Problem Set 5 Microeconomics

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This version: October 2020

Problem 1. (Multinomial Probit) Consider three alternatives indexed by $0, 1, 2$. Random utilities for this three alterantives are

$$U_{ij} = \mu_j + \epsilon_{ij}$$

for $j \in \{0, 1, 2\}$.

Consider the following multivariate normal distribution of preference shocks:

$$\begin{pmatrix} \epsilon_{i0} \\ \epsilon_{i1} \\ \epsilon_{i2} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} & \sigma_{02} \\ \sigma_{10} & \sigma_1^2 & \sigma_{12} \\ \sigma_{20} & \sigma_{21} & \sigma_2^2 \end{bmatrix}\right).$$

Denote the pdf of the multivariate normal by $f(\epsilon_0, \epsilon_1, \epsilon_2)$.

Let P_j be the probability that this agent with random utilities chooses j, for instance

$$P_2 = Pr(U_2 > U_1, U_2 > U_0).$$

(a) Show that under the assumption of multivariate normal shocks you obtain the following multivariate probit model of choice:

$$P_2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\epsilon_2+\mu_2-\mu_1} \int_{-\infty}^{\epsilon_2+\mu_2-\mu_0} f(\epsilon_0, \epsilon_1, \epsilon_2) d\epsilon_0 d\epsilon_1 d\epsilon_2.$$

Proof. $P_2 = Pr(U_2 > U_1, U_2 > U_0)$

$$= Pr(\mu_2 + \epsilon_2 > \mu_1 + \epsilon_1, \mu_2 + \epsilon_2 > \mu_0 + \epsilon_0)$$

$$= Pr(\epsilon_1 < \epsilon_2 + \mu_2 - \mu_1, \epsilon_0 < \epsilon_2 + \mu_2 - \mu_0)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\epsilon_2+\mu_2-\mu_1} \int_{-\infty}^{\epsilon_2+\mu_2-\mu_0} f(\epsilon_0, \epsilon_1, \epsilon_2) d\epsilon_0 d\epsilon_1 d\epsilon_2.$$

□

(b) Show that for the case of $J = 1, 2$, with no correlation among shocks, and assuming $U_1 = 0$, and variances set to 1, we have

$$P_2 = Pr(U_2 > U_1) = \Phi(\mu_2),$$

where Φ is the univariate normal CDF.

Proof.

$$P_2 = Pr(\mu_2 + \epsilon_2 > 0),$$

then

$$P_2 = Pr(\epsilon > -\mu_2),$$

since the standard normal distribution is symmetric then:

$$P_2 = \Phi(\mu_2).$$

□

(c) Show that for the case of two alternatives $J = 1, 2$, both both Probit and Logit, the following is true:

$$\mu_2 \geq \mu_1 \iff P_2 \geq P_1.$$

(d) Show that Probit may fail IIA.

Problem 2. Aggregation. (You need to refer to Chapter 4 of the MWG for some literals of this question. Read that chapter.). Consider the following random consumer. There are two commodities, $L = 2$, and prices and income will be strictly positive $p >> 0$, $w > 0$. There is a continuum of individuals and their choices will satisfy Walras' law. Consider the random consumption X_1, this is a well-behaved absolutely continuous, random variable, with finite first and second moment. The random variable X_1 has a well-behaved distribution function $F(x_1)$ defined over $x_1 \in \mathbb{R}_+$, the density function is $f(x_1) > 0$ only if $x_1 \in \mathbb{R}_+$. The distribution of random consumption variable X_1 does not depend on prices or income. Analogous to X_1, the consumption of good 2 is governed by the random variable X_2. With probability 1, for any realization of X_1, X_2, it must be that:

$$Prob(p_1 X_1 + p_2 X_2 = w) = 1.$$

The particular model we consider is one, where X_1 is realized, taking values on the interval $0 \leq x_1 \leq \frac{w}{p_1} = \boldsymbol{x}_1^{max}$. Then the realizations of X_2 are fully determined as it must be that $x_2 = \frac{w - p_1 x_1}{p_2}$.

Mean demand is given by the right-truncated mean formula:

$$x_1^m(p,w) = E[X_1 | 0 \leq X_1 \leq \boldsymbol{x}_1^{max}] = \int_0^{\boldsymbol{x_1^{max}}} x_1 \frac{f(x_1)}{F(\boldsymbol{x_1^{max}})} dx_1.$$

$$x_2^m(p,w) = \frac{w - x_1^m(p,w) p_1}{p_2}.$$

(1) Show the **mean demand function** is homogeneous of degree zero in (p, w).

Proof. Notice that $x_1^m(p,w)$ only depends on p, w through $\boldsymbol{x_1^{max}} = \frac{w}{p_1}$, then we also notice that $x_1^{max}(p,w)$ is homogeneous of degree zero in p, w, then the mean demand function is homogeneous of degree zero.

□

(2) Show that the mean demand function has a Slutsky matrix function that is NSD. (Hint: compute the Slutsky matrix for the mean demand using the Leibniz rule for derivation of integrals, and noticing that x_1^{max} is a function of (p, w). First derive with respect to x^{max} then use chain rule: $x_1^m(p,w) = \int_0^{\boldsymbol{x_1^{max}}(p,w)} x_1 \frac{f(x_1)}{F(\boldsymbol{x_1^{max}}(p,w))} dx_1$.)

