Before you turn this problem in, make sure everything runs as expected. First, **restart the kernel** (in the menubar, select Kernel→Restart) and then **run all cells** (in the menubar, select Cell→Run All).

Make sure you fill in any place that says `YOUR CODE HERE` or "YOUR ANSWER HERE", as well as your name and collaborators below:

In [1]:
```
NAME = "Lin Tun Naing"
ID = "st122403"
```

Lab 05: Optimization Using Newton's Method

In this lab, we'll explore an alternative to gradient descent for nonlinear optimization problems: Newton's method.

Newton's method in one dimension

Consider the problem of finding the *roots* $\mathbf{x}$ of a nonlinear function $f : \mathbb{R}^N \rightarrow \mathbb{R}$. A root of f is a point $\mathbf{x}$ that satisfies $f(\mathbf{x}) = 0$.

In one dimension, Newton's method for finding zeroes works as follows:

1. Pick an initial guess x_0
2. Let $x_{i+1} = x_i + \frac{f(x_i)}{f'(x_i)}$
3. If not converged, go to #2.

Convergence occurs when $|f(x_i)| < \epsilon_1$ or when $|f(x_{i+1}) - f(x_i)| < \epsilon_2$.

Let's see how this works in practice.

Example 1: Root finding for a cubic polynomial

Let's begin by using Newton's method to find roots of a simple cubic polynomial

$$f(x) = x^3 + x^2.$$

In [2]:
```
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
import pandas as pd
```

Here's a function to evaluate a polynomial created with Numpy's `poly1d` function at a particular point x:

In [3]:
```
def fx(x, p):
    f_x = np.polyval(p, x)
    return f_x
```

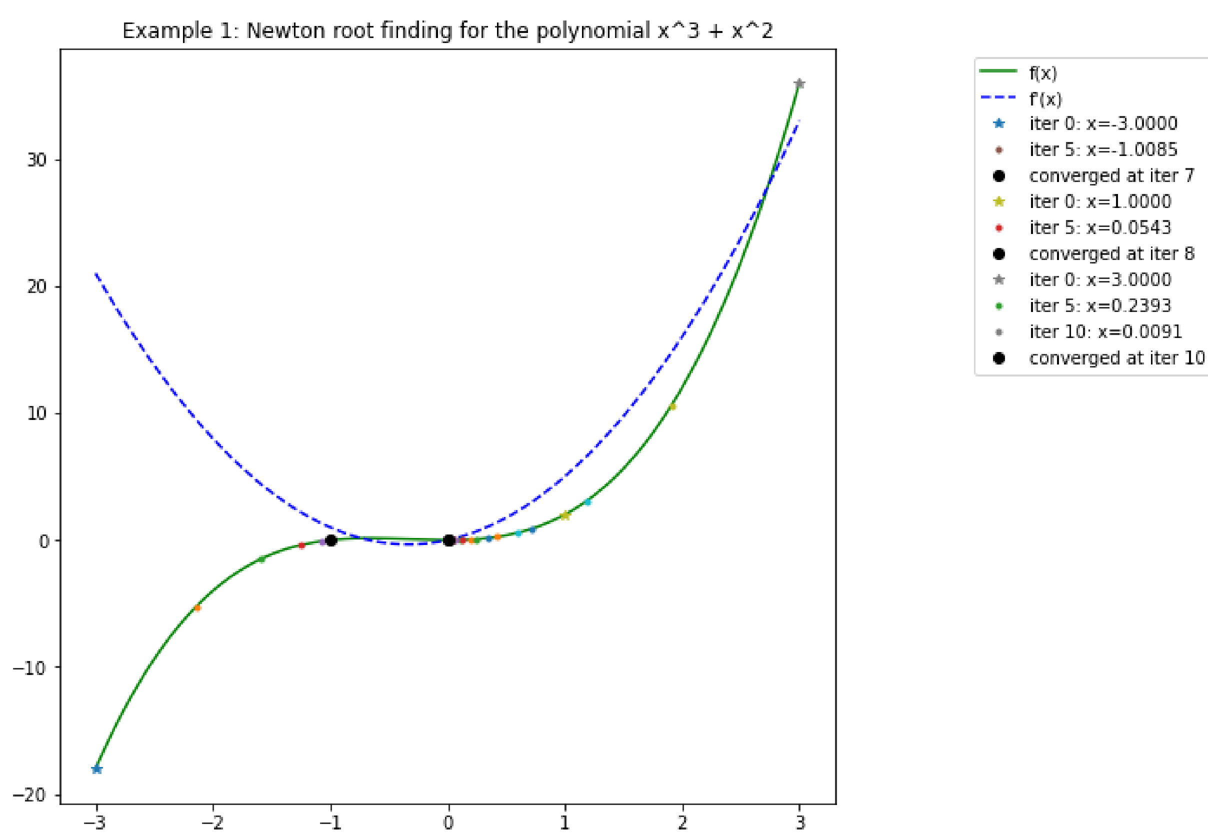
And here's some code to create the polynomial $x^3 + x^2$, get its derivative, and evalute the derivative at 200 points along the x axis:;

In [4]:
```
# Create the polynomial f(x) = x^3 + x^2
p = np.poly1d([1, 1, 0, 0]) # [1 * x^3, 1 * x^2, 0 * x^1, 0 * 1]

# Get f'(x) (the derivative of f(x) in polynomial form)
# We know it's 2x^2 + 2x, which is [3, 2, 0] in poly1d form
p_d = np.polyder(p)

print('f(x):')
print('----------')
print(p)
print('----------')
print("f'(x):")
print('----------')
print(p_d)
print('----------')

# Get 200 points along the x axis between -3 and 3
n = 200
x = np.linspace(-3, 3, n)

# Get values for f(x) and f'(x) in order to graph them later
y = fx(x, p)
y_d = fx(x,p_d)
```

```
f(x):
----------
   3     2
1 x + 1 x
----------
f'(x):
----------
   2
3 x + 2 x
----------
```

Next, let's try three possible guesses for x_0: -3, 1, and 3, and in each case, run Newton's root finding method from that initial guess.

In [5]:
```
# Initial guesses
x0_arr = [-3.0, 1.0, 3.0]

# Parameters for Newton: number of iterations,
# threshold for identifying a point as a zero
max_iters = 30
threshold = 0.0001

# Set up plot
fig1 = plt.figure(figsize=(8,8))
ax = plt.axes()
plt.plot(x, y, 'g-', label='f(x)')
plt.plot(x, y_d, 'b--', label="f'(x)")

roots = []
for x0 in x0_arr:
    i = 0
    xi = x0
    fxi = fx(xi, p)
    # Plot initial data point
    plt.plot(xi, fxi, '*', label=("iter 0: x=%.4f" % x0))
    while i < max_iters:
        # x_i+1 = x_i - f(x_i)/f'(x_i)
        xi = xi - fx(xi, p) / fx(xi, p_d)
        fxi = fx(xi, p)
        # Plot (xi, fxi) and add a legend entry every 5 iterations
        if (i+1) % 5 == 0:
            plt.plot(xi, fxi, '.', label=("iter %d: x=%.4f" % (i+1, xi)))
        else:
            plt.plot(xi, fxi, '.')
        # Check if |f(x)| < threshold
        if np.abs(fxi) < threshold:
            roots.append(xi)
            break
        i = i + 1
    plt.plot(xi, fx(xi, p), 'ko', label=("converged at iter %d" % (i+1)))

plt.legend(bbox_to_anchor=(1.5, 1.0), loc='upper right')
plt.title('Example 1: Newton root finding for the polynomial x^3 + x^2')

plt.show()
```



Example 2: Root finding for the sine function

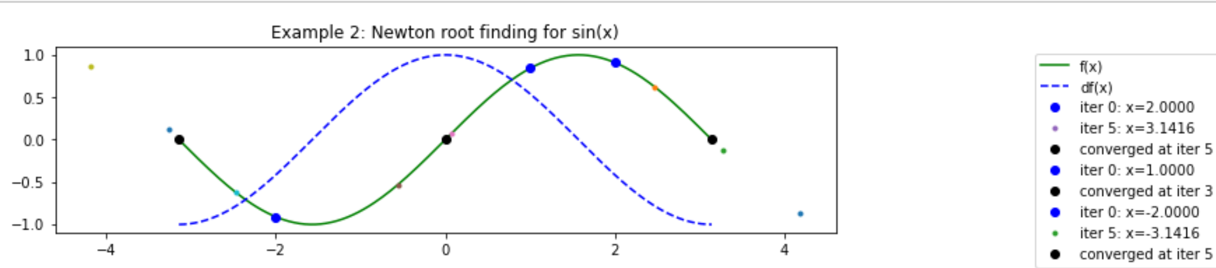
Next, consider the function $f(x) = \sin(x)$:

In [6]:
```python
def fx_sin(x):
    return np.sin(x)

def fx_dsin(x):
    return np.cos(x)
```

Let's get 200 points in the range $[-\pi, \pi]$ for plotting:

In [7]:
```python
# Get f(x)=sin(x) and f'(x) at 200 points for plotting
n = 200
x = np.linspace(-np.pi, np.pi, n)
y = fx_sin(x)
y_d = fx_dsin(x)
```

In [8]:
```python
# Initial guesses
x0_arr = [2.0, 1.0, -2.0]

# Parameters for Newton: number of iterations,
# threshold for identifying a point as a zero
max_iters = 30
threshold = 0.0001

# Set up plot
fig1 = plt.figure(figsize=(10,10))
ax = plt.axes()
ax.set_aspect(aspect='equal', adjustable='box')
plt.plot(x, y, 'g-', label='f(x)')
plt.plot(x, y_d, 'b--', label='df(x)')

roots = []
for x0 in x0_arr:
    i = 0;
    xi = x0
    fxi = fx_sin(xi)
    # Plot initial data point
    plt.plot(xi, fxi, 'bo', label=("iter 0: x=%.4f" % x0))
    while i < max_iters:
        # x_i+1 = x_i - f(x_i)/f'(x_i)
        xi = xi - fx_sin(xi) / fx_dsin(xi)
        fxi = fx_sin(xi)
        # Plot (xi, fxi) and add a legend entry every 5 iterations
        if (i+1) % 5 == 0:
            plt.plot(xi, fxi, '.', label=("iter %d: x=%.4f" % (i+1, xi)))
        else:
            plt.plot(xi, fxi, '.')
        # Check if |f(x)| < threshold
        if np.abs(fxi) < threshold:
            roots.append(xi)
            break
        i = i + 1
    plt.plot(xi, fx_sin(xi), 'ko', label=("converged at iter %d" % (i+1)))

plt.legend(bbox_to_anchor=(1.5, 1.0), loc='upper right')
plt.title('Example 2: Newton root finding for sin(x)')

plt.show()

print('Roots: %f, %f, %f' % (roots[0], roots[1], roots[2]))
```




```
Roots: 3.141593, -0.000096, -3.141593
```

Notice that we get some extreme values of x for some cases. For example, when $x_0 = -2$, where the slope is pretty close to 0, the next iteration gives a value less than -4.

