01_LG(TakeHome)

August 23, 2021

0.0.1 Take-home exercise (40 points)

Find an interesting dataset for linear regression on Kaggle. Implement the normal equations and gradient descent then evaluate your model's performance.

Write a brief report on your experiments and results in the form of a Jupyter notebook.

Explain the dataset which you get and which rows which you use. How many data in your dataset?

Write down your all code at below. Show the results, goodness of fit and plot cost graph

[1]:
```
NAME = "Kanawut Kaewnoparat"
ID = "st122109"
```

[2]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

[3]:
```
data = pd.read_csv("insurance.csv")
```

[4]:
```
data.head()
```

[4]:

	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

[5]:
```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1338 entries, 0 to 1337
Data columns (total 7 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   age       1338 non-null   int64
 1   sex       1338 non-null   object
 2   bmi       1338 non-null   float64
```

```
 3   children  1338 non-null   int64
 4   smoker    1338 non-null   object
 5   region    1338 non-null   object
 6   charges   1338 non-null   float64
dtypes: float64(2), int64(2), object(3)
memory usage: 73.3+ KB
```

[6]:
```
data.dtypes
```

[6]:
```
age           int64
sex          object
bmi         float64
children      int64
smoker       object
region       object
charges     float64
dtype: object
```

[7]:
```
data.isnull().sum()
```

[7]:
```
age         0
sex         0
bmi         0
children    0
smoker      0
region      0
charges     0
dtype: int64
```

[8]:
```
data.select_dtypes(include = [int, float]).head()
```

[8]:

	age	bmi	children	charges
0	19	27.900	0	16884.92400
1	18	33.770	1	1725.55230
2	28	33.000	3	4449.46200
3	33	22.705	0	21984.47061
4	32	28.880	0	3866.85520

[9]:
```
data.select_dtypes(exclude= [int, float]).columns
```

[9]:
```
Index(['sex', 'smoker', 'region'], dtype='object')
```

[10]:
```
data['sex'].unique()
```

[10]:
```
array(['female', 'male'], dtype=object)
```

[11]:
```
categorical_col = data.select_dtypes(exclude= [int, float]).columns
for i in categorical_col:
    print(f"Unique valeus in {i}: {data[i].unique()}")
print(categorical_col)
```

```
Unique valeus in sex: ['female' 'male']
Unique valeus in smoker: ['yes' 'no']
Unique valeus in region: ['southwest' 'southeast' 'northwest' 'northeast']
Index(['sex', 'smoker', 'region'], dtype='object')
```

[12]:
```
numerical_col = data.select_dtypes(include= [int, float]).columns
print(numerical_col)
```

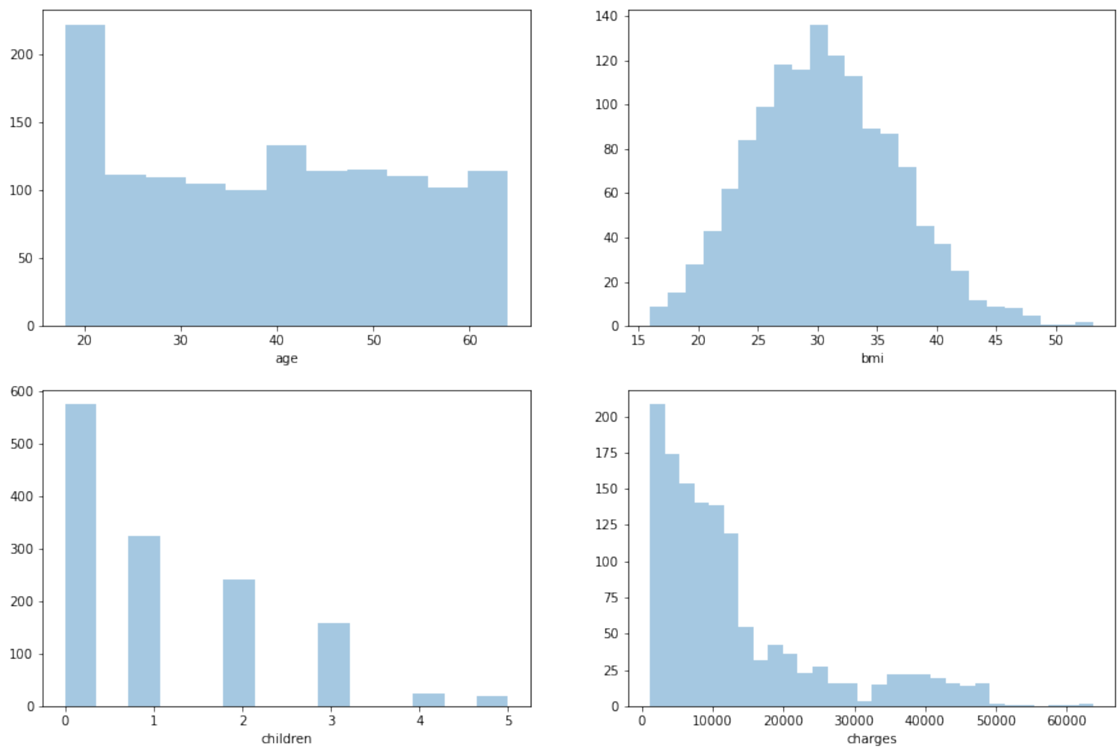
```
Index(['age', 'bmi', 'children', 'charges'], dtype='object')
```

[13]:
```
for index, value in enumerate(numerical_col):
    print(index, value)
```

```
0 age
1 bmi
2 children
3 charges
```

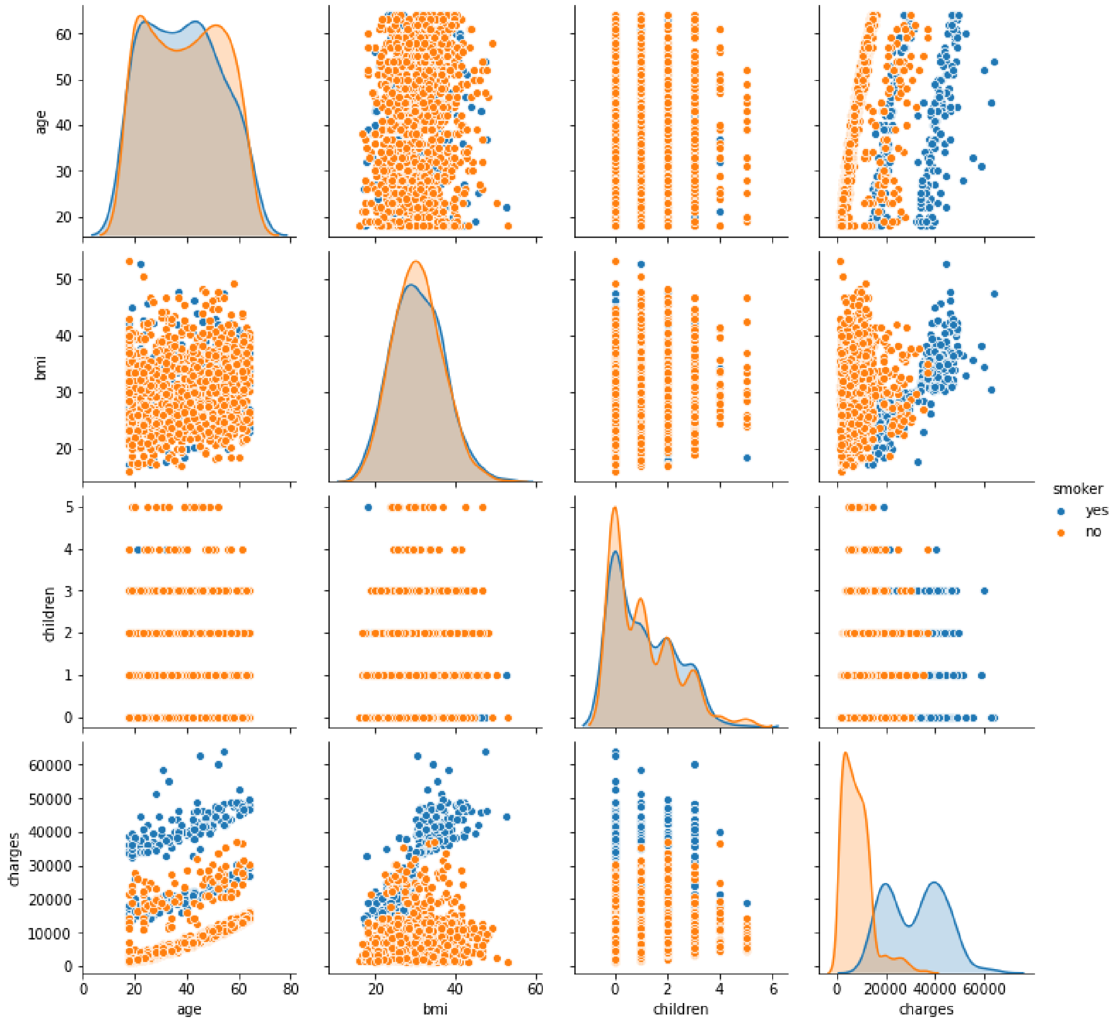
[14]:
```
fig, axes = plt.subplots(2,2, figsize = (15,10))
sns.distplot(a = data['age'], kde = False, ax = axes[0,0] )
sns.distplot(a = data['bmi'], kde = False, ax = axes[0,1] )
sns.distplot(a = data['children'], kde = False, ax = axes[1,0] )
sns.distplot(a = data['charges'], kde = False, ax = axes[1,1] )
```

