

Problem Set 8

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Problem 1

Consider a pure-exchange economy with two kinds of goods and two consumers. Consumer 1 is endowed with one unit of good 1 and none of good 2; consumer 2 is endowed with none of good 1 and one unit of good 2. Their consumption sets are each \mathbb{R}_+^2 , and their utility functions u_1 and u_2 are defined by, for each consumption bundle $(x, y) \in \mathbb{R}_+^2$

$$u_1(x, y) := \min \left\{ x + y, \frac{1}{2}y + 1 \right\}$$

$$u_2(x, y) := xy$$

a. On a coordinate system where the horizontal axis is x and vertical axis y , graph the following indifference curves for consumer 1 and, for each indifference curve, write down the explicit coordinates of the x - and y -intercepts of the curve (if they exist) and the point(s) at which the slope of the curve changes discontinuously (if such points exist).

i. $\{(x, y) \in \mathbb{R}_+^2 : u_1(x, y) = 1\}$

ii. $\{(x, y) \in \mathbb{R}_+^2 : u_1(x, y) = 3/2\}$

iii. $\{(x, y) \in \mathbb{R}_+^2 : u_1(x, y) = 2\}$

b. For each of the following items, write down the explicit solution for this economy, in set-theoretic notations, and label the solution in a diagram with clearly marked axes, origins, coordinates and indifference maps:

i. The set of all Pareto optimal allocations

- ii. The set of all Walras equilibrium allocations (be sure to provide the supporting price vectors)
- iii. The core

Problem 2

Consider a 2-consumer 2-good exchange economy where free disposal is impossible. A consumption bundle is denoted by (x, y) , meaning x units of X and y units of Y . Everyone's consumption set is \mathbb{R}_+^2 . The endowment for consumer i is denoted e_i , given as:

$$e_1 := (4, 4), \quad e_2 := (2, 0)$$

The utility function of consumption bundle (x, y) for consumer i is denoted u_i , defined by:

$$u_1(x, y) := y - (x - 4)^2$$

$$u_2(x, y) := 2x + y$$

a. On a clearly labeled Edgeworth box, first locate the coordinate of the endowment point and label it by E , then graph the indifference curves corresponding to the following equations (and be precise about the coordinate position of each graph).

i. $u_1(x_1, y_1) = 1$ for Consumer 1

ii. $u_2(x_2, y_2) = 1$ for Consumer 2

b. For each of the following items, write down the explicit solution for this economy, in set-theoretic notations, and label the solution in a diagram with clearly marked axes, origins, coordinates and indifference maps:

- i. The set of all Pareto optimal allocations
- ii. The set of all Walras equilibrium allocations (be sure to specify the supporting price vectors)
- iii. The set of allocations that can be supported as price equilibrium with possible transfers
- iv. The core