Newton's method for optimization

Now, consider the problem of minimizing a scalar function $J : \mathbb{R}^n \mapsto \mathbb{R}$. We would like to find

$$\theta^* = \text{argmin}_\theta J(\theta)$$

We already know gradient descent:

$$\theta^{(i+1)} \leftarrow \theta^{(i)} - \alpha \nabla_J(\theta^{(i)}).$$

But Newton's method gives us a potentially faster way to find θ^* as a zero of the system of equations

$$\nabla_J(\theta^*) = \mathbf{0}.$$

In one dimension, to find the zero of $f'(x)$, obviously, we would apply Newton's method to $f'(x)$, obtaining the iteration

$$x_{i+1} = x_i - f'(x_i)/f''(x_i).$$

The multivariate extension of Newton's optimization method is

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \mathtt{H}_f^{-1}(\mathbf{x}_i)\nabla_f(\mathbf{x}_i),$$

where $\mathtt{H}_f(\mathbf{x})$ is the *Hessian* of f evaluated at $\mathbf{x}$:

$$\mathtt{H}_f(\mathbf{x}) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 x_n} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n x_1} & \frac{\partial^2 f}{\partial x_n x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

This means, for the minimization of $J(\theta)$, we would obtain the update rule

$$\theta^{(i+1)} \leftarrow \theta^{(i)} - \mathtt{H}_J^{-1}(\theta^{(i)})\nabla_J(\theta^{(i)}).$$

Application to logistic regression

Let's create some difficult sample data as follows:

Class 1: Two features x_1 and x_2 jointly distributed as a two-dimensional spherical Gaussian with parameters

$$\mu = \begin{bmatrix} x_{1c} \\ x_{2c} \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}.$$

Class 2: Two features x_1 and x_2 in which the data are generated by first sampling an angle θ according to a uniform distribution, sampling a distance d according to a one-dimensional Gaussian with a mean of $(3\sigma_1)^2$ and a variance of $(\frac{1}{2}\sigma_1)^2$, then outputting the point

$$\mathbf{x} = \begin{bmatrix} x_{1c} + d\cos\theta \\ x_{2c} + d\sin\theta \end{bmatrix}.$$

Generate 100 samples for each of the classes, guided by the following exercises.

Exercise 1.1 (5 points)

Generate data for class 1 with 100 samples:

$$\mu = \begin{bmatrix} x_{1c} \\ x_{2c} \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}.$$

Hint:

In [9]:
```
mu_1 = np.array([1.0, 2.0])
sigma_1 = 1
num_sample = 100

cov_mat = np.array([[sigma_1, 0], [0,sigma_1]])
X1 = np.random.multivariate_normal(mu_1, cov_mat, num_sample)
```

In [10]:
```
print(X1[:5])

# Test function: Do not remove
assert X1.shape == (100, 2), 'Size of X1 is incorrect'
assert cov_mat.shape == (2, 2), 'Size of x_test is incorrect'
count = 0
for i in range(2):
    for j in range(2):
        if i==j and cov_mat[i,j] != 0:
            if cov_mat[i,j] == sigma_1:
                count += 1
        else:
            if cov_mat[i,j] == 0:
                count += 1
assert count == 4, 'cov_mat data is incorrect'

print("success!")
# End Test function
```

```
[[1.20843043 2.10696693]
 [1.14318802 2.00814591]
 [1.47051804 0.68384279]
 [0.51191179 3.27545311]
 [1.02435511 2.10586304]]
success!
```

Expected result (or something similar):
[[-0.48508229 2.65415886]
[1.17230227 1.61743589]
[-0.61932146 3.53986541]
[0.70583088 1.45944356]
[-0.93561505 0.2042285]]

Exercise 1.2 (5 points)

Generate data for class 2 with 100 samples:

$$\mathbf{x} = \begin{bmatrix} x_{1c} + d\cos\theta \\ x_{2c} + d\sin\theta \end{bmatrix}$$

where θ is sampled uniformly from $[0, 2\pi]$ and d is sampled from a one-dimensional Gaussian with a mean of $(3\sigma_1)^2$ and a variance of $(\frac{1}{2}\sigma_1)^2$.

Hint:

In [11]:
```
# 1. Create sample angle from 0 to 2pi with 100 samples
angle = np.random.uniform(0, 2*np.pi, 100)
# 2. Create sample with normal distribution of d with mean and variance
mean = (3 * sigma_1)**2
variance = (sigma_1/2)**2
d =  np.random.normal(mean, variance, 100)
# 3 Create X2
X2 = np.array([X1[:,0] +( d* (np.cos(angle))), X1[:,1] + (d * (np.sin(angle)))]).

# YOUR CODE HERE
# raise NotImplementedError()
X2.shape
```

Out[11]: (100, 2)

In [12]:
```
print('angle:',angle[:5])
print('d:', d[:5])
print('X2:', X2[:5])

# Test function: Do not remove
assert angle.shape == (100,) or angle.shape == (100,1) or angle.shape == 100, 'Si
assert d.shape == (100,) or d.shape == (100,1) or d.shape == 100, 'Size of d is i
assert X2.shape == (100,2), 'Size of X2 is incorrect'
assert angle.min() >= 0 and angle.max() <= 2*np.pi, 'angle generate incorrect'
assert d.min() >= 8 and d.max() <= 10, 'd generate incorrect'
assert X2[:,0].min() >= -13 and X2[:,0].max() <= 13, 'X2 generate incorrect'
assert X2[:,1].min() >= -10 and X2[:,1].max() <= 13.5, 'X2 generate incorrect'

print("success!")
# End Test function
```

```
angle: [3.55181342 6.05906779 4.62503987 2.31340236 4.51858335]
d: [8.67847388 8.84772542 8.9436972  8.81147448 9.27273754]
X2: [[-6.75001479e+00  9.76963726e+00]
 [ 6.90287128e-01 -5.44649620e+00]
 [-7.61524646e-01 -4.95410181e+00]
 [-2.90878884e+00  6.14898900e-03]
 [-9.88422141e+00 -5.20561188e+00]]
success!
```

Expected result (or something similar):

angle: [4.77258271 3.19733552 0.71226709 2.11244845 6.06280915]
d: [9.13908279 8.84218552 9.24427852 8.74831667 8.85727588]
X2: [[0.064701 -6.46837219]
[-7.65614929 1.12480234]
[6.37750805 9.58147629]
[-3.80438416 8.95550952]
[7.70745021 -1.73194274]]

Exercise 1.3 (5 points)

Combine X1 and X2 into single dataset

In [13]:
```
# 1. concatenate X1, X2 together
X = np.concatenate((X1,X2), axis = 0)
# 2. Create y with class 1 as 0 and class 2 as 1
y = np.concatenate((np.zeros((100,)), np.ones((100,))), axis = 0)

# YOUR CODE HERE
# raise NotImplementedError()
# print(y.shape)
```

In [14]:
```
print("shape of X:", X.shape)
print("shape of y:", y.shape)

# Test function: Do not remove
assert X.shape == (200, 2), 'Size of X is incorrect'
assert y.shape == (200,) or y.shape == (200,1) or y.shape == 200, 'Size of y is i
assert y.min() == 0 and y.max() == 1, 'class type setup is incorrect'

print("success!")
# End Test function
```

```
shape of X: (200, 2)
shape of y: (200,)
success!
```

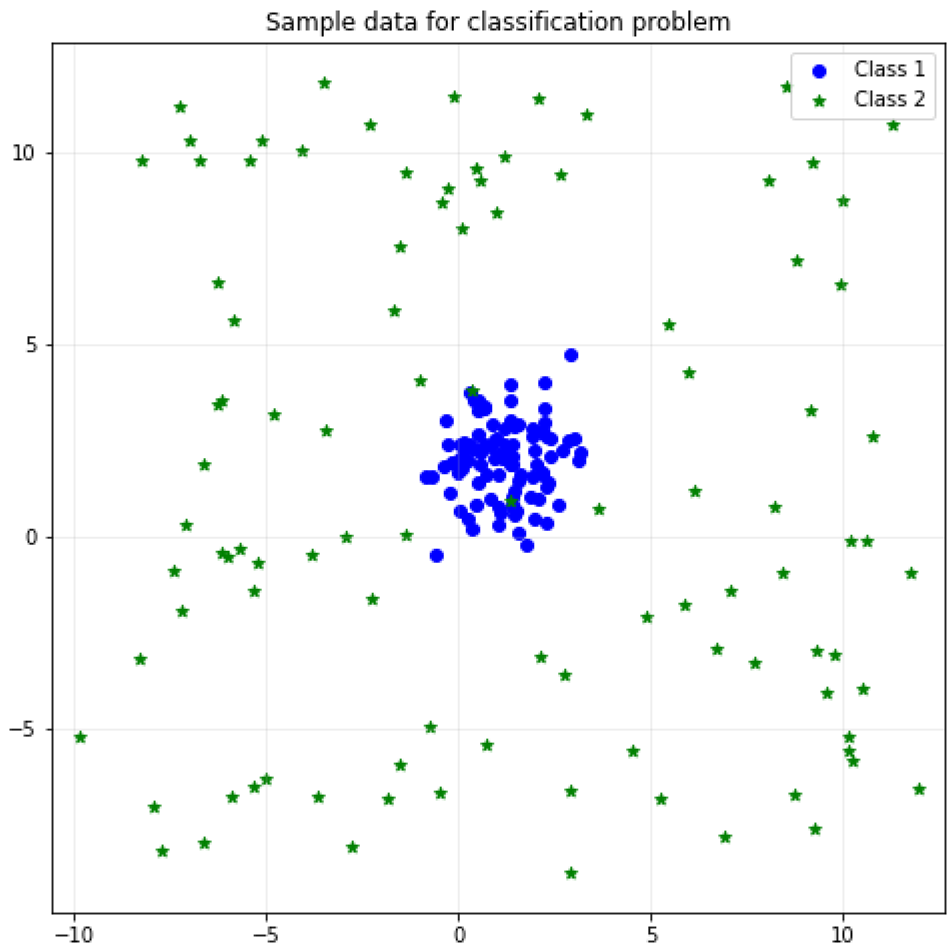
Expect result (or looked alike):

shape of X: (200, 2)
shape of y: (200, 1)

Exercise 1.4 (5 points)

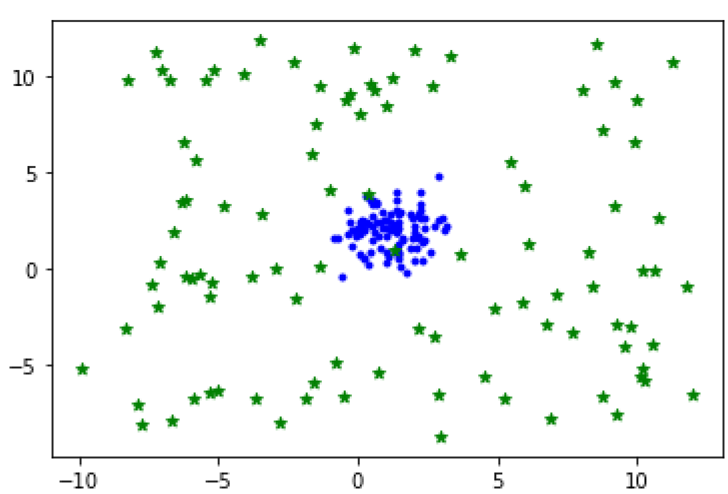
Plot the graph between class1 and class2 with **difference color and point style**.

