1)\mathbf{V}_2\end{bmatrix}\right) \tag{18}$$

$$= \text{rank}\left(\begin{bmatrix}\mathbf{V}_1 & \mathbf{0} \\ \mathbf{0}' & \mathbf{V}_2\end{bmatrix}\right) \tag{19}$$

$$= \min\left(\text{rank}\left(\mathbf{V}_1\right) + \text{rank}\left(\mathbf{V}_2\right), n\right) \tag{20}$$

$$\overset{(b)}{=} \min\left(\min\left(n - \mathcal{F}\left(\bar{\mathbf{H}}\right), d_v\right) + \min\left(\mathcal{F}\left(\bar{\mathbf{H}}\right), d_v\right), n\right) \tag{21}$$

$$\overset{(c)}{=} \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right), \tag{22}$$

where, $\mathbf{0}$ and $\mathbf{0}'$ are the all zero matrix with the similar size of $\mathbf{V}_1$ and $\mathbf{V}_2$ respectively. The equality of (a) comes from subtracting α_1 times of the first d_v columns of the matrix $\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]$ from the last d_v columns. The equality of (b) comes from the fact that $\mathbf{V}_1$ has the size of $\left(n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right) \times d_v$ and the size of $\mathbf{V}_2$ is $\mathcal{F}\left(\bar{\mathbf{H}}\right) \times d_v$. The equality of (c) comes from the fact that if $\mathcal{F}\left(\bar{\mathbf{H}}\right) \leq d_v$, then $\min\left(\mathcal{F}\left(\bar{\mathbf{H}}\right), d_v\right) = \mathcal{F}\left(\bar{\mathbf{H}}\right)$ and from definition 6, since $\mathcal{F}\left(\bar{\mathbf{H}}\right) \geq n - \mathcal{F}\left(\bar{\mathbf{H}}\right)$ the value of $\min\left(n - \mathcal{F}\left(\bar{\mathbf{H}}\right), d_v\right) = n - \mathcal{F}\left(\bar{\mathbf{H}}\right)$ and subsequently in this case

$\min\left(\min\left(n-\mathcal{F}\left(\bar{\mathbf{H}}\right),d_v\right)+\min\left(\mathcal{F}\left(\bar{\mathbf{H}}\right),d_v\right),n\right) = n = \min\left(d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}\right)\right),n\right)$. It is obvious if $\mathcal{F}\left(\bar{\mathbf{H}}\right)>d_v$, then:

$$\begin{aligned}&\min\left(\min\left(\mathcal{F}\left(\bar{\mathbf{H}}\right),d_v\right)+\min\left(n-\mathcal{F}\left(\bar{\mathbf{H}}\right),d_v\right),n\right)\\&\qquad=\min\left(d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}\right)\right),n\right). \qquad (23)\end{aligned}$$

So, we can conclude that when the number the states of the matrix $\bar{\mathbf{H}}$ is one or two, the result of the first lemma is right. Now we analyze 3 different cases as follows:

Case 1: Let $\mathcal{F}\left(\bar{\mathbf{H}}\right)\geq\frac{n}{2}$, by basic operations on the matrix $\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]$ and displacing the rows in the block with the length of $\mathcal{F}\left(\bar{\mathbf{H}}\right)$ to the first rows. We have:

$$\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)=\operatorname{rank}\left(\left[\mathbf{V}'\ \bar{\mathbf{H}}'\mathbf{V}'\right]\right). \qquad (24)$$

where $\left[\mathbf{V}'\ \bar{\mathbf{H}}'\mathbf{V}'\right]$ is a matrix which is generated from the basic operation on the matrix $\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]$ (displaced rows matrix). The column matrix $\bar{\mathbf{H}}'$ can be equivalently expressed by multiplying two matrices of $\bar{\mathbf{H}}'_1$ and $\bar{\mathbf{H}}'_2$ ($\bar{\mathbf{H}}'=\bar{\mathbf{H}}'_1\bar{\mathbf{H}}'_2$) where $\bar{\mathbf{H}}'_2$ has two states and $\mathcal{F}\left(\bar{\mathbf{H}}'_2\right)=\mathcal{F}\left(\bar{\mathbf{H}}'\right)=\mathcal{F}\left(\bar{\mathbf{H}}\right)$. So, we have:

$$\begin{aligned}\operatorname{rank}\left(\left[\mathbf{V}'\ \bar{\mathbf{H}}'\mathbf{V}'\right]\right)&=\operatorname{rank}\left(\left[\mathbf{V}'\ \bar{\mathbf{H}}'_{\mathbf{1}}\bar{\mathbf{H}}'_{\mathbf{2}}\mathbf{V}'\right]\right) &(25)\\&\overset{(a)}{\geq}\operatorname{rank}\left(\left[\mathbf{V}'\ \bar{\mathbf{H}}'_{\mathbf{2}}\mathbf{V}'\right]\right) &(26)\\&\overset{(b)}{=}\min\left(d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}'_2\right)\right),n\right) &(27)\\&\overset{(c)}{=}\min\left(d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}\right)\right),n\right). &(28)\end{aligned}$$

The inequality of (a) comes from this fact that $\bar{\mathbf{H}}'_1$ has random elements. Similar to (17)-(22), (b) comes from the fact that $\bar{\mathbf{H}}'_2$ has two states and (c) comes from $\mathcal{F}\left(\bar{\mathbf{H}}\right)=\mathcal{F}\left(\bar{\mathbf{H}}'_2\right)$. Now we want to generalize the previous conclusion for more general form of the matrix $\bar{\mathbf{H}}$. In other hand in (14) we show that $\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)\leq\min\left(d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}\right)\right),n\right)$.

So, from (14) we can conclude that $\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)=\min\left(d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}\right)\right),n\right)$ and in this case the proof has been completed.

Case 2: Let $\mathcal{F}\left(\bar{\mathbf{H}}\right)<\frac{n}{2}$ and $d_v\geq\frac{n}{2}$. Now assume $\bar{\mathbf{H}}'$ is a matrix with $\frac{n}{2}\leq\mathcal{F}\left(\bar{\mathbf{H}}'\right)\leq\frac{n+1}{2}$ and two states. In this case we have:

$$\begin{aligned}\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)&\overset{(a)}{\geq}\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}'\mathbf{V}\right]\right) &(29)\\&\overset{(b)}{=}\min\left(d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}'\right)\right),n\right) &(30)\\&\overset{(c)}{=}n. &(31)\end{aligned}$$

The inequality of (a) comes from this fact that $\mathcal{F}\left(\bar{\mathbf{H}}\right)\leq\mathcal{F}\left(\bar{\mathbf{H}}'\right)$. The equality (b) comes from (22) and (c) comes from that $d_v+\min\left(d_v,n-\mathcal{F}\left(\bar{\mathbf{H}}'\right)\right)>n$. In other hand $\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)\leq n$, so we have $\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right)=n$. Therefore in this case the proof has been completed.

Case 3: Let $\mathcal{F}\left(\bar{\mathbf{H}}\right) < \frac{n}{2}$ and $d_v < \frac{n}{2}$. Similar to previous case, assume $\bar{\mathbf{H}}'$ is a matrix with $\frac{n}{2} \leq \mathcal{F}\left(\bar{\mathbf{H}}'\right) \leq \frac{n+1}{2}$ and two states. In this case we have:

$$\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) \geq \operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}'\mathbf{V}\right]\right) \tag{32}$$

$$\overset{(a)}{=} \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}'\right)\right), n\right) \tag{33}$$

$$\overset{(b)}{=} 2d_v. \tag{34}$$

where (a) comes from (22) and (b) comes from that $n - \mathcal{F}\left(\bar{\mathbf{H}}'\right) > d_v$. In other hand from the inequality of (14) $\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) \leq \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right) = 2d_v$, so we can conclude that $\operatorname{rank}\left(\left[\mathbf{V}\ \bar{\mathbf{H}}\mathbf{V}\right]\right) = 2d_v$. Therefore in this case the proof has been completed. ■

A. Example of IA for 3-user IC

Consider table I, in this table we have 6 cross channels with the changing patterns which have clearly been expressed at the second column. In this case we design precoders at different transmitters with the following steps:
Step 1: Finding all the sets of $\mathcal{C}'_{\mathcal{K}_r}$ from the third definition with $2 < |\mathcal{K}_r| \leq |\mathcal{K}|$ as follows:

$$\mathcal{C}'_{\mathcal{K}} = \bigcup_{i \neq j} \mathcal{C}^{[ij]} = \{5, 7, 8, 10\}.\ i, j \in \mathcal{K} \tag{35}$$

In this example since since we have three users, $\mathcal{K}_r = \mathcal{K}$.
Step 2: From definition 4, finding all the sets of $\mathcal{B}_t^{\mathcal{K}_r}$ for $2 < |\mathcal{K}_r| \leq |\mathcal{K}|$ as follows:

$$\begin{aligned} &\mathcal{B}_1^{\mathcal{K}} = \{1, 2, 3, 4\},\ \mathcal{B}_2^{\mathcal{K}} = \{5, 6\}, \\ &\mathcal{B}_3^{\mathcal{K}} = \{7\},\ \mathcal{B}_4^{\mathcal{K}} = \{8, 9\},\ \mathcal{B}_5^{\mathcal{K}} = \{10\}. \end{aligned} \tag{36}$$

