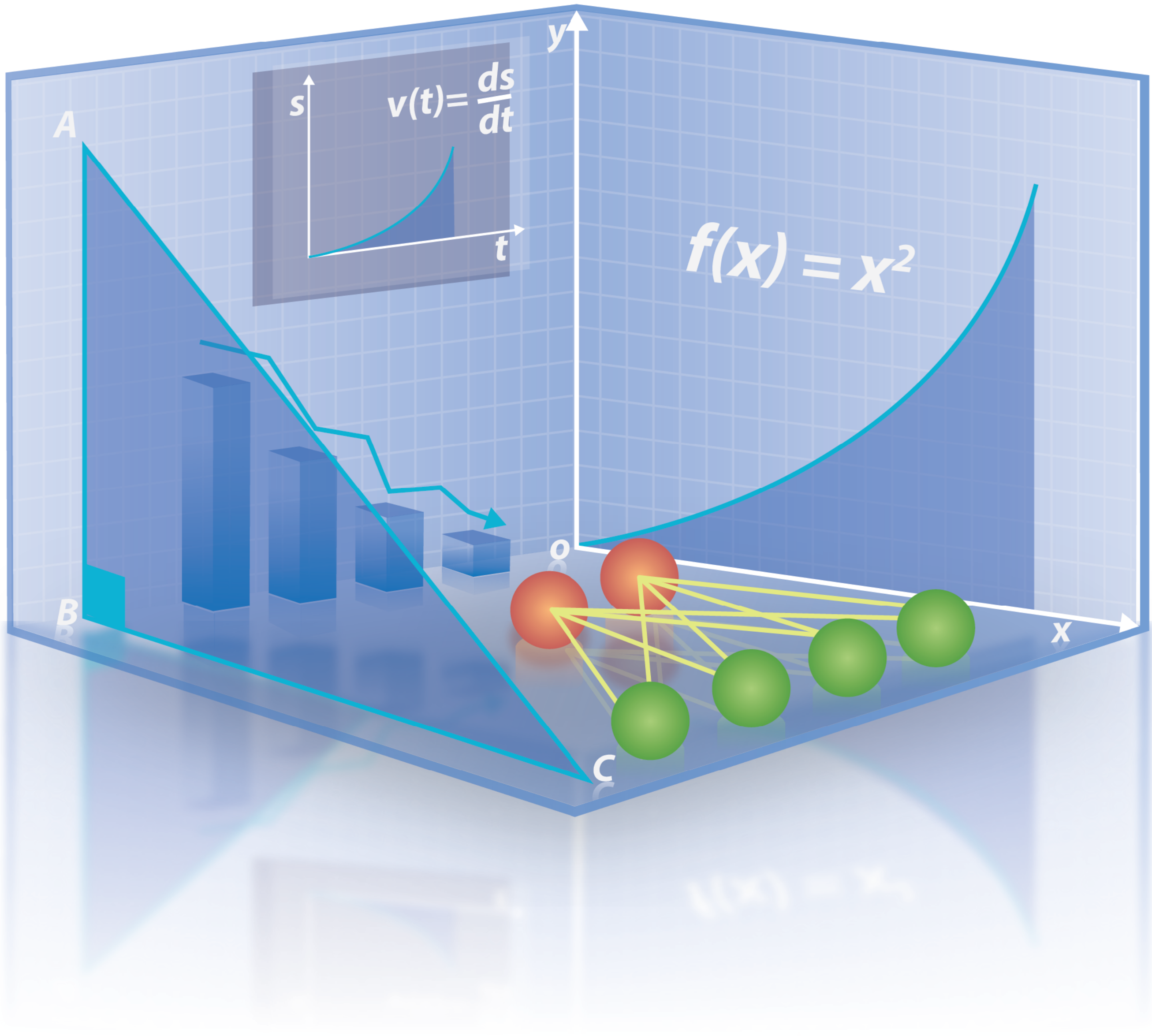
Vedantu's Tatva Practice Book

Mathematics

Volume 2

Class 11 JEE



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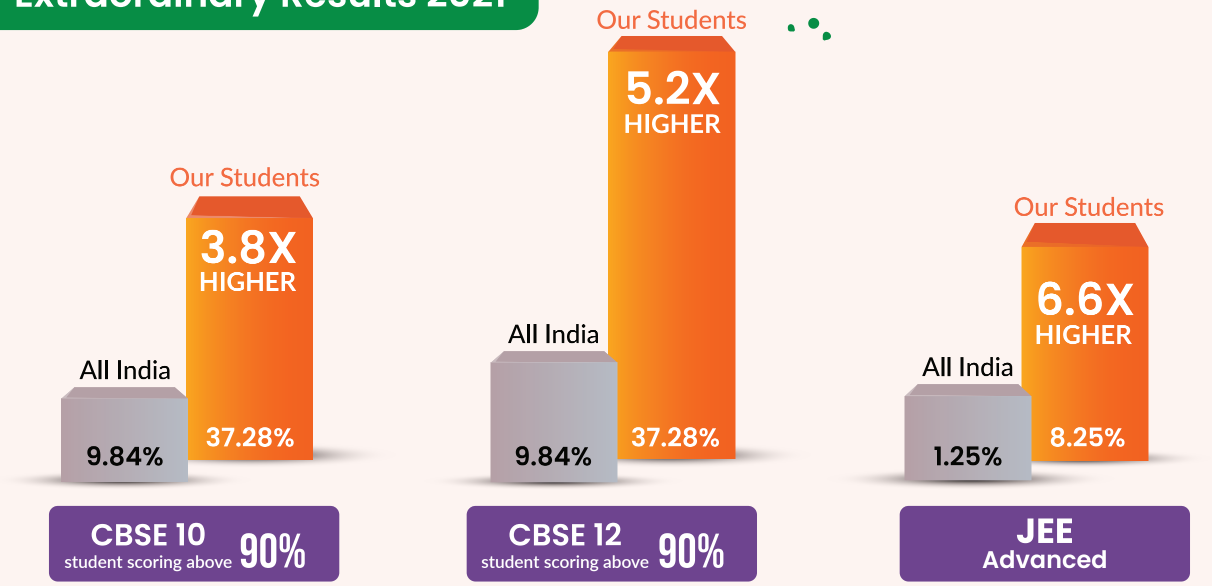

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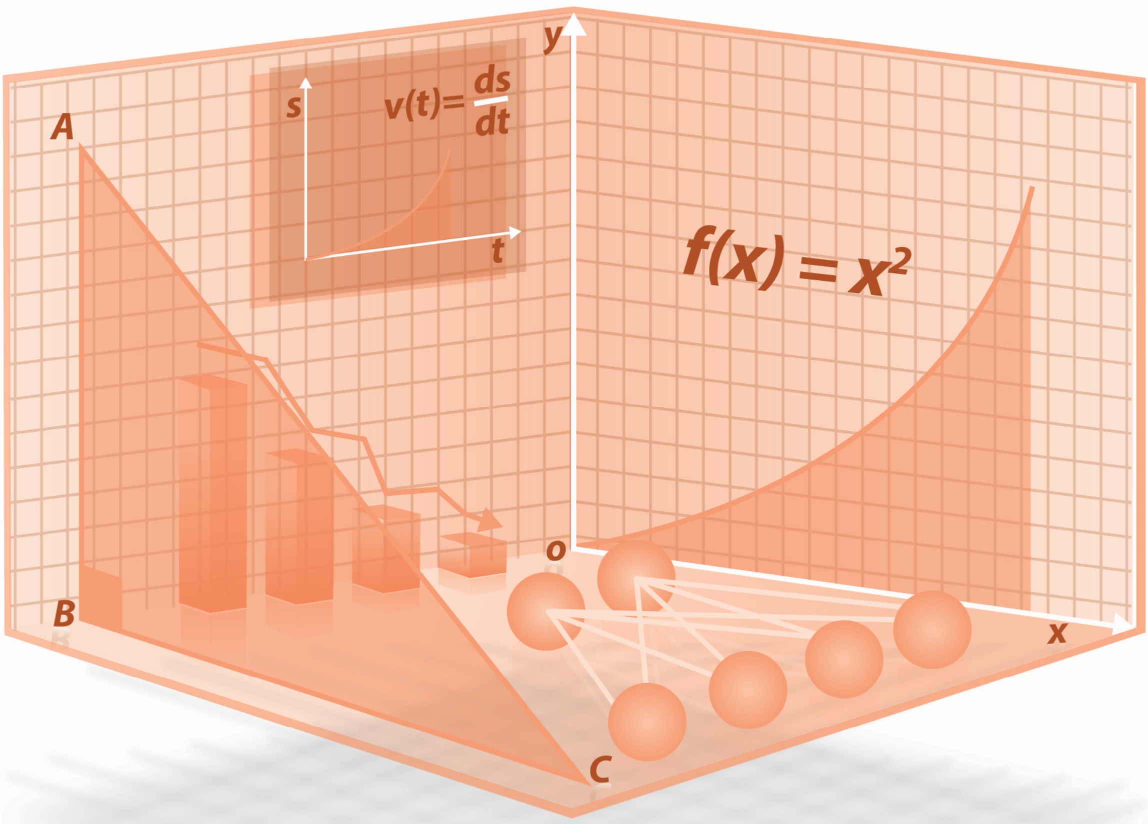

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How to use your Tatva Practice Book

4. Fundamental Principle of Counting :
If an event can occur in 'm' different ways following which another event can occur in 'n' different ways following which another event can occur in 'p' different ways then

SCAN CODE
Permutations

1

Scan the QR Code in each chapter's theory section to view micro concept videos related to the chapter, on the Vedantu app.

Exercise - 2:

1. If every element of a third order determinant of value Δ is multiplied by 5, then the value of new determinant is: **(JEE 2019)**

(a) Δ (b) 5Δ
(c) 25Δ (d) 125Δ

2

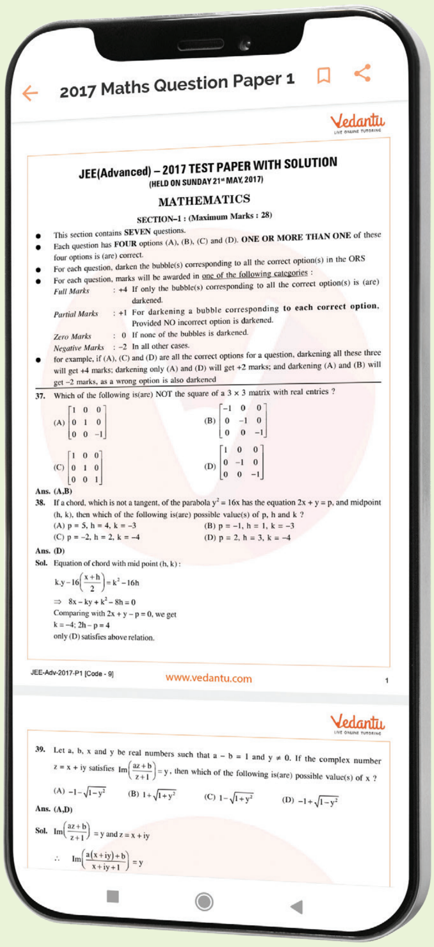
Solve all types of exercise questions based on the latest JEE pattern.

Answer Key

CHAPTER-1: DETERMINANTS
Exercise-1: Basic Objective

3

Scan the QR code in the Answers Section to view detailed solutions for all exercise questions.



For extra exam preparation content, visit the Vedantu app. You can find previous years' JEE papers with solutions and detailed analysis.

Founder's Message

Dear Student,

I am delighted to present to you a Ready Reckoner and an amazing book to guide you for your exams-'TATVA'. Tatva—which means the 'Core' is fully aligned with the culture, the mission, and the vision of Vedantu and therefore it gives me immense pleasure and joy to share this book with you. We at Vedantu have always believed in revolutionizing the teaching and learning process and always speedily progressed in the direction of bringing superior quality education and content to your table. Tatva is a step forward in this direction. This book is your guide, your practice guru, and your companion in moving towards your dreams. The book is a result of the consistent effort, diligence, and research by our experienced team of subject experts and teachers.

This book has been customized with curated content to suit the needs of JEE aspirants like you and guide you on the right path to cracking JEE and optimizing your efficiency. Tatva is a comprehensive amalgamation of important concepts, theories, derivations, definitions, solved examples, concept videos, practice questions, and important questions. We have ensured that high-quality content and the right ingredients are in place in this booklet to help you climb up the success ladder.

A few guiding points to optimally use Tatva with a planned approach:

Tatva equips you with Theory, Concept Videos, and Solved examples to help you revise concepts, mark your notes, walk you through the entire summary, and eventually makes you capable of clearing all your conceptual doubts all by yourself.

We suggest revision of theory followed by practice of solved examples.

Practice relevant questions daily after finishing Vedantu lectures and session assignments. We believe that a daily dose of Tatva will keep all your exam blues at bay.

Use the Tatva booklet to mark notes so that it always comes in handy for last-minute revision sessions before your exams. Notes should include key points of theory, solved examples, and some questions which you couldn't solve in the first attempt.

Exercise 1 and Exercise 2 of JEE Tatva deal with basic questions and those which can be asked or already asked in JEE Main. Similarly, Exercise 3 and Exercise 4 deal with JEE Advanced level questions. We recommend you solve basic JEE Main questions before moving to JEE Advanced level questions.

Before wrapping up, the practice mantra: "Don't practice until you get it right. Practice until you can't get it wrong."

We strongly believe in you and your capabilities. So believe in yourself because success is only one step away. Wishing that your talent shines bright. All the very best!

Anand Prakash
Founder and Academic Head, Vedantu

Anand Prakash Sir has been a pioneer in producing Top Ranks in JEE/NEET and Olympiads. He has personally taught and mentored AIR 1, 6, 7 (JEE Advanced), AIR-1, 7, 9(AIIMS), and thousands of more students who have successfully cleared these competitive exams in the last few years.

Credits

"Happiness lies in the joy of achievement and the thrill of creative effort."

—Franklin D. Roosevelt

Tatva is the brainchild of a group of creative Vedans who have strived tirelessly to weave success stories for you. We extend our heartfelt gratitude to the superb team of Vedans who give wings to the vision of Vedantu, starting with our leaders who have been guiding and encouraging us at every step of the way:
Vamsi Krishna Sir, Anand Prakash Sir and Pulkit Jain Sir

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Mathematics Team

We truly appreciate all the Master Teachers of Vedantu whose relentless efforts helped us translate this vision into reality. Our heartfelt gratitude to our creative content developers and the typesetting team, who have put in their hard work, insight, and eagerness to nurture and execute Tatva into 'your ready handbook' and bring a positive learning experience to you.

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The journey of bringing Tatva to life, from an idea to the book you are holding, would not have been possible without the extensive support of our diligent Operations Team, our amazing Academic Team, our dedicated team of Teachers, and our talented Tech Team.

TABLE OF CONTENTS

SETS, RELATIONS & FUNCTION

LIMITS AND DERIVATIVES

TRIGONOMETRY

STATISTICS

MATHEMATICAL REASONING

05

SETS, RELATIONS & FUNCTION

SETS, RELATIONS & FUNCTION

SETS

1. SET

A set is a collection of well-defined and well distinguished objects.

1.1 Notations

The sets are usually denoted by capital letters A, B, C, etc. and the members or elements of the set are denoted by lower-case letters a, b, c, etc. If x is a member of the set A, we write $x \in A$ (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin A$ (read as 'x does not belong to A,). If x and y both belong to A, we write $x, y \in A$.

2. REPRESENTATION OF A SET

Usually, sets are represented in the following two ways :

(i) Roster form or Tabular form

(ii) Set Builder form or Rule Method

2.1 Roster Form

In this form, we list all the member of the set within braces (curly brackets) and separate these by commas. For example, the set A of all odd natural numbers less that 10 in the Roster form is written as :

$A = \{1, 3, 5, 7, 9\}$

NOTES :

(i) In roster form, every element of the set is listed only once.

(ii) The order in which the elements are listed is immaterial.

For example, each of the following sets denotes the same set $\{1, 2, 3\}$, $\{3, 2, 1\}$, $\{1, 3, 2\}$

2.2 Set-Builder Form

In this form, we write a variable (say x) representing any member of the set followed by a property satisfied by each member of the set.

For example, the set A of all prime numbers less than 10 in the set-builder form is written as

$A = \{x \mid x \text{ is a prime number less that } 10\}$

The symbol '|' stands for the words 'such that'. Sometimes, we use the symbol ':' in place of the symbol '|'.

3. TYPES OF SETS

3.1 Empty Set or Null Set

A set which has no element is called the null set or empty set. It is denoted by the symbol ϕ or { }.

For example, each of the following is a null set :

(a) The set of all real numbers whose square is -1.

(b) The set of all rational numbers whose square is 2.

(c) The set of all those integers that are both even and odd.

A set consisting of atleast one element is called a non-empty set.

3.2 Singleton Set

A set having only one element is called singleton set.

For example, $\{0\}$ is a singleton set, whose only member is 0.

3.3 Finite and Infinite Set

A set which has finite number of elements is called a finite set. Otherwise, it is called an infinite set.

For example, the set of all days in a week is a finite set whereas the set of all integers, denoted by $\{...-2, -1, 0, 1, 2,...\}$ or $\{x \mid x \text{ is an integer}\}$, is an infinite set.

An empty set is a finite set.

3.4 Cardinal Number

The number of elements in finite set is represented by n(A), and is known as Cardinal number of set A.

3.5 Equal Sets

Two sets A and B are said to be equals, written as $A = B$, if every element of A is in B and every element of B is in A.

3.6 Equivalent Sets

Two finite sets A and B are said to be equivalent, if $n(A) = n(B)$. Clearly, equal sets are equivalent but equivalent sets need not be equal.

For example, the sets $A = \{4, 5, 3, 2\}$ and $B = \{1, 6, 8, 9\}$ are equivalent but are not equal.

4. SUBSET

Let A and B be two sets. If every elements of A is an element of B, then A is called a subset of B and we write $A \subset B$ or $B \supset A$ (read as 'A is contained in B' or 'B contains A'). B is called superset of A.

NOTES :

(i) Every set is a subset and a superset of itself.

(ii) If A is not a subset of B, we write $A \not\subset B$.

(iii) The empty set is the subset of every set.

(iv) If A is a set with $n(A) = m$, then the number of subsets of A are 2^m and the number of proper subsets of A are $2^m - 1$.

For example, let $A = \{3, 4\}$, then the subsets of A are ϕ, $\{3\}$, $\{4\}$. $\{3, 4\}$. Here, $n(A) = 2$ and number of subsets of $A = 2^2 = 4$. Also, $\{3\} \subset \{3,4\}$ and $\{2,3\} \not\subset \{3, 4\}$

4.1 Power Set

The set of all subsets of a given set A is called the power set of A and is denoted by P(A).

For example, if $A = \{1, 2, 3\}$, then

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\} \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Clearly, if A has n elements, then its power set P (A) contains exactly 2^n elements.

5. OPERATIONS ON SETS

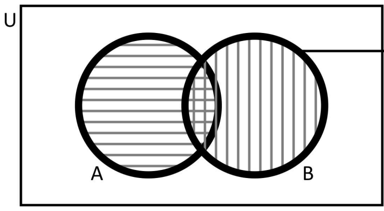
5.1 Union of Two Sets

The union of two sets A and B, written as $A \cup B$ (read as 'A union B'), is the set consisting of all the elements which are either in A or in B or in both. Thus,

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Clearly, $x \in A \cup B \Rightarrow x \in A$ or $x \in B$, and

$x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$.



For example, if $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$, then $A \cup B = \{a, b, c, d, e, f\}$

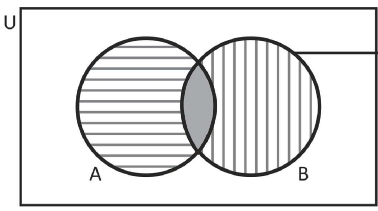
5.2 Intersection of Two sets

The intersection of two sets A and B, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B. Thus,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Clearly, $x \in A \cap B \Rightarrow x \in A$ and $x \in B$, and

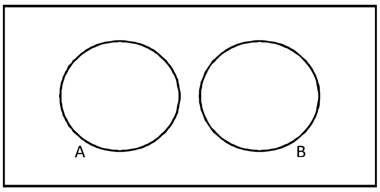
$x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$.



For example, if $A = \{a, b, c, d)$ and $B = \{c, d, e, f\}$, then $A \cap B = \{c, d\}$.

5.3 Disjoint Sets

Two sets A and B are said to be disjoint, if $A \cap B = \phi$, i.e. A and B have no element in common.



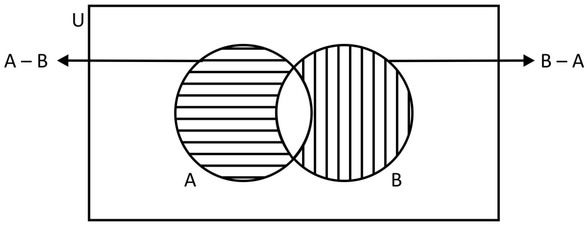
For example, if A = {1, 3, 5} and B = {2, 4, 6}, then $A \cap B = \phi$, so A and B are disjoint sets.

5.4 Difference of Two Sets

If A and B are two sets, then their difference A - B is defined as :

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.



For example, if A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9} then A - B = {2, 4} and B - A = {7, 9}.

Important Results

(a) $A - B \neq B - A$

(b) The sets A - B , B - A and $A \cap B$ are disjoint sets

(c) $A - B \subseteq A$ and $B - A \subseteq B$

(d) $A - \phi = A$ and $A - A = \phi$

5.5 Symmetric Difference of Two Sets

The symmetric difference of two sets A and B , denoted by $A \Delta B$, is defined as

$A \Delta B = (A - B) \cup (B - A)$.

For example, if A = {1,2,3,4,5} and B = {1, 3,5,7,9} then

$A \Delta B = (A - B) \cup (B - A) = \{2,4\} \cup \{7,9\} = \{2,4,7,9\}$.

5.6 Complement of a Set

If U is a universal set and A is a subset of U, then the complement of A is the set which contains those elements of U, which are not contained in A and is denoted by A' or A^c. Thus,

$A^c = \{x : x \in U \text{ and } x \notin A\}$

For example, if U = {1,2,3,4 ...} and A ={2,4,6,8,...}, then, $A^c = \{1,3,5,7, ...\}$

Important Results

(a) $U^c = \phi$ (b) $\phi^c = U$

(c) $A \cup A^c = U$ (d) $A \cap A^c = \phi$

6. ALGEBRA OF SETS

1. For any set A , we have

   (a) $A \cup A = A$ (b) $A \cap A = A$

2. For any set A, we have

   (a) $A \cup \phi = A$ (b) $A \cap \phi = \phi$

   (c) $A \cup U = U$ (d) $A \cap U = A$

3. For any two sets A and B, we have

   (a) $A \cup B = B \cup A$ (b) $A \cap B = B \cap A$

4. For any three sets A, B and C, we have

   (a) $A \cup (B \cup C) = (A \cup B) \cup C$

   (b) $A \cap (B \cap C) = (A \cap B) \cap C$

5. For any three sets A, B and C, we have

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. If A is any set, we have $(A^c)^c = A$.

7. DeMorgan's Laws For any two sets A and B, we have

   (a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

Important Results on Operations on Sets

(i) $A \subseteq A \cup B, B \subseteq A \cup B, A \cap B \subseteq A, A \cap B \subseteq B$

(ii) $A - B = A \cap B^c$

(iii) $(A - B) \cup B = A \cup B$

(iv) $(A - B) \cap B = \phi$ (v) $A \subseteq B \Leftrightarrow B^c \subseteq A^c$

(vi) $A - B = B^c - A^c$

(vii) $(A \cup B) \cap (A \cup B^c) = A$

(viii) $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

(ix) $A - (A - B) = A \cap B$

(x) $A - B = B - A \Leftrightarrow A = B$

(xi) $A \cup B = A \cap B \Leftrightarrow A = B$

(xii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

7. CARDINALITY

If A, B and C are finite sets and U be the finite universal set, then

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A - B) = n(A) - n(A \cap B)$
3. $n(A \Delta B)$ = Number of elements which belong to exactly one of A or B

   $= n((A - B) \cup (B - A))$

   $= n(A - B) + n(B - A)$

   [$\because$ (A - B) and (B - A) are disjoint]

   $= n(A) - n(A \cap B) + n(B) - n(A \cap B) = n(A) + n(B) - 2n(A \cap B)$
4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
5. Number of elements in exactly two of the sets A,B,C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
6. Number of elements in exactly one of the sets A,B,C $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$
7. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
8. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

RELATIONS

1. CARTESIAN PRODUCT OF SETS

Definition : Given two non-empty sets P & Q. The cartesian product $P \times Q$ is the set of all ordered pairs of elements from P & Q i.e.

$$P \times Q = \{(p, q); p \in P; q \in Q\}$$

2. RELATIONS

2.1 Definition

Let A & B be two non-empty sets. Then any subset 'R' of $A \times B$ is a relation from A to B.

If $(a, b) \in R$, then we write it as a R b which is read as a is related to b' by the relation R', 'b' is also called image of 'a' under R.

2.2 Domain and Range of a Relation

If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. symbolically.

Domain of R = $\{x : (x, y) \in R\}$

Range of R = $\{y : (x, y) \in R\}$

The set B is called co-domain of relation R.

Note that range $\subset$ co-domain.

NOTES :

Total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and total number of relations is 2^{pq}.

2.3 Inverse of a Relation

Let A, B be two sets and let R be a relation from a set A to set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

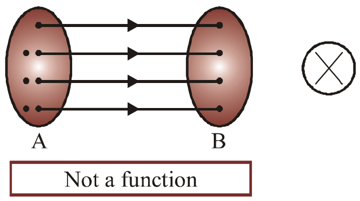
Also, Domain (R) = Range (R^{-1}) and Range (R) = Domain (R^{-1}).

3. FUNCTIONS

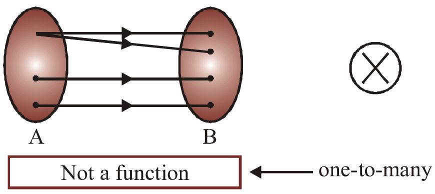
3.1 Definition

A relation 'f' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Relations which can not be catagorized as a function

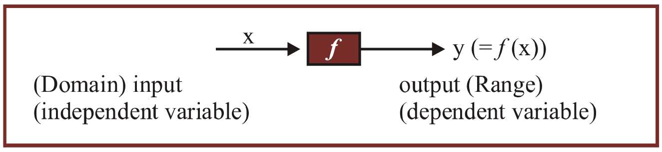


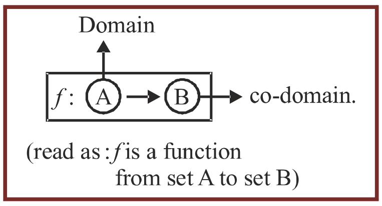
As not all elements of set A are associated with some elements of set B.

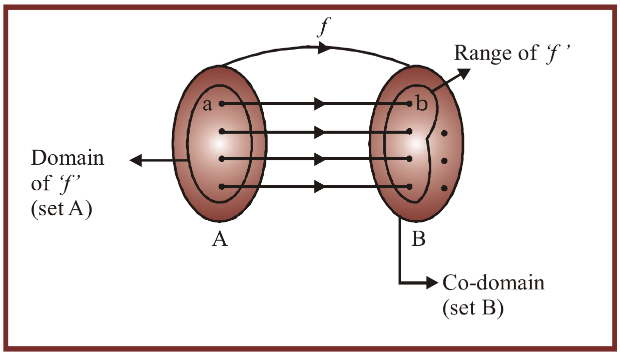


An element of set A is not associated with a unique element of set B.

Notations







3.2 Domain, Co-domain and Range of a Function

Domain : When we define $y = f(x)$ with a formula and the domain is not stated explicitly, the domain is assumed to be the largest set of x–values for which the formula gives real y–values.

The domain of $y = f(x)$ is the set of all real x for which $f(x)$ is defined (real).

Rules for finding Domain

(i) Expression under even root (i.e. square root, fourth root etc.) should be non–negative.

(ii) Denominator $\neq 0$.

(iii) $\log_a x$ is defined when $x > 0$, $a > 0$ and $a \neq 1$.

(iv) If domain of $y = f(x)$ and $y = g(x)$ are D_1 and D_2 respectively, then the domain of $f(x) \pm g(x)$ or $f(x) . g(x)$ is $D_1 \cap D_2$. While

domain of $\dfrac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{x : g(x) = 0\}$.

Range : The set of all f-images of elements of A is known as the range of f & denoted by $f(A)$.

Range $= f(A) = \{f(x) : x \in A\}$;

$f(A) \subseteq B$ {Range $\subseteq$ Co-domain}.

Rules for finding Range

First of all find the domain of $y = f(x)$

(i) If domain $\in$ finite number of points

$\Rightarrow$ range $\in$ set of corresponding $f(x)$ values.

(ii) If domain $\in$ R or R – {some finite points}

Put $y = f(x)$

Then express x in terms of y. From this find y for x to be defined. (i.e., find the values of y for which x exists).

(iii) If domain $\in$ a finite interval, find the least and greater value for range using monotonocity.

NOTES :

1. Question of format :

$$\left(y = \frac{Q}{Q};\ y = \frac{L}{Q};\ y = \frac{Q}{L}\right) \quad \begin{matrix} Q \rightarrow \text{quadratic} \\ L \rightarrow \text{Linear} \end{matrix}$$

Range is found out by cross-multiplying & creating a quadratic in 'x' & making $D \geq 0$ (as $x \in R$)

2. Questions to find range in which-the given expression $y = f(x)$ can be converted into x (or some function of x) = expression in 'y'.

Do this & apply method (ii).

NOTES :

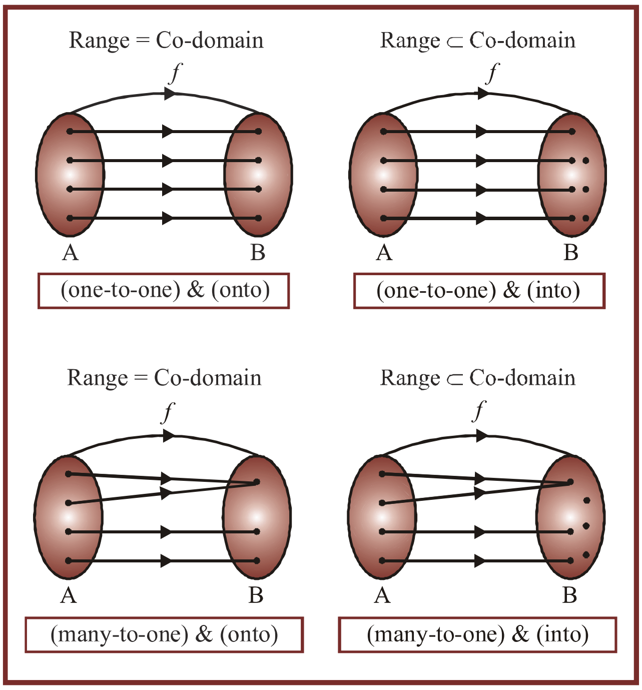
Two functions f & g are said to be equal iff

1. Domain of f = Domain of g

2. Co-domain of f = Co-domain of g

3. $f(x) = g(x)\ \forall\ x \in$ Domain.

3.3 Classification of Functions



NOTES :

(a) One-to-One functions are also called Injective functions.

(b) Onto functions are also called Surjective

(c) (one-to-one) & (onto) functions are also called Bijective Functions.

Methods to check one-one mapping

1. Theoretically : If $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$, then $f(x)$ is one-one.

2. Graphically : A function is one-one, iff no line parallel to x-axis meets the graph of function at more than one point.

3. By Calculus : For checking whether $f(x)$ is One-One, find whether function is only increasing or only decreasing in their domain. If yes, then function is one-one, i.e. if $f'(x) \geq 0,\ \forall\ x \in$ domain or i.e., if $f'(x) \leq 0,\ \forall\ x \in$ domain, then function is one-one.

Methods to check into/onto mapping

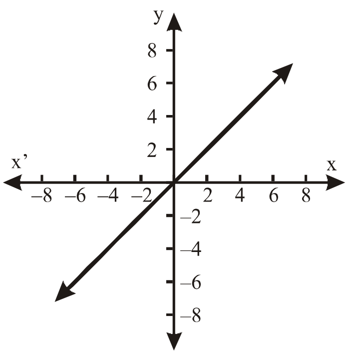
Find the range of f(x) and compare with co-domain. If range equals co-domain then function is onto, otherwise it is into.

3.4 Some standard real functions & their graphs

3.4.1 Identity Function :

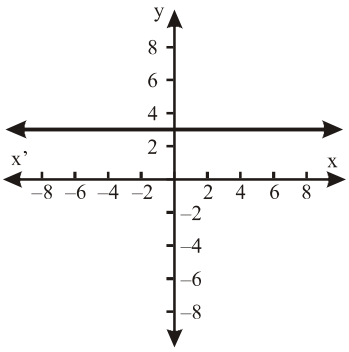
The function $f : R \rightarrow R$ defined by

$y = f(x) = x\ \forall\ x \in R$ is called identity function.



$f(x) = x$

3.4.2 Constant Function : The function $f : R \to R$ defined by $y = f(x) = c,\ \forall\ x \in R$ where c is a constant is called constant function

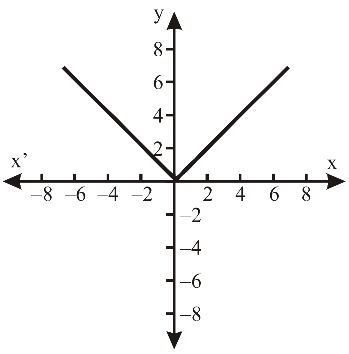


$f(x) = 3$

3.4.3 Modulus Function : The function $f : R \to R$ defined by

$$f(x) = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$$

is called modulus function. It is denoted by $y = f(x) = |x|$.



$f(x) = |x|$

Its also known as "Absolute value function'.

Properties of Modulus Function :

The modulus function has the following properties :

1. For any real number x, we have $\sqrt{x^2} = |x|$

2. $|x\ y| = |x|\,|y|$, $\left|\dfrac{x}{y}\right| = \dfrac{|x|}{|y|}$

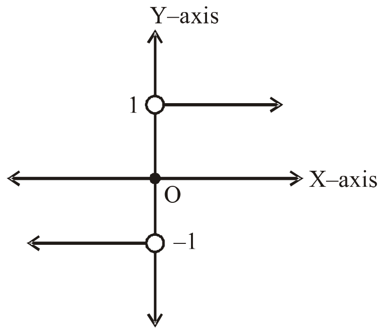
3. $\big||x| - |y|\big| \leq |x + y| \leq |x| + |y|$
4. $\big||x| - |y|\big| \leq |x - y| \leq |x| + |y|$

$\Big\}$ triangle inequality

3.4.4 Signum Function : The function $f : R \to R$ defined by

$$f(x) = \begin{cases} 1; & x > 0 \\ 0; & x = 0 \\ -1; & x < 0 \end{cases}$$

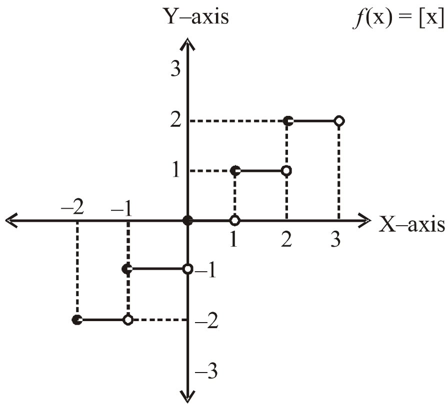
is called signum function. It is usually denoted by $y = f(x) = \text{sgn}(x)$.



NOTES :

$$\text{Sgn}(x) = \begin{cases} \dfrac{|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

3.4.5 Greatest Integer Function : The function $f : R \to R$ defined as the greatest integer less than or equal to x. It is usually denoted as $y = f(x) = [x]$



Properties of Greatest Integer Function :

If n is an integer and x is any real number between n and n + 1, then the greatest integer function has the following properties :

(1) $[-n] = -[n]$

(2) $[x+n] = [x]+n$

(3) $[-x] = -[x]-1$

(4) $[x] + [-x] = \begin{cases} -1, & \text{if } x \notin I \\ 0, & \text{if } x \in I \end{cases}$

NOTES :

Fractional part of x, denoted by $\{x\}$ is given by $x-[x]$. So,

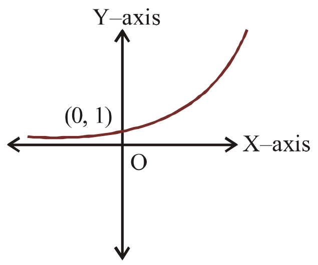
$$\{x\} = x - [x] = \begin{cases} x-1; & 1 \le x < 2 \\ x\ ; & 0 \le x < 1 \\ x+1; & -1 \le x < 0 \end{cases}$$

3.4.6 Exponential Function :

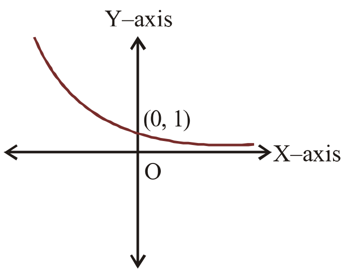
$f(x) = a^x$, $a > 0$, $a \neq 1$

Domain : $x \in R$

Range : $f(x) \in (0, \infty)$



$f(x) = a^x$, when $a > 1$



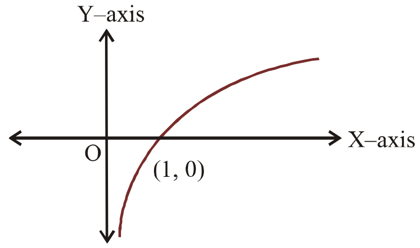
$f(x) = a^x$, when $0 < a < 1$

3.4.7 Logarithm Function :

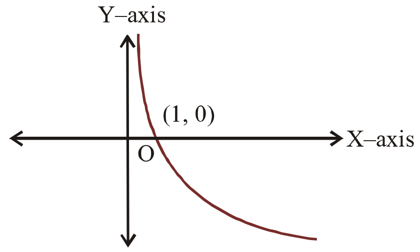
$f(x) = \log_a x$, $a > 0$, $a \neq 1$

Domain : $x \in (0, \infty)$

Range : $y \in R$



$f(x) = \log_a x$, when $a > 1$



$f(x) = \log_a x$, when $0 < a < 1$

(a) The Principal Properties of Logarithms

Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$.

(i) $\log_b a = c \quad \Rightarrow \quad a = b^c$

(ii) $\log_a (M \cdot N) = \log_a M + \log_a N$

(iii) $\log_a (M/N) = \log_a M - \log_a N$

(iv) $\log_a M^N = N \log_a M$

(v) $\log_b a = \dfrac{\log_c a}{\log_c b}$, $c > 0$, $c \neq 1$.

(vi) $a^{\log_c b} = b^{\log_c a}$, $a, b, c > 0$, $c \neq 1$.

NOTES :

(a) $\log_a a = 1$

(b) $\log_b a \cdot \log_c b \cdot \log_a c = 1$

(c) $\log_a 1 = 0$

(d) $e^{x \ln a} = e^{\ln a^x} = a^x$

(b) Properties of Monotonocity of Logarithm

(i) If $a > 1$, $\log_a x < \log_a y \Rightarrow 0 < x < y$
(ii) If $0 < a < 1$, $\log_a x < \log_a y \Rightarrow x > y > 0$
(iii) If $a > 1$ then $\log_a x < p \Rightarrow 0 < x < a^p$
(iv) If $a > 1$ then $\log_a x > p \Rightarrow x > a^p$
(v) If $0 < a < 1$ then $\log_a x < p \Rightarrow x > a^p$
(vi) If $0 < a < 1$ then $\log_a x > p \Rightarrow 0 < x < a^p$

NOTES :

If the exponent and the base are on same side of the unity, then the logarithm is positive.

If the exponent and the base are on different sides of unity, then the logarithm is negative.

4. ALGEBRA OF REAL FUNCTION

4.1 Addition of two real functions

Let $f : X \to R$ and $g : X \to R$ by any two real functions, where $X \subset R$. Then, we define $(f + g) : X \to R$ by

$(f + g)(x) = f(x) + g(x)$, for all $x \in X$.

4.2 Subtraction of a real function from another

Let $f : X \to R$ be any two any two real functions, where $X \subset R$. Then, we define $(f - g) : X \to R$ by

$(f - g)(x) = f(x) - g(x)$, for all $x \in X$.

4.3 Multiplication by a scalar

Let $f : X \to R$ be a real valued function and α be a scalar. Here by scalar, we mean a real number. Then the product αf is a function from X to R defined by $(\alpha f)(x) = \alpha f(x)$, $x \in X$.

4.4 Multiplication of two real functions

The product (or multiplication) of two real functions $f : X \to R$ and $g : X \to R$ is a function $fg : X \to R$ defined by $(fg)(x) = f(x)\, g(x)$, for all $x \in X$.

This is also called *pointwise multiplication*.

4.5 Quotient of two real functions

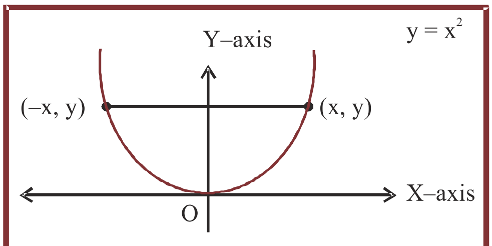
Let f and g be two real functions defined from $X \to R$ where $X \subset R$. The quotient of f by g denoted by f/g is a function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ provided } g(x) \neq 0, x \in X.$$

5. EVEN AND ODD FUNCTIONS

Even Function : $f(-x) = f(x)$, $\forall\ x \in$ Domain

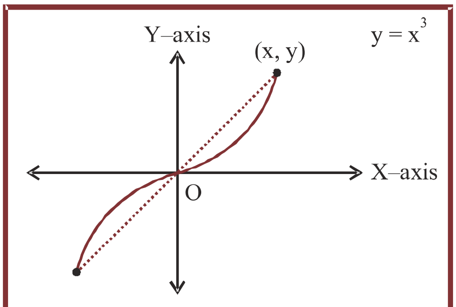
The graph of an even function $y = f(x)$ is symmetric about the y–axis. i.e., (x, y) lies on the graph $\Leftrightarrow$ $(-x, y)$ lies on the graph.



Graph of an even function

Odd Function : $f(-x) = -f(x)$, $\forall\ x \in$ Domain

The graph of an odd function $y = f(x)$ is symmetric about origin i.e. if point (x, y) is on the graph of an odd function, then $(-x, -y)$ will also lie on the graph.

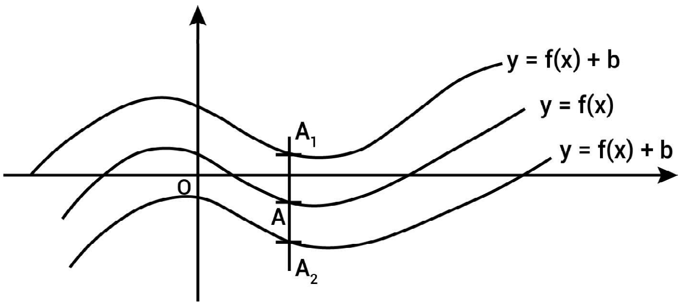


Graph of an odd function

6. GRAPHICAL TRANSFORMATION

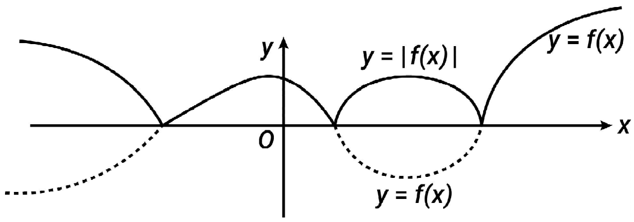
6.1 Drawing graph of $y = f(x) \pm b, b \in R^+$ from known graph of $y = f(x)$

It is obvious that the domain of f(x) and f(x) + b are the same. The graph of f(x) + b can be obtained by translating the graph of f(x) in the positive direction on y-axis and the graph of f(x) - b can be obtained by translating the graph of f(x) in the negative direction on y-axis.



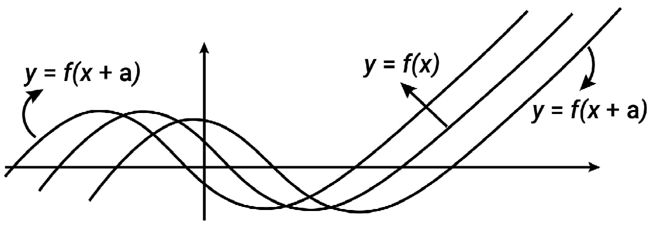
6.2 Drawing graph of $y = |f(x)|$ from known graph of $y = f(x)$

We have $|f(x)| = f(x)$ if $f(x) \geq 0$ and $|f(x)| = -f(x)$ if $f(x) < 0$ which means that the graph of f(x) and |f(x)| would concide if $f(x) \geq 0$ and the sections, where $f(x) < 0$, get inverted in the upwards direction. Figure depicts the procedure.



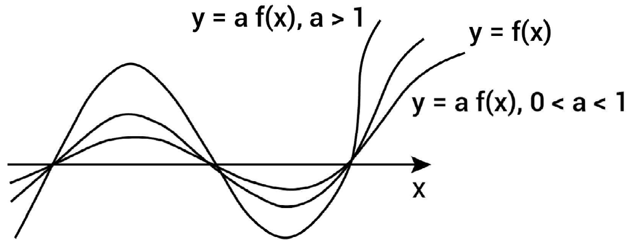
6.3 Drawing graph of $y = f(x \pm a), a \in R^+$ from known graph of $y = f(x)$

The graph of f(x - a) can be obtained by translating the graph of f(x) in the positive direction on x-axis and the graph of f(x + a) can be obtained by translating the graph of f(x) in the negative direction on x-axis. The procedure is depicted in figure.



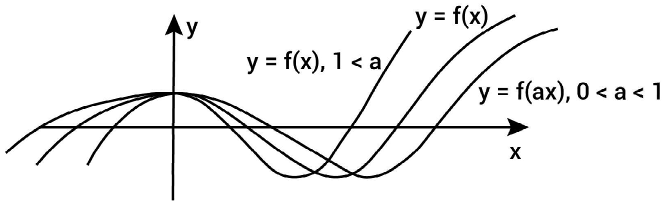
6.4 Drawing graph of $y = af(x), a \in R^+$ from known graph of $y = f(x)$

We know that the corresponding points (points with the same x-coordinates) have their ordinates in the ratio of 1:a (where $a > 0$). Figure depicts the procedure.



6.5 Drawing graph of $y = f(ax), a \in R^+$ from known graph of $y = f(x)$

If $0 < a < 1$, then f(x) will stretch by a times along x - axis, and if $a > 1$, then f(x) will compress by a times along x - axis. Figure depicts the procedure.



7. PERIODIC FUNCTION

Definition :

A function $f(x)$ is said to be periodic function, if there exists a positive real number T, such that $f(x+T) = f(x)$, $\forall x \in$ domain of f(x). Then, $f(x)$ is a perio dic function where least positive value of T is called fundamental period.

Graphically, if the graph repeats at fixed interval, then function is said to be periodic and its period is the width of that interval.

Some standard results on periodic functions

	Functions	Periods
(i)	$\sin^n x, \cos^n x, \sec^n x, \text{cosec}^n x$	π ; if n is even.
		2π ; (if n is odd or fraction)
(ii)	$\tan^n x, \cot^n x$	π ; n is even or odd.
(iii)	$\|\sin x\|, \|\cos x\|, \|\tan x\|$ $\|\cot x\|, \|\sec x\|, \|\text{cosec } x\|$	π

(iv)	$x-[x]$, [.] represents greatest integer function	1
(v)	Algebraic functions e.g., $\sqrt{x}$, x^2, x^3+5, ...etc.	period does not exist

Properties of Periodic Function

(i) If $f(x)$ is periodic with period T, then

(a) $c.f(x)$ is periodic with period T.

(b) $f(x \pm c)$ is periodic with period T.

(c) $f(x) \pm c$ is periodic with period T.

where c is any constant.

(ii) If $f(x)$ is periodic with period T, then

$kf(cx+d)$ has period $T/|c|$,

i.e. Period is only affected by coefficient of x where k, c, d are constants.

(iii) If $f_1(x), f_2(x)$ are periodic functions with periods T_1, T_2 respectively, then $h(x) = af_1(x) \pm bf_2(x)$ has period as, LCM of $\{T_1, T_2\}$

NOTES :

(a) LCM of $\left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{\text{LCM of }(a, c, e)}{\text{HCF of }(b, d, f)}$

(b) LCM of rational and rational always exists.

LCM of irrational and irrational sometime exists.

But LCM of rational and irrational never exists.

e.g., LCM of $(2\pi, 1, 6\pi)$ is not possible as $2\pi, 6\pi \in$ irrational and $1 \in$ rational.

SOLVED EXAMPLES

Example – 1

Write the set of all positive integers whose cube is odd.

Sol. The elements of the required set are not even.

[$\because$ Cube of an even integer is also an even integer]

Moreover, the cube of a positive odd integer is a positive odd integer.

$\Rightarrow$ The elements of the required set are all positive odd integers.

Hence, the required set, in the set builder form, is :

$$\{2k+1 : k \geq 0, \ k \in Z\}.$$

Example – 2

Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}\right\}$ in the set builder form.

Sol. In each element of the given set the denominator is one more than the numerator.

Also the numerators are from 1 to 7.

Hence the set builder form of the given set is :

$$\{x : x = n/n+1, \ n \in N \ \text{and} \ 1 \leq n \leq 7\}.$$

Example – 3

Write the set $\{x : x$ is a positive integer and $x^2 < 30\}$ in the roster form.

Sol. The squares of positive integers whose squares are less than 30 are : 1, 2, 3, 4, 5.

Hence the given set, in roster form, is $\{1, 2, 3, 4, 5\}$.

Example – 4

Write the set $\{0, 1, 4, 9, 16, ...\}$ in set builder form.

Sol. The elements of the given set are squares of integers :

$0, \pm 1, \pm 2, \pm 3, \pm 4, ...$

Hence the given set, in set builder form, is $\{x^2 : x \in Z\}$.

Example – 5

State which of the following sets are finite and which are infinite

(i) $A = \{x : x \in N \text{ and } x^2 - 3x + 2 = 0\}$

(ii) $B = \{x : x \in N \text{ and } x^2 = 9\}$

(iii) $C = \{x : x \in N \text{ and } x \text{ is even}\}$

(iv) $D = \{x : x \in N \text{ and } 2x - 3 = 0\}$.

Sol. (i) $A = \{1, 2\}$.

[$\because x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1, 2$]

Hence A is finite.

(ii) $B = \{3\}$.

[$\because x^2 = 9 \Rightarrow x = \pm 3$. But $3 \in N$]

Hence B is finite.

(iii) $C = \{2, 4, 6, ...\}$

Hence C is infinite.

(iv) $D = \phi$. $\left[\because 2x - 3 = 0 \Rightarrow x = \frac{3}{2} \notin N\right]$

Hence D is finite.

Example – 6

Which of the following are empty (null) sets ?

(i) Set of odd natural numbers divisible by 2

(ii) $\{x : 3 < x < 4, x \in N\}$

(iii) $\{x : x^2 = 25$ and x is an odd integer$\}$

(iv) $[x : x^2 - 2 = 0$ and x is rational$]$

(v) $\{x : x$ is common point of any two parallel lines$\}$.

Sol. (i) Since there is no odd natural number, which is divisible by 2.

$\therefore$ it is an empty set.

(ii) Since there is no natural number between 3 and 4.

$\therefore$ it is an empty set.

(iii) Now $x^2 = 25 \Rightarrow x = \pm 5$, both are odd.

$\therefore$ The set $\{-5, 5\}$ is non-empty.

(iv) Since there is no rational number whose square is 2,

$\therefore$ the given set is an empty set.

(v) Since any two parallel lines have no common point,

$\therefore$ the given set is an empty set.

Example – 7

Find the pairs of equal sets from the following sets, if any, giving reasons :

$A=\{0\}, B=\{x : x>15 \text{ and } x<5\}$,

$C=\{x : x-5=0\}, D=\{x : x^2=25\}$,

E = {x : x is a positive integral root of the equation $x^2-2x-15=0\}$.

Sol. Here we have,

$A=\{0\}$

$B=\phi$

[$\because$ There is no number, which is greater than 15 and less than 5]

$C=\{5\}$ [$\because x-5=0 \Rightarrow x=5$]

$D=\{-5, 5\}$ [$\because x^2=25 \Rightarrow x=\pm 5$]

and $E=\{5\}$.

[$\because x^2-2x-15=0 \Rightarrow (x-5)(x+3)=0 \Rightarrow x=5,-3$. Out of these two,

5 is positive integral]

Clearly $C=E$.

Example – 8

Are the following pairs of sets equal ? Give reasons.

(i) $A=\{1, 2\}, B=\{x : x$ is a solution of $x^2+3x+2=0\}$

(ii) A = {x : x is a letter in the word FOLLOW},

B = {y : y is a letter in the word WOLF}.

Sol. (i) $A=\{1, 2\}, B=\{-2, -1\}$

[$\because x^2+3x+2=0 \Rightarrow (x+2)(x+1)=0 \Rightarrow x=-2, -1$]

Clearly $A \neq B$.

(ii) A = {F, O, L, L, O, W} = {F, O, L, W}

B = {W, O, L, F} = {F, O, L, W}.

Clearly A = B.

Example – 9

Let $U=\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A=\{1, 2, 3, 4\}$, $B=\{2, 4, 6, 8\}$. Find :

(i) A^C (ii) B^C (iii) $(A^C)^C$ (iv) $(A \cup B)^C$

Sol. (i) A^C = Set of those elements of U, which are not in $A=\{5, 6, 7, 8, 9\}$.

(ii) B^C = Set of those elements of U, which are not in $B=\{1, 3, 5, 7, 9\}$.

(iii) $(A^C)^C$ = Set of those elements of U, which are not in $A'=\{1, 2, 3, 4\}=A$.

(iv) $A \cup B=\{1,2,3,4\} \cup \{2,4,6,8\}=\{1,2,3,4,6,8\}$.

$\therefore$ $(A \cup B)^C$ = Set of those elements of U, which are not in $(A \cup B)=\{5, 7, 9\}$.

Example – 10

If U = {x : x is a letter in English alphabet}, A = {x : x is a vowel in English alphabet}.

Find A^C and $(A^C)^C$.

Sol. (i) Since A = {x : x is a letter in English alphabet},

$\therefore$ A^C is the set of those elements of U, which are not vowels

= {x : x is a consonant in English alphabet}.

(ii) $(A^C)^C$ is the set of those elements of U, which are not consonants = {x : x is a vowel in English alphabet} = A.

Hence $(A^C)^C=A$.