In [15]:
```
fig1 = plt.figure(figsize=(8,8))
ax = plt.axes()
plt.title('Sample data for classification problem')
plt.grid(axis='both', alpha=.25)
# plot graph here
# YOUR CODE HERE
# raise NotImplementedError()
markers = ["o","*"]
colors = ['b', 'g']

for i, c in enumerate(np.unique(y)):
    plt.scatter(X[y == c, 0], X[y == c, 1], c = colors[i], marker = markers[i], l
# end plot graph
plt.legend()
plt.axis('equal')
plt.show()
```

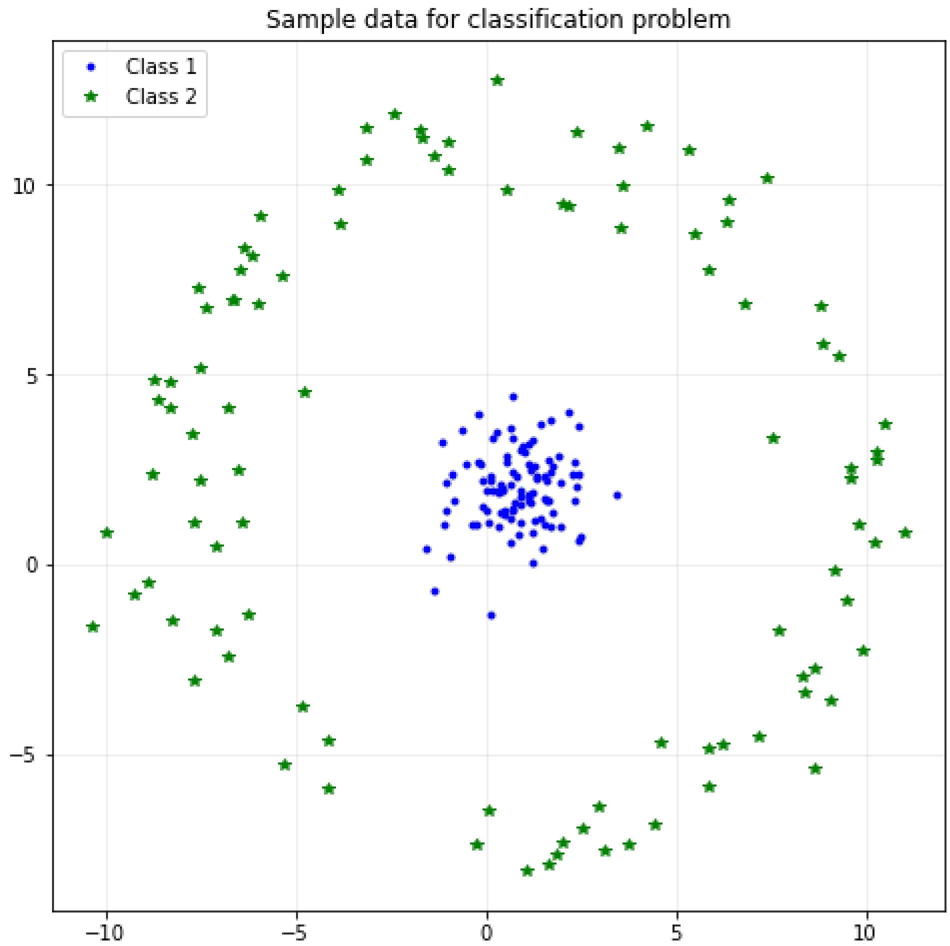


In [16]:
```
plt.scatter(X1[:,0], X1[:,1], c='b', marker='.')
plt.scatter(X2[:,0], X2[:,1], c='g', marker='*')
```

Out[16]: <matplotlib.collections.PathCollection at 0x7fe701761160>



Expect result (or looked alike):



Exercise 1.5 (5 points)

Split data into training and test datasets with 80% of training set and 20% of test set

In [17]:
```
import random

train_size = 0.8
m = X.shape[0]
idx = np.arange(0, m)
random.shuffle(idx)
train_length = int(train_size * m)

idx_train = idx[:train_length]
idx_test = idx[train_length:]

X_train = X[idx_train, :]
X_test = X[idx_test, :]
y_train = y[idx_train]
y_test = y[idx_test]

# YOUR CODE HERE
# raise NotImplementedError()
# print(y)
```

In [18]:
```
print('idx_train:', idx_train[:10])
print("train size, X:", X_train.shape, ", y:", y_train.shape)
print("test size,  X:", X_test.shape,  ", y:", y_test.shape)

# Test function: Do not remove
assert X_train.shape == (160, 2), 'Size of X_train is incorrect'
assert y_train.shape == (160,) or y_train.shape == (160,1) or y.shape == 160, 'Si
assert X_test.shape == (40, 2), 'Size of X_test is incorrect'
assert y_test.shape == (40,) or y_test.shape == (40,1) or y.shape == 40, 'Size of

print("success!")
# End Test function
```

```
idx_train: [ 83 117  89  24 104 192  98  41 164 135]
train size, X: (160, 2) , y: (160,)
test size,  X: (40, 2) , y: (40,)
success!
```

Expected reult (or something similar):
idx_train: [78 61 28 166 80 143 6 76 98 133]
train size, X: (160, 2) , y: (160, 1)
test size, X: (40, 2) , y: (40, 1)

Exercise 1.6 (5 points)

Write a function to normalize your X data

Practice yourself (No grade, but has extra score 3 points)

Try to use Jupyter notebook's LaTeX equation capabilities to write the normalization equations for your dataset.

YOUR ANSWER HERE

In [19]:
```
def normalization(X):
    """
    Take in numpy array of X values and return normalize X values,
    the mean and standard deviation of each feature
    """
    means = np.mean(X, axis = 0)
    stds = np.std(X, axis = 0)
    X_norm = (X - means) / stds
    # YOUR CODE HERE
#     raise NotImplementedError()

    return X_norm
```

In [20]:
```
XX = normalization(X)

X_train_norm = XX[idx_train]
X_test_norm = XX[idx_test]

# Add 1 at the first column of training dataset (for bias) and use it when traini
X_design_train = np.insert(X_train_norm,0,1,axis=1)
X_design_test = np.insert(X_test_norm,0,1,axis=1)

m,n = X_design_train.shape

print(X_train_norm.shape)
print(X_design_train.shape)
print(X_test_norm.shape)
print(X_design_test.shape)

# Test function: Do not remove
assert XX[:,0].min() >= -2.5 and XX[:,0].max() <= 2.5, 'Does the XX is normalized
assert XX[:,1].min() >= -2.5 and XX[:,1].max() <= 2.5, 'Does the XX is normalized

print("success!")
# End Test function
```

```
(160, 2)
(160, 3)
(40, 2)
(40, 3)
success!
```

Exercise 1.7 (10 points)

define class for logistic regression: batch gradient descent

The class includes:

- **Sigmoid** function

$$sigmoid(z) = \frac{1}{1 + e^{-z}}$$

- **Softmax** function

$$softmax(z) = \frac{e^{z_i}}{\sum_n e^z}$$

- **Hyperthesis (h)** function

$$\hat{y} = h(X;\theta) = softmax(\theta.X)$$

- **Gradient (Negative likelihood)** function

$$gradient = -X.\frac{y - \hat{y}}{n}$$

- **Cost** function

$$cost = \frac{\sum((-y \log \hat{y}) - ((1 - y) \log (1 - \hat{y})))}{n}$$

- **Gradient ascent** function
- **Prediction** function
- **Get accuracy** funciton

In [21]:
```
class Logistic_BGD:
    def __init__(self):
        pass

    def sigmoid(self,z):
        s = 1/(1+ np.exp(-z))
        # YOUR CODE HERE
#         raise NotImplementedError()
        return s

    def softmax(self, z):
        sm = np.exp(z)/(np.sum(np.exp(z)))
        # YOUR CODE HERE
#         raise NotImplementedError()
        return sm

    def h(self,X, theta):
        hf = self.sigmoid(X @ theta)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return hf

    def gradient(self, X, y, y_pred):
        grad = np.dot(X.T, (y_pred - y))/ len(y)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return grad

    def costFunc(self, theta, X, y):
        y_hat = self.h(X, theta)
        cost = (np.sum(-y * np.log(y_hat) - (1 - y) * np.log(1 - y_hat)))/ X.shap
        grad = self.gradient(X, y, y_hat)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return cost, grad

    def gradientAscent(self, X, y, theta, alpha, num_iters):
        m = len(y)
        J_history = []
        theta_history = []
        for i in range(num_iters):
            # 1. calculate cost, grad function
            cost, grad = self.costFunc(theta, X, y)
            # 2. update new theta
            theta = theta - alpha * grad
            # YOUR CODE HERE
#             raise NotImplementedError()


            J_history.append(cost)
            theta_history.append(theta)
        J_min_index = np.argmin(J_history)
        print("Minimum at iteration:",J_min_index)
        return theta_history[J_min_index] , J_history

    def predict(self,X, theta):
        labels=[]
        # 1. take y_predict from hyperthesis function
        # 2. classify y_predict that what it should be class1 or class2
        # 3. append the output from prediction
        # YOUR CODE HERE
        y_predict = self.h(X, theta)
        for i in range(X.shape[0]):
            if y_predict[i] >= 0.5:
                labels.append(1)
            else:
                labels.append(0)

        labels=np.asarray(labels)
        return labels

    def getAccuracy(self,X,y,theta):
        predict_y = self.predict(X, theta)
```

```
        correct = 0
        for i in range(len(predict_y)):
            if predict_y[i] == y[i]:
                correct += 1
        percent_correct = correct/ len(y) * 100
        # YOUR CODE HERE
#         raise NotImplementedError()
        return percent_correct
```