Considering above sets, our transmission time is divided among 5 different transmission blocks. As an example, the first and second transmission blocks are consisted of 4 and 2 time snapshots.
Step 3: In each transmission block, at different transmitters we design all the precoder matrices as follows:

$$\text{at TX}_1\text{: } \bar{\mathbf{V}}^{[1]}(t) = \left[\mathbf{v}_1, \ldots, \mathbf{v}_{d_1(t)}\right], \tag{37}$$

$$\text{at TX}_2\text{: } \bar{\mathbf{V}}^{[2]}(t) = \left[\mathbf{v}_1, \ldots, \mathbf{v}_{d_2(t)}\right], \tag{38}$$

$$\text{at TX}_3\text{: } \bar{\mathbf{V}}^{[3]}(t) = \left[\mathbf{v}_1, \ldots, \mathbf{v}_{d_3(t)}\right], \tag{39}$$

where t indicates the block number. All the basis vectors $\mathbf{v}_j$ at t^{th} transmission block are nonzero random column vectors with the size of $|\mathcal{B}_t^{\mathcal{K}}| \times 1$. In this scheme, all the basis vectors at different transmitters with the same indexes are equivalent and all the basis vectors with different indexes are linearly independent.
Step 4: In each transmission block, we try to find 3-tuple $(d_1(t), d_2(t), d_3(t))\,, 1 \leq t \leq 5$ such that the sum DoF of $d_{\text{sum}} = d_1(t) + d_2(t) + d_3(t)$ at each transmission block of t, subjected to the two following constraints becomes maximized:

$$d_i(t) + \max_{j=1:3, j \neq i} d_j(t) \overset{(a)}{\leq} |\mathcal{B}_t^{\mathcal{K}}| \tag{40}$$

$$d_i(t) \overset{(b)}{\leq} |\mathcal{B}_t^{\mathcal{K}}| - \mathcal{F}\left(\bar{\mathbf{H}}_t^{[ii]}\right), \tag{41}$$

where $\bar{\mathbf{H}}_t^{[ii]}$ represents a diagonal matrix with elements extracted from the time snapshots of t^{th} block of the diagonal elements of the matrix $\bar{\mathbf{H}}^{[ii]}$. The inequality (a) comes from this fact that, since at RX_i in each transmission block of $\mathcal{B}_t^{\mathcal{K}}$ all the cross channels are diagonal matrices with the constant diagonal elements, all the interference signals are aligned and the space spanned by the interference signals has the dimension of $\max_{j\neq i}\left(\mathrm{rank}\left(\bar{\mathbf{V}}^{[j]}(t)\right)\right) = \max_{j\neq i} d_j(t)$. So, the desired and interference signal dimensions should not be greater than the number of time snapshots of each transmission block or $|\mathcal{B}_t^{\mathcal{K}}|$. The inequality (b) comes from the result of the first lemma, which indicates that the number of desired signal at its corresponding receiver should not be greater than $|\mathcal{B}_t^{\mathcal{K}}| - \mathcal{F}\left(\bar{\mathbf{H}}^{[ii]}\right)$; otherwise, the desired signal space can not be linearly independent from interference signals.

So, for our specific problem at the first transmission block ($t = 1$) and $|\mathcal{B}_1^{\mathcal{K}}| = 4$, we can upper bound the values of $d_i(1), i \in \{1, 2, 3\}$ as follows:

$$d_1(1) \leq |\mathcal{B}_1^{\mathcal{K}}| - \mathcal{F}\left(\bar{\mathbf{H}}_1^{[11]}\right) = 4 - 2 = 2, \tag{42}$$

$$d_2(1) \leq |\mathcal{B}_1^{\mathcal{K}}| - \mathcal{F}\left(\bar{\mathbf{H}}_1^{[22]}\right) = 4 - 3 = 1, \tag{43}$$

$$d_3(1) \leq |\mathcal{B}_1^{\mathcal{K}}| - \mathcal{F}\left(\bar{\mathbf{H}}_1^{[33]}\right) = 4 - 2 = 2. \tag{44}$$

Setting $(d_1(1), d_2(1), d_3(1)) = (2, 1, 2)$, we can show that the second constrain in (41) has been satisfied.

Remark 1: For the blocks with $|\mathcal{K}_r| \leq 2$, since we cannot use the benefits of the IA scheme (we have one or two transceivers and there is no chance to align any interference signals), we can choose one or two random transmitters TX_{i_1} and TX_{i_2} to set the 2-tuples $(d_{i_1}(t), d_{i_2}(t))$ as follows:

$$d_{i_1}(t) + d_{i_2}(t) = 1. \tag{45}$$

Step 5: Finding proper precoders at transmitters to satisfy previous step constrains of (42)-(44). As an example for our specific problem, at the first transmission block we design $\bar{\mathbf{V}}^{[1]}(1)$, $\bar{\mathbf{V}}^{[2]}(1)$ and $\bar{\mathbf{V}}^{[3]}(1)$ such that:

$$\bar{\mathbf{V}}^{[1]}(1) = [\mathbf{v}_1, \mathbf{v}_2], \tag{46}$$

$$\bar{\mathbf{V}}^{[2]}(1) = [\mathbf{v}_1], \tag{47}$$

$$\bar{\mathbf{V}}^{[3]}(1) = [\mathbf{v}_1, \mathbf{v}_2], \tag{48}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are two column vectors with random elements and the size of $|\mathcal{B}_1^{\mathcal{K}}| \times 1$. Now at the first transmission block let's analyze the IA conditions at different receivers as follows [**?**], [3]:

$$\begin{aligned}
\text{at RX}_1\text{: span}\left(\bar{\mathbf{H}}^{[12]}\bar{\mathbf{V}}^{[2]}(1)\right) &\in \text{span}\left(\bar{\mathbf{H}}^{[13]}\bar{\mathbf{V}}^{[3]}(1)\right)\\
\mathrm{rank}\left(\left[\bar{\mathbf{H}}^{[11]}\bar{\mathbf{V}}^{[1]}(1)\ \bar{\mathbf{H}}^{[13]}\bar{\mathbf{V}}^{[3]}(1)\right]\right) &= 4,\\
\text{at RX}_2\text{: span}\left(\bar{\mathbf{H}}^{[21]}\bar{\mathbf{V}}^{[1]}(1)\right) &\in \text{span}\left(\bar{\mathbf{H}}^{[23]}\bar{\mathbf{V}}^{[3]}(1)\right)\\
\mathrm{rank}\left(\left[\bar{\mathbf{H}}^{[22]}\bar{\mathbf{V}}^{[2]}(1)\ \bar{\mathbf{H}}^{[23]}\bar{\mathbf{V}}^{[3]}(1)\right]\right) &= 4,\\
\text{at RX}_3\text{: span}\left(\bar{\mathbf{H}}^{[32]}\bar{\mathbf{V}}^{[2]}(1)\right) &\in \text{span}\left(\bar{\mathbf{H}}^{[31]}\bar{\mathbf{V}}^{[1]}(1)\right)\\
\mathrm{rank}\left(\left[\bar{\mathbf{H}}^{[33]}\bar{\mathbf{V}}^{[3]}(1)\ \bar{\mathbf{H}}^{[31]}\bar{\mathbf{V}}^{[1]}(1)\right]\right) &= 4.
\end{aligned} \tag{49}$$

So, at the first transmission block the first and third users get 2 resources while the second user gets 1 resource. Therefore, at the fist transmission block we can achieve 5 resources from 4 transmission snapshots. Consequently, at the first transmission block we get $\frac{5}{4}$ sum DoF. From the first remark, for other transmission blocks $|\mathcal{B}_t^{\mathcal{K}}| \leq 2,\ t > 1$, we cannot use the benefits of IA scheme. So, in this example the achievable sum DoF can be calculated as follows:

$$\frac{d_{\text{sum}}}{n} = \frac{\sum_{t=1}^{5}\sum_{i=1}^{3} d_i(t)}{n} = \frac{5+2+1+2+1}{10} = \frac{11}{10}. \tag{50}$$

Therefore, without accessing channel coefficients we can achieve $d_{\text{sum}} > 1$.

B. Optimal search algorithm:

As a standard searching algorithm for the problem of IA using channel changing pattern, in this subsection, we generalize the solution of previous example. It is obvious that for the case of $K = 2$ the optimum solution is to set $d_{\text{sum}} = 1$. The interesting problem is when $K > 2$. In the general case we divide the transmission time (consisting of n time snapshots) among different transmission blocks and in each transmission block we activate a subset of transmitters $\mathcal{K}_r$.