[14]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7f186f2c9b20>
```



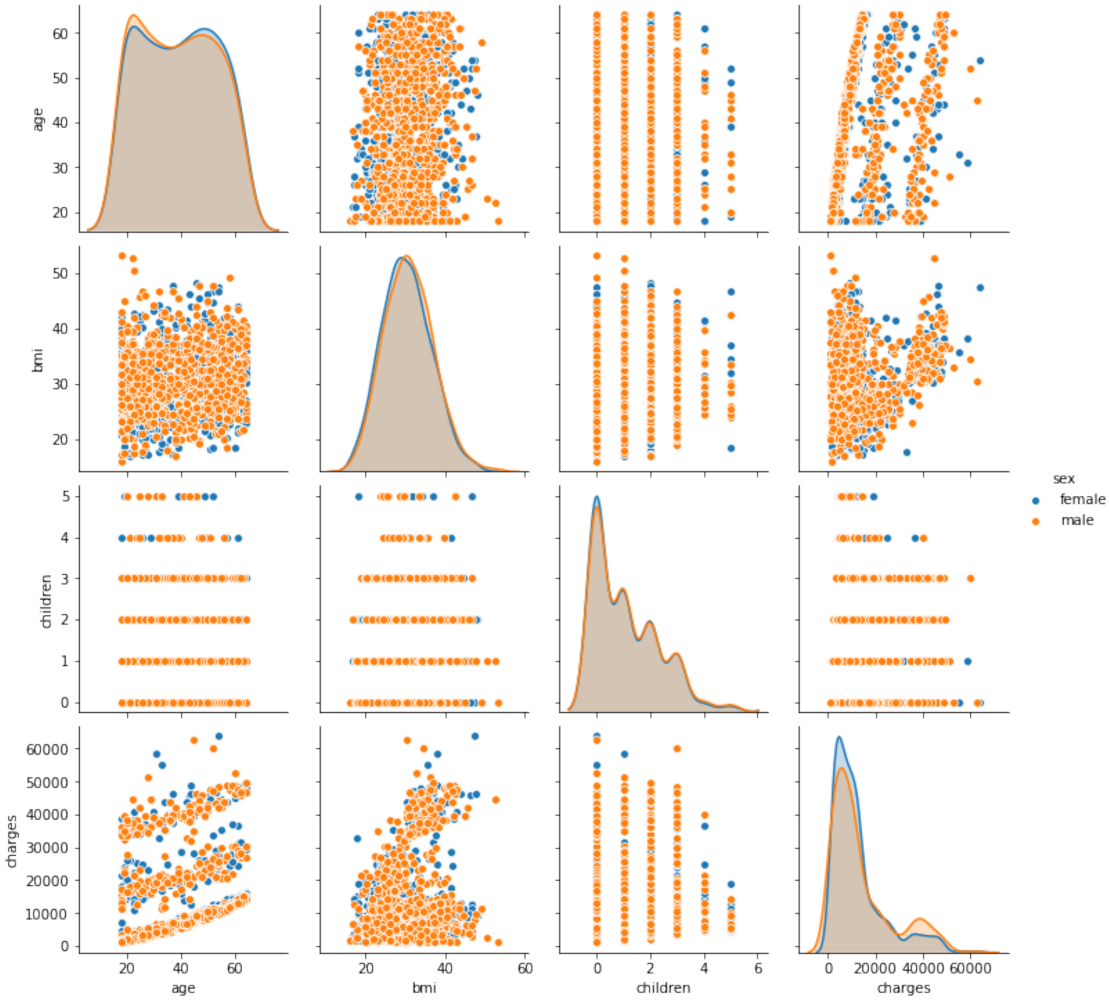
[15]:
```
sns.pairplot(data, hue = 'smoker')
```

[15]: <seaborn.axisgrid.PairGrid at 0x7f186ee50f10>



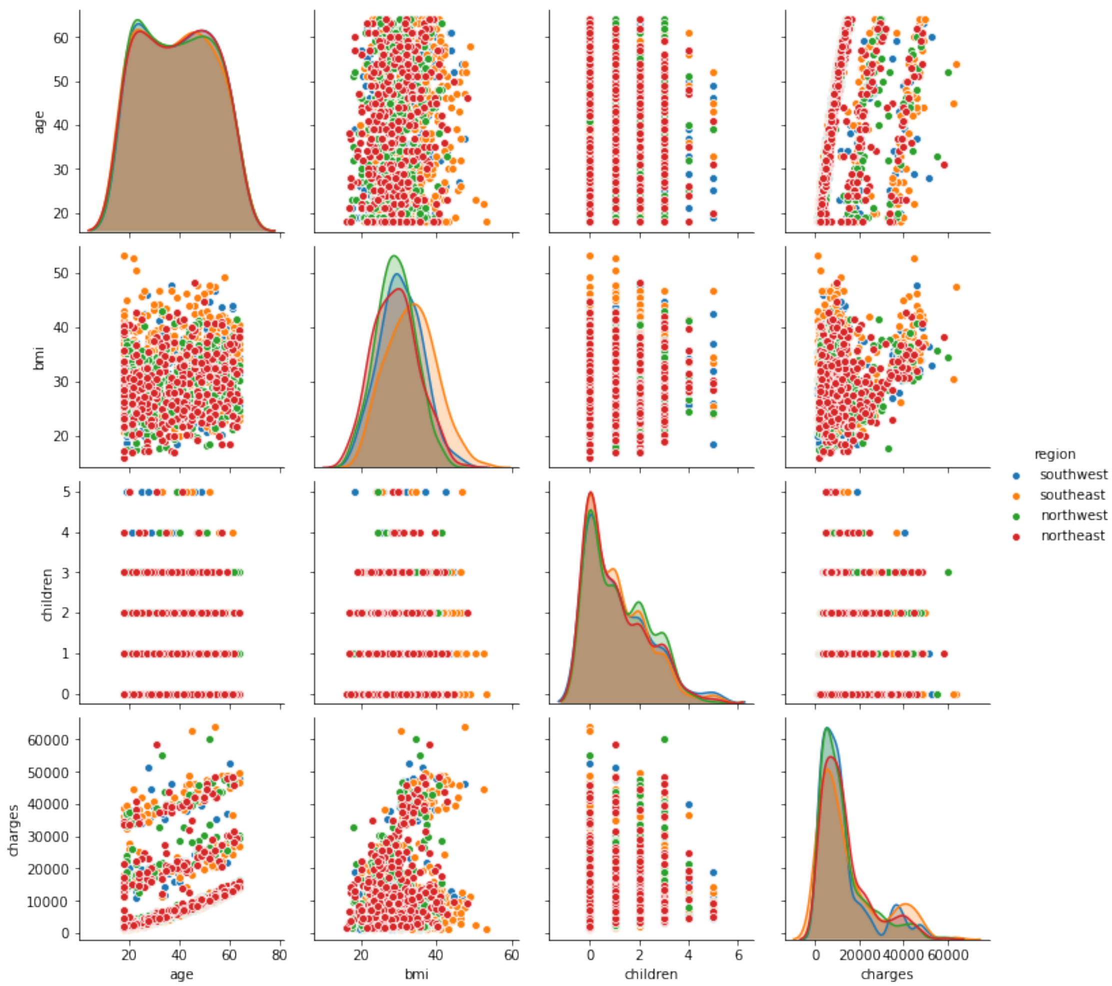
[16]:
```
sns.pairplot(data, hue = 'sex')
```

[16]: <seaborn.axisgrid.PairGrid at 0x7f186e920a30>



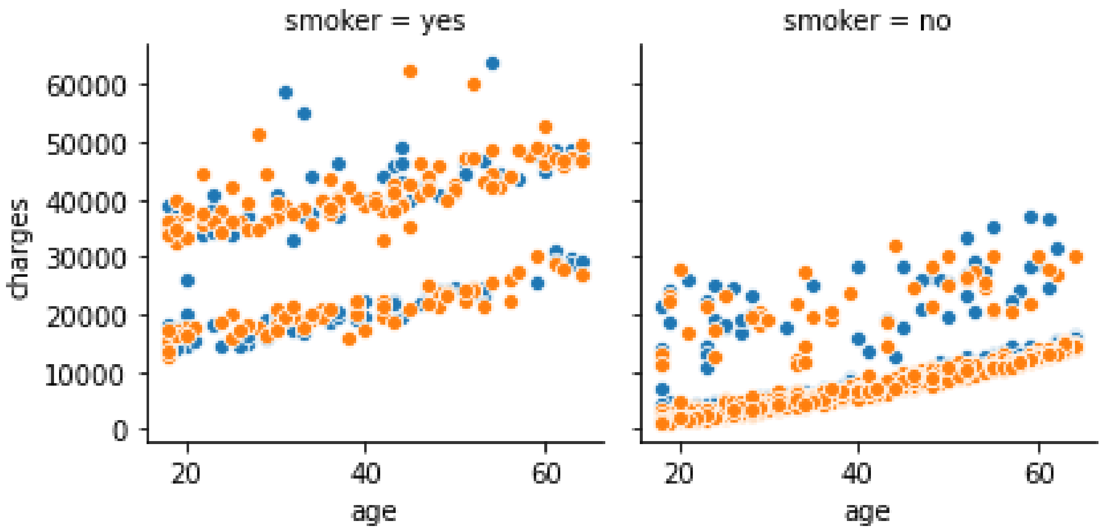

