Problem Set 4 Microeconomics

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Problem 1. (Expected Utility) Consider the following "portfolio choice" problem. The consumer (with preferences that admit an expected utility representation) has wealth w and a Bernoulli utility $u(x) = ln(x)$ there is a safe asset that has net real return of zero. There is also a risky asset with a random net return that has only two possible returns R_1 with probability q and R_0 with probability $1 - q$.

Let A be the amount invested in the risky asset so $w - A$ is the amount invested in the safe asset.

(i) Find A as a function of w. Does the consumer buys more or less of the risky asset as w increases?

(ii) A second (expected utility) consumer has a Bernoulli utility function $u(x) = -exp(-x)$. How does his consumption of the risky asset changes with wealth?

(iii) Compute the coefficient of absolute risk aversion $r(x) = -\frac{u''(x)}{u'(x)}$ for the first and second consumers. How these coefficients depend on wealth? How does this coefficient explains the answers in (i) and (ii).

Problem 2. Non-expected utility theory. (Modeling)

From class we had the Allais paradox behavior:

Z is finite in fact $N = 3$,

First Prize	Second Price	Third Price
2.5 millions	0.5 millions	0

Two choice sets:

The first,

$L_1 = (0, 1, 0)$ and $L_1' = (0.10, 0.89, 0.01)$.

The second,

$L_2 = (0, 0.11, 0.89)$ and $L_2' = (0.10, 0, 0.90)$.

Imagine an individual that choose $L_1 \succ L_1'$ and $L_2' \succ L_2$.

This is inconsistent with expected utility.

If $L_1 \succ L_1'$ then

$$u_{0.5} > (0.10)u_{2.5} + (0.89)u_{.5} + (0.01)u_0.$$

Adding, $(0.89)u_0 - (0.89)u_{0.5}$ to both sides we get,

$$(0.11)u_{0.5} + (0.89)u_0 > (0.1)u_{2.5} + (0.9)u_0,$$

and thus $L_2 \succ L_2'$.

Now consider a model with weighted probabilities.

$$V(p) = \sum_{z \in Z} \phi(p(z))u(z),$$

where $\phi(0) = 0$, $\phi(1) = 1$ and $\phi(t) \geq 0$. Find a ϕ that explains the Allais paradox and show it can explain the choices $L_1 \succ L_1'$ and $L_2' \succ L_2$ assuming that $u(z) = z$ for simplicity.

Does your new model satisfies first order stochastic dominance monotonicity, $p \geq_{FOSD} q \implies V(p) \geq V(q)$?

Advanced question: How can you modify the probability weighting model to satisfy FOSD. (Optional for 10% to the Midterm).