Definition 7: We define the set $\mathcal{G}_u$ as follows:

$$\mathcal{G}_u = \{\mathcal{K}_{u_1}, \mathcal{K}_{u_2}, \ldots, \mathcal{K}_{u_{l_u}}\} \tag{51}$$

where $\mathcal{K}_{u_t} \subseteq \mathcal{K} = \{1, \ldots, K\}, 1 \leq t \leq l_u$.

In our proposed method, depending on the set $\mathcal{G}_u$, the transmission time is divided into $|\mathcal{G}_u| = l_u$ distinct blocks and each block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}, 1 \leq t \leq l_u$ contains $|\mathcal{B}_t^{\mathcal{K}_{u_t}}| \leq n$ time snapshots. In each transmission block we activate a subset of transmitters $\mathcal{T}^{(u_t)} = \{\mathrm{TX}_{k_1^{u_t}}, \ldots, \mathrm{TX}_{k_r^{u_t}}\},\ 1 \leq r \leq K$ to transmit their messages to their corresponding receivers. The set of active transceivers $\{k_1^{u_t}, \ldots, k_r^{u_t}\}$ in the transmission block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}, 1 \leq t \leq l_u$ is equivalent to the set of $\mathcal{K}_{u_t}$ in $\mathcal{G}_u$. In other words, $\mathcal{K}_{u_t} = \{k_1^{u_t}, \ldots, k_r^{u_t}\}$ shows the indexes of the active transceivers at the transmission block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}, 1 \leq t \leq l_u$. The transmission block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}$ for every set of $\mathcal{G}_u$ is defined as follows:

$$\mathcal{B}_1^{\mathcal{K}_{u_1}} = \{i | i \in [1:m], m \notin \mathcal{C}'_{\mathcal{K}_{u_1}}\} \tag{52}$$

$$\mathcal{B}_t^{\mathcal{K}_{u_t}} = \{i | i \in \left[\max\left(\mathcal{B}_{t-1}^{\mathcal{K}_{u_{t-1}}}\right) + 1 : m\right], m \notin \mathcal{C}'_{\mathcal{K}_{u_t}}\}, \tag{53}$$

where all the members of the set $\mathcal{B}_t^{\mathcal{K}_{u_t}}$ are consecutive. In other words, $\max \mathcal{B}_t^{\mathcal{K}_{u_t}} - \min \mathcal{B}_t^{\mathcal{K}_{u_t}} = |\mathcal{B}_t^{\mathcal{K}_{u_t}}| - 1$.

As an example consider we have $K = 4$ and $\mathcal{G}_1 = \{\{1,3,4\}, \{1,2,3,4\}\}$ where $u = 1$. In this case we have two transmission blocks of $\mathcal{B}_1^{\mathcal{K}_{1_1}}$ and $\mathcal{B}_2^{\mathcal{K}_{1_2}}$ which indicate that at the first and the second transmission block the active transmitters have been selected from $\mathcal{T}^{1_1} = \{\mathrm{TX}_1, \mathrm{TX}_3, \mathrm{TX}_4\}$ and $\mathcal{T}^{1_2} = \{\mathrm{TX}_1, \mathrm{TX}_2, \mathrm{TX}_3, \mathrm{TX}_4\}$, respectively. Similarly, if $\mathcal{G}_2 = \{\{3,4\}, \{1,2,4\}, \{1\}\}$, since $|\mathcal{G}_2| = 3$, we have 3 transmission block. At the first transmission block we have 2 active transmitters of TX_3 and TX_4, at the second transmission block we have 3 active transmitters of TX_1, TX_2 and TX_4, and at the third transmission block we only have one active transmitter of TX_1.

Generally for the transmission time consisted of n time snapshots, the set $\mathcal{G}_u$ with $|\mathcal{G}_u| \leq n$ can be chosen from n^Q different sets where the value of Q is calculated as follows:

$$Q \overset{(a)}{=} 2^K - 1. \tag{54}$$

The equality of (a) comes from this fact that in each transmission snapshot we can activate a subset of transmitters from the set of $\mathcal{K}$ except $\varnothing$, consequently, we should have, $u \leq n^Q$.

Remark 2: From definition 4, the length of each transmission block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}$ should be selected such that all the cross-links among transceivers with the indexes in the set of $\mathcal{K}_{u_t}$ have not any altering point. In other words, there is not any changing point in t^{th} transmission block of cross channels among active users.

The maximum value of d_{sum}, mathematically can be calculated from the following optimization problem:

$$\max_{\mathcal{G}_u, d_i(t)} \sum_{t=1}^{|\mathcal{G}_u|} \sum_i d_i(t), \; 1 \leq u \leq n^Q, i \in \mathcal{K}_{u_t}, \tag{55}$$

subject to the following constraints at each block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}$:

$$d_i(t) + \max_{j \in \mathcal{K}_{u_t}, j \neq i} d_j(t) \leq |\mathcal{B}_t^{\mathcal{K}_{u_t}}| \tag{56}$$

$$d_i(t) \leq |\mathcal{B}_t^{\mathcal{K}_{u_t}}| - \mathcal{F}\left(\bar{\mathbf{H}}_t^{[ii]}\right), \tag{57}$$

Lemma 2: For the $K-$user IC in a random block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}$, there exists $\bar{\mathbf{V}}^{[k_1^{u_t}]}, \ldots, \bar{\mathbf{V}}^{[k_r^{u_t}]}$ such that all the conditions of (56) *and* (57) *are satisfied.*

Proof: We proof this lemma by finding proper precoder matrices at transmitters. Let at the block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}$ we have:

$$\begin{aligned} \bar{\mathbf{V}}^{[k_1^{u_t}]}(t) &= \left[\mathbf{v}_1 \; \ldots \; \mathbf{v}_{d_{k_1^{u_t}}}\right], \\ &\vdots \qquad\qquad \vdots \\ \bar{\mathbf{V}}^{[k_r^{u_t}]}(t) &= \left[\mathbf{v}_1 \; \ldots \; \mathbf{v}_{d_{k_r^{u_t}}}\right], \end{aligned} \tag{58}$$

where, $\mathbf{v}_j$ is a random vector with nonzero elements and the size of $|\mathcal{B}_t^{\mathcal{K}_{u_t}}| \times 1$. Now, assume $\bar{\mathbf{V}}^{[k_m^{u_t}]}(t)$ be a precoder matrix with the maximum rank of $d_{k_m^{u_t}}$ among the active transmitters with the indexes in the set of $\mathcal{K}_{u_t}$. In other words, $\max_{j \in \mathcal{K}_{u_t}} \text{rank}\left(\bar{\mathbf{V}}^{[j]}(t)\right) = d_{k_m^{u_t}}$.

Since at each transmission block all the cross-links have the constant elements, all the shared basis vectors at different receivers of $\text{RX}_j, j \in \mathcal{K}_{u_t}, j \neq k_m^{u_t}$ are in the space spanned by the matrix $\bar{\mathbf{V}}^{[k_m^{u_t}]}(t)$. Similarly, at the $\text{RX}_{k_m^{u_t}}$, since all the cross channels among active transceivers have constant values, all the interference signals are aligned with each other. Now, we should show that all the desired signal are linearly independent of the interference signals. Considering the first lemma, from (56) and (57) we can conclude that the desired signal space at each active receiver is linearly independent of the interference signals. So, the proof has been completed. ■

The optimum vector for the $K-$tuple $(d_1, \ldots, d_K)$ to maximized d_{sum} can be obtained from a simple integer programming and searching algorithm (see algorithm 1). Although, this searching algorithm shows a new scheme to use the channel changing pattern to partially or perfectly align interference signals, this algorithm cannot give us enough intuition to find out how it can be efficient. In the next section we find the average achievable sum DoF to analyze the performance of our technique.

Algorithm 1 A search algorithm for finding optimum value of $K-$tuple $(d_1, \ldots, d_K)$

Input: $K, n,\ \mathcal{C}^{[ij]},\ i,j \in \mathcal{K}$
  Set : $Q = 2^K - 1,\ d^* = 0$
for $u = 1$ to 2^Q **do**
  $\mathcal{B}_1^{\mathcal{K}_{u_1}} = \{i | i \in [1:m], m \notin \mathcal{C}'_{\mathcal{K}_{u_1}}\}$
  for $t = 2$ to $|\mathcal{G}_u|$ **do**
    $\mathcal{B}_t^{\mathcal{K}_{u_t}} = \{i | i \in \left[\max\left(\mathcal{B}_{t-1}^{\mathcal{K}_{u_{t-1}}}\right) + 1 : m\right], m \notin \mathcal{C}'_{\mathcal{K}_{u_t}}\}$
    $M_t = 0$
    for $k = 1$ to $k = K$ **do**
      $E_k(t) = |\mathcal{B}_t^{\mathcal{K}_{u_t}}| - \mathcal{F}\left(\bar{\mathbf{H}}_t^{[kk]}\right)$
    end for
    $\mathbf{E}(t) = (E_1(t), \ldots, E_k(t))$
    for $(e_1(t), \ldots, e_K(t)) = (0, \ldots, 0)$ to $(n, \ldots, n)$ **do**
      if $\sum_{j=1}^{K} e_j > M_t$, $(e_1(t), \ldots, e_K(t)) \leq \mathbf{E}(t)$ and $e_j + \max\limits_{k \in \mathcal{K}_{u_t} - j} e_k \leq |\mathcal{B}_t^{\mathcal{K}_{u_t}}|$ **then**
        $(d_1(t), \ldots, d_K(t)) = (e_1(t), \ldots, e_K(t))$
        $M_t = \sum_{j \in K_{u_t}} d_j(t)$
      end if
    end for
  end for
  if $\sum_{t=1}^{|\mathcal{G}_u|} M_t \geq d^*$ **then**
    $d^* = \sum_{t=1}^{|\mathcal{G}_u|} M_t$
    $\mathbf{d}(t) = (d_1(t), \ldots, d_K(t))$
  end if
end for
Output: $\mathbf{d}(t)$