Example – 11

Let $A=\{1, 2, 3, 4, 5\}, B=\{3, 4, 5, 6, 7\}, C=\{6, 7, 8, 9\}$ and $D=\{7, 8, 9, 10\}$. Find :

(a) (i) $A \cup B$ (ii) $B \cup D$

(iii) $A \cup B \cup C$ (iv) $B \cup C \cup D$

(b) (i) $A \cap B$ (ii) $B \cap D$ (iii) $A \cap B \cap C$.

Sol. (a) (i) $A \cup B=\{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7\}$

$=\{1, 2, 3, 4, 5, 6, 7\}$.

(ii) $B \cup D=\{3, 4, 5, 6, 7\} \cup \{7, 8, 9, 10\}$

$=\{3, 4, 5, 6, 7, 8, 9, 10\}$.

(iii) $A \cup B \cup C=\{1,2,3,4,5\} \cup \{3,4,5,6,7\} \cup \{6,7,8,9\}$.

$=\{1,2,3,4,5,6,7\} \cup \{6,7,8,9\}=\{1,2,3,4,5,6,7,8,9\}$.

(iv) $B \cup C \cup D=\{3,4,5,6,7\} \cup \{6,7,8,9\} \cup \{7,8,9,10\}$.

$=\{3,4,5,6,7,8,9\} \cup \{7,8,9,10\}=\{3,4,5,6,7,8,9,10\}$.

(b) (i) $A \cap B=\{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\}=\{3, 4, 5\}$.

(ii) $B \cap D=\{3,4,5,6,7\} \cap \{7,8,9,10\}=\{7\}$.

(iii) $A \cap B \cap C=\{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} \cap \{6, 7, 8, 9\}$
$=\{3, 4, 5\} \cap \{6, 7, 8, 9\}=\phi$.

Example – 12

If $A_1 = \{2, 3, 4, 5\}, A_2 = \{3, 4, 5, 6\}, A_3 = \{4, 5, 6, 7\}$, find $\cup A_i$ and $\cap A_i$, where $i = \{1, 2, 3\}$.

Sol. (i) $\cup A_i = A_1 \cup A_2 \cup A_3 = \{2,3,4,5\} \cup \{3,4,5,6\} \cup \{4,5,6,7\}$

$= \{2,3,4,5\} \cup \{3,4,5,6,7\} = \{2,3,4,5,6,7\}$.

(ii) $\cap A_i = A_1 \cap A_2 \cap A_3 = \{2,3,4,5\} \cap \{3,4,5,6\} \cap \{4,5,6,7\}$

$= \{2,3,4,5\} \cap \{4,5,6\} = \{4,5\}$.

Example – 13

Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7, 8\}$. Find $(A-B) \cup (B-A)$.

Sol. We have, $A = \{1,2,3,4,5,6\}$ and $B = \{3,4,5,6,7,8\}$.

$\therefore A-B = \{1,2\}$ and $B-A = \{7,8\}$

$\therefore (A-B) \cup (B-A) = \{1,2\} \cup \{7,8\} = \{1,2,7,8\}$.

Example – 14

Prove that :

$A \cap (B-C) = (A \cap B) - (A \cap C)$

Sol. Let x be an arbitrary element of $A \cap (B-C)$.

Then $x \in A \cap (B-C)$

$\Rightarrow$ $x \in A$ and $x \in (B-C)$

$\Rightarrow$ $x \in A$ and ($x \in B$ and $x \notin C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) and ($x \in A$ and $x \notin C$)

$\Rightarrow$ $x \in (A \cap B)$ and $x \notin (A \cap C)$

$\Rightarrow$ $x \in \{(A \cap B) - (A \cap C)\}$

$\therefore$ $A \cap (B-C) \subseteq (A \cap B) - (A \cap C)$ ... (1)

Let y be an arbitrary element of $(A \cap B) - (A \cap C)$.

Then $y \in (A \cap B) - (A \cap C)$

$\Rightarrow$ $y \in (A \cap B)$ and $y \notin (A \cap C)$

$\Rightarrow$ ($y \in A$ and $y \in B$) and ($y \in A$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and ($y \in B$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and $y \in (B-C)$

$\Rightarrow$ $y \in A \cap (B-C)$

$\therefore$ $(A \cap B) - (A \cap C) \subseteq A \cap (B-C)$ ... (2)

Combining (1) and (2).

$A \cap (B-C) = (A \cap B) - (A \cap C)$.

Example – 15

Prove the following :

$A \subset B \Leftrightarrow B^c \subset A^c$

Sol. Let $x \in B^c$, where x is arbitrary.

Now $x \in B^c$

$\Rightarrow$ $x \notin B$

$\Rightarrow$ $x \notin A$ [$\because A \subset B$]

$\Rightarrow$ $x \in A^c$

$\therefore$ $B^c \subset A^c$ ... (1)

Conversely : Let $x \in A$, where x is arbitrary.

Now $x \in A$

$\Rightarrow$ $x \notin A^c$

$\Rightarrow$ $x \notin B_c$ [$\because B^c \subset A^c$]

$\Rightarrow$ $x \in B$

$\therefore$ $A \subset B$

Combining (1) and (2), $A \subset B \Leftrightarrow B^c \subset A^c$.

Example – 16

Prove the following :

$A - B = A - (A \cap B)$

where U is the universal set.

Sol. Let $x \in (A-B)$, where x is arbitrary.

Now $x \in (A-B)$

$\Leftrightarrow$ $x \in A$ and $x \notin B$

$\Leftrightarrow$ ($x \in A$ and $x \in A$) and $x \notin B$

[Note this step]

$\Leftrightarrow$ $x \in A$ and ($x \in A$ and $x \notin B$)

[Associative Law]

$\Leftrightarrow$ $x \in A$ and $x \notin (A \cap B)$

$\Leftrightarrow$ $x \in A - (A \cap B)$

Hence $A - B \subset A - (A \cap B)$.

Now Let $y \in A - (A \cap B)$

$\Rightarrow y \in A$ and $y \notin (A \cap B)$

$\Rightarrow y \in A$ and $y \notin B$

$\Rightarrow y \in A - B$.

So, $A - B = A - (A \cap B)$.

Example – 17

If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(a) $A = C$ (b) $B = C$

(c) $A \cap B = \phi$ (d) $A = B$

Ans. (b)

Sol. Let $x \in C$

Suppose $x \in A \Rightarrow x \in A \cap C$

$\Rightarrow x \in A \cap B$ ($\because A \cap C = A \cap B$)

Thus $x \in B$

Again suppose $x \notin A \Rightarrow x \in C \cup A$

$\Rightarrow x \in B \cup A \Rightarrow x \in B$

Thus in both cases $x \in C \Rightarrow x \in B$

Hence $C \subseteq B$ ..(*i*)

Similarly we can show that $B \subseteq C$ ...(*ii*)

Combining (i) and (ii) we get B = C.

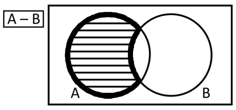
Example – 18

If A and B are any two sets, prove using Venn Diagrams

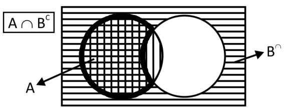
(i) $A - B = A \cap B^C$ (ii) $(A - B) \cup B = A \cup B$.

Sol.

(i)

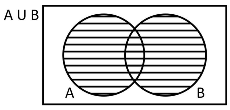


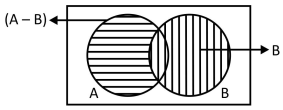
Shaded region in both are the same.



The common shaded area represents the intersection.

(ii)





Combination of all the shaded regions represents the union.

Example – 19

If $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$ and $C = \{7, 8, 9\}$, verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Sol. We have, $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$ and $C = \{7, 8, 9\}$.

$\therefore$ $A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$ ...(1)

$A \cup C = \{1, 2, 3\} \cup \{7, 8, 9\}$

$= \{1, 2, 3, 7, 8, 9\}$ ...(2)

and $B \cap C = \{4, 5, 6\} \cap \{7, 8, 9\} = \phi$ ...(3)

Now $A \cup (B \cap C) = \{1, 2, 3\} \cup \phi = \{1, 2, 3\}$ ...(4)

and $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 7, 8, 9\}$

$= \{1, 2, 3\}$ ...(5)

From (4) and (5), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, which verifies the result.

Example – 20

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that

(i) $(A \cup B)^C = A^C \cap B^C$ (ii) $(A \cap B)^C = A^C \cup B^C$.

Sol. We have, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$.

(i) $A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$

$\therefore$ $= \{2, 3, 4, 5, 6, 7, 8\}$

$\therefore$ $(A \cup B)^C = \{1, 9\}$ ...(1)

Also $A^C = \{1, 3, 5, 7, 9\}$

and $B^C = \{1, 4, 6, 8, 9\}$

$\therefore$ $A^C \cap B^C = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\}$

$= \{1, 9\}$ ...(2)

From (1) and (2), $(A \cup B)^C = A^C \cap B^C$, which verifies the result.

(ii) $A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$

$\therefore$ $(A \cap B)^C = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(3)

and $A^C \cup B^C = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(4)

From (3) and (4), $(A \cap B)^C = A^C \cup B^C$, which verifies the result.

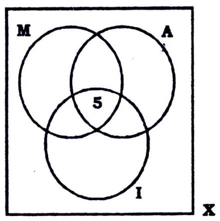
Example – 21

In a class of 200 students who appeared in a certain examination. 35 students failed in MHTCET, 40 in AIEEE, 40 in IIT, 20 failed in MHTCET and AIEEE, 17 in AIEEE and IIT, 15 in MHTCET and IIT and 5 failed in all three examinations. Find how many students

(i) Did not fail in any examination.

(ii) Failed in AIEEE or IIT.

Sol.



$n(M) = 35, n(A) = 40, n(I) = 40$

$n(M \cap A) = 20, n(A \cap I) = 17,$

$n(I \cap M) = 15, n(M \cap A \cap I) = 5$

$n(X) = 200$

$n(M \cup A \cup I) = n(M) + n(A) + n(I) -$

$n(M \cap A) - n(A \cap I) - n(M \cap I) + n(M \cap A \cap I)$

$= 35 + 40 + 40 - 20 - 17 - 15 + 5 = 68$

(i) Number of students passed in all three examination

$= 200 - 68 = 132$

(ii) Number of students failed in IIT or AIEEE

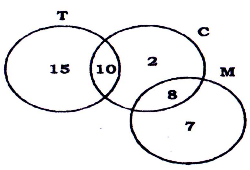
$= n(I \cup A) = n(I) + n(A) - n(I \cap A)$

$= 40 + 40 - 17 = 63$

Example – 22

In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee, 8 students take both milk and coffee. None of the them take tea and milk both and everyone takes atleast one beverage, find the number of students in the hostel.

Sol.



Let the sets, T, C and M are the students who drink tea, coffee and milk respectively. This problem can be solved by Venn diagram.

$n(T) = 25; n(C) = 20; n(M) = 15$

$n(T \cap C) = 10; n(M \cap C) = 8$

Number of students in hostel

$= n(T \cup C \cup M)$

$\therefore n(T \cup C \cup M) = 15 + 10 + 2 + 8 + 7 = 42$

Example – 23

If $A = \{1, 2\}$, find $A \times A \times A$

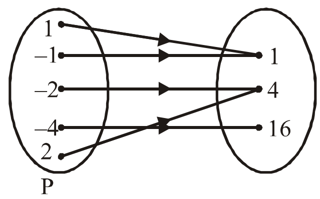
Sol. $A \times A \times A = \{(x, y, z), x \in A, y \in A, z \in A\}$

First find A×A than find A×A×A

so, $A \times A \times A = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1),$
$(2, 2, 2), (2, 2, 1), (2, 1, 2), (1, 2, 2)\}$

Example – 24

Following figure shows a relation between sets P and Q. Write this relation in (i) set builder form, (ii) roster form



Sol. It is clear, that relation R is "y is the square of x".

(i) In set builder form, $R = \{(x, y) : y = x^2, x \in P, y \in Q\}$

(ii) In roster form,

$R = \{(1, 1), (-1, 1), (2, 4), (-2, 4) (-4, 16)\}$

Example – 25

Let R be the relation on Z defined by R = {(a, b); a, b, ∈ Z, a – b is an integer}. Find domain and range of R.

Sol. As for any two integers a & b, a – b is an integer hence domain and range is all integers.

Example – 26

Determine domain and range of :-

$$R = \left\{\left(x + 4, \frac{2 + x}{2 - x}\right) : 4 \le x \le 6,\ x \in N\right\}$$

Sol. $R = \left\{(8, -3), \left(9, -\frac{7}{3}\right), (10, -2)\right\}$

By taking x = 4, 5,6

so, domain $= \{8, 9, 10\}$

range $= \left\{-3, -\frac{7}{3}, -2\right\}$

Example – 27

Let $A = \{1, 2\}$. List all the relations on A.

Sol. Given $A = \{1, 2\}$

$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$

Since a relation R from set A to set A is a subset of $A \times A$

$\therefore$ All the relations on A are :

ϕ, $\{(1, 1)\}$, $\{(1, 2)\}$, $\{(2, 1)\}$, $\{(2, 2)\}$, $\{(1, 1), (1, 2)\}$, $\{(1, 1), (2, 1)\}$, $\{(1, 1), (2, 2)\}$, $\{(1, 2), (2, 1)\}$, $\{(1, 2), (2, 2)\}$, $\{(2, 1), (2, 2)\}$, $\{(1, 1), (1, 2), (2, 1)\}$, $\{(1, 1), (1, 2), (2, 2)\}$, $\{(1, 1), (2, 1), (2, 2)\}$, $\{(1, 2), (2, 1), (2, 2)\}$, $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$.

Since $n(A \times A) = 4$, the number of all relations on the set $A = 2^4$ i.e., 16.

(As number of subsets of a set with n elements is 2^n)

Example – 28

The solution set of $x^2 + 2 \le 3x \le 2x^2 - 5$ is

(a) ϕ (b) $[1, 2]$

(c) $(-\infty, -1) \cup [5/2, \infty)$ (d) none

Ans. (a)

Sol. $x^2 + 2 \le 3x \le 2x^2 - 5$

$x^2 + 2 \le 3x$ *and* $3x \le 2x^2 - 5$

$x^2 - 3x + 2 \le 0$ *and* $2x^2 - 3x - 5 \ge 0$

$(x-1)(x-2) \le 0$ *and* $(2x-5)(x+1) \ge 0$

$\Rightarrow x \in [1, 2]$ *and* $x \in (-\infty, -1] \cup \left[\frac{5}{2}, \infty\right)$

$\Rightarrow x \in \phi$

Example – 29

Find the set of values of 'x' for which the given conditions are true :

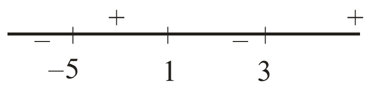
(a) $-(x-1)(x-3)(x+5) < 0$

(b) $\dfrac{(x-1)\,(x-2)}{(x-3)} \le 0$

Ans. (a) $(-5, 1) \cup (3, \infty)$ (b) $(-\infty, 1] \cup [2, 3)$

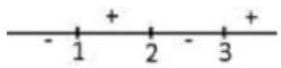
Sol. (a) $-(x-1)(x-3)(x+5) < 0$

$\Rightarrow (x-1)(x-3)(x+5) > 0$



$\Rightarrow x \in (-5, 1) \cup (3, \infty)$

(b) $\dfrac{(x-1)(x-2)}{(x-3)} \le 0$



$x \in (-\infty, 1] \cup [2, 3)$

Example – 30

The number of integral solutions of $\dfrac{x+2}{x^2+1} > \dfrac{1}{2}$ is

(a) 4 (b) 5

(c) 3 (d) none of these

Ans. (c)

Sol. $\dfrac{x+2}{x^2+1} > \dfrac{1}{2}$

$2x + 4 > x^2 + 1 \; (\because x^2 + 1 > 0)$

$x^2 - 2x - 3 < 0$

$\Rightarrow (x-3)(x+1) < 0$

$\Rightarrow x \in (-1, 3)$

Number of integer values $= 3$

Example – 31

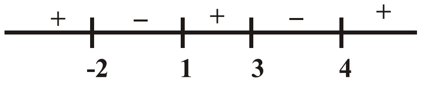
Find the domain of definition of the following

function : $f(x)=\sqrt{\dfrac{(x-1)(x+2)}{(x-3)(x-4)}}$

Sol. For $f(x)$ to be defined $\dfrac{(x-1)(x+2)}{(x-3)(x-4)} \ge 0$ and $x \ne 3, 4$

By wavy – curve method the domain of definition of $f(x)$ is the set

$x \in (-\infty, -2] \cup [1, 3) \cup (4, \infty)$.



Example – 32

Find domain for $f(x)=\sqrt{\cos(\sin x)}$.

Sol. $f(x)=\sqrt{\cos(\sin x)}$ is defined, if

$\cos(\sin x) \ge 0$

As, we know

$-1 \le \sin x \le 1$ for all x

$\cos\theta \ge 0$

(Here, $\theta = \sin x$ lies in the 1st and 4th quadrants)

i.e. $\cos(\sin x) \ge 0$, for all x

i.e. $x \in R$.

Thus, domain $f(x) \in R$

Example – 33

A function f is defined on the set {1, 2, 3, 4, 5} as follows :

$$f(x)=\begin{cases}1+x & \text{if } 1 \le x < 2\\ 2x-1 & \text{if } 2 \le x < 4\\ 3x-10 & \text{if } 4 \le x < 6\end{cases}$$

(i) Find the domain of the function.

(ii) Find the range of the function.

(iii) Find the values of $f(2), f(3), f(4), f(6)$.

Sol. (i) Domain : {1, 2, 3, 4, 5}

(ii) Range :

$f(1)=1+1=2$ $f(4)=3(4)-10=2$

$f(2)=2(2)-1=3$ $f(5)=3(5)-10=5$

$f(3)=2(3)-1=5$

So, range is {2, 3, 5}

(iii) $f(2)=3, f(4)=2, f(3)=5$, f(6) is not defined as 6 is not in domain.

Example – 34

Let A = {1, 2}. List all the relations on A.

Sol. Given A = {1, 2}

$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

Since a relation R from set A to set A is a subset of $A \times A$

$\therefore$ All the relations on A are :

ϕ, {(1, 1)}, {(1, 2)}, {(2, 1)}, {(2, 2)}, {(1, 1), (1, 2)}, {(1, 1), (2, 1)}, {(1, 1), (2, 2)}, {(1, 2), (2, 1)}, {(1, 2), (2, 2)}, {(2, 1), (2, 2)}, {(1, 1), (1, 2), (2, 1)}, {(1, 1), (1, 2), (2, 2)}, {(1, 1), (2, 1), (2, 2)}, {(1, 2), (2, 1), (2, 2)}, {(1, 1), (1, 2), (2, 1), (2, 2)}.

Since $n(A \times A) = 4$, the number of all relations on the set $A = 2^4$ i.e., 16.

(As number of subsets of a set with n elements is 2^n)

Example – 35

Find the domain and range of the following functions

(i) $\left\{\left(x, \dfrac{x^2-1}{x-1}\right) : x \in R, x \ne 1\right\}$

(ii) $\left\{\left(x, \dfrac{1}{1-x^2}\right) : x \in R, x \ne \pm 1\right\}$

Sol. (i) Let $f(x)=\left\{\left(x, \dfrac{x^2-1}{x-1}\right) : x \in R, x \ne 1\right\}$

Clearly, f is not defined when $x = 1$

$\therefore$ f is defined for all real values of x except $x = 1$

$\therefore$ Domain $= R - \{1\}$

Let $y=\dfrac{x^2-1}{x-1}=x+1\,(\text{as } x \ne 1)$

$\therefore$ $x = y - 1$

Clearly $y \ne 2$ as $x \ne 1$

$\therefore$ Range $= R - \{2\}$.

(ii) Let $f(x)=\left\{\left(x, \frac{1}{1-x^2}\right): x \in R, x = \pm 1\right\}$

Clearly, $f(x)=\frac{1}{1-x^2}$ is not defined when $1-x^2=0$

i.e., when $x=\pm 1$

$\therefore$ Domain $= R-\{1,-1\}$

Further, $y=\frac{1}{1-x^2}$ Since $x \neq \pm 1$

$\Rightarrow \quad (1-x^2)=\frac{1}{y} \qquad \Rightarrow \quad x=\pm\sqrt{\left(1-\frac{1}{y}\right)}=\pm\sqrt{\frac{y-1}{y}}$

$\therefore$ x is defined when $y \in (-\infty, 0) \cup [1, \infty)$.

$\Rightarrow$ $y \in (-\infty, 0) \cup [1, \infty)$

$\therefore$ Range $= (-\infty, 0) \cup [1, \infty)$.

Example – 36

Find the range of the following function :

$$f(x)=ln\sqrt{x^2+4x+5}$$

Sol. Here $f(x)=\ln\sqrt{x^2+4x+5}=\ln\sqrt{(x+2)^2+1}$

i.e. x^2+4x+5 takes all values in $[1, \infty)$

$\Rightarrow$ $f(x)$ will take all values in $[0, \infty)$.

Hence range of $f(x)$ is $[0, \infty)$.

Example – 37

Find the range of the function $y=\frac{1}{2-\sin 3x}$

Sol. Clearly, as Denominator $(2-\sin 3x) \neq 0$

$\Rightarrow$ Domain : $x \in R$

We have, $\quad y=\frac{1}{2-\sin 3x}$

Note : (sin 3x) can be seperated & written as a function of y

$\Rightarrow \quad 2-\sin 3x=\frac{1}{y}$

$\Rightarrow \quad \sin 3x=\frac{2y-1}{y}$

for x to be real

$\Rightarrow \quad -1 \le \frac{2y-1}{y} \le 1$ (since, $-1 \le \sin 3x \le 1$)

$-1 \le \frac{2y-1}{y} \le 1$

$\frac{2y-1}{y}+1 \ge 0 \cap \frac{2y-1}{y}-1 \le 0$

$\frac{3y-1}{y} \ge 0 \cap \frac{y-1}{y} \le 0$

$\Rightarrow \quad y \ge \frac{1}{3} \cap y \le 1$

$\Rightarrow \quad$ Range : $y \in \left[\frac{1}{3}, 1\right] \leftarrow$

Alternate Method :

$$y=\frac{1}{2-\sin 3x}$$

we know, $\quad -1 \le \sin 3x \le 1$

$\Rightarrow \quad 1 \ge -\sin 3x \ge -1$

$\Rightarrow \quad 1 \le 2-\sin 3x \le 3$

$\Rightarrow \quad \frac{1}{1} \ge \left(\frac{1}{2-\sin 3x}\right) \ge \frac{1}{3}$

$\Rightarrow \quad$ Range $y \in \left[\frac{1}{3}, 1\right]$

Inequality changes upon reciprocating as all expressions across inequality are (positive).

Example – 38

Let $f, g : R \to R$ be defined respectively by $f(x)=x+1$, $g(x)=2x-3$. Find $f+g$, $f-g$ and f/g.

Sol. Let $\quad f(x)=x+1, g(x)=2x-3$

$\therefore \quad f+g=f(x)+g(x)=(x+1)+(2x-3)$

$=3x-2$

$f-g=f(x)-g(x)=(x+1)-(2x-3)$

$=x+1-2x+3=-x+4$

$\frac{f}{g}=\frac{f(x)}{g(x)}=\frac{x+1}{2x-3}; \ x \neq \frac{3}{2}$

Example – 39

Check whether the function :

$f(x) = 2x^3 + 3x^2 + 6x + 5$ is

one-to-one or many-to-one

Sol. $f(x) = 2x^3 + 3x^2 + 6x + 5$

$f'(x) = 6(x^2 + x + 1) > 0 \ \forall \ x \in R$

as $(a > 0 \ \& \ D < 0)$ for $x^2 + x + 1$

$\Rightarrow$ $f(x)$ is increasing function on its entire domain

$\Rightarrow$ one-to-one function.

Example – 40

Let $A = \{x : -1 \le x \le 1\} = B$ for a mapping $f : A \to B$. For the following functions from A to B, find whether it is surjective or bijective.

$f(x) = |x|$

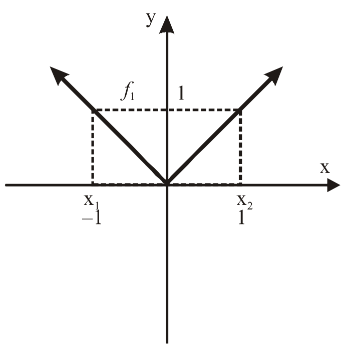
Sol. $f(x) = |x|$

Graphically we can see that for $x \in [-1, 1]$

$y = |x| \in [0, 1]$

Since, Range $([0, 1]) \subset$ co-domain $(B = [-1, 1])$

many-to-one



$\Rightarrow$ into function

$\Rightarrow$ $f : [-1, 1] \to [-1, 1], \ f(x) = |x|$

is many-to-one & into

Example – 41

Solve $(x + 1)^2 + (x^2 + 3x + 2)^2 = 0$

Sol. Here, $(x + 1)^2 + (x^2 + 3x + 2)^2 = 0$ if and only if each term is zero simultaneously,

$(x + 1) = 0$ and $(x^2 + 3x + 2) = 0$

i.e., $x = -1$ and $x = -1, -2$

The common solution is $x = -1$

Hence, solution of above equation is $x = -1$

Example – 42

Solve $\dfrac{|x+3| + x}{x+2} > 1$

Sol. $\dfrac{|x+3| + x}{x+2} - 1 > 0$

$\Rightarrow \dfrac{|x+3| + x - x - 2}{x+2} > 0$

$\Rightarrow \dfrac{|x+3| - 2}{x+2} > 0$ ...(i)

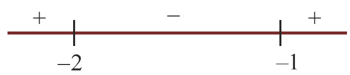
Now two cases arises :

Case I : When $x + 3 \ge 0$ ...(ii)

$\Rightarrow \dfrac{x + 3 - 2}{x + 2} > 0$

$\Rightarrow \dfrac{x + 1}{x + 2} > 0$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$ using number line rule as shown in figure.



But $x \ge -3$ {from (ii)}

$\Rightarrow x \in [-3, -2) \cup (-1, \infty)$ ...(a)

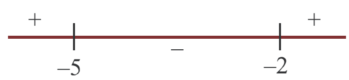
Case II : When $x + 3 < 0$ ...(iii)

$\Rightarrow \dfrac{-(x + 3) - 2}{x + 2} > 0$

$\Rightarrow \dfrac{-(x + 5)}{(x + 2)} > 0$

$\Rightarrow \dfrac{(x + 5)}{(x + 2)} < 0$

$\Rightarrow x \in (-5, -2)$ using number line rule as shown in figure.



But $x < -3$ {from (iii)}

$\therefore \quad x \in (-5, -3)$ ...(b)

Thus from (a) and (b), we have;

$x \in [-3, -2) \cup (-1, \infty) \cup (-5, -3)$

$\Rightarrow \quad x \in (-5, -2) \cup (-1, \infty)$

Example – 43

The value of x if $|x+3| > |2x-1|$ is

(a) $\left(-\frac{2}{3}, 4\right)$ (b) $\left(-\frac{2}{3}, \infty\right)$

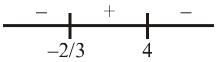
(c) (0, 1) (d) None of these

Sol. Squaring both sides, we get

$|x+3|^2 > |2x-1|^2$

or $\{(x+3)-(2x-1)\}\{(x+3)+(2x-1)\} > 0$

$\Rightarrow \{(-x+4)(3x+2)\} > 0$



$\Rightarrow \quad x \in \left(-\frac{2}{3}, 4\right)$

Hence, (a) is the correct answer.

Example – 44

Solve for x

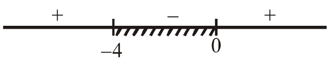
$$|x| + |x+4| = 4$$

Sol. $|x| + |x+4| = 4$

As we know, $|x| + |y| = |x-y|$, iff $xy \le 0$

$x(x+4) \le 0$

Using number line rule,



$\Rightarrow \quad x \in [-4, 0]$

Example – 45

Solve $\left|\frac{x}{x-1}\right| + |x| = \frac{x^2}{|x-1|}$

Sol. Let $f(x) = \frac{x}{x-1}$ and $g(x) = x$

$\therefore \quad f(x) + g(x) = \frac{x}{x-1} + x = \frac{x^2}{x-1}$

Using, $|f(x)| + |g(x)| = |f(x) + g(x)|$

i.e. $f(x) \,.\, g(x) \ge 0$

$\Rightarrow \quad \frac{x}{x-1}.x \ge 0 \Rightarrow \frac{x^2}{x-1} \ge 0$

$\Rightarrow \quad x \in \{0\} \cup (1, \infty)$

Example – 46

If y =3[x] +1 =2[x – 3] +5, then find the value of [x + y], where [.] represents greatest integer function.

Sol. We are given that $3[x] + 1 = 2([x]-3) + 5$

$\Rightarrow \quad [x] = -2$

$\Rightarrow \quad y = 3(-2) + 1 = -5$

Hence $[x+y] = [x] + y = -2 - 5 = -7$

Example – 47

Solve the equation $|2x-1| = 3[x] + 2\{x\}$ for x.

where [.] represents greatest integer function and {} represents fraction part function.

Sol. **Case I :** For $x < \frac{1}{2}, |2x-1| = 1-2x$

$\Rightarrow \quad 1-2x = 3[x] + 2\{x\}.$

$\Rightarrow \quad 1-2x = 3(x-\{x\}) + 2\{x\}.$

$\Rightarrow \quad \{x\} = 5x - 1.$

Now $0 \le \{x\} < 1$

$\Rightarrow \quad 0 \le 5x - 1 < 1.$

$\Rightarrow \quad \frac{1}{5} \le x < \frac{2}{5} \Rightarrow [x] = 0$

$\Rightarrow \quad x = \{x\} \Rightarrow x = 5x - 1$

$\Rightarrow \quad x = \frac{1}{4}$, which is a solution.

Case II : For $x \ge \frac{1}{2}, |2x-1| = 2x-1$

$\Rightarrow \quad 2x-1 = 3[x] + 2\{x\}.$

$\Rightarrow \quad 2x-1 = 3(x-\{x\}) + 2\{x\}.$

$\{x\} = x + 1$

Now $0 \leq \{x\} < 1$

$\Rightarrow \quad 0 \leq x + 1 < 1. \Rightarrow \quad -1 \leq x < 0.$

which is not possible since $x \geq \frac{1}{2}$.

Hence $x = \frac{1}{4}$ is the only solution.

Example – 48

For a real number x, $[x]$ denotes the integral part of x. The value of

$$\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \left[\frac{1}{2} + \frac{2}{100}\right] + \ldots + \left[\frac{1}{2} + \frac{99}{100}\right] \text{is}$$

(a) 49 (b) 50

(c) 48 (d) 51

Ans. (b)

Sol. $$\underbrace{\left[\frac{1}{2} + \frac{1}{100}\right] + \ldots\ldots \left[\frac{1}{2} + \frac{49}{100}\right]}_{each\ has\ value = 0} +$$

$$\underbrace{\left[\frac{1}{2} + \frac{50}{100}\right] + \ldots\ldots \left[\frac{1}{2} + \frac{99}{100}\right]}_{each\ has\ value = 1}$$

$= 50$

Example – 49

Find the domain of definition of the following

function : $f(x) = \sqrt{\log_{\frac{1}{2}}(2x - 3)}$

Sol. For f(x) to be defined $\log_{1/2}(2x - 3) \geq 0$

$\Rightarrow \quad 2x - 3 \leq 1$

$\Rightarrow \quad x \leq 2 \qquad ....(1)$

Also $2x - 3 > 0$

$\Rightarrow \quad x > \frac{3}{2}. \qquad ....(2)$

Combining (1) and (2) we get the required values of x.

Hence the domain of definition of f(x) is the set $\left(\frac{3}{2}, 2\right]$

Example – 50

Find the domain of the function;

$$f(x) = \frac{1}{\log_{10}(1 - x)} + \sqrt{(x + 2)}$$

Sol. $f(x) = \frac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$

{as we know; $\log_a x$ is defined when x and a > 0 and a ≠ 1 also $\log_a 1 = 0$}

Thus, $\log_{10}(1 - x)$ exists when, $1 - x > 0 \qquad ...(i)$

also $\frac{1}{\log_{10}(1 - x)}$ exists when, $1 - x > 0$

and $\quad 1 - x \neq 1 \qquad ...(ii)$

$\Rightarrow \quad x < 1 \quad$ and $\quad x \neq 0 \qquad ...(iii)$

also we have $\sqrt{x + 2}$ exists when $x + 2 \geq 0$

or $\quad x \geq -2 \qquad ....(iv)$

Thus, $f(x) = \frac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$ exists when (iii) and (iv) both holds true.

$\Rightarrow \quad -2 \leq x < 1 \quad$ and $\quad x \neq 0$

$\Rightarrow \quad x \in [-2, 0) \cup (0, 1)$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Types of sets & Subset

1. The set of intelligent students in a class is

(a) a null set

(b) a singleton set

(c) a finite set

(d) not a well defined collection

2. Which of the following is the empty set?

(a) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$

b) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$

(c) $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$

(d) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

3. Let $A = \{2, 3, 4\}$ and $X = \{0, 1, 2, 3, 4\}$, then which of the following statements is correct

(a) $\{0\} \in A^c$ w.r.t. X (b) $\phi \in A^c$ w.r.t. X

(c) $\{0\} \subset A^c$ w.r.t.X (d) $0 \subset A^c$ w.r.t. X.

Operation on sets

4. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, \ldots\ldots, 18\}$ and N is the universal set, then $A^c \cup ((A \cup B) \cap B^c)$ is

(a) A (b) N

(c) B (d) None of these

5. Let $A = \{x : x \text{ is a multiple of } 3\}$ and $B = \{x : x \text{ is a multiple of } 5\}$. Then $A \cap B$ is given by

(a) $\{3, 6, 9 \ldots\}$ (b) $\{5, 10, 15, 20, \ldots\}$

(c) $\{15, 30, 45, \ldots\}$ (d) None of these

6. If $Y \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$, then

(a) The smallest set of Y is $\{3, 5, 9\}$

(b) The smallest set of Y is $\{2, 3, 5, 9\}$

(c) The largest set of Y is $\{1, 2, 3, 5\}$

(d) The largest set of Y is $\{2, 3, 5, 9\}$

7. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $[A \cup (B \cap C)]$ is

(a) $\{1, 2, 3, 4, 5, 6\}$ (b) $\{1, 2, 4, 5\}$

(c) $\{1, 2, 3, 4\}$ (d) $\{3\}$

8. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to

(a) B^c (b) A^c

(c) B (d) A

9. If $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$ and $C = \{3, 4, 6, 7\}$, then

(a) $A - (B \cap C) = \{1, 3, 4\}$ (b) $A - (B \cap C) = \{1, 2, 4\}$

(c) $A - (B \cup C) = \{2, 3\}$ (d) $A - (B \cup C) = \{1, 2\}$

Classification of function

10. Let $A = [-1, 1]$ and $f : A \to A$ be defined as $f(x) = x|x|$ for all $x \in A$, then $f(x)$ is

(a) many-one into function

(b) one-one into function

(c) many-one onto function

(d) one-one onto function

11. The function $f : R \to R$ defined by

$f(x) = (x-1)(x-2)(x-3)$ is

(a) one-one but not onto

(b) onto but not one-one

(c) both one-one and onto

(d) neither one-one nor onto

12. Let $f : R \to R$ be defined by $f(x) = \dfrac{|x| - 1}{|x| + 1}$ then f is:

(a) both one - one and onto

(b) one - one but not onto

(c) onto but not one - one

(d) neither one - one nor onto.

13. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{when n is odd} \\ -\dfrac{n}{2}, & \text{when n is even} \end{cases} \quad \text{is}$$

(a) onto but not one-one

(b) one-one and onto both

(c) neither one-one nor onto

(d) one-one but not onto

Domain of a function

14. Find the domain of $f(x)=\frac{1}{\sqrt{x-5}}+x^2+\frac{1}{\sqrt{x+7}}$

(a) $x \in [-7, 5]$ (b) $x \in (5, \infty)$

(c) $x \in (-\infty, 7)$ (d) none of these

15. Find the domain $y=\sqrt{1-x}+\sqrt{x-5}$

(a) $x \in \phi$ (b) $y \in (-\infty, 1]$

(c) $x \in (-\infty, 1] \cup [5, \infty)$ (d) none of these

16. The domain of the function

$f(x)=\sqrt{x-3-2\sqrt{x-4}}-\sqrt{x-3+2\sqrt{x-4}}$ is

(a) $[4, \infty)$ (b) $(-\infty, 4]$

(c) $(4, \infty)$ (d) $(-\infty, 4)$

17. If $f(x)=\frac{1}{\sqrt{|x|-x}}$, then domain of $f(x)$ is

(a) $(-\infty, 0)$ (b) $(-\infty, 2)$

(c) $(-\infty, \infty)$ (d) None of the above

Modulus functions

18. $|3x+7| < 5$, then x belongs to

(a) $(-4, -3)$ (b) $(-4, -2/3)$

(c) $(-5, 5)$ (d) $(-5/3, 5/3)$

19. Solution of $|3x-2| \geq 1$ is

(a) $[1/3, 1]$ (b) $(1/3, 1)$

(c) $\{1/3, 1\}$ (d) $\left(-\infty, \frac{1}{3}\right] \cup [1, \infty)$

20. If $-5 < x < 4$, then :

(a) $0 \leq |x| < 4$ (b) $4 < |x| < 5$

(c) $0 \leq |x| < 5$ (d) none of these

21. $|2x-3| < |x+5|$, then x belongs to

(a) $(-3, 5)$ (b) $(5, 9)$

(c) $(-2/3, 8)$ (d) $(-8, 2/3)$

22. $\left|\frac{x^2+6}{5x}\right| \geq 1$

(a) $(-\infty, -3)$

(b) $(-\infty, -3) \cup (3, \infty)$

(c) R

(d) $(-\infty, -3] \cup [-2, 0) \cup (0, 2] \cup [3, \infty)$

23. Solution of $\left|x+\frac{1}{x}\right| < 4$ is

(a) $\left(2-\sqrt{3}, 2+\sqrt{3}\right) \cup \left(-2-\sqrt{3}, -2+\sqrt{3}\right)$

(b) $R-\left(2-\sqrt{3}, 2+\sqrt{3}\right)$

(c) $R-\left(-2-\sqrt{3}, -2+\sqrt{3}\right)$

(d) none of these

Greatest integer functions

24. The domain of the function

$$f(x)=\frac{1}{\sqrt{[x]^2-[x]-6}}$$

where [ ] denotes greatest integer function

(a) $R-[-2, 4)$ (b) $R-\{-3, 2\}$

(c) R (d) $R-\{2, 3\}$

25. If $[x]^2=[x+2]$, where $[x]$ = the greatest integer less than or equal to x, then x must be such that

(a) $x = 2, -1$ (b) $x \in [2, 3)$

(c) $x \in [-1, 0)$ (d) none of these

26. The domain of the function $f(x)=\log_e(x-[x])$, where [.] denotes the greatest integer function, is

(a) R (b) R–Z

(c) $(0, +\infty)$ (d) None of these

Logarithmic functions

27. Let $f(x)=\log_{x^2}25$ and $g(x)=\log_x 5$ then $f(x)=g(x)$ holds for x belonging to

(a) R (b) $(0, 1) \cup (1, +\infty)$

(c) ϕ (d) None of these

28. The domain of the function $f(x)=\log_2(\log_3(\log_4 x))$ is

(a) $(-\infty, 4)$ (b) $(4, \infty)$

(c) $(0, 4)$ (d) $(1, \infty)$

29. The value of x, $\log_{1/2} x \geq \log_{1/3} x$ is

(a) $(0, 1]$ (b) $(0, 1)$

(c) $[0, 1)$ (d) none

30. Indicate the correct alternative : The number $\log_2 7$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a prime number

Range of a function

31. Find the Range $y = \dfrac{2x+1}{x-5}$

(a) $R - \{2\}$ (b) $x \neq 5$
(c) $R - \{5\}$ (d) none of these

32. Range of the function $f(x) = \dfrac{x}{1+x^2}$ is

(a) $(-\infty, \infty)$ (b) $[-1, 1]$
(c) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (d) $\left[-\sqrt{2}, \sqrt{2}\right]$

33. The range of the function $f(x) = x^2 + \dfrac{1}{x^2+1}$

(a) $[1, \infty)$ (b) $[2, \infty)$
(c) $\left[\dfrac{3}{2}, \infty\right)$ (d) None of these

Wavy Curve Method

34. The set of values of x satisfying the inequalities $(x-1)(x-2) < 0$ and $(3x-7)(2x-3) > 0$ is

(a) $(1, 2)$ (b) $(2, 7/3)$
(c) $(1, 7/3)$ (d) $(1, 3/2)$

35. If $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0$, then

(a) $x > 3$ (b) $x < 3$
(c) $3 < x < 4$ (d) $x = 7/2$

36. Find the set of values of 'x' for which the given condition is true $(x-1)(x-3)(x+5) > 0$

(a) $(-5, 1) \cup (3, \infty)$ (b) $(-1, 5)$
(c) $[-5, 1] \cup [3, \infty)$ (d) none of these

37. The value of x for which $12x - 6 < 0$ and $12 - 3x < 0$

(a) ϕ (b) R
(c) $R - \{0\}$ (d) none of these

38. The value of x for which $\dfrac{x-3}{4} - x < \dfrac{x-1}{2} - \dfrac{x-2}{3}$ and $2 - x > 2x - 8$

(a) $[-1, 10/3]$ (b) $(-1, 10/3)$
(c) R (d) none of these

39. If $c < d$, $x^2 + (c+d)x + cd < 0$, then x belongs to.

(a) $(-d, -c]$ (b) $(-d, -c)$
(c) R (d) ϕ

40. Solution of $\dfrac{x-7}{x+3} > 2$ is

(a) $(-3, \infty)$ (b) $(-\infty, -13)$
(c) $(-13, -3)$ (d) none of these

41. The set of values of x which satisfy the inequations $5x + 2 < 3x + 8$ and $\dfrac{x+2}{x-1} < 4$ is

(a) $(-\infty, 1)$ (b) $(2, 3)$
(c) $(-\infty, 3)$ (d) $(-\infty, 1) \cup (2, 3)$

42. If $x^2 - 1 \leq 0$ and $x^2 - x - 2 \geq 0$, then x lies in the interval set

(a) $(1, -1)$ (b) $(-1, 1)$
(c) $(1, 2)$ (d) $\{-1\}$

43. The solution set of $\dfrac{x^2 - 3x + 4}{x+1} > 1$, $x \in R$ is

(a) $(3, \infty)$ (b) $(-1, 1) \cup (3, \infty)$
(c) $[-1, 1] \cup [3, \infty)$ (d) none

44. If $\dfrac{1}{a} < \dfrac{1}{b}$, then :

(a) $|a| > |b|$ (b) $a < b$
(c) $a > b$ (d) none of these

45. If $-2 < x < 3$, then :

(a) $4 < x^2 < 9$ (b) $0 \leq |x| < 5$
(c) $0 \leq x^2 < 9$ (d) None of these

46. $x > \sqrt{2 - x^2}$

(a) $x \in (1, \infty)$ (b) $x \in (-\infty, -1) \cup (1, \infty)$
(c) $x \in \left(1, \sqrt{2}\right]$ (d) $x \in \left[\sqrt{2}, \infty\right)$

Misc examples-sets-functions

47. A function whose graph is symmetrical about the y-axis is given by

(a) $f(x) = \log_e\left(x + \sqrt{x^2+1}\right)$

(b) $f(x + y) = f(x) + f(y)$ for all $x, y \in R$

(c) $f(x) = \cos x + \sin x$

(d) None of these

48. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

(a) $f(x) = f(-x)$ (b) $f(2+x) = f(2-x)$

(c) $f(x+2) = f(x-2)$ (d) $f(x) = -f(-x)$

49. If $A = \{1, 2, 3\}$, $B = \{a, b\}$, then $A \times B$ is given by

(a) $\{(1, a), (2, b), (3, b)\}$

(b) $\{(1, b), (2, a)\}$

(c) $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

(d) $\{(1, a), (2, a), (2, b), (3, b)\}$

50. Solve for x : $3^{(x^2-2)} < \left(\frac{1}{3}\right)^{\left(1-\frac{3}{2}|x|\right)}$

(a) $\left(-\sqrt{2}, -1\right)$ (b) $\left(-\sqrt{2}, 2\right)$

(c) $\left(-2, -\sqrt{2}\right)$ (d) None of these

51. The largest interval among the following for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

(a) $-4 < x \le 0$ (b) $0 < x < 1$

(c) $-100 < x < 100$ (d) $-\infty < x < \infty$

52. If $f(x) = x^2 - 3x + 1$ and $f(2\alpha) = 2f(\alpha)$, then α is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ (d) none of these

Numerical Value Type Questions

53. If $A = \{x : x = 4n + 1, 2 \le n \le 5\}$, $n \in N$ then number of subsets of A is

54. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \ne +1\}$. Then the number of elements in the power set of R is:

55. In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, percentage of persons travelling by car or bus is

56. X and Y are two sets such that n (X) = 17, n(Y) = 23, $n(X \cup Y) = 38$ then $n(X \cap Y)$ is

57. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, then number of elements $S \cup T$ has

58. In a committee 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. The number of persons speaking at least one of these two languages is

59. In a group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English .Then number of persons who can speak Hindi only is

60. In a statistical investigation of 1,003 families of Calcutta, it was found that 63 families had neither a radio nor a T.V, 794 families had a radio and 187 had a T.V. The number of families in that group having both a radio and a T.V is

61. If A has 3 elements and B has 6 elements, then the minimum number of elements in the set $A \cup B$ is

62. If the value for which $\frac{(x-1)}{x} \ge 2$ is [–k, 0), then the value of k is

63. If the domain of the function $f(x) = \sqrt{\log_{10}\frac{3-x}{x}}$ is (a, b] then a + 2b equals

64. The number of real solutions of

$$\sqrt{x^2 - 4x + 3} = \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$$ is

65. The number of real solutions of the equation $e^x = x$ is

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. Let $S = \{x \in R : x \geq 0$ and $2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\}$ Then S : **(2018)**

(a) Contain exactly four element

(b) is an empty set.

(c) contain exactly one element

(d) contains exactly two elements.

2. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, when $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_2(x-y)$ equals : **(8-04-2019/Shift-2)**

(a) $2f_1(x)f_1(y)$ (b) $2f_1(x+y)f_1(x-y)$

(c) $2f_1(x)f_2(y)$ (d) $2f_1(x+y)f_2(x-y)$

3. The domain of the definition of the function

$f(x) = \dfrac{1}{4-x^2} + \log_{10}\left(x^3 - x\right)$ is:

(9-04-2019/Shift-2)

(a) $(-1,0)\cup(1,2)\cup(3,\infty)$

(b) $(-2,-1)\cup(-1,0)\cup(2,\infty)$

(c) $(-1,0)\cup(1,2)\cup(2,\infty)$

(d) $(1,2)\cup(2,\infty)$

4. Two newspapers A and B are published in city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is: **(9-04-2019/Shift-2)**

(a) 13.9 (b) 12.8

(c) 13 (d) 13.5

5. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true ? **(12-04-2019/Shift-2)**

(a) $B \cap C \neq \phi$

(b) If $(A - B \subseteq C)$, then $A \subseteq C$

(c) $(C \cup A) \cap (C \cup B) = C$

(d) If $(A - C) \subseteq B$, then $A \subseteq B$

6. Let A = $\{x \in R$: x is not a positive integer$\}$. Define a function f: $A \to R$ as $f(x) = \dfrac{2x}{x-1}$, then f is: **(9-01-2019/Shift-2)**

(a) not injective

(b) neither injective nor surjective

(c) surjective but not injective

(d) injective but not surjective

7. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is: **(10-01-2019/Shift-1)**

8. Let N be the set of natural numbers and two functions f and g be defined as $f, g : \mathbf{N} \to \mathbf{N}$ such that

$$f(n) = \begin{cases} \dfrac{n+1}{2} & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is: **(10-01-2019/Shift-2)**

(a) onto but not one-one

(b) one-one but not onto

(c) both one-one and onto

(d) neither one-one nor onto

9. Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}, x \in R$.

Then the range of f is : **(11-01-2019/Shift-1)**

(a) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (b) $R - [-1, 1]$

(c) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $(-1, 1) - \{0\}$

10. Let $S = \{1, 2, 3, \ldots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is : **(12-01-2019/Shift-1)**

(a) $2^{100} - 1$ (b) $2^{50}\left(2^{50} - 1\right)$

(c) $2^{50} - 1$ (d) $2^{50} + 1$

11. Let Z be the set of integers.

If $A = \left\{x \in Z : 2^{(x+2)\left(x^2-5x+6\right)} = 1\right\}$ and

$B = \{x \in Z : -3 < 2x - 1 < 9\}$ then the number of subsets of the set A x B, is **(12-01-2019/Shift-2)**

(a) 2^{15} (b) 2^{18}

(c) 2^{12} (d) 2^{10}

12. If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is : **(2-9-2020/Shift-1)**

(a) $\{-1, 0, 1\}$ (b) $\{-2, -1, 1, 2\}$

(c) $\{0, 1\}$ (d) $\{-2, -1, 0, 1, 2\}$

13. Let [t] denote the greatest integer $\leq$ t. Then the equation in x, $[x]^2 + 2[x+2] - 7 = 0$ has : **(4-09-2020/Shift-1)**

(a) exactly four integral solutions

(b) infinitely many solutions

(c) no integral solution

(d) exactly two solution

14. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x % of the people read both the newspapers, then a possible value of x can be : **(4-09-2020/Shift-1)**

(a) 37 (b) 29

(c) 65 (d) 55

15. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be: **(5-09-2020/Shift-1)**

(a) 63 (b) 54

(c) 38 (d) 36

16. Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _____.

(6-09-2020/Shift-1)

17. Let $X = \{n \in N : 1 \leq n \leq 50\}$. If $A = \{n \in X : n$ is a multiple of 2$\}$ and $B = \{n \in X : n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is _______. **(7-01-2020/Shift-2)**

18. Let $f : (1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{x^2+1}$,

where $[x]$ denotes the greatest integerd $\leq x$ Then the range of f is: **(8-01-2020/Shift-2)**

(a) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$ (b) $\left(\frac{2}{5}, \frac{4}{5}\right]$

(c) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (d) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

19. If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x - 2| \geq 3\}$ then :

(9-1-2020/Shift-2)

(a) $A - B = [-1, 2]$ (b) $B - A = R - (-2, 5)$

(c) $A \cup B = R - (2, 5)$ (d) $A \cap B = (-2, -1)$

20. Let $A = \{n \in N : n \text{ is a } 3-\text{digit number}\}$

$B = \{9k + 2 : k \in N\}$ and

$C = \{9k + l : k \in N\}$ for some $l(0 < l < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to ______

(24-02-2021/Shift-1)

21. The number of elements in the set

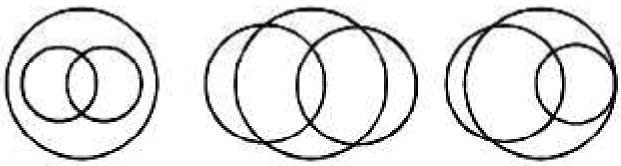
$\{x \in R : (|x| - 3)|x + 4| = 6\}$ is equal to

(16-03-2021/Shift-1)

(a) 1 (b) 3

(c) 2 (d) 4

22. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement ?



P Q R

(17-03-2021/Shift-1)

(a) P and R (b) None of these

(c) Q and R (d) P and Q

23. Let $A = \{n \in N \mid n^2 \le n + 10{,}000\}, B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ______.

(27-07-2021/Shift-2)

24. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If $K\%$ of them are suffering from both ailments, then K cannot belong to the set:

(26-08-2021/Shift-1)

(a) $\{80, 83, 86, 89\}$ (b) $\{79, 81, 83, 85\}$

(c) $\{84, 87, 90, 93\}$ (d) $\{84, 86, 88, 90\}$

25. If $A = \{x \in R : |x - 2| > 1\}$, $B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\}$, $C = \{x \in R : |x - 4| \ge 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _______.

(27-08-2021/Shift-1)

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

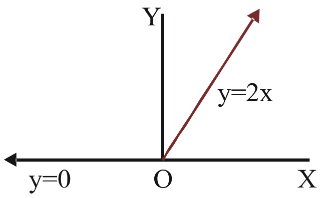
1. The solution set of $log_2 |4-5x| > 2$ is

(a) $\left(8/5, \infty\right)$ (b) $\left(4/5, 8/5\right)$

(c) $(-\infty, 0)\cup\left(8/5, \infty\right)$ (d) none

2. The graph of a real-valued function $f(x)$ is the following.

The function is



(a) $f(x) = x-|x|$ (b) $f(x) = x+|x|$

(c) $f(x) = 2x$ (d) None of these

3. Solution of the inequality $x > \sqrt{(1-x)}$ is given by

(a) $\left(-\infty, \left(-1-\sqrt{5}\right)/2\right)$

(b) $\left(\left(\sqrt{5}-1\right)/2, \infty\right)$

(c) $\left(-\infty, \left(-1-\sqrt{5}\right)/2\right)\cup\left(\left(\sqrt{5}-1\right)/2, \infty\right)$

(d) $\left(\left(\sqrt{5}-1\right)/2, 1\right]$

4. If for $x \in R$, $\frac{1}{3} \le \frac{x^2-2x+4}{x^2+2x+4} \le 3$, then $\frac{9.3^{2x}+6.3^x+4}{9.3^{2x}-6.3^x+4}$ lies b/w

(a) 1 and 2 (b) 1/3 and 3

(c) 0 and 4 (d) none of these

5. The domain of the function $f(x) = \sqrt{x-\sqrt{1-x^2}}$ is

(a) $\left[-1, -\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}}, 1\right]$

(b) $[-1, 1]$

(c) $\left(-\infty, -\frac{1}{2}\right]\cup\left[\frac{1}{\sqrt{2}}, \infty\right)$

(d) $\left[\frac{1}{\sqrt{2}}, 1\right]$

6. The function $f: R \to R$ defined by $f(x) = 6^x + 6^{|x|}$ is

(a) one-one and onto (b) many-one and onto

(c) one-one and into (d) many-one and into

7. If $|x-1|+|x|+|x+1| \ge 6$; then x lies in

(a) $(-\infty, 2]$ (b) $(-\infty, -2]\cup[2, \infty)$

(c) R (d) ϕ

8. Solution of $|1/x-2| < 4$ is

(a) $\left(-\infty, -1/2\right)$ (b) $\left(1/6, \infty\right)$

(c) $\left(-1/2, 1/6\right)$ (d) $\left(-\infty, -1/2\right)\cup\left(1/6, \infty\right)$

9. Solution of $2^x + 2^{|x|} \ge 2\sqrt{2}$ is

(a) $\left(-\infty, \log_2\left(\sqrt{2}+1\right)\right)$

(b) $(0, \infty)$

(c) $\left(\frac{1}{2}, \log_2\left(\sqrt{2}-1\right)\right)$

(d) $\left(-\infty, \log_2\left(\sqrt{2}-1\right)\right]\cup\left[1/2, \infty\right)$

10. If $f(x) = \cos[\pi]x + \cos[\pi x]$, where $[y]$ is the greatest integer function of y then $f(\pi/2)$ is equal to

(a) cos 3 (b) 0

(c) cos 4 (d) none of these

11. The domain of the function $f(x) = \sqrt{x^2 - [x]^2}$, where $[x]$ = the greatest integer less than or equal to x is

(a) R (b) $[0, +\infty)$

(c) $(-\infty, 0]$ (d) None of these

12. Let $f(x) = [x]$ = the greatest integer less than or equal to x and $g(x) = x - [x]$. Then for any two real numbers x and y.

