Problem Set 9

Victor H. Aguiar

This version: Fall 2020

Problem 1. (30 points) Expected Completion time: 20 minutes) Consider an economy with two goods, two consumers

$u_1(x_{11}, x_{21}) = x_{11} - \frac{1}{8}(x_{21})^{-8}$ and $u_2(x_{12}, x_{22}) = -\frac{1}{8}(x_{12})^{-8} + x_{22}$ with
$\omega_1 = (2, r)$ and $\omega_2 = (r, 2)$, with $r = 2^{8/9} - 2^{1/9}$.

(a) Compute the excess demand for any of the 2 goods. (You can assume nonnegativity of the prices $p >> 0$). And obtain all equilibria analytically.

(b) Is the economy regular? Describe formally what is a regular economy and then find out whether this economy is regular.

(c) Can you intuitively describe what means that the economy is regular? What would happen if the economy is not regular if I perturb the endowments by a bit. In particular, what would happen with the multiple equilibria?

(d) Prove the following statement:

Any regular (normalized) equilibrium price vector is locally isolated (or locally unique). That is, there is an $\epsilon > 0$ such that if $p' \neq p$, $p'_2 = p_L = 1$ (where p_L, p'_L are the prices of the last good L in vector p, p' respectively), and $||p' - p|| < \epsilon$, then $z(p') \neq 0$. Moreover, if the economy is regular, then the number of normalized equilibrium price vectors is finite.

e) Does the aggregate excess demand function satisfy the gross substitutes property?

Problem 2. (40 points) Modeling the "Endowment Effect". I am going to present to you a summary of an experiment showing a behavioral bias called the Endowment Effect. The task is to write a decision model algorithm to explain the Endowment Effect.

The Endowment Effect Experiment.

Kahneman, Knetch and Thaler [1990], did an experiment with the following features:

- 44 subjects
- 22 subjects given mugs
- The other 22 subjects given nothing
- Subjects who owned mugs asked to announce the price at which they would be prepared to sell mug
- Subjects who did not own mug announced price at which they are prepared to buy mug
- Experimenter figured out prices at which supply of mugs equals demand.

If our subjects are rational consumers then the prediction in this market is:

- Prediction for rationality: As mugs are distributed randomly, we should expect half the mugs (11) to get traded.
- Explanation: Consider the group of mug lovers (i.e. those that have valuation above the median), of which there are 22.
  - Half of these should have mugs, and half should not.
  - The 11 mug haters that have mugs should trade with the 11 mug lovers that do not.

The **Experiment Outcome** however differs from the **Prediction**, and we call this the **Endowment Effect.**

- In 4 sessions, the number of trades was 4,1,2 and 2 (respectively per session).
- Median seller valued mug at $5.25
- Median buyer valued mug at $2.75
- Willingness to pay/willingness to accept gap
- **Subject's preferences seem to be affected by whether or not their reference point was owning the mug.**

1. Explain why the Endowment Effect is not consistent with a traditional model of rationality. Hint: The answer has less to do with the idea of revealed preferences, but more with the idea of stability of preferences. Write down the model of trade formally.

2. Propose a decision algorithm that explains the endowment effect given the trading scheme of the experiment.

Grading Criterion

1. Work group is not allowed, if I find out that several of you have essentially the same model, you will receive the total grade of this question divided by the number of students that share the same response.

2. Every question indicates how much points they are worth out of 100.

3. The last question will be graded on the following 5 items: Logical consistency of the decision algorithm. Clarity of the explanation of the decision algorithm. Degree of success in explaining the endowment effect. Clarity in the explanation of the endowment effect using the decision algorithm. Creativity and degree to which you have included topics that we covered in class (limited attention, reference dependence, among others).

Hint:

Consider now two time periods $t = 0, 1$, $t = 0$ means that this is before the experiments happens, and $t = 1$ is after the treatment is assigned.

The valuation of a mug for subject i at t is $v_{i,t}$.

- Assumption: Under Rationality, assume that $v_{i,t}$ is i.i.d. from some uniform distribution on $[0, 1]$, and pretty much $v_{i0} = v_{i1}$ (i.e., this means the subject is not changing her preferences).

Problem 3. (30 points) Computational Robinson Crusoe.

Robinson Crusoe is trapped in an island and he spends his days eating coconuts (c) or napping on the beach -getting Vitamin D-(d). His utility funciton is

$$U(c, d) = \alpha ln(c) + (1 - \alpha) ln(d) \tag{1}$$

with $\alpha \in [0, 1]$ a parameter. Coconuts c are measured in kg in a week so $c \in \mathbb{R}^L_+$, and $d \in [0, \overline{L}]$ is measured as the total of hours in a week Robinson spends in the beach, where $\overline{L}$ is the total amount of hours in a week.

Coconuts need to be harvested from the Island's trees. The time Robinson spends collecting coconuts from trees is measured in ours $l \in [0, \overline{L}]$. Since the island is very isolated we assume that

$$l + d = \overline{L}. \tag{2}$$

Hence, Robinson can only work or nap in the beach and the time he spends consuming the coconuts is negligible.

The productivity of Robinson is given by

$$f(l) = Al^{\beta}, \tag{3}$$

where $A > 0$ and $\beta \in (0, 1)$ are fixed parameters. We assume, that Robinson cannot consume coconuts he has not collected himself so:

$$c = f(l). \tag{4}$$

Since Robinson wants to be happy his consumption and working decision are the solution to the following problem:

Centralized problem:

$$\max_{c,l} \alpha ln(c) + (1 - \alpha) ln(\overline{L} - l)$$

$$s.t.$$

$$c = Al^{\beta}.$$

1) Let $\overline{L} = 100$. Show this centralized problem has a solution, and also show that this solution is unique.

2) Compute the close-form solution of the consumption and labor decisions of Robinson Crusoe's centralized problem.

3) Write down the equivalent descentralized general equilibrium problem associated to the centralized problem above. In other words, break down the problem of Robinson into a consumer problem and a firm problem. Show that both problems produce the same solution. Show that the problem has a solution using a fixed-point argument.

Check the notes for this. In the descentralized problem you need to find the allocations c^*, l^*, assume the price of coconuts is set to 1 ($p = 1$) and the wage for labor l is w. Profits of the firm are denoted by π.

4) Calibration. Imagine an observer (Friday) has collected the following dataset that corresponds to the behavior of the descentralized economy in 3).

variable	
c^*	60.056
l^*	6.250
w	0.961
p	1
π^*	54.01
A	50

Use this dataset to compute the implied values α and β using 3).

5) Implement the descentralized general equilibrium problem in Julia, with the calibrated α, β from 4) and solve it numerically. Verify the solutions of your sytem of equations reproduce the dataset Friday collected.

6) Now Friday, decides to impose a tax on the production of coconuts that produces revenue equal to $\tau f(l)$ for $\tau \in (0, 1)$. This tax revenue is not redistributed, as is consumed completely by Friday. What is the revenue maximizing tax rate τ?