IV. Average Achievable Sum DoF and Numerical Results

A. Average achievable sum DoF:

In this section we analyze the average achievable sum DoF $\bar{d}_{\text{sum}}$. We can model all signaling channels with the state diagram which has been shown in Fig. 1. In this figure in each time snapshot the state of the channel remains in its previous state with the probability of p_{ij} and changes with the probability of $1 - p_{ij}$. It is clear that if $p_{i_1 i_2} = p_{i_1 i_1}$, all the direct and the cross channels have the same statistical characteristic. From Definition 4, at the transmission block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}$ all the cross channels have a constant value. The probability that we have a transmission block of $\mathcal{B}_t^{\mathcal{K}_{u_t}}$ with the n^* time snapshots can be calculated from the following relation:

$$P\left(|\mathcal{B}_t^{\mathcal{K}_{u_t}}| = n^*\right) = \prod_{k_i \neq k_j} p_{k_i k_j}^{(n^*-1)}, k_i, k_j \in \mathcal{K}_{u_t}, \tag{59}$$

where (n^*-1) comes from the fact that during n^* time snapshots we have n^*-1 transition points. During these transition points, all the cross channels should have a constant value. In (59), k_i and k_j indicate the indexes of transmitters and receivers in the set of $\mathcal{K}_{u_t}$. Let's define the vector $\mathbf{E}_{\mathcal{K}_{u_t}}$ as follows:

$$\mathbf{E}_{\mathcal{K}_{u_t}} = \left(e_{k_1}, e_{k_2}, \ldots, e_{k_r}\right), \tag{60}$$

where $e_{k_i} = n^* - \mathcal{F}\left(\bar{\mathbf{H}}_t^{[k_i k_i]}\right),\ k_i \in \mathcal{K}_{u_t}$. In fact, e_{k_i} indicates the second constraint of (57) on the achievable DoF of d_{k_i}, in other words we should have $d_{k_i} \leq e_{k_i} = n^* - \mathcal{F}\left(\bar{\mathbf{H}}_t^{[k_i k_i]}\right),\ k_i \in \mathcal{K}_{u_t}$ or equivalently:

$$\mathcal{F}\left(\bar{\mathbf{H}}_t^{[k_i k_i]}\right) \leq n^* - d_{k_i},\ k_i \in \mathcal{K}_{u_t}. \tag{61}$$

Now, we want to calculate the probability that $\mathcal{F}\left(\bar{\mathbf{H}}_t^{[k_i k_i]}\right)$ has lower or equivalent value to the term of $n^* - d_{k_i},\ k_i \in \mathcal{K}_{u_t}$. In this case the states of the diagonal channel matrix $\bar{\mathbf{H}}_t^{[k_i k_i]}$ can be modeled with the vector $\mathbf{S}^{[k_i]} = \left(s_1^{[k_i]}, \ldots, s_{L^{[k_i]}}^{[k_i]}\right)$ where $1 \leq s_i^{[k_i]} \leq n^*$, $0 \leq L^{[k_i]} \leq n^*$. The value of $s_i^{[k_i]}$ indicates the number of time snapshots which is occupied by the i^{th} state of the channel between TX_{k_i} and RX_{k_i}. The value of $L^{[k_i]}$ in the vector $\mathbf{S}^{[k_i]} = \left(s_1^{[k_i]}, \ldots, s_{L^{[k_i]}}^{[k_i]}\right)$ indicates the number of blocks in which the diagonal matrix of $\bar{\mathbf{H}}_t^{[k_i k_i]}$ during n^* time snapshots has constant value. It is clear that the vector $\mathbf{S}^{[k_i]}$ has the following relation with the channel changing pattern of $\mathcal{C}^{[k_i k_i]}$:

$$s_1^{[k_i]} = c_1^{[k_i k_i]} - 1, \tag{62}$$

$$s_l^{[k_i]} = c_l^{[k_i k_i]} - c_{l-1}^{[k_i k_i]}, \tag{63}$$

and $s_{L^{[k_i]}}^{[k_i]} = n^* - c_{L^{[k_i]}-1}^{[k_i k_i]} + 1$. As an example, for the channel matrix of $\bar{\mathbf{H}}^{[11]}$ in the table I, we have:

$$\mathbf{S}^{[11]} = (1,1,2,2,2,2),\ L^{[1]} = 6 \tag{64}$$

$$\mathbf{S}^{[22]} = (1,3,2,1,3),\ L^{[2]} = 5 \tag{65}$$

$$\mathbf{S}^{[33]} = (1,2,2,3,2),\ L^{[3]} = 5. \tag{66}$$

Now we wish to calculate the probability that during a transmission block consisted of n^* time snapshots the direct channel of $\bar{\mathbf{H}}_t^{[k_i k_i]}$ has $L^{[k_i]}$ states can be calculated as follows:

$$\begin{aligned} & p\left(|\mathbf{S}^{[k_i k_i]}| = L^{[k_i]}\right) \\ & = \binom{n^*-1}{L^{[k_i]}-1} p_{k_i k_i}^{\sum_{l=1}^{L^{[k_i]}} s_l^{[k_i k_i]} - 1} \left(1 - p_{k_i k_i}\right)^{L^{[k_i]}-1} \qquad (67) \\ & \stackrel{(a)}{=} \binom{n^*-1}{L^{[k_i]}-1} p_{k_i k_i}^{n^* - L^{[k_i]}} \left(1 - p_{k_i k_i}\right)^{L^{[k_i]}-1}, \qquad (68) \end{aligned}$$

where $|\mathbf{S}^{[k_i k_i]}| = L^{[k_i]}$ indicates the number of blocks in the direct channel of $\bar{\mathbf{H}}_t^{[k_i k_i]}$ and (a) comes from the fact that at each transmission block, the value of $\sum_{l=1}^{L^{[k_i]}} s_l^{[k_i k_i]}$ equals to the number of time snapshots of n^*. If d_{k_i} is the sum DoF of the k_i-th user, from (57) we should have $d_{k_i} \leq n^* - \mathcal{F}\left(\bar{\mathbf{H}}_t^{[k_i k_i]}\right)$. It is clear if $L^{[k_i]}$ is the number of channel states in $\mathbf{S}^{[k_i]} = \left(s_1^{[k_i]}, \ldots, s_{L^{[k_i]}}^{[k_i]}\right)$, the value of $\mathcal{F}\left(\bar{\mathbf{H}}_t^{[k_i k_i]}\right)$ can be calculated from the vector $\mathbf{S}^{[k_i]}$ with the following relation:

$$\mathcal{F}\left(\bar{\mathbf{H}}_t^{[k_i k_i]}\right) = \max_{1 \leq l \leq L^{[k_i]}} s_l^{[k_i k_i]}, \tag{69}$$

where $\max_{1\leq l\leq L^{[k_i]}} s_l^{[k_ik_i]}$ indicates the maximum block length in which the channel matrix $\bar{\mathbf{H}}_t^{[k_ik_i]}$ has the constant value. From (68), we can conclude that the probability of occurance all the state vectors with $L^{[k_i]}$ states are similar and equals to $\binom{n^*-1}{L^{[k_i]}-1} p_{k_ik_i}^{n^*-L^{[k_i]}} \left(1-p_{k_ik_i}\right)^{L^{[k_i]}-1}$.