(a) $f(x+y) = f(x) + f(y)$

(b) $g(x+y) = g(x) + g(y)$

(c) $f(x+y) = f(x) + f(y + g(x))$

(d) none of these

13. The domain of $f(x) = \sqrt{\log_{x^2-1}(x)}$ is

(a) $(\sqrt{2}, +\infty)$ (b) $(0, \infty)$

(c) $(1, +\infty)$ (d) None of these

14. The domain of the real-valued function $f(x) = \log_e |\log_e x|$ is

(a) $(1, +\infty)$ (b) $(0, +\infty)$

(c) $(e, +\infty)$ (d) None of these

15. If $x = \log_a (bc)$, $y = \log_b (ca)$ and $z = \log_c (ab)$ then which of the following is equal to 1?

(a) $x + y + z$ (b) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

(c) xyz (d) none of these

16. If $f(n+1) = \dfrac{2f(n)+1}{2}$, n = 1, 2, ... and $f(1) = 2$, then $f(101)$ equals

(a) 52 (b) 49

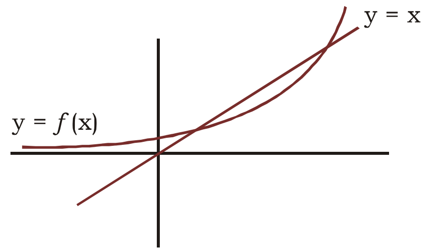
(c) 48 (d) 51

17. The domain of function $f(x) = \dfrac{1}{\sqrt{x^2 - \{x\}^2}}$, where $\{x\}$ denotes fraction part of x.

(a) $R - [0, 1)$ (b) $R - \left[\frac{1}{2}, 1\right]$

(c) $(-\infty, \frac{1}{2}] \cup (1, \infty)$ (d) none of these

18. If graph of $y = f(x)$ is



Then $f(x)$ can be

(a) $y = 2e^x$ (b) $y = 4e^x$

(c) $y = e^{x+\frac{1}{2}}$ (d) $y = \frac{1}{4}e^x$

19. The domain of definition of the function y(x) is given by the equation $2^x + 2^y = 2$ is

(a) $0 < x \le 1$ (b) $0 \le x \le 1$

(c) $-\infty < x \le 0$ (d) $-\infty < x < 1$

20. Solution set of the inequality : $\dfrac{1}{2^x - 1} > \dfrac{1}{1 - 2^{(x-1)}}$ is

(a) $(1, \infty)$ (b) $\left(0, \log_2\left(4/3\right)\right)$

(c) $(-1, \infty)$ (d) $\left(0, \log_2\left(4/3\right)\right) \cup (1, \infty)$

21. If $a^2 + b^2 + c^2 = 1$, then ab + bc + ca lies in the interval

(a) $\left[-\frac{1}{2}, 1\right]$ (b) $\left[0, \frac{1}{2}\right]$

(c) $[0, 1]$ (d) $[1, 2]$

22. Let f: {x, y, z} → {a, b, c} be a one-one function and only one of the conditions (i) $f(x) \neq b$, (ii) $f(y)$, = b (iii) $f(z) \neq a$ is true then the function f is given by the set

(a) {(x, a), (y, b), (z, c)} (b) {(x, a), (y, c), (z, b)}

(c) {(x, b), (y, a), (z, c)} (d) {(x, c), (y, b), (z, a)}

23. The equation $||x - 1| + a| = 4$ can have real solutions for x if **'a'** belongs to the interval

(a) $(-\infty, 4]$ (b) $(-\infty, -4]$

(c) $(4, +\infty)$ (d) $[-4, 4]$

24. If $x^4 f(x) - \sqrt{1 - \sin 2\pi x} = |f(x)| - 2f(x)$, then f(-2) equals:

(a) $\frac{1}{17}$ (b) $\frac{1}{11}$

(c) $\frac{1}{19}$ (d) 0

25. If $0 < x < 1000$ and $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31}{30}x$, where $[x]$ is the greatest integer less than or equal to x, the number of possible values of x is

(a) 34 (b) 32
(c) 33 (d) none of these

26. The domain of the function $y = \underbrace{\log_{10} \log_{10} \log_{10} \ldots \log_{10}}_{\text{n times}} x$ is

(a) $[10^n, +\infty)$ (b) $(10^{n-1}, +\infty)$
(c) $[10^{n-2}, +\infty)$ (d) None of these

27. If [.] denotes the greatest integer function then the domain of the real-valued function $\log_{[x+1/2]}|x^2 - x - 2|$ is

(a) $\left[\frac{3}{2}, +\infty\right)$ (b) $\left[\frac{3}{2}, 2\right) \cup (2, +\infty)$

(c) $\left(\frac{1}{2}, 2\right) \cup (2, +\infty)$ (d) None of these

Objective Questions II [One or more than one correct option]

28. If $\log_k x . \log_5 k = \log_x 5$, $k \neq 1$, $k > 0$, then x is equal to

(a) k (b) $\frac{1}{5}$
(c) 5 (d) None of these

29. If $\frac{1}{2} \leq \log_{0.1} x \leq 2$ then

(a) the maximum value of x is $\frac{1}{\sqrt{10}}$

(b) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(c) x does not lie between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(d) the minimum value of x is $\frac{1}{100}$

30. Which of the following functions is not injective ?

(a) $f(x) = |x + 1|, x \in [-1, 0]$
(b) $f(x) = x + 1/x,\ x \in (0, \infty)$
(c) $f(x) = x^2 + 4x - 5$
(d) $f(x) = e^{-x},\ x \in [0, \infty)$

31. If f is an even function defined on the interval $(-5, 5)$ then a value of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ is

(a) $\frac{-1+\sqrt{5}}{2}$ (b) $\frac{-3+\sqrt{5}}{2}$

(c) $\frac{-1-\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$

Numerical Value Type Questions

32. If $f\left(x+\frac{1}{x}\right) = x^3 + x^{-3}$ then f (5) must be equal to

33. The range of the function $\sqrt{x-6} + \sqrt{12-x}$ is an interval of length $2\sqrt{3} - \sqrt{k}$ then k must be

34. The least period of the function

$\cos(\cos x) + \sin(\cos x) + \sin 4x$ is $k\frac{\pi}{2}$

then value of k must be

Assertion & Reason

(A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(C) If ASSERTION is true, REASON is false.

(D) If ASSERTION is false, REASON is true.

35. **Assertion :** The function $\frac{ax + b}{cx + d}$, $(ad - bc \neq 0)$ cannot attain the value $\frac{a}{c}$.

Reason : The domain of the function $g(y) = \frac{b - dy}{cy - a}$ is all the reals except a/c.

(a) A (b) B
(c) C (d) D

36. **Assertion** : The domain of a function $y = f(x)$ will be all reals if for every real x there exist real y.

Reason : The range of a function $y = f(x)$ will be all reals if for every real y there exists a real x such that $f(x) = y$.

(a) A (b) B

(c) C (d) D

37. **Assertion** : Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Then there is a bijective mapping from A to B.

Reason : An onto function is not necessarily one-one.

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

38.

Column–I	Column–II
(A) odd function	**(P)** $x-[x]$
(B) even function	**(Q)** $\log\left(x+\sqrt{1+x^2}\right)$
(C) neither even nor odd	**(R)** $x\log\frac{1+x}{1-x}$
	(S) $\frac{2^{x/2}}{1+2^{x/2}}$

The Correct Matching is :

(a) (A–R); (B–Q); (C–P,S)

(b) (A–P,S); (B–R); (C–R)

(c) (A–Q); (B–P,S); (C–R)

(d) (A–Q); (B–R); (C–P,S)

39.

Column–I	Column–II
(A) $f(x+y)=f(x)+f(y)$	**(P)** $\log_3 x$
(B) $f(xy)=f(x)+f(y)$	**(Q)** $\tan^{-1}x$
(C) $f(x+y)=f(x).f(y)$	**(R)** $3x$
(D) $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$	**(S)** 3^x

The Correct Matching is :

(a) (A–P); (B–R); (C–S); (D–Q)

(b) (A–R); (B–P); (C–S); (D–Q)

(c) (A–Q); (B–S); (C–P); (D–R)

(d) (A–R); (B–S); (C–Q); (D–P)

Using the following passage, solve Q.40 to Q.42

Passage

A rational function is defined as quotient of two polynomials, $p(x)$ and $q(x)$. The domain of the rational function must be all reals except the roots of the equation $q(x) = 0$. The range of rational function can be found by finding minimum and maximum values of the function. In case $p(x)$ and $q(x)$ have a common factor $x-\beta$. Then after cancelling the common factor, the rational function must assume a value at $x = \beta$ which should be deleted from the found range since β is not there in the domain of the rational function.

40. The range of the rational function $f(x) = \frac{3x+1}{2x+1}$ must be

(a) $R-\left\{-\frac{1}{2}\right\}$ (b) $R-\left\{-\frac{1}{3}\right\}$

(c) $R-\left\{\frac{3}{2}\right\}$ (d) R

41. The range of the rational function $f(x) = \frac{(2x+1)}{2x^2+5x+2}$ must be

(a) $R-\{0\}$ (b) $R-\{-2\}$

(c) $R-\{0,-2\}$ (d) $R-\left\{0,\frac{2}{3}\right\}$

42. The range of the rational function $f(x) = \dfrac{2x^2 + 5x + 2}{2x + 1}$ must be

(a) $R - \{0\}$ (b) $R - \{-2\}$

(c) $R - \left\{0, -2, \dfrac{2}{3}\right\}$ (d) $R - \left\{\dfrac{3}{2}\right\}$

Text

43. Find all real numbers x which satisfy the equation,

$2\, log_2 log_2 x + log_{1/2} log_2 \left(2\sqrt{2}\, x\right) = 1.$

44. Find the values of x satisfying the equation $|x - 1|^A = (x - 1)^7$ where $A = log_3 x^2 - 2\, log_x 9$.

45. Find all real numbers x which satisfy the equation, $\log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2.$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. Number of solutions of $\log_4(x-1) = \log_2(x-3)$ is **(2001)**

(a) 3 (b) 1

(c) 2 (d) 0

2. If $f : [0, \infty) \to [0, \infty)$ and $f(x) = \dfrac{x}{1+x}$, then f is **(2003)**

(a) one-one and onto

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

3. Range of the function $f(x) = \dfrac{x^2+x+2}{x^2+x+1}; x \in R$ is **(2003)**

(a) $(1, \infty)$ (b) $(1, 11/7)$

(c) $(1, 7/3]$ (d) $(1, 7/5)$

4. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is **(2012)**

(a) one-one and onto

(b) onto but not one-one

(c) one-one but not onto

(d) neither one-one nor onto

Match the Following

The question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching. For question, choose the option corresponding to the correct matching.

Match the conditions/expressions in Column I with statement in Column II.

5. Let $f(x) = \dfrac{x^2-6x+5}{x^2-5x+6}$

	Column I	**Column II**
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	**(P)** $0 < f(x) < 1$
(B)	If $1 < x < 2$, then $f(x)$ satisfies	**(Q)** $f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	**(R)** $f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	**(S)** $f(x) < 1$

(2007)

The Correct Matching is

(a) (A–P; B–Q; C–Q; D–P)

(b) (A–Q; B–P; C–Q; D–P)

(c) (A–P; B–P; C–Q; D–Q)

(d) (A–Q; B–Q; C–P; D–P)

Find Answer Key and Detailed Solutions at the end of this book

SETS, RELATIONS & FUNCTION

Please share your valuable feedback by scanning the QR code.

06

LIMITS AND DERIVATIVES

LIMITS AND DERIVATIVES

1. INTRODUCTION

Calculus is the mathematics of motion and change, while algebra, geometry, and trigonometry are more static in nature. The development of calculus in the 17th century by Newton, Leibnitz and others grew out of attempts by these and earlier mathematicians to answer certain fundamental questions about dynamic real-world situations. These investigations led to two fundamental procedures- differentiation and integration; which can be formulated in terms of a concept called- limit.

In a very real sense, the concept of limit is the threshold to modern mathematics. You are about to cross that threshold, and beyond lies the fascinating world of calculus.

2. LIMIT OF A FUNCTION

The notation :

$$\boxed{\lim_{x \to c} f(x) = L}$$

is read "the limit of f(x) as x approaches c is L" and means that the functional values f(x) can be made arbitrarily close to a unique number L by choosing x sufficiently close to c (but not equal to c).

2.1 One-Sided Limits

2.1.1 Right-hand Limit (RHL) : We write

$\lim_{x \to a^+} f(x) = \lim_{h \to 0^+} f(a+h) = l_1$ if we can make the number f(x) as close to l_1 as we please by choosing x sufficiently close to a on a small interval (a, b) immediately to the right of a.

2.1.2 Left-hand limit (LHL) : We write

$\lim_{x \to a^-} f(x) = \lim_{h \to 0^+} f(a-h) = l_2$ if we can make the number f(x) as close to l_2 as we please by choosing x sufficiently close to a on a small interval (c, a) immediately to the left of a.

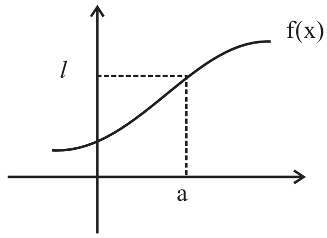
2.1.3 Limit of a function f(x) is said to exist as x → a (x approaches a) when ;

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \ell$ (finite quantity)

$$\Rightarrow \lim_{x \to a} f(x) = \ell$$

Limits From Graph

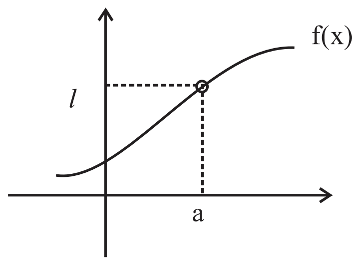
1.



LHL = l, RHL = l

$$\Rightarrow \lim_{x \to a} f(x) = l$$

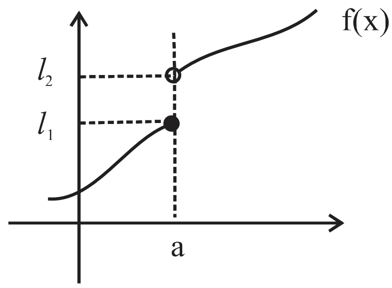
2.



LHL = l, RHL = l

$$\Rightarrow \lim_{x \to a} f(x) = l$$

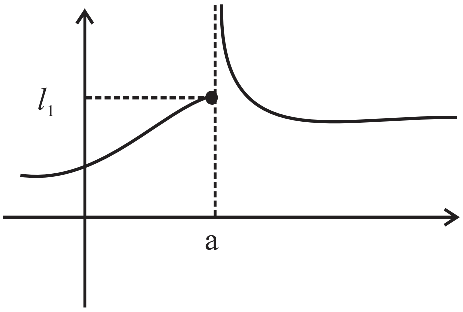
3.



LHL = l_1, RHL = l_2

$\because LHL \neq RHL$

$\Rightarrow \lim_{x \to a} f(x)$ does not exist

4. 

$\text{LHL} = l_1$

$\text{RHL} = \text{Not defined (approaches } \infty)$

So $\lim\limits_{x\to a} f(x)$ does not exist.

3. ALGEBRA OF LIMITS

Let f and g be two real functions with domain D. We define four new functions $f \pm g$, fg, f/g on domain D by setting $(f \pm g)(x) = f(x) \pm g(x)$, $(fg)(x) = f(x) \times g(x)$
$(f/g)(x) = f(x)/g(x)$, if $g(x) \neq 0$ for any $x \in D$.
Following are some results concerning the limits of these functions.

Let both $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exist and equal l and m respectively, then

1. $\lim\limits_{x\to a}(f \pm g)(x) = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x) = l \pm m$

2. $\lim\limits_{x\to a}(f\,g)(x) = \lim\limits_{x\to a} f(x) \times \lim\limits_{x\to a} g(x) = lm$

3. $\lim\limits_{x\to a}\left(\dfrac{f}{g}\right)(x) = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \dfrac{l}{m}$, provided $m \neq 0$.

4. $\lim\limits_{x\to a} k\,f(x) = k.\lim\limits_{x\to a} f(x)$, where k is constant.

5. $\lim\limits_{x\to a}\left(f(x)\right)^{g(x)} = l^m$; (provided $\lim\limits_{x\to a} f(x) > 0$)

6. $\lim\limits_{x\to a} fog(x) = f\left(\lim\limits_{x\to a} g(x)\right) = f(m)$; provided $f(x)$ is continuous at $g(x) = m$.

In particular

(a) $\lim\limits_{x\to a} \log f(x) = \log\left(\lim\limits_{x\to a} f(x)\right) = \log l$; (provided $l > 0$)

(b) $\lim\limits_{x\to a} e^{f(x)} = e^{\lim\limits_{x\to a} f(x)} = e^{l}$.

7. If $\lim\limits_{x\to a} f(x) = +\infty$ or $-\infty$, then $\lim\limits_{x\to a} \dfrac{1}{f(x)} = 0$.

4. EVALUATION OF ALGEBRAIC LIMITS

Let $f(x)$ be an algebraic function and 'a' be a real number. Then $\lim\limits_{x\to a} f(x)$ is known as an algebraic limit.

E.g. $\lim\limits_{x\to 1}\dfrac{x^3-1}{x-1}$, $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$, $\lim\limits_{x\to 2}\dfrac{x^5-32}{x-2}$ etc.

are algebraic limits.

4.1 Direct substitution method

If by direct substitution of the point in the given expression we get a finite number, then the number obtained is the limit of the given expression.

If upon substituting the point in the given expression, we get the following forms. :

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 0\times\infty, \infty^0, 0^0, 1^\infty$$

(Indeterminate Forms)

Then we can't find the value of limit by direct substitution. Following methods are followed to find the limit of the function.

4.2 Factorisation method

Consider $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$. If by putting $x = a$ the rational function $\dfrac{f(x)}{g(x)}$ takes the form $\dfrac{0}{0}, \dfrac{\infty}{\infty}$ etc, then (x–a) is a factor of both $f(x)$ & $g(x)$. In such a case we factorise the numerator and denominator, and then cancel out the common factor (x–a). After cancelling out the common factor (x–a), we again put x = a in the given expression and see whether we get a meaningful number or not. This process is repeated till we get a meaningful number.

4.3 Rationalisation method

This is particularly used when either numerator or denominator, or both involve expressions consisting of square roots (radical signs)

NOTE : Sometimes, it is easier to convert limit to a new variable h that tends to 0. For this, we can use substitution $x = a + h$ or $x = a - h$.

4.4 Method of evaluating algebraic limits when $x \to \infty$

To evaluate this type of limits we follow the following procedure.

Step–1 : Write down the given expression in the form of a rational function, i.e., $\frac{f(x)}{g(x)}$, if it is not so.

Step–2 : If k is the highest power of x in numerator and denominator both, then divide each term in numerator and denominator by x^k.

Step–3 : Use the result $\lim\limits_{x\to\infty} \frac{1}{x^n} = 0$, where $n > 0$.

Important Results :

If m, n are positive integers and a_0, $b_0 \neq 0$ are non-zero numbers, then $\lim\limits_{x\to\infty} \frac{a_0x^m + a_1x^{m-1} + ... + a_{m-1}x + a_m}{b_0x^n + b_1x^{n-1} + ... + b_{n-1}x + b_n}$

$$= \begin{cases} \frac{a_0}{b_0}, & \text{if} \quad m = n \\ 0, & \text{if} \quad m < n \\ \infty, & \text{if} \quad m > n,\ (a_0 \times b_0) > 0 \\ -\infty, & \text{if} \quad m > n,\ (a_0 \times b_0) < 0 \end{cases}$$

NOTES :

(1) Sometimes such questions can be solved using substitution $x = \frac{1}{t}$.

(2) Questions with variable tending to $-\infty$ can be solved using substitution $x = -t$.

5. STANDARD LIMITS

5.1 Trigonometric Limits

To evaluate trigonometric limits the following results are very useful.

1. $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

2. $\lim\limits_{x\to 0} \frac{\tan x}{x} = 1$

(in 1 & 2, x is measured in radians)

3. $\lim\limits_{x\to 0} \frac{\sin^{-1} x}{x} = 1$

4. $\lim\limits_{x\to 0} \frac{\tan^{-1} x}{x} = 1$

5. $\lim\limits_{x\to 0} \frac{\sin x^o}{x} = \frac{\pi}{180}$

6. $\lim\limits_{x\to 0} \cos x = 1$

7. $\lim\limits_{x\to a} \frac{\sin(x-a)}{(x-a)} = 1$

8. $\lim\limits_{x\to a} \frac{\tan(x-a)}{(x-a)} = 1$

5.2 Exponential & Logarithmic Limits

To evaluate the exponential and logarithmic limits we use the following results.

1. $\lim\limits_{x\to 0} \frac{a^x - 1}{x} = \log_e a$

2. $\lim\limits_{x\to 0} \frac{e^x - 1}{x} = 1$

3. $\lim\limits_{x\to 0} \frac{\log(1+x)}{x} = 1$

4. $\lim\limits_{x\to a} \left(\frac{x^n - a^n}{x-a}\right) = n.a^{n-1}$

6. EXPANSIONS TO EVALUATE LIMITS

1. $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+....$

   $(|x|<1)$

2. $e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...$

3. $a^x = 1+x(\log_e a)+\frac{x^2}{2!}(\log_e a)^2+....(a>0)$

4. $\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...(-1<x\le 1)$

5. $\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-....$

6. $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-....$

7. $\tan x = x+\frac{x^3}{3}+\frac{2}{15}x^5+....$

8. $\sin^{-1}x = x+\frac{1}{2}.\frac{x^3}{3}+\frac{1}{2}.\frac{3}{4}.\frac{x^5}{5}+\frac{1}{2}.\frac{3}{4}.\frac{5}{6}.\frac{x^7}{7}+...$

9. $\tan^{-1}x = x-\frac{1}{3}x^3+\frac{1}{5}x^5-...$

7. 1^∞ FORM

To evaluate the exponential limits of the form 1^∞ we use the following results.

1. If $\lim_{x\to a} f(x)=0$ and $\lim_{x\to a} g(x)\to\infty$, then

   $\lim_{x\to a}[1+f(x)]^{g(x)} = e^{\lim_{x\to a} f(x)\times g(x)}$,

2. If $\lim_{x\to a} f(x)=1$ and $\lim_{x\to a} g(x)\to\infty$, then

   $\lim_{x\to a}[f(x)]^{g(x)} = \lim_{x\to a}[1+(f(x)-1)]^{g(x)} = e^{\lim_{x\to a}(f(x)-1)g(x)}$

7.1 Particular Cases

1. $\lim_{x\to 0}(1+x)^{1/x} = e$

2. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

3. $\lim_{x\to 0}(1+\lambda x)^{1/x} = e^\lambda$

4. $\lim_{x\to\infty}\left(1+\frac{\lambda}{x}\right)^x = e^\lambda$

8. L-HOSPITAL'S RULE

If $f(x)$ and $g(x)$ be two functions of x such that

1. $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$

2. both are continuous at $x = a$,

3. both are differentiable at $x = a$,

4. $f'(x)$ and $g'(x)$ are continuous at the point $x = a$, then

   $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$, provided that $g(a)\ne 0$.

The above rule is also applicable if $\lim_{x\to a} f(x)\to\infty$ and $\lim_{x\to a} g(x)\to\infty$.

Generalisation : If $\lim_{x\to a}\frac{f'(x)}{g'(x)}$ assumes the indeterminate form $(0/0)$ or (∞/∞) and $f'(x), g'(x)$ satisfy all the conditions embodied in L-Hospitals rule, we can repeat the application of this rule on $\frac{f'(x)}{g'(x)}$ to get

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} = \lim_{x\to a}\frac{f''(x)}{g''(x)} = ...$$

Sometimes it may be necessary to repeat this process a number of times till our goal of evaluating the limit is achieved.

9. $0 \times \infty$, 0^0 AND ∞^0 FORMS

9.1 $0 \times \infty$ form

If $\lim\limits_{x \to a} f(x) = 0$ and $\lim\limits_{x \to a} g(x) \to \infty$, then

$\lim\limits_{x \to a} \left(f(x) \times g(x) \right)$ can be converted to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form by writing limit as

$$\lim_{x \to a} \frac{f(x)}{1/g(x)} \ or \ \lim_{x \to a} \frac{g(x)}{1/f(x)}$$

It can now be solved using L-Hopital's rule.

9.2 0° and ∞° form

we convert these to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form using log.

If $\lim\limits_{x \to a} f(x) = 0 \ or \ \infty$ and $\lim\limits_{x \to a} g(x) = 0$, then

$$l = \lim_{x \to a} \left(f(x) \right)^{g(x)} \Rightarrow \log l = \lim_{x \to a} g(x) . \log f(x)$$

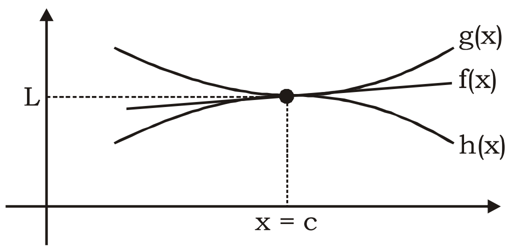
$$\Rightarrow \log l = \lim_{x \to a} \frac{g(x)}{1/\log f(x)}$$

$$\Rightarrow l = e^{\lim\limits_{x \to a} \frac{g(x)}{1/\log f(x)}}$$

10. SQUEEZE RULE/SANDWICH RULE

If $g(x) \le f(x) \le h(x)$ on an open interval containing 'c', and if :

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L, \text{ then } \lim_{x \to c} f(x) = L$$



In words : If a function can be squeezed/sandwiched between two functions whose limits at a particular point c have the same value L, then that function must also have limit L at x = c.

DERIVATIVES

11. DEFINITION

The rate of change of one quantity with respect to some another quantity has a great importance. For example, the rate of change of displacement of a particle with respect to time is called its velocity and the rate of change of velocity is called its acceleration.

The rate of change of a quantity 'y' with respect to another quantity 'x' is called the derivative or differential coefficient of y with respect to x.

Derivative at a Point

The derivative of a function at a point x = a is defined by

$f'(a) = \lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$ (provided the limit exists and is finite)

The above definition of derivative is also called derivative by first principle.

(1) Geometrical meaning of derivatives at a point : Consider the curve y = f(x). Let f(x) be differentiable at x = c. Let P(c,f(c)) be a point on the curve and Q(x, f(x)) be a neighbouring point on the curve. Then,

Slope of the chord $PQ = \frac{f(x) - f(c)}{x - c}$. Taking limit as $Q \to P$, i.e., we get

$$\lim_{Q \to P} \left(Slope \ of \ the \ chord \ PQ \right) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \quad ..(i)$$

As $Q \to P$, chord PQ becomes tangent at P.

Therefore from (i), we have

slope of the tangent at $P = \lim\limits_{x \to c} \frac{f(x) - f(c)}{x - c} = \left(\frac{df(x)}{dx} \right)_{x=c}$

NOTES :

Thus, the derivatives of a function at point x = c is the slope of the tangetn to curve , y = f(x) at point (c, f(c)).

(2) Physical interpretation at a point : Let a particle moves in a straight line OX starting from O towards X. Clearly, the position of the particle at any instant would depend upon the time elapsed. In other words, the distance of the particle from O will be some function f of time t.

Let at any time $t = t_0$, the particle be at P and after a further time h, it is at Q so that $OP = f(t_0)$ and $OQ = f(t_0 + h)$.

hence, the average speed of the particle during the journey from P to Q is $\frac{PQ}{h}$, i.e., $\frac{f(t_0+h) - f(t_0)}{h}$. Taking its limit as $h \to 0$, we get its instantaneous speed to be $\lim_{h\to 0} \frac{f(t_0+h) - f(t_0)}{h}$, which is simply $f'(t_0)$. Thus, if $f(t)$ gives the distance of a moving particle at time t, then the derivative of f at $t = t_0$ represents the instantaneous speed of the particle at the point P, i.e., at time $t = t_0$.

Important Tips

* $\frac{dy}{dx}$ is $\frac{d}{dx}(y)$ in which $\frac{d}{dx}$ is simply a symbol of operation and 'd' divided by dx.

12. DERIVATIVE OF STANDARD FUNCTION

(i) $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

(ii) $\frac{d}{dx}(e^x) = e^x$

(iii) $\frac{d}{dx}(a^x) = a^x \cdot \ln a \ (a > 0)$

(iv) $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$

(v) $\frac{d}{dx}(\log_a |x|) = \frac{1}{x}\log_a e$

(vi) $\frac{d}{dx}(\sin x) = \cos x$

(vii) $\frac{d}{dx}(\cos x) = -\sin x$

(viii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(ix) $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

(x) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cdot \cot x$

(xi) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(xii) $\frac{d}{dx}(\text{constant}) = 0$

13. THEOREMS ON DERIVATIVES

If u and v are derivable functions of x, then,

(i) Term by term differentiation : $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(ii) Multiplication by a constant $\frac{d}{dx}(K u) = K\frac{du}{dx}$, where K is any constant

(iii) **"Product Rule"** $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

In general,

If $u_1, u_2, u_3, u_4, \ldots, u_n$ are the functions of x, then

$$\frac{d}{dx}\left(u_1 . u_2 . u_3 . u_4 \ldots . u_n\right)$$

$$=\left(\frac{du_1}{dx}\right)\left(u_2\, u_3\, u_4 \ldots u_n\right)+\left(\frac{du_2}{dx}\right)\left(u_1\, u_3\, u_4 \ldots u_n\right)$$

$$+\left(\frac{du_3}{dx}\right)\left(u_1\, u_2\, u_4 \ldots u_n\right)+\left(\frac{du_4}{dx}\right)\left(u_1\, u_2\, u_3\, u_5 \ldots u_n\right)$$

$$+\ldots+\left(\frac{du_n}{dx}\right)\left(u_1\, u_2\, u_3 \ldots u_{n-1}\right)$$

(iv) **"Quotient Rule"** $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\left(\frac{du}{dx}\right)-u\left(\frac{dv}{dx}\right)}{v^2}$ where $v \neq 0$

(b) **Chain Rule :** If $y = f(u)$, $u = g(w)$, $w = h(x)$

then $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dw}.\frac{dw}{dx}$

or $\frac{dy}{dx}=f'(u).g'(w).h'(x)$

SOLVED EXAMPLES

Example – 1

Evaluate $\lim\limits_{x\to 2}\dfrac{x^3-6x^2+11x-6}{x^2-6x+8}$.

Sol. When $x = 2$, the expression

$\dfrac{x^3-6x^2+11x-6}{x^2-6x+8}$ is of the form $\dfrac{0}{0}$.

Now, $\lim\limits_{x\to 2}\dfrac{x^3-6x^2+11x-6}{x^2-6x+8}\left(\dfrac{0}{0}\text{form}\right)$

$$=\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}\left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 2}\frac{(x-1)(x-3)}{(x-4)}=\frac{(2-1)(2-3)}{(2-4)}=\frac{1}{2}.$$

Example – 2

Evaluate the following limits :

$$\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{3-x}-3^{x/2}}.$$

Sol. $L=\lim\limits_{x\to 2}\dfrac{3^x+3^{3-x}-12}{3^{3-x}-3^{x/2}}$

$$\Rightarrow\quad L=\lim_{x\to 2}\frac{3^x+\dfrac{27}{3^x}-12}{\dfrac{27}{3^x}-3^{x/2}}$$

$$\Rightarrow\quad L=\lim_{x\to 2}\frac{3^{2x}-12.3^x+27}{-\left(3^{\frac{3x}{2}}-3^3\right)}$$

$$\Rightarrow\quad L=-\lim_{x\to 2}\frac{(3^x-9)\,(3^x-3)}{\left(3^{x/2}-3\right)\left(3^x+9+3.3^{x/2}\right)}$$

$$\Rightarrow\quad L=\lim_{x\to 2}-\frac{(3^{x/2}+3)\,(3^x-3)}{\left(3^x+3.3^{x/2}+9\right)}$$

$$\Rightarrow\quad L=\frac{-6.6}{9+3.3+9}=\frac{-36}{27}=\frac{-4}{3}.$$

Example – 3

Evaluate $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$.

Sol. When $x = a$, $\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ assumes the indeterminate form $\dfrac{0}{0}$.

Now, $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

$$=\lim_{x\to a}\frac{\left(\sqrt{a+2x}-\sqrt{3x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(\sqrt{3a+x}-2\sqrt{x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$=\lim_{x\to a}\frac{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{3(a-x)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$=\lim_{x\to a}\frac{\left(\sqrt{3a+x}+2\sqrt{x}\right)}{3\left(\sqrt{a+2x}+\sqrt{3x}\right)}=\frac{4\sqrt{a}}{3\left(2\sqrt{3a}\right)}=\frac{2}{3\sqrt{3}}.$$

Example – 4

The value of $\lim\limits_{x\to 3}\left(\log_a \dfrac{x-3}{\sqrt{x+6}-3}\right)$ is

(a) $log_a 6$ (b) $log_a 3$

(c) $log_a 2$ (d) None of these

Ans. (a)

Sol. $\lim\limits_{x\to 3}\log_a \dfrac{x-3}{\sqrt{x+6}-3}$

$$= \lim_{x\to 3}\log_a \frac{(x-3)\left(\sqrt{x+6}+3\right)}{\left(\sqrt{x+6}-3\right)\left(\sqrt{x+6}+3\right)}$$

$$= \lim_{x\to 3}\log_a \frac{(x-3)\left(\sqrt{x+6}+3\right)}{(x-3)}$$

$$= \log_a 6$$

Example – 5

Evaluate the following limit :

$$\lim_{n\to\infty}\frac{1^2+2^2+3^2+.....+n^2}{n^3}.$$

($x \to \infty$ type problem)

Sol. $\lim\limits_{n\to\infty}\dfrac{1^2+2^2+.....+n^2}{n^3} = \lim\limits_{n\to\infty}\dfrac{n(n+1)(2n+1)}{6n^3}$

$$= \frac{1}{6}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = \frac{1}{6}(1+0)(2+0) = \frac{1}{3}.$$

Example – 6

$\lim\limits_{n\to\infty}\left\{\dfrac{1}{1-n^2}+\dfrac{2}{1-n^2}+...\dfrac{n}{1-n^2}\right\}$ is equal to :

(a) 0 (b) $-\dfrac{1}{2}$

(c) $\dfrac{1}{2}$ (d) none of these

Ans. (b)

Sol. $= \lim\limits_{n\to\infty}\left\{\dfrac{1}{1-n^2}+\dfrac{2}{1-n^2}+.......+\dfrac{n}{1-n^2}\right\}$

$$= \lim_{n\to\infty}\left\{\frac{1+2+3+....+n}{1-n^2}\right\}$$

$$= \lim_{n\to\infty}\frac{n(n+1)}{2(1-n^2)} = \lim_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)}{2n^2\left(\frac{1}{n^2}-1\right)} = \frac{-1}{2}.$$

Example – 7

Evaluate the following limits :

$$\lim_{x\to a}\frac{e^{\sqrt{x}}-e^{\sqrt{a}}}{x-a}.$$

$$\left(\text{using } \lim_{x\to 0}\frac{a^x-1}{x} = \log a\right)$$

Sol. $\lim\limits_{x\to a}\dfrac{e^{\sqrt{x}}-e^{\sqrt{a}}}{x-a} = \lim\limits_{x\to a}\dfrac{e^{\sqrt{a}}\left(e^{\sqrt{x}-\sqrt{a}}-1\right)}{x-a}$

$$= e^{\sqrt{a}}\lim_{x\to a}\frac{e^{\sqrt{x}-\sqrt{a}}-1}{\sqrt{x}-\sqrt{a}}\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{x-a}$$

$$= e^{\sqrt{a}}(1)\lim_{x\to a}\frac{(x-a)}{(x-a)\left(\sqrt{x}+\sqrt{a}\right)} = \frac{e^{\sqrt{a}}}{2\sqrt{a}}$$

Example – 8

Evaluate the following limits :

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)\cos x}{\sin x - \operatorname{cosec} x}.$$

$$\left(\text{using } \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

Sol. $\lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)\cos x}{\sin x - \operatorname{cosec} x}$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)\sin x\cos x}{\sin^2 x - 1}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)\sin x}{-\cos x}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)}{(\cos x)} \lim_{x\to\frac{\pi}{2}} \frac{(\sin x)}{-1}$$

$= -1.$

Example – 9

Solve : $\lim_{x\to\pi/3} \frac{\tan^3 x - 3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}$.

Sol. Let $L = \lim_{x\to\pi/3} \frac{\tan^3 x - 3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}$ and $x - \frac{\pi}{3} = t$

$$\Rightarrow \quad L = \lim_{t\to 0} \frac{\tan^3\left(t+\frac{\pi}{3}\right) - 3\tan\left(t+\frac{\pi}{3}\right)}{\cos\left(t+\frac{\pi}{2}\right)}$$

$$\text{using } \tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$$

$$\Rightarrow \quad L = \lim_{t\to 0} \frac{\tan(3t+\pi)\left[3\tan^2\left(t+\frac{\pi}{3}\right)-1\right]}{-\sin t}$$

$$\Rightarrow \quad L = \lim_{t\to 0} \frac{+\tan(3t)}{-\sin t} . \lim_{t\to 0}\left[3\tan^2\left(t+\frac{\pi}{3}\right)-1\right]$$

$$\Rightarrow \quad L = -3\lim_{t\to 0}\frac{\tan(3t)}{3t} \times \lim_{t\to 0}\frac{t}{\sin t} \times \lim_{t\to 0}\left[3\tan^2\left(t+\frac{\pi}{3}\right)-1\right]$$

$$\Rightarrow \quad L = -3 \times 1 \times 1 \times 8 = -24.$$

Example – 10

$$\lim_{x\to\frac{\pi}{2}} \frac{\left[1-\tan\left(\frac{x}{2}\right)\right](1-\sin x)}{\left[1+\tan\left(\frac{x}{2}\right)\right](\pi-2x)^3} \text{ is}$$

(a) $\frac{1}{8}$ (b) 0

(c) $\frac{1}{32}$ (d) ∞

Ans. (c)

Sol. $\lim_{x\to\frac{\pi}{2}} \frac{\left[1-\tan\frac{x}{2}\right](1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{(\pi-2x)^3}$$

$$Put\ x = \frac{\pi}{2}+h$$

$$= \lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4}-\frac{\pi}{4}-\frac{h}{2}\right)(1-\cos h)}{(\pi-\pi-2h)^3}$$

$$= \lim_{h\to 0} \frac{\tan\left(-\frac{h}{2}\right).2\sin^2\frac{h}{2}}{-8h^3}$$

$$= \frac{1}{4}.\lim_{h\to 0}\frac{\tan\frac{h}{2}}{\frac{h}{2}\times 2}.\lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4}$$

$$= \frac{1}{4}\times\frac{1}{2}\times\frac{1}{4} = \frac{1}{32}$$

Example – 11

$$\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}=\ldots$$

Sol. $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$

$$=\lim_{x\to 1}(1-x)\cot\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)$$

$$=\lim_{x\to 1}\frac{1-x}{\tan\left[\frac{\pi}{2}(1-x)\right]}=\lim_{x\to 1}\frac{\frac{\pi}{2}(1-x)}{\tan\left[\frac{\pi}{2}(1-x)\right]}\times\frac{2}{\pi}=\frac{2}{\pi}$$

Example – 12

$$\lim_{x\to-\infty}\left[\frac{\left(x^4\sin\left(\frac{1}{x}\right)+x^2\right)}{(1+|x|^3)}\right]=\ldots.$$

Sol. $\lim_{x\to-\infty}\left[\frac{x^4\sin\frac{1}{x}+x^2}{1+|x|^3}\right]$

$$=\lim_{x\to-\infty}\left[\frac{x^4\sin\frac{1}{x}+x^2}{1-x^3}\right]$$

$$=\lim_{x\to-\infty}\frac{x^4\left(\sin\frac{1}{x}+\frac{1}{x^2}\right)}{x^3\left(\frac{1}{x}-1\right)}$$

$$=\lim_{x\to-\infty}\frac{x\sin\frac{1}{x}}{-1}=\lim_{x\to-\infty}\frac{-\sin\frac{1}{x}}{\frac{1}{x}}=-1.$$

Example – 13

$\operatorname{Limit}_{n\to\infty} n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{\pi}{4n}\right)$ has the value equal to

(a) $\pi/3$ (b) $\pi/4$

(c) $\pi/6$ (d) none

Ans. (b)

Sol. $Let\ n=\frac{1}{x}\Rightarrow as\ n\to\infty,\ x\to 0$

$$\lim_{x\to 0}\frac{1}{x}\sin\frac{\pi x}{4}\cos\frac{\pi x}{4}=\lim_{x\to 0}\frac{1}{2x}\times 2\sin\frac{\pi x}{4}\cos\frac{\pi x}{4}$$

$$=\lim_{x\to 0}\frac{\sin\frac{\pi x}{2}}{\frac{\pi x}{2}}\times\frac{\pi}{4}=\frac{\pi}{4}$$

Example – 14

$\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x-\sin x}$ is equal to

(a) 1 (b) –1

(c) 2 (d) 0

Ans. (c)

Sol. $\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x-\sin x}$

$$=\lim_{x\to 0}\frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\right)-\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\right)-2x}{x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots\right)}$$

$$\lim_{x\to 0}\frac{2x^3\left(\frac{1}{3!}+\frac{x^2}{5!}+\ldots\right)}{x^3\left(\frac{1}{3!}-\frac{x^2}{5!}+\ldots\right)}=\frac{\frac{2}{3!}}{\frac{1}{3!}}=2$$

Example – 15

If $\lim\limits_{x\to 0}\dfrac{\sin 2x + a\sin x}{x^3}$ be finite, then the value of a and the limit are given by

(a) –2, 1 (b) –2, –1

(c) 2, 1 (d) 2, –1

Ans. (b)

Sol. $\lim\limits_{x\to 0}\dfrac{\sin 2x + a\sin x}{x^3}$

$$= \lim_{x\to 0}\frac{2\sin x\cos x + a\sin x}{x^3}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\frac{(2\cos x + a)}{x^2} = \lim_{x\to 0}\frac{a + 2\cos x}{x^2}$$

$$= \lim_{x\to \infty}\frac{a + 2\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}.......\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{a + 2 + x^2\left(-\frac{2}{2!} + \frac{2x^2}{4!}....\right)}{x^2}$$

for limit to be finite

$a + 2 = 0 \Rightarrow a = -2$

So, value of limit = –1

Example – 16

$$\underset{x\to 0}{\text{Limit}}\ \frac{e^{-x^2/2} - \cos x}{x^3 \sin x} =$$

(a) 1/4 (b) 1/6

(c) 1/12 (d) 1/8

Ans. (c)

Sol. $\lim\limits_{x\to 0}\dfrac{e^{-\frac{x^2}{2}} - \cos x}{x^3 \sin x}$

$$= \lim_{x\to 0}\frac{\left(1 - \frac{x^2}{2} + \frac{x^4}{4.2!}....\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}....\right)}{x^3\left(x - \frac{x^3}{3!} + ...\right)}$$

$$= \lim_{x\to 0}\frac{x^4\left(\frac{1}{4.2!} - \frac{1}{4!}\right) + x^6\left(-\frac{1}{8.3!} + \frac{1}{6!}\right) + ...}{x^4}$$

$$= \frac{1}{8} - \frac{1}{24} = \frac{1}{12}$$

Example – 17

Evaluate : $\lim\limits_{x\to 0}\dfrac{\sin 3x^2}{l n\cos(2x^2 - x)}$ using LH rule.

($\frac{0}{0}$ type of indeterminate form)

Sol. Let $\lim\limits_{x\to 0}\dfrac{\sin 3x^2}{l n\cos(2x^2 - x)}$ [$\frac{0}{0}$ form]

Apply LH rule to get :

$$L = \lim_{x\to 0}\frac{-6x\cos 3x^2\cos(2x^2 - x)}{(4x - 1)\sin(2x^2 - x)}$$

$$= -6\lim_{x\to 0}\frac{\cos 3x^2\cos(2x^2 - x)}{4x - 1}\lim_{x\to 0}\frac{x}{\sin(2x^2 - x)}$$

The limit of first factor is computed directly, the limit of the second one, which represents an indeterminate form of the 0/0 is found with the aid of the L'Hospital's rule. Again consider.

$$L = -6\lim_{x\to 0}\frac{\cos 3x^2\cos(2x^2 - x)}{4x - 1}\cdot\lim_{x\to 0}\frac{x}{\sin(2x^2 - x)}$$

$$\Rightarrow\quad L = -6.\frac{1.1}{-1}\lim_{x\to 0}\frac{1}{(4x - 1)\cos(2x^2 - x)}$$

$$\Rightarrow\quad L = -6.\frac{1}{-1.(-1)} = -6$$

Example – 18

If $\lim\limits_{x\to 2}\dfrac{\tan(x-2)\{x^2+(k-2)x-2k\}}{x^2-4x+4}=5$, then k is equal to:

(a) 0 (b) 1

(c) 2 (d) 3

Ans. (d)

Sol. $\lim\limits_{x\to 2}\dfrac{\tan(x-2)\left\{x^2+(k-2)x-2k\right\}}{x^2-4x+4}=5$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{\tan(x-2)}{(x-2)}\dfrac{\left\{x^2+(k-2)x-2k\right\}}{(x-2)}=5$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{2x+(k-2)}{1}=5$ (L'H Rule)

$\Rightarrow 4+k-2=5\Rightarrow k=3$

Example – 19

If function f (x) is differentiable at x = a, then $\lim\limits_{x\to a}\dfrac{x^2f(a)-a^2f(x)}{x-a}$

(a) $2a\,f(a)+a^2f'(a)$ (b) $-a^2f'(a)$

(c) $af(a)-a^2f'(a)$ (d) $2af(a)-a^2f'(a)$

Ans. (d)

Sol. $\lim\limits_{x\to a}\dfrac{x^2f(a)-a^2f(x)}{x-a}$

$\lim\limits_{x\to a}\dfrac{2xf(a)-a^2f'(x)}{1}$ (L'H Rule)

$=2a\,f(a)-a^2f'(a)$

Example – 20

The value of $\lim\limits_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ is

(a) $2a\sin a+a^2\cos a$ (b) $2a\sin a-a^2\cos a$

(c) $2a\cos a+a^2\sin a$ (d) None of these

Ans. (a)

Sol. $\lim\limits_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}\left(\dfrac{0}{0}\text{ form}\right)$

Apply L'H Rule

$\lim\limits_{h\to 0}\dfrac{2(a+h)\sin(a+h)+(a+h)^2\cos(a+h)}{1}$

$=2a\sin a+a^2\cos a$

Example – 21

$\lim\limits_{h\to 0}\dfrac{\log(1+2h)-2\log(1+h)}{h^2}=\ldots.$

Sol. $\lim\limits_{h\to 0}\dfrac{\log(1+2h)-2\log(1+h)}{h^2}$ $\left(\frac{0}{0}\text{ form}\right)$

Apply L'H rule

$=\lim\limits_{h\to 0}\dfrac{\dfrac{2}{1+2h}-\dfrac{2}{1+h}}{2h}=\lim\limits_{h\to 0}\dfrac{2(1+h-1-2h)}{2h(1+h)(1+2h)}$

$=\lim\limits_{h\to 0}\dfrac{-2h}{2h(1+h)(1+2h)}$

$=-1$

Example – 22

Evaluate : $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$

[1^∞ type of indeterminate form]

Sol. Since, $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)=1$ and $\lim\limits_{x\to a}\left(\dfrac{\pi x}{2a}\right)=\infty.$

$\Rightarrow$ $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$ of the form (1^∞)

Hence, $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$

$=e^{\lim\limits_{x\to a}\left(2-\frac{a}{x}-1\right)\times\tan\frac{\pi x}{2a}}.$

$$= e^{\lim_{x\to a}\left(\frac{x-a}{x}\right)\times\tan\left(\frac{\pi x}{2a}\right)}.$$

$$= e^{\lim_{h\to 0}\frac{h}{a+h}\times\tan\left(\frac{\pi}{2}+\frac{\pi h}{2a}\right)}$$ (putting, $x-a=h$)

$$= e^{-\lim_{h\to 0}\frac{h}{a+h}\times\cot\left(\frac{\pi h}{2a}\right)}$$ $\left(\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta\right)$

$$= e^{-\lim_{h\to 0}\frac{(\pi h/2a)}{\tan(\pi h/2a)}\times\lim_{h\to 0}\frac{(2a)}{(a+h)\pi}}$$

$\therefore\ e^{-2/\pi}$.

Example – 23

$$\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$$ is equal to

(a) e^4 (b) e^2

(c) e^3 (d) e

Ans. (a)

Sol. $$\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x \quad (1^\infty\ form)$$

$$= e^{\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}-1\right).x}$$

$$= e^{\lim_{x\to\infty}\left(\frac{4x+1}{x^2+x+2}\right).x}$$

$$= e^{\lim_{x\to\infty}\frac{x^2\left(4+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x}+\frac{2}{x^2}\right)}}$$

$= e^4$

Example – 24

Evaluate : $\lim_{x\to 1}\left(\frac{1}{l n x}-\frac{1}{x-1}\right)$ using LH rule.

[$\infty-\infty$ type of indeterminate form]

Sol. Let $L=\lim_{x\to 1}\left(\frac{1}{l n x}-\frac{1}{x-1}\right)$ [$\infty-\infty$ form]

Let us reduce it to an indeterminate form of the type 0/0.

$$L=\lim_{x\to 1}\frac{x-1-l n x}{(x-1)l n x}$$ [0/0 form]

Apply LH rule to get,

$$L=\lim_{x\to 1}.\frac{1-1/x}{l n x+1-1/x}$$

$$\Rightarrow\quad L=\lim_{x\to 1}.\frac{x-1}{x l n x+x-1}$$

Apply LH rule again,

$$\Rightarrow\quad L=\lim_{x\to 1}.\frac{1}{\ln x+1+1}=\frac{1}{2}$$

Example – 25

Evaluate the left hand and right hand *l*imits of the function

$$f(x)=\begin{cases}\frac{\sqrt{(x^2-6x+9)}}{(x-3)}, & x\neq 3\\ 0, & x=3\end{cases}$$

at x = 3 and hence comment on the existence of *l*imit at x=3.

Sol. The given function can be written as

$$f(x)=\begin{cases}\frac{|x-3|}{(x-3)}, & x\neq 3\\ 0, & x=3\end{cases}$$

$\therefore$ L.H.L. $=\lim_{x\to 3^-} f(x)$

$=\lim_{h\to 0} f(3-h)$

$$= \lim_{h\to 0}\frac{|3-h-3|}{(3-h-3)}$$

$$= \lim_{h\to 0}\frac{|-h|}{(-h)} = \lim_{h\to 0}\frac{h}{-h} = -1$$

& R.H.L. $= \lim_{x\to 3^+} f(x)$

$$= \lim_{h\to 0} f(3+h)$$

$$= \lim_{h\to 0}\frac{|3+h-3|}{(3+h-3)}$$

$$= \lim_{h\to 0}\frac{|h|}{h} = \lim_{h\to 0}\frac{h}{h} = 1$$

Hence left hand limit and right hand limit of $f(x)$ at $x = 3$ are -1 and 1 respectively.

As left Hand Limit $\neq$ Right Hand Limit

i.e. $\lim_{x\to 3} f(x)$ does not exist.

Example – 26

If $f(x) = \begin{cases} \dfrac{\tan[x]}{[x]}, & [x]\neq 0 \\ 0, & [x]=0 \end{cases}$ where

$[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x\to 0} f(x)$ equals

(a) 1 (b) –1

(c) 0 (d) Does not exist

Ans. (d)

Sol. <u>LHL</u>:

Put $x = 0 - h$

$$\lim_{h\to 0}\frac{\tan[0-h]}{[0-h]} = \frac{\tan 1}{1}$$

<u>RHL</u>:

Put $x = 0 + h$

$$\lim_{h\to 0}\frac{\tan[0+h]}{[0+h]} = \frac{\tan 0}{0} = \text{indeterminate}$$

$\Rightarrow$ Limit does not exist.

Example – 27

Evaluate the left hand and right hand limits of the function defined by

$$f(x) = \begin{cases} 1+x^2, \text{ if } 0\le x\le 1 \\ 2-x, \text{ if } \quad x>1 \end{cases} \text{ at } x = 1.$$

Also, show that $\lim_{x\to 1} f(x)$ does not exist.

Sol. We have,

(LHL of f(x) at x = 1)

$$= \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$$

$$= \lim_{h\to 0} 1+(1-h)^2 = \lim_{h\to 0} 2-2h+h^2 = 2.$$

and,

(RHL of f(x) at x = 1)

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} 2-(1+h) = \lim_{h\to 0} 1-h = 1$$

Clearly, $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$.

So, $\lim_{x\to 1} f(x)$ does not exist.

Example – 28

If $f(x)=\begin{cases}\frac{x-|x|}{x}, & x\neq 0\\ 2, & x=0\end{cases}$ show that $\lim\limits_{x\to 0} f(x)$ does not exist.

Sol. We have,

(LHL of f(x) at x = 0)

$= \lim\limits_{x\to 0^-} f(x)$

$= \lim\limits_{h\to 0} f(0-h) = \lim\limits_{h\to 0}\frac{-h-|-h|}{(-h)}$

$= \lim\limits_{h\to 0}\frac{-h-h}{-h} = \lim\limits_{h\to 0}\frac{-2h}{-h} = \lim\limits_{h\to 0} 2 = 2$

(RHL of f(x) at x = 0)

$= \lim\limits_{x\to 0^+} f(x)$

$= \lim\limits_{h\to 0} f(0+h) = \lim\limits_{h\to 0}\frac{h-|h|}{h}$

$= \lim\limits_{h\to 0}\frac{h-h}{h} = \lim\limits_{h\to 0}\frac{0}{h} = \lim\limits_{h\to 0} 0 = 0$

Clearly, $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$

So, $\lim\limits_{x\to 0} f(x)$ does not exist.

Example – 29

If $f(x)=\begin{cases} mx^2+n, x<0\\ nx+m, 0\le x\le 1\\ nx^3+m, x>1\end{cases}$

For what values of integers m, n does the limits $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to 1} f(x)$ exist.