```
[17]: sns.pairplot(data, hue = 'region')
```

```
[17]: <seaborn.axisgrid.PairGrid at 0x7f186cb13d90>
```



[18]:
```
grid = sns.FacetGrid(data= data, col = 'smoker', hue = 'sex')
grid.map(sns.scatterplot, 'age', 'charges')
```

[18]: <seaborn.axisgrid.FacetGrid at 0x7f186e978190>



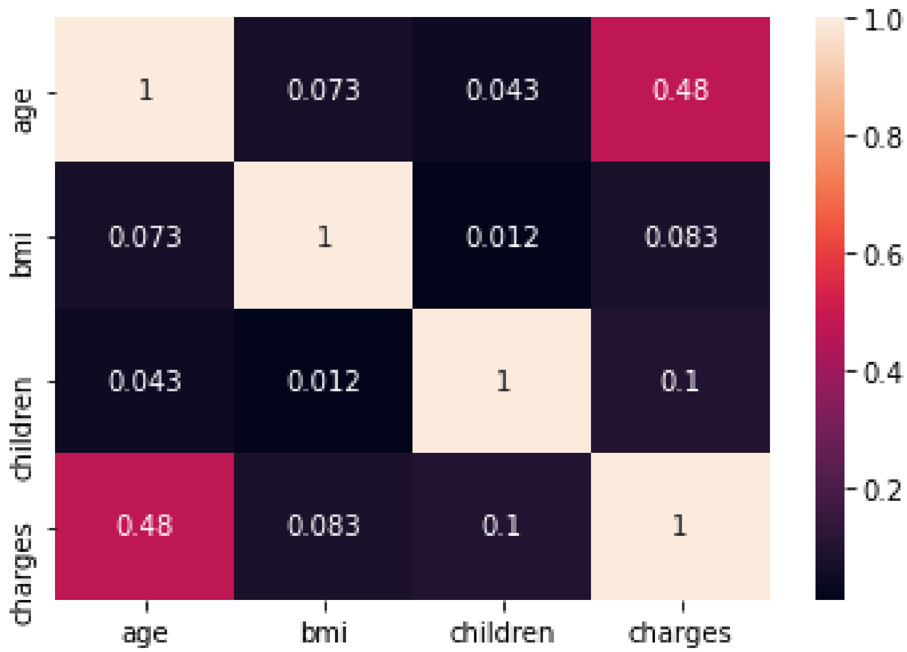
[19]:
```
data[numerical_col].corr(method = 'kendall')
```

[19]:

	age	bmi	children	charges
age	1.000000	0.073273	0.043253	0.475302
bmi	0.073273	1.000000	0.011562	0.082524
children	0.043253	0.011562	1.000000	0.103107
charges	0.475302	0.082524	0.103107	1.000000

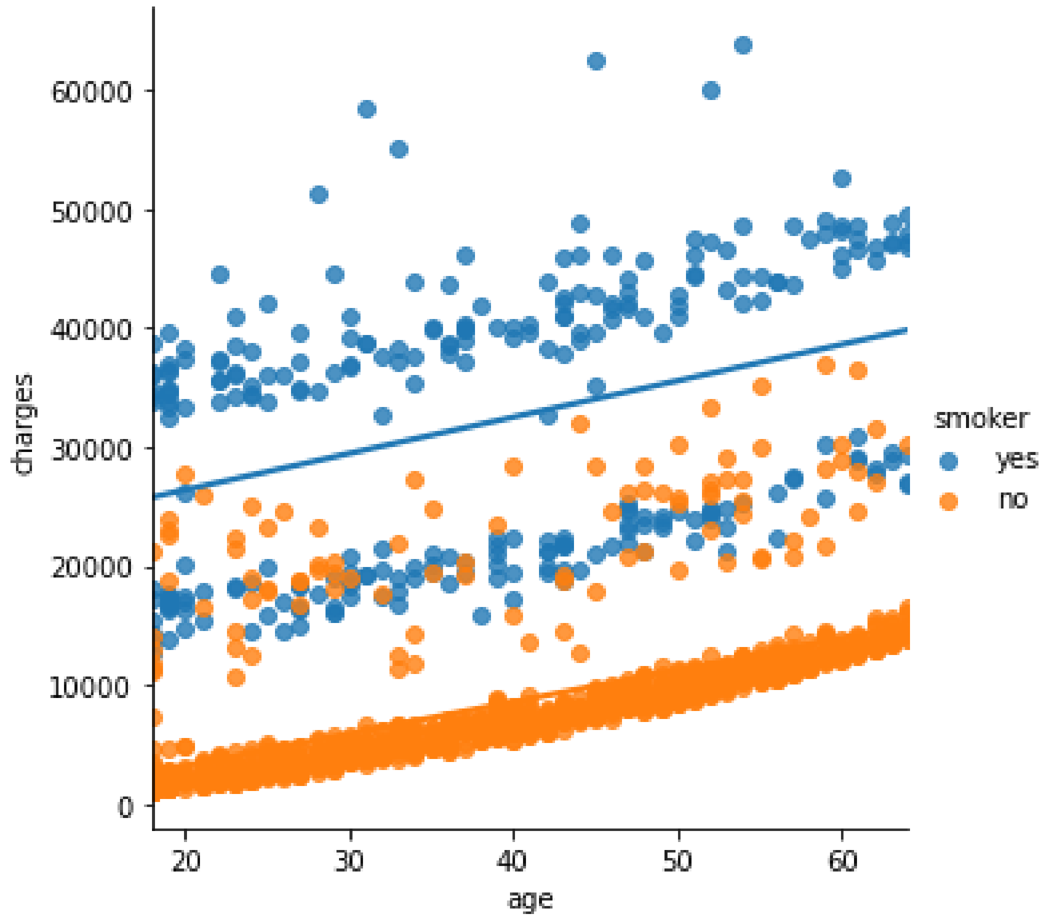
[20]:
```
sns.heatmap(data[numerical_col].corr(method = 'kendall'), annot = True)
```

[20]: <matplotlib.axes._subplots.AxesSubplot at 0x7f186c463c40>



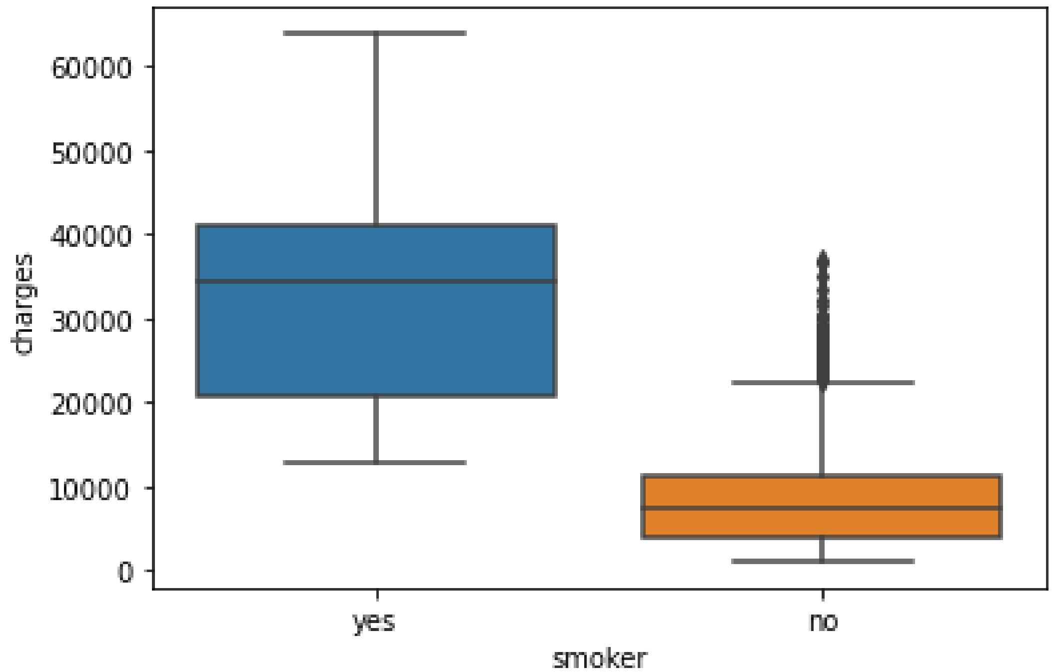

```
[21]: sns.lmplot(x = 'age', y='charges', data =data, ci = 0.95, hue = 'smoker')
```

