Micro I: Problem Set 6.

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Exercise 1. (Cobb-Douglas Production Function) Consider the production function (Cobb-Douglas) $f(z_1, z_2) = z_1^{\alpha} z_2^{\beta}$. The output level is denoted by $q \in \mathbb{R}_+$ and $0 < \alpha, \beta$ as is sold at prices $p \in \mathbb{R}_{++}$, the prices of inputs are $w = [w_1 \quad w_2]'$.

(i) What conditions over α, β guarantees decreasing returns, constant and increasing returns to scale (respectively)?

(ii) Solve the cost minimization problem and compute the cost function.

(iii) Solve the profit maximization problem and obtain the profit function as well as the supply function when the firm has decreasing returns to scale.

(iv) (Keeping the assumptions in (iii)) Obtain the factor demand $z(w, q)$.

Exercise 2. (Profit Function) Prove that f the production set Y exhibits nondecreasing returns to scale, then either $\pi(p) \leq 0$ or (exclusive) $\pi(p) = +\infty$.

Exercise 3. (Cost minimization problem). The **cost minimization problem:**

$$Min_{z \geq 0} w'z$$

$$s.t \quad f(z) \geq q.$$

And $c(w, q) = min_{z \geq 0, f(z) \geq q} w'z$ is the cost function and $z(w, q) = argmin_{z \geq 0, f(z) \geq q} w'z$. Assume that $f(z)$ is strictly concave. Prove the following proposition.

Proposition 1. *The properties of the $c(w, q)$ and the $z(w, q)$ are,*

(i) c is HD1 in w and nondecreasing in q.

(ii) c is a concave function.

(iii) z is HD0 in w

(iv) Sheppard's lemma. If $z(\overline{w}, q)$ consists of a single point then $c(\cdot)$ is differentiable with respect to w at $\overline{w}$ and $\nabla_w c(\overline{w}, q) = z(\overline{w}, q)$.

(v) If z is differentiable at $\overline{w}$, then $D_w z(\overline{w}, p) = D_w^2 c(\overline{w}, q)$ is symmetric and NSD with $D_w z(\overline{w}, q)\overline{w} = 0$.

(vi) $c(w, q)$ is a convex function of q (in particular, marginal costs are nondecreasing in q) (Hint: recall f is strictly concave).