Sol. It is given that

$\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to 1} f(x)$ both exist

$\Leftrightarrow \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$\Leftrightarrow \lim\limits_{h\to 0} f(0-h) = \lim\limits_{h\to 0} f(0+h)$ and $\lim\limits_{h\to 0} f(1-h) = \lim\limits_{h\to 0} f(1+h)$

$\Leftrightarrow \lim\limits_{h\to 0} m(-h)^2+n = \lim\limits_{h\to 0} n(h)+m$ and

$\lim\limits_{h\to 0} n(1-h)+m = \lim\limits_{h\to 0} n(1+h)^3+m$

$\Leftrightarrow n=m$ and $n+m=n+m$

$\Leftrightarrow m=n$

Hence, $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to 1} f(x)$ both exist for $n=m$.

Example – 30

If $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$. For what value (s) of a does $\lim\limits_{x\to a} f(x)$ exist ?

Sol. We have,

$$f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}-x+1, & x<0\\ 0, & x=0\\ x-1, & x>0\end{cases}$$

$$\left[\because |x|=\begin{cases}x, x\ge 0\\ -x, x<0\end{cases}\right]$$

Clearly, $\lim\limits_{x\to a} f(x)$ exists for all $a\neq 0$.

So, let us see whether $\lim\limits_{x\to 0} f(x)$ exist or not.

We have,

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0} f(0-h) = \lim\limits_{h\to 0} -(-h)+1 = 1$

$\lim\limits_{x\to 0+} f(x) = \lim\limits_{x\to 0} f(0+h) = \lim\limits_{h\to 0} h-1 = -1$

$\therefore \lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$

So, $\lim\limits_{x\to 0} f(x)$ does not exist. Hence, $\lim\limits_{x\to a} f(x)$ exists for all $a\neq 0$.

Example – 31

Suppose $f(x)=\begin{cases} a+bx & , \quad x<1 \\ 4 & , \quad x=1 \\ b-ax & , \quad x>1 \end{cases}$

and if $\lim\limits_{x\to1} f(x)=f(1)$. What are possible values of a and b ?

Sol. We have,

$\lim\limits_{x\to1} f(x)=f(1)$

$\Leftrightarrow \quad \lim\limits_{x\to1^-} f(x)=\lim\limits_{x\to1^+} f(x)=f(1)$

$\Leftrightarrow \quad \lim\limits_{x\to1^-} f(x)=f(1)$ and $\lim\limits_{x\to1^+} f(x)=f(1)$

$\Leftrightarrow \quad \lim\limits_{h\to0} f(1-h)=4$ and $\lim\limits_{h\to0} f(1+h)=4$

$\Leftrightarrow \quad \lim\limits_{h\to0} \{a+b(1-h)\}=4$ and $\lim\limits_{h\to0} \{b-a(1+h)\}=4$

$\Leftrightarrow \quad a+b=4$ and $b-a=4$

$\Leftrightarrow \quad a=0, b=4$

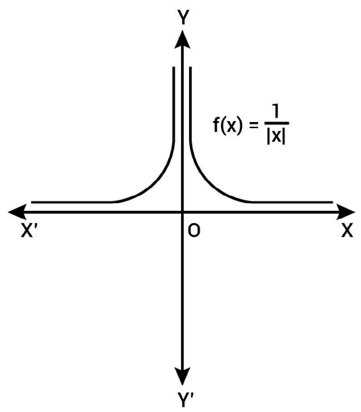
Example – 32

Discuss the existence of each of the following limits :

(i) $\lim\limits_{x\to0} \frac{1}{x}$ (ii) $\lim\limits_{x\to0} \frac{1}{|x|}$

Sol.(i) The graph of $f(x)=\frac{1}{x}$ is as shown in Fig. We observe that as x approaches to 0 from the LHS i.e. x is negative and very close to zero, then the values of $\frac{1}{x}$ are negative and very large in magnitude.

$\therefore \quad \lim\limits_{x\to0^-} \frac{1}{x} \to -\infty$



Similarly, when x approaches to 0 from the right i.e. x is positive and very close to 0, then the values of $\frac{1}{x}$ are very large and positive.

$\therefore \quad \lim\limits_{x\to0^+} \frac{1}{x} \to \infty$ Thus we have, $\lim\limits_{x\to0^-} \frac{1}{x} \neq \lim\limits_{x\to0^+} \frac{1}{x}$

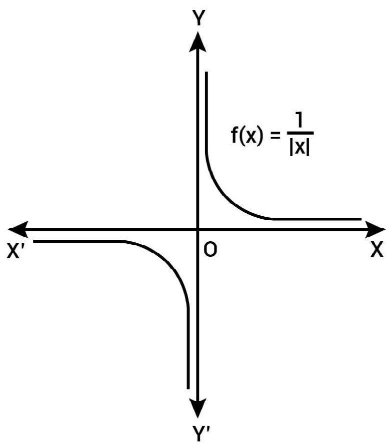
Hence, $\lim\limits_{x\to0} \frac{1}{x}$ does not exist.

(ii) The graph of $f(x)=\frac{1}{|x|}$ is shown in Fig.. We observe that as x approaches to 0 from LHS i.e. x is negative and close to 0, then $|x|$ is close to zero and is positive. Consequently, $\frac{1}{|x|}$ is large and positive.

$\therefore \quad \lim\limits_{x\to0^-} \frac{1}{|x|} \to \infty$

Also, if x approaches to 0 from RHS i.e. x is positive and close to 0, then $|x|$ is close to zero and is positive.

Consequently, $\frac{1}{|x|}$ is large and positive.



$\therefore \quad \lim\limits_{x\to0^+} \frac{1}{|x|} \to \infty$

Thus, we have

$$\lim_{x\to0^-} \frac{1}{|x|} = \lim_{x\to0^+} \frac{1}{|x|}$$

Hence, L.H.L = R.H.L $\neq$ finite value (limit does not exist)

Example – 33

Find the derivative of sin x at x = 0.

Sol. Let f (x) = sin x. Then,

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\sin h-\sin 0}{h}=\lim_{h\to 0}\frac{\sin h}{h}=1$$

Example – 34

Differentiate $\sqrt{x}$ with respect to x from first principle

Sol. Let $f(x)=\sqrt{x}$. Then, $f(x+h)=\sqrt{x+h}$

$$\therefore \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$=\lim_{h\to 0}\frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

$$\therefore \frac{d}{dx}\left(\sqrt{x}\right)=\frac{1}{2\sqrt{x}}$$

Example – 35

Differentiate $\frac{ax^2+bx+c}{\sqrt{x}}$ with respect to x.

Sol. $$\frac{d}{dx}\left\{\frac{ax^2+bx+c}{\sqrt{x}}\right\}$$

$$=\frac{d}{dx}\left\{\frac{ax^2}{\sqrt{x}}+\frac{bx}{\sqrt{x}}+\frac{c}{\sqrt{x}}\right\}$$

$$=\frac{d}{dx}\left\{ax^{3/2}+bx^{1/2}+cx^{-1/2}\right\}$$

$$=\frac{d}{dx}\left\{ax^{3/2}\right\}+\frac{d}{dx}\left\{bx^{1/2}\right\}+\frac{d}{dx}\left\{cx^{-1/2}\right\}$$

$$=a\frac{d}{dx}\left\{x^{3/2}\right\}+b\frac{d}{dx}\left\{x^{1/2}\right\}+c\frac{d}{dx}\left\{x^{-1/2}\right\}$$

$$=a\left(\frac{3}{2}x^{1/2}\right)+b\left(\frac{1}{2}x^{-1/2}\right)+c\left(-\frac{1}{2}x^{-3/2}\right)$$

$$=\frac{3a}{2}x^{1/2}+\frac{b}{2}x^{-1/2}-\frac{c}{2}x^{-3/2}$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Concept of LHL & RHL

1. $\lim_{x\to -1}\{[x]+|x|\}$, where [.] denotes the greatest integer function,

(a) is 0 (b) is 1

(c) does not exist (d) none of these

2. $\lim_{x\to 0}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}=$

(a) 0 (b) 1

(c) -1 (d) Does not exist

3. If $f(x)=\begin{cases}\dfrac{\sin[x]}{[x]}, & [x]\neq 0\\ 0, & [x]=0\end{cases}$, where [.] denotes the greatest integer function, then $\lim_{x\to 0}f(x)$ is equal to

(a) 1 (b) 0

(c) -1 (d) Does not exist

Evaluating algebraic limits

4. $\lim_{x\to 1^+}\dfrac{\sqrt{x^2-1}+\sqrt{x-1}}{\sqrt{x^2-1}}$

(a) $\dfrac{1}{2}$ (b) $\sqrt{2}+1$

(c) 1 (d) $1+\dfrac{1}{\sqrt{2}}$

5. $\lim_{x\to\sqrt{3}}\dfrac{x^2-3}{x^2+3\sqrt{3}x-12}$

(a) $\dfrac{1}{5}$ (b) $\dfrac{2}{5}$

(c) $\dfrac{\sqrt{3}}{5}$ (d) $\dfrac{3}{5}$

6. $\lim_{x\to 1}\dfrac{\left(\sqrt{x}-1\right)\left(2x-3\right)}{2x^2+x-3}$ is equal to

(a) $\dfrac{1}{10}$ (b) $\dfrac{-1}{10}$

(c) 1 (d) None of these

7. The value of $\lim_{x\to 7}\dfrac{2-\sqrt{x-3}}{x^2-49}$ is

(a) 2/9 (b) -2/49

(c) 1/56 (d) -1/56

8. $\lim_{x\to b}\dfrac{\sqrt{x-a}-\sqrt{b-a}}{x^2-b^2}$

(a) $\dfrac{1}{4b\sqrt{a-b}}$ (b) $\dfrac{1}{4b\sqrt{b-a}}$

(c) $\dfrac{1}{4a\sqrt{a-b}}$ (d) $\dfrac{1}{b\sqrt{b-a}}$

9. $\lim_{x\to 2}\dfrac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$

(a) $\dfrac{1}{8\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{3}}$

(c) $8\sqrt{3}$ (d) $\sqrt{3}$

10. $\lim_{x\to\infty}\dfrac{(2+x)^{40}\,(4+x)^{5}}{(2-x)^{45}}$

(a) -1 (b) 1

(c) 16 (d) 32

11. $\text{Limit}_{n\to\infty}\dfrac{(n+2)!+(n+1)!}{(n+3)!}$, $n\in N=$

(a) 0 (b) 1

(c) 2 (d) -1

12. $\underset{n\to\infty}{\text{Limit}}\dfrac{5^{n+1}+3^{n}-2^{2n}}{5^{n}+2^{n}+3^{2n+3}}=$

(a) 5 (b) 3

(c) 1 (d) zero

13. $\lim\limits_{x\to\infty}\sqrt{\dfrac{x-\sin x}{x+\cos^2 x}}$

(a) 0 (b) ∞

(c) 1 (d) none of these

14. $\lim\limits_{n\to\infty}(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\text{to n terms}\right)}$ is equal to

(a) 2 (b) 4

(c) 8 (d) 0

15. $\lim\limits_{x\to-\infty}3x+\sqrt{9x^2-x}$

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{6}$

(c) $-\dfrac{1}{6}$ (d) $-\dfrac{1}{3}$

16. $\lim\limits_{x\to3}\left([x-3]+[3-x]-x\right)$, where [.] denotes the greatest integer function, is equal to

(a) 4 (b) –4

(c) 0 (d) Does not exist

17. $\underset{x\to1}{\text{Limit}}\left(1-x+[x-1]+[1-x]\right)=$ where [x] denotes greatest integer function

(a) 0 (b) 1

(c) – 1 (d) does not exist

Trigonometric limits

18. $\lim\limits_{x\to1}(1-x)\tan\left(\dfrac{\pi x}{2}\right)$

(a) $\dfrac{\pi}{2}$ (b) $\pi+2$

(c) $\dfrac{2}{\pi}$ (d) none of these

19. $\lim\limits_{h\to0}\dfrac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}$

(a) -2/3 (b) -3/4

(c) -2$\sqrt{3}$ (d) 4/3

20. $\lim\limits_{\theta\to0}\dfrac{4\theta(\tan\theta-\sin\theta)}{(1-\cos2\theta)^2}$ is

(a) $\dfrac{1}{\sqrt{2}}$ (b) 1/2

(c) 1 (d) 2

21. $\lim\limits_{x\to0}\dfrac{\cos ecx-\cot x}{x}$ is equal to

(a) $\dfrac{-1}{2}$ (b) 1

(c) $\dfrac{1}{2}$ (d) 1

22. $\lim\limits_{x\to0}\dfrac{1-\cos x}{x\,(2^x-1)}$ is equal to

(a) $\dfrac{1}{2}\log_2 e$ (b) $\dfrac{1}{2}\log_e 2$

(c) 1 (d) none of these

23. The value of $\lim\limits_{n\to\infty}\cos\left(\dfrac{x}{2}\right)\cos\left(\dfrac{x}{4}\right)\cos\left(\dfrac{x}{8}\right)\ldots\cos\left(\dfrac{x}{2^n}\right)$ is

(a) 1 (b) $\dfrac{\sin x}{x}$

(c) $\dfrac{x}{\sin x}$ (d) None of these

24. The value of $\underset{x\to0}{\text{Limit}}\dfrac{\cos(\sin x)-\cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6

(c) 1/4 (d) 1/2

25. $\lim_{\theta \to \frac{\pi}{2}} (\sec\theta - \tan\theta) =$

(a) 0 (b) 1
(c) –1 (d) 2

26. $\lim_{x \to \infty} x.\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) π (d) 0

27. $\lim_{x \to 1} (1+\cos\pi x)\cot^2 \pi x$

(a) –1 (b) $\frac{1}{2}$
(c) 1 (d) none of these

Logarithmic and exponential limits

28. $\lim_{x \to 0} \frac{3^x - 1}{\sqrt{x+1}-1}$ is equal to

(a) $\log_e 9$ (b) $\log_e 3$
(c) 0 (d) 1

29. $\lim_{n \to \infty} \frac{4^{1/n}-1}{3^{1/n}-1}$ is equal to

(a) $\log_4 3$ (b) 1
(c) $\log_3 4$ (d) none of these

30. $\lim_{x \to 0} \frac{3^x - 2^x}{4^x - 3^x}$ is equal to

(a) 1 (b) –1
(c) 0 (d) none of these

31. If α, β are the roots of $x^2 - ax + b = 0$, then $\lim_{x \to \alpha} \frac{e^{x^2-ax+b}-1}{x-\alpha}$ is

(a) $\beta - \alpha$ (b) $\alpha - \beta$
(c) 2α (d) 2β

32. $\lim_{x \to 0} \frac{2^x - 1}{(1+x)^{\frac{1}{2}} - 1} =$

(a) log2 (b) log4
(c) $\log\sqrt{2}$ (d) None of these

33. The value of $\lim_{x \to 3}\left(\log_a \frac{x-3}{\sqrt{x+6}-3}\right)$ is

(a) $\log_a 6$ (b) $\log_a 3$
(c) $\log_a 2$ (d) None of these

34. $\text{Limit}_{x \to 2} \frac{\sin\left(e^{x-2}-1\right)}{\ln(x-1)} =$

(a) 0 (b) – 1
(c) 2 (d) 1

One power infinity

35. $\lim_{x \to 0}\left\{\tan\left(\frac{\pi}{4}-x\right)\right\}^{1/x}$ is equal to

(a) 1 (b) e
(c) e^2 (d) e^{-2}

36. $\lim_{x \to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ is equal to

(a) e (b) $e^{1/2}$
(c) e^{-2} (d) none of these

37. $\lim_{x \to \infty}\left(\frac{x-1}{x+1}\right)^{x+2}$ is equal to

(a) e (b) e^{-1}
(c) e^{-2} (d) none of these

38. $\text{Limit}_{x \to 0^+}\left(1+\tan^2\sqrt{x}\right)^{5/x} =$

(a) e^5 (b) e^2
(c) e (d) none

39. The $\lim_{x \to 0}(\cos x)^{\cot x}$ is

(a) - 1 (b) 0
(c) 1 (d) None of these

40. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b are

(a) $a\in R, b\in R$ (b) $a=1, b\in R$

(c) $a\in R, b=2$ (d) $a=1, b=2$

L-Hopital rule

41. Let $f(2)=4$ and $f'(2)=4$. Then, $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ is given by

(a) 2 (b) –2

(c) –4 (d) 3

42. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is

(a) 0 (b) –1/3

(c) 2/3 (d) –2/3

43. The value of $\lim_{x\to 0}\frac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}$ is equal to

(a) 0 (b) -3

(c) -1 (d) infinity

44. $\lim_{x\to 0}\left\{\frac{\sin x-x+\frac{x^3}{6}}{x^5}\right\}=$

(a) 1/120 (b) -1/120

(c) 1/20 (d) None of these

45. The value of $\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}$ is

(a) 1/2 (b) 0

(c) 1 (d) None of these

46. If $f(9)=9$ and $f'(9)=1$, then $\lim_{x\to 9}\frac{3-\sqrt{f(x)}}{3-\sqrt{x}}$ is equal to

(a) 0 (b) 1

(c) –1 (d) None of these

47. $\text{Limit}_{x\to 0}|x|^{\sin x}=$

(a) 0 (b) 1

(c) –1 (d) none

Numerical Value Type Questions

48. $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$ is equal to

49. $\lim_{x\to 1}\frac{1-x^{-2/3}}{1-x^{-1/3}}$ is equal to

50. $\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$ is equal to

51. The value of $\lim_{x\to\infty}\frac{(x+1)(3x+4)}{x^2(x-8)}$ is equal to

52. $\lim_{n\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots+\frac{n}{1-n^2}\right)$ is equal to

53. The value of $\lim_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2+100}$ is equal

54. $\lim_{n\to\infty}\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^n}$ equals

55. $\text{Limit}_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}$ is equal to

56. $\lim_{x\to 0} \frac{2\sin^2 3x}{x^2} =$

57. $\lim_{x\to 0} \frac{x^2 \sin\left(\frac{1}{x}\right)}{\sin x}$ is equal to ____

58. $\lim_{x\to \frac{\pi}{2}} (\tan x) \log \sin x =$

59. The limiting value of $(\cos x)^{1/\sin x}$ as $x \to 0$ is

60. If $\frac{d}{dx}\left(\frac{\sec x + \tan x}{\sec x - \tan x}\right) = \frac{n \cos x}{(1-\sin x)^2}$, then value of n is

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to: **(2015)**

(a) 2 (b) $\frac{1}{2}$

(c) 4 (d) 3

2. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{\sin^2 x}$ is equal to : **(2015/Online Set–1)**

(a) 2 (b) 3

(c) $\frac{5}{4}$ (d) $\frac{3}{2}$

3. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is **(2015/Online Set–2)**

(a) 2 (b) -2

(c) 1/2 (d) -1/2

4. Let $p=\lim_{x\to 0+}(1+\tan^2\sqrt{x})^{\frac{1}{2x}}$ then log p is equal to : **(2016)**

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2

5. If $\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, then 'a' is equal to: **(2016/Online Set–1)**

(a) 2 (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{4}$

6. $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x-x\tan 2x}$ is : **(2016/Online Set–2)**

(a) -2 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) 2

7. $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals : **(2017)**

(a) $\frac{1}{24}$ (b) $\frac{1}{16}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

8. $\lim_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to : **(2017/Online Set–1)**

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$

9. For each $t\in R$, let [t] be the greatest integer less than or equal to t. Then

$\lim_{x\to 0+}x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)$ **(2018)**

(a) does not exist (in R) (b) is equal to 0.

(c) is equal to 15. (d) is equal to 120.

10. $\lim_{x\to 0}\frac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals : **(2018/Online Set–3)**

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $-\frac{1}{6}$ (d) $\frac{1}{6}$

11. $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals : (8-04-2019/Shift-1)

(a) $4\sqrt{2}$ (b) $\sqrt{2}$

(c) $2\sqrt{2}$ (d) 4

12. Let $f:R\to R$ be a differentiable function satisfying $f'(3)+f'(2)=0$. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to : (8-04-2019/Shift-2)

(a) 1 (b) e^{-1}

(c) e (d) e^2

13. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is:

(10-04-2019/Shift-1)

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$

(c) $\frac{3}{2}$ (d) $\frac{4}{3}$

14. If $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$ then a + b is equal to:

(10-04-2019)

(a) -4 (b) 5

(c) -7 (d) 1

15. $\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is _____.

(12-04-2019/Shift-2)

(a) 6 (b) 2

(c) 3 (d) 1

16. Let $f(x)=5-|x-2|$ and , $g(x)=|x-1|, x\in R$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim_{x\to \alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to ______. (12-04-2019/Shift-2)

(a) $\frac{1}{2}$ (b) $\frac{-3}{2}$

(c) $\frac{-1}{2}$ (d) $\frac{3}{2}$

17. $\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ (9-01-2019/Shift-1)

(a) exists and equals $\frac{1}{4\sqrt{2}}$

(b) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(c) exists and equals $\frac{1}{2\sqrt{2}}$

(d) does not exist

18. For each $x\in$R, let [x] be greatest integer less than or equal to x. Then $\lim_{x\to 0}\frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to:

(9-01-2019/Shift-2)

(a) – sin 1 (b) 1

(c) sin 1 (d) 0

19. For each $t\in$ R, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1^+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(10-1-2019/Shift-1)

(a) equals 1 (b) equals 0

(c) equals –1 (d) does not exist

20. Let [x] denote the greatest integer less than or equal to x.

Then $\lim_{x\to 0}\frac{\tan\left(\pi\sin^2 x\right)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$

(11-01-2019/Shift-1)

(a) does not exist (b) equals π

(c) equals π + 1 (d) equals 0

21. $\lim_{x\to 0}\frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to (11-01-2019/Shift-2)

(a) 0 (b) 2

(c) 4 (d) 1

22. $\lim_{x\to\frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is **(12-01-2019/Shift-1)**

(a) 4 (b) $4\sqrt{2}$

(c) $8\sqrt{2}$ (d) 8

23. $\lim_{x\to 1-} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$ is equal to **(12-01-2019/Shift-2)**

(a) $\frac{1}{\sqrt{2\pi}}$ (b) $\sqrt{\frac{2}{\pi}}$

(c) $\sqrt{\frac{\pi}{2}}$ (d) $\sqrt{\pi}$

24. If $\lim_{x\to 1} \frac{x+x^2+x^3+....+x^n-n}{x-1} = 820, (n \in N)$ then the value of n is equal to : **(2-09-2020/Shift-1)**

25. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to : **(2-09-2020/Shift-2)**

(a) e (b) e^2

(c) 2 (d) 1

26. Let [t] denote the greatest integer ≤ t. If for some $\lambda \in R-\{0, 1\}$ $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then L is equal to : **(3-09-2020/Shift-1)**

(a) 0 (b) 2

(c) $\frac{1}{2}$ (d) 1

27. If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$, then the value of k is .... **(3-09-2020/Shift-1)**

28. $\lim_{x\to a} \frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}} (a \neq 0)$ is equal to : **(3-09-2020/Shift-2)**

(a) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (b) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(c) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (d) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

29. Let $f:(0,\infty)\to(0,\infty)$ be a differentiable function such that f(1) = e and $\lim_{t\to x}\frac{t^2 f^2(x)-x^2 f^2(t)}{t-x} = 0$. If f(x) = 1, then x is equal to: **(4-9-2020/Shift-2)**

(a) e (b) 2e

(c) $\frac{1}{e}$ (d) $\frac{1}{2e}$

30. If α is positive root of the equation, p(x) = x^2-x-2=0, then $\lim_{x\to\alpha^+}\frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to: **(5-09-2020/Shift-1)**

(a) $\frac{1}{2}$ (b) $\frac{3}{\sqrt{2}}$

(c) $\frac{3}{2}$ (d) $\frac{1}{\sqrt{2}}$

31. $\lim_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$ **(5-09-2020/Shift-2)**

(a) is equal to $\sqrt{e}$ (b) is equal to 1

(c) is equal to 0 (d) does notexist

32. $\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{\frac{-x}{2}}-3^{1-x}}$ is equal to **(7-01-2020/Shift-1)**

33. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to: **(8-01-2020/Shift-1)**

(a) e (b) $\frac{1}{e^2}$

(c) $\frac{1}{e}$ (d) e^2

34. $\lim_{x\to 0}\frac{\int_0^x t\sin(10t)\,dt}{x}$ is equal to **(8-01-2020/Shift-2)**

(a) 0 (b) $\frac{1}{10}$

(c) $-\frac{1}{10}$ (d) $-\frac{1}{5}$

35. $\lim_{n\to\infty}\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ______.

(24-02-2021/Shift-1)

36. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+....+\frac{1}{n}}{n^2}\right)^n$ is equal to:

(25-02-2021/Shift-1)

(a) 1 (b) 0

(c) $\frac{1}{e}$ (d) $\frac{1}{2}$

37. If $\lim_{x\to 0}\frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of $a-2b$ is: **(25-02-2021/Shift-2)**

38. The value of $\lim_{h\to 0}2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cos h-\sin h)}\right\}$ is :

(26-02-2021/Shift-1)

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{3}{4}$

39. Let $f(x)$ be a differentiable function at $x=a$, such that $f'(a)=2$, $f(a)=4$. Then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals:

(26-02-2021/Shift-2)

(a) $4-2a$ (b) $2a+4$

(c) $2a-4$ (d) $a+4$

40. If $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then $a+b+c$ is equal to ______. **(16-03-2021/Shift-1)**

41. The value of $\lim_{x\to 0^+}\frac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3}$, where $[x]$ denotes the greatest integer $\le x$ is :

(17-03-2021/Shift-1)

(a) $\frac{\pi}{4}$ (b) 0

(c) $\frac{\pi}{2}$ (d) π

42. The value of the limit $\lim_{\theta\to 0}\frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ is equal to :

(17-03-2021/Shift-2)

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{4}$ (d) 0

43. The value of $\lim\limits_{n\to\infty}\dfrac{[r]+[2r]+......+[nr]}{n^2}$, where r is a non-zero real number and $[r]$ denotes the greatest integer less than or equal to r, is equal to : **(17-03-2021/Shift-2)**

(a) r (b) $\dfrac{r}{2}$

(c) 0 (d) 2r

44. If $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of $(6L+1)$ is : **(18-03-2021/Shift-1)**

(a) $\dfrac{1}{6}$ (b) 6

(c) 2 (d) $\dfrac{1}{2}$

45. Let $f:R\to R$ satisfy the equation $f(x+y)=f(x).f(y)$ all $x, y\in R$ and $f(x)\neq 0$ for any $x\in$. If the function f is differentiable at $x=0$ and $f'(0)=3$, then $\lim\limits_{h\to 0}\dfrac{1}{h}(f(h)-1)$ is equal _______. **(18-03-2021/Shift-2)**

46. If the value of $\lim\limits_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to____. **(20-07-2021/Shift-1)**

47. If $\lim\limits_{x\to 0}\dfrac{\alpha xe^x-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2 x}=10,\ \alpha,\beta,\gamma\in R$, then the value of $\alpha+\beta+\gamma$ is ______? **(20-07-2021/Shift-2)**

48. The value of $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to: **(27-07-2021/Shift-2)**

(a) –1 (b) –4

(c) 0 (d) 4

49. Let $f:R\to R$ be a function such that $f(2)=4$ and $f'(2)=1$. Then, the value of $\lim\limits_{x\to 2}\dfrac{x^2f(2)-4f(x)}{x-2}$ is equal to: **(27-07-2021/Shift-1)**

(a) 4 (b) 8

(c) 16 (d) 12

50. If α,β are the distinct roots of $x^2+bx+c=0$, then $\lim\limits_{x\to\beta}\dfrac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to: **(27-08-2021/Shift-1)**

(a) b^2+4c (b) $2(b^2-4c)$

(c) b^2-4c (d) $2(b^2+4c)$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. $\underset{x\to 3}{\text{Limit}} \dfrac{(x^3+27)\ln(x-2)}{(x^2-9)} =$

(a) – 8 (b) 8
(c) 9 (d) – 9

2. If $\underset{x\to 0}{\text{Lim}} \dfrac{x^3}{\sqrt{a+x}(bx-\sin x)} = 1, a > 0$, then $a + 2b$ is equal to

(a) 36 (b) 37
(c) 38 (d) 40

3. For $x \in R$, $\lim\limits_{x\to\infty}\left(\dfrac{x-3}{x+2}\right)^x$ is equal to

(a) e (b) e^{-1}
(c) e^{-5} (d) e^5

4. If $f(n+1) = \dfrac{1}{2}\left\{f(n)+\dfrac{9}{f(n)}\right\}$, $n \in N$ and $f(n) > 0$ for all $n \in N$ then $\lim\limits_{n\to\infty} f(n)$ is equal to

(a) 3 (b) – 3
(c) 1/2 (d) None of these

5. $\lim\limits_{n\to\infty}\dfrac{1}{n}\left(1+e^{1/n}+e^{2/n}+...+e^{\frac{n-1}{n}}\right)$ is equal to

(a) e (b) –e
(c) e – 1 (d) 1 – e

6. $\lim\limits_{h\to 0}\dfrac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$ is

(a) sin a (b) –sin a
(c) cos a (d) –cos a

7. If α, β are the roots of the quadratic equation $ax^2+bx+c=0$ then,

$\underset{x\to\alpha}{\text{Limit}}\dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2} =$

(a) 0 (b) $\dfrac{1}{2}(\alpha-\beta)^2$
(c) $\dfrac{a^2}{2}(\alpha-\beta)^2$ (d) $-\dfrac{a^2}{2}(\alpha-\beta)^2$

8. $\lim\limits_{h\to 0}\dfrac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cos h-\sin h\right)}$ is equal to

(a) 4/3 (b) – 4/3
(c) 2/3 (d) 3/4

9. $\underset{x\to -1}{\text{Limit}}\dfrac{\cos 2-\cos 2x}{x^2-|x|} =$

(a) 2 cos 2 (b) – 2 cos 2
(c) 2 sin 2 (d) – 2 sin 2

10. The value of $\lim\limits_{x\to 0}\left(\left[\dfrac{100x}{\sin x}\right]+\left[\dfrac{99\sin x}{x}\right]\right)$, where [.] represents greatest integral function is

(a) 199 (b) 198
(c) 0 (d) None of these

11. $\underset{x\to 0}{\text{Limit}}\dfrac{(4^x-1)^3}{\sin\left(\frac{x}{p}\right)\ln\left(1+\frac{x^2}{3}\right)} =$

(a) 9 p (log 4) (b) 3 p $(\log 4)^3$
(c) 12 p $(\log 4)^3$ (d) 27 p $(\log 4)^2$

12. $\underset{x\to 2}{\text{Limit}}\dfrac{\sin(e^{x-2}-1)}{\ln(x-1)} =$

(a) 0 (b) – 1
(c) 2 (d) 1

13. The limiting value of $(\cos x)^{1/\sin x}$ as $x \to 0$ is

(a) 1 (b) e

(c) 0 (d) none

14. The value of

$$\lim_{x\to\frac{\pi}{2}}\left[1^{1/\cos^2 x}+2^{1/\cos^2 x}+...+n^{1/\cos^2 x}\right]^{\cos^2 x} \text{ is}$$

(a) 0 (b) n

(c) ∞ (d) $\frac{n(n+1)}{2}$

15. $\text{Limit}_{x\to 5}\frac{x^2-9x+20}{x-[x]}$ where [x] is the greatest integer not greater than x

(a) is equal to 1 (b) 0

(c) 4 (d) none

16. $\text{Limit}_{x\to 0}\sin^{-1}(\sec x)$

(a) is equal to $\pi/2$ (b) is equal to 1

(c) is equal to zero (d) none of these

17. $\text{Lim}_{x\to 0}|x(x-1)|^{[\cos 2x]}$, where [.] denotes greatest integer function, is equal to

(a) 1 (b) 0

(c) e (d) Does not exists

18. If [x] denotes the greatest integer $\le$ x, then

$\text{Limit}_{n\to\infty}\frac{1}{n^4}\left(\left[1^3x\right]+\left[2^3x\right]+...+\left[n^3x\right]\right)$ equals

(a) x/2 (b) x/3

(c) x/6 (d) x/4

19. $$\text{Limit}_{y\to 0}\left(\text{Limit}_{x\to\infty}\frac{\exp\left(x\ln\left(1+\frac{ay}{x}\right)\right)-\exp\left(x\ln\left(1+\frac{by}{x}\right)\right)}{y}\right)=$$

(a) a + b (b) a – b

(c) b – a (d) – (a + b)

20. The value of $\lim_{x\to a^-}\sqrt{a^2-x^2}\cot\left(\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right)$ is

(a) $\frac{2a}{\pi}$ (b) $-\frac{2a}{\pi}$

(c) $\frac{4a}{\pi}$ (d) $-\frac{4a}{\pi}$

21. The value of $\text{Limit}_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6

(c) 1/4 (d) 1/2

22. The values of a, b and c such that

$$\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2 \text{ are}$$

(a) a = 1, b = –2, c = 1 (b) a = 1, b = 2, c = –1

(c) a = 1, b = 2, c = 1 (d) a = –1, b = 2, c = 1

23. $$\lim_{x\to 0}\frac{\sin x^4-x^4\cos x^4+x^{20}}{x^4\left(e^{2x^4}-1-2x^4\right)} \text{ is equal to}$$

(a) 0 (b) –1/6

(c) 1/6 (d) does not exist

24. If $f(a)=2, f'(a)=1, g(a)=-1, g'(a)=2$, then

$$\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a} \text{ is equal to}$$

(a) 3 (b) 5

(c) –3 (d) 0

25. $$\lim_{x\to\frac{\pi}{2}}\frac{\sin x-(\sin x)^{\sin x}}{1-\sin x+\ln\sin x} \text{ equals}$$

(a) 1 (b) 2

(c) 3 (d) 4

Objective Questions II [One or more than one correct option]

26. The value of a for which

$$\lim_{x\to 0}\frac{(e^x-1)^4}{\sin\left(\frac{x^2}{a^2}\right)\log_e\left\{1+\frac{x^2}{2}\right\}}=8,\text{ is}$$

(a) -2 (b) -1

(c) 1 (d) 2

27. If $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$, then

(a) $b=\frac{-3}{2}$ (b) $a=\frac{5}{2}$

(c) $b=\frac{-1}{2}$ (d) $a=\frac{-5}{2}$

28. If $\lim_{x\to 0}(\cos x+a\sin bx)^{1/x}=e^2$, then the values of a and b are

(a) $a=1, b=2$ (b) $a=2, b=1/2$

(c) $a=2\sqrt{2},\ b=\frac{1}{\sqrt{2}}$ (d) $a=4, b=2$

29. If $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\lambda/x}$, $(a, b, c, \lambda>0)$ is equal to

(a) 1, if $\lambda=1$ (b) abc, if $\lambda=1$

(c) abc, if $\lambda=3$ (d) $(abc)^{2/3}$, if $\lambda=2$

30. The limit of sequence $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}$ .... is

(a) a rational number (b) 2

(c) is an irrational number (d) $2\sqrt{2}$

31. If x is a real number in [0, 1] then the value of $\lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\pi x)]$ is given by, where [x] represents greatest integer $\le x$.

(a) 2 if x is rational (b) 1 for all x

(c) 1 if x is irrational (d) 2 for all x

Numerical Value Type Questions

32. The value of $\lim_{x\to 0}\frac{\sin x-\tan x}{\tan^{-1}x-\sin^{-1}x}$ is

Assertion & Reason

(A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(C) If ASSERTION is true, REASON is false.

(D) If ASSERTION is false, REASON is true.

33. **Assertion :** If a and b are positive and [x] denotes greatest integer $\le x$, then

$$\lim_{x\to 0^+}\frac{x}{a}\left[\frac{b}{x}\right]=\frac{b}{a}$$

Reason : $\lim_{x\to\infty}\frac{\{x\}}{x}=0$ where $\{x\}$ denotes fractional part of x.

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

34.

Column–I	Column–II
(A) $\lim_{x\to\infty}x\cos\frac{\pi}{8x}.\sin\frac{\pi}{8x}=$	**(P)** $\frac{\pi}{8}$
(B) $\lim_{x\to 0}\frac{\tan\left[-\pi^2\right]x^2-\left[-\pi^2\right]x^2}{\sin^2(x)}=$	**(Q)** $\sqrt{2}$
(C) $\lim_{x\to\infty}\sqrt{\frac{2x-\sin x+\cos x}{x+\cos^2 x+\sin^2 x}}=$	**(R)** $\frac{8}{\pi}$
(D) $\lim_{x\to 1}\left(\frac{x^n-1}{n(x-1)}\right)^{\frac{1}{x-1}}=$	**(S)** $e^{\frac{n-1}{2}}$
	(T) 0

The correct matching is

(a) (A–P; B – T; C – Q; D –S)

(b) (A–T; B – P; C – Q; D –S)

(c) (A–P; B – Q; C – T; D –S)

(d) (A–S; B – T; C – Q; D –P)

Using the following passage, solve Q.35 to Q.37

Passage – 1

AP is a diameter of a unit circle with centre at O. Let AC be an arc of this circle, which subtends angle θ radian at centre O. A tangent line is drawn to the circle at the point A and a segement AB on this tangent is laid off whose length is equal to that of the arc AC. A straight line BC is drawn to intersect the extension of the diameter AP at Q. CD is the perpendicular that fall from the point C upon the diameter AP.

35. The area of the trapezoid ABCD is

(a) $\dfrac{1-\cos\theta}{\theta-\sin\theta}$ (b) $(\theta+\sin\theta)\sin^2\dfrac{\theta}{2}$

(c) $2\cos^2\dfrac{\theta}{2}(\theta-\sin\theta)$ (d) $\theta(\theta+\sin\theta)$

36. The length AQ equal to

(a) $\dfrac{\theta(1-\cos\theta)}{\theta-\sin\theta}$ (b) $\dfrac{\theta(1-\cos\theta)}{\theta+\sin\theta}$

(c) $\dfrac{\theta(1+\cos\theta)}{\theta-\sin\theta}$ (d) $\dfrac{\theta(1+\cos\theta)}{\theta+\sin\theta}$

37. The value of the $\lim\limits_{\theta\to0^+} AQ$ is

(a) 0 (b) 1

(c) 2 (d) 3

Using the following passage, solve Q.38 to Q.40

Passage – 2

Consider two functions $f(x) = \lim\limits_{n\to\infty}\left(\cos\dfrac{x}{\sqrt{n}}\right)^n$ and $g(x) = -x^{4b}$ where $b = \lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$ Then.

38. $f(x)$ is

(a) e^{-x^2} (b) $e^{\frac{-x^2}{2}}$

(c) e^{x^2} (d) $e^{\frac{x^2}{2}}$

39. $g(x)$ is

(a) $-x^2$ (b) x^2

(c) x^4 (d) $-x^4$

40. Number of solutions of $f(x)+g(x)=0$ is

(a) 2 (b) 4

(c) 0 (d) 1

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. Let $f: R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$, then

$\lim\limits_{x\to 0}\left(\dfrac{f(1+x)}{f(1)}\right)^{1/x}$ equals : **(2002)**

(a) 1 (b) $e^{\frac{1}{2}}$

(c) e^2 (d) e^3

2. If $\lim\limits_{x\to 0}\dfrac{\{(a-n)\,nx-\tan x\}\sin nx}{x^2}=0$, where n is non zero real number, then a is equal to **(2003)**

(a) 0 (b) $\dfrac{n+1}{n}$

(c) n (d) $n+\dfrac{1}{n}$

3. The value of $\lim\limits_{x\to 0}((\sin x)^{1/x}+(1/x)^{\sin x})$, where $x > 0$ is **(2006)**

(a) 0 (b) –1

(c) 1 (d) 2

4. If $\lim\limits_{x\to 0}[1+x\log(1+b^2)]^{\frac{1}{x}}=2b\sin^2\theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is **(2011)**

(a) $\pm\dfrac{\pi}{4}$ (b) $\pm\dfrac{\pi}{3}$

(c) $\pm\dfrac{\pi}{6}$ (d) $\pm\dfrac{\pi}{2}$

5. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4$, then **(2012)**

(a) $a = 1, b = 4$ (b) $a = 1, b = -4$

(c) $a = 2, b = -3$ (d) $a = 2, b = 3$

6. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation

$$\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0,$$

where $a > -1$. Then, $\lim\limits_{a\to 0^+}\alpha(a)$ and $\lim\limits_{a\to 0^+}\beta(a)$ are **(2012)**

(a) $-\dfrac{5}{2}$ and 1 (b) $-\dfrac{1}{2}$ and -1

(c) $-\dfrac{7}{2}$ and 2 (d) $-\dfrac{9}{2}$ and 3

7. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **(2013)**

(a) $-\dfrac{1}{4}$ (b) $\dfrac{1}{2}$

(c) 1 (d) 2

Objective Questions II [One or more than one correct option]

8. Let $L=\lim\limits_{x\to 0}\dfrac{a-\sqrt{a^2-x^2}-\dfrac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then **(2009)**

(a) $a = 2$ (b) $a = 1$

(c) $L=\dfrac{1}{64}$ (d) $L=\dfrac{1}{32}$

9. Let $f(x)=\dfrac{1-x(1+|1-x|)}{|1-x|}\cos\left(\dfrac{1}{1-x}\right)$ for $x \neq 1$. then **(2017)**

(a) $\lim\limits_{x\to 1^+}f(x)=0$

(b) $\lim\limits_{x\to 1^-}f(x)=0$

(c) $\lim\limits_{x\to 1^+}f(x)$ does not exist

(d) $\lim\limits_{x\to 1^-}f(x)$ does not exist

10. Let $f : R \to R$ be a function we say that f has

PROPERTY 1: if $\lim_{h\to 0} \frac{f(h)-f(0)}{\sqrt{|h|}}$ exist and is finite and

PROPERTY 2: if $\lim_{h\to 0} \frac{f(h)-f(0)}{h^2}$ exist and is finite.

Then which of the following options is/are correct? **(2019)**

(a) $f(x) = x^{2/3}$ has PROPERTY 1

(b) $f(x) = \sin x$ has PROPERTY 2

(c) $f(x) = |x|$ has PROPERTY 1

(d) $f(x) = x|x|$ has PROPERTY 2

Numerical Value Type Questions

11. The largest value of the non-negative integer a for which

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$ is **(2014)**

12. Let m and n be two positive integers greater than 1. If $\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is **(2015)**

13. Let $\alpha, \beta \in R$ be such that $\lim_{x\to 0}\frac{x^2\sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha+\beta)$ equals **(2016)**

14. Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit

$$\lim_{x\to 0^+}\frac{(1-x)^{\frac{1}{x}}-e^{-1}}{x^a}$$ is equal to a nonzero real number, is ………. **(2020)**

15. The value of the limit

$$\lim_{x\to\frac{\pi}{2}}\frac{4\sqrt{2}(\sin 3x+\sin x)}{\left(2\sin 2x\sin\frac{3x}{2}+\cos\frac{5x}{2}\right)-\left(\sqrt{2}+\sqrt{2}\cos 2x+\cos\frac{3x}{2}\right)}$$

is____ **(2020)**

Find Answer Key and Detailed Solutions at the end of this book

07

TRIGONOMETRY

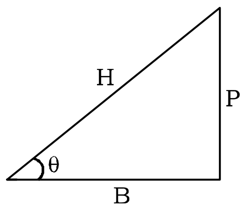
TRIGONOMETRY

TRIGONOMETRIC RATIOS & IDENTITIES

1. THE MEANING OF TRIGONOMETRY

Tri	Gon	Metron
↓	↓	↓
3	sides	Measure

Hence, this particular branch in Mathematics was developed in ancient past to measure 3 sides, 3 angles and 6 elements of a triangle. In today's time–trigonometric functions are used in entirely different shapes. The 2 basic functions are sine and cosine of an angle in a right–angled triangle and there are 4 other derived functions.



sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$	$\frac{B}{P}$	$\frac{H}{B}$	$\frac{H}{P}$

Trigonometric Ratios of Standard Angles

T–Ratio ↓	Angle (θ) 0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.
cot	N.D.	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D.
cosec	N.D.	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

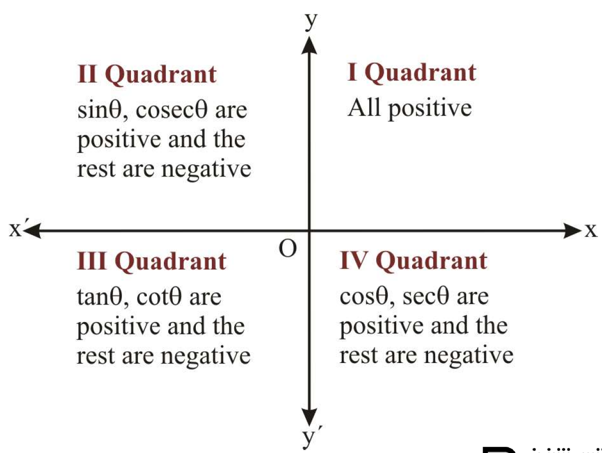
2. BASIC TRIGONOMETRIC IDENTITIES

(a) $\sin^2\theta + \cos^2\theta = 1 : -1 \le \sin\theta \le 1; -1 \le \cos\theta \le 1 \;\forall\; \theta \in R$

(b) $\sec^2\theta - \tan^2\theta = 1 : |\sec\theta| \ge 1 \;\forall\; \theta \in R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\}$

(c) $\text{cosec}^2\theta - \cot^2\theta = 1 : |\text{cosec}\theta| \ge 1 \;\forall\; \theta \in R - \{n\pi, n \in Z\}$

The sign of the trigonometric ratios in different quadrants are as under :



3. TRIGONOMETRIC RATIOS OF ALLIED ANGLES

Using trigonometric ratio of allied angles, we could find the trigonometric ratios of angles of any magnitude.

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$ $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\tan(-\theta) = -\tan\theta$ $\cot(-\theta) = -\cot\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$ $\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$ $\sec(-\theta) = \sec\theta$

$\sec\left(\frac{\pi}{2}-\theta\right) = \operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta$

$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$ $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$\sin(\pi-\theta) = \sin\theta$ $\cos(\pi-\theta) = -\cos\theta$

$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$ $\cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta$

$\tan(\pi-\theta) = -\tan\theta$ $\cot(\pi-\theta) = -\cot\theta$

$\sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right) = \sec\theta$

$\sec(\pi-\theta) = -\sec\theta$ $\operatorname{cosec}(\pi-\theta) = \operatorname{cosec}\theta$

$\sin(\pi+\theta) = -\sin\theta$ $\cos(\pi+\theta) = -\cos\theta$

$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta$ $\cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta$

$\tan(\pi+\theta) = \tan\theta$ $\cot(\pi+\theta) = \cot\theta$

$\tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta$ $\cot\left(\frac{3\pi}{2}-\theta\right) = \tan\theta$

$\sec(\pi+\theta) = -\sec\theta$ $\operatorname{cosec}(\pi+\theta) = -\operatorname{cosec}\theta$

$\sec\left(\frac{3\pi}{2}-\theta\right) = -\operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{3\pi}{2}-\theta\right) = -\sec\theta$

$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$ $\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$

$\sin(2\pi-\theta) = -\sin\theta$ $\cos(2\pi-\theta) = \cos\theta$

$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$ $\cot\left(\frac{3\pi}{2}+\theta\right) = -\tan\theta$

$\tan(2\pi-\theta) = -\tan\theta$ $\cot(2\pi-\theta) = -\cot\theta$

$\sec\left(\frac{3\pi}{2}+\theta\right) = \operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{3\pi}{2}+\theta\right) = -\sec\theta$

$\sec(2\pi-\theta) = \sec\theta$ $\operatorname{cosec}(2\pi-\theta) = -\operatorname{cosec}\theta$

$\sin(2\pi+\theta) = \sin\theta$ $\cos(2\pi+\theta) = \cos\theta$

$\tan(2\pi+\theta) = \tan\theta$ $\cot(2\pi+\theta) = \cot\theta$

$\sec(2\pi+\theta) = \sec\theta$ $\operatorname{cosec}(2\pi+\theta) = \operatorname{cosec}\theta$

4. TRIGONOMETRIC FUNCTIONS OF SUM OR DIFFERENCE OF TWO ANGLES

(a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(b) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

(c) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(d) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

(e) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(f) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(g) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(f) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(h) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) \,.\, \sin(A-B)$

(i) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B) \,.\, \cos(A-B)$

(j) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

5. MULTIPLE ANGLES AND HALF ANGLES

(a) $\sin 2A = 2 \sin A \cos A$; $\sin\theta = 2 \sin\dfrac{\theta}{2} \cos\dfrac{\theta}{2}$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$;

$2\cos^2\dfrac{\theta}{2} = 1 + \cos\theta, \; 2\sin^2\dfrac{\theta}{2} = 1 - \cos\theta$

(c) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$; $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

(d) $\sin 2A = \dfrac{2\tan A}{1 - \tan^2 A}$; $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(e) $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$

(g) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

6. TRANSFORMATION OF PRODUCTS INTO SUM OR DIFFERENCE OF SINES & COSINES

(a) $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

(b) $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

(c) $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

(d) $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

7. FACTORISATION OF THE SUM OR OF TWO DIFFERENCE SINES OR COSINES

(a) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

8. IMPORTANT TRIGONOMETRIC RATIOS

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$ where $n \in Z$

(b) $\sin 15^\circ$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$ or $\cos\frac{5\pi}{12}$;

$\cos 15^\circ$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$ or $\sin\frac{5\pi}{12}$;

$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$;

$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$

(c) $\sin\frac{\pi}{10}$ or $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ &

$\cos 36^\circ$ or $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

9. CONDITIONAL IDENTITIES

If $A+B+C=\pi$ then :

(i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(ii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(iv) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(vi) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(vii) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$

(viii) $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$

10. RANGE OF TRIGONOMETRIC EXPRESSION

$E = a\sin\theta + b\cos\theta$

$E = \sqrt{a^2+b^2}\sin(\theta+\alpha)$, $\left(\text{where } \tan\alpha = \frac{b}{a}\right)$

$E = \sqrt{a^2+b^2}\cos(\theta-\beta)$, $\left(\text{where } \tan\beta = \frac{a}{b}\right)$

Hence for any real value of θ, $-\sqrt{a^2+b^2} \le E \le \sqrt{a^2+b^2}$

11. SINE AND COSINE SERIES

(a) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + ... + \sin(\alpha+\overline{n-1}\beta)$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha+\frac{n-1}{2}\beta\right)$$

(b) $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + ... + \cos(\alpha+\overline{n-1}\beta)$

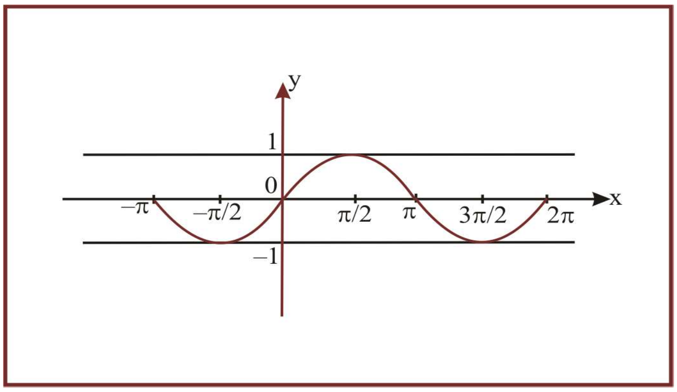
$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha+\frac{n-1}{2}\beta\right)$$

(c) $\cos\theta . \cos 2\theta . \cos 4\theta .... \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n\sin\theta}$

12. GRAPHS OF TRIGONOMETRIC FUNCTIONS

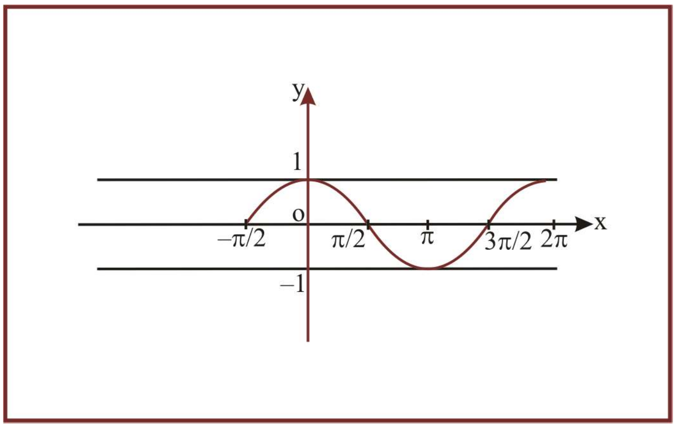
(a) $y = \sin x$,

$x \in R$; $y \in [-1, 1]$



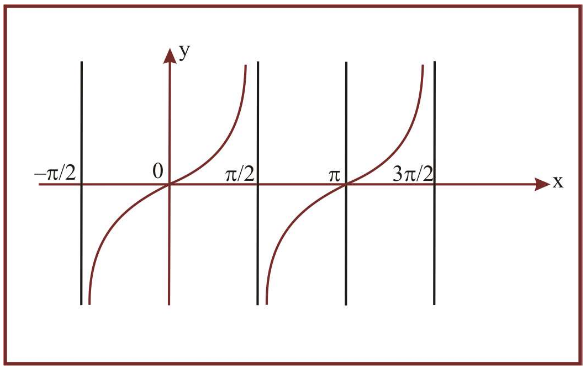
(b) $y = \cos x$,

$x \in R\,;\, y \in [-1, 1]$



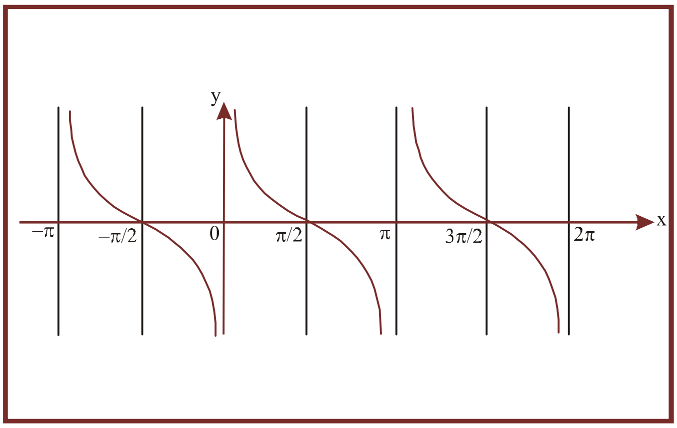
(c) $y = \tan x$,

$$x \in R - \left\{(2n+1)\frac{\pi}{2};\, n \in Z\right\};\; y \in R$$



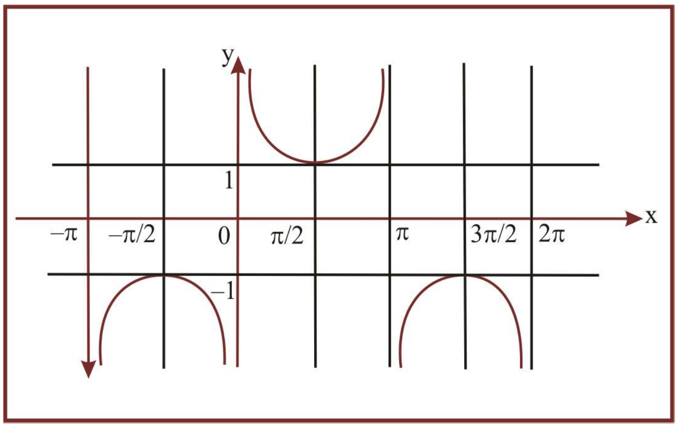
(d) $y = \cot x$,

$x \in R - \{n\pi;\, n \in z\};\, y \in R$



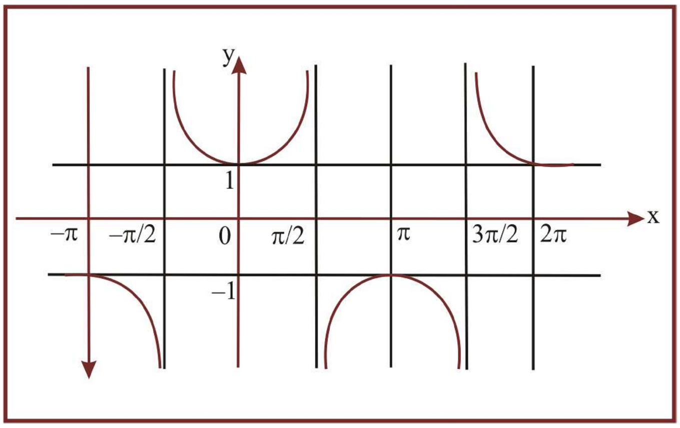
(e) $y = \text{cosec}\, x$,

$x \in R - \{n\pi;\, n \in Z\};\;\, y \in (-\infty, -1] \cup [1, \infty)$



(f) $y = \sec x$,

$$x \in R - \left\{(2n+1)\frac{\pi}{2};\, n \in Z\right\};\; y \in (-\infty, -1] \cup [1, \infty)$$



TRIGONOMETRIC EQUATIONS

13. TRIGONOMETRIC EQUATIONS

The equations involving trigonometric functions of unknown angles are known as Trigonometric equations.

e.g., $\cos\theta = 0$, $\cos^2\theta - 4\cos\theta = 1$.