In [22]:

```
# Test function: Do not remove
lbgd = Logistic_BGD()
test_x = np.array([[1,2,3,4,5]]).T
out_x1 = lbgd.sigmoid(test_x)
out_x2 = lbgd.sigmoid(test_x.T)
print('out_x1', out_x1.T)
assert np.array_equal(np.round(out_x1.T, 5), np.round([[0.73105858, 0.88079708, 0
assert np.array_equal(np.round(out_x2, 5), np.round([[0.73105858, 0.88079708, 0.9
out_x1 = lbgd.softmax(out_x1)
out_x2 = lbgd.softmax(out_x2)
print('out_x1', out_x1.T)
assert np.array_equal(np.round(out_x1.T, 5), np.round([[0.16681682, 0.19376282, 0
assert np.array_equal(np.round(out_x2, 5), np.round([[0.16681682, 0.19376282, 0.2
test_t = np.array([[0.3, 0.2]]).T
test_x = np.array([[1,2,3,4,5, 6], [2, 9, 4, 3, 1, 0]]).T
test_y = np.array([[0,1,0,1,0,1]]).T
test_y_p = lbgd.h(test_x, test_t)
print('test_y_p', test_y_p.T)
assert np.array_equal(np.round(test_y_p.T, 5), np.round([[0.66818777, 0.9168273,
test_g = lbgd.gradient(test_x, test_y, test_y_p)
print('test_g', test_g.T)
assert np.array_equal(np.round(test_g.T, 5), np.round([[0.9746016, 0.73165696]],
test_c, test_g = lbgd.costFunc(test_t, test_x, test_y)
print('test_c', test_c.T)
assert np.round(test_c, 5) == np.round(0.87192491, 5), "costFunc function is inco
test_t_out , test_j = lbgd.gradientAscent(test_x, test_y, test_t, 0.001, 3)
print('test_t_out', test_t_out.T)
print('test_j', test_j)
assert np.array_equal(np.round(test_t_out.T, 5), np.round([[0.29708373, 0.1978115
assert np.round(test_j[2], 5) == np.round(0.86896665, 5), "gradientAscent functio
test_l = lbgd.predict(test_x, test_t)
print('test_l', test_l)
assert np.array_equal(np.round(test_l, 1), np.round([1,1,1,1,1,1], 1)), "gradient
test_a = lbgd.getAccuracy(test_x,test_y,test_t)
print('test_a', test_a)
assert np.round(test_a, 1) == 50.0, "getAccuracy function is incorrect"

print("success!")
# End Test function
```

```
out_x1 [[0.73105858 0.88079708 0.95257413 0.98201379 0.99330715]]
out_x1 [[0.16681682 0.19376282 0.20818183 0.21440174 0.21683678]]
test_y_p [[0.66818777 0.9168273  0.84553473 0.85814894 0.84553473 0.85814894]]
test_g [[0.9746016  0.73165696]]
test_c 0.8719249134773479
Minimum at iteration: 2
test_t_out [[0.29708373 0.19781153]]
test_j [0.8719249134773479, 0.870441756946089, 0.8689666485816598]
test_l [1 1 1 1 1 1]
test_a 50.0
success!
```

Expected result:

out_x1 [[0.73105858 0.88079708 0.95257413 0.98201379 0.99330715]]

out_x1 [[0.16681682 0.19376282 0.20818183 0.21440174 0.21683678]]

test_y_p [[0.66818777 0.9168273 0.84553473 0.85814894 0.84553473 0.85814894]]

test_g [[0.9746016 0.73165696]]

test_c [0.87192491]

Minimum at iteration: 2

test_t_out [[0.29708373 0.19781153]]

test_j [array([0.87192491]), array([0.87044176]), array([0.86896665])]
test_l [1 1 1 1 1 1]
test_a 50.0

Exercise 1.8 (5 points)

Training the data using Logistic_BGD class.

- Input: X_design_train
- Output: y_train
- Use 50,000 iterations

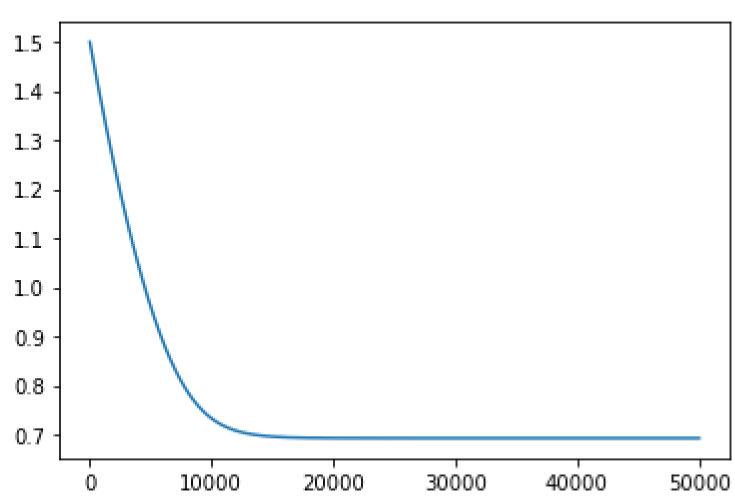
Find the initial_theta yourself

In [23]:
```
alpha = 0.001
iterations = 50000

BGD_model = Logistic_BGD()
initial_theta = np.random.randn(X_design_train.shape[1],)
bgd_theta, bgd_cost = BGD_model.gradientAscent(X_design_train, y_train, initial_t

# YOUR CODE HERE
# raise NotImplementedError()
```

```
Minimum at iteration: 49999
```

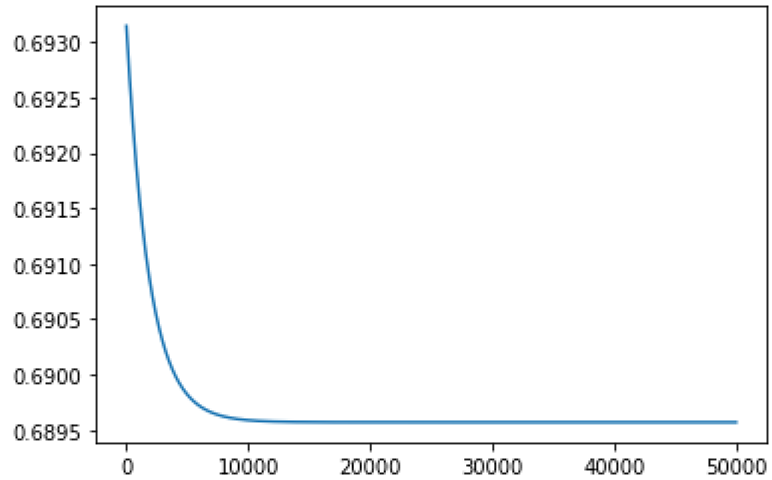
In [24]:
```
print(bgd_theta)
print(len(bgd_cost))

print(bgd_cost[0])
plt.plot(bgd_cost)
plt.show()

# Test function: Do not remove
assert bgd_theta.shape == (X_train.shape[1] + 1,1) or bgd_theta.shape == (X_trair
assert len(bgd_cost) == iterations, "cost data size is incorrect"

print("success!")
# End Test function
```

```
[ 0.05114452 -0.01338292 -0.01763408]
50000
1.499471045823371
```




```
success!
```

Expected result (or look alike):
[[-0.07328673]
[-0.13632896]
[0.05430939]]
50000



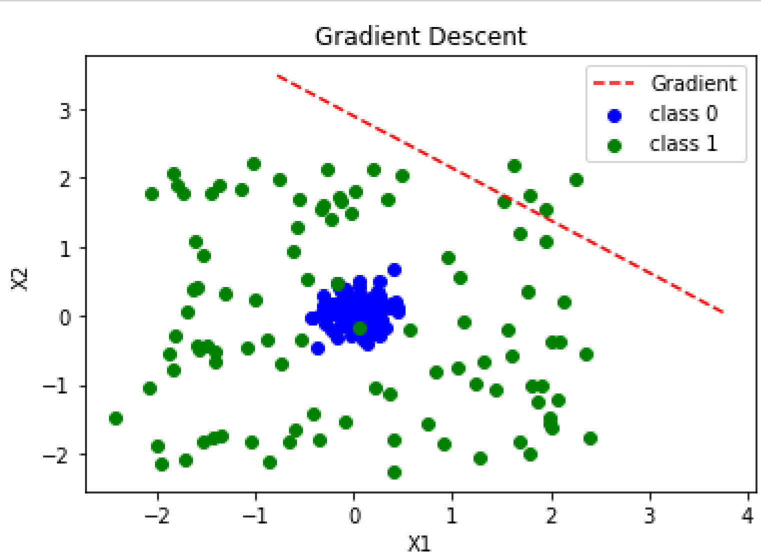
In lab exercises

1. Verify that the gradient descent solution is correct. Plot the optimal decision boundary you obtain.
2. Write a new class that uses Newton's method for the optmization rather than simple gradient descent.
3. Verify that you obtain a similar solution with Newton's method. Plot the optimal decision boundary you obtain.
4. Compare the number of iterations required for gradient descent vs. Newton's method. Do you observe other issues with Newton's method such as a singular or nearly singular Hessian matrix?

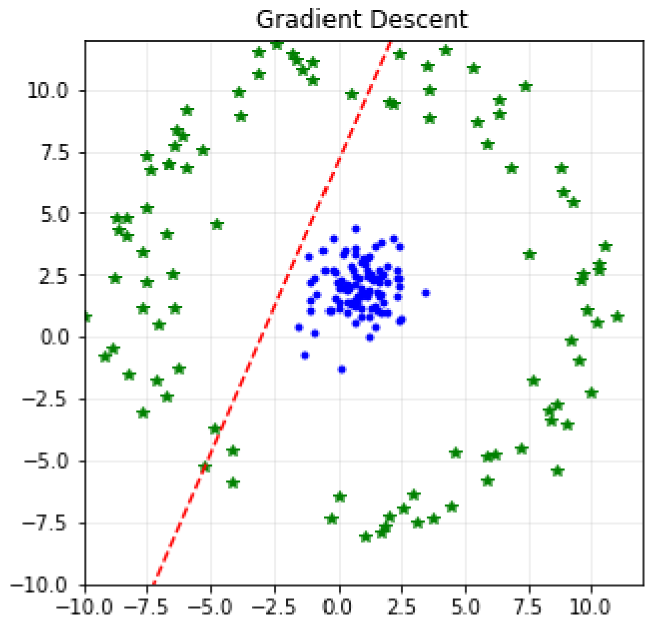
Exercise 1.9 (5 points)

Plot the optimal decision boundary of gradient ascent

In [25]:
```python
# YOUR CODE HERE
def boundary_points(X, theta):
    theta = theta.reshape(-1,1)
    v_orthogonal = np.array([[theta[1,0]],[theta[2,0]]])
    v_ortho_length = np.sqrt(v_orthogonal.T @ v_orthogonal)
    dist_ortho = theta[0,0] / v_ortho_length
    v_orthogonal = v_orthogonal / v_ortho_length
    v_parallel = np.array([[-v_orthogonal[1,0]],[v_orthogonal[0,0]]])
    projections = X @ v_parallel
    proj_1 = min(projections)
    proj_2 = max(projections)
    point_1 = proj_1 * v_parallel - dist_ortho * v_orthogonal
    point_2 = proj_2 * v_parallel - dist_ortho * v_orthogonal
    return point_1, point_2

x_df = pd.DataFrame(X, columns=['X0', 'X1'])

x_df['y'] = y

x_df['X0'] = normalization(x_df.X0)
x_df['X1'] = normalization(x_df.X1)
linX = x_df[['X0','X1']].values
linX = np.insert(linX, 0, 1, axis=1)

X_train = linX[idx_train]
X_test = linX[idx_test]
y_train = y[idx_train]
y_test = y[idx_test]

y0_df = x_df[x_df.y == 0]
y1_df = x_df[x_df.y == 1]


point_1, point_2 = boundary_points(linX[:,1:], bgd_theta)

plt.title('Gradient Descent')
plt.scatter(y0_df.X0, y0_df.X1, c='b', label='class 0')
plt.scatter(y1_df.X0, y1_df.X1, c='g', label='class 1')
plt.legend()
plt.xlabel('X1')
plt.ylabel('X2')

# plot the boundaries for both methods
plt.plot([point_1[0,0], point_2[0,0]],[point_1[1,0], point_2[1,0]], 'r--', label=
plt.legend(loc=0)
plt.show()
```



Expected result (or look alike):