Using (57) and (68), we want to calculate the probability that all the values of the $s_l^{[k_ik_i]}, 1 \leq l \leq L^{[k_i]}$, have lower value than $n^* - d_{k_i}$. We define the set of events $\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)$ as follows:

$$\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right) = \left\{ s_l^{[k_ik_i]} \mid \max_l s_l^{[k_ik_i]} \leq n^* - d_{k_i}, \sum_{l=1}^{L^{[k_i]}} s_l^{[k_ik_i]} = n^*, s_l \geq 1 \right\}. \tag{70}$$

From [18], we can easily show that the cardinality of the set $\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)$ has the following value:

$$|\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)| = \sum_{j=0}^{L^{[k_i]}} (-1)^j \binom{L^{[k_i]}}{j} \binom{n^* - j(n^* - d_{k_i}) - 1}{L^{[k_i]} - 1}. \tag{71}$$

Let $\mathcal{Z}^{[k_ik_i]}\left(n^*, L^{[k_i]}\right)$ be defined as follows:

$$\mathcal{Z}^{[k_ik_i]}\left(n^*, L^{[k_i]}\right) = \left\{ s_l^{[k_ik_i]} \mid \sum_{l=1}^{L^{[k_i]}} s_l^{[k_ik_i]} = n^*, s_l \geq 1 \right\}, \tag{72}$$

similar to (71) from [18], the cardinality of the set $\mathcal{Z}^{[k_ik_i]}\left(n^*, L^{[k_i]}\right)$ has the value of $\binom{n^*-1}{L^{[k_i]}-1}$. From (68), all the events in the set of $\mathcal{Z}^{[k_ik_i]}\left(n^*, L^{[k_i]}\right)$ have the same probability of occurrence. So, the probability of occurring each member of the set $\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)$ can be calculated by dividing the cardinality of the set $\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)$ to the cardinality of the set $\mathcal{Z}^{[k_ik_i]}\left(n^*, L^{[k_i]}\right)$ multiplied by the probability that we have $L^{[k_i]}$ states (68). In other words, we have:

$$\begin{aligned} p\left(\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)\right) &= p\left(|\mathbf{S}^{[k_ik_i]}| = L^{[k_i]}\right) \frac{|\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)|}{|\mathcal{Z}^{[k_ik_i]}\left(n^*, L^{[k_i]}\right)|} && (73) \\ &= p_{k_ik_i}^{n^*-L^{[k_i]}} \left(1-p_{k_ik_i}\right)^{L^{[k_i]}-1} |\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)|, && (74) \end{aligned}$$

So, from (74) and (71) we have:

$$p\left(\mathcal{X}^{[k_ik_i]}\left(n^*, L^{[k_i]}, d_{k_i}\right)\right) = p_{k_ik_i}^{n^*-L^{[k_i]}} \left(1-p_{k_ik_i}\right)^{L^{[k_i]}-1} \sum_{j=0}^{L^{[k_i]}} (-1)^j \binom{L^{[k_i]}}{j} \binom{n^* - j(n^* - d_{k_i}) - 1}{L^{[k_i]} - 1}. \tag{75}$$

The probability that in a transmission block consisted of n^* time snapshots, the $|\mathcal{K}_{u_t}|$-tuple of $(d_{k_1}, \ldots, d_{k_r})$ from the active transceivers with the indexes in the set of $\mathcal{K}_{u_t}$ lies in the achievable DoF region of our proposed method can be derived as follows:

$$p\left(d_{k_1},\ldots,d_{k_r}|\mathcal{K}_{u_t},|\mathcal{B}_t^{\mathcal{K}_{u_t}}|=n^*\right)=$$
$$\prod_{k_i\in\mathcal{K}_{u_t}}\sum_{L^{[k_i]}=1}^{n^*}p\left(\mathcal{X}^{[k_ik_i]}\left(n^*,L^{[k_i]},d_{k_i}\right)\right)\ i,j\in\mathcal{K}_{u_t}. \tag{76}$$

Therefore, from (59) we have:

$$p\left(d_{k_1},\ldots,d_{k_r}|\mathcal{K}_{u_t}\right)=$$
$$\prod_{i\neq j}p_{ij}^{(n^*-1)}\prod_{k_i\in\mathcal{K}_{u_t}}\sum_{L^{[k_i]}=1}^{n^*}p\left(\mathcal{X}^{[k_ik_i]}\left(n^*,L^{[k_i]},d_{k_i}\right)\right)\ i,j\in\mathcal{K}_{u_t}, \tag{77}$$

where the term $\prod_{i\neq j}p_{ij}^{(n^*-1)}$ indicates the probability of $P\left(|\mathcal{B}_t^{\mathcal{K}_{u_t}}|\right)$ as it has been calculated in (59). Therefore, the average transmission sum DoF for our proposed method can be calculated as follows:

$$\bar{d}_{\text{sum}}=\sum_{1\leq n^*\leq\infty,\mathcal{K}_{u_t}\subseteq\mathcal{K}}\left(p\left(d_{k_1},\ldots,d_{k_r}|\mathcal{K}_{u_t}\right)\frac{1}{n^*}\sum_{k_i\in\mathcal{K}_{u_t}}d_{k_i}\right)$$
$$,1\leq n^*\leq\infty, \tag{78}$$

where

$$d_{k_i}+\max_{k_j\in\mathcal{K}_{u_t},k_j\neq k_i}d_{k_j}\leq n^*. \tag{79}$$

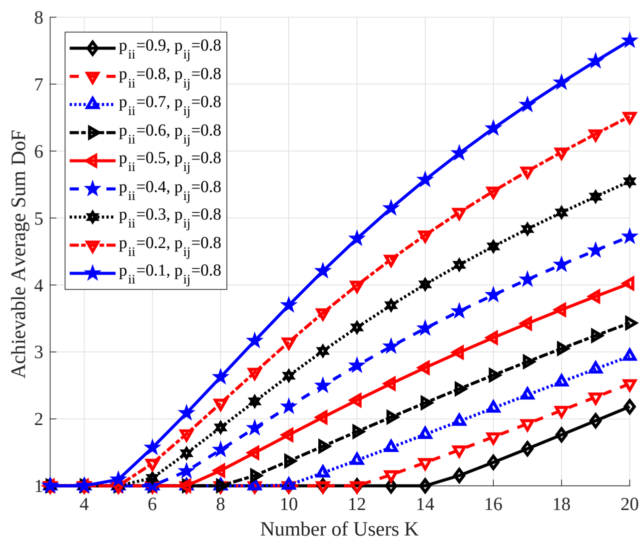

Fig. 4. This figure shows the average achievable sum DoF as a function of number of user. As the number of users increases the chance of finding proper channel conditions increase and we achieve to higher average achievable sum DoF.

Fig. 4 shows the numerical results for the case where the probability of channel changing pattern for the cross channels are 0.8 and the probability of channel changing pattern for the direct channels ranges from 0.1 to 0.9. As it has been indicated when the direct channel has higher fluctuations and variability, we can achieve higher average achievable sum DoF. In the next section we propose a blind interference alignment scheme that does not access to the channel changing pattern.

V. Blind Opportunistic Interference Alignment

In the previous section we assumed that all the transmitters have access to all the channel changing patterns. Although this information needs lower feedback rate than previous schemes, but it is necessary to allocate a part of channel bandwidth for the feedback link. In addition before data transmission the transmitters need to access channel changing pattern which is not practical.

In this section, we consider the case where the transmitters have not access to the channel state information and the transmitters by the aid of superposition coding try to maximize their average transmission rate. At all of the transmitters $\mathrm{TX}_k, 1 \leq k \leq K$, we use a multi-layer encoding scheme consists of M distinct layers of $l_i^{[k]}, 1 \leq i \leq M$. In this coding procedure, every layers of $l_i^{[k]}, 1 \leq i \leq M$ are represented by a Gaussian random variable $X_i^{[k]},\ 1 \leq i \leq M$. Without accessing channel state information at transmitters, we are going to maximize the average transmission rate. These transmission layers are designed such that at the receiver the decoder based on the state of channel changing pattern structure can recover some parts of the transmitted data.

A. Encoding method

1) Let $X_i^{[k]}, 1 \leq i \leq M$ be M Gaussian distributed continuous random variables with $X_i^{[k]} \sim \mathcal{N}\left(0, P_i^{[k]}\right)$, where $P_i^{[k]}$[1] is the transmission power assigned for i^{th} layer at TX_k. Therefore, we have:
$$\sum_{i=1}^{M} P_i^{[k]} = P_t^{[k]}, 1 \leq k \leq K, \tag{80}$$
where $P_t^{[k]}$ is the total transmission power. We assume all the transmitters use the same transmission power of, e. g. $P_t^{[1]} = P_t^{[2]} = \cdots = P_t^{[K]} = P_t$, therefore for simplification of the notation we omit the index of k from our formulation and we represent the transmission power of each transmitter by P_t.
2) At TX_k, we partition our message $W^{[k]}$ into M sub-messages of $\left(W_1^{[k]}, \cdots, W_M^{[k]}\right)$, and we send each sub-message via a single transmission layer.
For each layer of $l_i^{[k]}, 1 \leq i \leq M$ consisted of n time snapshots, we consider $\left(2^{\frac{n}{2}R_i^{[k]}}, \frac{n}{2}\right)$ code for the Gaussian channel with power constraint of $P_i^{[k]}$ where $W_i^{[k]} \in \{1, \ldots, 2^{\frac{n}{2}R_i^{[k]}}\}$ and we generate codewords of $x_{i1}^{[k]\frac{n}{2}}(1), \ldots, x_{i1}^{[k]\frac{n}{2}}(2^{\frac{n}{2}R_i^{[k]}})$ for the i^{th} layer with the following constraint:
$$\sum_{j=1}^{\frac{n}{2}} x_{ij}^{[k]}(w_i) \leq nP_i^{[k]}, w_i \in \{1, \ldots, 2^{\frac{n}{2}R_i^{[k]}}\} \tag{81}$$
where j and R_i indicate the index of time and the transmission rate of the i^{th} layer, respectively.
3) In the j-th transmission time, TX_k computes $X_j^{[k]} = \sum_{i=1}^{M} x_{ij}^{[k]\frac{n}{2}}(w_i)$ of M independent random variables $X_{1j}^{[k]}, \cdots, X_{Mj}^{[k]}$.
4) The transmitters use a random full rank matrix of $\bar{\mathbf{V}}$ with the size of $n \times \frac{n}{2}$ as their precoder matrices.

