Micro Summer: Problem Set 1.

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Exercise 1. Say X is finite and $\mathcal{B} \subseteq 2^X \setminus \emptyset$ does not contain some pairs and triples (i.e., there are some missing menus that of cardinality 2 and 3). Prove that there exists some choice correspondences $c : \mathcal{B} \to 2^X \setminus \emptyset$ that cannot be rationalized. (Hint: Do it by means of a counterexample).

Exercise 2. Prove that a demand function x that satisfies Walras' law, HD0 and the Compensated Law of Demand has to satisfy WARP.

Exercise 3. Consider a choice set X, and a consumer with preference defined over X, $\succeq \subseteq X \times X$, such that the preferences are a preorder (but not necessarily complete).

a) Assume in this literal that X is finite, prove that the preferences $\succeq \subseteq X \times X$ defined above, in general, cannot be represented by a utility function $u : X \mapsto \mathbb{R}$. Formally, find a counterexample to the statement: There exists a $u : X \mapsto \mathbb{R}$ such that for any $a, b \in X$, $a \succeq b \iff u(a) \geq u(b)$.

b) Assume in this literal that X is finite, prove that the preferences $\succeq \subseteq X \times X$ defined above, have a multiple utility representation, i.e., there is a set of utility functions $\mathcal{U}$ such that for any $a, b \in X$ with $a \succeq b \iff u(a) \geq u(b)$ for all $u \in \mathcal{U}$. (Hint: the set of utilities $\mathcal{U}$ can be finite. Also you could use the idea of a binary relation closure).

c) Maintain the assumptions in b). Let $C^{\succeq}(A) = \{a \in A | \text{there is no } b \in A, b \succ a\}$. Consider the data set $\{C^{\succeq}(A)\}_{A \in \mathcal{A}}$, where $\mathcal{A} \equiv 2^X \setminus \emptyset$, show (by means of a counterexample) that the data set may fail GARP.

d) Maintain the assumptions in b), and consider the data set $\{C(A)\}_{A \in \mathcal{A}}$, where $\mathcal{A} \equiv 2^X \setminus \emptyset$. Prove that the following axiom is a necessary condition for the data set $\{C(A)\}_{A \in \mathcal{A}}$ to be generated by the preferences in b) (i.e., $C(A) \equiv \{a \in A | \text{there is no } b \in A, b \succ a\}\}$.

Weak Axiom of Revealed Non-Inferiority (WARNI). For any $A \in \mathcal{A}$ and $y \in A$, if for every $x \in C(A)$ there exists a $B \in \mathcal{A}$ with $y \in C(B)$ and $x \in B$, then $y \in C(A)$.