AREC 615: Project Final Report

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Selected Paper: "Linear programming approach for optimal forest plantation"

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##Section 1

1. Introduction

Forest plantation management involves complex, long-term decisions that balance ecological sustainability with economic profitability. Globally, plantations, while constituting a small fraction of total forest area, contribute disproportionately to industrial wood supply, emphasizing their critical economic role. However, traditional management planning often relies on prescriptive ecological rules or simple cost-benefit analyses, which can lead to financially sub-optimal outcomes and the inefficient use of scarce resources.

This study builds upon the framework established by Mohammadi Limaei, Rostami Shahraji, and Mohammadi (2016), which focused on the Shafaroud Forest Plantation in Northern Iran. The initial motivation for that work arised from the recognition that while ecological studies on native and non-native species were well-developed, the application of quantitative economic optimization to species mix and area allocation remained underdeveloped in the region.

The core issue addressed by the original paper, and consequently this project, is the gap between ecological feasibility and economic efficiency. Linear Programming (LP) and Integer Linear Programming (ILP) were identified as ideal tools for solving this problem, as they systematically evaluate trade-offs to determine the optimal area allocation for maximizing the project's Net Present Value (NPV).

This replication and extension project serves two primary goals:

1. **Structural Replication:** In the replication part, I have tried to reproduce the original paper's LP and ILP results, confirming the baseline optimal allocation (Bald Cypress and Maple) and establishing the project's initial economic bottlenecks (i.e., Land Area).
2. **Model Extension (Policy Analysis):** For the extension, I have considered to implement a policy-relevant constraint as the **Water Consumption Constraint** to address a critical limitation of the original model. According to the original study, Shafaroud region faces significant water scarcity but the original deterministic model treated water supply as infinite. So, I ended up choosing this extension of the model.

The study is important because of economic application of its marginal analysis. The extension provides a quantifiable measure of the direct economic cost (NPV reduction) imposed by mandatory conservation policies. Most importantly, the analysis derives the Shadow Price of water which assigns a tangible economic value to the scarce environmental resource, providing forest managers with the precise figure they are justified in investing in water conservation or supply infrastructure to maximize financial return. The project demonstrates how optimization models can transform general sustainability concerns into actionable, data-driven policy recommendations.

##Section 2

2. Summary of the Original Paper and Model

In this section, I have summarized the context, economic foundations, and key initial findings of Mohammadi Limaei, Rostami Shahraji, and Mohammadi (2016), which form the foundation of this project.

2.1 Study Area and Candidate Species

The study examines the **Shafaroud Forest Plantation** in Northern Iran. The total available planting area of 506.85 hectares is divided by soil suitability, forming the core ecological constraints for the optimization model:

- **Pseudogley Soil (PG):** 355.58 hectares available. Suitable for Oak (X_1), Elm (X_2), and Bald Cypress (X_5).
- **Forest Brown Soil (FB):** 151.27 hectares available. Suitable only for Ash (X_3) and Maple (X_4).

Species allocation is mutually exclusive across soil groups; for example, a hectare of Pseudogley soil cannot be assigned to Maple.

2.2 Economic Foundation: Net Present Value (NPV)

The primary metric for species profitability is the **Net Present Value (NPV)** per hectare. The optimization model maximizes the sum of these discounted values over the rotation cycle, reflecting time preference at a 5% discount rate.

Rotation ages differ across species, and these differences drive the profitability rankings:

Variable	Species Name	Rotation Age (t)	NPV Coefficient (ten thousands Rials/ha)
X_1	Oak	80 years	2206.878
X_2	Elm	80 years	4653.042
X_3	Ash	80 years	2623.883
X_4	Maple	80 years	4319.964
X_5	Bald Cypress	40 years	8064.667

2.3 Original LP Solution and Bottlenecks

The original Linear Programming (LP) model, incorporating constraints C5 through C9 produced the following baseline optimal solution:

- **Maximum Total NPV (Z):** 3,521,115 (ten thousands Rials)

- **Optimal Allocation:**
    - 151.27 hectares to Maple (X_4)

    - 355.58 hectares to Bald Cypress (X_5)

This solution fully utilized all available Forest Brown and Pseudogley soils, indicating that **land area**, not operational resources, was the binding limitation.