[1]For simplifying the notation all the superscript in [.] indicates the indexes of the users.

5) The output of the encoding function at TX_k, $\bar{\mathbf{X}}^{[k]}$, is an $n \times 1$ column vector with the following relation:

$$\bar{\mathbf{X}}^{[k]} = \bar{\mathbf{V}}\bar{\mathbf{x}}^{[k]} \tag{82}$$

where $\bar{\mathbf{x}}^{[k]}$ is a column matrix as follows:

$$\bar{\mathbf{x}}^{[k]} = \left[X_1^{[k]}, \dots, X_{\frac{n}{2}}^{[k]}\right]^T. \tag{83}$$

B. Model of received signal

Fig. 4, shows a simple model for reception space at a receiver for 3-user interference channel. In this case we assume that $p_{ij} << p_{ii}, i \neq j$, it means that the probability of changing channel value for the cross channels is much lower than direct channels. As it is indicated in this figure all the interference signals are aligned at the receiver, while depend on the value of $\mathcal{F}\left(\bar{\mathbf{H}}\right)$ the direct channel signal space has some free interference dimensions which are linearly independent from the interference signals. The decoder decodes the transmitted signal using following zero forcing matrix:

$$\bar{\mathbf{D}} = \left(\bar{\mathbf{I}} - \bar{\mathbf{V}}(\bar{\mathbf{V}}^H\bar{\mathbf{V}})^{-1}\bar{\mathbf{V}}^H\right) \tag{84}$$

Since all the transmitters use the same precoder of $\bar{\mathbf{V}}$, for the encoding scheme of the previous subsection where all the transmitted signals have the same number of dimension of $\frac{n}{2}$, the number of free interference subspace dimensions can be calculated from the first lemma as follows:

$$\text{rank}\left([\mathbf{V}\ \mathbf{H}\mathbf{V}]\right) - \text{rank}\left([\mathbf{V}]\right) = \min\left(d_v + \min\left(d_v, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right) - d_v, \tag{85}$$

Substituting $d_v = \frac{n}{2}$ in the above equation we have:

$$\begin{aligned}
\text{rank}\left([\mathbf{V}\ \mathbf{H}\mathbf{V}]\right) - \text{rank}\left([\mathbf{V}]\right) &= \min\left(\frac{n}{2} + \min\left(\frac{n}{2}, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), n\right) - \frac{n}{2} &(86)\\
&= \min\left(\min\left(\frac{n}{2}, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right), \frac{n}{2}\right) &(87)\\
&= \min\left(\frac{n}{2}, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right) &(88)
\end{aligned}$$

C. Decoding method

For simplicity of our notation we consider the decoding operation at the first receiver and we omit the index of k (transmitter-receiver index) from our formulation. Also, we consider all the channel matrices are known at their corresponding receivers and the channel matrices give no gain to the received signals and we can omit their gains at all the receivers.

After applying zero forcing matrix, let y^n be the received sequence at the first receiver, then based on the following successive decoding strategy steps we can decode a part of transmitted data:

1) First layer decoding: The decoder finds that $\widehat{w}_1$ is sent if there exists a unique message such that $\left(x_1^{[1]\frac{n}{2}}(\widehat{w}_1), y^n\right) \in \mathcal{T}_\epsilon^n$, otherwise it declares error in the first decoding step.
2) Second layer decoding: After decoding the first layer and if such $\widehat{w}_1$ is found, the decoder tries to find $\widehat{w}_2$ such that the 3-tuple of $\left(x_1^{[1]\frac{n}{2}}(\widehat{w}_1), x_2^{[1]\frac{n}{2}}(\widehat{w}_2), y^n\right) \in \mathcal{T}_\epsilon^n$, otherwise it declares error in the second decoding stage.

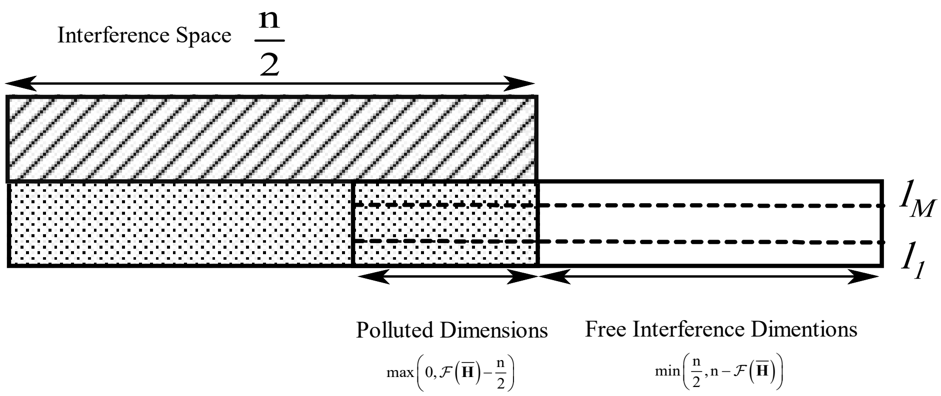

Fig. 5. The desired signal space at one of the receivers. As it indicated in this figure the reception space at a receiver consisted of three different subspaces, the desired space which is free of interference signal and has the dimension of $\min\left(\frac{n}{2}, n-\mathcal{F}(\bar{\mathbf{H}})\right)$, the desired signal space which is polluted by interference signals and has the dimension of $\max\left(0, \mathcal{F}(\bar{\mathbf{H}})-\frac{n}{2}\right)$.

3) i-th layer decoding: After finding unique messages $\widehat{w}_1, \cdots, \widehat{w}_{i-1}$ in the previous steps, the decoder finds $\widehat{w}_i$ such that $\left(x_1^{[1]\frac{n}{2}}(\widehat{w}_1), \ldots, x_i^{[1]\frac{n}{2}}(\widehat{w}_i), y^n\right) \in \mathcal{T}_\epsilon^n$, otherwise it declares error in the i-th decoding step.

D. Error probability analysis

In this subsection, we show that we can set the following transmission rate for i-th transmission layer:

$$R_i \leq I\left(X_i^{[1]}; Y|X_1^{[1]}, \ldots, X_{i-1}^{[1]}\right), \tag{89}$$

which guaranties its decodability at the receiver.

Let $\mathcal{E}_i$ be an event that decoding the i-th layer is erroneous. Thanks to the symmetry of the random codebook generation,

$$\begin{aligned} P\left(\mathcal{E}_i\right) &= P\left(\mathcal{E}_i|w_1 = 1, \ldots, w_M = 1\right) \\ &= P\left(\widehat{w}_i \neq 1|w_1 = 1, \ldots, w_M = 1\right), \end{aligned} \tag{90}$$

where $\widehat{w}_i$ indicates the i-th decoded message at the receiver. The decoder in the first step of decoding makes an error if one or both of the following events occur:

$$\mathcal{E}_{11} = \{\left(x_1^{[1]\frac{n}{2}}(1), y^n\right) \notin \mathcal{T}_\epsilon^n\}, \tag{91}$$

$$\mathcal{E}_{12} = \{\exists \widehat{w}_1 \neq 1: \ \left(x_1^{[1]\frac{n}{2}}(\widehat{w}_1), y^n\right) \in \mathcal{T}_\epsilon^n\}. \tag{92}$$

By the law of large numbers (LLN), the probability of the first term $P(\mathcal{E}_{11})$ tends to zero as $n \to \infty$. The second event $P(\mathcal{E}_{12})$ with the aid of packing lemma tends to zero as n goes to infinity if $R_1 \leq I\left(X_1; Y\right)$ [**?**]. So, if $R_1 \leq I\left(X_1; Y\right)$ and $n \to \infty$ the value of $P(\mathcal{E}_1) = P(\mathcal{E}_{11} \cup \mathcal{E}_{12})$ goes to zero. Now, assume that the decoder recovers the messages $w_1, \ldots, w_{i-1}$ correctly. In the i-th step, the decoder makes an error if one or both of the following events occur:

$$\mathcal{E}_{i1} = \{ \left(x_1^{[1]\frac{n}{2}}(1), \ldots, x_i^{[1]\frac{n}{2}}(1), y^n \right) \notin \mathcal{T}_\epsilon^n \}, \tag{93}$$

$$\mathcal{E}_{i2} = \{ \exists \widehat{w}_i \neq 1 : \left(x_1^{[1]\frac{n}{2}}(1), \ldots, x_i^{[1]\frac{n}{2}}\left(\widehat{w}_i\right), y^n \right) \in \mathcal{T}_\epsilon^n \}. \tag{94}$$

