Micro Summer: Problem Set 2.

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Exercise 1. Find the Marshallian demand from the following utility functions: (i) (Leontief) $u(x_1, x_2) = min\{\alpha_1 x_1, \alpha_2 x_2\}$ and (ii) (linear) $u(x_1, x_2) = x_1 + x_2$, subject to $p_1 x_1 + p_2 x_2 = w$ for strictly positive prices and positive wealth. (Hint: you cannot use the KKT conditions for (i) because the Leontief function is non-differentiable).

Exercise 2. Consider the following problem, the consumer has a continuous utility function $u : X \mapsto \mathbb{R}$ defined over $X \subseteq \mathbb{R}^L_{++}$, his budget constraint is given by a continuous function $b : X \times \mathbb{R}^L_{++} \mapsto \mathbb{R}^L_{++}$ such that $b(x, p) \leq w$ where w is a positive amount of wealth. (i) Show that the Utility Maximization Problem with the non-linear budget constraint defined before has a solution. (ii) Describe sufficient conditions over the function b (i.e., convexity/concavity with respect to x) and over u such that the solution is unique. (iii) What is the Slutsky matrix in this case (Hint: Recall that a Slutsky demand is $x^{slutsky}(p, q) = x(p, p'q)$ in the linear budget constraint case when $b(x, p) = p'x$, assume that $b(x, p) = w$ in this part).

Exercise 3. Consider the Cobb-Douglas case $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$ for the UMP problem with linear constraint $p_1 x_1 + p_2 x_2 = w$. (i) Solve the UMP, (ii) solve the associated EMP. (iii) Compute the indirect utility, the expenditure function. (iv) Verify Roy's Identity and Sheppard's lemma. (v) Verify that $h(p, v(p, w)) = x(p, w)$ and that $w = e(p, v(p, w))$. (vi) Verify the Slutsky Equation.