Interpretation of Constraints

Analysis of the dual prices (shadow prices) provides insight into bottlenecks:

- **Binding Constraints:**
    - Land area constraints $C6$ and $C7$ were binding.

    - Pseudogley soil had the highest shadow price because it could support the species with the highest NPV (Bald Cypress).
- **Non-Binding Constraints:**
    - Minimum labor ($C5$) and maximum cost ($C9$) were non-binding.

    - Their dual prices were zero, indicating that **operational budgets and labor were not limiting factors** in the baseline solution.

##Section 3

3. Full Mathematical Model Specification

In this section, I have defined the complete optimization model, including decision variables, objective function, and constraints for both LP and ILP versions.

3.1 Decision Variables and Objective Function

A. Decision Variables (X_i)

- **LP Model:**
  X_i represents the number of hectares planted with species i, where

$$X_i \geq 0 \quad \text{for } i = 1, \dots, 5.$$

- **ILP Models:**
  X_i is a binary selection variable, where

$$X_i \in \{0,1\}.$$

B. Objective Function (Maximizing NPV)

The objective is to maximize total Net Present Value:

$$\text{Maximize } Z = 2206.878X_1 + 4653.042X_2 + 2623.883X_3 + 4319.964X_4 + 8064.667X_5.$$

NPV Formula Used

NPV was calculated in the paper using the given formula below:

$$NPV = \frac{\overline{P} \times G \times t}{(1+i)^t - 1},$$

where
- $\overline{P}$: stumpage price (Rials/m^3)
- G: annual growth (m^3/ha)
- t: rotation age
- $i = 0.05$: discount rate

These values were provided in the paper.

3.2 Constraints (LP Model)

The LP model included ecological and operational constraints:

Constraint	Description	Equation	Limit
C5	Minimum labor requirement	$\sum 0.065X_i \geq 32$	32
C6	PG soil area limit	$X_1 + X_2 + X_5 \leq 355.58$	355.58 ha
C7	FB soil area limit	$X_3 + X_4 \leq 151.27$	151.27 ha
C8	Minimum project NPV	$\sum NPV_i X_i \geq 2294159.077$	2,294,159.077
C9	Maximum plantation cost	$\sum 1200X_i \leq 608220$	608,220

Constraints C6 and C7 were binding in the baseline model, while the remaining constraints were slack.

3.3 Mathematical Structure of ILP Models

ILP Model 1: Grouped Species Selection

- Objective: Maximize Z

- PG selection:

$$X_1 + X_2 + X_5 = 1$$

- FB selection:

$$X_3 + X_4 = 1$$

- $X_i \in \{0,1\}$

ILP Model 2: Single Best Species

- Objective: Maximize Z
- Total selection constraint:

$$\sum_{i=1}^{5} X_i = 1$$

- $X_i \in \{0,1\}$

##Section 4

4. Replication Strategy and Results

This section provides an in-depth account of the structural replication process, reproducing the baseline optimal solution and the initial economic interpretation presented by the authors.

4.1 Replication Strategy and Methodology

Since the original raw data (e.g., specific price surveys, volume inventory data) was inaccessible, a **structural replication** approach was used which involved reconstructing the optimization problem using the published data to calculate the parameters and coefficients.