A solution of a trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g., $\sin\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4}$ or $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{11\pi}{4}, \ldots$

Thus, the trigonometric equation may have infinite number of solutions and can be classified as :

SCAN CODE
Trigonometry

(i) Principal solution: that lie in $[0, 2\pi)$

(ii) General solution

14. GENERAL SOLUTION

Since, trigonometric functions are periodic, a solution is generalised by means of periodicity of the trigonometrical functions. The solution consisting of all possible solutions of a trigonometric equation is called its general solution.

14.1 Results

1. $\sin\theta = 0 \Leftrightarrow \theta = n\pi$
2. $\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\dfrac{\pi}{2}$
3. $\tan\theta = 0 \Leftrightarrow \theta = n\pi$
4. $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\alpha$, where $\alpha \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
5. $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi \pm \alpha$, where $\alpha \in [0, \pi]$.
6. $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$, where $\alpha \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
7. $\sin^2\theta = \sin^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$.
8. $\cos^2\theta = \cos^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$.
9. $\tan^2\theta = \tan^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$.
10. $\sin\theta = 1 \Leftrightarrow \theta = (4n+1)\dfrac{\pi}{2}$.
11. $\cos\theta = 1 \Leftrightarrow \theta = 2n\pi$.
12. $\cos\theta = -1 \Leftrightarrow \theta = (2n+1)\pi$.
13. $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi + \alpha$.

NOTES :

1. Every where in this chapter 'n' is taken as an integer, if not stated otherwise.
2. The general solution should be given unless the solution is required in a specified interval or range.
3. α is taken as the principal value of the angle. (i.e., Numerically least angle is called the principal value).

SOLUTION OF TRIANGLE

1. **Sine Rule :** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

2. **Cosine Rule :**

(i) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

(ii) $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

(iii) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

3. **Projection Formula :**

(i) $a = b\cos C + c\cos B$

(ii) $b = c\cos A + a\cos C$

(iii) $c = a\cos B + b\cos A$

4. **Napier's Analogy - tangent rule :**

(i) $\tan\dfrac{B-C}{2} = \dfrac{b-c}{b+c}\cot\dfrac{A}{2}$

(ii) $\tan\dfrac{C-A}{2} = \dfrac{c-a}{c+a}\cot\dfrac{B}{2}$

(iii) $\tan\dfrac{A-B}{2} = \dfrac{a-b}{a+b}\cot\dfrac{C}{2}$

5. **Trigonometric Functions of Half Angles :**

(i) $\sin\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{b\,c}}$; $\sin\dfrac{B}{2} = \sqrt{\dfrac{(s-c)(s-a)}{ca}}$

$\sin\dfrac{C}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$

(ii) $\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$; $\cos\dfrac{B}{2} = \sqrt{\dfrac{s(s-b)}{ca}}$;

$$\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}$$

(iii) $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\frac{\Delta}{s(s-a)}$ where

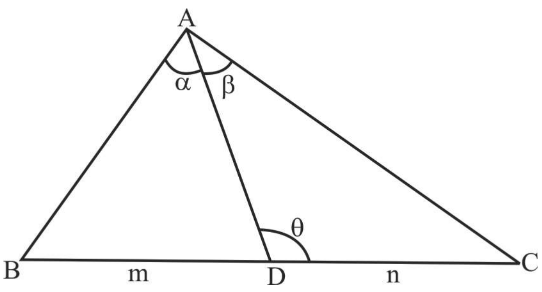
$s=\frac{a+b+c}{2}$ is semi peremetre of triangle.

6. m-n Rule :

If BD : DC = m : n, then

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$= n\cot B - m\cot C$



7. Area of Triangle (Δ) :

$$\Delta=\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A=\frac{1}{2}ca\ \sin B$$

$$=\sqrt{s(s-a)(s-b)(s-c)}$$

8. Radius of Circumcircle :

$$R=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}=\frac{abc}{4\Delta}$$

9. Radius of The Incircle :

(i) $r=\frac{\Delta}{s}$

(ii) $r=(s-a)\tan\frac{A}{2}=(s-b)\tan\frac{B}{2}=(s-c)\tan\frac{C}{2}$

(iii) $r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

10. Radius of the Ex-Circles :

(i) $r_1=\frac{\Delta}{s-a}:r_2=\frac{\Delta}{s-b}:r_3=\frac{\Delta}{s-c}$

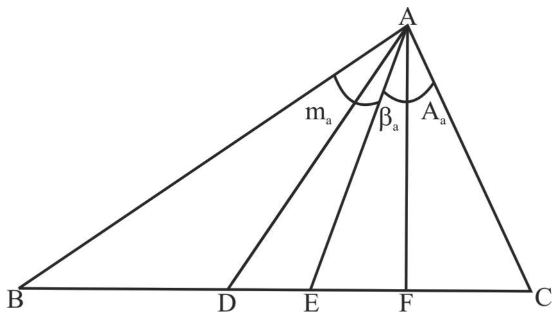
(ii) $r_1=s\tan\frac{A}{2};r_2=s\tan\frac{B}{2};r_3=s\tan\frac{C}{2}$

(iii) $r_1=4R\sin\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}$

11. Length of Angle Bisectors, Medians & Altitudes :

(i) Length of an angle bisector from the angle A

$$\beta_a=\frac{2bc\cos\frac{A}{2}}{b+c};$$



(ii) Length of the median from the angle A

$$=m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}\ \ \&$$

(iii) Length of altitude from the angle A

$$A_a=\frac{2\Delta}{a}$$

12. The distance of the special points from vertices and sides of triangle :

(i) circumcentre (O) : OA = R and O_a = R cos A

(ii) Incentre (I) : IA = r cosec(A/2) and $I_a = r$

(iii) Excentre (I_1) : $I_1A = r_1$ cosec(A/2)

(iv) Orthocentre : HA = 2R cos A and H_a = 2R cos B cos C

(v) Centroid (G) : $GA = \frac{1}{3}\sqrt{2b^2 + 2c^2 - a^2}$ and

$G_a = 2\Delta / 3a$

13. Pedal Triangle :

The triangle formed by joining the feet of the altitudes is called the Pedal Triangle.

(i) Its angles are $\pi - 2A, \pi - 2B$ and $\pi - 2C$.

(ii) The sides are a cos A = R sin 2A, a cos B = R sin 2B, a cos C = R sin 2C

(iii) Circum radii of the triangle PBC, PCA, PAB and ABC are equal.

SOLVED EXAMPLES

Example – 1

The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as

(a) $\sin A \cos A + 1$ (b) $\sec A \operatorname{cosec} A + 1$

(c) $\tan A + \cot A$ (d) $\sec A + \operatorname{cosec} A$

Ans. (b)

Sol. $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$

$$=\frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)}$$

$$=\frac{\sin^3 A-\cos^3 A}{(\sin A-\cos A)\cos A\sin A}$$

$$=\frac{(\sin A-\cos A)\left(\sin^2 A+\sin A\cos A+\cos^2 A\right)}{(\sin A-\cos A)\sin A\cos A}$$

$$=\frac{1+\sin A\cos A}{\sin A\cos A}=1+\sec A\operatorname{cosec} A$$

Example – 2

Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x)-f_6(x)$ equals :

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Ans. (a)

Sol. $f_k(x)=\frac{1}{K}\left(\sin^k x+\cos^k x\right)$

$$f_4(x)=\frac{1}{4}\left(\sin^4 x+\cos^4 x\right)$$

$$=\frac{1}{4}\left(1-2\sin^2 x\cos^2 x\right)$$

$$=\frac{1}{4}-\frac{1}{2}\sin^2 x\cos^2 x$$

$$f_6(x)=\frac{1}{6}\left(\sin^6 x+\cos^6 x\right)$$

$$=\frac{1}{6}\left(1-3\sin^2 x\cos^2 x\right)$$

$$f_4(x)-f_6(x)=\frac{1}{4}-\frac{1}{6}$$

$$=\frac{1}{12}$$

Example – 3

If $\sec\alpha$ and $\operatorname{cosec}\alpha$ are the roots of $x^2-px+q=0$, then show $p^2=q(q+2)$.

Sol. Since, $\sec\alpha$ and $\operatorname{cosec}\alpha$ are roots of $x^2-px+q=0$

$\therefore$ $\sec\alpha+\operatorname{cosec}\alpha=p$ and $\sec\alpha.\operatorname{cosec}\alpha=q$

$\therefore$ $\sin\alpha+\cos\alpha=p\sin\alpha.\cos\alpha$ and $\sin\alpha.\cos\alpha=\frac{1}{q}$

$\therefore$ $\sin\alpha+\cos\alpha=\frac{p}{q}.$

Squaring both sides, we get

$$\sin^2\alpha+\cos^2\alpha+2\sin\alpha.\cos\alpha=\frac{p^2}{q^2}$$

$$1+2\sin\alpha.\cos\alpha=\frac{p^2}{q^2}$$

or $1+\frac{2}{q}=\frac{p^2}{q^2}$ $\Rightarrow$ $p^2=q(q+2)$.

Example – 4

If $2\cos\theta + \sin\theta = 1\left(\theta \neq \frac{\pi}{2}\right)$ then $7\cos\theta + \sin\theta$ is equal to

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{11}{2}$ (d) $\frac{46}{5}$

Ans. (b)

Sol. $2\cos\theta + \sin\theta = 1$

$\Rightarrow 2\cos\theta = 1 - \sin\theta$

$\Rightarrow 4\cos^2\theta = 1 + \sin^2\theta - 2\sin\theta$

$\Rightarrow 4 - 4\sin^2\theta = 1 + \sin^2\theta - 2\sin\theta$

$\Rightarrow 5\sin^2\theta - 2\sin\theta - 3 = 0$

$\Rightarrow 5\sin^2\theta - 5\sin\theta + 3\sin\theta - 3 = 0$

$\Rightarrow (5\sin\theta + 3)(\sin\theta - 1) = 0$

$\because \theta \neq \frac{\pi}{2} \Rightarrow \sin\theta \neq 1$

$\Rightarrow \sin\theta = -\frac{3}{5} \Rightarrow \cos\theta = -\frac{4}{5}$ *or* $\frac{4}{5}$

If $\cos\theta = -\frac{4}{5}$ then

$7\cos\theta + 6\sin\theta = -\frac{28}{5} - \frac{18}{5} = -\frac{46}{5}$

If $\cos\theta = \frac{4}{5}$ *then*

$7\cos\theta + 6\sin\theta = +\frac{28}{5} - \frac{18}{5} = 2$

Example – 5

Let A and B denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

(a) A is true and B is false (b) A is false and B is true

(c) both A and B are true (d) both A and B are false

Ans. (c)

Sol. $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -3/2$

$\Rightarrow (\cos\beta\cos\gamma + \sin\beta\sin\gamma) + (\cos\gamma\cos\alpha + \sin\gamma\sin\alpha)$

$+(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = -3/2$

$\Rightarrow 2(\cos\beta\cos\gamma + \cos\gamma\cos\alpha + \cos\alpha\cos\beta)$

$+2(\sin\beta\sin\gamma + \sin\gamma\sin\alpha + \sin\alpha\sin\beta) + 3 = 0$

$\Rightarrow \{\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2(\cos\alpha\cos\beta + \cos\beta$

$\cos\gamma + \cos\gamma\cos\alpha)\} + \{\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2$

$(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha)\} = 0$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

Which yields simultaneously

$\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

Example – 6

If α, β and γ are in A.P., show that

$\cot\beta = \frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$.

Sol. Since, α, β and γ are in A.P.

$2\beta = \alpha + \gamma$

$\Rightarrow \cot\beta = \cot\left(\frac{\alpha + \gamma}{2}\right)$

$\Rightarrow \cot\beta = \frac{\cos\left(\frac{\alpha+\gamma}{2}\right)}{\sin\left(\frac{\alpha+\gamma}{2}\right)}$

Multiplying and dividing by 2 sin $\left(\frac{\alpha-\gamma}{2}\right)$, we get

$$\cot\beta = \frac{2\cos\left(\frac{\alpha+\gamma}{2}\right).\sin\left(\frac{\alpha-\gamma}{2}\right)}{2\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\alpha-\gamma}{2}\right)} = \frac{\sin\alpha-\sin\gamma}{\cos\gamma-\cos\alpha}$$

$$\Rightarrow \cot\beta = \frac{\sin\alpha-\sin\gamma}{\cos\gamma-\cos\alpha}.$$

Example – 7

$\cos^2 A\,(3-4\cos^2 A)^2 + \sin^2 A\,(3-4\sin^2 A)^2 =$

(a) cos 4A (b) sin 4A

(c) 1 (d) None of these

Ans. (c)

Sol. $\cos^2 A\left(3-4\cos^2 A\right)^2 + \sin^2 A\left(3-4\sin^2 A\right)^2$

$= \left(4\cos^3 A-3\cos A\right)^2 + \left(3\sin A-4\sin^3 A\right)^2$

$= \cos^2 3A + \sin^2 3A$

$= 1$

Example – 8

Prove that :

$\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan 3A$

Sol. We have,

LHS $= \tan A + \tan(60° + A) - \tan(60° - A)$

$$\Rightarrow \text{LHS} = \tan A + \frac{\sqrt{3}+\tan A}{1-\sqrt{3}\tan A} - \frac{\sqrt{3}-\tan A}{1+\sqrt{3}\tan A}$$

$$\Rightarrow \text{LHS} = \tan A + \frac{8\tan A}{1-3\tan^2 A}$$

$$\Rightarrow \text{LHS} = \frac{9\tan A-3\tan^3 A}{1-3\tan^2 A}$$

$$\Rightarrow \text{LHS} = 3\left(\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}\right) = 3\tan 3A = \text{RHS}$$

Example – 9

Prove that :

$$\cos^3 A + \cos^3(120° + A) + \cos^3(240° + A) = \frac{3}{4}\cos 3A$$

Sol. We know that

$$\cos 3A = 4\cos^3 A - 3\cos A \Rightarrow \cos^3 A = \frac{1}{4}(\cos 3A + 3\cos A)$$

$$\therefore \text{LHS} = \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}\{\cos(360° + 3A) + 3\cos(120° + A)\} + \frac{1}{4}\{\cos(720 + 3A) + 3\cos(240° + A)\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}\{\cos 3A + 3\cos(120° + A)\} + \frac{1}{4}\{\cos 3A + 3\cos(240° + A)\}$$

$$\Rightarrow \text{LHS} = \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A + \cos(120° + A) + \cos(240° + A)\}$$

$$\Rightarrow \text{LHS} = \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A + 2\cos(180° + A)\cos 60°\}$$

$$\Rightarrow \text{LHS} = \frac{3}{4}\cos 3A + \frac{3}{4}\left\{\cos A - 2\cos A \times \frac{1}{2}\right\} = \frac{3}{4}\cos 3A = \text{RHS}$$

Example – 10

Prove that : $\sin 3A \sin^3 A + \cos 3A \cos^3 A = \cos^3 2A$

Sol. We have,

$$\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$$

$\therefore$ LHS $= \sin 3A \sin^3 A + \cos 3A \cos^3 A$

$$\Rightarrow \text{LHS} = \sin 3A\left\{\frac{3\sin A - \sin 3A}{4}\right\} + \cos 3A\left\{\frac{\cos 3A + 3\cos A}{4}\right\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{3(\cos A\cos 3A + \sin A\sin 3A) + (\cos^2 3A - \sin^2 3A)\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{3\cos(3A - A) + \cos 2(3A)\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{3\cos 2A + \cos 3(2A)\}$$

$\Rightarrow$ LHS $= \frac{1}{4}\{3\cos 2A + (4\cos^3 2A - 3\cos 2A)\}$

$= \cos^3 2A = $ RHS

Example – 11

Prove that

$\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$.

Sol. L.H.S. $= \tan A + 2\tan 2A + 4\tan 4A + 8\left(\frac{1-\tan^2 4A}{2\tan 4A}\right)$

$= \tan A + 2\tan 2A + \left(\frac{4\tan^2 4A + 4 - 4\tan^2 4A}{\tan 4A}\right)$

$= \tan A + 2\tan 2A + 4\cot 4A$

$= \tan A + 2\tan 2A + 4\left(\frac{1-\tan^2 2A}{2\tan 2A}\right)$

$= \tan A + \left[\frac{2\tan^2 2A + 2 - 2\tan^2 2A}{\tan 2A}\right]$

$= \tan A + 2\cot 2A$

$= \tan A + 2\left(\frac{1-\tan^2 A}{2\tan A}\right) = \frac{\tan^2 A + 1 - \tan^2 A}{\tan A}$

$= \cot A = $ R.H.S.

Note: Students are adviced to learn above result as formulae.
i.e., $\tan A + 2\cot 2A = \cot A$

Example – 12

If $\cot(\theta-\alpha)$, $3\cot\theta$, $\cot(\theta+\alpha)$ are in A.P., and θ is not an integral multiple of $\frac{\pi}{2}$, then $\sin\theta\,\mathrm{cosec}\,\alpha$ is equal to :

(a) $\pm\sqrt{2}$ (b) $\pm\sqrt{\frac{3}{2}}$

(c) $\pm\sqrt{\frac{2}{3}}$ (d) none of these

Ans. (b)

Sol. $\cot(\theta-\alpha), 3\cot\theta, \cot(\theta+\alpha) \to$ A.P

$\Rightarrow 6.\cot\theta = \cot(\theta-\alpha) + \cot(\theta+\alpha) = \frac{\sin 2\theta}{\sin^2\theta - \sin^2\alpha}$

$\Rightarrow 3\sin^2\theta - 3\sin^2\alpha = \sin^2\theta$

$\Rightarrow 3 - 3\sin^2\alpha \cdot cosec^2\theta = 1$

$\Rightarrow 3\sin^2\alpha \cdot cosec^2\theta = 2$

$\Rightarrow \sin\alpha \cdot cosec\theta = \pm\sqrt{\frac{3}{2}}$

Example – 13

Let α, β be such that $\pi < \alpha - \beta < 3\pi$.

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is

(a) $\frac{6}{65}$ (b) $\frac{3}{\sqrt{130}}$

(c) $-\frac{3}{\sqrt{130}}$ (d) $\frac{-6}{65}$

Ans. (c)

Sol. $\sin\alpha + \sin\beta = \frac{21}{65}$

and $\cos\alpha + \cos\beta = -\frac{27}{65}$

by squaring and adding we get

$2(1 + \cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{(21)^2 + (27)^2}{(65)^2}$

$2[1 + \cos(\alpha-\beta)] = \frac{1170}{(65)^2}$

$\cos^2\frac{(\alpha-\beta)}{2} = \frac{1170}{4\times 65\times 65} = \frac{130\times 9}{(130)\times(130)} = \frac{9}{130}$

$\therefore \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}}$

As $\pi < \alpha-\beta < 3\pi$ then $\cos\left(\frac{\alpha-\beta}{2}\right) = $ negative

Example – 14

Prove that : tan 6° tan 42° tan 66° tan 78° = 1

Sol. We have,

$$\text{LHS}=\frac{\sin 6°\sin 42°\sin 66°\sin 78°}{\cos 6°\cos 42°\cos 66°\cos 78°}$$

$$\Rightarrow \quad \text{LHS}=\frac{(2\sin 66°\sin 6°)(2\sin 78°\sin 42°)}{(2\cos 66°\cos 6°)(2\cos 78°\cos 42°)}$$

$$\Rightarrow \quad \text{LHS}=\frac{(\cos 60°-\cos 72°)(\cos 36°-\cos 120°)}{(\cos 60°+\cos 72°)(\cos 36°+\cos 120°)}$$

$$\Rightarrow \quad \text{LHS}=\frac{(\cos 60°-\sin 18°)(\cos 36°+\sin 30°)}{(\cos 60°+\sin 18°)(\cos 36°-\sin 30°)}$$

$$\Rightarrow \quad \text{LHS}=\frac{\left(\frac{1}{2}-\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}+\frac{1}{2}\right)}{\left(\frac{1}{2}+\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}-\frac{1}{2}\right)}$$

$$=\frac{(3-\sqrt{5})(3+\sqrt{5})}{(\sqrt{5}+1)(\sqrt{5}-1)}=\frac{9-5}{5-1}=1=\text{RHS}$$

Example – 15

Prove that : $4\sin 27°=\sqrt{(5+\sqrt{5})}-\sqrt{(3-\sqrt{5})}$

Sol. We have,

$16\sin^2 27° = 8(1-\cos 54°)$

$\Rightarrow \quad 16\sin^2 27° = 8(1-\sin 36°)$

$$\Rightarrow \quad 16\sin^2 27°=8\left(1-\frac{\sqrt{10-2\sqrt{5}}}{4}\right)$$

$$\Rightarrow \quad 16\sin^2 27°=2\left(4-\sqrt{10-2\sqrt{5}}\right)$$

$$\Rightarrow \quad 16\sin^2 27°=8-2\sqrt{10-2\sqrt{5}}$$

$$\Rightarrow \quad 16\sin^2 27°=(5+\sqrt{5})+(3-\sqrt{5})-2\sqrt{(5+\sqrt{5})(3-\sqrt{5})}$$

$\Rightarrow \quad 16\sin^2 27° =$

$$\left(\sqrt{5+\sqrt{5}}\right)^2+\left(\sqrt{3-\sqrt{5}}\right)^2-2\sqrt{(5+\sqrt{5})(3-\sqrt{5})}$$

$$\Rightarrow \quad 16\sin^2 27°=\left\{\sqrt{5+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right\}^2$$

$$\Rightarrow \quad 4\sin 27°=\sqrt{5+\sqrt{5}}-\sqrt{3-\sqrt{5}}$$

Example – 16

The equation $a\sin x+b\cos x=c$ where $|c|>\sqrt{a^2+b^2}$ has

(a) a unique solution

(b) infinite number of solutions

(c) no solution

(d) None of the above

Ans. (c)

Sol. We know

$$-\sqrt{a^2+b^2}\le a\sin x+b\cos x\le\sqrt{a^2+b^2}$$

But $|c|>\sqrt{a^2+b^2}$

$\Rightarrow$ there is no solution for

$a\sin x+b\cos x=c$

Example – 17

If $y=\sin^2\theta+\operatorname{cosec}^2\theta$, $\theta\ne 0$, then

(a) $y=0$ (b) $y\le 2$

(c) $y\ge -2$ (d) $y\ge 2$

Ans. (d)

Sol. $\because y=\sin^2\theta+\operatorname{cosec}^2\theta$

$\Rightarrow y=(\sin\theta-\operatorname{cosec}\theta)^2+2$

$\Rightarrow y\ge 2$

Example – 18

If $A=\sin^2 x+\cos^4 x$, then for all real x

(a) $\frac{13}{16}\le A\le 1$ (b) $1\le A\le 2$

(c) $\frac{3}{4}\le A\le\frac{13}{16}$ (d) $\frac{3}{4}\le A\le 1$

Ans. (d)

Sol. $A=\sin^2 x+\cos^4 x$

We have $\cos^4 x\le\cos^2 x$

$\sin^2 x = \sin^2 x$

Adding $\sin^2 x + \cos^4 x \le \sin^2 x + \cos^2 x$

$\because A \le 1.$

Again $A = t + (1-t)^2 = t^2 - t + 1$, where $t = \sin^2 x$

& so $0 \le t \le 1$

minimum is $3/4$

Thus $\frac{3}{4} \le A \le 1$

Example – 19

Prove that $\frac{\tan 3x}{\tan x}$ never lies between $\frac{1}{3}$ and 3.

Sol. Let $y = \frac{\tan 3x}{\tan x}$. Then,

$$y = \frac{3\tan x - \tan^3 x}{\tan x(1 - 3\tan^2 x)}$$

$$\Rightarrow \quad y = \frac{3 - \tan^2 x}{1 - 3\tan^2 x}$$

$\Rightarrow \quad (3y - 1)\tan^2 x = y - 3$

$$\Rightarrow \quad \tan^2 x = \frac{y-3}{3y-1}$$

Now,

$\tan^2 x \ge 0$ for all x

$$\therefore \quad \frac{y-3}{3y-1} \ge 0$$

$\Rightarrow \quad y < \frac{1}{3}$ or, $y \ge 3$ (Using wavy curve method)

$\Rightarrow$ y does not lie between 1/3 and 3.

Example – 20

Evaluate :

$\cos 12° \cos 24° \cos 36°. \cos 48°. \cos 72°. \cos 84°.$

Sol. $\cos 12° \cos 24° \cos 36° \cos 48°. \cos 72°. \cos 84°.$

$\Rightarrow \quad \cos 12° \cos 24° \cos 48° . \cos (180° – 96°).$

$\cos 36° . \cos 72°$

$\Rightarrow \quad -(\cos 12° . \cos 24° . \cos 48°. \cos 96°) .$

$(\cos 36° . \cos 72°)$

$$\Rightarrow \quad -\frac{\sin(2^4.12°)}{2^4.\sin(12°)} \cdot \frac{\sin(2^2.36°)}{2^2.\sin(36°)}$$

using, $\cos A \cos 2A \ldots\ldots \cos 2^{n-1} A = \frac{\sin(2^n A)}{2^n \sin A}$

$$\Rightarrow \quad -\frac{\sin(192°)}{16.\sin(12°)} \cdot \frac{\sin(144°)}{4.\sin(36°)}$$

$$\Rightarrow \quad -\frac{\sin(180° + 12°).\sin(180° - 36°)}{64.\sin 12°.\sin 36°}$$

$$\Rightarrow \quad \frac{\sin 12°.\sin 36°}{64 \sin 12°.\sin 36°} = \frac{1}{64}.$$

Example – 21

If $A + B + C = \pi$, then prove the following

(i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A . \sin B . \sin C$

(ii) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A . \cos B. \cos C$

(iii) $\frac{\cos A - \cos B + \cos C + 1}{\cos A + \cos B + \cos C - 1} = \cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right)$

Sol. **(i)** L.H.S.

$= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2 \sin C \cos C$

$= 2 \sin (A + B) . \cos (A – B) + 2 \sin C [–\cos (A + B)]$

$= 2 \sin C . \cos (A – B) – 2\sin C . \cos (A + B)$

$= 2 \sin C [\cos (A – B) – \cos (A + B)]$

$= 2 \sin C \times 2 \sin A \sin B$

$= 4 \sin A \sin B \sin C.$

= R.H.S.

(ii) L.H.S.

$= \sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2 A + \frac{1-\cos 2B}{2} + \frac{1-\cos 2C}{2}$

$= \frac{1}{2}[2 + 2\sin^2 A – (\cos 2B + \cos 2C)]$

$= \frac{1}{2}$ $[2 + 2\sin^2 A - 2\cos\left(\frac{2B+2C}{2}\right)$

$\cos\left(\frac{2B-2C}{2}\right)]$

$= 1 + \sin^2 A - \cos(B + C) \cdot \cos(B - C)$

$= 2 - \cos^2 A + \cos A \cdot \cos(B - C)$

$= 2 + \cos A[-\cos A + \cos(B - C)]$

$= 2 + \cos A[\cos(B + C) + \cos(B - C)]$

$= 2 + \cos A \times 2\cos B \cdot \cos C$

$= 2 + 2\cos A \cdot \cos B \cdot \cos C$

(iii) L.H.S.

$$= \frac{\cos A - \cos B + \cos C + 1}{\cos A + \cos B + \cos C - 1}$$

$$= \frac{(\cos A + \cos C) + (1 - \cos B)}{(\cos A + \cos C) - (1 - \cos B)}$$

$$= \frac{2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) + 2\sin^2\left(\frac{B}{2}\right)}{2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) - 2\sin^2\left(\frac{B}{2}\right)}$$

$$= \frac{2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) + 2\cos^2\left(\frac{A+C}{2}\right)}{2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) - 2\cos^2\left(\frac{A+C}{2}\right)}$$

$$= \frac{2\cos\left(\frac{A+C}{2}\right)\left[\cos\left(\frac{A-C}{2}\right) + \cos\left(\frac{A+C}{2}\right)\right]}{2\cos\left(\frac{A+C}{2}\right)\left[\cos\left(\frac{A-C}{2}\right) - \cos\left(\frac{A+C}{2}\right)\right]}$$

$$= \frac{\cos\left(\frac{A-C}{2}\right) + \cos\left(\frac{A+C}{2}\right)}{\cos\left(\frac{A-C}{2}\right) - \cos\left(\frac{A+C}{2}\right)}$$

$$= \frac{2\cos\left(\frac{A}{2}\right).\cos\left(\frac{C}{2}\right)}{2\sin\left(\frac{A}{2}\right).\sin\left(\frac{C}{2}\right)}$$

$$= \cot\left(\frac{A}{2}\right).\cot\left(\frac{C}{2}\right)$$

$=$ R.H.S.

Example – 22

$2\cos x \cos 2x = \cos x.$

Sol. The given equation is equivalent to the equation $\cos x\,(2\cos 2x - 1) = 0.$

This equation is equivalent to the collection of equations.

$$\left[\begin{array}{l}\cos x = 0,\\ \cos 2x = \frac{1}{2},\end{array}\right. \Rightarrow \left[\begin{array}{ll}x = \frac{\pi}{2} + \pi n, & n \in Z,\\ 2x = \pm\frac{\pi}{3} + 2\pi k, \text{ i.e. } x = \pm\frac{\pi}{6} + \pi k, & k \in Z.\end{array}\right.$$

Answer : $\frac{\pi}{2} + \pi n,\ \pm\frac{\pi}{6} + \pi k \quad (n, k \in Z)$

Example – 23

The number of value of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 4 (b) 6

(c) 1 (d) 2

Ans. (a)

Sol. $2\sin^2 x + 5\sin x - 3 = 0 \Rightarrow (2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = \frac{1}{2}, \sin x \neq -3$

there $\sin x = \frac{1}{2}$ we know that each trigonometric function assumes same value twice in $0 \le x \le 360$

in our problem $0^0 \le x \le 540^0$, So number of value are 4 like $30^0, 150^0, 390^0, 510^0$

Example – 24

$3\cos^2 x - 10\cos x + 3 = 0.$

Sol. Assume $\cos x = y$. The given equation assumes the form $3y^2 - 10y + 3 = 0.$

Solving it, we find that $y_1 = \frac{1}{3}$, $y_2 = 3.$

The value $y_2 = 3$ does not satisfy the condition since $|\cos x| \le 1$.

Consequently, $\cos x = \frac{1}{3}, x = \pm\cos^{-1}\frac{1}{3} + 2\pi n,\ n \in Z$

Answer : $\pm\cos^{-1}\left(\frac{1}{3}\right) + 2\pi n\ (n \in Z)$.

Equations of the form

$a_0 \sin^n x + a_1 \sin^{n-1} x \cos x + a_2 \sin^{n-2} x \cos^2 x + ... + a_{n-1} \sin x \cos^{n-1} x + a_n \cos^n x = 0$,

where $a_0, a_1, ..., a_n$ are real numbers, are said to be homogeneous with respect to sin x and cos x.

Example – 25

All solutions of the equation, $2\sin\theta + \tan\theta = 0$ are obtained by taking all integral values of m and n in :

(a) $2n\pi + \frac{2\pi}{3}$

(b) $n\pi \ \&\ 2m\pi \pm \frac{2\pi}{3}$

(c) $n\pi \ \&\ m\pi \pm \frac{\pi}{3}$

(d) $n\pi \ \&\ 2m\pi \pm \frac{\pi}{3}$

Ans. (b)

Sol. $2\sin\theta + \tan\theta = 0$

$\Rightarrow 2\sin\theta + \frac{\sin\theta}{\cos\theta} = 0$

$\Rightarrow \sin\theta = 0 \text{ or } \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = n\pi \text{ or } \theta = 2m\pi \pm \frac{2\pi}{3}$

Example – 26

$\frac{\sin 3\theta}{2\cos 2\theta + 1} = \frac{1}{2}$ if

(a) $\theta = n\pi + \frac{\pi}{6}$

(b) $\theta = 2n\pi - \frac{\pi}{6}$

(c) $\theta = n\pi + (-1)^n \frac{\pi}{6}$

(d) $\theta = n\pi - \frac{\pi}{6}$

Ans. (c)

Sol. $\frac{\sin 3\theta}{2\cos 2\theta + 1} = \frac{1}{2}$

$\Rightarrow 2\sin 3\theta = 2\cos 2\theta + 1$

$\Rightarrow 2\left(3\sin\theta - 4\sin^3\theta\right) = 2\left(1 - 2\sin^2\theta\right) + 1$

$\Rightarrow 8\sin^3\theta - 4\sin^2\theta - 6\sin\theta + 3 = 0$

$\Rightarrow (2\sin\theta - 1)\left(4\sin^2\theta - 3\right) = 0$

$\Rightarrow \sin\theta = \frac{1}{2} \text{ or } \sin^2\theta = \frac{3}{4}$

$\Rightarrow \theta = n\pi + (-1)^n \cdot \frac{\pi}{6} \text{ or } \sin^2\theta = \sin^2\frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

Example – 27

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 1$

Sol. $\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x = 1, \cos\left(x - \frac{\pi}{6}\right) = 1,$

$x - \frac{\pi}{6} = 2\pi n\ (n \in Z),\ x = \frac{\pi}{6} + 2\pi n\ (n \in Z).$

Answer : $\frac{\pi}{6} + 2\pi n\ (n \in Z)$.

Example – 28

$\cos 3x + \sin 2x - \sin 4x = 0$

Sol. $\cos 3x + (\sin 2x - \sin 4x) = 0$

Transforming the expression in brackets by formula

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

we obtain

$\cos 3x + (-2\sin x \cos 3x) = 0,$

$\cos 3x(1 - 2\sin x) = 0.$

The last equation is equivalent to the collection of equation

$\cos 3x = 0, \sin x = \frac{1}{2};$

consequently, $x = \frac{\pi}{6} + \frac{\pi}{3}n, x = (-1)^k\frac{\pi}{6} + \pi k\ (n, k \in Z)$

The set of solution $x = (-1)^k \frac{\pi}{6} + \pi k$ $(k \in Z)$ belongs entirely to the set of solution $x = \frac{\pi}{6}+\frac{\pi n}{3}$ $(n \in Z)$. Therefore, this set alone remains as a set of solutions.

Answer : $\frac{\pi}{6}+\frac{\pi}{3}n$ $(n \in Z)$.

Example – 29

$\sin 5x \cos 3x = \sin 6x \cos 2x.$

Sol. We apply formula $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha-\beta) + \sin(\alpha+\beta))$ to both sides of the equation :

$$\frac{1}{2}(\sin 8x + \sin 2x) = \frac{1}{2}(\sin 8x + \sin 4x),$$

$$\sin 2x - \sin 4x = 0$$

Using formula $\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$,

we obtain $-2\sin x\cos 3x = 0$.

$$\Rightarrow \left[\begin{array}{l}\sin x = 0,\\ \cos 3x = 0,\end{array}\right. \Rightarrow \left[\begin{array}{ll}x = \pi n, & n \in Z,\\ 3x = \frac{\pi}{2} + \pi k,\ x = \frac{\pi}{6}+\frac{\pi}{3}k, & k \in Z.\end{array}\right.$$

Answer : $\frac{\pi}{6}+\frac{\pi}{3}k$ $(n, k \in Z)$.

Example – 30

$\sin^2 x + \sin^2 2x = 1$

Sol. $\frac{1-\cos 2x}{2}+\frac{1-\cos 4x}{2} = 1 \Rightarrow \cos 2x + \cos 4x = 0$

$\Rightarrow 2\cos 3x\cos x = 0.$

The last equation is equivalent to the collection of two equations.

(a) $\cos 3x = 0,\ 3x = \frac{\pi}{2} + \pi n,\ x = \frac{\pi}{6}+\frac{\pi}{3}n,\ n \in Z$

(b) $\cos x = 0,\ x = \frac{\pi}{2} + \pi k,\ k \in Z$

The set of solutions of equation (b) is a subset of the set of solutions of (a) and, therefore, in the answer we write only roots equation (a).

Answer : $\frac{\pi}{6}+\frac{\pi n}{3}$ $(n \in Z)$.

Example – 31

$\cos x - 2\sin^2\frac{x}{2} = 0.$

Sol. $\cos x - (1-\cos x) = 0 \Rightarrow 2\cos x - 1 = 0$

$\Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3} + 2\pi n\ (n \in Z)$

Answer : $\pm\frac{\pi}{3} + 2\pi n$ $(n \in Z)$.

Example – 32

$2\sin\frac{x}{2}\cos^2 x - 2\sin\frac{x}{2}\sin^2 x = \cos^2 x - \sin^2 x.$

Sol. On the left-hand side of the equation we put the factor $2\sin\frac{x}{2}$ before the parentheses :

$2\sin\frac{x}{2}(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x.$

Replacing the expression $\cos^2 x - \sin^2 x$ by $\cos 2x$ according to formula (2), we get

$$2\sin\frac{x}{2}\cos 2x = \cos 2x,$$

or $$2\sin\frac{x}{2}\cos 2x - \cos 2x = 0$$

$$\Rightarrow \cos 2x\left(2\sin\frac{x}{2} - 1\right) = 0$$

$$\Rightarrow \left[\begin{array}{l}\cos 2x = 0,\\ \sin\frac{x}{2} = \frac{1}{2},\end{array}\right. \Rightarrow \left[\begin{array}{ll}x = \frac{\pi}{4}+\frac{\pi}{2}n, & n \in Z,\\ x = (-1)^k\frac{\pi}{3} + 2\pi k, & k \in Z.\end{array}\right.$$

Answer : $\frac{\pi}{4}+\frac{\pi}{2}n,\ (-1)^k\frac{\pi}{3} + 2\pi k$ $(n, k \in Z)$.

(a) Equations of the form $P(\sin x \pm \cos x, \sin x\cos x) = 0$, where $P(y, z)$ is a polynomial, can be solved by the change.

$$\cos x \pm \sin x = t \Rightarrow 1 \pm 2 \sin x \cos x = t^2.$$

Let us consider an example.

Example – 33

If the equation tan θ + tan 2θ + tan θ tan 2θ =1, then θ is equal to

(a) $\frac{n\pi}{3}-\frac{\pi}{6}$ (b) $\frac{n\pi}{3}+\frac{\pi}{12}$

(c) $\frac{n\pi}{3}+\frac{\pi}{2}$ (d) None of these

Ans. (b)

Sol. $\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$

$$\Rightarrow \tan\theta + \tan 2\theta = 1 - \tan\theta \cdot \tan 2\theta$$

$$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \cdot \tan 2\theta} = 1$$

$$\Rightarrow \tan 3\theta = 1 \Rightarrow 3\theta = n\pi + \frac{\pi}{4}, n \in Z$$

$$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12} \cdot n \in Z$$

Example – 34

The value of cos y cos (π/2 –x) – cos (π/2 –y) cos x + sin y cos (π/2 –x) + cos x sin (π/2 –y) is zero if

(a) x = 0 (b) y = 0

(c) x = y + π/4 (d) y = x – 3π/4

Ans. (d)

Sol. $\cos y \cdot \cos\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - y\right) \cdot \cos x + \sin y \cos\left(\frac{\pi}{2} - x\right)$

$$+ \cos x \sin\left(\frac{\pi}{2} - y\right) = 0$$

$$\Rightarrow (\sin x \cdot \cos y - \cos x \cdot \sin y)$$

$$+(\sin x \cdot \sin y + \cos x \cdot \cos y) = 0$$

$$\Rightarrow \sin(x - y) + \cos(x - y) = 0$$

$$\Rightarrow \tan(x - y) = -1 \Rightarrow x - y = n\pi - \frac{\pi}{4}$$

$$\Rightarrow y = x - \frac{3\pi}{4} \text{ (for n = 1)}$$

Example – 35

sin x + cos x = 1 + sin x cos x.

Sol. We introduce the designation sin x + cos x = t.

Then $(\sin x + \cos x)^2 = t^2$, $1 + 2 \sin x \cos x = t^2$,

$$\sin x \cos x = \frac{t^2 - 1}{2}.$$

In the new designations the initial equation looks like

$$t = 1 + \frac{t^2 - 1}{2} \text{ or } t^2 - 2t + 1 = 0, (t-1)^2 = 0, t = 1,$$

i.e.,

$$\sin x + \cos x = 1, \sqrt{2}\left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x\right) = 1,$$

$$\cos\frac{\pi}{4} \cos x + \sin\frac{\pi}{4} \sin x = \frac{1}{\sqrt{2}},$$

$$\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$x - \frac{\pi}{4} = \pm\frac{\pi}{4} + 2\pi n, \ n \in Z,$$

$$x = \frac{\pi}{4} \pm \frac{\pi}{4} + 2\pi n, \ n \in Z.$$

Answer : $\frac{\pi}{2} + 2\pi n, 2\pi n \ (n \in Z)$.

(b) Equations of the form a sin x + b cos x + d = 0, where a, b, and d are real numbers, and a, b ≠ 0, can be solved by the change.

$$\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \ \sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}},$$

$$x \neq \pi + 2\pi n \quad (n \in Z)$$

Example – 36

$6 \sin^2 x - \sin x \cos x - \cos^2 x = 3.$

Sol. $6 \sin^2 x - \sin x \cos x - \cos^2 x - 3(\sin^2 x + \cos^2 x) = 0.$

Removing the brackets and collecting like terms, we get

$$3 \sin^2 x - \sin x \cos x - 4 \cos^2 x = 0.$$

Since the values $x = \frac{\pi}{2} + \pi n$ are not roots of the equation and $\cos x \neq 0$,

we divide both sides of the equation by $\cos^2 x$

$3\tan^2 x - \tan x - 4 = 0$,

whence $\tan x = -1, x = -\frac{\pi}{4} + \pi n, \quad n \in Z$

and $\tan x = \frac{4}{3}, x = \tan^{-1}\frac{4}{3} + \pi k, \quad k \in Z$

Answer : $-\frac{\pi}{4} + \pi n, \ \tan^{-1}\frac{4}{3} + \pi k \ (n, \ k \in Z)$

Example – 37

$3\cos x + 4\sin x = 5$.

Sol. $$3\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + 4\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = 5,$$

$$3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} = 5 + 5\tan^2\frac{x}{2},$$

$$4\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + 1 = 0, \quad \left(2\tan\frac{x}{2} - 1\right)^2 = 0$$

$$\tan\frac{x}{2} = \frac{1}{2}, x = 2\tan^{-1}\frac{1}{2} + 2\pi n, n \in Z$$

Answer : $2\tan^{-1}\frac{1}{2} + 2\pi n, (n \in Z)$.

(c) Many equations can be solved by introducing a new variable.

$f(x) = \sqrt{\varphi(x)}$

Example – 38

$\tan(p\pi/4) = \cot(q\pi/4)$ if

(a) $p + q = 0$

(b) $p + q = 2n + 1$

(c) $p + q = 2n$

(d) $p + q = 2(2n + 1)$ where n is any integer

Ans. (d)

Sol. $$\tan\left(\frac{p\pi}{4}\right) = \cot\left(\frac{q\pi}{4}\right)$$

$$\Rightarrow \tan\left(\frac{p\pi}{4}\right) = \tan\left(\frac{\pi}{2} - \frac{q\pi}{4}\right)$$

$$\Rightarrow \frac{p\pi}{4} = n\pi + \frac{\pi}{2} - \frac{q\pi}{4}$$

$$\Rightarrow (p+q)\frac{\pi}{4} = \frac{\pi}{2}(2n+1)$$

$$\Rightarrow (p+q) = 2(2n+1), n \in Z$$

Example – 39

$$\left(\cos\frac{x}{4} - 2\sin x\right)\sin x + \left(1 + \sin\frac{x}{4} - 2\cos x\right) \times \cos x = 0.$$

Sol. $$\cos\frac{x}{4}\sin x - 2\sin^2 x + \cos x + \sin\frac{x}{4}\cos x - 2\cos^2 x = 0.$$

$$\sin\left(x + \frac{x}{4}\right) + \cos x - 2(\sin^2 x + \cos^2 x) = 0, \sin\frac{5x}{4} + \cos x = 2.$$

Since the functions $\sin\frac{5x}{4}$ and $\cos x$ have the greatest value equal to 1, their sum is equal to 2 if $\sin\frac{5x}{4} = 1$ and $\cos x = 1$ simultaneously, i.e.

$$\Rightarrow \begin{cases} \sin\frac{5x}{4} = 1, \\ \cos x = 1, \end{cases} \Rightarrow \begin{cases} \frac{5x}{4} = \frac{\pi}{2} + 2\pi n, \\ x = 2\pi k \ (n, k \in Z); \end{cases}$$

$$2\pi k = \frac{2\pi}{5} + \frac{8\pi}{5}n, \quad k = \frac{1+4n}{5}$$

Since $k \in Z$, it follows that $n = 1 + 5m$ ($m \in Z$), and then $x = 2\pi + 8\pi m, m \in Z$

Answer : $2\pi + 8\pi m, m \in Z$

Example – 40

$$\sqrt{1-\cos x} = \sin x, x \in [\pi, 3\pi]$$

Sol. $\begin{cases} 1-\cos \geq 0, \\ \sin x \geq 0. \end{cases}$

Under the condition that both sides of the equation are nonnegative, we square them:

$$1 - \cos x = \sin^2 x, \quad 1 - \cos x = 1 - \cos^2 x,$$

$$\cos^2 x - \cos x = 0, \quad \cos x (\cos x - 1) = 0.$$

(1) $\cos x = 0, x = \frac{\pi}{2} + \pi n, n \in Z,$

(2) $\cos x = 1, x = 2\pi k, k \in Z$. But since $\sin x \geq 0$

and $x \in [\pi, 3\pi]$, we leave $x = 2\pi, \frac{5\pi}{2}$.

Answer : $2\pi, \frac{5\pi}{2}$.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Trigonometric functions

1. If $\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha}=\lambda$ then $\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$ is equal to

(a) $\frac{1}{\lambda}$ (b) λ

(c) $1-\lambda$ (d) $1+\lambda$

2. If sin θ and cos θ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation :

(a) $a^2 + b^2 + 2ac = 0$ (b) $a^2 - b^2 - 2ac = 0$

(c) $a^2 + c^2 - 2ab = 0$ (d) $a^2 - b^2 + 2ac = 0$

3. If $3\sin\theta - 5\cos\theta = a$, then $5\sin\theta + 3\cos\theta$ is equal to

(a) $1 - a$ (b) $\sqrt{1-a^2}$

(c) $\sqrt{34-a^2}$ (d) $\sqrt{34-a^2}$ or $-\sqrt{34-a^2}$

4. Let $0\le\theta\le\frac{\pi}{2}$ and $x = X\cos\theta + Y\sin\theta$,

$y = X\sin\theta - Y\cos\theta$ such that $x^2 + 4xy + y^2 = aX^2 + bY^2$, where a, b are constants. Then

(a) $a = -1, b = 3$ (b) $\theta=\frac{\pi}{6}$

(c) $a = 4, b = -2$ (d) $\theta=\frac{\pi}{3}$

5. If $\tan\theta = -4/3$, then $\sin\theta$ is

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $\frac{-4}{5}$ (d) none of these

6. If $\sqrt{\frac{1-\sin A}{1+\sin A}}+\frac{\sin A}{\cos A}=\frac{1}{\cos A}$, for all permissible values of A, then A may belongs to

(a) First Quadrant

(b) Second Quadrant

(c) Third Quadrant

(d) Fourth Quadrant or First quadrant

7. Which of the following is correct –

(a) $\sin 1^\circ > \sin 1$ (b) $\sin 1^\circ < \sin 1$

(c) $\sin 1^\circ = \sin 1$ (d) $\sin 1^\circ=\frac{\pi}{180}\sin 1$

8. Which of the following is correct ?

(a) $\cos 1 > \cos 2$ (b) $\cos 1 < \cos 2$

(c) $\cos 1 = \cos 2$ (d) none of these

Compound angles formulae

9. If $\sec A=\frac{17}{8}$ and $\text{cosec } B=\frac{5}{4}$; if A, B lies in first quadrant then sec (A + B) is equal to

(a) $\frac{85}{36}$ (b) $-\frac{85}{36}$

(c) $-\frac{85}{84}$ (d) $\frac{85}{84}$

10. The two legs of a right triangle are

$\sin\theta+\sin\left(\frac{3\pi}{2}-\theta\right)$ and $\cos\theta-\cos\left(\frac{3\pi}{2}-\theta\right)$. The length of its hypotenuse is

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) some function of θ

11. The sines of two angles of a triangle are equal to $\frac{5}{13}$ & $\frac{99}{101}$. The cosine of the third angle can be :

(Assume that sum of all angles in a triangle are supplementary)

(a) $\frac{245}{1313}$ (b) $\frac{255}{1313}$

(c) $\frac{735}{1313}$ (d) $\frac{765}{1313}$

12. If tanx. tany = a and x + y = π/6, then tanx and tany satisfy the equation

(a) $x^2-\sqrt{3}(1-a)x+a=0$

(b) $\sqrt{3}x^2-(1-a)x+a\sqrt{3}=0$

(c) $x^2+\sqrt{3}(1+a)x-a=0$

(d) $\sqrt{3}x^2+(1+a)x-a\sqrt{3}=0$

13. If $\alpha+\beta=\frac{\pi}{2}$ and $\beta+\gamma=\alpha$, then tan α equals

(a) $\tan\beta+2\tan\gamma$ (b) $2\tan\beta+\tan\gamma$

(c) $\tan\beta+\tan\gamma$ (d) none of these

14. tan 5x tan 3x tan 2x = ...