```
[21]: <seaborn.axisgrid.FacetGrid at 0x7f186c376190>
```



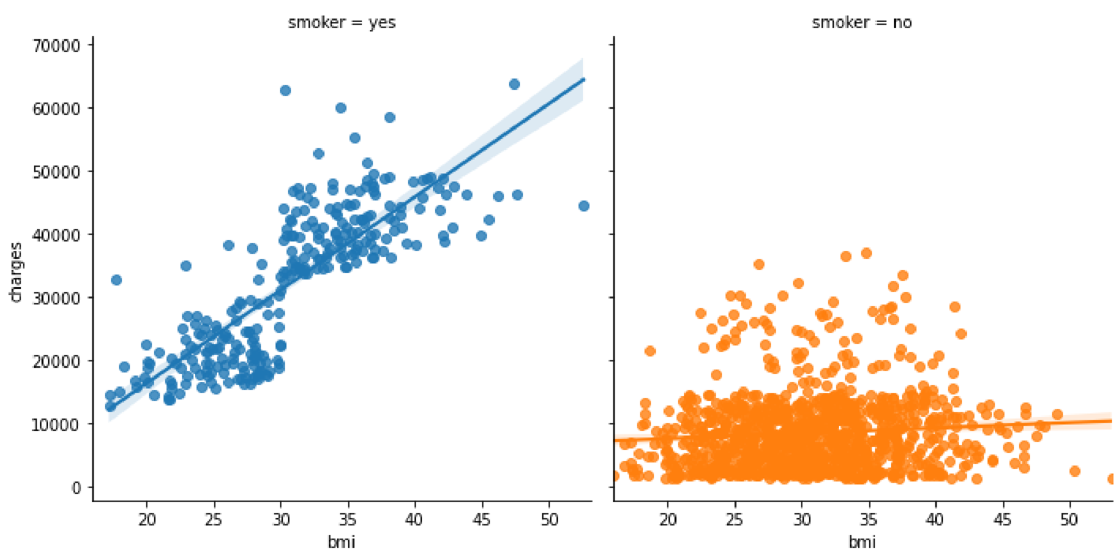
[22]:
```
sns.boxplot(x = 'smoker', y='charges', data =data)
```

[22]: <matplotlib.axes._subplots.AxesSubplot at 0x7f186c2f37c0>



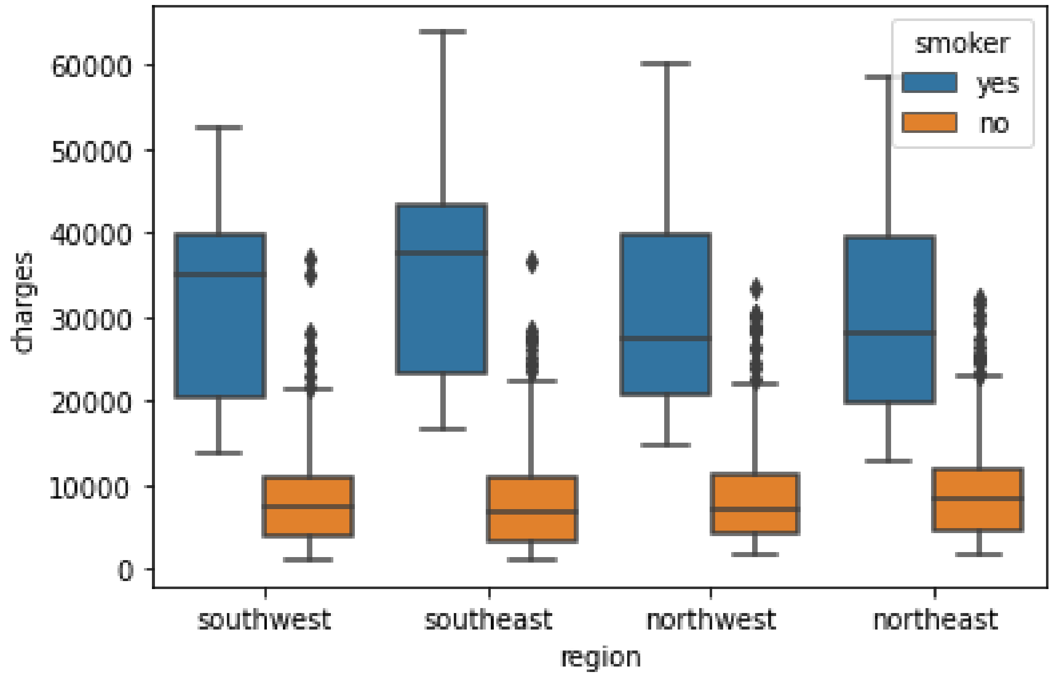
[23]:
```
sns.lmplot(x = 'bmi', y ='charges', hue ='smoker', col = 'smoker', data = data)
```

[23]: <seaborn.axisgrid.FacetGrid at 0x7f186c255520>



[24]:
```
sns.boxplot(x = 'region', y ='charges', data = data, hue = 'smoker')
```

[24]: <matplotlib.axes._subplots.AxesSubplot at 0x7f186c1e62e0>



[25]:
```
data[data['smoker'] == 'no'].describe()
```

[25]:

	age	bmi	children	charges
count	1064.000000	1064.000000	1064.000000	1064.000000
mean	39.385338	30.651795	1.090226	8434.268298
std	14.083410	6.043111	1.218136	5993.781819
min	18.000000	15.960000	0.000000	1121.873900
25%	26.750000	26.315000	0.000000	3986.438700
50%	40.000000	30.352500	1.000000	7345.405300
75%	52.000000	34.430000	2.000000	11362.887050
max	64.000000	53.130000	5.000000	36910.608030

[26]:
```
data['region'].value_counts(normalize = True)
```

[26]:
```
southeast    0.272048
northwest    0.242900
southwest    0.242900
northeast    0.242152
Name: region, dtype: float64
```

[27]:
```
data['sex'].value_counts(normalize = True)
```

[27]:
```
male      0.505232
female    0.494768
Name: sex, dtype: float64
```

[28]:
```
data['smoker'].value_counts(normalize = True)
```

[28]:
```
no     0.795217
yes    0.204783
Name: smoker, dtype: float64
```

0.1 SUMMARY FROM EXPLORING DATA

- This dataset contains 1,338 records, with 6 independent features and an invididual medical costs charged by health insurance as dependent variable
- This dataset contains no null values in any column
- Out of 6 features, 3 are numerical features: age, bmi and number of children, and
- Out of 6 features, 3 are categorical features: sex, smoker status and region
- Graphing multiple pairplots, we see that the feature with most impact on medical cost/ charges is the smoker status
- For smoker, the interquartile cost ranges from 20k to 40k, with the median of around 36k
- For non-smoker, the interquartile cost ranges from only 4k to 11k, with the median of around 7k
- This mean that the medical cost for smokers could be as high as 5 times the cost for non-smokers!
- The slight problem from this summary might be the fact that sample size of smokers accounts for only 20% of entire dataset

[29]:
```
X = data.iloc[:, data.columns != 'charges']
X.head()
```

[29]:

	age	sex	bmi	children	smoker	region
0	19	female	27.900	0	yes	southwest
1	18	male	33.770	1	no	southeast
2	28	male	33.000	3	no	southeast
3	33	male	22.705	0	no	northwest
4	32	male	28.880	0	no	northwest

[30]:
```