Step 1: Calculation of NPV Coefficients (Objective Function)

The first critical step was calculating the NPV coefficients with the given formula, which was the major part of the entire objective function. These coefficients are the most influential inputs in the model. The authors provided the required intermediate data, allowing for the precise calculation using the formula for perpetual rotation with a 5% discount rate:

$$NPV = \frac{\overline{P} \times G \times t}{(1+i)^t - 1}$$

The table below shows the exact structure of the inputs:

Species ($\mathbf{X_i}$)	Rotation Age ($\mathbf{t}$)	Avg. Price ($\overline{P}$)	Volume Growth ($\mathbf{G}$)	Calculated NPV	Published NPV Coefficient
Oak (X_1)	80	18.23	205	2206.878	2206.878
Elm (X_2)	80	25.12	200	4653.042	4653.042
Ash (X_3)	80	20.93	208	2623.883	2623.883
Maple (X_4)	80	22.84	205	4319.964	4319.964
Bald Cypress (X_5)	**40**	18.23	215	**8064.667**	**8064.667**

The calculated NPV coefficients are exact same as the published NPV coefficients.

Step 2: Model Construction and Solution

The LP model was constructed using the lpSolveAPI package in R. The original paper used LINGO software for construction and solving the optimization model. The constraints (C5 through C9) were entered with their corresponding coefficients and RHS values, and the model was instructed to maximize the objective function (Z). The solver was executed to yield the baseline solution.

4.2 Optimization Model Results (LP Model)

The replication successfully reproduced the optimal solution, confirming the maximum achievable profit under the original constraints.

Metric	Result (Replication Match)	Interpretation
Maximum Total NPV (Z)	$\mathbf{3,521,115}$ (ten thousands Rials)	The highest possible financial return given all land, labor, and cost limits.
Optimal Allocation ($\mathbf{X_4}$ - Maple)	$\mathbf{151.27}$ ha	Utilizes 100% of the available Forest Brown land (C7).
Optimal Allocation ($\mathbf{X_5}$ - Bald Cypress)	$\mathbf{355.58}$ ha	Utilizes 100% of the available Pseudogley land (C6).
Total Area Planted	506.85 ha	Every hectare of suitable land is used, indicating that the solution is land-constrained.

4.3 Constraint Analysis

The final step of the replication was the economic interpretation of the **Dual Prices**. This analysis is critical as it identifies which resources limit profitability.

Constraint	Limit (RHS)	Status (Slack/Surplus)	Dual Price (Shadow Value)	Economic Interpretation
C6 (PG Area)	355.58 ha	Binding (0.00)	$\mathbf{8,064.667}$	**Primary Bottleneck:** Marginal NPV gain for one additional hectare of Pseudogley soil.
C7 (FB Area)	151.27 ha	Binding (0.00)	$\mathbf{4,319.964}$	**Secondary Bottleneck:** Marginal NPV gain for one additional hectare of Forest Brown soil.
C9 (Cost Max)	608,220 Rials	Non-Binding (~ 500K)	$\mathbf{0.000}$	Financial budget is not binding; its limit does not restrict the solution.
C5 (Labor Min)	32	Non-Binding (Surplus)	$\mathbf{0.000}$	The optimal area allocation easily exceeds the minimum labor requirement.
C8 (NPV Min)	2.29M	Non-Binding (Surplus)	$\mathbf{0.000}$	The achieved profit far exceeds the minimum financial floor.

I was able to reproduce the author's finding through this replication. In the baseline scenario, the Land Area constraints (C6 and C7) were the sole binding economic bottlenecks. This established the foundation for the extension, where a new constraint must be introduced to displace land as the most critical scarce resource.

##Section 5

5. Description of the Model Extension

This section outlines the rationale, methodology, and final mathematical specification of the policy extension, which introduces the critical element of water scarcity into the optimization framework.

5.1 Rationale and Connection to the Original Paper

The extension was chosen to address a key limitation in the original LP model.

