Mock exam on Mathematics for Economists, April 2nd, 2012.
Time allowed 120 minutes.
Students should answer all of the following eight questions. Calculators are not permitted in the exam. Marks will be deducted for insufficient explanations within your answers.
Section A: 10 points each question.

1. Determine whether the function $f(x, y) = \ln(5x + y) - 5(x + y)^2$ is convex (concave up), concave (concave down), strictly convex, strictly concave or neither.

2. Solve the differential equation $y^{(4)} - y = \cos(2x)$. The $y^{(4)}$ denotes the forth derivative of y.

3. Consider the monopolist producing two distinct goods. The cost function is given by $TC(q_1, q_2) = q_1 + kq_2$, where the constant $k \in (0; 1)$. And the demand functions are given by $q_1(p_1, p_2) = q_2(p_1, p_2) = (p_1p_2)^{-3}$.

   (a) Find the optimal production bundle for the monopolist.

   (b) For which values of k one of the product is priced under marginal costs?

4. The density of a standard normal random variable X is given by $f(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$. Taking the fact that $\int_{-\infty}^{\infty} f(x)\,dx = 1$ for granted calculate $E(X^2)$, $E(X^4)$.

   Hint: if you don't remember, $E(X^n) = \int_{-\infty}^{+\infty} x^n f(x)\,dx$

5. The point elasticity of demand for a good is given by $\varepsilon = p^2/(p^2 + 4p + 3)$. Find the demand function $q(p)$ given the initial condition $q(1) = 1$.

6. Find the values a and b such that the function f is homogeneous:

$$f(x, y) = 2x^{b-a}y^{b+2} + y^{a+1}x^{-3b} + y^{7b}x^{-2a}$$

   For the values of a and b you have found expand the function

$$h(x) = \sqrt{1 + f(x, x)} \cdot (1 - \cos(f(x, 2x))$$

   as a power series up to x^4. State the range for x where your expansion is correct.

Section B: 20 points each question.

1. It is known that $x_0 = 0$, $x_{100} = 50k$ where $k \in \mathbb{N}$ is constant and for any $n \in \{2, 3, \ldots 100\}$ the following difference equation is satisfied:

$$x_n - 2x_{n-1} + x_{n-2} = -1$$

   (a) Find the particular solution

   (b) Find the maximum value of x_n for $n \in \{0, \ldots, 100\}$ as a function of k.

2. Using the Lagrange multiplier method find the minimum of the function

$$f(x, y, z) = 2x^2 + 4y^2 + xy + 8z^2 + 2yz$$

   subject to $x + y + 1.5z \geqslant 1.2$ and $x + y + z = 1$.