(a) tan 5x – tan 3x – tan 2x

(b) $\frac{\sin 5x-\sin 3x-\sin 2x}{\cos 5x-\cos 3x-\cos 2x}$

(c) 0

(d) None of these

15. If $\tan\alpha=\frac{m}{m+1}$, $\tan\beta=\frac{1}{2m+1}$, then α + β is equal to

(a) π/4 (b) π/3

(c) $\tan^{-1}\frac{m}{2m+1}$ (d) $\tan^{-1}\frac{m+1}{2m+1}$

16. If A + C = B, then tan A tan B tan C is

(a) tan A tan B + tan C

(b) tan B – tan C – tan A

(c) tan A + tan C – tan B

(d) –(tan A tan B + tan C)

17. If tanα, tanβ are the roots of the equation $x^2+px+q=0\ (p\neq 0)$, then

(a) $\sin(\alpha+\beta)=-p$ (b) $\tan(\alpha+\beta)=p/(q-1)$

(c) $\cos(\alpha+\beta)=1-q$ (d) none of these

Multiple angle formulae

18. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) 1

(c) $\frac{1}{2}$ (d) $\sqrt{3}$

19. Which of the following when simplified does not reduce to unity ?

(a) $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$

(b) $\frac{\sin(\pi-\alpha)}{\sin\alpha-\cos\alpha\tan\frac{\alpha}{2}}+\cos(\pi-\alpha)$

(c) $\frac{1}{4\sin^2\alpha\cos^2\alpha}-\frac{(1-\tan^2\alpha)^2}{4\tan^2\alpha}$

(d) $\frac{1+\sin 2\alpha}{2(\sin\alpha+\cos\alpha)^2}$

20. If $\pi<2\theta<\frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ is equal to

(a) –2cosθ (b) –2sinθ

(c) 2cosθ (d) 2sinθ

21. If $\tan\frac{\alpha}{2}$ and $\tan\frac{\beta}{2}$ are the roots of the equation $8x^2-26x+15=0$ then cos (α + β) is equal to

(a) $-\frac{627}{725}$ (b) $\frac{627}{725}$

(c) –1 (d) none of these

22. For all real values of θ, cot θ – 2 cot 2θ is equal to

(a) tan 2θ (b) tan θ

(c) –cot 3θ (d) none of these

23. If cos 20° – sin 20° = p then cos 40° is equal to

(a) $-p\sqrt{2-p^2}$ (b) $p\sqrt{2-p^2}$

(c) $p+\sqrt{2-p^2}$ (d) none of these

24. If $x+\frac{1}{x}=2\cos\theta$, then $x^3+\frac{1}{x^3}=$

(a) $\cos 3\theta$ (b) $2\cos 3\theta$

(c) $\frac{1}{2}\cos 3\theta$ (d) $\frac{1}{3}\cos 3\theta$

25. $\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\cos\frac{7\pi}{8}$ is equal to

(a) 1/2 (b) $\frac{1-\sqrt{2}}{2\sqrt{2}}$

(c) 1/8 (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

26. The value of $\sin 78° - \sin 66° - \sin 42° + \sin 6°$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) -1 (d) none of these

27. The value of

$\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{7\pi}{14}.\sin\frac{9\pi}{14}.\sin\frac{11\pi}{14}.\sin\frac{13\pi}{14}$

is equal to

(a) 1 (b) $\frac{1}{16}$

(c) $\frac{1}{64}$ (d) none of these

28. The value of

$\cos 12°.\cos 24°.\cos 36°.\cos 48°.\cos 72°.\cos 84°$ is

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$

(c) $\frac{1}{16}$ (d) $\frac{1}{128}$

Max-min values of trigonometric expression

29. If $x \in R$ and $x \neq 0$, then which of the following is not possible ?

(a) $2\sin\theta = x+\frac{1}{x}$ (b) $2\cos\theta = x+\frac{1}{x}$

(c) $2\sin\theta = x-\frac{1}{x}$ (d) $\sin\theta = x+\frac{1}{x}$

30. Maximum value of $\sin x + \cos x$ is

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

31. Minimum value of $\sin^6\theta + \cos^6\theta$ is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

32. If $A = 2\sin^2\theta - \cos 2\theta$, then A lies in the interval

(a) $[-1, 3]$ (b) $[1, 2]$

(c) $[-2, 4]$ (d) none of these

Trigonometric equations

33. If $4\sin^2\theta = 1$, then the values of θ are

(a) $2n\pi \pm \frac{\pi}{3}, n \in Z$ (b) $n\pi \pm \frac{\pi}{3}, n \in Z$

(c) $n\pi \pm \frac{\pi}{6}, n \in Z$ (d) $2n\pi \pm \frac{\pi}{6}, n \in Z$

34. The general solution of $\tan\left(\frac{\theta}{2}\right) = 0$ is

(a) $2n\pi;\ n \in I$ (b) $n\pi;\ n \in I$

(c) $(2n+1)\frac{\pi}{2};\ n \in I$ (d) None of these

35. A solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$, lies in the interval

(a) $(-\pi/4, \pi/4)$ (b) $(\pi/4, 3\pi/4)$

(c) $(3\pi/4, 5\pi/4)$ (d) $(5\pi/4, 7\pi/4)$

36. In $2\cos^2\theta + 3\sin\theta = 0$, then the general value of θ is–

(a) $n\pi + (-1)^n\frac{\pi}{6};\ n \in I$ (b) $2n\pi \pm \frac{\pi}{6};\ n \in I$

(c) $n\pi + (-1)^{n+1}\frac{\pi}{6};\ n \in I$ (d) None of these

37. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is:

(a) 6 (b) 4

(c) 3 (d) 1

38. A value of θ satisfying $\cos\theta + \sqrt{3}\sin\theta = 2$ is

(a) $\frac{5\pi}{3}$ (b) $\frac{4\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

39. The number of solutions of $\cos\theta + \sqrt{3}\sin\theta = 5, 0 \le \theta \le 5\pi$, is

(a) 4 (b) 0

(c) 5 (d) None of these

40. General solution of equation $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ is

(a) $n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}; n \in I$

(b) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}; n \in I$

(c) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}; n \in I$

(d) None of these

41. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are

(a) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (b) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$ (d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

42. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is

(a) ϕ

(b) $\{\pi/4\}$

(c) $\{n\pi + \pi/4 \mid n = 1, 2, 3.....\}$

(d) $\{2n\pi + \pi/4 \mid n = 1, 2, 3.....\}$

Solution of triangles

43. In a ΔABC, $\tan\frac{A}{2} = \frac{5}{6}, \tan\frac{C}{2} = \frac{2}{5}$, then

(a) a, c, b are in AP (b) a, b, c are in AP

(c) b, a c are in AP (d) a, b, c are in GP

44. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

(a) $\sqrt{\frac{x^3}{8}}$ (b) $\frac{1}{2}x^2$

(c) πx^2 (d) $\frac{3}{2}x^2$

45. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is

(a) $a\cot\left(\frac{\pi}{n}\right)$ (b) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$

(c) $a\cot\left(\frac{\pi}{2n}\right)$ (d) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

46. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is

(a) there is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(b) there is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(c) there is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(d) there is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

47. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(c) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

48. In a triangle ABC, $2ca\sin\left(\frac{A-B+C}{2}\right)$ is equal to

(a) $a^2 + b^2 - c^2$ (b) $c^2 + a^2 - b^2$

(c) $b^2 - c^2 - a^2$ (d) $c^2 - a^2 - b^2$

49. The sides of triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is

(a) 120° (b) 90°

(c) 60° (d) 150°

50. If in a triangle ABC

$$a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2},$$

then the sides a, b and c

(a) are in AP (b) are in GP

(c) are in HP (d) satisfy $a+b=c$

51. In a trangle ABC, if $\cot A\cot B\cot C>0$, then the triangle is

(a) acute angled (b) right angled

(c) obtuse angled (d) does not exist

52. In a triangle PQR, if $\angle R=\frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2+bx+c=0$, $a\neq 0$ then

(a) $b=a+c$ (b) $b=c$

(c) $c=a+b$ (d) $a=b+c$

53. In a Δ PQR, if $3\sin P+4\cos Q=6$ and $4\sin Q+3\cos P=1$, then the angle R is equal to

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

Heights and distances

54. A person standing on the bank of river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is

(a) 40 m (b) 30 m

(c) 20 m (d) 60 m

55. The upper $\left(\frac{3}{4}\right)$th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40m from the foot. A possible height of the vertical pole is

(a) 20 m (b) 40 m

(c) 60 m (d) 80 m

56. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (=a) subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is

(a) $\frac{2a}{\sqrt{3}}$ (b) $2a\sqrt{3}$

(c) $\frac{a}{\sqrt{3}}$ (d) $\sqrt{3}$

57. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is

(a) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}+1}\right)$m (b) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}-1}\right)$m

(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (d) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m

58. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is :

(a) $20(\sqrt{3}-1)$ (b) $40(\sqrt{2}-1)$

(c) $40(\sqrt{3}-\sqrt{2})$ (d) $20\sqrt{2}$

Numerical Value Type Questions

59. If the perimeter of a sector of a circle, of area 25π sq. cms. is 20 cms then area of a sector in sq cm is

60. Number of sides of regular polygon of interior angle $\frac{3\pi}{4}$ is

61. If $5\sin\theta = 3$, then $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta}$ is equal to

62. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x - 1$ is equal to

63. If $f(x) = 3\left[\sin^4\left(\frac{3\pi}{2} - x\right) + \sin^4(3\pi + x)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + x\right) + \sin^6(5\pi - x)\right]$ then, for all permissible values of x, f (x) is equal to

64. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ$ is

65. $\cos 24^\circ + \cos 5^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ =$

66. The value of $\tan\frac{\pi}{8}\tan\frac{3\pi}{8}$ is

67. If $0 \le \beta \le \alpha \le \frac{\pi}{4}$, $\cos(\alpha+\beta) = \frac{3}{5}$ and $\cos(\alpha-\beta) = \frac{4}{5}$ then $\sin 2\alpha$ is equal to

68. If $A - B = \frac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B) =$

69. If $\cos 2x + 2\cos x = 1$ then $\sin^2 x(2 - \cos^2 x)$ is equal to

70. Minimum value of $5\sin^2\theta + 4\cos^2\theta$ is

71. Minimum value of $\sec^2\theta + \mathrm{cosec}^2\theta$ is

72. The maximum value of $12\sin\theta - 9\sin^2\theta$ is

73. The numerical value of $8\sin\frac{\pi}{18}.\sin\frac{5\pi}{18}.\sin\frac{7\pi}{18}$ is equal to

74. If $(2^n + 1)\theta = \pi$, then $2^n \cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta =$

75. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. If the angles of elevation of the top of a tower from three collinear points A, B and C on a line leading to the foot of the tower, are 30^0, 45^0 and 60^0 respectively, then the ratio, AB : BC, is: **(2015)**

(a) $1:\sqrt{3}$ (b) 2 : 3

(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

2. In a $\Delta ABC, \frac{a}{b}=2+\sqrt{3}$ and $\angle C = 60^o$. Then the ordered pair $(\angle A, \angle B)$ is equal to : **(2015/Online Set–1)**

(a) $(45^o, 75^o)$ (b) $(75^o, 45^o)$

(c) $(105^o, 15^o)$ (d) $(15^o, 105^o)$

3. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta = \frac{1}{2}$ and θ is the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to: **(2015/Online Set–2)**

(a) $\frac{3}{5}$ (b) $\frac{7}{5}$

(c) $\frac{4}{5}$ (d) $\frac{8}{5}$

4. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is : **(2016)**

(a) 5 (b) 7

(c) 9 (d) 3

5. The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x+18\cos^4 x}-\sqrt{2\cos^4 x+18\sin^2 x}\right|=1$ is : **(2016/Online Set–1)**

(a) 2 (b) 4

(c) 6 (d) 8

6. If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{6}$, then the minimum value of $\tan A + \tan B$ is : **(2016/Online Set–2)**

(a) $\sqrt{3}-\sqrt{2}$ (b) $2-\sqrt{3}$

(c) $4-2\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$

7. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is **(2017)**

(a) $-\frac{3}{5}$ (b) $\frac{1}{3}$

(c) $\frac{2}{9}$ (d) $-\frac{7}{9}$

8. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to: **(2017)**

(a) $\frac{6}{7}$ (b) $\frac{1}{4}$

(c) $\frac{2}{9}$ (d) $\frac{4}{9}$

9. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to : **(2018)**

(a) $\frac{20}{9}$ (b) $\frac{2}{3}$

(c) $\frac{13}{9}$ (d) $\frac{8}{9}$

10. If tanA and tanB are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\sin^2(A+B) - 10\sin(A+B).\cos(A+B) - 25\cos^2(A+B)$ is : **(2018/Online Set–1)**

(a) -10 (b) 10

(c) -25 (d) 25

11. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60^0 from a point on the ground. If, after five seconds, its elevation from the same point, is 30^0, then the speed (in km/hr) of the aeroplane, is : **(2018/Online Set–1)**

(a) 1500 (b) 1440

(c) 750 (d) 720

12. The number of solutions of sin 3x = cos 2x, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is : **(2018/Online Set–2)**

(a) 1 (b) 2

(c) 3 (d) 4

13. A tower T_1 of height 60m is located exactly opposite to a tower T_2 of height 80m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is : **(2018/Online Set–2)**

(a) $10\sqrt{2}$ (b) $10\sqrt{3}$

(c) $20\sqrt{3}$ (d) $20\sqrt{2}$

14. If an angle A of a ΔABC satisfies

5 cos A + 3 = 0, then the roots of the quadratic equation, $9x^2+27x+20=0$ are : **(2018/Online Set–3)**

(a) sec A, cot A (b) sin A, sec A

(c) sec A, tan A (d) tan A, cos A

15. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 minute for the angle of depression of the car to change from 30^0 to 45^0; then after this, the time taken (in minute) by the car to reach the foot of the tower, is : **(2018/Online Set–3)**

(a) $9(1+\sqrt{3})$ (b) $18(1+\sqrt{3})$

(c) $18(\sqrt{3}-1)$ (d) $\frac{9}{2}(\sqrt{3}-1)$

16. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is: **(8-04-2019/Shift-2)**

(a) 5 : 9 : 13 (b) 6 : 5 : 4

(c) 3 : 4 : 5 (d) 5 : 6 : 7

17. Let $S=\{\theta\in[-2\pi,2\pi]:2\cos^2\theta+3\sin\theta=0\}$. Then the sum of the elements of S is: **(9-04-2019/Shift-1)**

(a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$

(c) 2π (d) π

18. Two poles standing on a horizontal ground are of heights 5m and 10m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is: **(9-04-2019/Shift-2)**

(a) $5(2+\sqrt{3})$ (b) $5(\sqrt{3}+1)$

(c) $\frac{5}{2}(2+\sqrt{3})$ (d) $10(\sqrt{3}-1)$

19. The value of $\sin 10°\sin 30°\sin 50°\sin 70°$ is: **(9-04-2019/Shift-2)**

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$

(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

20. ABC is a triangular park with AB= AC= 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is : **(10-04-2019/Shift-1)**

21. The angles A, B and C of a triangle ABC are in A.P. and $a:b=1:\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is: **(10-4-2019/Shift-2)**

(a) $\frac{2}{\sqrt{3}}$ (b) $4\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\frac{4}{\sqrt{3}}$

22. The number of solutions of the equation $1+\sin^4 x=\cos^2 3x,\ x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$ is **(12-04-2019/Shift-1)**

23. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is: **(12-04-2019/Shift-2)**

(a) $15\left(3+\sqrt{3}\right)$ (b) $15\left(5-\sqrt{3}\right)$

(c) $15\left(3-\sqrt{3}\right)$ (d) $15\left(1+\sqrt{3}\right)$

24. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to : **(12-04-2019/Shift-2)**

(a) R (b) [1, 4]

(c) [3, 7] (d) [2, 6]

25. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals: **(9-01-2019/Shift-1)**

(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(b) $13 - 4\cos^6\theta$

(c) $13 - 4\cos^2\theta + 6\cos^4\theta$

(d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^4\theta$

26. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is **(9-01-2019/Shift-2)**

(a) 3 (b) 1

(c) 4 (d) 2

27. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is: **(10-1-2019/Shift-1)**

(a) π (b) $\frac{5\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{8}$

28. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ **(10-01-2019/Shift-2)**

(a) $\frac{1}{512}$ (b) $\frac{1}{1024}$

(c) $\frac{1}{256}$ (d) $\frac{1}{2}$

29. With the usual notation, in ΔABC, if $\angle A + \angle B = 120^\circ, a = \sqrt{3}+1$ and $b = \sqrt{3}-1$ then the ratio $\angle A : \angle B$, is: **(10-01-2019/Shift-2)**

(a) 7 : 1 (b) 5 : 3

(c) 9 : 7 (d) 3 : 1

30. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for $k = 1, 2, 3, \ldots$ Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to **(11-01-2019/Shift-1)**

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$

(c) $\frac{-1}{12}$ (d) $\frac{5}{12}$

31. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value: **(11-01-2019/Shift-2)**

(a) (7 , 19 ,25) (b) (3 , 4 ,5)

(c) (5 , 12 ,13) (d) (19 , 7 ,25)

32. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is : **(12-01-2019/Shift-1)**

(a) $\sqrt{19}$ (b) $\frac{\sqrt{79}}{2}$

(c) $\sqrt{34}$ (d) $\sqrt{31}$

33. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is: **(12-01-2019/Shift-2)**

(a) 60 (b) 50

(c) 45 (d) 42

34. If the equation $\cos^4\theta+\sin^4\theta+\lambda=0$ has real solutions for θ, then λ lies in the interval : **(2-09-2020/Shift-2)**

(a) $\left(-\frac{1}{2},-\frac{1}{4}\right]$ (b) $\left[-1,-\frac{1}{2}\right]$

(c) $\left[-\frac{3}{2},-\frac{5}{4}\right]$ (d) $\left(-\frac{5}{4},-1\right)$

35. Two vertical poles AB = 15m and CD = 10m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is : **(4-9-2020/Shift-1)**

(a) 5 (b) 20/3

(c) 10/3 (d) 6

36. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30º. If the angle of depression of the image of C in the lake from the point P is 60º,then PC (in m)is equal to: **(4-09-2020/Shift-2)**

(a) $200\sqrt{3}$ (b) $400\sqrt{3}$

(c) 400 (d) 100

37. If $L=\sin^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$ and

$M=\cos^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$ then: **(5-09-2020/Shift-2)**

(a) $M=\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$ (b) $M=\frac{1}{4\sqrt{2}}+\frac{1}{4}\cos\frac{\pi}{8}$

(c) $L=-\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$ (d) $L=\frac{1}{4\sqrt{2}}-\frac{1}{4}\cos\frac{\pi}{8}$

38. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is ____. **(6-09-2020/Shift-1)**

39. The angle of elevation of the summit of a mountain from a point on the ground is 45º. After climbing up one km towards the summit at an inclination of 30º from the ground, the angle of elevation of the summit is found to be 60º. Then the height (in km) of the summit from the ground is : **(6-09-2020/Shift-2)**

(a) $\frac{1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{1}{\sqrt{3}-1}$

40. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos2\beta}{2}}=\frac{1}{\sqrt{10}},\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$

then $\tan(\alpha+2\beta)$ is equal to **(8-01-2020/Shift-2)**

41. The value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8}+\sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$ is **(9-01-2020/Shift-1)**

(a) $\frac{1}{4}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

42. The number of distinct solutions of the equation $\log_{\frac{1}{2}}|\sin x|=2-\log_{\frac{1}{2}}|\cos x|$ in the interval $[0,2\pi]$, is ____________. **(9-01-2020/Shift-1)**

43. If in a triangle ABC, AB = 5 units, $\angle B=\cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is : **(20-07-2021/Shift-1)**

(a) $6+8\sqrt{3}$ (b) $8+2\sqrt{2}$

(c) $4+2\sqrt{3}$ (d) $10+6\sqrt{2}$

44. Let in a right-angled triangle, the smallest angle be θ. If a triangle formed by taking reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to ?

(20-07-2021/Shift-2)

(a) $\frac{\sqrt{5}+1}{4}$ (b) $\frac{\sqrt{5}-1}{2}$

(c) $\frac{\sqrt{2}-1}{2}$ (d) $\frac{\sqrt{5}-1}{4}$

45. A spherical gas balloon of radius 16 meter subtends an angle $60°$ at the eye of the observe A while the angle of elevation of its center from the eye of A is $75°$. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is ?

(25-07-2021/Shift-1)

(a) $8\left(\sqrt{2}+2+\sqrt{3}\right)$ (b) $8\left(\sqrt{6}+\sqrt{2}+2\right)$

(c) $8\left(2+2\sqrt{3}+\sqrt{2}\right)$ (d) $8\left(\sqrt{6}-\sqrt{2}+2\right)$

46. The sum of all values of x in $[0,2\pi]$, for which $\sin x+\sin 2x+\sin 3x+\sin 4x=0$, is equal to ?

(25-07-2021/Shift-1)

(a) 11π (b) 12π

(c) 8π (d) 9π

47. If $\sin\theta+\cos\theta=\frac{1}{2}$, then

$16\left(\sin(2\theta)+\cos(4\theta)+\sin(6\theta)\right)$ is equal to :

(27-07-2021/Shift-2)

(a) 27 (b) –27

(c) –23 (d) 23

48. Let $\alpha=\max\limits_{x\in R}\left\{8^{2\sin 3x}.4^{4\cos 3x}\right\}$ and $\beta=\min\limits_{x\in R}\left\{8^{2\sin 3x}.4^{4\cos 3x}\right\}$

If $8x^2+bx+c=0$ is a quadratic equation whose roots are $\alpha^{\frac{1}{5}}$ and $\beta^{\frac{1}{5}}$, then the value of $c-b$ is equal to:

(27-07-2021/Shift-2)

(a) 43 (b) 42

(c) 50 (d) 47

49. The value of $\cot\frac{\pi}{24}$ is: **(25-07-2021/Shift-2)**

(a) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$ (b) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

(c) $\sqrt{2}-\sqrt{3}+2-\sqrt{6}$ (d) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

50. If n is the number of solutions of the equation

$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$ and

S is the sum of all these solutions, then the order pair (n,S) is ? **(01-09-2021/Shift-2)**

(a) $\left(3,\frac{13\pi}{9}\right)$ (b) $\left(2,\frac{8\pi}{9}\right)$

(c) $\left(3,\frac{5\pi}{3}\right)$ (d) $\left(2,\frac{2\pi}{3}\right)$

51. The value of

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

is: **(26-08-2021/Shift-2)**

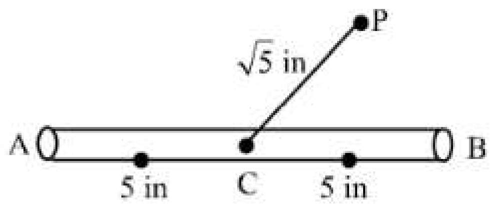
(a) $\frac{1}{8}$ (b) $\frac{1}{8\sqrt{2}}$

(c) $\frac{1}{4\sqrt{2}}$ (d) $\frac{1}{4}$

52. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of the table such that $PC=\sqrt{5}$ inches and $\angle PCB=\tan^{-1}(2)$.

The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is:

(26-08-2021/Shift-2)



(a) $\tan^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$

(c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $\tan^{-1}(1)$

53. Let $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite to these angles are a, b, c respectively, then:

(27-08-2021/Shift-1)

(a) c^2, a^2, b^2 in A.P. (b) $b^2-a^2=a^2+c^2$

(c) b^2, c^2, a^2 are in A.P. (d) a^2, b^2, c^2 are in A.P.

54. The sum of solutions of the equation

$\frac{\cos x}{1+\sin x}=|\tan 2x|,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)-\left\{\frac{\pi}{4},-\frac{\pi}{4}\right\}$ is:

(26-08-2021/Shift-1)

(a) $-\frac{11\pi}{30}$ (b) $-\frac{7\pi}{30}$

(c) $-\frac{\pi}{15}$ (d) $\frac{\pi}{10}$

55. Two poles, AB of length a metres and CD of length $(a+b),(b\neq a)$ metres are erected at the same horizontal level with bases at B and D. If $BD=x$ and $\tan\angle ACB=\frac{1}{2}$, then: **(27-08-2021/Shift-2)**

(a) $x^2+2(a+2b)x+a(a+b)=0$

(b) $x^2-2ax+b(a+b)=0$

(c) $x^2+2(a+2b)x-b(a+b)=0$

(d) $x^2-2ax+a(a+b)=0$

56. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta=0$ in $[0,4\pi]$. The $\frac{8S}{\pi}$ is equal to ____. **(27-08-2021/Shift-2)**

57. A vertical pole fixed to the horizontal ground is divided in the ratio $3:7$ by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18m away from the base of the pole, then the height of the pole (in meters) is?

(31-08-2021/Shift-1)

(a) $12\sqrt{15}$ (b) $8\sqrt{10}$

(c) $6\sqrt{10}$ (d) $12\sqrt{10}$

58. $\text{cosec}\,18°$ is a root of the equation :

(31-08-2021/Shift-1)

(a) $x^2-2x-4=0$ (b) $x^2-2x+4=0$

(c) $4x^2+2x-1=0$ (d) $x^2+2x-4=0$

59. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of $2R+r$ (in cm) is equal to ____.

(16-03-2021/Shift-2)

60. If for $x\in\left(0,\frac{\pi}{2}\right)$, $\log_{10}\sin x+\log_{10}\cos x=-1$ and $\log_{10}(\sin x+\cos x)=\frac{1}{2}(\log_{10}n-1), n>0$, then the value of n is equal to **(16-03-2021/Shift-1)**

(a) 20 (b) 9

(c) 6 (d) 12

61. Let $\tan\alpha, \tan\beta$ and $\tan\gamma;\ \alpha,\beta,\gamma\neq\frac{(2n-1)\pi}{2}, n\in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha+\cos 3\beta+\cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2$ is equal to .... **(17-03-2021/Shift-2)**

62. In a triangle ABC, if $|\overline{BC}| = 8$, $|\overline{CA}| = 7$, $|\overline{AB}| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to : **(18-03-2021/Shift-2)**

(a) $\frac{127}{20}$ (b) $\frac{85}{14}$

(c) $\frac{25}{4}$ (d) $\frac{115}{16}$

63. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to **(18-03-2021/Shift-2)**

(a) 400 (b) 250

(c) 350 (d) 500

64. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the part be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to: **(18-03-2021/Shift-2)**

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2\sqrt{3}}{3}$

(c) $2\sqrt{3}$ (d) $\sqrt{3}$

65. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$ is equal to : **(18-03-2021/Shift-2)**

(a) $3\sqrt{2}$ (b) $2\sqrt{2}$

(c) $\frac{9}{\sqrt{2}}$ (d) $7\sqrt{2}$

66. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is .... **(18-03-2021/Shift-1)**

67. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is : **(24-02-2021/Shift-2)**

(a) $2400\sqrt{3}$ m (b) $1800\sqrt{3}$ m

(c) $1200\sqrt{3}$ m (d) $3600\sqrt{3}$ m

68. If $e^{\left(\cos^2 x + \cos^4 x + \cos^6 x + ...\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is **(24-02-2021/Shift-1)**

(a) $\frac{1}{2}$ (b) $\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\frac{3}{2}$

69. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is **(24-02-2021/Shift-1)**

(a) 25 (b) $25\sqrt{3}$

(c) 30 (d) $20\sqrt{3}$

70. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to : **(25-02-2021/Shift-2)**

(a) $\frac{1+\sqrt{3}}{2}$ (b) $\frac{1-\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

71. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in: **(25-02-2021/Shift-1)**

(a) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(b) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(c) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(d) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

72. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is: **(25-02-2021/Shift-1)**

(a) $10\left(\sqrt{3}-1\right)$ (b) $10\left(\sqrt{3}+1\right)$

(c) 10 (d) $10\sqrt{3}$

73. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is _____. **(26-02-2021/Shift-1)**

74. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is _____ . **(26-02-2021/Shift-1)**

75. The number of solutions of $\sin^7 x + \cos^7 = 1, x \in [0, 4\pi]$ is equal to: **(22-07-2021/Shift-2)**

(a) 5 (b) 9

(c) 11 (d) 7

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. If $\sin x + \cos y = a$ and $\cos x + \sin y = b$, then $\tan \dfrac{x-y}{2}$ is equal to

(a) $a + b$ (b) $a - b$

(c) $\dfrac{a+b}{a-b}$ (d) $\dfrac{a-b}{a+b}$

2. If $6 \cos 2\theta + 2 \cos^2\left(\dfrac{\theta}{2}\right) + 2 \sin^2 \theta = 0, -\pi < \theta < \pi$, then $\theta =$

(a) $\pi/6$ (b) $\pi/3, \cos^{-1}(3/5)$

(c) $\cos^{-1}(3/5)$ (d) $\pi/3, \pi - \cos^{-1}(3/5)$

3. If $\cos A = \dfrac{3}{4}$ then the value of $\sin \dfrac{A}{2} \sin \dfrac{5A}{2}$ is

(a) 1/32 (b) 11/8

(c) –89/8 (d) 11/16

4. Let $\cos(\alpha + \beta) = \dfrac{4}{5}$ and let $\sin(\alpha - \beta) = \dfrac{5}{13}$, where $0 \le \alpha$, $\beta \le \dfrac{\pi}{4}$. Then $\tan 2\alpha$ is equal to

(a) $\dfrac{25}{16}$ (b) $\dfrac{56}{33}$

(c) $\dfrac{19}{12}$ (d) $\dfrac{20}{7}$

5. If $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in G.P. then roots of the equation $x^2 + 2x \cot\beta + 1 = 0$ are always

(a) equal (b) real

(c) imaginary (d) greater than 1

6. Which of the following statements are possible with a, b, m and n being non-zero real numbers :

(a) $4 \sin^2\theta = 5$

(b) $(a^2 + b^2) \cos\theta = 2ab$

(c) $(m^2 + n^2) \operatorname{cosec}\theta = m^2 - n^2$

(d) none of these

7. The equation $k \cos x - 3 \sin x = k + 1$ is solvable only if k belongs to the interval

(a) $[k, +\infty]$ (b) $[-4, 4]$

(c) $(-\infty, 4]$ (d) None of these

8. $\sin\theta(\sin\theta + \sin 3\theta)$ is

(a) ≥ 0 for all θ (b) ≥ 0 only when $\theta \ge 0$

(c) ≤ 0 for all θ (d) ≤ 0 only when $\theta \le 0$

9. The least value of $\cos^2\theta - (6 \sin\theta \cdot \cos\theta) + 3 \sin^2\theta + 2$ is

(a) $4+\sqrt{10}$ (b) $4-\sqrt{10}$

(c) 0 (d) 4

10. If $a = \sin\dfrac{\pi}{18} \sin\dfrac{5\pi}{18} \sin\dfrac{7\pi}{18}$, and x is the solution of the equation $y = 2[x] + 2$ and $y = 3[x-2]$, where $[x]$ denotes the integral part of x, then a is equal to

(a) $[x]$ (b) $\dfrac{1}{[x]}$

(c) $2[x]$ (d) $[x]^2$

11. If $x \in \left[-\dfrac{5\pi}{2}, \dfrac{5\pi}{2}\right]$, the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is

(a) π (b) 2π

(c) $5\pi/2$ (d) none of these

12. The number of real solutions of

$\sin e^x \cdot \cos e^x = 2^{x-2} + 2^{-x-2}$ is

(a) zero (b) one

(c) two (d) infinite

13. The value of θ satisfying

$3 \cos^2\theta - 2\sqrt{3} \sin\theta \cos\theta - 3 \sin^2\theta = 0$ are $(n \in I)$

(a) $n\pi - \dfrac{2\pi}{3}, n\pi + \dfrac{\pi}{6}$ (b) $n\pi - \dfrac{\pi}{3}, n\pi + \dfrac{\pi}{6}$

(c) $2n\pi - \dfrac{\pi}{3}, n\pi$ (d) $2n\pi + \dfrac{\pi}{3}, n\pi$

14. If r > 0, $-\pi \le \theta \le \pi$ and r, θ satisfy $r \sin\theta = 3$ and $r = 4(1+\sin\theta)$, then the number of possible solutions of the pair (r, θ) is

(a) 2 (b) 4

(c) 0 (d) inifinite

15. If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation, $\sin 7x + \sin 4x + \sin x = 0$ is :

(a) 3 (b) 5

(c) 6 (d) None

16. General solution of the equation, $\cot 3\theta - \cot\theta = 0$ is $(n \in I)$

(a) $\theta = (2n-1)\frac{\pi}{2}$ (b) $\theta = (2n-1)\frac{\pi}{4}$

(c) $\theta = (2n-1)\frac{\pi}{3}$ (d) none

17. If m and n (> m) are positive integers, the number of solutions of the equation $n|\sin x| = m|\cos x|$ in $[0, 2\pi]$ is

(a) m (b) n

(c) mn (d) none of these

18. If $2\sin x + 1 \ge 0$ and $x \in [0, 2\pi]$, then the solution set for x is

(a) $\left[0, \frac{7\pi}{6}\right]$ (b) $\left[0, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]$

(c) $\left[\frac{11\pi}{6}, 2\pi\right]$ (d) None of these

19. If $\cos x - \sin x \ge 1$ and $0 \le x \le 2\pi$ then the solution set for x is

(a) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$ (b) $\left[\frac{3\pi}{2}, \frac{7\pi}{4}\right] \cup \{0\}$

(c) $\left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$ (d) none of these

20. If $|\tan x| \le 1$ and $x \in [-\pi, \pi]$ then the solution set for x is

(a) $\left[-\pi, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

(b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

(c) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(d) none of these

21. If $4\sin^2 x - 8\sin x + 3 \le 0$, $0 \le x \le 2\pi$, then the solution set for x is

(a) $\left[0, \frac{\pi}{6}\right]$ (b) $\left[0, \frac{5\pi}{6}\right]$

(c) $\left[\frac{5\pi}{6}, 2\pi\right]$ (d) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

22. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals :

(a) $2(\tan\beta + \tan\gamma)$ (b) $\tan\beta + \tan\gamma$

(c) $\tan\beta + 2\tan\gamma$ (d) $2\tan\beta + \tan\gamma$

23. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution, is :

(a) 4 (b) 8

(c) 10 (d) 12

24. Given both θ and ϕ are acute angles $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to

(a) $\left(\frac{\pi}{3}, \frac{\pi}{6}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (d) $\left(\frac{5\pi}{6}, \pi\right]$

25. The number of ordered pairs (α, β), where $\alpha, \beta \in (-\pi, \pi)$ satisfying $\cos(\alpha - \beta) = 1$ and $\cos(\alpha+\beta) = \frac{1}{e}$ is : $(e > 1)$

(a) 0 (b) 1

(c) 2 (d) 4

26. If $k_1 = \tan 27\theta - \tan\theta$ and

$$k_2 = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}, \text{ then}$$

(a) $k_1 = 2k_2$ (b) $k_1 = k_2 + 4$

(c) $k_1 = k_2$ (d) none of these

27. If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta+\alpha) + 2\tan\alpha$ is

(a) 3 (b) 2

(c) 1 (d) 0.

28. If $\cos\alpha = \dfrac{2\cos\beta - 1}{2 - \cos\beta}$, $(0<\alpha<\pi, 0<\beta<\pi)$, then $\tan\dfrac{\alpha}{2}\cot\dfrac{\beta}{2}$ is equal to

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) none of these

29. $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) =$

(a) $\sin 2\alpha$ (b) $\cos 2\beta$

(c) $\cos 2\alpha$ (d) $\sin 2\beta$

30. $\dfrac{1}{\sin 3\alpha}\left[\sin^3\alpha + \sin^3\left(\dfrac{2\pi}{3}+\alpha\right) + \sin^3\left(\dfrac{4\pi}{3}+\alpha\right)\right]$ is equal to

(a) $\dfrac{4}{3}$ (b) $\dfrac{3}{4}$

(c) $-\dfrac{3}{4}$ (d) none of these

31. If in a triangle ABC, $\sin^2A + \sin^2B + \sin^2C = 2$, then the triangle is always

(a) isosceles triangle (b) right angled

(c) acute angled (d) obtuse angled

32. If in Δ ABC if $A > B$, $\tan A + \tan B + \tan C = 6$ and $\tan A \tan B = 2$, then $\sin^2 A : \sin^2 B : \sin^2 C$ is

(a) 8 : 9 : 5 (b) 8 : 5 : 9

(c) 5 : 9 : 8 (d) 5 : 8 : 5

33. The number of solution in $[0, \pi/2]$ of the equation $\cos 3x \tan 5x = \sin 7x$ is

(a) 5 (b) 7

(c) 6 (d) none of these

34. If $\sin^4x + \cos^4y + 2 = 4\sin x\cos y$, and $0 \le x, y \le \pi/2$ then $\sin x + \cos y$ is equal to :

(a) –2 (b) 0

(c) 2 (d) none of these

35. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $0 < k < 1$, then the measure of angle C is :

(a) $\pi/3$ (b) $\pi/2$

(c) $2\pi/3$ (d) $5\pi/6$

Objective Questions II [One or more than one correct option]

36. If $\sin\theta + \sin\phi = a$ and $\cos\theta + \cos\phi = b$, then

(a) $\cos\left(\dfrac{\theta-\phi}{2}\right) = \pm\dfrac{1}{2}\sqrt{(a^2+b^2)}$

(b) $\cos\left(\dfrac{\theta-\phi}{2}\right) = \pm\dfrac{1}{2}\sqrt{(a^2-b^2)}$

(c) $\tan\left(\dfrac{\theta-\phi}{2}\right) = \pm\sqrt{\left(\dfrac{4-a^2-b^2}{a^2+b^2}\right)}$

(d) $\cos(\theta-\phi) = \dfrac{a^2+b^2-2}{2}$

37. If $\sin x + 7\cos x = 5$ then $\cos(x-\phi) = 1/\sqrt{2}$ if

(a) $\phi = \cos^{-1} 7/\sqrt{50}$ (b) $\phi = \sin^{-1} 1/\sqrt{50}$

(c) $\phi = \cos^{-1} 1/7$ (d) $\phi = \sin^{-1} 5/7$

38. Which of following functions have the maximum value unity?

(a) $\sin^2x - \cos^2x$

(b) $\sqrt{\dfrac{6}{5}}\left(\dfrac{1}{\sqrt{2}}\sin x + \dfrac{1}{\sqrt{3}}\cos x\right)$

(c) $\cos^6x + \sin^6x$

(d) $\cos^2x + \sin^4x$

39. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if

(a) $\cos 12x = \cos 14x$ (b) $\sin 13x = 0$

(c) $\sin x = 0$ (d) $\cos x = 0$

40. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by :

(a) $x = n\pi + (-1)^{n+1}(\pi/6)$

(b) $x = n\pi + (-1)^n(\pi/6)$

(c) $x = n\pi + (-1)^n(\pi/3)$

(d) $x = n\pi - (-1)^n(\pi/6)$ where $n \in Z$

41. $4\sin^4x + \cos^4x = 1$ if

(a) $x = n\pi$ (b) $n\pi \pm \sin^{-1}\sqrt{\dfrac{2}{5}}$

(c) $x = n\pi/2$ (d) none

42. $\cos 15x = \sin 5x$ if $(n \in I)$

(a) $x = -\frac{\pi}{20} + \frac{\pi}{5}n$ (b) $x = \frac{\pi}{40} + \frac{\pi}{10}n$

(c) $x = \frac{3\pi}{20} + \frac{\pi}{5}n$ (d) $x = -\frac{3\pi}{40} + \frac{\pi}{10}n$

43. $\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$ if

(a) $\tan x = 3$ (b) $\tan x = -1$

(c) $x = n\pi + \pi/4$ (d) $x = n\pi + \tan^{-1}(-3)$

44. $5\sin^2 x + \sqrt{3}\sin x\cos x + 6\cos^2 x = 5$ if $(n \in I)$

(a) $\tan x = -1/\sqrt{3}$ (b) $\sin x = 0$

(c) $x = n\pi + \pi/2$ (d) $x = n\pi + \pi/6$

45. The positive values of x satisfy the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+......\infty} = 4^3$ will be (where $|\cos x| < 1$)

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ (d) None of these

46. The equation $|\cot x| = \cot x + \frac{1}{\sin x}$, $(n \in Z)$

(a) has a general solution $\frac{2\pi}{3}(3n+1)$

(b) has a general solution $\frac{2\pi}{3}(3n-1)$

(c) is not defined if $x = n\pi$

(d) cannot have a solution if cot x is positive

47. If $\left(\frac{\sin\theta}{\sin\phi}\right)^2 = \frac{\tan\theta}{\tan\phi} = 3$, then

(a) $\tan\phi = 1/\sqrt{3}$ (b) $\tan\phi = -1/\sqrt{3}$

(c) $\tan\theta = \sqrt{3}$ (d) $\tan\theta = -\sqrt{3}$

48. Let $f_n(\theta) = \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)............(1+\sec 2^n\theta)$, then

(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$

(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

49. In a triangle ABC

(a) $\sin A\sin B\sin C \le \frac{3\sqrt{3}}{8}$

(b) $\sin^2 A + \sin^2 B + \sin^2 C \le \frac{9}{4}$

(c) sin A sin B sin C is always positive

(d) $\sin^2 A + \sin^2 B = 1 + \cos C$

50. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$ if

(a) $\cos x = -1/2$ (b) $\sin 2x = \cos 2x$

(c) $x = n\pi/2 + \pi/8$ (d) $x = 2n\pi \pm 2\pi/3$ $(n \in I)$

Assertion & Reason

(A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(C) If ASSERTION is true, REASON is false

(D) If ASSERTION is false, REASON is true

(E) If both ASSERTION and REASON are false.

51. **Assertion :** The numbers sin 18° and – sin 54° are the roots of same quadratic equation with integer co-efficients.

Reason : If x = 18°, then 5x = 90°, if y = – 54°, then 5y = –270°

(a) A (b) B

(c) C (d) D

(e) E

52. **Assertion :** The function $f(x) = \min\{\sin x, \cos x\}$ takes the value $\frac{4}{5}$ twice when x varies from $\frac{20\pi}{3}$ to $\frac{43\pi}{6}$.

Reason : The periods of sin x and cos x are equal to 2π.

(a) A (b) B

(c) C (d) D

(e) E

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

53.

	Column - I		Column - II
(A)	$\cos^2 52\frac{1°}{2} - \sin^2 22\frac{1°}{2}$	**(P)**	1
(B)	$\cos^2\frac{3\pi}{5} + \cos^2\frac{4\pi}{5}$	**(Q)**	$\frac{3-\sqrt{3}}{4\sqrt{2}}$
(C)	$\sin 24° + \cos 6°$	**(R)**	$\frac{3}{4}$
(D)	$\sin^2 50 + \cos^2 130°$	**(S)**	$\frac{\sqrt{15}+\sqrt{3}}{4}$

The Correct Matching is

(a) A – Q; B– R; C–S; D–P

(b) A – R; B– Q; C–S; D–P

(c) A – P; B– R; C–S; D–Q

(d) A – S; B– P; C–Q; D–R

54. Match the following for the trigonometric equation $\left|x+\frac{1}{4}\right| - \left|x-\frac{1}{4}\right| = \cos \pi x$, (n is an integer) :

	Column - I		Column - II
(A)	Over $\left(-\infty, -\frac{1}{4}\right)$	**(P)**	$\left\{\frac{1}{3}\right\} \cup \left\{2n \pm \frac{1}{3}, n > 0\right\}$
(B)	Over $\left[-\frac{1}{4}, \frac{1}{4}\right]$	**(Q)**	$\left\{-\frac{2}{3}\right\} \cup \left\{2n \pm \frac{2}{3}, n < 0\right\}$
(C)	Over $\left(\frac{1}{4}, \infty\right)$	**(R)**	No solution

The Correct Matching is

(a) A–Q; B–R; C–P

(b) A–R; B–Q; C–P

(c) A–P; B–R; C–Q

(d) A–P; B–Q; C–R

Using the following passage, solve Q.55 to Q.57

Passage – 1

Given $\cos 2^m\theta \cos 2^{m+1}\theta$ .... $\cos 2^n\theta$

$$= \frac{\sin 2^{n+1}\theta}{2^{n-m+1}\sin 2^m\theta}, \text{ where } 2^m\theta \neq k\pi, n, m, k \in I$$

Solve the following :

55. $\sin\frac{9\pi}{14} . \sin\frac{11\pi}{14} \sin\frac{13\pi}{14} =$

(a) $\frac{1}{64}$ (b) $-\frac{1}{64}$

(c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

56. $\cos 2^3\frac{\pi}{10} \cos 2^4\frac{\pi}{10} \cos 2^5\frac{\pi}{10}$ .... $\cos 2^{10}\frac{\pi}{10} =$

(a) $\frac{1}{128}$ (b) $\frac{1}{256}$

(c) $\frac{1}{512}\sin\frac{\pi}{10}$ (d) $\frac{\sqrt{5}-1}{512}\sin\frac{3\pi}{10}$

57. $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}$ .... $\cos\frac{11\pi}{11} =$

(a) $-\frac{1}{32}$ (b) $\frac{1}{512}$

(c) $\frac{1}{1024}$ (d) $-\frac{1}{2048}$

Using the following passage, solve Q.58 to Q.60

Passage – 2

If $P_n = \sin^n\theta + \cos^n\theta$ where $n \in W$ (whole number) and $\theta \in R$ (real number)

58. If $P_1 = m$, then the value of $4(1 - P_6)$ is

(a) $3(m-1)^2$ (b) $3(m^2-1)^2$

(c) $3(m+1)^2$ (d) $3(m^2+1)^2$

59. The value of $2P_6 - 3P_4 + 10$ is

(a) 0 (b) 6

(c) 9 (d) 15

60. If $P_{n-2} - P_n = \sin^2\theta\cos^2\theta\, P_\lambda$, then the value of λ is

(a) n – 1 (b) n – 2

(c) n – 3 (d) n – 4

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. The set of value of θ satisfying the inequation $2\sin^2\theta - 5\sin\theta + 2 > 0$, where $0 < \theta < 2\pi$, is: **(2006)**

(a) $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},2\pi\right)$ (b) $\left[0,\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6},2\pi\right]$

(c) $\left[0,\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3},2\pi\right]$ (d) none of these

2. Let $\theta\in\left(0,\frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then **(2006)**

(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$

(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

3. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0$ & $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is **(2007)**

(a) zero (b) one

(c) two (d) four

4. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then **(2011)**

(a) $P \subset Q$ and $Q - P \neq \phi$ (b) $Q \not\subset P$

(c) $P \not\subset Q$ (d) $P = Q$

5. In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is **(2014)**

(a) $\frac{3y}{2x(x+c)}$ (b) $\frac{3y}{2c(x+c)}$

(c) $\frac{3y}{4x(x+c)}$ (d) $\frac{3y}{4c(x+c)}$

6. For $x \in (0,\pi)$, then eqation $\sin x + 2\sin 2x - \sin 3x = 3$ has **(2014)**

(a) infinitely many solutions (b) three solutions

(c) one solution (d) no solution

7. Let $S = \left\{x\in(-\pi,\pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solution of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to **(2016)**

(a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$

(c) 0 (d) $\frac{5\pi}{9}$

8. The value of $\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to **(2016)**

(a) $3-\sqrt{3}$ (b) $2(3-\sqrt{3})$

(c) $2(\sqrt{3}-1)$ (d) $2(2+\sqrt{3})$

Objective Questions II [One or more than one correct option]

9. If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then **(2009)**

(a) $\tan^2 x = \frac{2}{3}$ (b) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(c) $\tan^2 x = \frac{1}{3}$ (d) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

10. For $0 < \theta < \frac{\pi}{2}$, the solution(s) of

$$\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$$ is/are **(2009)**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

11. Let $\theta, \phi \in [0, 2\pi]$ be such that $2 \cos \theta (1 - \sin \phi) = \sin^2\theta$ $\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi-1$, $\tan(2\pi - \theta) > 0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$. Then, ϕ cannot satisfy **(2012)**

(a) $0<\phi<\frac{\pi}{2}$ (b) $\frac{\pi}{2}<\phi<\frac{4\pi}{3}$

(c) $\frac{4\pi}{3}<\phi<\frac{3\pi}{2}$ (d) $\frac{3\pi}{2}<\phi<2\pi$

12. In a ΔPQR, P is the largest angle and $\cos P=\frac{1}{3}$. Further in circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then, possible length(s) of the side(s) of the triangle is (are) **(2013)**

(a) 16 (b) 18

(c) 24 (d) 22

13. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s = x + y + z$. If $\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then **(2016)**

(a) area of the triangle XYZ is $6\sqrt{6}$

(b) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(c) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}=\frac{4}{35}$

(d) $\sin^2\left(\frac{X+Y}{2}\right)=\frac{3}{5}$

14. Let α and β be non zero real numbers such that $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$. Then which of the following is/are true? **(2017)**

(a) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(b) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

(c) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

15. In a non-right-angled triangle ΔPQR, Let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p=\sqrt{3}$, $q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct ? **(2019)**

(a) length of $RS=\frac{\sqrt{7}}{2}$

(b) Area of $\Delta SOE=\frac{\sqrt{3}}{12}$

(c) Radius of incircle of $\Delta PQR=\frac{\sqrt{3}}{2}(2-\sqrt{3})$

(d) Length of $OE=\frac{1}{6}$

16. For non-negative integer n, let

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0,\pi]$ which of the following options is/are correct? **(2019)**

(a) $f(4)=\frac{\sqrt{3}}{2}$

(b) If $\alpha=\tan(\cos^{-1}f(6))$, then $\alpha^2+2\alpha-1=0$

(c) $\sin(7\cos^{-1}f(5))=0$

(d) $\lim_{n\to\infty} f(n)=\frac{1}{2}$

17. Let x, y and z be positive numbers. Suppose x, y and z are lengths of the sides of a triangle opposite to its angles X, Y and Z, respectively. If $\tan\frac{X}{2}+\tan\frac{Z}{2}=\frac{2y}{x+y+z}$, then which of the following statements is/are TRUE ? **(2020)**

(a) $2Y=X+Z$ (b) $Y=X+Z$

(c) $\tan\frac{X}{2}=\frac{x}{y+z}$ (d) $x^2+z^2-y^2=xz$

18. Consider a triangle PQR having side of lengths p, q and r opposite to the angles P, Q and R respectively. Then which of the following statements is (are) TRUE ? **(2021)**

(a) $\cos P\geq 1-\frac{p^2}{2qr}$

(b) $\cos R\geq\left(\frac{q-r}{p+q}\right)\cos P+\left(\frac{p-r}{p+q}\right)\cos Q$

(c) $\frac{q+r}{p}<2\frac{\sqrt{\sin Q\sin R}}{\sin P}$

(d) If $p<q$ and $p<r$, the $\cos Q>\frac{p}{r}$ and $\cos R>\frac{p}{q}$

Numerical Value Type Questions

19. The number of all possible values of θ, where $0<\theta<\pi$, for which the system of equations

$(y+z)\cos 3\theta=(xyz)\sin 3\theta$

$$x\sin 3\theta=\frac{2\cos 3\theta}{y}+\frac{2\sin 3\theta}{z}$$

and $(xyz)\sin 3\theta=(y+2z)\cos 3\theta+y\sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0z_0\neq 0$, is ..... **(2010)**

20. The number of distinct solution of the equation

$\frac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x=2$ in the interval $[0, 2\pi]$ is **(2015)**

21. Let $f:[0, 2]\to R$ be the function defined by

$$f(x)=(3-\sin(2\pi x))\sin\left(\pi x-\frac{\pi}{4}\right)-\sin\left(3\pi x+\frac{\pi}{4}\right)$$

If $\alpha,\beta\in[0, 2]$ are such that

$\{x\in[0, 2]: f(x)\geq 0\}=[\alpha,\beta]$, then the value of $\beta-\alpha$ is .... **(2020)**

22. In a triangle ABC, let $AB=\sqrt{23}$, and $BC=3$ and $CA=4$.

Then the value of $\frac{\cot A+\cot C}{\cot B}$ is ____. **(2021)**

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

23. Let f(x) = sin(πcosx) and g(x) = cos(2π sinx) be two functions defined for x > 0. Define the following sets whose elements are written in increasing order

$X=\{x:f(x)=0\}, Y=\{x:f'(x)=0\}$

$Z=\{x:g(x)=0\}, W=\{x:g'(x)=0\}$

List-I contains sets X, Y, Z and W List-II contains some information regarding these set. **(2019)**

	List - I	**List - II**
(I)	X	**(P)** $\supseteq\left\{\frac{\pi}{2},\frac{3\pi}{2},4\pi,7\pi\right\}$
(II)	Y	**(Q)** an arithmetic progression
(III)	Z	**(R)** NOT an arithmetic progression
(IV)	W	**(S)** $\supseteq\left\{\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}\right\}$
		(T) $\supseteq\left\{\frac{\pi}{3},\frac{2\pi}{3},\pi\right\}$
		(U) $\supseteq\left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$

Which of the following is the only correct combination?

(a) IV – (Q), (T) (b) III – (R), (U)

(c) III – (P), (Q), (U) (d) IV – (P), (R), (S)

24. Let f(x) = sin(πcosx) and g(x) = cos(2π sinx) be two functions defined for x > 0. Define the following sets whose elements are written in increasing order

$X=\{x:f(x)=0\}, Y=\{x:f'(x)=0\}$

$Z=\{x:g(x)=0\}, W=\{x:g'(x)=0\}$

List-I contains sets X, Y, Z and W List-II contains some information regarding these set. **(2019)**

	List - I	**List - II**
(I)	X	**(P)** $\supseteq\left\{\frac{\pi}{2},\frac{3\pi}{2},4\pi,7\pi\right\}$
(II)	Y	**(Q)** an arithmetic progression
(III)	Z	**(R)** NOT an arithmetic progression
(IV)	W	**(S)** $\supseteq\left\{\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}\right\}$
		(T) $\supseteq\left\{\frac{\pi}{3},\frac{2\pi}{3},\pi\right\}$
		(U) $\supseteq\left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$

Which is the following is only CORRECT combination?

(a) I – (Q), (U) (b) I – (P), (R)

(c) II – (Q), (T) (d) II – (R), (S)

Text

25. In any triangle prove that

$$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$ **(2000)**

Find Answer Key and Detailed Solutions at the end of this book

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08

STATISTICS

STATISTICS

1. MEASURES OF CENTRAL TENDENCY

An average or a central value of a statistical series is the value of the variable which describes the characteristics of the entire distribution.

The following are the five measures of central tendency.

(1) Arithmetic Mean

(2) Geometric Mean

(3) Harmonic Mean

(4) Median

(5) Mode

1.1 Arithmetic Mean

Arithmetic mean is the most important among the mathematical mean.

According to Horace Secrist,

"The arithmetic mean is the amount secured by dividing the sum of values of the items in series by their number".

(1) Simple arithmetic mean in individual series (Ungrouped data)

(i) Direct method : If the series in this case be $x_1, x_2, x_3, ..., x_n$, then the arithmetic mean $\bar{x}$ is given by

$$\bar{x} = \frac{\text{Sum of the series}}{\text{Number of terms}}, \text{i.e., } \bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

(ii) Short cut method : Arithmetic mean $(\bar{x}) = A + \frac{\Sigma d_i}{n}$,

Where, A = assumed mean, d_i = deviation from assumed mean = x_i - A, where x_i is the individual item, Σd_i = sum of deviations and n= number of items.

(2) Simple arithmetic mean in continuous series (Grouped data)

(i) Direct method : If the terms of the given series be $x_1, x_2, ...x_n$ and the corresponding frequencies be $f_1, f_2, f_3...f_n$, then the arithmetic mean $\bar{x}$ is given by,

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ...f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

(ii) Short cut method : Arithmetic mean $(\bar{x}) = A + \frac{\Sigma f_i (x_i - A)}{\Sigma f_i}$

Where A = assumed mean, f_i = frequency and $x_i - A$ = deviation of each item from the assumed mean.

(3) Properties of arithmetic mean

(i) Algebraic sum of the deviations of a set of values from their arthmetic mean is zero. If x_i/f_i, i = 1, 2, ..., n is the frequency distribution, then

$\sum_{i=1}^{n} f_i (x_i - \bar{x}) = 0, \bar{x}$ being the mean of the distribution.

(ii) The sum of the squares of the deviations of a set of values is minimum when taken about mean.

(iii) Mean of the composite series : If $\bar{x}_i$ (i=1,2,...,k) are the means of k-component series of sizes n_i, (i = 1, 2, ..., k) respectively, then the mean $\bar{x}_i$ of the composite series obtained on combining the component series is given by

the formula $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + ... + n_k\bar{x}_k}{n_1 + n_2 + ... + n_k} = \frac{\sum_{i=1}^{n} n_i \bar{x}_i}{\sum_{i=1}^{n} n_i}$

1.2 Meometric Mean

(1) Ungrouped Data :

If $x_1, x_2, x_3, ...x_n$ are n values of a variate x, none of them being zero, then geometric mean (G.M.) is given by G.M. = $(x_1 x_2 x_3 ....x_n)^{1/n}$

$$\Rightarrow \log(\text{G.M.}) = \frac{1}{n}(\log x_1 + \log x_2 + ... + \log x_n).$$

(2) Grouped Data : G.M. of n values $x_1, x_2, ..., x_n$ of a variate x occuring with frequency $f_1, f_2,..., f_n$ is given by G.M.

$$=\left(x_1^{f_1}.x_2^{f_2}....x_n^{f_n}\right)^{\frac{1}{N}}, \text{ where } N = f_1 + f_2 + ... + f_n.$$

NOTES :

If G_1 and G_2 are geometric means of two series containing n_1 and n_2 positive values respectively and G is geometric mean of their combined series, then $G = \left(G_1^{n_1} \times G_2^{n_2}\right)^{\frac{1}{n_1+n_2}}$

1.3 Harmonic Mean

(1) Ungrouped Data :

The harmonic mean of n items $x_1, x_2,...., x_n$ is defined as

$$\text{H.M.} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+......+\frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

(2) Grouped Data :

If the frequency distribution is $f_1, f_2, f_3,......,f_n$ respectively,

$$\text{then H.M.} = \frac{f_1+f_2+f_3+.....+f_n}{\left(\frac{f_1}{x_1}+\frac{f_2}{x_2}+.......+\frac{f_n}{x_n}\right)} = \frac{N}{\sum_{i=1}^{n}\frac{f_i}{x_i}}$$

Note : A.M. gives more weightage to larger values whereas G.M. given more weightage to smaller values.