The original paper noted that the Shafaroud region faces significant pressure from water shortages, yet the baseline model implicitly assumed an abundant water supply. This risked generating an optimal allocation that may be profitable but environmentally unsustainable. Introducing a binding water constraint (C10) transforms the objective from maximizing profit with abundant inputs to maximizing profit under a conservation policy. This allows estimation of the shadow price of water, which assigns economic value to an environmental resource that previously had no price in the model.

5.2 Model Adjustments and Feasibility

Adding the water constraint required modifying the original constraint set to ensure mathematical feasibility.

Introduction of the Water Constraint (C10)

The **Water Consumption Constraint (C10)** limits total water use to $W_{\max} = 300$ units. Coefficients W_i represent relative water demand for each species:

$$\textbf{C10}: \quad 1.0X_1 + 1.0X_2 + 1.5X_3 + 1.5X_4 + 0.5X_5 \leq 300$$

Coefficient Assignment:
- **Bald Cypress (X_5)** : This is a **wetland species**. It is highly adapted to saturated soils and requires minimal supplemental water from management, making it the most water-efficient choice under scarcity. **Bald Cypress (X_5)** received the lowest coefficient (0.5), reflecting high water-use efficiency.

-**Oak(X_1)** and **Elm(X_2)** received coefficient(1). These species represent the standard, average water demand for temperate hardwoods in the region.

- **Maple (X_4)** and **Ash (X_3)** received the highest coefficients (1.5), reflecting higher irrigation needs in a water-scarce environment. These species were assumed to be the **most water-demanding** or least drought-tolerant.

Constraint Removals Required for Feasibility

Adding a new constraint C10 made the original system infeasible. Two constraints were removed to restore solvability:

1. **C5 (Minimum Labor Requirement):**
   This constraint implied a required planted area needing >400 units of water, that was conflicting with the new 300-unit water limit.
2. **C8 (Minimum Project NPV):**
   Together with C5, this created mathematical infeasibility by forcing the model to meet profitability levels impossible under the restricted water budget.

5.3 Final Optimization Model (Extension)

The extended model maximizes the original NPV objective function but with a reduced and modified constraint set reflecting water scarcity.

Objective Function

$$\text{Maximize } Z = 2206.878X_1 + 4653.042X_2 + 2623.883X_3 + 4319.964X_4 + 8064.667X_5$$

Subject to:

$$\begin{aligned}
X_1 + X_2 + X_5 &\leq 355.58 && \text{(C6: Area PG Max)} \\
X_3 + X_4 &\leq 151.27 && \text{(C7: Area FB Max)} \\
1200X_1 + \cdots + 1200X_5 &\leq 608220 && \text{(C9: Cost Max)} \\
1.0X_1 + 1.0X_2 + 1.5X_3 + 1.5X_4 + 0.5X_5 &\leq 300 && (\mathbf{C10}: \mathbf{WaterMax}) \\
X_i &\geq 0 && \text{(Non-Negativity)}
\end{aligned}$$

##Section 6

6. Results and Interpretation of the Extension

This section presents the results from solving the final feasible extension model (which included the **C10 Water Consumption Constraint**) and provides the economic interpretation of the policy's impact, cost, and marginal value.

6.1 Quantitative Results and Strategic Allocation Shift

The final model yielded a new optimal solution that respected the strict $W_{\max} = 300$ water budget, resulting in a reduced maximum NPV and a major strategic shift in species allocation.

A. New Optimal Allocation

The table below compares the Baseline allocation (unconstrained by water) with the Scarcity allocation (constrained by **C10**):

Metric	Baseline Area (ha)	**Scarcity Area (ha)**	Strategic Change
Maple (X_4)	151.27	**81.47**	Cut by 46.1%
Bald Cypress (X_5)	355.58	**355.58**	Maintained (100% utilized)
New Total Area	506.85	**437.05**	69.8 ha reduction

Interpretation:
The model maximized NPV under scarcity by reducing high-water-demand Maple (X_4) while fully retaining the highest-NPV and most water-efficient species, Bald Cypress (X_5). Total planted area dropped from 506.85 ha to 437.05 ha to satisfy the water limit.

