Micro I: Problem Set 6.

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November 17, 2020

Exercise 1. Consider a continuum of agents $u(x,m) = \phi(x) + m = A^{\frac{1}{\alpha}} \frac{\alpha}{(\alpha-1)} x^{1-\frac{1}{\alpha}} + m$ with A a constant $A \in (0,1]$ where the endowment of numeraire is denoted by ω_m and is distributed with $\omega_m \sim F_m(\cdot)$ and $\alpha \sim U[1,2]$ is distributed uniformly. As usual, p is the price of the consumption good denoted by x and the price of the numeraire is normalized to 1.

We also have a continuum of firms each of them with the cost function $c(q) = \frac{1}{1+\frac{1}{\beta}} q^{1+\frac{1}{\beta}}$ with $\beta \sim U[1,2]$, that denotes the amount of consumption good produced with numeraire. It is a private ownership economy so each consumer indexed by (α, ω_m) has a share over all firms indexed by β. Formally, the share for any firm are $1 = \int \theta(\alpha, \omega_m, \beta) dF(\alpha, \omega_m)$ with $F(\alpha, \omega_m)$ the joint distribution over (α, ω_m).

(Hint: Market Clearance has to hold only in expectation and you can assume interior solutions and existence of the equilibrium).

(1) Prove that the effect of the numeraire distribution $\omega_m \sim F_m(\cdot)$ on the equilibrium of the consumption good is null.

(2) Solve the partial equilibrium in a closed form. Find the individual demands $x(\alpha, p^*)$, the individual supply $q(\alpha, p^*)$ and the equilibrium price p^*.

Assume that the endowment of the numeraire is constant $\omega_m > 0$ and positive.

(3) What is the average numeraire demand? (You can write just the formula, for general distributions, no need to integrate).

Proof. (i) By consumer utility maximization

$x(\alpha, p) = Ap^{-\alpha}$

(ii) By firms profit maximizing

$q(\beta, p) = p^{\beta}$

(iii) Market cleareance means

$\frac{A}{plogp} - \frac{A}{p^2 logp} = -\frac{p}{log(p)} + \frac{p^2}{log(p)}$

Then the only real solution is $p^* = A^{1/3}$, that for $A \in (0,1]$ gives $p \in (0,1]$.

Then the individual demands and supplies are:

$x(\alpha, p^*) = A^{1-\frac{\alpha}{3}}$

$q(\beta, p^*) = A^{\beta/3}$.

□

(iv)

Proof. $E[m] = \frac{3(A^{1/9} - A^{1/3})}{log(A)} - E[\omega_m] - \int \int \theta(\alpha, \omega_m, \beta)[\pi(\beta, p)] dF(\alpha, \omega_m) dF(\beta)$

□

Exercise 2. Estimation of Demand in a Partial Equilibrium Situation.

We consider a sequence of markets, in each market consumers and firms maximize static utility and static profits (agents are myopic so there is no dynamics).

Consider the following aggregate demand of good 1, (good 2 is a numeraire) for market in time t:

$$x_t^d = \alpha_o + \alpha_1 p_t + u_t,$$

α_o, α_1 are deterministic parameters but u_t is realization of a random variable $\boldsymbol{u}_t$ representing exogenous shocks to preferences per-time period.

Consider the following aggregate supply function of good 1,

$$x_t^s = \beta_o + \beta_1 p_t + v_t$$

β_o, β_1 are deterministic parameter but v_t is a realization of a random variable $\boldsymbol{v}_t$ that are shocks to productivity.

The market clearing conditions per realization of shocks are:

$$x_t^s = x_t^d.$$

(1) For a realization of (u_t, v_t) find the equilibrium quantity and the equilibrium price of a given economy time t.

Proof. $p_t^* = \frac{\beta_0-\alpha_0}{\alpha_1-\beta_1} + \frac{v_t-u_t}{\alpha_1-\beta_1}$

$x_t^* = \frac{\alpha_1\beta_0-\alpha_0\beta_1}{\alpha_1-\beta_1} + \frac{\alpha_1 v_t-\beta_1 u_t}{\alpha_1-\beta_1}$. □

Notice that, under the assumption that the random variables $\boldsymbol{u}_t, \boldsymbol{v}_t$ is i.i.d. in time, observed prices and quantities are also random with realizations of prices and quantities in equilibrium p_t^* and x_t^* for the given realizations of the u_t, v_t.