In [26]:
```
print("Accuracy =",BGD_model.getAccuracy(X_design_test,y_test,bgd_theta))
```

```
Accuracy = 45.0
```

Exercise 2.1 (10 points)

Write Newton's method class

In [27]:
```
class Logistic_NM: #logistic regression for newton's method

    def __init__(self):
        pass

    def sigmoid(self,z):
        s = 1/(1+ np.exp(-z))
        # YOUR CODE HERE
#         raise NotImplementedError()
        return s

    def h(self,X, theta):
        hf = self.sigmoid(X @ theta)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return hf

    def gradient(self, X, y, y_pred):
        grad = np.dot(X.T, (y_pred - y))/ len(y)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return grad

    def hessian(self, X, y, theta):
        y_hat = self.h(X, theta)
        # YOUR CODE HERE
        hess_mat = ((y_hat).T @ (1-y_hat)) * (X.T @ X)/ len(y_hat)
#         raise NotImplementedError()
        return hess_mat

    def costFunc(self, theta, X, y):
        y_hat = self.h(X, theta)
        cost = (np.sum(-y * np.log(y_hat) - (1 - y) * np.log(1 - y_hat)))/ X.shap
        grad = self.gradient(X, y, y_hat)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return cost, grad

    def newtonsMethod(self, X, y, theta, num_iters):
        m = len(y)
        J_history = []
        theta_history = []

        for i in range(num_iters):
            hessian_mat = np.zeros((X.shape[1], X.shape[1]))
            hmat_xi = self.hessian(X, y, theta)
            hessian_mat += hmat_xi
            cost, grad = self.costFunc(theta, X,y)

            try :
                theta = theta - np.linalg.inv(hessian_mat) @ grad
            except Exception as e :
                error_msg = e
                found_sigular_matrix = True

                theta = theta - np.linalg.pinv(hessian_mat) @ grad

            J_history.append(cost)
            theta_history.append(theta)
        J_min_index = np.argmin(J_history)
        print("Minimum at iteration:", J_min_index)
        return theta_history[J_min_index] , J_history

    def predict(self,X, theta):
        labels=[]
        # 1. take y_predict from hyperthesis function
        # 2. classify y_predict that what it should be class1 or class2
        # 3. append the output from prediction
        # YOUR CODE HERE
        y_predict = self.h(X, theta)
        for i in range(X.shape[0]):
            if y_predict[i] >= 0.5:
                labels.append(1)
```

```
            else:
                labels.append(0)

        labels=np.asarray(labels)
        return labels

    def getAccuracy(self,X,y,theta):
        predict_y = self.predict(X, theta)
        correct = 0
        for i in range(len(predict_y)):
            if predict_y[i] == y[i]:
                correct += 1
        percent_correct = correct/ len(y) * 100
        # YOUR CODE HERE
#         raise NotImplementedError()
        return percent_correct
```

In [28]:

```
# Test function: Do not remove
lbgd = Logistic_NM()
test_x = np.array([[1,2,3,4,5]]).T
out_x1 = lbgd.sigmoid(test_x)
out_x2 = lbgd.sigmoid(test_x.T)
print('out_x1', out_x1.T)
assert np.array_equal(np.round(out_x1.T, 5), np.round([[0.73105858, 0.88079708, 0
assert np.array_equal(np.round(out_x2, 5), np.round([[0.73105858, 0.88079708, 0.9
test_t = np.array([[0.3, 0.2]]).T
test_x = np.array([[1,2,3,4,5, 6], [2, 9, 4, 3, 1, 0]]).T
test_y = np.array([[0,1,0,1,0,1]]).T
test_y_p = lbgd.h(test_x, test_t)
print('test_y_p', test_y_p.T)
assert np.array_equal(np.round(test_y_p.T, 5), np.round([[0.66818777, 0.9168273,
test_g = lbgd.gradient(test_x, test_y, test_y_p)
print('test_g', test_g.T)
assert np.array_equal(np.round(test_g.T, 5), np.round([[0.9746016, 0.73165696]],
test_h = lbgd.hessian(test_x, test_y, test_t)
print('test_h', test_h)
assert test_h.shape == (2, 2), "hessian matrix function is incorrect"
assert np.array_equal(np.round(test_h.T, 5), np.round([[12.17334371, 6.55487738],
test_c, test_g = lbgd.costFunc(test_t, test_x, test_y)
print('test_c', test_c.T)
assert np.round(test_c, 5) == np.round(0.87192491, 5), "costFunc function is inco
test_t_out , test_j = lbgd.newtonsMethod(test_x, test_y, test_t, 3)
print('test_t_out', test_t_out.T)
print('test_j', test_j)
assert np.array_equal(np.round(test_t_out.T, 5), np.round([[0.14765747, 0.1560701
assert np.round(test_j[2], 5) == np.round(0.7534506190845247, 5), "newtonsMethod
test_l = lbgd.predict(test_x, test_t)
print('test_l', test_l)
assert np.array_equal(np.round(test_l, 1), np.round([1,1,1,1,1,1], 1)), "gradient
test_a = lbgd.getAccuracy(test_x,test_y,test_t)
print('test_a', test_a)
assert np.round(test_a, 1) == 50.0, "getAccuracy function is incorrect"

print("success!")
# End Test function
```

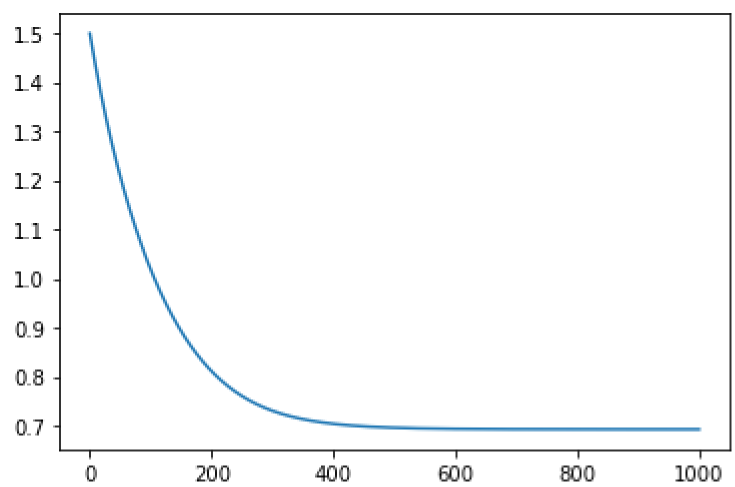
```
out_x1 [[0.73105858 0.88079708 0.95257413 0.98201379 0.99330715]]
test_y_p [[0.66818777 0.9168273  0.84553473 0.85814894 0.84553473 0.85814894]]
test_g [[0.9746016  0.73165696]]
test_h [[12.17334371  6.55487738]
 [ 6.55487738 14.84880387]]
test_c 0.8719249134773479
Minimum at iteration: 2
test_t_out [[0.14765747 0.15607017]]
test_j [0.8719249134773479, 0.7967484437157274, 0.7534506190845246]
test_l [1 1 1 1 1 1]
test_a 50.0
success!
```

Expect result: out_x1 [[0.73105858 0.88079708 0.95257413 0.98201379 0.99330715]]
test_y_p [[0.66818777 0.9168273 0.84553473 0.85814894 0.84553473 0.85814894]]
test_g [[0.9746016 0.73165696]]

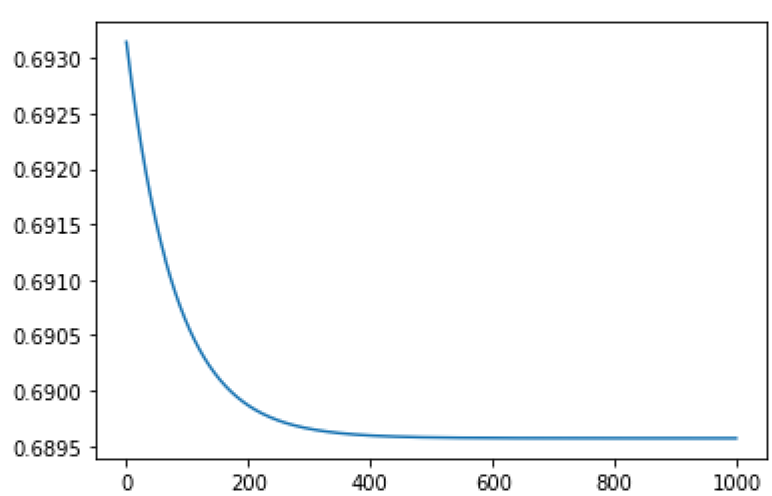
test_h [[12.17334371 6.55487738]
[6.55487738 14.84880387]]
test_c 0.8719249134773479
Minimum at iteration: 2
test_t_out [[0.14765747 0.15607017]]
test_j [0.8719249134773479, 0.7967484437157274, 0.7534506190845247]
test_l [1 1 1 1 1 1]
test_a 50.0

In [29]:
```
NM_model = Logistic_NM()

iterations = 1000

nm_theta, nm_cost = NM_model.newtonsMethod(X_design_train, y_train, initial_theta
print("theta:",nm_theta)

print(nm_cost[0])
plt.plot(nm_cost)
plt.show()
```

```
Minimum at iteration: 999
theta: [ 0.04893522 -0.01596414 -0.02354655]
1.499471045823371
```



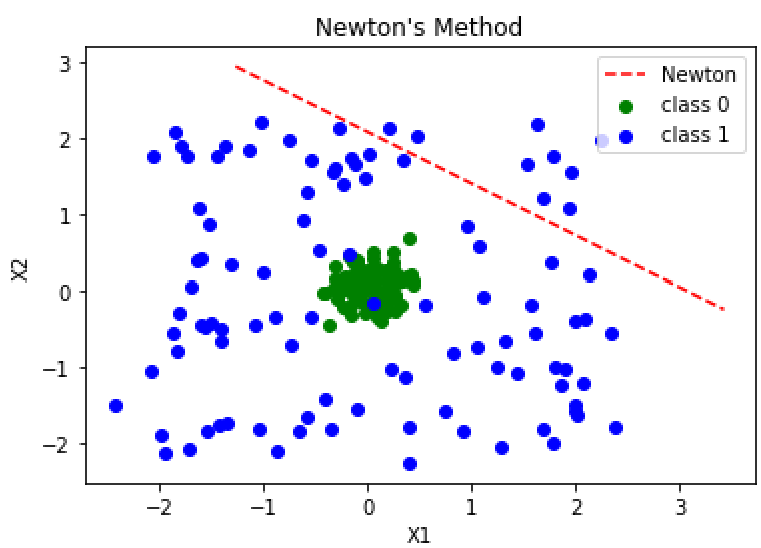
Expected result (or look alike):
Minimum at iteration: 999
theta: [[-0.07313861]
[-0.13605172]
[0.05419746]]
0.6931471805599453