Proof. $\partial_{x_1^{max}} E[X_1 | 0 \leq X_1 \leq \boldsymbol{x}_1^{max}] = x_1^{max} \frac{f(x_1^{max})}{F(\boldsymbol{x_1^{max}}(p,w))} + \int_0^{x_1^{max}(p,w)} x_1 (-\frac{f(x_1^{max})}{F(x_1^{max})} \frac{f(x_1)}{F(x_1^{max})}) =$

$$\frac{f(x_1^{max})}{F(\boldsymbol{x_1^{max}}(p,w))}(x_1^{max} - x_1^m(p,w)) > 0$$

$\partial_{p_1} x_1^{max}(p,w) = -\frac{w}{p_1^2}$, $\partial_w x_1^{max}(p,w) x_1^m(p,w) = \frac{x_1^m(p,w)}{p_1}$
$\partial_{p_1} x_1^{max}(p,w) - \partial_w x_1^{max}(p,w) x_1^m(p,w) = -\frac{x_1^{max} - x_1^m}{p_1} < 0.$

Finally, notice that:

$\partial_{p_1} x_1^m(p,w) = \partial_{x_1} E[X_1 | 0 \leq X_1 \leq \boldsymbol{x}_1^{max}] \frac{\partial x_1^{max}}{\partial p_1}$
$\partial_w x_1^m(p,w) x_1^m(p,w) = \partial_{x_1} E[X_1 | 0 \leq X_1 \leq \boldsymbol{x}_1^{max}] \frac{\partial x_1^{max}}{w} x_1^m(p,w).$

This implies:

$S(p,w)_{11} = \partial_{x_1^{max}} E[X_1|0 \leq X_1 \leq \boldsymbol{x}_1^{max}](\partial_{p_1} x_1^{max}(p,w) - \partial_w x_1^{max}(p,w) x_1^m(p,w)) < 0$, because of the signs of each factor $(+)(-)$.

By homogeneity of degree zero and two goods, this is enough to conclude that $S(p,w)$ is NSD.

□

(3) Show that the Slutsky matrix function of the mean demand function is symmetric.

Proof. Trivial from HD0.

□

(4) Does this mean demand function admit a positive representative consumer representation?

Proof. Yes, follows from 1-3.

□

5. Does this mean demand function admit a normative representative consumer representation?

Proof. No.

□

Problem 3. (Too much choice/Choice overload)

- Propose a model for consumer behavior that captures the following stylized fact. Consumer are more likely to choose an outside option (that we equate in this problem to not choosing) when the size of the menu is too big. Hint: The choice set is $X \cup \{o\}$ where o is an outside option (not in X). Let X be finite. Menus are of the form $A \cup \{o\}$ where $A \subseteq X$, i.e., the outside option is always available to the consumer.

- You should describe with words what is the consumer choice algorithm, then you should describe what is "new" in your model and how that relates to some behavioral or bounded rationality feature we learned in class, finally you should write the behavioral maximization problem and show how your new model predicts the fact that when the menu size is large the consumer chooses the outside option.

- You should try to explain the model to me in the best way you can, by using words, diagrams and examples. However, you should have some formality, make the effort to write down the model using math, this is the only way to get full credit.

1 Hint: What is Choice Overload?

1.1 Choice Environment

We let X be a finite choice set, and o and outside option (that we equate in this case with not choosing). The main feature of $o \notin X$.

Every menu that the consumer will face will be of the form $A \cup \{o\}$ where $A \subseteq X$, when $A \neq \emptyset$. The collection of this menus is going to be called $\mathcal{D}$.

1.2 Dataset of primitive

We consider a consumer that produces a choice correspondence:

$$c(A \cup \{o\}) \subseteq A \cup \{o\}.$$

We force the choice to be non-empty, $c(A \cup \{o\}) \neq \emptyset$.

We imagine that the consumer was in an experiment where she chose from every possible menu and generated the dataset:

$$(c(A \cup \{o\})_{A \in \mathcal{D}}.$$

1.3 Choice overload

Let $|A| = n$ the cardinality of a set. We consider two sets, $A \subseteq B$, so $|B| \geq |A|$. We can say that B is a big menu and A is a small menu.

Now, choice overload means that:

$$(i) \quad o \notin c(A \cup \{o\})$$

$$(ii) \quad o \in c(B \cup \{o\}).$$

1.4 Choice Overload with Probabilities

You may instead choose a different environment, in this case, we keep the choice set-up the same but we modify the data or primitive:

$$p(a, A \cup \{o\}) \in [0, 1]$$

$\sum_{a\in A} p(a, A \cup \{o\}) + p(o, A \cup \{o\}) = 1.$

Now, choice overload is:

$$(i) \quad p(o, B) > p(o, A)$$

$$(ii) \quad A \subset B, |A| < |B|.$$

1. You could use the attention filter, this will work.
2. Luce model with attention filter.