Again by LLN, $P(\mathcal{E}_{i1}) \to 0$ as $n \to \infty$, and by using packing lemma if $R_i^{[1]} \leq I\left(X_i^{[1]}; Y | X_1^{[1]}, \ldots, X_{i-1}^{[1]} \right)$, $P(\mathcal{E}_{i2})$ tends to zero as n goes to infinity. So, if $R_i^{[1]} \leq I\left(X_i^{[1]}; Y | X_1^{[1]}, \ldots, X_{i-1}^{[1]} \right)$ and $n \to \infty$ the value of $P(\mathcal{E}_i) = P(\mathcal{E}_{i1} \cup \mathcal{E}_{i2})$ goes to zero. Also, we can assume that for decoding each layer of i we have i free dimensions at the receiver, therefore without losing generality of our problem we can assume that the number of layers is $\frac{n}{2}$ ($M = \frac{n}{2}$). For the encoding strategy of $R_i \leq I\left(X_i^{[1]}; Y | X_1^{[1]}, \ldots, X_{i-1}^{[1]} \right)$ we have:

$$R_i^{[1]} \leq I\left(X_i^{[1]}; Y | X_1^{[1]}, \ldots, X_{i-1}^{[1]} \right) \tag{95}$$

$$\leq \frac{i}{2} \log \left(1 + \frac{P_i^{[1]}}{N + \sum_{j=i+1}^{M} P_j^{[1]}} \right), \; M = \frac{n}{2}. \tag{96}$$

where N is the noise power and we treat interference as noise at receiver, also the value of i indicates the number of free interference dimension at the receiver.

Note: We assume that the channel matrices and the zero-forcing operation at the receivers give no gain at decoding operation or have the same effect on the received signals and the additive noise. Therefore, without losing generality and for simplicity we can omit their effect on the received signal.

Depend on different states of the channel at receiver the receiver try to decode a part of transmitted data. If the number of free interference space at the receiver is l_s we can except that the receiver can decode the transmission layers from the first layer to $l_s - th$ layer. In the next subsection we try to divide the total transmission power among $M = \frac{n}{2}$ layers to maximize the average transmission power.

E. Maximize average transmission rate:

Referring previous subsection, the average transmission rate per transmission time can be calculated from the following relation:

$$\bar{R}_{\text{avg}} = \frac{1}{n} \sum_{i=1}^{\frac{n}{2}} \frac{i}{2} \log \left(1 + \frac{P_i^{[1]}}{N + \sum_{j=i+1}^{M} P_j^{[1]}} \right) F_I\left(\frac{n}{2} - i \right) \tag{97}$$

where $F_I\left(\frac{n}{2} - i\right) = p\left(\min\left(\frac{n}{2}, n - \mathcal{F}\left(\bar{\mathbf{H}}\right)\right) \geq i\right)$. When the value of n has enough large value, the value of $\frac{i}{n}, 1 \leq i \leq \frac{n}{2}$ and the function $P_i^{[1]}$ can be represented by the new parameters of $0 \leq z \leq \frac{1}{2}$ and $\rho(z)dz$, respectively.

In this case we consider that $P(1/2 - z) = \int_z^{\frac{1}{2}} \rho(x)dx$. So, $P(0) = \int_{\frac{1}{2}}^{\frac{1}{2}} \rho(x)dx = 0$ and $P(z = 1/2) = \int_0^{\frac{1}{2}} \rho(x)dx = P_t^{[1]}$. Therefore, when the value of n has enough large value, the equation 97 can be represented as follows:

$$\bar{R}_{\text{avg}} = \frac{1}{2\ln 2} \int_0^{\frac{1}{2}} \frac{z\rho(1/2 - z) F_Z\left(1/2 - z\right)}{N + P(1/2 - z)} dz \tag{98}$$

where $F_Z\left(\frac{1}{2}-z\right)=F_I\left(\frac{n}{2}-\lfloor nz\rfloor\right),\ 0\leq z\leq\frac{1}{2}$ and $\rho(z)=\frac{dP(z)}{dz}$. The following theorem help us to maximize the above average transmission rate. For simplifying our notation we subside $\frac{1}{2}-z$ with u and therefore we have:

$$\bar{R}_{\text{avg}}=\frac{1}{2\ln 2}\int_{\frac{1}{2}}^{0}\frac{-(1/2-u)\rho(u)F_U\left(u\right)}{N+P(u)}du \tag{99}$$

$$=\frac{1}{2\ln 2}\int_{0}^{\frac{1}{2}}\frac{(1/2-u)\rho(u)F_U\left(u\right)}{N+P(u)}du \tag{100}$$

Theorem 1: A necessary condition for the function $y(u)$ to be an extremum of:

$$\int_{z_1}^{z_2}D\left(y,y^{'},u\right)du, \tag{101}$$

is that y satisfies the following Euler differential equation:

$$D_y-\frac{dD_{y'}}{du}=0,\ \ z_1\leq u\leq z_2. \tag{102}$$

where the subscripts denote the partial derivatives with respect to corresponding arguments [19], [20].

In our problem we have:

$$D(y,y',u)=\frac{1}{2\ln 2}\frac{(1/2-u)\rho(u)F_U\left(u\right)}{N+P(u)} \tag{103}$$

where $y(u)=P(u)$ and $y'(u)=\rho(u)$. In this case we have:

$$D_y=\frac{1}{2\ln 2}\frac{-(1/2-u)\rho(u)F_U(u)}{\left(N+P(u)\right)^2} \tag{104}$$

$$D_{y'}=\frac{1}{2\ln 2}\frac{(1/2-u)F_U(u)}{N+P(u)}. \tag{105}$$

So, we have:

$$\frac{dD_{y'}}{du}=\frac{1}{2\ln 2}\frac{(-F_U(u)+(1/2-u)f_U(u))(N+P(u))}{(N+P(u))^2}, \tag{106}$$

where $f_U(u)=\frac{dF_U(u)}{du}$, and from the first theorem we have:

$$(1/2-u)\rho(u)F_U(u)+(-F_U(u)+(1/2-u)f_U(u))(N+P(u))=0 \tag{107}$$

So, we have:

$$\frac{dP(u)}{N+P(u)}=-\frac{(-F_U(u)+(1/2-u)f_U(u))du}{(1/2-u)F_U(u)}, \tag{108}$$

and finally:

$$\ln\left(N+P(u)\right)=-\ln\left((1/2-u)F_U(u)\right)+C. \tag{109}$$

Therefore:

$$P(1/2-z)=\frac{C}{zF_Z(1/2-z)}-N. \tag{110}$$

where $P(1/2-z=0)=0$. So, the value of C can be calculated by setting $z=\frac{1}{2}$ from the above equation as follows:

$$C=\frac{N}{2}F_Z\left(0\right), \tag{111}$$

and the function $P(z)$ can be calculated as follows:

$$P(z)=\frac{N}{2(1/2-z)F_Z(z)}F_Z\left(0\right)-N. \tag{112}$$

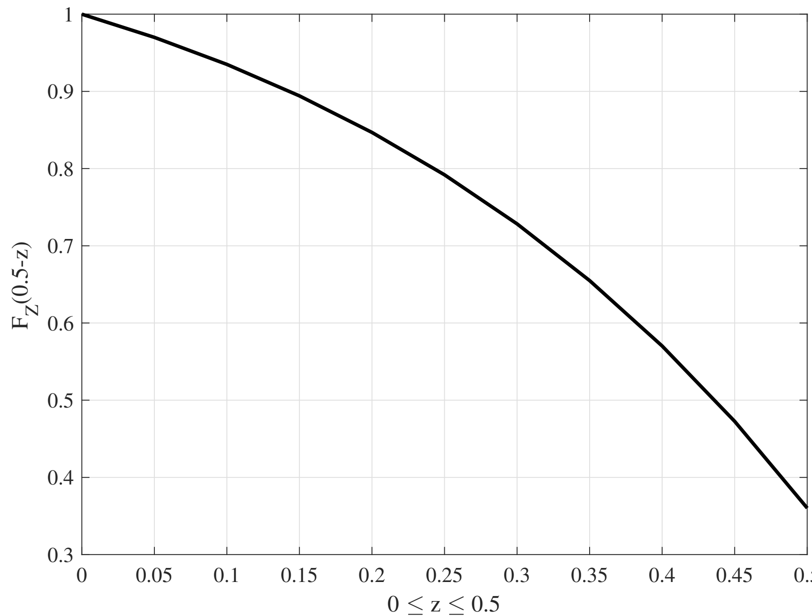

Fig. 6. Finding the cumulative distribution of $F_Z(z)$ from $F_I(i)$. The value of the function $F_I(i)$ can be calculated from $F_I(i) = \sum_{L=1}^{n} p\left(\mathcal{X}^{[11]}\left(n, L, n-i\right)\right)$.

