Advanced Micro Winter 2020: Problem Set 3

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Exercise 1. Solve the Sparse max consumer problem with respect to quantities $smax_{x|p^s}$ for the following $x \in B(p,w) = \{x \in X | p'x = w\}$, with p^s the perceived prices.

(a) Quasilinear: $u(x_1, x_2) = x_1^{1/2} + x_2$ (consider only the unique interior solutions)

(b) Constant Elasticity of Substitution (CES): $u(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)$.

Solve the Sparse max consumer problem with respect to expenditure/budget-share $smax_{\omega|p^s}$ where $\omega_l = p_l q_l$ for $x \in B(p,w) = \{x \in X | \sum_{l=1}^{L} \omega_l = w\}$, with p^s the perceived prices.

(c) Quasilinear: $u(x_1, x_2) = x_1^{1/2} + x_2$ (consider only interior solutions).

(d) Constant Elasticity of Substitution (CES): $u(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)$.

(a) Step 1: solve

$\mathcal{L}^s = x_1^{1/2} + x_2 + \lambda^s(w - p_1^s x_1 - p_2^s x_2)$

FOC:

$\frac{1}{2} x_1^{-\frac{1}{2}} = \lambda^s p_1^s$

$x_1 = (2\lambda^s p_1^s)^{-2}$

$1 = \lambda^s p_2^s$

This gives:

$x_1 = (2\frac{p_1^s}{p_2^s})^{-2}$

Step 2: Compute x_2

$x_1 = (2\frac{p_1^s}{p_2^s})^{-2}$ and by the budget constraint $x_2 = w/p_2 - p_1 x_1 / p_2$

$x_2 = \frac{w}{p_2} - \frac{p_1 (2\frac{p_1^s}{p_2^s})^{-2}}{p_2}$.

(b)

From problem set 2, we have that the rational CES demand is

$x_1^r(p,w) = w(p_1 + p_1^\sigma p_2^{1-\sigma})^{-1} = \frac{w}{p_1^\sigma (p_1^{1-\sigma} + p_2^{1-\sigma})}$

$x_2^r(p,w) = \left(\frac{p_1}{p_2}\right)^\sigma x_1 = \left(\frac{p_1}{p_2}\right)^\sigma \frac{w}{p_1^\sigma (p_1^{1-\sigma} + p_2^{1-\sigma})} = \frac{w}{p_2^\sigma (p_1^{1-\sigma} + p_2^{1-\sigma})}$.

Using the notes, and the fact that the CES demand is linear in wealth we know that

$x(p,w)=\frac{x^r(p^s,w)}{p'x^r(p^s,1)}$ where $p'x^rp(p^s,1)=\frac{p_1}{p_1^{s,\sigma}(p_1^{s,1-\sigma}+p_2^{s,1-\sigma})}+\frac{p_2}{p_2^{s,\sigma}(p_1^{s,1-\sigma}+p_2^{s,1-\sigma})}=\frac{p_1/p_1^{s,\sigma}+p_2/p_2^{s,\sigma}}{(p_1^{s,1-\sigma}+p_2^{s,1-\sigma})}$

and $x_i^r(p^s,w)=\frac{w}{p_i^{s,\sigma}(p_1^{s,1-\sigma}+p_2^{s,1-\sigma})}$

$x(p,w)=\frac{w}{p_i^{s,\sigma}}\frac{1}{(p_1/p_1^{s,\sigma}+p_2/p_2^{s,\sigma})}$.

(c) Step 1:

$\mathcal{L}^s=(\omega_1/p_1^s)^{1/2}+(\omega_2/p_2^s)+\lambda^s(w-\omega_1-\omega_2)$

FOC:

$\frac{1}{2}(x_1)^{-1/2}\frac{1}{p_1^s}=\lambda^s$, $\frac{1}{2}(x_1)^{-1/2}=\lambda^sp_1^s$

$\frac{1}{p_2^s}=\lambda^s$

So we have $x_1=(2\frac{p_1^s}{p_2^s})^{-2}=\frac{p_2^{s,2}}{4p_1^{s,2}}$ and the solution is $\omega_1=\frac{p_2^{s,2}}{4p_1^s}$

Step 2:

$\omega_2=w-\omega_1=w-\frac{p_2^{s,2}}{4p_1^s}$

(d) We know by the notes that $\omega_l=p_lx(p,w)=p_l^sx_l^r(p^s,w)$, so we use problem set 2 to compute

$p_l^sx_l^r(p^s,w)=\frac{p_l^{s,1-\sigma}w}{(p_1^{s,1-\sigma}+p_2^{s,1-\sigma})}$ for $l=1,2$.

We check that it satisfies the constraint.

$\frac{p_1^{s,1-\sigma}w}{(p_1^{s,1-\sigma}+p_2^{s,1-\sigma})}+\frac{p_2^{s,1-\sigma}w}{(p_1^{s,1-\sigma}+p_2^{s,1-\sigma})}=w.$

Exercise 2. Download the replication package of Aguiar Kashaev (2020, Restud) from .