y= data.iloc[:, data.columns == 'charges']
y.head()
```

[30]:

	charges
0	16884.92400
1	1725.55230
2	4449.46200
3	21984.47061
4	3866.85520

[31]:
```
X_numerical_col = X.select_dtypes(include =[int, float]).columns
X_categorical_col = X.select_dtypes(exclude =[int, float]).columns

print("Numerical column name: {}".format(X_numerical_col))
print("Numerical column name: {}".format(X_categorical_col))
```

```
Numerical column name: Index(['age', 'bmi', 'children'], dtype='object')
Numerical column name: Index(['sex', 'smoker', 'region'], dtype='object')
```

0.2 Preprocessing Categorical Columns with OneHotEncoder

[32]:
```
from sklearn.preprocessing import OneHotEncoder
encoder = OneHotEncoder(sparse = False)
encoded = encoder.fit_transform(X[X_categorical_col ])
print(encoder.get_feature_names())

#create new encoded dataframe
X_categorical = pd.DataFrame(encoded, columns = encoder.get_feature_names())
X_categorical.head()
```

```
['x0_female' 'x0_male' 'x1_no' 'x1_yes' 'x2_northeast' 'x2_northwest'
 'x2_southeast' 'x2_southwest']
```

[32]:
```
   x0_female  x0_male  x1_no  x1_yes  x2_northeast  x2_northwest  \
0        1.0      0.0    0.0     1.0           0.0           0.0
1        0.0      1.0    1.0     0.0           0.0           0.0
2        0.0      1.0    1.0     0.0           0.0           0.0
3        0.0      1.0    1.0     0.0           0.0           1.0
4        0.0      1.0    1.0     0.0           0.0           1.0

   x2_southeast  x2_southwest
0           0.0           1.0
1           1.0           0.0
2           1.0           0.0
3           0.0           0.0
4           0.0           0.0
```

[33]:
```
processed_X = X.drop(columns = ['sex', 'smoker', 'region'])
```

[34]:
```
processed_X = pd.concat([processed_X, X_categorical], axis = 1)
processed_X.head()
```

[34]:
```
   age     bmi  children  x0_female  x0_male  x1_no  x1_yes  x2_northeast  \
0   19  27.900         0        1.0      0.0    0.0     1.0           0.0
1   18  33.770         1        0.0      1.0    1.0     0.0           0.0
2   28  33.000         3        0.0      1.0    1.0     0.0           0.0
3   33  22.705         0        0.0      1.0    1.0     0.0           0.0
```

```
4   32   28.880          0         0.0        1.0     1.0      0.0              0.0

   x2_northwest  x2_southeast  x2_southwest
0           0.0           0.0           1.0
1           0.0           1.0           0.0
2           0.0           1.0           0.0
3           1.0           0.0           0.0
4           1.0           0.0           0.0
```

0.3 Preprocessing wiht TrainTestSplit

[35]:
```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(processed_X, y, test_size =␣
↪0.3)
```

[36]:
```
print(X_train.shape)
print(y_train.shape)
```