It is obvious when $P(z_0) \geq P_t^{[1]}$ then $P(z > z_0) = P_t^{[1]}$. From (71) the value of the function $F_Z(z)$ for the precoder length of $n = 20$ and the probability of changing direct channel of $p_{11} = 0.9$, has been depicted in Fig. 5. By finding $P(z)$ the transmission power for each transmission layer can be calculated as follows:

$$P_j^{[1]} = P_Z\left(\frac{j-1}{n}\right) - P_Z\left(\frac{j}{n}\right). \tag{113}$$

For a case where $P_t^{[1]} = 100$, $n = 20$ and $N = 1$ the total transmission power for each transmission layer has been depicted in Fig. 6.

F. Numerical Results:

In this subsection, we consider an interference channel in an enough low block length transmission time which is consisted of $n = 10$ time snapshots. Because of the deployed antenna structure of Fig. 1, we can divide the transmission time into enough small blocks in which all the cross links have a constant value. In this example, we assume that the value of transmission block length of $n = 10$ satisfies our assumption that all the cross channels remain constant. Fig. 8 shows the average achievable transmission rate for different values of p_{11} using multi-layer encoding strategy of section V. This figure indicates that when the value of p_{11} has lower value the chance of finding proper channel states to received desired signal linearly independent from interference signals increases.

VI. Conclusion

In this paper, we investigate the problem of interference alignment for the $K-$user IC. The proposed scheme allows transceivers to achieve higher average sum rate without knowledge of channel state information at trans-

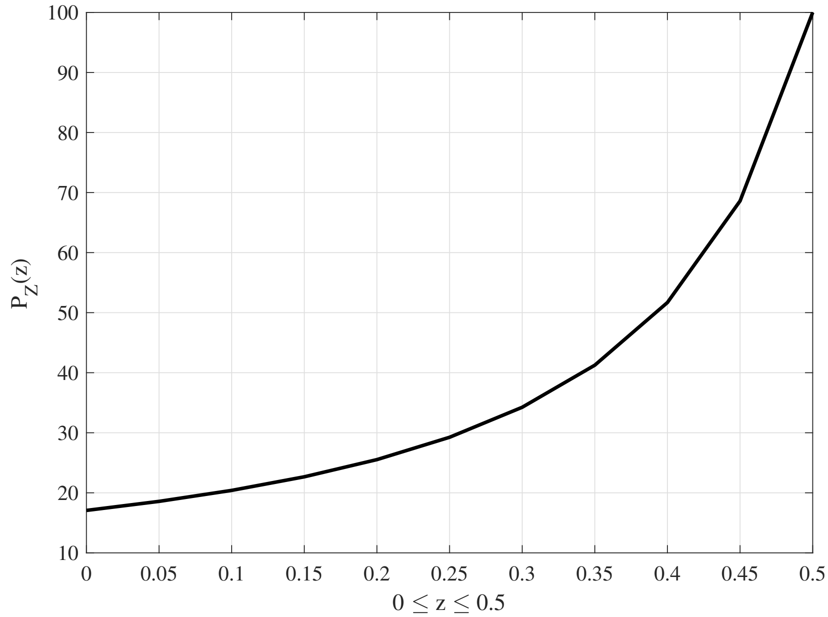

Fig. 7. The value of the $P_Z(z)$ as a function of $0 \leq z \leq \frac{1}{2}$. In this figure we assume $F_Z(z)$ has the figure like the previous figure in which $p_{11} = 0.9$ and $P_t^{[1]} = 100$.

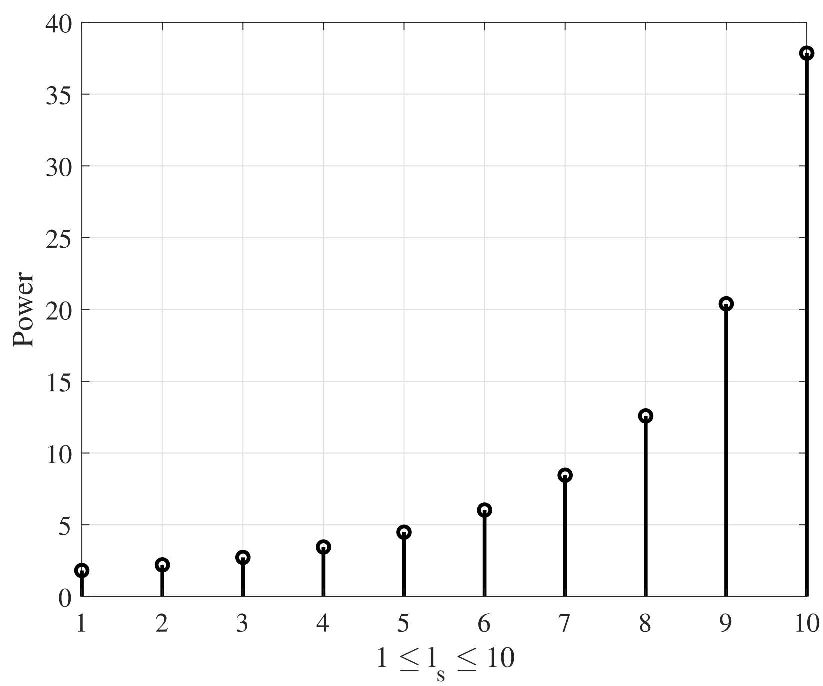

Fig. 8. This figure indicates the devoted power for each transmission layer of the proposed scheme. In this figure we assume that $n = 20$ and $p_{11} = 0.9$.

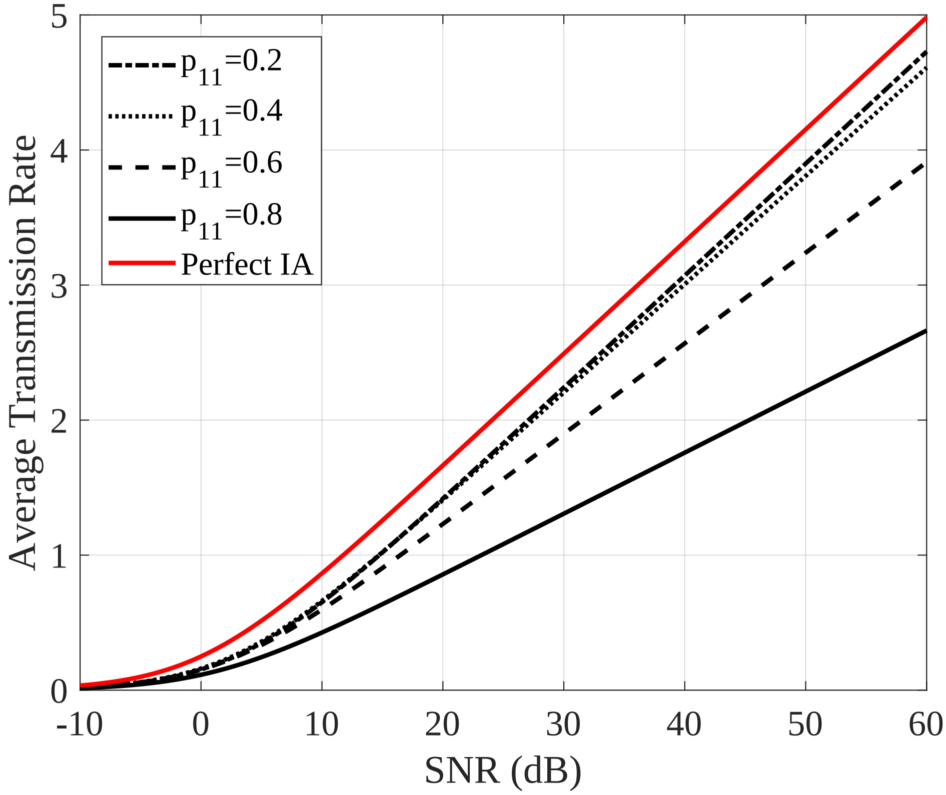

Fig. 9. Average achievable rate for each user in the $K-$user IC, and 4 different cases of $p_{11} = \{0.2, 0.4, 0.6, 0.8\}$ and $n = 20$. In this figure we assume that all the cross channels due to our implemented antenna structure have constant values. This figure shows that when the direct channels have higher probability of changing we can achieve higher average transmission rate.

mitters. In fact, in the proposed scheme of this paper, all the transmitters in a hunting mode strategy wait for proper channel conditions to transmit their data with the highest available sum DoF. We showed that with an antenna structure deployed at transmitters one can generate conditions to increase the chance of alignment conditions. For a more practical scenario in which the transmitters have not accessed to the channel variation time snapshots, we proposed a multi-layer encoding strategy. In this method based on cumulative distribution of $F_I\left(\frac{n}{2}-i\right) = p\left(\min\left(\frac{n}{2}, n-\mathcal{F}\left(\bar{\mathbf{H}}\right)\right) \geq i\right)$, we divide the total transmission power among many layers to achieve highest achievable sum rate. At the receivers based on the channel conditions, the decoder can decode a part of transmitted information from the desired transmitter.

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