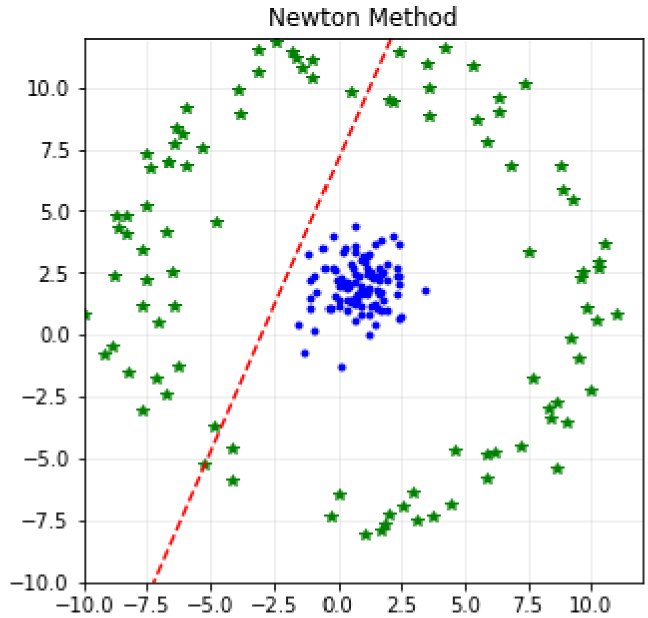
Exercise 2.2 (5 points)

Plot the optimal decision boundary of Newton method

In [30]:
```
# YOUR CODE HERE
# raise NotImplementedError()
x_df = pd.DataFrame(X, columns=['X0', 'X1'])

x_df['y'] = y

x_df['X0'] = normalization(x_df.X0)
x_df['X1'] = normalization(x_df.X1)
linX = x_df[['X0','X1']].values
linX = np.insert(linX, 0, 1, axis=1)

X_train = linX[idx_train]
X_test = linX[idx_test]
y_train = y[idx_train]
y_test = y[idx_test]

y0_df = x_df[x_df.y == 0]
y1_df = x_df[x_df.y == 1]


point_1n, point_2n = boundary_points(linX[:,1:], nm_theta)

plt.title("Newton's Method")
plt.scatter(y0_df.X0, y0_df.X1, c='g', label='class 0')
plt.scatter(y1_df.X0, y1_df.X1, c='b', label='class 1')
plt.legend()
plt.xlabel('X1')
plt.ylabel('X2')
# plot the boundaries for both methods
plt.plot([point_1n[0,0], point_2n[0,0]],[point_1n[1,0], point_2n[1,0]], 'r--', la
plt.legend(loc=0)
plt.show()
```



Expected result (or look alike):



In [31]:
```
print("Accuracy =",NM_model.getAccuracy(X_design_test,y_test,nm_theta))
```

```
Accuracy = 45.0
```

Exercise 2.3 (5 points)

Compare the number of iterations required for gradient descent vs. Newton's method. Do you observe other issues with Newton's method such as a singular or nearly singular Hessian matrix?

Gradient Descent requires 10000 iterations to reach minimal cost where Newton's Method only requires around 300 iterations to reach the minimum. Sometimes singular or nearly singular Hessian Matrix can be observed.

Take-home exercises

1. Perform a *polar transformation* on the data above to obtain a linearly separable dataset. (5 points)
2. Verify that you obtain good classification accuracy for logistic regression with GD or Netwon's method after the polar transformation (10 points)
3. Apply Newton's method to the dataset you used for the take home exercises in Lab 03. (20 points)

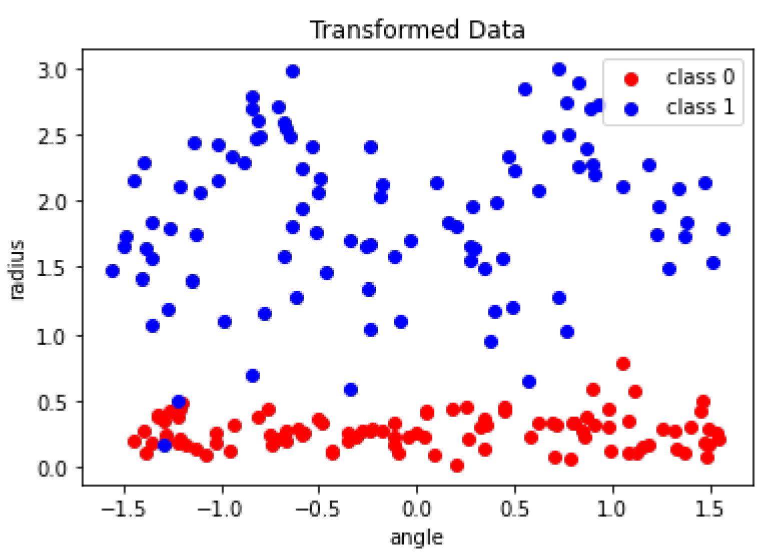
In [32]:
```
class Logistic_NM: #logistic regression for newton's method

    def __init__(self):
        pass

    def sigmoid(self,z):
        s = 1/(1+ np.exp(-z))
        # YOUR CODE HERE
#         raise NotImplementedError()
        return s

    def h(self,X, theta):
        hf = self.sigmoid(X @ theta)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return hf

    def gradient(self, X, y, y_pred):
        grad = np.dot(X.T, (y_pred - y))/ len(y)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return grad

    def hessian(self, X, y, theta):
        y_hat = self.h(X, theta)
        # YOUR CODE HERE
        hess_mat = ((y_hat).T @ (1-y_hat)) * (X.T @ X)/ len(y_hat)
#         raise NotImplementedError()
        return hess_mat

    def costFunc(self, theta, X, y):
        y_hat = self.h(X, theta)
        cost = (np.sum(-y * np.log(y_hat) - (1 - y) * np.log(1 - y_hat)))/ X.shap
        grad = self.gradient(X, y, y_hat)
        # YOUR CODE HERE
#         raise NotImplementedError()
        return cost, grad

    def newtonsMethod(self, X, y, theta, num_iters):
        m = len(y)
        J_history = []
        theta_history = []

        for i in range(num_iters):
            hessian_mat = np.zeros((X.shape[1], X.shape[1]))
            hmat_xi = self.hessian(X, y, theta)
            hessian_mat += hmat_xi
            cost, grad = self.costFunc(theta, X,y)

#             try :
#                 theta = theta - np.linalg.inv(hessian_mat) @ grad
#             except Exception as e :
#                 error_msg = e
#                 found_sigular_matrix = True

            theta = theta - np.linalg.pinv(hessian_mat) @ grad

            J_history.append(cost)
            theta_history.append(theta)
        J_min_index = np.argmin(J_history)
        print("Minimum at iteration:", J_min_index)
        return theta_history[J_min_index] , J_history

    def predict(self,X, theta):
        labels=[]
        # 1. take y_predict from hyperthesis function
        # 2. classify y_predict that what it should be class1 or class2
        # 3. append the output from prediction
        # YOUR CODE HERE
        y_predict = self.h(X, theta)
        for i in range(X.shape[0]):
            if y_predict[i] >= 0.5:
                labels.append(1)
```

```
            else:
                labels.append(0)

        labels=np.asarray(labels)
        return labels

    def getAccuracy(self,X,y,theta):
        predict_y = self.predict(X, theta)
        correct = 0
        for i in range(len(predict_y)):
            if predict_y[i] == y[i]:
                correct += 1
        percent_correct = correct/ len(y) * 100
        # YOUR CODE HERE
#         raise NotImplementedError()
        return percent_correct
```

In [33]:
```
# Prepare data
# Convert X to angle and radius
df = pd.DataFrame(X, columns=['X0', 'X1'])
df['angles'] = np.arctan(df.X1 / df.X0)
df['radius'] = np.sqrt(df.X0 ** 2 + df.X1 ** 2)
df['y'] = y

newX = df[['angles', 'radius']].values
newX = np.insert(newX, 0, 1, axis=1)
X_train = newX[idx_train]
X_test = newX[idx_test]
y_train = y[idx_train]
y_test = y[idx_test]
```

In [34]:
```
y0_df = df[df.y == 0]
y1_df = df[df.y == 1]

plt.title('Transformed Data')
plt.scatter(y0_df.angles, y0_df.radius, c='r', label='class 0')
plt.scatter(y1_df.angles, y1_df.radius, c='b', label='class 1')
plt.legend()
plt.xlabel('angle')
plt.ylabel('radius')
```

Out[34]: Text(0, 0.5, 'radius')



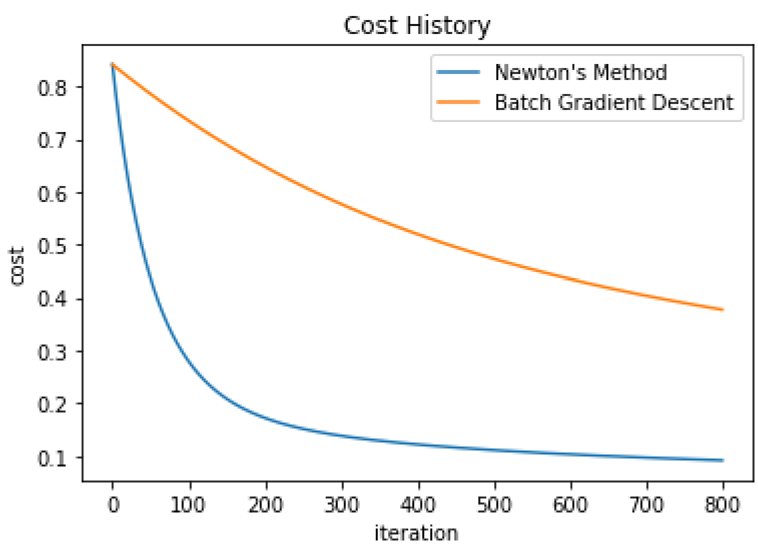
In [35]:
```
alpha = 0.01
iterations = 10000
init_theta = np.ones(newX.shape[1])

lg = Logistic_BGD()
theta, j_hist = lg.gradientAscent(X_train, y_train, init_theta, alpha, iterations
```

```
Minimum at iteration: 9999
```

In [36]:
```
iterations = 800
init_theta = np.ones(newX.shape[1])

ln = Logistic_NM()
theta_n, j_hist_n = ln.newtonsMethod(X_train, y_train, init_theta, iterations)
# print(theta_n)
```

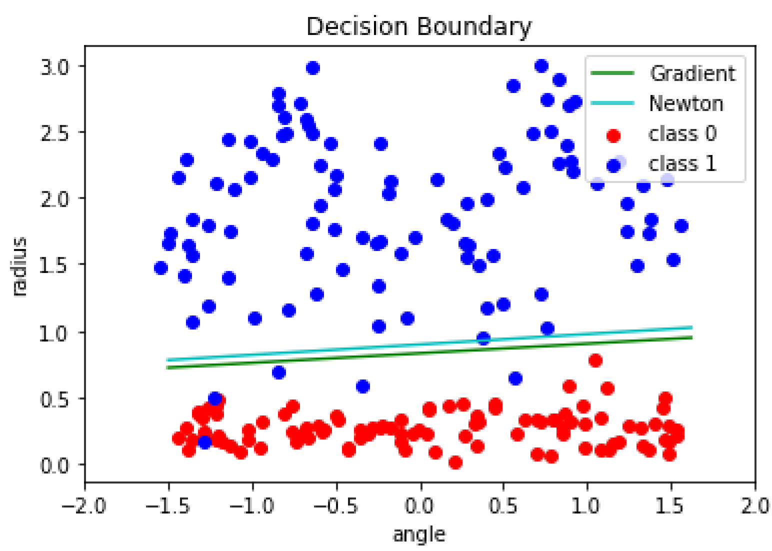
```
Minimum at iteration: 799
```

In [37]:
```
plt.plot(j_hist_n, label='Newton\'s Method')
plt.plot(j_hist[:iterations], label='Batch Gradient Descent')
plt.title('Cost History')
plt.xlabel('iteration')
plt.ylabel('cost')
plt.legend()
plt.show()

print('Minimum Cost for each method from polar transformation')
print('Gradient Descent:', np.min(j_hist))
print('Newton\'s Method :', np.min(j_hist_n))
```




```
Minimum Cost for each method from polar transformation
Gradient Descent: 0.1010223915711268
Newton's Method : 0.0914865419900052
```