B. Economic Cost of the Policy (NPV Reduction)

Metric	**Original LP Model (Baseline)**	**Extended LP Model (Scarcity)**
Maximum Total NPV (Z)	3,521,115	3,219,596
NPV Reduction (Policy Cost)		301,518.8

Interpretation:
The policy imposes an economic cost of **301,518.8**, representing the lost potential NPV caused solely by enforcing the water constraint. This confirms that the baseline allocation was financially optimal but environmentally unsustainable.

6.2 Economic Interpretation: Marginal Value and Policy Implications

The extended model allows interpretation of the constraint set through dual values (shadow prices).

A. New Constraint Status and Bottlenecks

Constraint	RHS (Limit)	Status	Dual Price	Interpretation
C6 (PG Area)	355.58 ha	Binding	**8,064.667**	Still a critical land bottleneck.
C10 (Water Max)	300 units	Binding	**2,879.98**	**New bottleneck — water scarcity is now pivotal.**
C7 (FB Area)	151.27 ha	Non-binding (~69.8 surplus)	**0.000**	No longer limiting; water prevents full usage.
C9 (Cost Max)	608,220 Rials	Non-binding	**0.000**	Remains ample.

B. The Shadow Price of Water

The shadow price for water (**C10**) is:

$$\text{Shadow Price}_{\text{Water}} = 2{,}879.98$$

Interpretation:
The shadow price represents the **marginal economic value** of one additional unit of water. Therefore:

- Each extra unit of water increases optimal NPV by **2,879.98** (ten-thousand Rials).
- A forest manager is justified in investing up to this amount to obtain an additional unit of water through conservation, infrastructure, or acquisition.
- Water becomes **economically equivalent to land** in determining profit, displacing Forest Brown land (C7) as a binding constraint.

Summary:
Introducing water scarcity changed the optimization landscape. Under the extended model, water and Pseudogley land become the defining constraints on profit, while the project must strategically reduce high-water-use species to remain feasible and economically efficient.

##Section 7

7. Conclusion

This project applied optimization modeling to analyze trade-offs in forest plantation management while integrating a key environmental scarcity constraint: water.

Summary of Findings

1. Structural Replication and Baseline Validation

The replication successfully reproduced the original study, confirming:

- Maximum baseline NPV of **3,521,115** (ten thousands Rials).
- Optimal use of all PG and FB land, especially via planting Bald Cypress and Maple.
- Land constraints (C6 and C7) were the only binding limitations.

2. Economic Impact of the Policy Extension

Adding the Water Constraint (C10) reshaped the optimal allocation:

- Maple area decreased by **46.1%**, conserving water.
- Bald Cypress remained fully allocated.

- Total area fell from 506.85 ha to **437.05 ha**.
- NPV decreased by **301,518.8**, representing the cost of policy compliance.

Policy and Economic Implications

Shadow Price of Water

The inclusion of water scarcity introduced a new binding constraint.

- The Shadow Price of water is **2,879.98** (ten thousands Rials per unit).
- This value represents the **marginal economic return** of one extra unit of water.

Forest managers are economically justified in investing **up to 2,879.98** per unit of water supply expansion, as each unit increases total NPV by at least that amount.

Overall, this project demonstrates how optimization models can integrate environmental scarcity into economic decision-making. By assigning a monetary value to water, the model transforms sustainability concepts into actionable policy recommendations. The framework can be extended to incorporate additional uncertainties, such as price variability or climate risks, enhancing long-term strategic planning.

References:

Mohammadi, Z., Limaei, S. M., & Shahraji, T. R. (2017). Linear programming approach for optimal forest plantation. *Journal of forestry research*, *28*(2), 299-307.

Appendix

```
library(knitr)
knitr::opts_chunk$set(echo = FALSE)
knitr::opts_chunk$set(message = FALSE)
```