```
(936, 11)
(936, 1)
```

0.4 Preprocessing Numerical Columns with StandardScaler

The reasons to standardize data AFTER the split is to prevent data leakage from test set!

[37]:
```
print(f"The numerical columns to be standardized is {X_numerical_col}")
```

```
The numerical columns to be standardized is Index(['age', 'bmi', 'children'],
dtype='object')
```

[38]:
```
X_mean = X_train[X_numerical_col].mean(axis = 0)
X_std = X_train[X_numerical_col].std(axis = 0)
```

[39]:
```
X_train.head()
```

[39]:
```
      age    bmi  children  x0_female  x0_male  x1_no  x1_yes  x2_northeast  \
888    22  39.50         0        0.0      1.0    1.0     0.0           0.0
660    37  46.53         3        0.0      1.0    1.0     0.0           0.0
934    32  37.18         2        0.0      1.0    1.0     0.0           0.0
1000   30  22.99         2        0.0      1.0    0.0     1.0           0.0
97     55  38.28         0        0.0      1.0    1.0     0.0           0.0

      x2_northwest  x2_southeast  x2_southwest
888            0.0           0.0           1.0
660            0.0           1.0           0.0
934            0.0           1.0           0.0
1000           1.0           0.0           0.0
```

```
97              0.0           1.0           0.0
```

[40]:
```
X_train[X_numerical_col] = (X_train[X_numerical_col] - X_mean) / X_std
```

```
/opt/conda/lib/python3.8/site-packages/pandas/core/frame.py:2963:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  self[k1] = value[k2]
```

[41]:
```
X_train.head()
```

[41]:
```
           age       bmi  children  x0_female  x0_male  x1_no  x1_yes  \
888  -1.216077  1.400738 -0.920626        0.0      1.0    1.0     0.0
660  -0.166162  2.551015  1.572255        0.0      1.0    1.0     0.0
934  -0.516134  1.021130  0.741295        0.0      1.0    1.0     0.0
1000 -0.656122 -1.300696  0.741295        0.0      1.0    0.0     1.0
97    1.093737  1.201116 -0.920626        0.0      1.0    1.0     0.0

      x2_northeast  x2_northwest  x2_southeast  x2_southwest
888            0.0           0.0           0.0           1.0
660            0.0           0.0           1.0           0.0
934            0.0           0.0           1.0           0.0
1000           0.0           1.0           0.0           0.0
97             0.0           0.0           1.0           0.0
```

[42]:
```
y_train.mean()
```

[42]:
```
charges    13332.657562
dtype: float64
```

[43]:
```
#scaling on y
y_mean = y_train.mean()
y_std = y_train.std()
```

[44]:
```
y_train = (y_train - y_mean) / y_std
```

[45]:
```
y_train.head()
```

[45]:
```
       charges
888  -0.944175
660  -0.558968
934  -0.701787
1000  0.326538
```

```
97   -0.251755
```

0.5 Create Linear Regression Model

Convert all the dataframe into numpy

[46]:
```
X_train_ = X_train.to_numpy()
print(X_train_.shape)

y_train_ = y_train.to_numpy()
print(y_train_.shape)
```

```
(936, 11)
(936, 1)
```

[47]:
```
#add intercept on the first index
X_train_ = np.insert(X_train_, 0,1 ,axis =1)
print(X_train_.shape)
```

```
(936, 12)
```

[48]:
```
theta = np.zeros(X_train_.shape[1])
print(theta.shape)
```

```
(12,)
```

[49]:
```
#must flatten the y_train so that it will become (m,) instead of (m,1)
y_train_ = y_train_.flatten()
```

[50]:
```
predicted = np.dot(X_train_, theta)
print(predicted.shape)

error = predicted - y_train_
print(error.shape)
```

```
(936,)
(936,)
```

[51]:
```
#To facilitate the computation, the threshold is set to early stop the gradient␣
↪descent when the delta between loss < threshold

theta = theta = np.zeros(X_train_.shape[1])
threshold = 0.00000001
loss_record = [100,10]
count=0
r2_record = []
for i in range(100000):
    alpha = 0.0001
```

```
    predicted = np.dot(X_train_, theta)
    error = predicted - y_train_

    loss  = np.sum(error **2) / 2
    gradient = np.dot(X_train_.T, error)
    theta = theta - (alpha * gradient)
    loss_record.append(loss)
    r2 = 1 - np.sum((y_train_ - predicted) **2) / np.sum((y_train_ - np.
 ↪mean(y_train_)) **2)
    r2_record.append(r2)
    count +=1
    if np.abs(loss_record[-1] - loss_record[-2]) < threshold:
        break
    else:
        pass


    if count % 50 ==0:
        print(f'{count}, loss: {loss}')


print(count)
```

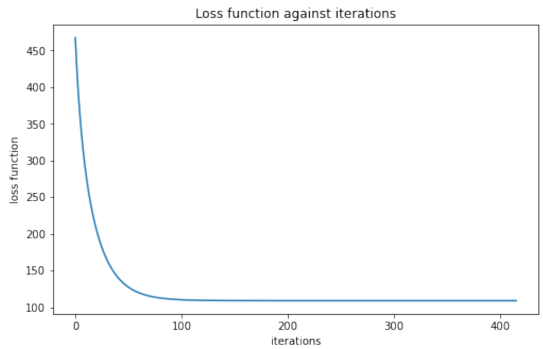
```
50, loss: 128.41181349876595
100, loss: 110.29430994962246
150, loss: 109.08926523294164
200, loss: 109.00529897602448
250, loss: 108.99908625853412
300, loss: 108.99859178493294
350, loss: 108.99854921015694
400, loss: 108.99854526134013
416
```

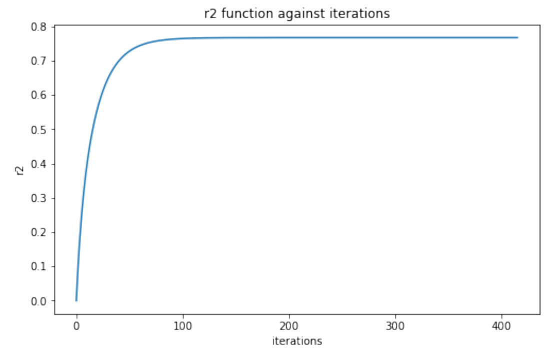
0.6 Plot loss function against iteration for training data

[52]:
```
fig,axes = plt.subplots(1,2, figsize = (18,5))
axes[0].plot(np.arange(count), loss_record[2:])
axes[0].set_xlabel('iterations')
axes[0].set_ylabel('loss function')
axes[0].set_title('Loss function against iterations')

axes[1].plot(np.arange(count), r2_record)
axes[1].set_xlabel('iterations')
axes[1].set_ylabel('r2')
axes[1].set_title('r2 function against iterations')
```

[52]:
```
Text(0.5, 1.0, 'r2 function against iterations')
```





[53]:
```
theta
```

[53]:
```
array([ 0.26594193,  0.29631842,  0.1631706 ,  0.04390204,  0.1315308 ,
        0.13441112, -0.85820643,  1.12414835,  0.13653553,  0.05243814,
        0.0451931 ,  0.03177515])
```

[54]:
```
r2
```

[54]:
```
0.7668480320081696
```

0.7 Summary on performing linear regression on training data

- with alpha at 0.0001 and early stop in place, the model iterates for 416 times at the loss delta < 0.0001
- Using r2 as the metric, the score of this model is around 77%

0.8 Perform on the test data

[55]:
```
#must standardize the data using the mean and std of training dataset
X_test[X_numerical_col] = (X_test[X_numerical_col] - X_mean)  / X_std
```

