Micro I: Problem Set 6.

Professor: Victor H. Aguiar

October 22, 2021

Exercise 1. (Cobb-Douglas Production Function) Consider the production function (Cobb-Douglas) $f(z_1, z_2) = z_1^{\alpha} z_2^{\beta}$. The output level is denoted by $q \in \mathbb{R}_+$ and $0 < \alpha, \beta$ as is sold at prices $p \in \mathbb{R}_{++}$, the prices of inputs are $w = [w_1 \quad w_2]'$.

(i) What conditions over α, β guarantees decreasing returns, constant and increasing returns to scale (respectively)?

(ii) Solve the cost minimization problem and compute the cost function.

(iii) Solve the profit maximization problem and obtain the profit function as well as the supply function when the firm has decreasing returns to scale.

(iv) (Keeping the assumptions in (iii)) Obtain the factor demand $z(w, q)$.

Exercise 2. (Profit Function) Prove that f the production set Y exhibits nondecreasing returns to scale, then either $\pi(p) \leq 0$ or (exclusive) $\pi(p) = +\infty$.