In [38]:
```
y0_df = df[df.y == 0]
y1_df = df[df.y == 1]


point_1, point_2 = boundary_points(newX[:,1:], theta)
point_1n, point_2n = boundary_points(newX[:,1:], theta_n)

plt.title('Decision Boundary')
plt.scatter(y0_df.angles, y0_df.radius, c='r', label='class 0')
plt.scatter(y1_df.angles, y1_df.radius, c='b', label='class 1')
plt.legend()
plt.xlabel('angle')
plt.ylabel('radius')
# plot the boundaries for both methods
plt.plot([point_1[0,0], point_2[0,0]],[point_1[1,0], point_2[1,0]], 'g-', label='
plt.plot([point_1n[0,0], point_2n[0,0]],[point_1n[1,0], point_2n[1,0]], 'c-', lab
plt.legend(loc=0)
# plt.ylim(0,14)
plt.xlim(-2,2)
plt.show()
```



In [39]:
```
print(X_train.shape)
print(theta.shape)
```

```
(160, 3)
(3,)
```

In [40]:
```
yg_pred = lg.predict(X_train, theta)
yn_pred = ln.predict(X_train, theta_n)
g_acc = lg.getAccuracy(X_train, y_train, theta)
n_acc = lg.getAccuracy(X_train, y_train, theta_n)
print(n_acc)
print("Train accuracy for polar transformation")
print('Gradient Accuracy : ', g_acc)
print('Newton Accuracy : ', n_acc)

yg_pred = lg.predict(X_test, theta)
yn_pred = ln.predict(X_test, theta_n)
g_acc = lg.getAccuracy(X_test, y_test, theta)
n_acc = lg.getAccuracy(X_test, y_test, theta_n)

print("Test accuracy for polar transformation")
print('Gradient Accuracy : ', g_acc)
print('Newton Accuracy : ', n_acc)
```

```
97.5
Train accuracy for polar transformation
Gradient Accuracy :  97.5
Newton Accuracy :  97.5
Test accuracy for polar transformation
Gradient Accuracy :  97.5
Newton Accuracy :  97.5
```

In [41]:
```
# Import Pandas. You may need to run "pip3 install pandas" at the console if it's

import pandas as pd

# Import the data

data_train = pd.read_csv('train_LoanPrediction.csv')
data_test = pd.read_csv('test_LoanPrediction.csv')

# Start to explore the data

print('Training data shape', data_train.shape)
print('Test data shape', data_test.shape)

print('Training data:\n', data_train)
```

```
Training data shape (614, 13)
Test data shape (367, 12)
Training data:
       Loan_ID  Gender Married Dependents     Education Self_Employed  \
0    LP001002    Male      No          0      Graduate            No
1    LP001003    Male     Yes          1      Graduate            No
2    LP001005    Male     Yes          0      Graduate           Yes
3    LP001006    Male     Yes          0  Not Graduate            No
4    LP001008    Male      No          0      Graduate            No
..        ...     ...     ...        ...           ...           ...
609  LP002978  Female      No          0      Graduate            No
610  LP002979    Male     Yes         3+      Graduate            No
611  LP002983    Male     Yes          1      Graduate            No
612  LP002984    Male     Yes          2      Graduate            No
613  LP002990  Female      No          0      Graduate           Yes

     ApplicantIncome  CoapplicantIncome  LoanAmount  Loan_Amount_Term  \
0               5849                0.0         NaN             360.0
1               4583             1508.0       128.0             360.0
2               3000                0.0        66.0             360.0
3               2583             2358.0       120.0             360.0
4               6000                0.0       141.0             360.0
..               ...                ...         ...               ...
609             2900                0.0        71.0             360.0
610             4106                0.0        40.0             180.0
611             8072              240.0       253.0             360.0
612             7583                0.0       187.0             360.0
613             4583                0.0       133.0             360.0

     Credit_History Property_Area Loan_Status
0               1.0         Urban           Y
1               1.0         Rural           N
2               1.0         Urban           Y
3               1.0         Urban           Y
4               1.0         Urban           Y
..              ...           ...         ...
609             1.0         Rural           Y
610             1.0         Rural           Y
611             1.0         Urban           Y
612             1.0         Urban           Y
613             0.0     Semiurban           N

[614 rows x 13 columns]
```

In [42]:
```
# Check for missing values in the training and test data

print('Missing values for train data:\n------------------------\n', data_train.is
print('Missing values for test data \n ------------------------\n', data_test.isn
```

```
Missing values for train data:
------------------------
 Loan_ID              0
Gender               13
Married               3
Dependents           15
Education             0
Self_Employed        32
ApplicantIncome       0
CoapplicantIncome     0
LoanAmount           22
Loan_Amount_Term     14
Credit_History       50
Property_Area         0
Loan_Status           0
dtype: int64
Missing values for test data
 ------------------------
 Loan_ID              0
Gender               11
Married               0
Dependents           10
Education             0
Self_Employed        23
ApplicantIncome       0
CoapplicantIncome     0
LoanAmount            5
Loan_Amount_Term      6
Credit_History       29
Property_Area         0
dtype: int64
```

In [43]:
```
# Compute ratio of each category value
# Divide the missing values based on ratio
# Fillin the missing values
# Print the values before and after filling the missing values for confirmation

print(data_train['Married'].value_counts())

married = data_train['Married'].value_counts()
print('Elements in Married variable', married.shape)
print('Married ratio ', married[0]/sum(married.values))

def fill_martial_status(data, yes_num_train, no_num_train):
    data['Married'].fillna('Yes', inplace = True, limit = yes_num_train)
    data['Married'].fillna('No', inplace = True, limit = no_num_train)

fill_martial_status(data_train, 2, 1)
print(data_train['Married'].value_counts())
```

```
Yes    398
No     213
Name: Married, dtype: int64
Elements in Married variable (2,)
Married ratio  0.6513911620294599
Yes    400
No     214
Name: Married, dtype: int64
```

In [44]:
```
print(data_train['Dependents'].value_counts())
dependent = data_train['Dependents'].value_counts()

print('Dependent ratio 1 ', dependent['0'] / sum(dependent.values))
print('Dependent ratio 2 ', dependent['1'] / sum(dependent.values))
print('Dependent ratio 3 ', dependent['2'] / sum(dependent.values))
print('Dependent ratio 3+ ', dependent['3+'] / sum(dependent.values))

def fill_dependent_status(num_0_train, num_1_train, num_2_train, num_3_train, nur
    data_train['Dependents'].fillna('0', inplace=True, limit = num_0_train)
    data_train['Dependents'].fillna('1', inplace=True, limit = num_1_train)
    data_train['Dependents'].fillna('2', inplace=True, limit = num_2_train)
    data_train['Dependents'].fillna('3+', inplace=True, limit = num_3_train)
    data_test['Dependents'].fillna('0', inplace=True, limit = num_0_test)
    data_test['Dependents'].fillna('1', inplace=True, limit = num_1_test)
    data_test['Dependents'].fillna('2', inplace=True, limit = num_2_test)
    data_test['Dependents'].fillna('3+', inplace=True, limit = num_3_test)

fill_dependent_status(9, 2, 2, 2, 5, 2, 2, 1)

print(data_train['Dependents'].value_counts())

# Convert category value "3+" to "4"

data_train['Dependents'].replace('3+', 4, inplace = True)
data_test['Dependents'].replace('3+', 4, inplace = True)
```

```
0     345
1     102
2     101
3+     51
Name: Dependents, dtype: int64
Dependent ratio 1  0.5759599332220368
Dependent ratio 2  0.17028380634390652
Dependent ratio 3  0.1686143572621035
Dependent ratio 3+  0.08514190317195326
0     354
1     104
2     103
3+     53
Name: Dependents, dtype: int64
```

In [45]:
```
print(data_train['LoanAmount'].value_counts())

LoanAmt = data_train['LoanAmount'].value_counts()

print('mean loan amount ', np.mean(data_train["LoanAmount"]))

loan_amount_mean = np.mean(data_train["LoanAmount"])

data_train['LoanAmount'].fillna(loan_amount_mean, inplace=True, limit = 22)
data_test['LoanAmount'].fillna(loan_amount_mean, inplace=True, limit = 5)
```

```
120.0    20
110.0    17
100.0    15
187.0    12
160.0    12
         ..
570.0     1
300.0     1
376.0     1
117.0     1
311.0     1
Name: LoanAmount, Length: 203, dtype: int64
mean loan amount  146.41216216216216
```

In [46]:
```
print(data_train['Gender'].value_counts())
gender_sum = data_train['Gender'].value_counts()

male_ratio = gender_sum['Male']/sum(gender_sum.values)
female_ratio = gender_sum['Female']/ sum(gender_sum.values)
# print(male_ratio)
# print(female_ratio)

def fill_gender(male_train, female_train, male_test, female_test):
    data_train['Gender'].fillna('Male', inplace= True, limit= male_train)
    data_train['Gender'].fillna('Female', inplace= True, limit= female_train)
    data_test['Gender'].fillna('Male', inplace = True, limit = male_test)
    data_test['Gender'].fillna('Female', inplace = True, limit = female_test)

print(data_train.isnull().sum()['Gender'])
print(data_test.isnull().sum()['Gender'])

train_na = data_train.isnull().sum()['Gender']
test_na = data_test.isnull().sum()['Gender']
male_train = int((male_ratio * train_na).round())
female_train = int((female_ratio * train_na).round())
male_test = int((male_ratio * test_na).round())
female_test = int((female_ratio * test_na).round())

fill_gender(male_train, female_train, male_test, female_test)

# print(male_train)
# print(male_test)
# print(female_train)
# print(female_test)



print(data_train['Gender'].value_counts())
print(data_test['Gender'].value_counts())
```

```
Male      489
Female    112
Name: Gender, dtype: int64
13
11
Male      500
Female    114
Name: Gender, dtype: int64
Male      295
Female     72
Name: Gender, dtype: int64
```