It contains the dataset from:

- Ahn, D., Choi, S., Gale, D., & Kariv, S. (2014). Estimating ambiguity aversion in a portfolio choice experiment. Quantitative Economics, 5(2), 195-223. [Replication files] <http://qeconomics.org/supp/243/code

(i) Use the file ReplicationAK/Data_all/rationalitydata3goods.csv to test GARP for each individual in the experiment. Provide the Julia code name_lastname_ps3.jl and the results in written form in your submission. Hint: you can use the file /ReplicationAK/SecondApp/Deterministic_test/2App_dt.jl in Julia 1.0 or 1.1, or if you use another version of Julia you can use it as a basis for your code.

(ii) Compute the Afriat's efficiency index for the same data set. Provide the Julia code name_lastname_ps3.jl and the results in written form in your submission, for this part you can mimic the figure we saw in class (without the randomly generated dotted curve).

Exercise 3. Zero price effect. You read, for problem set 2, the following chapter from Predictably Irrational from Dan Ariely:

http://christophe.heintz.free.fr/bgt/Ariely-Predicably-Irrational-Ch3.pdf

from pages 55-60.

Definition. Zero Price Effect. The zero price effect is an experimental finding that shows that even when relative price differences are the same among two goods, choices change significantly when one of the prices is zero. In particular, consumers may choose one good when both prices are positive, to a free good while keeping the price difference the same.

More formally, let there be two items $a, b \in X$, where a is a Lindt truffle and b is a Hershey Kiss.

- In the first experiment, the price of a, was $p_a^1 = 15$ cents per unit and $p_b^1 = 1$ cent per unit. In that case most consumers chose a over b.
- Then in the second experiment the prices where changed, keeping the price difference constant, to $p_a^2 = 14$ and $p_b^2 = 0$. In that case, most consumer chose b over a.

Incompatibility of the Zero Price Effect with Rationality

We can use the tools of revealed preferences to answer the question of whether this experimental findings are consistent with rationality.

- In particular, note that in experiment 1, given that a subject bought a at $p_a^1 = 15$, she could have buy b at $p_b^1 = 1$. That means that a is revealed preferred to b strictly.
- In experiment 2, given the experiment 1, the subject can afford both the a and b, because both are cheaper, moreover the price difference is the same as in experiment 1, however b is chosen over a.

This means that the subject cannot be rational. We can add a bit more detail to the model, consider $u(a) = g(a) - c(a)$ where the utility of a is the benefit of a versus the cost of a, for simplicity let $c(a) = p_a$. In experiment 1, we get that $u(a) = g(a) - 15$ and $u(b) = g(b) - 1$, then when the subject chooses a over b, we conclude that $u(a) > u(b)$ which implies $g(a) - g(b) \geq 14$.

Then in experiment 2, we get that $u(a) = g(a) - 14$ and $u(b) = g(b) - 0$, notice that the main idea behind a rational consumer is that preferences remain stable, so the benefits are the same.

Then it must be that $u(a) > u(b)$ because $g(a) - g(b) \geq 14$ by the first experiment.

Your Task:

You are required to provide a **decision algorithm** that accommodates/explains the zero price effect described above.

- You should use approximately 1 page for this.

- Describe the choice algorith in words, or diagrams, explaining step-by-step how the consumer will choose, when given two alternatives $\{a, b\}$ and two prices p_a, p_b (same environment that in the experiment). (If you feel comfortable, **also**, describing your model in mathematical terms, this is OK with me).
- Use your decision algorithm to accomodate and explain the zero price effect. Explain in one paragraph, what is the key feature of your model, that is different from the standard rational model that allows you to explain the zero price effect.

We consider a model of inattention to prices $p_j^s = p_j^m (p_j^d)^{1-m}$, we let $p_j^d = 1$ and we have the following, for the first experiment

$$g(a) - 15^m > g(b) - 1^m$$

From the second experiment:

$$g(a) - 14^m < g(b) - 0^m$$

Putting things together:

$$g(a) - 15^m + 1^m > g(b) > g(a) - 14^m$$

$$15^m - 1^m < 14^m$$

It turns out that for $m < 1$ the previous inequality works, let $m = \frac{1}{2}$ and we verify that:

$$g(a) = 3.3, g(b) = 0,$$

verify the inequalities from the experiment.

Grading Criterion

1. Work group is not allowed, if I find out that several of you have essentially the same model, you will receive the total grade of this question divided by the number of students that share the same response.
2. This question will be graded on the following 5 items: Logical consistency of the decision algorithm. Clarity of the explanation of the decision algorithm. Degree of success in explaining the zero price effect. Clarity in the explanation of the zero price effect using

the decision algorithm. Creativity and degree to which you have included topics that we covered in class (limited attention).