(2) Assume that an statistician that does not know microeconomics wants to find the demand elasticity with respect to prices using this data. He writes the following model:

$$x_t^* = \gamma_0 + \gamma_1 p_t^* + \epsilon_t,$$

Then he computes the OLS estimator of the price parameter: $\gamma_1^{ols} = \frac{cov(\boldsymbol{p}_t^*,\boldsymbol{x}_t^*)}{var(\boldsymbol{p}_t^*)}$. Show that this statistician will get the following quantity potentially very different from the price demand elasticity α_1: (Hint: assume $cov(\boldsymbol{u}_t, \boldsymbol{v}_t) = 0$).

$$\gamma_1^{ols} = \frac{\alpha_1 var(\boldsymbol{v}_t) + \beta_1 var(\boldsymbol{u}_t)}{var(\boldsymbol{v}_t) + var(\boldsymbol{u}_t)}.$$

Proof. Using the demand equation

$cov(\boldsymbol{p}_t, \boldsymbol{x}_t) = \alpha_1 var(\boldsymbol{p}_t) + cov(\boldsymbol{p_t}, \boldsymbol{u_t})$

Using the supply equation

$cov(\boldsymbol{p_t}, \boldsymbol{x_t}) = \beta_1 var(\boldsymbol{p}_t) + cov(\boldsymbol{p}_t, \boldsymbol{v_t})$

Now using the price equation

$p_t^* = \frac{\beta_0-\alpha_0}{\alpha_1-\beta_1} + \frac{v_t-u_t}{\alpha_1-\beta_1}$

the covariance

$cov(\boldsymbol{p}_t, \boldsymbol{u_t}) = -\frac{var(\boldsymbol{u}_t)}{\alpha_1-\beta_1}$

$cov(p_t, v_t) = \frac{var(v_t)}{\alpha_1-\beta_1}$

$cov(p_t, x_t) = \frac{1}{(\alpha_1-\beta_1)^2}(\alpha_1 var(\boldsymbol{v}_t) + \beta_1 var(\boldsymbol{u}_t))$

$var(p_t) = \frac{1}{(\alpha_1-\beta_1)^2}[var(v_t) + var(u_t) - 2cov(v_t, u_t)] =$

$\frac{1}{(\alpha_1-\beta_1)^2}[var(v_t) + var(u_t)]$

$\gamma^{ols} = \frac{\alpha_1 var(\boldsymbol{v}_t)+\beta_1 var(\boldsymbol{u}_t)}{var(\boldsymbol{v}_t)+var(\boldsymbol{u}_t)}$ □

(3) What condition over unobservables $\boldsymbol{u}_t$ and $\boldsymbol{v}_t$ would you have to impose for the statistician to miraculously get that:

$$\gamma^{ols} = \alpha_1.$$

Proof. Assume $\boldsymbol{u}_t = 0$ almost surely.

$\gamma^{ols} = \alpha_1$. □

(4) Now an econometrician that know microeconomics comes to the help of the statistician and notices the following, we cannot get demand price elasticities from market data because these quantities and prices are the result of the intersection of supply and demand. In order to estimate the demand function we need a demand shifter! Provide an example of a demand shifter z_t^d that satisfies the following $cov(p_t, z_t^d) \neq 0$ and $cov(u_t z_t^d) = 0$.

Show that the structural estimator is a consistent estimator of the elasticity of interest α_1:

$$\alpha_{IV} = \frac{cov(x_t, z_t^d)}{cov(p_t, z_t^d)} = \alpha_1.$$

Proof. We have $cov(z_t^d, x_t) = \alpha_1 cov(z_t^d, p_t) + cov(z_t^d, u_t)$ from the equation of demand, under the assumptions we solve for α_1 and this is well-defined. $\square$