1.4 Median

Median is defined as the value of an item or observation above or below which lies on an equal number of observation i.e., the median is the central value of the set of observations provided all the observations are arranged in the ascending or descending order.

(1) Calculation of median

(i) Individual series : If the data is raw, arrange in ascending or descending order. Let n be the number of observations.

If n is odd, Median = value of $\left(\frac{n+1}{2}\right)^{th}$ item.

If n is even, Median

$$=\frac{1}{2}\left[\text{value of}\left(\frac{n}{2}\right)^{th}\text{ item + value of}\left(\frac{n}{2}+1\right)^{th}\text{ item}\right]$$

(ii) Discrete series : In this case, we first find the cumulative frequencies of the variables arranged in ascending or descending order and the median is given by

If N is odd, Median = value of $\left(\frac{N+1}{2}\right)^{th}$ item.

If N is even, Median

$$=\frac{1}{2}\left[\text{value of}\left(\frac{N}{2}\right)^{th}\text{ item + value of}\left(\frac{N}{2}+1\right)^{th}\text{ item}\right]$$

where N is the cumulative frequency.

(iii) For grouped or continuous distributions : In this case, following formula can be used

(a) For series in ascending order: Median $= \ell + \frac{\left(\frac{N}{2}-C\right)}{f}\times i$

Where ℓ = Lower limit of the median class

f = Frequency of the median class

N = The sum of the all frequencies

i = The width of the median class

C = The cumulative frequency of the class precending to median class.

(b) For series in descending order

Median $= u - \frac{\left(\frac{N}{2}-C\right)}{f}\times i$, where u = upper limit of the median class.

As median divides a distribution into two equal parts, similarly the quartiles, quantiles, deciles and percentiles divide the distribution respectively into 4, 5, 10 and 100 equal part. The j^{th} quartile is given by

SCAN CODE
Statistics

$$Q_j = \ell + \left(\frac{j\frac{N}{4} - C}{f}\right) i; j = 1, 2, 3.$$ Q_1 is the lower quartile, Q_2 is the median and Q_3 is called the upper quartile.

(2) Lower quartile

(i) Discrete series : $Q_1 = \text{size of } \left(\frac{n+1}{4}\right)^{th}$ item

(ii) Continous series : $Q_1 = \ell + \frac{\left(\frac{N}{4} - C\right)}{f} \times i$

(3) Upper quartile

(i) Discrete series : $Q_3 = \text{size of } \left[\frac{3(n+1)}{4}\right]^{th}$ item

(ii) Continuous series : $Q_3 = \ell + \frac{\frac{3N}{4} - C}{f} \times i$

(4) Decile divides total frequencies N into ten equal parts

$$D_j = \ell + \frac{\frac{N \times j}{10} - C}{f} \times i \; [j = 1, 2, 3, 4, 5, 6, 7, 8, 9]$$

If $j = 5$, then $D_5 = \ell + \frac{\frac{N}{2} - C}{f} \times i$. Hence D_5 is also known as median.

(5) Percentile divides total frequencies N into hundred equal parts

$$P_k = \ell + \frac{\frac{N \times k}{100} - C}{f} \times i$$

where $k = 1, 2, 3, 4, 5, ..., 99.$

1.5 Mode

The mode or modal value of a distribution is that value of the variable for which the frequency is maximum. For continuous series, mode is calculated as, Mode

$$\text{Mode} = \ell_1 + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times i$$

Where, l = The lower limit of the modal class.

f_1 = The frequency of the modal class

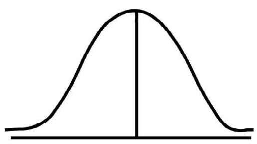
f_0 = The frequency of the class preceding the modal class

f_2 = The frequency of the class succeeding the modal class

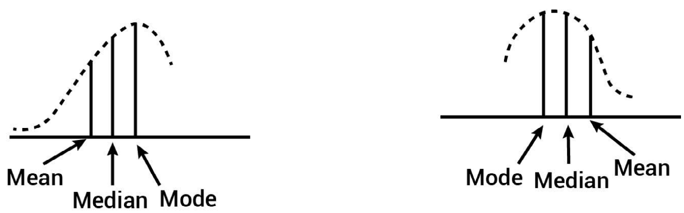
i = The size of the modal class.

2. RELATION BETWEEN MEAN, MEDIAN AND MODE

Symmetric distribution : A distribution is a symmetric distribution if the values of mean, mode and median coincide. In a symmetric distribution, frequencies are symmetrically distributed on both sides of the centre point of the frequency



Mean = Median = Mode



A distribution which is not symmetric is called skewed distribution. In a moderately asymmetric distribution, the interval between the mean and the median is approximately one-third of the interval between the mean and the mode i.e. we have the following empirical relation between them

Mean – Mode = 3(Mean – Median)

$\Rightarrow$ Mode = 3 Median – 2 Mean. It is known as Empirical relation.

NOTES :

Some points about arithmetic mean :

* Of all types of averages, the arithmetic mean is the most commonly used average.
* It is based upon all observations.
* If the number of observations is very large, it is more accurate and more reliable basis for comparison.

Some points about geometric mean :

* It is based on all items of the series
* It is most suitable for constructing index number, average ratios, precentages etc.
* G.M. cannot be calculated if the size of any of the item is zero or negative.

Some points about H.M. :

* It is based on all items of the series.
* This is useful in problems related with rates, ratios, time etc.
* A.M. $\geq$ G.M. $\geq$ H.M. and also $(G.M.)^2 = (A.M.)\,(H.M.)$

Some points about median :

* It is an appropriate average in dealing with qualitative data, like intelligence, wealth etc.
* The sum of the deviations of the items from median, ignoring algebraic signs, is less than the sum from any other point.

Some points about mode :

* It is not based on all items of the series.
* As compared to other averages mode is affected to a large extent by fluctuations of sampling.
* It is not suitable in a case where the relative importance of items have to be considered.

3. MEASURES OF DISPERSION

The degree to which numerical data tend to spread about an average value is called the dispersion of the data. The four measures of the dispersion are

(1) Range (2) Mean deviation

(3) Standard deviation (4) Square deviation

3.1 Range

It is the difference between the values of extreme items in a series. Range = $x_{max} - x_{min}$

The coefficient of range (scatter) = $\dfrac{x_{max} - x_{min}}{x_{max} + x_{min}}$

Range is not a measure of central tendency. Range is widely used in statistical series relating quality control in production.

(i) Inter–quartile range : We know that quartiles are the magnitudes of the items which divide the distribution into four equal parts. The inter-quartile range is found by taking the difference between third and first quartiles and is given by the formula.

Inter–quartile range = $Q_3 - Q_1$

Where, Q_1 = First quartile or lower quartile and Q_3 = Third quartile or upper quartile.

(ii) Percentile range : This is measured by the following formula.

Percentile range = $P_{90} - P_{10}$

Where, P_{90} = 90th percentile and P_{10} = 10th percentile

Percentile range is considered better than range as well as inter-quartile range.

(iii) Quartile deviation or semi inter–quartile range : It is one-half of the difference between the third quartile and first quartile i.e., $Q.D. = \dfrac{Q_3 - Q_1}{2}$ and coefficient of quartile deviation $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$.

Where, Q_3 is the third or upper quartile and Q_1 is the lowest or first quartile.

3.2 Mean deviation

The arithmetic average of the deviation (all taking positive) from the mean, median or mode is known as mean deviation.

(i) Mean deviation for ungerouped data (or individual series)

$$\text{Mean deviation} = \frac{\Sigma|x - M|}{n}$$

Where $|x - M|$ means the modulus of the deviation of the variate from the mean (mean, median or mode). M and n is the number of terms.

(ii) Mean deviation for continuous series : Here, first of all we find the mean from which deviation is to be taken. Then we find the deviation $dM = |x - M|$ of each variate from the mean M so obtained.

Next, we multiply these deviations by the corrsponding frequencies and find the product f. dM and then the sum ΣfdM of these products.

Lastly, we use the formula,

$$\text{Mean deviation} = \frac{\Sigma f|x - M|}{n} = \frac{\Sigma fdM}{n}, \text{ where } n = \Sigma f$$

3.3 Standard deviation

Standard deviation (or S.D.) is the square root of the arithmetic mean of the square of deviations of various values from their arithmetic mean and is generally denoted by σ (read as sigma).

(i) Coefficient of standard deivation : To compare the dispersion of two frequency distributions the relative measure of standard deviation is computed which is known as coefficient of standard deviation and is given by

$$\text{Coefficient of S.D.} = \frac{\sigma}{\bar{x}}, \text{ where } \bar{x} \text{ is the A.M.}$$

(ii) Standard deviation for individual series

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$$

where, $\bar{x}$ = The arithmetic mean of series

N = The total frequency.

(iii) Standard deviation for continuous series

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$$

where, $\bar{x}$ = Arithmetic mean of series

x_i = Mid value of the class

f_i = Frequency of the corresponding x_i

$N = \Sigma f$ = The total frequency

Short cut method :

(i) $\sigma = \sqrt{\dfrac{\Sigma fd^2}{N} - \left(\dfrac{\Sigma fd}{N}\right)^2}$ (ii) $\sigma = \sqrt{\dfrac{\Sigma d^2}{N} - \left(\dfrac{\Sigma d}{N}\right)^2}$

where, $d = x - A$ = Deviation from the assumed mean A

f = Frequency of the item

$N = \Sigma f$ = Sum of frequencies

3.4 Square deviation

(i) Root mean square deviation

$$S = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^2}$$

where A is any arbitrary number and S is called mean square deviation.

(ii) Relation between S.D. and root mean square deviation : If σ be the standard deivation and S be the root mean square deviation.

Then $S^2 = \sigma^2 + d^2$.

Obviously, S^2 will be least when $d = 0$ i.e. $\bar{x} = A$

Hence, mean square deviation and consequently root mean square deviation is least, if the deviations are taken from the mean.

4. VARIANCE

The square of standard deviation is called the variance.

(1) For Ungrouped Data :

$$\sigma_x^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\sigma_x^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2, \text{ where } d_i = x_i - A$$

(2) Grouped Data :

$$\sigma_x^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N}$$

$$\sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$$\sigma_u^2 = h^2 \left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] \text{ where } u_i = \frac{d_i}{h}$$

(3) Coefficient of variance :

Coefficient of variance = coefficient of S.D. $\times 100 = \frac{\sigma}{\bar{x}} \times 100$.

Variance of the combined series : If n_1; n_2 are the sizes, $\bar{x}_1; \bar{x}_2$, the means and $\sigma_1; \sigma_2$, the standard deviations of two series, then

$$\sigma^2 = \frac{1}{n_1 + n_2}\left[n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)\right]$$

Where, $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$, $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

(4) Properties of Standard Deviation (S.D.) and Variance :

1. The S.D. (and variance) is independent of change of origin i.e. If we add or subtract a constant number k to each observation of the data set, the resultant S.D. (and variance) remains same.
2. The S.D. (and variance) depends upon the change of scale.

(a) If we multiply/divide each observation by k, the resultant S.D. gets multiplied/divided by |k|.

(b) If we multiply/divide each observation by k, the resultant variance gets multiplied by k^2.

i.e. S.D. $(ax + b) = |a|$ S.D. (x)

and Var $(ax + b) = a^2 \cdot$ var (x)

NOTES :

* Range is widely used in satistical series relating to quality control in production.
* Standard deviation ≤ Range i.e., variance ≤ $(\text{Range})^2$.
* Empirical relation between measures of dispersion :

  Mean deviation = $\frac{4}{5}$ (standard deviation)

  Semi interquartile range = $\frac{2}{3}$ (Standard deviation)

  Semi interquartile range = $\frac{5}{6}$ (mean deviation)
* For a symmetrical distribution, the following relationships hold good.

  $\bar{X} \pm \sigma$ covers 68.72% items

  $\bar{X} \pm 2\sigma$ covers 95.45% items

  $\bar{X} \pm 3\sigma$ covers 99.74% items
* S.D. of first n natural number is $\sqrt{\frac{n^2 - 1}{12}}$.
* Range is not the measure of central tendency.

5. SKEWNESS

"Skewness" Measures the lack of symmetry. It is denoted

by γ_1 and is measured by $\gamma_1 = \dfrac{\Sigma(x_i - \mu)^3}{\left\{\Sigma\left(x_i - \mu^2\right)^{\frac{3}{2}}\right\}}$

The distribution is skewed if,

(i) Mean ≠ Median ≠ Mode

(ii) Quartiles are not equidistant from the median and

(iii) The frequency curve is stretched more to one side than to the other.

(1) Distribution : There are three types of distributions.

(i) Normal distribution : When $\gamma_1 = 0$, the distribution is said to be normal. In this case Mean = Median = Mode

(ii) Positively skewed distribution : When $\gamma_1 > 0$, the distribution is said to be positively skewed. In this case Mean > Median > Mode

(iii) Negatively skewed distribution : When $\gamma_1 < 0$, the distribution is said to be negatively skewed. In this case Mean < Median < Mode.

(2) Measures of skewness

Absolute measures of skewness : Various measures of skewness are

(a) $S_k = M - M_d$

(b) $S_k = M - M_O$

(c) $S_k = Q_3 + Q_1 - 2M_d$

Where, M_d = median , M_O = mode, M = mean

SOLVED EXAMPLES

Example – 1

If the mean of the distribution is 2.6, then the value of y is

Variate x	:	1	2	3	4	5
Frequency f of x	:	4	5	y	1	2

(a) 24 (b) 13 (c) 8 (d) 3

Ans. (c)

Sol. We know that, Mean $= \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

i.e. $2.6 = \dfrac{1\times4+2\times5+3\times y+4\times1+5\times2}{4+5+y+1+2}$

or $31.2+2.6y=28+3y$ or $0.4y=3.2 \Rightarrow y=8$

Example – 2

In a class of 100 students, there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class are 72, then what are the average marks of the girls.

(a) 73 (b) 65 (c) 68 (d) 74

Ans. (b)

Sol. Let the average marks of the girls students be x, then

$72 = \dfrac{70\times75+30\times x}{100}$ (Number of girls = 100 – 70 = 30)

i.e., $\dfrac{7200-5250}{30} = x$

$\therefore x = 65.$

Example – 3

Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is

(a) 48 (b) $82\frac{1}{2}$ (c) 50 (d) 80

Ans. (c)

Sol. Sum of 100 items = 49 × 100 = 4900

Sum of items added = 60 + 70 + 80 = 210

Sum of items replaced = 40 + 20 + 50 = 110

New sum = 4900 + 210 – 110 = 5000

$\therefore$ Correct mean $= \dfrac{5000}{100} = 50$

Example – 4

The harmonic mean of 4, 8, 16 is

(a) 6.4 (b) 6.7 (c) 6.85 (d) 7.8

Ans. (c)

Sol. H.M. of 4, 8, 16 $= \dfrac{3}{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}} = \dfrac{48}{7} = 6.85$

Example – 5

If the mean of the set of number $x_1, x_2, x_3, \ldots, x_n$ is $\bar{x}$, then the mean of the numbers $x_i + 2i$, $1 \le i \le n$ is

(a) $\bar{x}+2n$ (b) $\bar{x}+n+1$

(c) $\bar{x}+2$ (d) $\bar{x}+n$

Ans. (b)

Sol. We know that $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$ i.e., $\sum_{i=1}^{n} x_i = n\bar{x}$

$$\therefore \frac{\sum_{i=1}^{n}(x_i+2i)}{n} = \frac{\sum_{i=1}^{n} x_i + 2\sum_{i=1}^{n} i}{n} = \frac{n\bar{x}+2(1+2+\ldots n)}{n}$$

$$= \frac{n\bar{x}+2\frac{n(n+1)}{2}}{n} = \bar{x}+(n+1)$$

Example – 6

The average of n numbers $x_1, x_2, x_3, \ldots x_n$ is M. If x_n is replaced by x', then new average is

(a) $M - x_n + x'$ (b) $\frac{nM - x_n + x'}{n}$

(c) $\frac{(n-1)M + x'}{n}$ (d) $\frac{M - x_n + x'}{n}$

Ans. (b)

Sol. $M = \frac{x_1 + x_2 + x_3 \ldots x_n}{n}$

i.e., $nM = x_1 + x_2 + x_3 + \ldots x_{n-1} + x_n$

$nM - x_n = x_1 + x_2 + x_3 + \ldots x_{n-1}$

$$\frac{nM - x_n + x'}{n} = \frac{x_1 + x_2 + x_3 + \ldots + x_{n-1} + x'}{n}$$

$\therefore$ New average $= \frac{nM - x_n + x'}{n}$

Example – 7

Let $x_1, x_2, \ldots\ldots x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is

(a) 18 (b) 15

(c) 12 (d) 9

Ans. (a)

Sol. We know that,

Root mean square of numbers $\geq$ A.M. of the number

$$\Rightarrow \sqrt{\frac{400}{n}} \geq \frac{80}{n}$$

$$\Rightarrow \frac{20}{\sqrt{n}} \geq \frac{80}{n}$$

$$\Rightarrow \sqrt{n} \geq 4 \text{ or } n \geq 16$$

So, only possible option is '18'.

Example – 8

The following data gives the distribution of height of students

Height (in cm)	160	150	152	161	156	154	155
Number of students	12	8	4	4	3	3	7

The median of the distribution is

(a) 154 (b) 155 (c) 160 (d) 161

Ans. (b)

Sol. Arranging the data in ascending order of magnitude, we obtain

Height (in cm)	150	152	154	155	156	160	161
Number of students	8	4	3	7	3	12	4
Cumulative frequency	8	12	15	22	25	37	41

Here, total number of items is 41, i.e., an odd number.

Hence, the median is $\frac{41+1}{2}$ th i.e. 21st item

From cumulative frequency table, we find that median i.e., 21st item is 155.

(All items from 16 to 22nd are equal to = 155)

Example – 9

Compute the median from the following table

Marks obtained	No. of students
0-10	2
10-20	18
20-30	30
30-40	45
40-50	35
50-60	20
60-70	6
70-80	3

(a) 36.55 (b) 35.55

(c) 40.05 (d) None of these

Ans. (a)

Sol.

Marks obtained	No. of students	Cumulative Frequency
0-10	2	2
10-20	18	20
20-30	30	50
30-40	45	95
40-50	35	130
50-60	20	150
60-70	6	156
70-80	3	159

$N = \Sigma f = 159$, which is odd.

Median number $= \frac{1}{2}(N+1) = \frac{1}{2}(159+1) = 80$, which is in the class 30-40 (see the row of cumulative frequency 95, which contains 80).

Hence median class is 30-40.

$\therefore$ We have l = Lower limit of median class = 30

f = Frequency of median class = 45

C = Total of all frequency preceding median class = 50

i = Width of class interval of median class = 10

$\therefore$ Required median

$$= l + \frac{\frac{N}{2} - C}{f} \times i = 30 + \frac{\frac{159}{2} - 50}{45} \times 10 = 30 + \frac{295}{45} = 36.55$$

Example – 10

In a set of 2n distinct observations, each of the observations below the median is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations:

(a) increases by 1 (b) decreases by 1

(c) decreases by 2 (d) increases by 2

Ans. (a)

Sol. Mean $= \frac{x_1 + x_2 + x_3 + ... + x_n + x_{n+1} + ... + x_{2n}}{2n}$

Median $= \frac{x_n + x_{n+1}}{2}$

New Mean $= \frac{x_1 + x_2 + ... + x_n + 5n + x_{n+1} + ... + x_{2n} - 3n}{2n}$

$= \frac{x_1 + x_2 + ... + x_n + x_{n+1} + x_{2n}}{2n} + \frac{2n}{2n}$

= Original mean + 1

Example – 11

In a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

(a) 20.5 (b) 22.0

(c) 24.0 (d) 25.5

Ans. (c)

Sol. Given that

mean = 21 and median = 22

Using the Emperical Relation,

Mode = 3 median – 2 mean

$= 3(22) - 2(21)$

$= 66 - 42 = 24$

Example – 12

If the median and the range of four numbers [x, y, 2x + y, x – y], where $0 < y < x < 2y$, are 10 and 28 respectively, then the mean of the numbers is

(a) 18 (b) 10

(c) 5 (d) 14

Ans. (d)

Sol. Since $0 < y < x < 2y$

$\therefore\ y > \frac{x}{2} \Rightarrow x - y < \frac{x}{2}$

$\therefore\ x - y < y < x < 2x + y$

Hence median $= \frac{y + x}{2} = 10$

$\Rightarrow x + y = 20$ ...(i)

And range $= (2x + y) - (x - y) = x + 2y$

but range = 28

$\therefore\ x+2y=28 \qquad ...(ii)$

From equation (i) and (ii)

$x=12$ and $y=8$

$$\therefore\ \text{Mean}=\frac{(x-y)+y+x+(2x+y)}{4}$$

$$=\frac{4x+y}{4}$$

$$\Rightarrow\ \text{Mean}=\frac{4(12)+8}{4}=\frac{56}{4}=14$$

Example – 13

A batsman scores runs in 10 innings: 38, 70, 48, 34, 42, 55, 63, 46, 54, 44, then the mean deviation is

(a) 8.6 (b) 6.4 (c) 10.6 (d) 9.6

Ans. (a)

Sol. Arranging the given data in ascending order, we have

34, 38, 42, 44, 46, 48, 54, 55, 63, 70

Here, median $M=\frac{46+48}{2}=47$

($\because$ n = 10, median is the mean of 5th and 6th items)

$\therefore$ Mean deviation =

$$\frac{\Sigma|x_i-M|}{n}=\frac{\Sigma|x_i-47|}{10}=\frac{13+9+5+3+1+1+7+8+16+23}{10}=8.6$$

Example – 14

If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals

(a) 4 (b) 5

(c) 2 (d) 3

Ans. (a)

Sol. Median is the mean of 25th and 26th observation.

$$M=\frac{25a+26a}{2}=25.5a$$

$$\text{M.D.}=\frac{\sum|x_i-M|}{N}$$

$$50=\frac{1}{50}\{2|a|\times(0.5+1.5+....+24.5)\}$$

$$\Rightarrow\ 2500=2|a|\times\frac{25}{2}\times 25$$

$\therefore\ |a|=4$

Example – 15

If μ is the mean of distribution (y_i, f_i), then $\Sigma f_i(y_i-\mu)=$

(a) M.D. (b) S.D. (c) 0 (d) Relative frequency

Ans. (c)

Sol. We have $\Sigma f_i(y_i-\mu)=\Sigma f_iy_i-\mu\Sigma f_i=\mu\Sigma f_i-\mu\Sigma f_i=0$

$$\left[\therefore\ \mu=\frac{\Sigma f_iy_i}{\Sigma f_i}\right]$$

Example – 16

Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observations $x_i, i=1, 2, ..., n$. If each of the observations is increased by 5, then the new mean and the mean deviation about the new mean, respectively, are :

(a) $\overline{X}$, M.D. (b) $\overline{X}+5$, M.D.

(c) $\overline{X}$, M.D.+5 (d) $\overline{X}+5$, M.D.+5

Ans. (b)

Sol. $\overline{X}=\frac{x_1+x_2+x_3+......x_n}{n}$

$$\text{New Mean}=\frac{x_1+x_2+x_3+......+x_n+5n}{n}$$

$$\frac{x_1+x_2+......+x_n}{n}+\frac{5n}{n}$$

New Mean = Original Mean + 5

$$\text{M.D}=\frac{|(x_1-\overline{x})|+|(x_2-\overline{x})|+......+|(x_n-\overline{x})|}{n}$$

New M.D. about new mean =

$$\frac{|x_1+5-(\overline{x}+5)|+|x_2+5-(\overline{x}+5)|+......+|x_n+5-(\overline{x}+5)|}{n}$$

$=$ M.D.

Example – 17

The variance of first 50 even natural numbers is

(a) 833 (b) 437

(c) $\frac{437}{4}$ (d) $\frac{833}{4}$

Ans. (a)

Sol. $\sigma^2 = \left(\frac{\sum x_i^2}{n}\right) - (\bar{x})^2$

$$\bar{x} = \frac{2+4+6+......+100}{50} = 51$$

$$\text{Now, } \sigma^2 = \frac{2^2+4^2+6^2+......+100^2}{50} - (51)^2$$

$$= \frac{2^2[1^2+2^2+3^2+......+50^2]}{50} - (51)^2$$

$$= \frac{4\times(50)(51)(101)}{50\times 6} - (51)^2$$

$3434 - 2601 = 833$

Example – 18

In a series of 2n observations, half of them equals a and remaining half equals –a. If the standard deviation of the observations is 2, then |a| equals

(a) $\frac{\sqrt{2}}{n}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{n}$

Ans. (c)

Sol. a, a, ..., n times -a, –a, –a, ---- n times

i.e. mean $= 0$ and S.D.$= \sqrt{\frac{n(a-0)^2 + n(-a-0)^2}{2n}}$

$$2 = \sqrt{\frac{na^2+na^2}{2n}} = \sqrt{a^2}. \text{ Hence } |a| = 2.$$

Example – 19

What is the standard deviation of the following series

Measurements	0-10	10-20	20-30	30-40
Frequency	1	3	4	2

(a) 81 (b) 7.6 (c) 9 (d) 2.26

Ans. (c)

Sol.

Class	Frequency	y_1	$\mu_i = \frac{y_i - A}{10}, A = 25$	$f_i u_i$	$f_i u_i^2$
$0-10$	1	5	–2	–2	4
$10-20$	3	15	–1	–3	3
$20-30$	4	25	0	0	0
$30-40$	2	35	1	2	2
	10			–3	9

$$\sigma^2 = h^2\left[\frac{\Sigma f_i u_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)^2\right]$$

$$= 10^2\left[\frac{9}{10} - \left(\frac{-3}{10}\right)^2\right] = 90 - 9 = 81 \Rightarrow \sigma = 9$$

Example – 20

Consider the following statements :

(1) Mode can be computed from histogram

(2) Median is not independent of change of scale

(3) Variance is independent of change of origin and scale.

Which of these is/are correct ?

(a) only (1) and (2) (b) only (2)

(c) only (1) (d) (1), (2) and (3)

Ans. (a)

Sol. It is true that mode can be computed from histogram and median is not independent of change of scale.

But, variance is independent of change of origin and not of scale.

Example – 21

All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

(a) median (b) mode

(c) variance (d) mean

Ans. (c)

Sol. As we know,

Variance $(ax+b) = a^2$ Variance (x)

If we add a constant number 10 to each of the observations, then variance will not change.

Example – 22

S.D. of data is 6. When each observation is increased by 1, the S.D. of new data is

(a) 5 (b) 7 (c) 6 (d) 8

Ans. (c)

Sol. S.D. and variance of data is not changed, when each observation is increased (OR decreased) by the same constant.

Example – 23

Suppose a population A has 100 observations 101, 102, ..., 200, and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations, respectively, then V_A / V_B is

(a) 1 (b) 9/4

(c) 4/9 (d) 2/3

Ans. (a)

Sol. Since, variance is independent of change of origin. Therefore, variance of observations 101, 102, ..., 200 is same as variance of 151, 152, ..., 250

$$\Rightarrow V_A = V_B$$

$$\Rightarrow \frac{V_A}{V_B} = 1$$

Example – 24

The S.D. of a variate x is σ. The S.D. of the variate $\frac{ax+b}{c}$ where a, b, c are constants, is

(a) $\left(\frac{a}{c}\right)\sigma$ (b) $\left|\frac{a}{c}\right|\sigma$

(c) $\left(\frac{a^2}{c^2}\right)\sigma$ (d) None of these

Ans. (b)

Sol. Let $y = \frac{ax+b}{c}$ i.e., $y = \frac{a}{c}x + \frac{b}{c}$ i.e. $y = Ax + B$, where

$$A = \frac{a}{c}, B = \frac{b}{c}$$

$$\therefore \bar{y} = A\bar{x} + B$$

$$\therefore y - \bar{y} = A(x - \bar{x}) \Rightarrow (y - \bar{y})^2 = A^2 (x - \bar{x})^2$$

$$\Rightarrow \Sigma(y - \bar{y})^2 = A^2 \Sigma(x - \bar{x})^2 \Rightarrow n.\sigma_y^2 = A^2.n\sigma_x^2 \Rightarrow \sigma_y^2 = A^2\sigma_x^2$$

$$\Rightarrow \sigma_y = |A|\sigma_x \Rightarrow \sigma_y = \left|\frac{a}{c}\right|\sigma_x$$

Thus, new S.D. $= \left|\frac{a}{c}\right|\sigma$.

Example – 25

Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement 1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.

Statement 2 : Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(b) Statement 1 is true, Statement 2 is false

(c) Statement 1 is false, Statement 2 is true.

(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1

Ans. (b)

Sol. We know,

variance $(ax+b) = (a^2)$. var (x)

So, for statement-I

$a = 2, b = 0$

Var. $(2x_i) = 4(\sigma)^2$

Statement-I is true.

And A.M of $2x_1, 2x_2, ..., 2x_n$

$$= \frac{2(x_1 + x_2 + ... + x_n)}{n} = 2(\bar{x})$$

Statement-II is false.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Measure of central tendency

1. The mean of a set of observations is $\bar{x}$. If each observation is divided by $\alpha, \alpha \neq 0$ and then is increased by 10, then the mean of the new set is

(a) $\frac{\bar{x}}{\alpha}$ (b) $\frac{\bar{x}+10}{\alpha}$

(c) $\frac{\bar{x}+10\alpha}{\alpha}$ (d) $\alpha\bar{x}+10$

2. If the arithmetic mean of the numbers $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ is $\bar{x}$, then the arithmetic mean of numbers $ax_1 + b$, $ax_2 + b$, $ax_3 + b$, ..., $ax_n + b$, where a, b are two constants would be

(a) $\bar{x}$ (b) $na\bar{x} + nb$

(c) $a\bar{x}$ (d) $a\bar{x} + b$

3. Mean of 100 observations was found to be 45. It was later found that two observations 19 and 31 were incorrectly recorded as 91 and 13. The correct mean is

(a) 44.0 (b) 44.46

(c) 45.00 (d) 45.54

4. The A.M. of n observations is M. If the sum of n–4 observations is a, then the mean of remaining 4 observations is

(a) $\frac{nM-a}{4}$ (b) $\frac{nM+a}{2}$

(c) $\frac{nM-A}{2}$ (d) $nM + a$

5. The average weight of students in a class of 35 students is 40 kg. If the weight of the teacher be included, the average rises by $\frac{1}{2}$kg; the weight of the teacher is

(a) 40.5 kg (b) 50 kg

(c) 41 kg (d) 58 kg

6. The mean age of a combined group of men and women is 25 yrs. If the mean age of the group of men is 26 and that of the group of women is 21, then the ratio of men and women in the group is

(a) 1 : 4 (b) 4 : 1

(c) 3 : 1 (d) 1 : 3

7. **Statement–I:** The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is 80%.

Statement–II: Mean marks scored by the students of a class is 53. The mean marks of the girls is 55 and the mean marks of the boys is 50. The percentage of girls in the class is 64%.

(a) Only statement I is true (b) Only statement II is true

(c) Both statements are true(d) Both statements are false

8. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set is

(a) increased by 2

(b) decreased by 2

(c) two times the original median

(d) remains the same as that of original set

9. When tested, the lives (in hours) of 5 bulbs were noted as follows :

1357, 1090, 1666, 1494, 1623

The mean deviation (in hours) from their mean is

(a) 178 (b) 179

(c) 220 (d) 356

10. Mean deviation about the median for the data 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21 is

(a) 4.27 (b) 5.24

(c) 5.27 (d) 4.24

11. Consider any set of observations $x_1, x_2, x_3, \ldots, x_{101}$; such that $x_1 < x_2 < x_3 < \ldots < x_{101}$; then the mean deviation of this set of observations about a point k is minimum, when k equals

(a) x_1 (b) x_{51}

(c) $\frac{x_1+x_2+\ldots+x_{101}}{101}$ (d) x_{50}

Measure of dispersion

12. The number which indicates variability of data or observations, is called

(a) measure of central tendency

(b) mean

(c) median

(d) measure of dispersion

13. If v is the variance and σ is the standard deviation, then

(a) $v^2 = \sigma$ (b) $v = \sigma^2$

(c) $v = \frac{1}{\sigma}$ (d) $v = \frac{1}{\sigma^2}$

14. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(a) $\sqrt{\frac{52}{7}}$ (b) $\frac{52}{7}$

(c) $\sqrt{6}$ (d) 6

15. The mean of 100 observations is 50 and their standard deviation is 5. The sum of squares of all observations is

(a) 50000 (b) 250000

(c) 252500 (d) 255000

16. Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing, then the variance of all the five observations is

(a) 4 (b) 6

(c) 8 (d) 2

17. The variance of the first n natural numbers is

(a) $\frac{n^2-1}{12}$ (b) $\frac{n^2-1}{6}$

(c) $\frac{n^2+1}{6}$ (d) $\frac{n^2+1}{12}$

18. **Statement–I:** The mean and variance for first n natural numbers are $\frac{n+1}{2}$ and $\frac{n^2+1}{12}$, respectively.

Statement–II: The mean and variance for first 10 positive multiples of 3 are 16.5 and 74.25, respectively.

(a) Only statement I is true

(b) Only statement II is true

(c) Both statements are true

(d) Both statements are false

19. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ?

(a) $a = 3,\ b = 4$ (b) $a = 0, b = 7$

(c) $a = 5, b = 2$ (d) $a = 1, b = 6$

20. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(a) $\frac{5}{2}$ (b) $\frac{11}{2}$

(c) 6 (d) $\frac{13}{2}$

21. Let $x_1, x_2 ......, x_n$ be n observations. Let $\omega_i = \ell x_i + k$ for i = 1, 2,.........,n, where l and k are constants. If the mean of x_i's is 48 and their standard deviation is 12, the mean of ω_i's is 55 and standard deviation of ω_i's is 15. The values of l and k should be

(a) $l = 1.25, k = -5$ (b) $l = -1.25, k = 5$

(c) $l = 2.5, k = -5$ (d) $l = 2.5, k = 5$

22. If each observation of a raw data whose variance is σ^2, is increased by λ, then the variance of the new set is

(a) σ^2 (b) $\lambda^2\sigma^2$

(c) $\lambda + \sigma^2$ (d) $\lambda^2 + \sigma^2$

23. If each observation of a raw data whose variance is σ^2, is multiplied by λ, then the variance of the new set is

(a) σ^2 (b) $\lambda^2\sigma^2$

(c) $\lambda + \sigma^2$ (d) $\lambda^2 + \sigma^2$

24. Let a, b, c, d and e be the observations with mean m and standard deviation s. The standard deviation of the observations a + k, b + k, c + k, d + k and e + k is

(a) s (b) |k|s

(c) s + k (d) s/|k|

25. Let x_1, x_2, x_3, x_4 and x_5 be the observations with mean m and standard deviation s. Then, standard deviation of the observations kx_1, kx_2, kx_3, kx_4 and kx_5 is

(a) k + s (b) s/|k|

(c) |k|s (d) s

26. The variance of 20 observations is 5. If each observation is multiplied by 2, then the new variance of the resulting observations is

(a) $2^3 \times 5$ (b) $2^2 \times 5$

(c) 2×5 (d) $2^4 \times 5$

27. The mean of five observations is 4 and their variance is 5.2. If three observations are 1, 2, and 6, the other two are

(a) 2 and 9 (b) 3 and 8

(c) 4 and 7 (d) 5 and 6

28. In an experiment with 15 observations on x, the following results were available.

$$\sum x^2 = 2830, \sum x = 170$$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is

(a) 188.66 (b) 177.33

(c) 8.33 (d) 78.00

29. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later on, it was found that three observations were incorrect, which were recorded as 21, 21 and 18, then the mean and standard deviation, if the incorrect observations are omitted, are

(a) 21, 3.02 (b) 20, 3.02

(c) 21, 3.03 (d) 20, 3.03

Numerical Value Type Questions

30. The number of observations in a group is 40. If the average of first 10 is 4.5 and average of remaining 30 is 3.5, then the average of the whole group is

31. The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is

32. Following are the marks obtained by 9 students in a Mathematics test.

50, 69, 20, 33, 53, 39, 40, 65, 59

The mean deviation from the median is

33. The variance of the data 2, 4, 6, 8, 10 is

34. The mean and standard deviation of some data (in seconds) for the time taken to complete a test, calculated with the following results :

Number of observations = 25,

Mean = 18.2,

Standard deviation = 3.25

Further, another set of 15 observations $x_1, x_2, \ldots\ldots, x_{15}$ (also in seconds) is now available and we have $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524.$ The standard deviation of all 40 observations is

35. Consider the first 10 positive integers. If we multiply each number by (-1) and then add 1 to each number, the variance of the numbers so obtained is

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. The mean of the data set comprising of 16 observations is 16. If one of the observations valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data is : **(2015)**

(a) 16.0 (b) 15.8
(c) 14.0 (d) 16.8

2. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true ? **(2016)**

(a) $3a^2 - 23a + 44 = 0$ (b) $3a^2 - 26a + 55 = 0$
(c) $3a^2 - 32a + 84 = 0$ (d) $3a^2 - 34a + 91 = 0$

3. If the mean deviation of the numbers $1, 1 + d, \ldots\ldots, 1 + 100d$ from their mean is 255, then a value of d is **(2016/Online Set 1)**

(a) 10.1 (b) 5.05
(c) 20.2 (d) 10

4. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6; then the mean deviation from the mean of the data is **(2016/Online Set 2)**

(a) 2.8 (b) 2.6
(c) 8.4 (d) 2.4

5. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is **(2017/Online Set 1)**

(a) 25 (b) 35
(c) 30 (d) 40

6. The sum of 100 observations and the sum of their squares are 400 and 2474, respectively. Later on, three observations 3, 4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is **(2017/Online Set 2)**

(a) 8.00 (b) 8.25
(c) 9.00 (d) 8.50

7. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots\ldots, x_9$ is : **(2018)**

(a) 9 (b) 4
(c) 2 (d) 3

8. The mean of a set of 30 observations is 75. If each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to : **(2018/Online Set 1)**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{4}{3}$ (d) $\frac{10}{3}$

9. If the mean of the data : 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is : **(2018/Online Set 2)**

(a) $\frac{7}{8}$ (b) 1
(c) $\frac{9}{8}$ (d) 2

10. The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively. If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is: **(2018/Online Set 3)**

(a) 0 (b) 1
(c) 2 (d) 4

11. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is **(8-04-2019/Shift-1)**

12. A student scores the following marks in five tests: 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is : **(8-04-2019/Shift-2)**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{100}{3}$
(c) $\frac{10}{3}$ (d) $\frac{100}{\sqrt{3}}$

13. If the standard deviation of the numbers $-1, 0, 1, k$ is $\sqrt{5}$, where $k > 0$, then k is equal to: **(9-04-2019/Shift-1)**

(a) $2\sqrt{6}$ (b) $2\sqrt{\frac{10}{3}}$

(c) $4\sqrt{\frac{5}{3}}$ (d) $\sqrt{6}$

14. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to:

(9-04-2019/Shift-2)

(a) $\frac{9}{4}$ (b) $\frac{7}{2}$

(c) $\frac{8}{3}$ (d) $\frac{7}{3}$

15. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

(10-04-2019/Shift-1)

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(a) 3.2 (b) 3.0

(c) 2.5 (d) 2.8

16. If both the mean and the standard deviation of 50 observations $x_1, x_2, ... x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2, ...(x_{50}-4)^2$ is:

(10-04-2019/Shift-2)

(a) 400 (b) 380

(c) 525 (d) 480

17. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000 ; then the standard deviation of this data is: **(12-04-2019/Shift-1)**

18. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is: **(9-01-2019/Shift-1)**

19. A data consists of n observations:

$x_1, x_2, \ldots, x_n$, If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$

then the standard deviation of this data is:

(9-01-2019/Shift-2)

(a) 2 (b) $\sqrt{5}$

(c) 5 (d) $\sqrt{7}$

20. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then ratio of other two observations is:

(10-1-2019/Shift-1)

(a) 10 : 3 (b) 4 : 9

(c) 5 : 8 (d) 6 : 7

21. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations x_1, x_2, x_3, x_4, x_5 and –50 is equal to: **(10-01-2019/Shift-2)**

(a) 509.5 (b) 586.5

(c) 582.5 (d) 507.5

22. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}-d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}+d$. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals $\sqrt{k}$, then k is equal to: **(11-01-2019/Shift-1)**

23. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is :

(12-01-2019/Shift-1)

24. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations, is : **(12-01-2019/Shift-2)**

(a) 7 (b) 5

(c) 1 (d) 3

25. Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y = \{ax + b : x \in X \text{ and } a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to : **(2-9-2020/Shift-1)**

(a) – 27 (b) 7

(c) – 7 (d) 9

26. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, ..., b_{11}$ is 90, then the common difference of this A.P. is .... **(2-09-2020/Shift-2)**

27. For the frequency distribution :

Variate (x) :	x_1	x_2	$x_3 ..... x_{15}$
Frequency (f) :	f_1	f_2	$f_3 ...... f_{15}$

where $0 < x_1 < x_2 < x_3 < ..... < x_{15} \le 10$ and $\sum_{i=1}^{15} f_i = 0$, the standard deviation cannot be : **(3-09-2020/Shift-1)**

(a) 1 (b) 4

(c) 6 (d) 2

28. Let $x_i \ (1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in R$, then the standard deviation of these observations is : **(3-09-2020/Shift-2)**

(a) $\frac{7}{10}$ (b) $\frac{9}{10}$

(c) $\sqrt{\frac{3}{5}}$ (d) $\frac{4}{5}$

29. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is : **(4-09-2020/Shift-1)**

(a) 3 (b) 9

(c) 7 (d) 5

30. If the variance of the following frequency distribution :

Class:	10–20	20–30	30–40
Frequency:	2	X	2

is 50, then x is equal to .... **(4-09-2020/Shift-2)**

31. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2,4,10,12,14, then the absolute difference of the remaining two observations is: **(5-09-2020/Shift-1)**

(a) 1 (b) 4

(c) 3 (d) 2

32. If the mean and the standard deviation of the data 3,5,7,a,b are 5 and 2 respectively, then a and b are the roots of the equation: **(5-09-2020/Shift-2)**

(a) $x^2 - 20x + 18 = 0$ (b) $x^2 - 10x + 19 = 0$

(c) $2x^2 - 20x + 19 = 0$ (d) $x^2 - 10x + 18 = 0$

33. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na, (n, a > 1)$ then the standard deviation of n observations $x_1, x_2, ..., x_n$ is : **(6-09-2020/Shift-1)**

(a) $n\sqrt{a-1}$ (b) $\sqrt{na-1}$

(c) $a - 1$ (d) $\sqrt{a-1}$

34. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies respectively. ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$,

If the mean of this data is $\frac{728}{2^n}$, then n is equal to _____. **(6-09-2020/Shift-2)**

35. If variance of first n natural numbers is 10 and variance of first m even natural number is 16, m + n is equal to **(7-01-2020/Shift-1)**

36. If the mean and variance of eight numbers 3,7,9,12,13,20, x and y be 10 and 25 respectively then xy is equal to _________. **(7-01-2020/Shift-2)**

37. The mean and standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \ne 0$ and $q \ne 0$. if the new mean and standard deviation become half of their original values, then q is equal to: **(8-01-2020/Shift-1)**

(a) –20 (b) –5

(c) 10 (d) –10

38. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:

(8-01-2020/Shift-2)

(a) 4.01 (b) 3.99

(c) 3.98 (d) 4.02

39. Let the observations $x_i\,(1 \le i \le 10)$ satisfy the equations $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of observations $(x_1 - 3), (x_2 - 3) .... (x_{10} - 3)$, then the ordered pair (μ, λ) is equal to: **(9-01-2020/Shift-1)**

(a) (6,3) (b) (3,6)

(c) (3,3) (d) (6,6)

40. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:

(20-07-2021/Shift-1)

(a) 10, 11 (b) 8, 13

(c) 1, 20 (d) 3, 18

41. Consider the following frequency distribution:

Class:	Oct-20	20-30	30-40	40-50	50-60
Frequency:	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, $|\alpha - \beta|$ is equal to ___________ ? **(25-07-2021/Shift-1)**

42. If the mean and variance of the following data:

6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a - b)^2$ is equal to: **(27-07-2021/Shift-1)**

(a) 12 (b) 24

(c) 16 (d) 32

43. Let the mean and variance of the frequency distribution

x :	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f :	4	4	α	β

be 6 and 6.8 respectively. If is changed from 8 to 7, then the mean for the new data will be: **(27-07-2021/Shift-2)**

(a) $\frac{16}{3}$ (b) 4

(c) $\frac{17}{3}$ (d) 5

44. Consider the following frequency distribution :

Class:	0–6	6–12	12–18	18–24	24–30
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then value $(a - b)^2$ is equal to _________. **(22-07-2021/Shift-2)**

45. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is:

(25-07-2021/Shift-2)

(a) 5 (b) 8

(c) 4 (d) 6

46. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a - b|$ is equal to ? **(20-07-2021/Shift-2)**

(a) 7 (b) 11

(c) 9 (d) 1

47. Let X be a random variable with distribution

x	–2	–1	3	4	6
P (X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\sigma^2$ is equal to: **(01-09-2021/Shift-2)**

48. Let the mean and variance of four numbers 3, 7, x and $y(x > y)$ be 5 and 10 respectively. Then the mean of four numbers $3 + 2x, 7 + 2y, x + y$ and $x - y$ is ________.

(26-08-2021/Shift-2)

49. Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14, then n is equal to ______.

(27-08-2021/Shift-1)

50. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is: **(26-08-2021/Shift-1)**

(a) $(11, 25)$ (b) $(11, 26)$

(c) $(10.5, 26)$ (d) $(10.5, 25)$

51. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to__________. **(27-08-2021/Shift-2)**

52. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is:

(31-08-2021/Shift-2)

(a) $\frac{92}{5}$ (b) $\frac{134}{5}$

(c) $\frac{536}{25}$ (d) $\frac{112}{5}$

53. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to ____.

(16-03-2021/Shift-2)

54. Consider three observations a, b and c such that $b = a + c$. the standard deviation of $a + 2, b + 2, c + 2$ is d, then which of the following is true? **(16-03-2021/Shift-1)**

(a) $b^2 = 3(a^2 + c^2) - 9d^2$

(b) $b^2 = a^2 + c^2 + 3d^2$

(c) $b^2 = 3(a^2 + c^2) + 9d^2$

(d) $b^2 = 3(a^2 + c^2 + d^2)$

55. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ....

(17-03-2021/Shift-2)

Find Answer Key and Detailed Solutions at the end of this book

09

MATHEMATICAL REASONING

MATHEMATICAL REASONING

1. STATEMENT (Proposition)

In reasoning, we communicate our ideas or thoughts with the help of sentences in particular language. "A sentence is called a mathematically acceptable statement or proposition if it is either true or false but not both." A statement is assumed to be either true or false. A true statement is known as a valid statement and a false statement is known as an invalid statement.

2. TRUTH TABLE

Truth table is that which gives truth values of statements. It has a number of rows and columns. Note that for n statements, there are 2^n rows,

(i) Truth table for single statement p:

Number of rows = $2^1 = 2$

P
T
F

(ii) Truth table for two statements p and q :

p	q
T	T
T	F
F	T
F	F

Number of rows = $2^2 = 4$

(iii) Truth table for three statements p, q and r.

Number of rows = $2^3 = 8$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

3. NEGATION OF A STATEMENT

The denial of a statement p is called its negation and is written as ~p and read as 'not p'. Negation of any statement p is formed by writing "It is not the case that ....."

or "It is false that.....

or inserting the word "not" in p.

p	~p
T	F
F	T

Truth Table

4. COMPOUND STATEMENTS

If a statement is combination of two or more statements, then it is said to be a compound statement. Each statement which form a compound statement is known as its sub-statement or component statement.

5. BASIC CONNECTIVES

In the compound statement, two or more statements are connected by words like 'and', 'or', 'if.... then', 'only if', 'if and only if', 'there exists', 'for all' etc. These are called connectives. When we use these compound statements, it is necessary to understand the role of these words.

6. THE WORD " AND" (CONJUCTION)

Any two statements can be connected by the word "and" to form a compound statement. The compound statement with word "and" is true if all its component statements are true. The compound statement with word "and" is false if any or all of its component statements are false. The compound statement "p and q" is denoted by "$p \wedge q$".

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table

7. THE WORD "OR" (DISJUNCTION)

Any two statements can be connected by the word "OR" to form a compound statement. The compound statement with word "or" is true if any or all of its component statements are true. The compound statement with word "or" is false if all its component statements are false. The compound statement "p or q" is denoted by "$p \vee q$":

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table

8. TYPES OF "OR"

(i) Exclusive OR : If in statement $p \vee q$ i.e., p or q, happening of any one of p, q excludes the happening of the other then it is exclusive or. Here, both p and q cannot occur together. For example, in statement "I will go to delhi either by bus or by train", the use of 'or' is exclusive.

(ii) Inclusive OR : If in statement p or q, both p and q can also occur together then it is inclusive or. The statement 'In senior secondary exam you can take optional subject as physical education or computers' is an example of use of inclusive OR.

Implication

There are three types of implications which are "if... then", "Only if" and "if and only if"

9. CONDITIONAL CONNECTIVE 'IF... THEN'

If p and q are any two statements, then the compound statement in the form "if p then q" is called a conditional statement. The statement "If p then q" is denoted by $p \rightarrow q$ or $p \Rightarrow q$ (to be read as p implies q).In the implication $p \rightarrow q$, p is called the antecedent (or the hypothesis) and q the consequent (or the conclusion)

If p then q reveals the following facts :

(i) p is a sufficient condition for q

(ii) q is necessary condition for p

(iii) 'If p then q' has same meaning as that of 'p only if q'

(iv) $p \rightarrow q$ has same meaning as that of $\sim q \rightarrow \sim p$

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Truth Table

Examples:

(i) If $x = 4$, then $x^2 = 16$

(ii) If ABCD is a parallelogram, then AB = CD

(iii) If Mumbai is in England, then $2 + 2 = 5$

(iv) If Shikha works hard, then it will rain today.

10. CONTRAPOSITIVE, CONTRADICTION AND CONVERSE OF A CONDITIONAL STATEMENT

Let p and q are two statements such that $p \Rightarrow q$, then

(i) (Contrapositive of $p \Rightarrow q$) is ($\sim q \Rightarrow \sim p$)

(ii) Contradiction of $p \Rightarrow q$) is ($q \Rightarrow \sim p$)

(iii) (Converse of $p \Rightarrow q$) is ($q \Rightarrow p$)

NOTES :

A statement and its contrapositive convey the same meaning.

11. BICONDITIONAL CONNECTIVE "IF AND ONLY IF"

If p and q are any two statements, then the compound statement in the form of "p if and only if q" is called a biconditional statement and is written in symbolic form as $p \leftrightarrow q$ or $p \Leftrightarrow q$.

Statement $p \leftrightarrow q$ reveals the following facts:

(i) p if and only if q

(ii) q if and only if p

(iii) p is necessary and sufficient condition for q

(iv) q is necessary and sufficient condition for p

p	q	$p \leftrightarrow q$	$q \leftrightarrow p$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	T	T

Truth Table

For Example :

The following statements are biconditional statements :

(i) A number is divisible by 3 if and only if the sum of the digits forming the number is divisible by 3.

(ii) One is less then seven if and only if two is less then eight.

(iii) A triangle is equilateral if and only if it is equiangular.

12. TAUTOLOGY AND FALLACY/CONTRADICTION

(a) Tautology : This is a statement which is true for all truth values of its components. It is denoted by t. consider truth table of $p \vee \sim p$

p	~p	$p \vee \sim p$
T	F	T
F	T	T

We observe that last column is always true. Hence, $p \vee \sim p$ is a tautology.

(b) Fallacy (contradiction) : This is a statement which is false for all truth values of its components. It is denoted by f or c. Consider truth table of $p \wedge \sim p$

p	~p	$p \wedge \sim p$
T	F	F
F	T	F

We observe that last column is always false. Hence $p \wedge \sim p$ is a fallacy (contradiction).

13. LOGICALLY EQUIVALENT STATEMENTS

If truth values of statements p and q are same, then they are logically equivalent and written as $p \equiv q$.

14. ALGEBRA OF STATEMENTS

If p, q,r are any three statements and t is a tautology; c is a contradiction, then

(1) Commutative Law :

(i) $p \vee q \equiv q \vee p$ (ii) $p \wedge q \equiv q \wedge p$

(2) Associative Law :

(i) $p \vee (q \vee r) \equiv (p \vee q) \vee r$

(ii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

(3) Distributive Law :

(i) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(iii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$

(iv) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

(4) Identity Law :

(i) $p \vee t \equiv t$ (ii) $p \wedge t \equiv p$

(iii) $p \vee c \equiv p$ (iv) $p \wedge c \equiv c$

(5) Complement Law :

(i) $p \vee (\sim p) \equiv t$ (ii) $p \wedge (\sim p) \equiv c$

(iii) $\sim t \equiv c$ (iv) $\sim c \equiv t$

(v) $\sim(\sim p) \equiv p$

(6) Idempotent Law :

(i) $p \vee p \equiv p$ (ii) $p \wedge p \equiv p$

(7) De Morgan's Law:

(i) $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$

(ii) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

(8) Involution laws (or Double negation laws):

$$\sim(\sim p) \equiv p$$

(9) Contrapositive Laws : $p \rightarrow q \equiv \sim q \rightarrow \sim p$

15. NEGATION OF COMPOUND STATEMENTS

If p and q are two statements, then

(i) Negation of conjuction : $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(ii) Negation of disjunction : $\sim (p \vee q) \equiv \sim p \wedge \sim q$

(iii) Negation of conditional : $\sim(p \rightarrow q) \equiv p \wedge \sim q$

(iv) Negation of biconditional : $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ or $p \leftrightarrow \sim q$

We know that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\therefore \sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$

$\equiv \sim(p \to q) \vee \sim(q \to p) \equiv (p \wedge \sim q) \vee (q \wedge \sim q)$

Summary :

(i) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

(ii) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

(iii) $\sim(p \to q) \equiv \sim(\sim p \vee q) \equiv p \wedge (\sim q)$

(iv) $\sim(p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ or $p \Leftrightarrow \sim q$

16. DUALITY

Two compound statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

If a compound statement contains the special variable t (tautology) and c (contradiction), then to obtain its dual, we replace t by c and c by t in addition to replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

17. VALIDITY OF A STATEMENT

There are four methods to prove validity of a statement

(a) Direct method :

(i) To prove that "p and q" is true, show that both p and q are true.

(ii) To prove "p or q", show that any one of p or q is true

(iii) To prove $p \to q$, assume that p is true and show that q must be true.