```
/opt/conda/lib/python3.8/site-packages/pandas/core/frame.py:2963:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  self[k1] = value[k2]
```

[56]:
```
X_test.head()
```

[56]:
```
          age       bmi  children  x0_female  x0_male  x1_no  x1_yes  \
840 -1.286071  0.026295 -0.920626        0.0      1.0    1.0     0.0
772  0.323799  0.906593 -0.920626        1.0      0.0    1.0     0.0
```

```
683   0.953748 -1.083076 -0.920626        0.0       1.0     1.0      0.0
1039 -1.426060 -0.601203  0.741295        0.0       1.0     1.0      0.0
987   0.393793 -0.539026 -0.089666        1.0       0.0     1.0      0.0

      x2_northeast  x2_northwest  x2_southeast  x2_southwest
840            0.0           0.0           0.0           1.0
772            1.0           0.0           0.0           0.0
683            0.0           1.0           0.0           0.0
1039           0.0           1.0           0.0           0.0
987            0.0           1.0           0.0           0.0
```

[57]:
```python
y_test
```

[57]:
```
          charges
840    1526.31200
772   12797.20962
683    9863.47180
1039  22493.65964
987   28340.18885
…             …
1327   9377.90470
737    3484.33100
767    7050.64200
515   11362.75500
195    1639.56310

[402 rows x 1 columns]
```

[58]:
```python
y_test = (y_test - y_mean) / y_std
y_test.head()
```

[58]:
```
       charges
840  -0.956841
772  -0.043395
683  -0.281159
1039  0.742450
987   1.216280
```

[59]:
```python
X_test_ = X_test.to_numpy()
y_test_ = y_test.to_numpy().flatten()
print(X_test_.shape)
print(y_test_.shape)
```

```
(402, 11)
(402,)
```

[60]:
```
X_test_ = np.insert(X_test_, 0,1, axis =1)
print(X_test_.shape)
```

```
(402, 12)
```

[61]:
```
yhat = np.dot(X_test_, theta)
```

[62]:
```
r2_yhat = 1 - np.sum(y_test_ - yhat)**2 / np.sum((y_test_ - np.
↪mean(y_test_))**2)
print(r2_yhat)
```

```
0.8030244597975953
```

1 r2 of test model is at around 80%

[63]:
```
theta
```

[63]:
```
array([ 0.26594193,  0.29631842,  0.1631706 ,  0.04390204,  0.1315308 ,
        0.13441112, -0.85820643,  1.12414835,  0.13653553,  0.05243814,
        0.0451931 ,  0.03177515])
```

[64]:
```
processed_X.columns
```

[64]:
```
Index(['age', 'bmi', 'children', 'x0_female', 'x0_male', 'x1_no', 'x1_yes',
       'x2_northeast', 'x2_northwest', 'x2_southeast', 'x2_southwest'],
      dtype='object')
```

[65]:
```
pd.DataFrame(np.array([theta]), columns = ['intercept', 'age', 'bmi',␣
↪'children', 'x0_female', 'x0_male',
       'x1_no', 'x1_yes', 'x2_northeast', 'x2_northwest', 'x2_southeast',
       'x2_southwest'])
```

[65]:
```
   intercept       age       bmi  children  x0_female   x0_male     x1_no  \
0   0.265942  0.296318  0.163171  0.043902   0.131531  0.134411 -0.858206

     x1_yes  x2_northeast  x2_northwest  x2_southeast  x2_southwest
0  1.124148      0.136536      0.052438      0.045193      0.031775
```

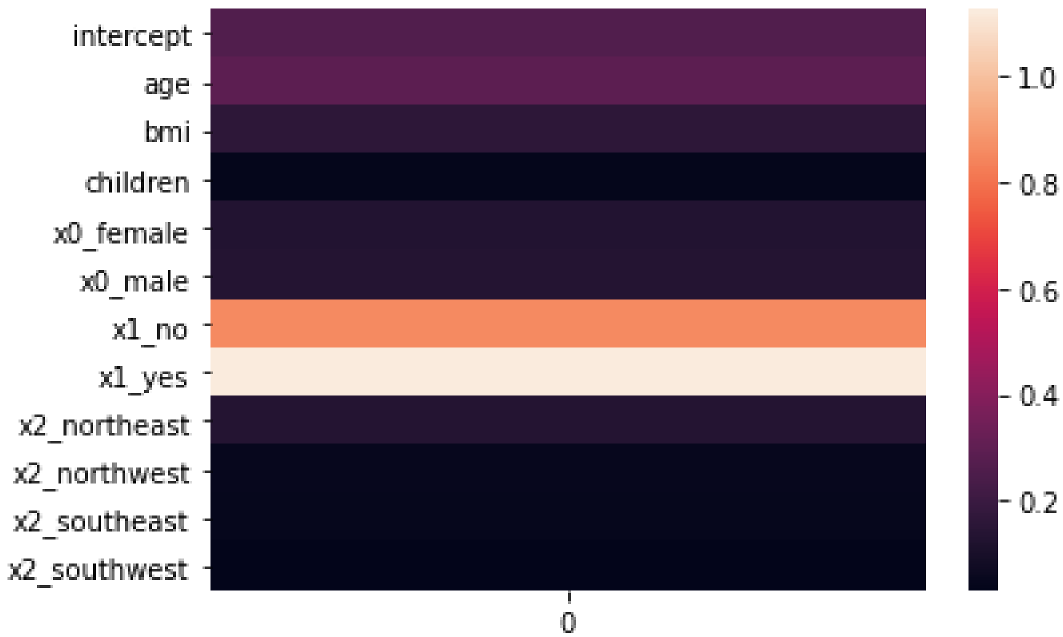
[66]:
```
pd.DataFrame(np.array([theta]), columns = np.insert(processed_X.columns.
↪to_numpy(), 0, 'intercept'))
```

[66]:
```
   intercept       age       bmi  children  x0_female   x0_male     x1_no  \
0   0.265942  0.296318  0.163171  0.043902   0.131531  0.134411 -0.858206

     x1_yes  x2_northeast  x2_northwest  x2_southeast  x2_southwest
0  1.124148      0.136536      0.052438      0.045193      0.031775
```

[67]:
```
sns.heatmap(np.abs(pd.DataFrame(np.array([theta]), columns = np.
↪insert(processed_X.columns.to_numpy(), 0, 'intercept'))).T)
```

[67]: <matplotlib.axes._subplots.AxesSubplot at 0x7f186c08b760>



The heatmap highlights the above-mentioned summary that 'smoker' status is the important feature to affect the medical costs

1.1 Now try analytical solution

I need to reset the first index 'intercept' to something close to 1 but 1 to avoid the multicollinearity that will return an error when trying to find the inverse matrix

I first tried with 1 as normal to find the repeated error of 'singular matrix' because in my X_train_, it looks like there are some columns that are linearly dependent on other columns

For example, for the 'intercept' column, the computer deems it dependent on the sum of One-hot-encoded male/ female or smoke / no smoke!!!

To avoid the singular error, I need to avoid assigning 'intercept' as the sum of any number of one-hot-encoded, so I used float number close to 1

The below is the old code that returns error from singular matrix (as reference to see how it blocks me from futher coding)

[280]:
```
#DO NOT RUN THIS CODE; it just to serves as reference for how the singular␣
↪matrix coud not be inverted
```

```
analytical_theta = np.linalg.inv(np.dot(X_train_.T, X_train_)) @ X_train_.T @
 ↪y_train_
```

```
     
↪---------------------------------------------------------------------------

       LinAlgError                               Traceback (most recent call
↪last)