In [47]:
```
se_train_na = data_train.isnull().sum()['Self_Employed']
se_test_na = data_test.isnull().sum()['Self_Employed']

se_sum = data_train['Self_Employed'].value_counts()
se_yes_ratio = se_sum['Yes']/ se_sum.values.sum()
se_no_ratio = se_sum['No']/ se_sum.values.sum()

# print(se_yes_ratio)
# print(se_no_ratio)

def fill_self_employed(train_yes, test_yes, train_no, test_no):
    data_train['Self_Employed'].fillna('Yes', inplace= True, limit = train_yes)
    data_train['Self_Employed'].fillna('No', inplace= True, limit = train_no)
    data_test['Self_Employed'].fillna('Yes', inplace= True, limit = test_yes)
    data_test['Self_Employed'].fillna('No', inplace= True, limit = test_no)

train_yes = int(round(se_yes_ratio * se_train_na))
train_no = int(round(se_no_ratio * se_train_na))
test_yes = int(round(se_yes_ratio * se_test_na))
test_no = int(round(se_no_ratio * se_test_na))
# print(train_yes)
# print(train_no)
# print(test_yes)
# print(test_no)
fill_self_employed(train_yes, test_yes, train_no, test_no)
```

In [48]:
```
LAT_train_na = data_train['Loan_Amount_Term'].isnull().sum()
LAT_test_na = data_test['Loan_Amount_Term'].isnull().sum()

LAT_count = data_train['Loan_Amount_Term'].value_counts()
# print(LAT_count)

v_ratio = {}
for i in LAT_count.index:
    v_ratio[i] = LAT_count[i]/ LAT_count.sum()

# (np.array(list(v_ratio.values())) * LAT_train_na).round()
# print(v_ratio)


for ind, v in v_ratio.items():
    v_ratio[ind] = int((v * LAT_train_na).round())

v_ratio_test = {}
for ind, v in v_ratio.items():
    v_ratio_test[ind] = int((v * LAT_test_na).round())

# print(v_ratio_test[480.0])

v360_train = v_ratio[360.0]
v180_train = v_ratio[180.0] + 1
v360_test = v_ratio_test[360.0]
v180_test = v_ratio[180.0]

def fill_Loan_Amount_Term(v360_train, v180_train, v360_test, v180_test):
    data_train['Loan_Amount_Term'].fillna(360.0, inplace= True, limit = v360_trai
    data_train['Loan_Amount_Term'].fillna(180.0, inplace= True, limit = v180_trai
    data_test['Loan_Amount_Term'].fillna(360.0, inplace= True, limit = v360_test)
    data_test['Loan_Amount_Term'].fillna(180.0, inplace= True, limit = v180_test)

fill_Loan_Amount_Term(v360_train, v180_train, v360_test, v180_test)
print(data_train.isnull().sum())
print(data_test.isnull().sum())
```

```
Loan_ID               0
Gender                0
Married               0
Dependents            0
Education             0
Self_Employed         0
ApplicantIncome       0
CoapplicantIncome     0
LoanAmount            0
Loan_Amount_Term      0
Credit_History       50
Property_Area         0
Loan_Status           0
dtype: int64
Loan_ID               0
Gender                0
Married               0
Dependents            0
Education             0
Self_Employed         0
ApplicantIncome       0
CoapplicantIncome     0
LoanAmount            0
Loan_Amount_Term      0
Credit_History       29
Property_Area         0
dtype: int64
```

In [49]:
```
CH_na = data_train['Credit_History'].isnull().sum()
CH_test_na = data_test['Credit_History'].isnull().sum()

total = data_train['Credit_History'].value_counts().sum()
one = data_train['Credit_History'].value_counts()[1.0]
zero = data_train['Credit_History'].value_counts()[0]

one_ratio = one/ total
zero_ratio = zero/ total

one_train = int((one_ratio * CH_na).round())
zero_train = int((zero_ratio * CH_na).round())
one_test = int((one_ratio * CH_test_na).round())
zero_test = int((zero_ratio * CH_test_na).round())

def fill_Credit_History(one_train, zero_train, one_test, zero_test):
    data_train['Credit_History'].fillna(1.0, inplace= True, limit = one_train)
    data_train['Credit_History'].fillna(0, inplace= True, limit = zero_train)
    data_test['Credit_History'].fillna(1.0, inplace= True, limit = one_test)
    data_test['Credit_History'].fillna(0, inplace= True, limit = zero_test)

fill_Credit_History(one_train, zero_train, one_test, zero_test)

print(data_train.isnull().sum())
print(data_test.isnull().sum())
```

```
Loan_ID              0
Gender               0
Married              0
Dependents           0
Education            0
Self_Employed        0
ApplicantIncome      0
CoapplicantIncome    0
LoanAmount           0
Loan_Amount_Term     0
Credit_History       0
Property_Area        0
Loan_Status          0
dtype: int64
Loan_ID              0
Gender               0
Married              0
Dependents           0
Education            0
Self_Employed        0
ApplicantIncome      0
CoapplicantIncome    0
LoanAmount           0
Loan_Amount_Term     0
Credit_History       0
Property_Area        0
dtype: int64
```

In [50]:
```
data_train['Gender'].replace('Male', 1, inplace = True)
data_train['Gender'].replace('Female', 2, inplace = True)
data_test['Gender'].replace('Male', 1, inplace = True)
data_test['Gender'].replace('Female', 2, inplace = True)

data_train['Married'].replace('Yes', 1, inplace = True)
data_train['Married'].replace('No', 0, inplace = True)
data_test['Married'].replace('Yes', 1, inplace = True)
data_test['Married'].replace('No', 0, inplace = True)

data_train['Self_Employed'].replace('Yes', 1, inplace = True)
data_train['Self_Employed'].replace('No', 0, inplace = True)
data_test['Self_Employed'].replace('Yes', 1, inplace = True)
data_test['Self_Employed'].replace('No', 0, inplace = True)

data_train['Education'].replace('Graduate', 1, inplace = True)
data_train['Education'].replace('Not Graduate', 0, inplace = True)
data_test['Education'].replace('Graduate', 1, inplace = True)
data_test['Education'].replace('Not Graduate', 0, inplace = True)

data_train['Property_Area'].replace('Urban', 1, inplace = True)
data_train['Property_Area'].replace('Rural', 2, inplace = True)
data_train['Property_Area'].replace('Semiurban', 3, inplace = True)
data_test['Property_Area'].replace('Urban', 1, inplace = True)
data_test['Property_Area'].replace('Rural', 2, inplace = True)
data_test['Property_Area'].replace('Semiurban', 3, inplace = True)

data_train['Loan_Status'].replace('Y', 1, inplace = True)
data_train['Loan_Status'].replace('N', 0, inplace = True)


data_train['Dependents'] = data_train['Dependents'].astype(int)
# print(data_train['Dependents'])
```

In [51]:
```
y = data_train['Loan_Status']
X = data_train.iloc[:, 1:-1]

y = np.array(y).reshape(-1,1)
X = np.array(X)

print(y.shape)
print(X.shape)
```

```
(614, 1)
(614, 11)
```

In [52]:
```
means = X.mean(axis = 0)
stds = X.std(axis = 0)

X_norm = (X - means)/ stds
X_norm = X_norm[:, (5,8,9)]
print(X_norm.shape)
```

```
(614, 3)
```

In [53]:
```
percent_train = .8

def partition(X, y, percent_train):
    idx = np.arange(0,y.shape[0])
    random.seed(1412)
    random.shuffle(idx)
    train_size = int(percent_train * len(idx))
    train_idx = idx[:train_size]
    test_idx = idx[train_size:]
    X_train = X[train_idx]
    y_train = y[train_idx]
    X_test = X[test_idx]
    y_test = y[test_idx]
    return idx, X_train, y_train, X_test, y_test
```

In [54]:
```
idx, X_train, y_train, X_test, y_test = partition(X_norm, y, percent_train)
print(X_train.shape)
print(X_test.shape)
print(y_train.shape)
print(y_test.shape)
```

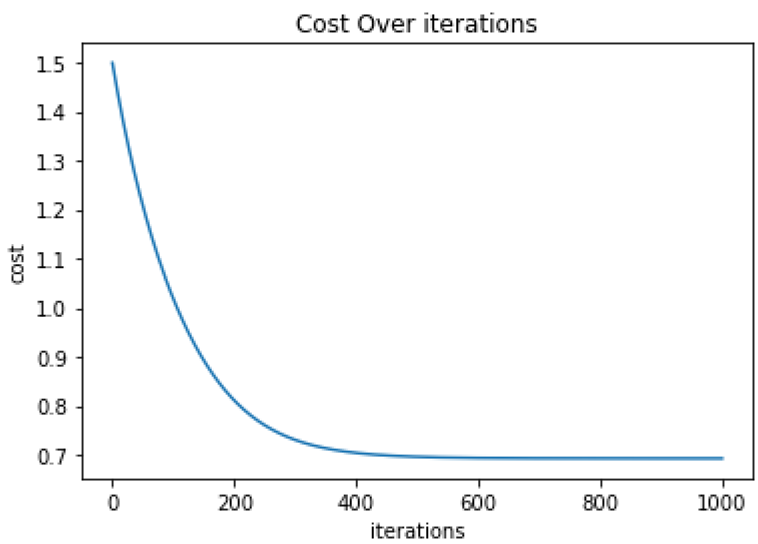
```
(491, 3)
(123, 3)
(491, 1)
(123, 1)
```

In [55]:
```
data_train.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 614 entries, 0 to 613
Data columns (total 13 columns):
 #   Column             Non-Null Count  Dtype  
---  ------             --------------  -----  
 0   Loan_ID            614 non-null    object 
 1   Gender             614 non-null    int64  
 2   Married            614 non-null    int64  
 3   Dependents         614 non-null    int64  
 4   Education          614 non-null    int64  
 5   Self_Employed      614 non-null    int64  
 6   ApplicantIncome    614 non-null    int64  
 7   CoapplicantIncome  614 non-null    float64
 8   LoanAmount         614 non-null    float64
 9   Loan_Amount_Term   614 non-null    float64
 10  Credit_History     614 non-null    float64
 11  Property_Area      614 non-null    int64  
 12  Loan_Status        614 non-null    int64  
dtypes: float64(4), int64(8), object(1)
memory usage: 62.5+ KB
```

In [56]:
```
NM_model = Logistic_NM()

iterations = 1000
theta_initial = np.ones((X_train.shape[1],1))

nm_theta1, nm_cost1 = NM_model.newtonsMethod(X_train, y_train, theta_initial, ite
print("theta:",nm_theta1)

print(nm_cost[0])
plt.plot(nm_cost)
plt.xlabel('iterations')
plt.ylabel('cost')
plt.title('Cost Over iterations')
plt.show()
```

```
Minimum at iteration: 999
theta: [[0.20574002]
 [0.12499897]
 [1.38603839]]
1.499471045823371
```



In [57]:
```
print("Accuracy = ",NM_model.getAccuracy(X_test, y_test, nm_theta1))
```

```
Accuracy =  84.5528455284553
```

The report

Write a brief report covering your experiments (both in lab and take home) and submit the Jupyter notebook via JupyterHub at https://puffer.cs.ait.ac.th (https://puffer.cs.ait.ac.th) before the next lab.

In your solution, be sure to follow instructions!

if the data is not easily linearly separable, both Newton's method and Linear Regression do not work well. They give us accuracy of around 57.5%. After polar transformation it seems worked better. Newton's method is faster to converge than linear regression model within 300 iterations.

For take home exercise, Newton's method converge in 400 iterations and the accuracy is 84.6%.