(iv) To prove $p \leftrightarrow q$, show that if p is true then q is true. Also show that if q is true, then p is true.

(b) Contrapositive Method:

To prove $p \to q$, assume that q is false and prove that p must be false.

(c) Contradiction Method:

To prove that a statement p is true, we assume that p is not true, then we arrive at some result which contradicts our assumption. Therefore, we conclude that p is true.

(d) Counter example Method :

To show that a statement is false, we give an example where the statement is not valid. Note that this method is used to disprove the statement. Giving examples in favour of a statement cannot prove that the given statement is valid.

18. VALIDITY OF AN ARGUMENT

An argument is an assertion that a given set of statements $s_1, s_2, \ldots s_n$ implies other statement 's'. In other words, an argument is an assertion that the statement 's' follows from statements $s_1, s_2, \ldots s_n$ which are called hypothesis. The statement 's' is called the conclusion.

We denote the argument containing hypotheses $s_1, s_2, \ldots s_n$ and conclusion 's' by

$s_1, s_2, \ldots s_n$; s or

$s_1, s_2, \ldots s_n$ /-s or

$s_1 \wedge s_2 \wedge \ldots \wedge s_n \to s$ or

The symbol "/-" is read as turnstile

An argument is said to be a valid argument if the conclusion 's' is true whenever all the hypotheses $s_1, s_2, \ldots s_n$ are true or equivalentry argument is valid when it is a tautology, otherwise the argument is called an invalid argument.

Method of testing the validity of argument :

Step I - Construct the truth table for conditional statement $s_1 \wedge s_2 \wedge s_3 \wedge \ldots \wedge s_n \to s$.

Step II - Check the last column of truth table. If the last column contains T only, then the given argument is valid otherwise it is an invalid argument.

SOLVED EXAMPLES

Example – 1

Which of the following is a logical statement ?

(a) Open the door

(b) What an intelligent student

(c) Are you going to Delhi

(d) All prime numbers are odd numbers

Ans. (d)

Sol. Options A, B and C are basic statements, But option D is a logical statement.

Example – 2

Which of the following is not a statement?

(a) Smoking is injurious to health

(b) $2+2=4$

(c) 2 is the only even prime number

(d) Come here

Ans. (d)

Sol. Option 'D' is an imperative sentence.

Hence, not a statement.

Example – 3

Write negation of the following statement:

"All cats scratch"

Sol. Some cats do not scratch

OR

There exists a cat which does not scratch

OR

At least one cat does not scratch.

Example – 4

Write negation of statement '$2+2=7$'

Sol. $2+2\neq 7$

Example – 5

Find the truth value of the statement "2 divides 4 and $3+7=8$"

Sol. 2 divides 4 is true and $3+7=8$ is false. So, given statement is false.

Example – 6

Write component statements of the statement "All living things have two legs and two eyes".

Sol. Component statements are :

All living things have two legs

All living things have two eyes

Example – 7

The negation of the statement "If I become a teacher, then I will open a school", is

(a) Neither I will become a teacher nor I will open a school.

(b) I will not become a teacher or I will open a school.

(c) I will become a teacher and I will not open a school.

(d) Either I will not become a teacher or I will not open a school.

Ans. (c)

Sol. Negation of $p \to q \equiv \sim((\sim p) \vee q)$

$\equiv p \wedge (\sim q)$

So, correct option is 'C'

Example – 8

Write the negation of the following compound statements:

(i) All the students completed their homework and the teacher is present.

(ii) Square of an integer is positive or negative

(iii) If my car is not in workshop, then I can go college.

(iv) ABC is an equilateral triangle if and only if it is equiangular.

Sol. (i) The component statements of the given statement are :

p : all the students completed their homework

q : The teacher is present.

The given statement is p and q, so its negation is ~p or ~q = Some of the students did not complete their home work or the teacher is not present.

(ii) The component statements of the given statement are:

p : Square of an integer is positive.

q : Square of an integer is negative.

The given statement is p or q, so its negation is ~p and ~q = There exists an integer whose square is neither positive nor negative.

(iii) Consider the following statements :

p : My car is not in workshop

q : I can go to college

The given statement in symbolic form is $p \to q$

Now, $\sim(p \to q) \equiv p \wedge (\sim q)$

$\Rightarrow \sim(p \to q)$: My car is not in workshop and I cannot go to college.

(iv) Consider the following statements:

p : ABC is an equilateral triangle.

q : It is equiangular

Clearly, the given statement in symbolic form is $p \leftrightarrow q$.

Now, $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow \sim(p \to q)$: Either ABC is an equilateral triangle and it is not equiangular or ABC is not an equilateral triangle and it is equiangular.

Example – 9

Let p and q stand for the statement 'Bhopal is in M.P.' and '3 + 4 = 7' respectively. Describe the conditional statement $\sim p \to \sim q$

Sol. $\sim p \to \sim q$: If Bhopal is not in M.P., then $3 + 4 \neq 7$

Example – 10

The statement p : For any real numbers x, y, if x = y, then $2x + a = 2y + a$, when $a \in Z$

(a) is true

(b) is false

(c) its contrapositive is not true

(d) None of the above

Ans. (a)

Sol. For any real numbers x and y,

given x = y

$\Rightarrow 2x = 2y$

$\Rightarrow 2x + a = 2y + a$ for some $a \in Z$

So, the statement is true.

Example – 11

Write the contrapositive of the following statement: "If Mohan is poet, then he is poor"

Sol. Consider the following statements :

p : Mohan is a poet

q : Mohan is poor

Clearly, the given statement in symbolic form is $p \to q$. Therefore, its contrapositive is given by $\sim q \to \sim p$.

Now, $\sim p$: Mohan is not a poet.

$\sim q$: Mohan is not poor

$\therefore \sim q \to \sim p$: If Mohan is not poor, then he is not a poet.

Hence, the contrapositive of the given statement is "If Mohan is not poor, then he is not a poet".

Example – 12

Write the converse and the contrapositive of the statement "If x is a prime number, then x is odd".

Sol. Given statement is : "If x is a prime number then x is odd".

Let p : x is prime number and q : x is odd

$\therefore$ Given statement is $p \to q$

The converse of $p \to q$ is $q \to p$ i.e., "If x is odd, then x is a prime number"

The contrapositive of $p \to q$ is $\sim q \to \sim p$ i.e., "If x is not odd, then x is not a prime number"

Example – 13

Write the contradiction of "If it rains, then I stay at home".

Sol. If I stay at home, then, it does not rain.

Example – 14

Find the truth values of $\sim p \vee q$

Sol.

p	q	~p	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Example – 15

Find the truth values of the compound statement $(p \vee \sim r) \wedge (q \vee \sim r)$

Sol.

p	q	r	$\sim r$	$p \vee \sim r$	$q \vee \sim r$	$(p \vee \sim r) \wedge (q \vee \sim r)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	F
F	T	T	F	F	T	F
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	F	F
F	F	F	T	T	T	T

Example – 16

Find the truth value of $(p \leftrightarrow \sim q) \leftrightarrow (q \to p)$

Sol.

p	q	$\sim q$	$p \leftrightarrow \sim q$	$q \to p$	$(p \leftrightarrow \sim q) \leftrightarrow (q \to p)$
T	T	F	F	T	F
T	F	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F

Example – 17

Show that $p \to (p \vee q)$ is a tautology

Sol.

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Example – 18

Consider :

Statement–1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement–2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

(a) Statement–1 is true, Statement–2 is true,

Statement–2 is not a correct explanation for Statement–1.

(b) Statement–1 is true, Statement–2 is false.

(c) Statement–1 is false, Statement–2 is true.

(d) Statement–1 is true, Statement–2 is true,

Statement–2 is a correct explanation for Statement–1.

Ans. (a)

Sol.

p	q	$p \Rightarrow q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Leftrightarrow \sim p)$
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Both statments are true, but they are independent of each other.

Example – 19

The proposition $\sim (p \vee \sim q) \vee \sim (p \vee q)$ is logically equivalent to :

(a) p (b) q

(c) ~p (d) ~ q

Ans. (c)

Sol.

p	q	$\sim p$	$\sim q$	$\sim (p \vee \sim q)$	$\sim (p \vee q)$	$\sim (p \vee \sim q) \vee \sim (p \vee q)$
T	T	F	F	F	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	T
F	F	T	T	F	T	T

So, correct option is c.

Example – 20

Show that the following argument is not valid : "If it rains, crops will be good. It did not rain. Therefore the crops were not good".

Sol. p : it rains

q : crops will be good

$S_1 : p \to q$, $S_2 : \sim p$, $S : \sim q$

p	q	S_1	S_2	S
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Not valid

Example – 21

Which of the following is logically equivalent to $(p \wedge q)$?

(a) $p \to \sim q$ (b) $\sim p \wedge \sim q$

(c) $\sim (p \to \sim q)$ (d) $\sim (\sim p \wedge \sim q)$

Ans. (c)

Sol. $p \to \sim q \equiv \sim p \vee \sim q \equiv \sim (p \wedge q)$

$(\because p \to q \equiv \sim p \vee q)$

$\therefore \sim (p \to \sim q) \equiv p \wedge q$

Example – 22

Find the negation of statement $p \wedge \sim q$

Sol. Negation of $(p \wedge \sim q) \equiv \sim (p \wedge \sim q)$

$\equiv \sim p \vee \sim\sim q \equiv \sim p \vee q$

Example – 23

By using laws of algebra of statements, show that $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$

Sol. $(p \vee q) \wedge \sim p \equiv (\sim p) \wedge (p \vee q)$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q)$

$\equiv f \vee (\sim p \wedge q)$

$\equiv \sim p \wedge q$

Example – 24

The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \leftrightarrow q)$ (b) $p \to (p \to q)$

(c) $p \to (p \vee q)$ (d) $p \to (p \wedge q)$

Ans. (c)

Sol. $p \to (q \to p) \equiv \sim p \vee (q \to p)$

$\equiv \sim p \vee (\sim q \vee p)$

$\equiv \sim p \vee p \vee (\sim q)$

$\equiv T$ (a tautology)

$p \to (p \vee q) \equiv \sim p \vee (p \vee q)$

$\equiv T$ (a tautology)

Example – 25

$\sim (p \vee q) \vee (\sim p \wedge q)$ is equivalent to

(a) p (b) $\sim p$

(c) q (d) $\sim q$

Ans. (b)

Sol. $\sim (p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge (\sim q \vee q)$

$\equiv \sim p \wedge t$

$\equiv \sim p$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Basic terms involved

1. Which of the following is a statement?

(a) x is a real number (b) Switch off the fan

(c) 6 is a natural number (d) Let me go

2. Which of the following is/are not a statement?

(I) Earth is a planet.

(II) Plants are living objects.

(III) $\sqrt{-3}$ is a rational number.

(IV) $x^2 - 5x + 6 < 0$, when $x \in R$.

(a) I and II (b) II and IV

(c) III and I (d) None of these

3. Consider the statement p : "New Delhi is a city". Which of the following is not negation of p?

(a) New Delhi is not a city

(b) It is false that New Delhi is a city

(c) It is not the case that New Delhi is not a city

(d) None of these

4. The negation of the statement '$\sqrt{2}$ is not a complex number' is

(a) $\sqrt{2}$ is a rational number

(b) $\sqrt{2}$ is an irrational number

(c) $\sqrt{2}$ is a real number

(d) $\sqrt{2}$ is a complex number

5. Let S be a non-empty subset of R. Consider the following statement :

p : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statement(s) is the negation of the statement p?

(a) There is a rational number $x \in S$ such that $x \le 0$

(b) There is no rational number $x \in S$ such that $x \le 0$

(c) Every rational number $x \in S$ satisfies $x \le 0$

(d) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

6. The component statements of the statement "The sky is blue or the grass is green" are

(a) p: The sky is blue, q : The sky is not blue

(b) p : The sky is blue, q : The grass is green

(c) Both (a) and (b)

(d) None of the above

7. For the compound statement:

'All prime numbers are either even or odd'. Which of the following is true?

(a) Both component statements are false

(b) Exactly one of the component statements is true

(c) Atleast one of the component statements is true

(d) Both the component statements are true.

8. The negation of the statement "Ramesh is cruel or he is strict" is

(a) Ramesh is neither cruel nor strict

(b) Ramesh is cruel or he is not strict

(c) Ramesh is not cruel or he is strict

(d) Ramesh is not cruel and he is strict

9. The negation of the statement

p : "for every positive real number x, the number x–1 is also positive" is

(a) "there exists atleast one positive real number x for which (x –1) is not positive"

(b) "for every positive real number x, the number (x + 1) is also positive"

(c) Both (a) and (b)

(d) Neither (a) nor (b)

10. Negation of the statement S : "There exists a number x such that $0 < x < 1$" is ...P... Here, P refers to

(a) there does not exist a number x such that $0 < x < 2$

(b)there does not exist a number x such that $0 < x < 1$

(c) Both (a) and (b)

(d) None of the above

11. The component statement of the compound statement "If you are born in some country, then you are a citizen of that country" are

p : You are born in some country

q : You are a citizen of that country

Then, the correct option is

(a) If p is true, then q must be true
(b) If p is true, then q is not true
(c) Both p and q are false
(d) None of the above

12. The statement 'x is an even number implies that x is divisible by 4' means the same as

(a) x is divisible by 4 is necessary condition for x to be an even number
(b) x is an even number is a necessary condition for x to divisible by 4
(c) x is divisible by 4 is a sufficient condition for x to be an even number
(d) x is divisible by 4 implies that x is not always an even number

13. The negation of the statement "If a quadrilateral is a square, then it is a rhombus".

(a) If a quadrilateral is not a square then it is a rhombus
(b) If a quadrilateral is a square then it is not a rhombus
(c) a quadrilateral is a square and it is not a rhombus
(d) a quadrilateral is not a square and it is a rhombus

14. Consider the statement

p : "It is necessary to have a password to log on to the server".

Then, "If-then" form of this statement is

(a) If you log on to sever, then you have a password
(b) If you log on to server, then you do not have a password
(c) Both (a) and (b)
(d) None of the above

15. The converse of the statement

"If $x > y$, then $x + a > y + a$" is

(a) if $x < y$, then $x + a < y + a$
(b) If $x + a > y + a$, then $x > y$
(c) If $x < y$, then $x + a > y + a$
(d) If $x > y$, then $x + a < y + a$

16. The statement "If x^2 is not even, then x is not even" is converse of the statement

(a) If x^2 is odd, then x is even
(b) If x is not even, then x^2 is not even
(c) If x is even, then x^2 is even
(d) If x is odd, then x^2 is even

17. The contrapositive of $(p \wedge q) \to r$ is

(a) $\sim r \to (p \vee q)$ (b) $r \to (p \vee q)$
(c) $\sim r \to (\sim p \vee \sim q)$ (d) $p \to (q \vee r)$

18. The contrapositive of "If two triangles are identical, then these are similar" is ...X...Here, X refers to

(a) If two triangles are not similar, then these are not identical
(b) If two triangles are not identical, then these are not similar
(c) Both (a) and (b)
(d) None of the above

19. If $x = 5$ and $y = -2$, then $x - 2y = 9$. The contrapositive of this statement is

(a) If $x - 2y = 9$, then $x = 5$ and $y = -2$
(b) If $x - 2y \neq 9$, then $x \neq 5$ and $y \neq -2$
(c) If $x - 2y \neq 9$, then $x \neq 5$ and $y \neq -2$
(d) If $x - 2y \neq 9$, then either $x \neq 5$ or $y \neq -2$

Truth table and logical equivalence

20. If p is true and q is false, then which of the following statement is not true?

(a) $p \wedge \sim q$ (b) $p \to q$
(c) $p \to \sim q$ (d) $q \to p$

21. Let p : "100 is a multiple of 4 and 5"

Statement I : p is true

Statement II: Both the component statements of p are true.

Choose the correct option

(a) Statement I is true. Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true; Statement II is true; Statement II is not a correct explanantion for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true.

22. Let q: "60 is a multiple of 3 or 5".

Statement I : q is true

Statement II : Both the component statements of q are true.

Choose the correct option

(a) Statement I is true. Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true; Statement II is true; Statement II is not a correct explanantion for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true.

23. Let p : 25 is multiple of 5.

q : 25 is multiple of 8

Statement I : The compound statement "p and q" is false.

Statement II: The compound statement "p or q" is false

Chose the correct option

(a) Only statement I is true

(b) Only statement II is true

(c) Both statements are true

(d) Both statements are false

24. **Statement–1 :** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement–2 : $\sim(p \leftrightarrow \sim q)$ is a tautology.

(a) Statement–1 is true, Statement–2 is true;

Statement–2 is not a corrrect explanation for Statement–1

(b) Statement–1 is true, Statement–2 is false

(c) Statement–1 is false, Statement–2 is true

(d) Statement–1 is true, Statement–2 is true;

Statement–2 is correct explanation for Statement–1

25. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is :

(a) $(p \vee q) \Rightarrow r$ (b) $(p \Rightarrow q) \vee (p \Rightarrow r)$

(c) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$ (d) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

26. The statement $(\sim(p \Leftrightarrow q)) \wedge p$ is equivalent to

(a) $p \wedge q$ (b) $q \Leftrightarrow p$

(c) $p \wedge \sim q$ (d) $\sim p \wedge q$

27. For two statements p and q, the logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to

(a) p (b) q

(c) ~p (d) ~q

28. Negation of the statement $p \rightarrow (q \wedge r)$ is

(a) $\sim p \rightarrow \sim(q \wedge r)$ (b) $\sim p \vee (q \wedge r)$

(c) $(q \wedge r) \rightarrow p$ (d) $p \wedge (\sim q \vee \sim r)$

29. Which of the following is logically equivalent to $\sim(p \leftrightarrow q)$

(a) $(\sim p) \leftrightarrow q$ (b) $(\sim p) \leftrightarrow (\sim q)$

(c) $p \rightarrow (\sim q)$ (d) $p \rightarrow q$

30. The negation of $(p \vee \sim q) \wedge q$ is

(a) $(\sim p \vee q) \wedge \sim q$ (b) $(p \wedge \sim q) \vee q$

(c) $(\sim p \wedge q) \vee \sim q$ (d) $(p \wedge \sim q) \vee \sim q$

31. If p and q are two statements, then $p \vee \sim(p \Rightarrow \sim q)$ is equivalent to

(a) $p \wedge q$ (b) p

(c) $p \wedge \sim q$ (d) $\sim p \wedge q$

32. Which of the following is always true?

(a) $(\sim p \vee \sim q) \equiv (p \wedge q)$

(b) $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$

(c) $\sim(p \rightarrow \sim q) \equiv (p \wedge \sim q)$

(d) $\sim(p \leftrightarrow q) \equiv (p \rightarrow q) \rightarrow (q \rightarrow p)$

33. The negation of the compound statement $(p \vee q) \wedge r$ is

(a) $(\sim p \vee \sim q) \wedge \sim r$ (b) $(\sim p \wedge \sim q) \vee \sim r$

(c) $\sim(p \vee q) \Rightarrow r$ (d) $p \wedge q$

34. The negation of $(\sim p \wedge q) \vee (p \wedge \sim q)$ is

(a) $(p \vee \sim q) \vee (\sim p \vee q)$ (b) $(p \vee \sim q) \wedge (\sim p \vee q)$

(c) $(p \wedge \sim q) \wedge (\sim p \vee q)$ (d) $(p \wedge \sim q) \wedge (p \vee \sim q)$

35. Let p : Maths is interesting and q : Maths is easy, then $p \Rightarrow (\sim p \vee q)$ is equivalent to

(a) If Maths is easy, then it is interesting

(b) Either Maths is interesting or it easy

(c) If Maths is interesting, then it is easy

(d) Maths is neither interesting nor easy

36. The converse of $p \Rightarrow (q \Rightarrow r)$ is

(a) $(q \wedge \sim r) \vee p$ (b) $(\sim q \vee r) \vee p$

(c) $(q \wedge \sim r) \wedge \sim p$ (d) $(q \wedge \sim r) \wedge p$

37. The contrapositive of $p \rightarrow (\sim q \rightarrow \sim r)$ is

(a) $(\sim q \wedge r) \rightarrow \sim p$ (b) $(q \rightarrow r) \rightarrow \sim p$

(c) $(q \vee \sim r) \rightarrow \sim p$ (d) None of these

38. $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is

(a) a tautology

(b) a contradiction

(c) both a tautology and a contradiction

(d) neither a tautology nor a contradiction

39. Let $\oplus$ and $\otimes$ are two mathematical operators. If $p \oplus (p \otimes q)$ is not a tautology, then $\oplus$ and $\otimes$ can be

(a) $\vee$ and $\Rightarrow$ respectively

(b) $\Rightarrow$ and $\vee$ respectively

(c) $\Rightarrow$ and $\wedge$ respectively

(d) None of these

40. If the compound statement $p \rightarrow (\sim p \vee q)$ is false, then the truth value of p and q are respectively

(a) T, T (b) T, F

(c) F, T (d) F, F

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. Consider the following statements : **(Online Set–2 2015)**

P : Suman is brilliant.

Q : Suman is rich.

R : Suman is honest.

The negation of the statement,

"Suman is brilliant and dishonest if and only if Suman is rich" can be equivalently expressed as :

(a) $\sim(Q \leftrightarrow \sim P \wedge R)$ (b) $\sim(Q \leftrightarrow \sim P \vee R)$

(c) $\sim(Q \leftrightarrow \sim P \vee \sim R)$ (d) $\sim(Q \leftrightarrow P \wedge \sim R)$

2. The contrapositive of the statement "If it is raining, then I will not come", is : **(Online Set–1 2015)**

(a) If I will come, then it is not raining.

(b) If I will not come, then it is raining.

(c) If I will not come, then it is not raining.

(d) If I will come, then it is raining.

3. The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to **(2016)**

(a) $p \vee \sim q$ (b) $\sim p \wedge q$

(c) $p \wedge q$ (d) $p \vee q$

4. Consider the following two statements : **(Online Set–1 2016)**

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals

(a) (F, F) (b) (F, T)

(c) (T, F) (d) (T, T)

5. The contrapositive of the following statement, "If the side of a square doubles, then its area increases four times", is **(Online Set–2 2016)**

(a) If the area of a square increases four times, then its side is not doubled.

(b) If the area of a square increases four times, then its side is doubled.

(c) If the area of a square does not increase four times, then its side is not doubled.

(d) If the side of a square is not doubled, then its area does not increase four times.

6. The following statement $(p \to q) \to [\sim p \to q) \to q]$ is **(2017)**

(a) equivalent to $\sim p \to q$ (b) equivalent to $p \to \sim q$

(c) a fallacy (d) a tautology

7. The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to **(Online Set–1 2017)**

(a) $p \wedge \sim q$ (b) $p \vee \sim q$

(c) $p \to \sim q$ (d) $q \to p$

8. Contrapositive of the statement 'If two numbers are not equal, then their squares are not equal' is **(Online Set–2 2017)**

(a) If the squares of two numbers are not equal, then the numbers are equal.

(b) If the squares of two numbers are equal, then the numbers are not equal.

(c) If the squares of two numbers are equal, then the numbers are equal.

(d) If the squares of two numbers are not equal, then the numbers are not equal.

9. If $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false, then the truth values of p, q and r are, respectively: **(2018/Online Set–1)**

(a) F, T, F (b) T, F, T

(c) T, T, T (d) F, F, F

10. If $p \to (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively : **(2018/Online Set–3)**

(a) F, F (b) T, F

(c) F, T (d) T, T

11. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to : **(2018)**

(a) $\sim q$ (b) $\sim p$

(c) p (d) q

12. Consider the following two statements :

Statement p :

The value of sin 120^0 can be derived by taking $\theta=240^0$ in the equation

$$2\sin\frac{\theta}{2}=\sqrt{1+\sin\theta}-\sqrt{1-\sin\theta}.$$

Statement q :

The angles A, B, C and D of any quadrilateral ABCD satisfy the equation

$$\cos\left(\frac{1}{2}(A+C)\right)+\cos\left(\frac{1}{2}(B+D)\right)=0$$

Then the truth values of p and q are respectively : **(2018/Online Set–2)**

(a) F, T (b) T, F

(c) T, T (d) F, F

13. For any two statements p and q, the negation of the expression $p\vee(\sim p\wedge q)$ is : **(9-04-2019/Shift-1)**

(a) $\sim p\wedge\sim q$ (b) $p\wedge q$

(c) $p\leftrightarrow q$ (d) $\sim p\vee\sim q$

14. If $p\Rightarrow(q\vee r)$ is false, then the truth values of p,q,r are respectively: **(9-04-2019/Shift-2)**

(a) F, T, T (b) T, F, F

(c) T, T, F (d) F, F, F

15. Which one of the following Boolean expressions is a tautology? **(10-04-2019)**

(a) $(p\wedge q)\vee(p\wedge\sim q)$ (b) $(p\vee q)\vee(p\vee\sim q)$

(c) $(p\vee q)\wedge(p\vee\sim q)$ (d) $(p\vee q)\wedge(\sim p\vee\sim q)$

16. The negation of the Boolean expression $\sim s\vee(\sim r\wedge s)$ is equivalent to: **(10-4-2019/Shift-2)**

(a) $\sim s\wedge\sim r$ (b) r

(c) $s\vee r$ (d) $s\wedge r$

17. If the truth value of the statement $p\rightarrow(\sim q\vee r)$ is false (F), then the truth values of the statements p, q, r are respectively. **(12-04-2019/Shift-1)**

(a) T, T, F (b) T, F, F

(c) T, F, T (d) F, T, T

18. The Boolean expression $\sim(p\Rightarrow(\sim q))$ is equivalent to ______. **(12-04-2019/Shift-2)**

(a) $p\wedge q$ (b) $q\Rightarrow\sim p$

(c) $p\vee q$ (d) $(\sim p)\Rightarrow q$

19. The logical statement $[\sim(\sim p\vee q)\vee(p\wedge r)]\wedge(\sim q\wedge r)$ is equivalent to: **(9-01-2019/Shift-2)**

(a) $(\sim p\wedge\sim q)\wedge r$ (b) $-p\vee r$

(c) $(p\wedge r)\wedge\sim q$ (d) $(p\wedge\sim q)\vee r$

20. Consider the following three statements:

p : 5 is a prime number

q : 7 is a factor of 192

r : L.C.M. of 5 and 7 is 35

Then the truth value of which one of the following statements is true? **(10-01-2019/Shift-2)**

(a) $(\sim p)\vee(q\wedge r)$ (b) $(p\wedge q)\vee(\sim r)$

(c) $(\sim p)\wedge(\sim q\wedge r)$ (d) $p\vee(\sim q\wedge r)$

21. If q is false and $p\wedge q\leftrightarrow r$ is true, then which one of the following statements is a tautology? **(11-01-2019/Shift-1)**

(a) $(p\vee r)\rightarrow(p\wedge r)$ (b) $(p\wedge r)\rightarrow(p\vee r)$

(c) $(p\wedge r)$ (d) $(p\vee r)$

22. The Boolean expression

$((p\wedge q)\vee(p\vee\sim q))\wedge(\sim p\wedge\sim q)$ is equivalent to : **(12-01-2019/Shift-1)**

(a) $p\wedge q$ (b) $p\wedge(-q)$

(c) $(\sim p)\wedge(\sim q)$ (d) $p\vee(-q)$

23. The expression $\sim(\sim p\rightarrow q)$ is logically equivalent to: **(12-01-2019/Shift-2)**

(a) $\sim p\wedge\sim q$ (b) $p\wedge\sim q$

(c) $\sim p\wedge q$ (d) $p\wedge q$

24. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :

(8-04-2019/Shift-1)

(a) If you are not a citizen of India, then you are not born in India.

(b) If you are a citizen of India, then you are born in India

(c) If you are born in India, then you are not a citizen of India

(d) If you are not born in India, then you are not a citizen of India.

25. Which one of the following statements is not a tautology?

(8-04-2019/Shift-2)

(a) $(p \vee q) \rightarrow (p \vee (\sim q))$ (b) $(p \wedge q) \Rightarrow (\sim p) \vee q$

(c) $p \rightarrow (p \vee q)$ (d) $(p \wedge q) \rightarrow p$

26. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is :

(11-01-2019/Shift-2)

(a) If the squares of two numbers are not equal, then the numbers are equal.

(b) If the squares of two numbers are equal, then the numbers are not equal.

(c) If the squares of two numbers are equal, then the numbers are equal.

(d) If the squares of two numbers are not equal, then the numbers are not equal.

27. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :

(2-9-2020/Shift-1)

(a) If I will catch the train, then I reach the station in time.

(b) If I do not reach the station in time, then I will catch the train.

(c) If I do not reach the station in time, then I will not catch the train.

(d) If I will not catch the train, then I do not reach the station in time.

28. Which of the following is a tautology ?

(2-09-2020/Shift-2)

(a) $(p \rightarrow q) \wedge (q \rightarrow p)$ (b) $(\sim p) \wedge (p \vee q) \rightarrow q$

(c) $(q \rightarrow p) \vee \sim (p \rightarrow q)$ (d) $(\sim q) \vee (p \wedge q) \rightarrow q$

29. The proposition $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to :

(3-09-2020/Shift-1)

(a) $(-p) \vee (\sim q)$ (b) $(\sim p) \wedge q$

(c) q (d) $(\sim p) \vee q$

30. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively : (3-09-2020/Shift-2)

(a) F, T, F (b) T, F, T

(c) T, T, F (d) T, T, T

31. Given the following two statements :

$(S_1): (q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology

$(S_2): \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then :

(4-09-2020/Shift-1)

(a) only (S_1) is correct

(b) both (S_1) and (S_2) are correct.

(c) only (S_2) is correct

(d) both (S_1) and (S_2) are not correct.

32. Contrapositive of the statement :

'If a function f is differentiable at a, then it is also continuous at a', is: (4-09-2020/Shift-2)

(a) If a function f is not continuous at a, then it is not differentiable at a.

(b) If a function f is continuous at a, then it is differentiable at a.

(c) If a function f is continuous at a, then it is not differentiable at a.

(d) If a function f is not continuous at a, then it is differentiable at a.

33. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to: (5-09-2020/Shift-1)

(a) $(x \wedge y) \wedge (\sim x \vee \sim y)$ (b) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(c) $(x \wedge \sim y) \vee (\sim x \wedge y)$ (d) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

34. The statement $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is:

(5-09-2020/Shift-2)

(a) equivalent to $(p \vee q) \wedge (\sim p)$

(b) equivalent to $(p \wedge q) \vee (\sim p)$

(c) a contradiction

(d) a tautology

35. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to: **(6-09-2020/Shift-1)**

(a) $p \wedge \sim q$ (b) $\sim p \vee \sim q$

(c) $\sim p \vee q$ (d) $\sim p \wedge \sim q$

36. Consider the statement : "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is : **(6-09-2020/Shift-2)**

(a) For an integer n, if n is even, then $n^3 - 1$ is even

(b) For an integer n, if n is odd, then $n^3 - 1$ is even

(c) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(d) For an integer n, if n is even, then $n^3 - 1$ is odd

37. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to **(7-01-2020/Shift-1)**

(a) $\sim p$ (b) p

(c) q (d) $\sim q$

38. Let A,B,C and D be four non-empty sets. The Contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is : **(7-01-2020/Shift-2)**

(a) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(b) If $A \nsubseteq C$, then $A \subseteq B$ and $B \subseteq D$

(c) If $A \nsubseteq C$, then $A \nsubseteq B$ and $B \subseteq D$

(d) If $A \nsubseteq C$, then $A \nsubseteq B$ or $B \nsubseteq D$

39. Which of the following is a tautology? **(8-01-2020/Shift-1)**

(a) $(P \wedge (P \to Q)) \to Q$ (b) $P \wedge (P \vee Q)$

(c) $(Q \to (P \wedge (P \to Q)))$ (d) $P \vee (P \wedge Q)$

40. Which of the following statements is a tautology? **(8-01-2020/Shift-2)**

(a) $\sim (p \wedge \sim q) \to (p \vee q)$ (b) $(\sim p \vee \sim q) \to (p \wedge q)$

(c) $p \vee (\sim q) \to (p \wedge q)$ (d) $\sim (p \vee \sim q) \to (p \vee q)$

41. Negation of the statement: "$\sqrt{5}$ is an integer or 5 is irrational" is: **(9-01-2020/Shift-1)**

(a) $\sqrt{5}$ is irrational or 5 is an integer

(b) $\sqrt{5}$ is not an integer or 5 is not irrational

(c) $\sqrt{5}$ is an integer and 5 is irrational

(d) $\sqrt{5}$ is not an integer and 5 is not irrational

42. If $p \to (p \wedge \sim q)$ is false. Then the truth values of p and q are respectively **(9-1-2020/Shift-2)**

(a) F, T (b) T, F

(c) F, F (d) T, T

43. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to: **(20-07-2021/Shift-1)**

(a) $\sim q \Rightarrow p$ (b) $p \Rightarrow q$

(c) $p \Rightarrow \sim q$ (d) $q \Rightarrow p$

44. Consider the following three statements:

(A) If $3+3=7$ then $4+3=8$.

(B) If $5+3=8$ then earth is flat.

(C) If both (A) and (B) are true then $5+6=17$.

Then, which of the following statements is correct? **(20-07-2021/Shift-2)**

(a) (A) and (C) are true while (B) is false

(b) (A) is true while (B) and (C) are false

(c) (A) is false, but (B) and (C) are true

(d) (A) and (B) are false while (C) is true

45. The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to ? **(25-07-2021/Shift-1)**

(a) q (b) $\sim q$

(c) p (d) $\sim p$

46. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to: **(27-07-2021/Shift-1)**

(a) $P \vee Q$ (b) $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

(c) $P \wedge \sim Q$ (d) $\sim (P \Rightarrow Q)$

47. Which of the following is the negation of the statement "for all $M > 0$, there exists $x \in S$ such that $x \geq M$"? **(27-07-2021/Shift-2)**

(a) there exists $M > 0$, such that for all $x \geq M$ for all $x \in S$

(b) there exists $M > 0$, there exists for $x \in S$ such that $x \geq M$

(c) there exists $M > 0$, such that $x < M$ for all $x \in S$

(d) there exists $M > 0$, there exists $x \in S$ such that $x < M$

48. Which of the following Boolean expressions is not a tautology? **(22-07-2021/Shift-2)**

(a) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(b) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(c) $(p \Rightarrow q) \vee (q \Rightarrow \sim p)$

(d) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

49. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following: **(25-07-2021/Shift-2)**

(a) The match will not be played and weather is not good and ground is wet.

(b) If the match will not be played, then either weather is not good or ground is wet.

(c) The match will not be played or weather is good and ground is not wet.

(d) The match will be played and weather is not good or ground is wet.

50. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$ **(01-09-2021/Shift-2)**

(a) $\sim (p \rightarrow \sim q)$ (b) $\sim p \rightarrow \sim q$

(c) $\sim (q \rightarrow p)$ (d) $\sim (p \rightarrow q)$

51. Consider the two statements:

$(S_1): (p \rightarrow q) \vee (\sim q \rightarrow p)$ is a tautology

$(S_2): (p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

(26-08-2021/Shift-2)

(a) only (S_1) is true

(b) only (S_2) is true

(c) both (S_1) and (S_2) are true

(d) both (S_1) and (S_2) are false

52. The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is : **(27-08-2021/Shift-1)**

(a) equivalent to $q \rightarrow \sim r$ (b) equivalent to $p \rightarrow \sim r$

(c) a fallacy (d) a tautology

53. If the truth value of the Boolean expression $((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)) \rightarrow (p \wedge q)$ is false, then the truth values of the statements p, q, r respectively can be: **(26-08-2021/Shift-1)**

(a) FFT (b) FTF

(c) TFT (d) TFF

54. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to: **(27-08-2021/Shift-2)**

(a) $(p \wedge q) \Rightarrow (r \wedge p)$ (b) $(q \wedge r) \Rightarrow (p \wedge q)$

(c) $(p \wedge q) \Rightarrow (r \wedge q)$ (d) $(p \wedge q) \Rightarrow (r \vee q)$

55. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then : **(31-08-2021/Shift-1)**

(a) $* = \vee, \square = \wedge$ (b) $* = \wedge, \square = \vee$

(c) $* = \wedge, \square = \wedge$ (d) $* = \vee, \square = \vee$

56. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is: **(31-08-2021/Shift-2)**

(a) $p \wedge q \wedge r$ (b) $\sim p \wedge q \wedge r$

(c) $p \wedge \sim q \wedge \sim r$ (d) $\sim p \wedge q \wedge \sim r$

57. Which of the following Boolean expressions is a tautology? **(16-03-2021/Shift-2)**

(a) $(p \wedge q) \vee (p \vee q)$ (b) $(p \wedge q) \rightarrow (p \rightarrow q)$

(c) $(p \wedge q) \wedge (p \rightarrow q)$ (d) $(p \wedge q) \vee (p \rightarrow q)$

58. If the Boolean expression $(p \wedge q)$ ›” $(p \otimes q)$ is a tautology, then ›” and $\otimes$ are respectively given by : **(17-03-2021/Shift-2)**

(a) $\wedge, \rightarrow$ (b) $\wedge, \vee$

(c) $\rightarrow, \rightarrow$ (d) $\vee, \rightarrow$

59. If the Boolean expression $(p \rightarrow q) \leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $(p * (\sim q))$ is equivalent to : **(17-03-2021/Shift-1)**

(a) $p \rightarrow \sim q$ (b) $p \rightarrow q$

(c) $q \rightarrow p$ (d) $\sim q \rightarrow p$

60. If P and Q are two statements, then which of the following compound statement is a tautology ? **(18-03-2021/Shift-2)**

(a) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(b) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(c) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

(d) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

61. The negation of the statement $\sim p \wedge (p \vee q)$ **(24-02-2021/Shift-2)**

(a) $\sim p \wedge \sim q$ (b) $\sim p \vee q$

(c) $\sim p \wedge q$ (d) $p \vee \sim q$

62. TFor the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct ? **(24-02-2021/Shift-2)**

(a) (a) and (b) both are tautologies.

(b) (b) is a tautologies but not (a).

(c) (a) is a tautology but not (b).

(d) (a) and (b) both are not tautologies.

63. The statement among the following that is a tautology is **(24-02-2021/Shift-2)**

(a) $B \rightarrow [A \wedge (A \rightarrow B)]$ (b) $A \vee (A \wedge B)$

(c) $[A \wedge (A \rightarrow B)] \rightarrow B$ (d) $A \wedge (A \vee B)$

64. The contrapositive of the statement " If you will work, you will earn money" is: **(25-02-2021/Shift-2)**

(a) To earn money, you need to work

(b) If you will not earn money, you will not work

(c) You will earn money, if you will not work

(d) If you will earn money, you will work

65. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to: **(25-02-2021/Shift-2)**

(a) $A \rightarrow (A \vee B)$ (b) $A \rightarrow (A \wedge B)$

(c) $A \rightarrow (A \leftrightarrow B)$ (d) $A \rightarrow (A \rightarrow B)$

Find Answer Key and Detailed Solutions at the end of this book

Answer Key

CHAPTER - 5 | SETS, RELATIONS & FUNCTION

EXERCISE - 1 :
BASIC OBJECTIVE QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (d)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)
6. (a)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (d)
11. (b)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (a)
16. (a)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)
21. (c)	**22.** (d)	**23.** (a)	**24.** (a)	**25.** (d)
26. (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (c)
31. (a)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (c)
36. (a)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (c)
41. (d)	**42.** (d)	**43.** (b)	**44.** (d)	**45.** (c)
46. (c)	**47.** (d)	**48.** (b)	**49.** (c)	**50.** (d)
51. (d)	**52.** (c)	**53.** (16)	**54.** (16)	**55.** (60)
56. (2)	**57.** (42)	**58.** (60)	**59.** (600)	**60.** (41)
61. (6)	**62.** (1)	**63.** (3)	**64.** (1)	**65.** (0)

EXERCISE - 2 :
PREVIOUS YEAR JEE MAIN QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (d)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (d)
6. (d)	**7.** (38)	**8.** (a)	**9.** (a)	**10.** (b)
11. (a)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (d)
16. (28)	**17.** (29)	**18.** (d)	**19.** (b)	**20.** (5)
21. (c)	**22.** (b)	**23.** (832)	**24.** (b)	**25.** (256)

CHAPTER - 5 | SETS, RELATIONS & FUNCTION

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (c)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (d)
6. (d)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (c)
11. (d)	**12.** (c)	**13.** (a)	**14.** (d)	**15.** (b)
16. (a)	**17.** (d)	**18.** (d)	**19.** (d)	**20.** (d)
21. (a)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (c)
26. (d)	**27.** (b)	**28.** (b,c)	**29.** (a,b,d)	**30.** (b,c)
31. (a,b,c,d)		**32.** (110)	**33.** (6)	**34.** (4)
35. (a)	**36.** (b)	**37.** (b)	**38.** (d)	**39.** (b)
40. (c)	**41.** (d)	**42.** (d)	**43.** (x=8)	
44. (x = 2 or 81)		**45.** (x = 3 or −3)		

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (b)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (a)

Answer Key

CHAPTER -6 | LIMITS AND DERIVATIVES

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

DIRECTION TO USE - Scan the QR code and check detailed solutions.

1. (c)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (b)
6. (b)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (a)
11. (a)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (b)
16. (b)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (b)
21. (c)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (a)
26. (a)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (d)
31. (b)	**32.** (b)	**33.** (a)	**34.** (d)	**35.** (d)
36. (d)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (b)
41. (c)	**42.** (c)	**43.** (b)	**44.** (a)	**45.** (a)
46. (b)	**47.** (b)	**48.** (3)	**49.** (2)	**50.** (3)
51. (0)	**52.** (−0.5)	**53.** (0.5)	**54.** (1)	**55.** (1)
56. (18)	**57.** (0)	**58.** (0)	**59.** (1)	**60.** (2)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

DIRECTION TO USE - Scan the QR code and check detailed solutions.

1. (a)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (b)
6. (a)	**7.** (b)	**8.** (b)	**9.** (d)	**10.** (c)
11. (a)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (b)
16. (a)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (a)
21. (d)	**22.** (d)	**23.** (b)	**24.** (40)	**25.** (b)
26. (b)	**27.** (8)	**28.** (c)	**29.** (c)	**30.** (b)
31. (b)	**32.** (36)	**33.** (b)	**34.** (a)	**35.** (1)
36. (a)	**37.** (5)	**38.** (c)	**39.** (a)	**40.** (4)
41. (c)	**42.** (a)	**43.** (b)	**44.** (c)	**45.** (3)
46. (3)	**47.** (3)	**48.** (b)	**49.** (d)	**50.** (b)

CHAPTER -6 | LIMITS AND DERIVATIVES

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (c)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (c)
6. (d)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (b)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (d)
16. (d)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (c)
21. (b)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (b)
26. (a,d)	**27.** (a,d)	**28.** (a,c)	**29.** (c,d)	**30.** (a,b)
31. (a,c)	**32.** (1)	**33.** (a)	**34.** (a)	**35.** (b)
36. (a)	**37.** (d)	**38.** (b)	**39.** (a)	**40.** (a)

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (c)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (b)
6. (b)	**7.** (d)	**8.** (a,c)	**9.** (b,c)	**10.** (a,c)
11. (0)	**12.** (2)	**13.** (7)	**14.** (1)	**15.** (8)

Answer Key

CHAPTER -7 | TRIGONOMETRY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

DIRECTION TO USE - Scan the QR code and check detailed solutions.

1. (b)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (b)
6. (d)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (c)
11. (b)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (a)
16. (b)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (d)
21. (a)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (c)
26. (b)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (c)
31. (d)	**32.** (a)	**33.** (c)	**34.** (a)	**35.** (d)
36. (c)	**37.** (c)	**38.** (d)	**39.** (b)	**40.** (b)
41. (a)	**42.** (a)	**43.** (b)	**44.** (b)	**45.** (b)
46. (c)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (a)
51. (a)	**52.** (c)	**53.** (b)	**54.** (c)	**55.** (b)
56. (c)	**57.** (c)	**58.** (a)	**59.** (25)	**60.** (8)
61. (4)	**62.** (0)	**63.** (1)	**64.** (0)	**65.** (0.5)
66. (1)	**67.** (1)	**68.** (2)	**69.** (1)	**70.** (4)
71. (4)	**72.** (4)	**73.** (1)	**74.** (1)	**75.** (0)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

DIRECTION TO USE - Scan the QR code and check detailed solutions.

1. (c)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (d)
6. (c)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (c)
11. (b)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (a)
16. (b)	**17.** (c)	**18.** (a)	**19.** (a)	**20.** (20)
21. (c)	**22.** (5)	**23.** (a)	**24.** (d)	**25.** (b)
26. (d)	**27.** (c)	**28.** (a)	**29.** (a)	**30.** (a)
31. (a)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (d)
36. (c)	**37.** (a)	**38.** 80	**39.** (d)	**40.** (1)
41. (b)	**42.** (8)	**43.** (a)	**44.** (b)	**45.** (b)
46. (d)	**47.** (c)	**48.** (b)	**49.** (d)	**50.** (a)
51. (a)	**52.** (b)	**53.** (c)	**54.** (a)	**55.** (b)
56. (56)	**57.** (d)	**58.** (a)	**59.** (15)	**60.** (d)
61. (144)	**62.** (b)	**63.** (b)	**64.** (c)	**65.** (c)
66. (1)	**67.** (c)	**68.** (a)	**69.** (b)	**70.** (a)
71. (d)	**72.** (b)	**73.** (11)	**74.** (1)	**75.** (a)

CHAPTER -7 | TRIGONOMETRY

EXERCISE - 3 :
ADVANCED OBJECTIVE QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (d)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (b)
6. (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (b)
11. (b)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (b)
16. (a)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (a)
21. (d)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (d)
26. (a)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (c)
31. (b)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (c)
36. (a,c,d)	**37.** (a,b)	**38.** (a,b,c,d)		
39. (a,b,c)	**40.** (a,d)	**41.** (a,b)	**42.** (a,b,c,d)	
43. (c,d)	**44.** (a,c)	**45.** (a,b)	**46.** (a,c,d)	
47. (a,b,c,d)		**48.** (a,b,c,d)		
49. (a,b,c)	**50.** (a,b,c,d)		**51.** (b)	**52.** (d)
53. (a)	**54.** (a)	**55.** (c)	**56.** (b)	**57.** (c)
58. (b)	**59.** (c)	**60.** (d)		

EXERCISE - 4 :
PREVIOUS YEAR JEE ADVANCED QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (a)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (b)
6. (d)	**7.** (c)	**8.** (c)	**9.** (a,b)	**10.** (c,d)
11. (a,c,d)	**12.** (b,d)	**13.** (a,c,d)	**14.** (a,c)	**15.** (a,c,d)
16. (a,b,c)	**17.** (b,c)	**18.** (a,b)	**19.** (3)	**20.** (8)
21. (1)	**22.** (2)	**23.** (d)	**24.** (c)	

Answer Key

CHAPTER - 8 | STATISTICS

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (c)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (d)
6. (b)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (c)
11. (b)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (c)
16. (a)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (b)
21. (a)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (c)
26. (b)	**27.** (c)	**28.** (d)	**29.** (d)	**30.** (3.75)
31. (2.57)	**32.** (12.67)	**33.** (8)	**34.** (3.87)	**35.** (8.25)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (c)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (b)
6. (c)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (c)
11. (48)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (d)
16. (a)	**17.** (2)	**18.** (20)	**19.** (b)	**20.** (b)
21. (d)	**22.** (2)	**23.** (31)	**24.** (a)	**25.** (c)
26. (3)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (4)
31. (d)	**32.** (b)	**33.** (d)	**34.** (6)	**35.** (18)
36. (54)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (a)
41. (164)	**42.** (c)	**43.** (c)	**44.** (4)	**45.** (c)
46. (d)	**47.** (781)	**48.** (12)	**49.** (13)	**50.** (c)
51. (25)	**52.** (c)	**53.** (5)	**54.** (a)	**55.** (68)

Answer Key

CHAPTER -9 | MATHEMATICAL REASONING

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (c)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (c)
6. (b)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (b)
11. (a)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (b)
16. (b)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (b)
21. (a)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (b)
26. (c)	**27.** (c)	**28.** (d)	**29.** (a)	**30.** (c)
31. (b)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (c)
36. (a)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (b)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

DIRECTION TO USE -
Scan the QR code and check detailed solutions.

1. (d)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)
6. (d)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (d)
11. (b)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (b)
16. (d)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (d)
21. (b)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (a)
26. (c)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (c)
31. (d)	**32.** (a)	**33.** (b)	**34.** (d)	**35.** (d)
36. (d)	**37.** (a)	**38.** (d)	**39.** (a)	**40.** (d)
41. (d)	**42.** (d)	**43.** (b)	**44.** (a)	**45.** (d)
46. (b)	**47.** (c)	**48.** (a)	**49.** (d)	**50.** (d)
51. (c)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (b)
56. (c)	**57.** (b)	**58.** (c)	**59.** (c)	**60.** (b)
61. (d)	**62.** (a)	**63.** (c)	**64.** (b)	**65.** (a)

MASTER INDEX

Creating Impact at Scale

65 Million+
Monthly
Youtube Views

49 Million+
Monthly
Web + App Visitors

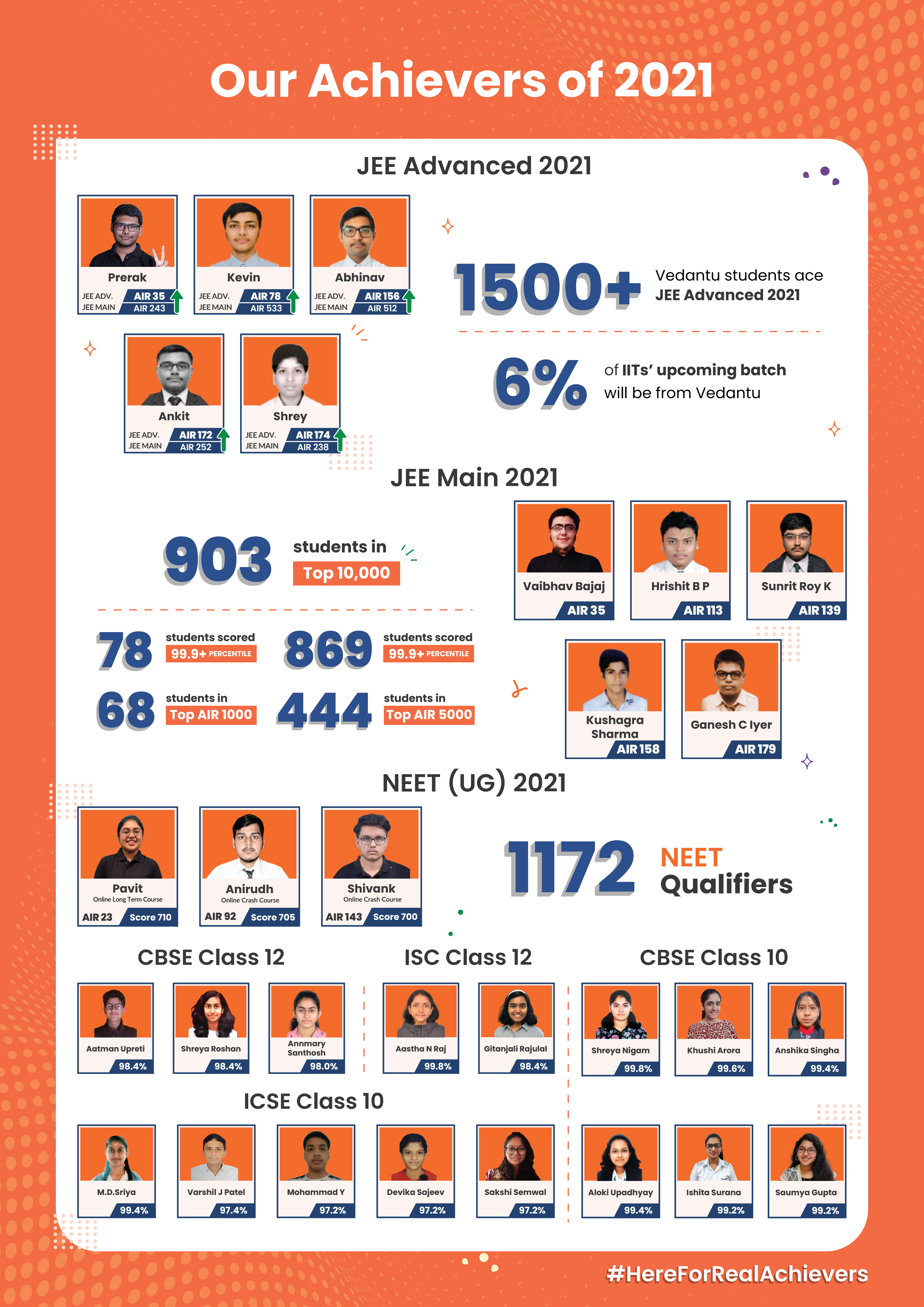
40+ Countries
Where Students
Take Live Classes

25 Million+
Monthly Active
Users

26 Million+
Hours of
Live Learning

25 MILLION+
Parents Trust Vedantu

43 MILLION+
Happy Students



Our Achievers of 2021
JEE Advanced 2021
Prerak
JEE ADV. AIR 35
JEE MAIN AIR 243
Kevin
JEE ADV. AIR 78
JEE MAIN AIR 533
Abhinav
JEE ADV. AIR 156
JEE MAIN AIR 512
Ankit
JEE ADV. AIR 172
JEE MAIN AIR 252
Shrey
JEE ADV. AIR 174
JEE MAIN AIR 238
1500+
Vedantu students ace
JEE Advanced 2021
6%
of IITs' upcoming batch
will be from Vedantu
JEE Main 2021
903
students in
Top 10,000
78
students scored
99.9+ PERCENTILE
869
students scored
99.9+ PERCENTILE
68
students in
Top AIR 1000
444
students in
Top AIR 5000
Vaibhav Bajaj
AIR 35
Hrishit B P
AIR 113
Sunrit Roy K
AIR 139
Kushagra Sharma
AIR 158
Ganesh C Iyer
AIR 179
NEET (UG) 2021
Pavit
Online Long Term Course
AIR 23
Score 710
Anirudh
Online Crash Course
AIR 92
Score 705
Shivank
Online Crash Course
AIR 143
Score 700
1172
NEET
Qualifiers
CBSE Class 12
Aatman Upreti
98.4%
Shreya Roshan
98.4%
Annmary Santhosh
98.0%
ISC Class 12
Aastha N Raj
99.8%
Gitanjali Rajulal
98.4%
CBSE Class 10
Shreya Nigam
99.8%
Khushi Arora
99.6%
Anshika Singha
99.4%
ICSE Class 10
M.D.Sriya
99.4%
Varshil J Patel
97.4%
Mohammad Y
97.2%
Devika Sajeev
97.2%
Sakshi Semwal
97.2%
Aloki Upadhyay
99.4%
Ishita Surana
99.2%
Saumya Gupta
99.2%
#HereForRealAchievers