       /tmp/ipykernel_167/574514158.py in <module>
   ----> 1 analytical_theta = np.linalg.inv(np.dot(X_train_.T, X_train_)) @
↪X_train_.T @ y_train_


       <__array_function__ internals> in inv(*args, **kwargs)


       /opt/conda/lib/python3.8/site-packages/numpy/linalg/linalg.py in inv(a)
       543     signature = 'D->D' if isComplexType(t) else 'd->d'
       544     extobj = get_linalg_error_extobj(_raise_linalgerror_singular)
   --> 545     ainv = _umath_linalg.inv(a, signature=signature, extobj=extobj)
       546     return wrap(ainv.astype(result_t, copy=False))
       547


       /opt/conda/lib/python3.8/site-packages/numpy/linalg/linalg.py in
↪_raise_linalgerror_singular(err, flag)
        86
        87 def _raise_linalgerror_singular(err, flag):
   ---> 88     raise LinAlgError("Singular matrix")
        89
        90 def _raise_linalgerror_nonposdef(err, flag):


       LinAlgError: Singular matrix
```

[286]:
```
#DO NOT RUN THIS CODE; it just serves as reference for how the singular matrix
 ↪coud not be inverted
#see that intercept columns ~  (X_female + X-male)
X_train.T @  X_train
```

[286]:
```
                  intercept           age           bmi      children  \
intercept      9.360000e+02  1.243450e-14  2.209344e-12  7.549517e-15
age            1.243450e-14  9.350000e+02  1.067586e+02  2.631458e+01
bmi            2.209344e-12  1.067586e+02  9.350000e+02  1.090044e+01
```

```
children      7.549517e-15  2.631458e+01  1.090044e+01  9.350000e+02
x0_female     4.470000e+02  2.541935e+01 -1.306959e+01 -2.031536e+01
x0_male       4.890000e+02 -2.541935e+01  1.306959e+01  2.031536e+01
x1_no         7.550000e+02 -1.732732e+00  3.482977e+00  1.824681e+00
x1_yes        1.810000e+02  1.732732e+00 -3.482977e+00 -1.824681e+00
x2_northeast  2.290000e+02  8.038445e-01 -5.544868e+01 -2.538266e+01
x2_northwest  2.500000e+02 -1.082511e+01 -5.513757e+01  1.164486e+01
x2_southeast  2.360000e+02  6.287851e+00  1.099656e+02 -5.703107e+00
x2_southwest  2.210000e+02  3.733411e+00  6.206967e-01  1.944091e+01

               x0_female     x0_male       x1_no      x1_yes  x2_northeast  \
intercept     447.000000  489.000000  755.000000  181.000000    229.000000
age            25.419350  -25.419350   -1.732732    1.732732      0.803845
bmi           -13.069588   13.069588    3.482977   -3.482977    -55.448679
children      -20.315360   20.315360    1.824681   -1.824681    -25.382659
x0_female     447.000000    0.000000  377.000000   70.000000    106.000000
x0_male         0.000000  489.000000  378.000000  111.000000    123.000000
x1_no         377.000000  378.000000  755.000000    0.000000    181.000000
x1_yes         70.000000  111.000000    0.000000  181.000000     48.000000
x2_northeast  106.000000  123.000000  181.000000   48.000000    229.000000
x2_northwest  120.000000  130.000000  208.000000   42.000000      0.000000
x2_southeast  112.000000  124.000000  183.000000   53.000000      0.000000
x2_southwest  109.000000  112.000000  183.000000   38.000000      0.000000

              x2_northwest  x2_southeast  x2_southwest
intercept       250.000000    236.000000    221.000000
age             -10.825106      6.287851      3.733411
bmi             -55.137573    109.965555      0.620697
children         11.644860     -5.703107     19.440906
x0_female       120.000000    112.000000    109.000000
x0_male         130.000000    124.000000    112.000000
x1_no           208.000000    183.000000    183.000000
x1_yes           42.000000     53.000000     38.000000
x2_northeast      0.000000      0.000000      0.000000
x2_northwest    250.000000      0.000000      0.000000
x2_southeast      0.000000    236.000000      0.000000
x2_southwest      0.000000      0.000000    221.000000
```

---

[323]:
```
X_train_[:,0] = 1.001
```

X_train

[324]:
```
X_train_
```

[324]:
```
array([[ 1.001     , -1.21607708,  1.40073753, …,  0.        ,
         0.        ,  1.        ],
```

```
       [ 1.001     , -0.16616181,  2.55101484, …,  0.        ,
         1.        ,  0.        ],
       [ 1.001     , -0.51613356,  1.02112966, …,  0.        ,
         1.        ,  0.        ],
       …,
       [ 1.001     , -0.44613921, -0.43676093, …,  0.        ,
         1.        ,  0.        ],
       [ 1.001     , -0.65612227, -1.47168325, …,  0.        ,
         0.        ,  0.        ],
       [ 1.001     ,  0.53378171,  0.22264413, …,  0.        ,
         0.        ,  1.        ]])
```

[325]:
```
np.linalg.inv(np.dot(X_train_.T ,X_train_)) @ X_train_.T @ y_train_
```

[325]:
```
array([-1.63208170e+13,  2.94971694e-01,  1.70423816e-01,  4.41698075e-02,
        1.63371378e+13,  1.63371378e+13,  5.43371206e-01,  2.50131628e+00,
       -1.41890509e-01, -2.51389241e-01, -2.48455862e-01, -3.10711286e-01])
```

[326]:
```
theta
```

[326]:
```
array([ 0.26594193,  0.29631842,  0.1631706 ,  0.04390204,  0.1315308 ,
        0.13441112, -0.85820643,  1.12414835,  0.13653553,  0.05243814,
        0.0451931 ,  0.03177515])
```

[327]:
```
analytical_theta = np.linalg.inv(np.dot(X_train_.T ,X_train_)) @ X_train_.T @␣
↪y_train_
```

1.1.1 Analytical method on train data

[328]:
```
analytical_yhat_train = np.dot(X_train_, analytical_theta)
analytical_yhat_r2 = 1 - np.sum((y_train_ - analytical_yhat_train)**2) / np.
↪sum((y_train_ - np.mean(y_train_))**2)
```

[329]:
```
sse = np.sum((y_train_ - analytical_yhat_train)**2)
tse = np.sum((y_train_ - np.mean(y_train_))**2)
print(f"sse: {sse}")
print(f"tse: {tse}")
print(f"r2: {analytical_yhat_r2}")
```

```
sse: 233.41438426728513
tse: 935.0
r2: 0.7503589473077165
```

1.1.2 Analytical method on test data

[330]:
```
X_test_[:,0] = 1.001
```

[331]:
```
analytical_yhat_test = np.dot(X_test_, analytical_theta)
analytical_yhat_r2_test = 1 - np.sum((y_test_ - analytical_yhat_test)**2) / np.
 ↪sum((y_test_ - np.mean(y_test_))**2)
```

[332]:
```
sse = np.sum((y_test_ - analytical_yhat_test)**2)
tse = np.sum((y_test_ - np.mean(y_test_))**2)
print(f"sse: {sse}")
print(f"tse: {tse}")
print(f"r2: {analytical_yhat_r2_test}")
```

```
sse: 108.09617334970144
tse: 352.78158601985206
r2: 0.6935889580596801
```

1.2 SUMMARY OF LINEAR REGRESSION USING GRADIENT DESCENT vs NORMAL EQUATIONS

- r2 of gradient descent method on training dat ~ 77% and ~80% on the test data
- r2 of analytical method on training data ~ 75% and ~69% on the test data
- the performance of gradient descent method is slightly better on training data, but noticeably higher than