New SEIR with age compartments

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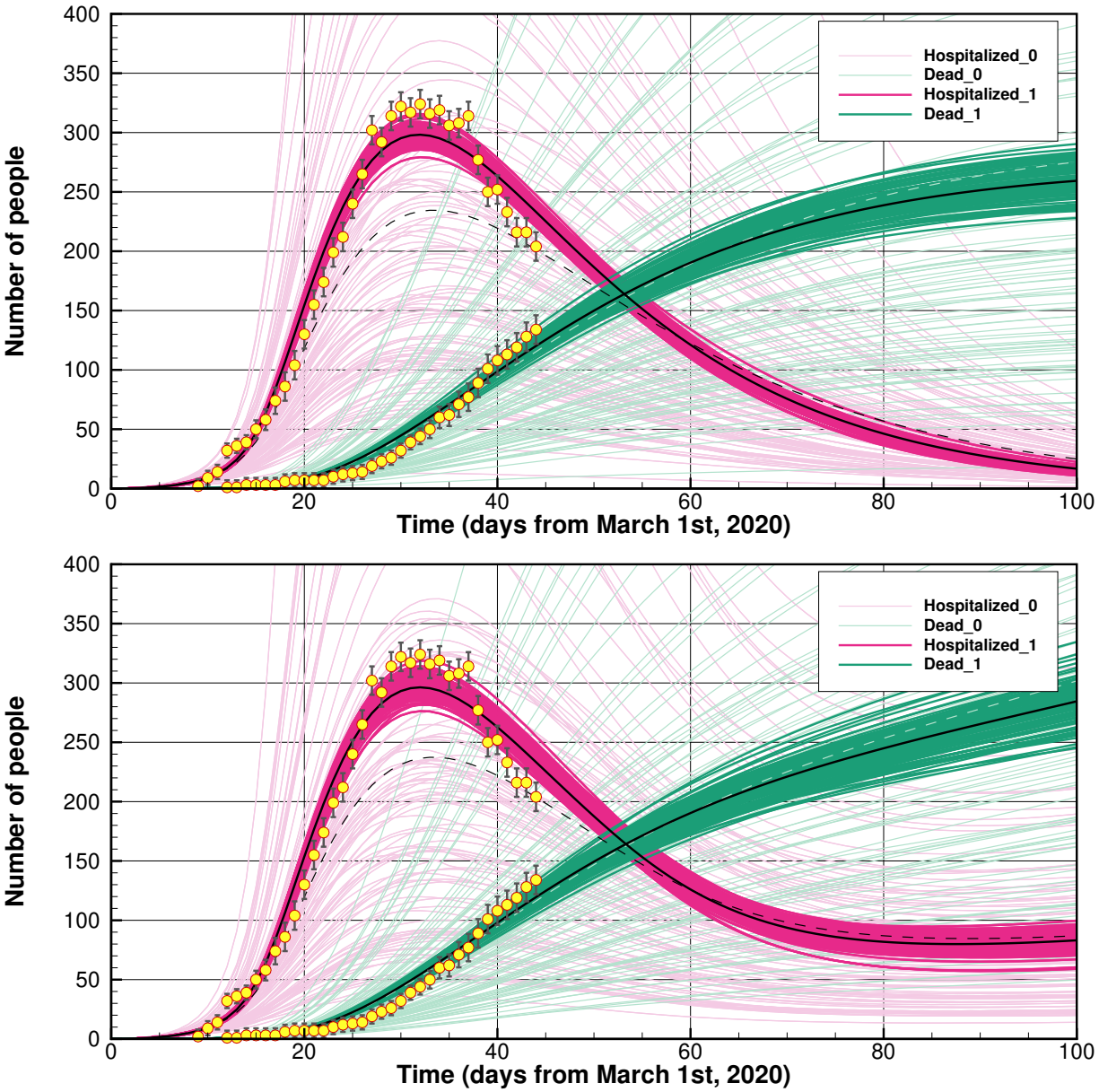
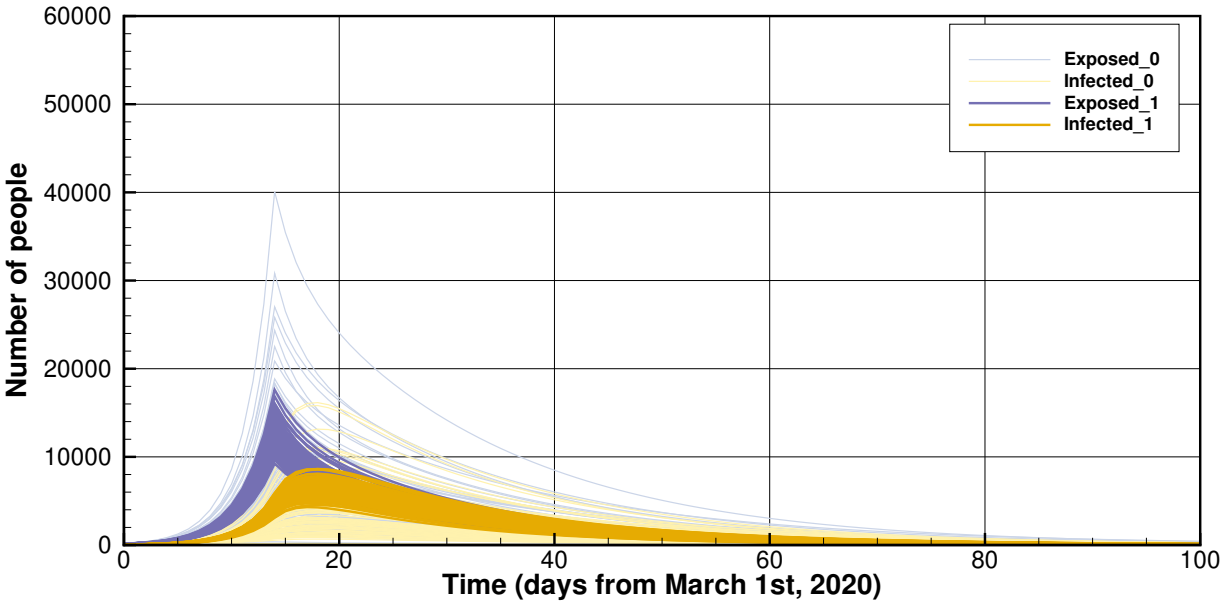
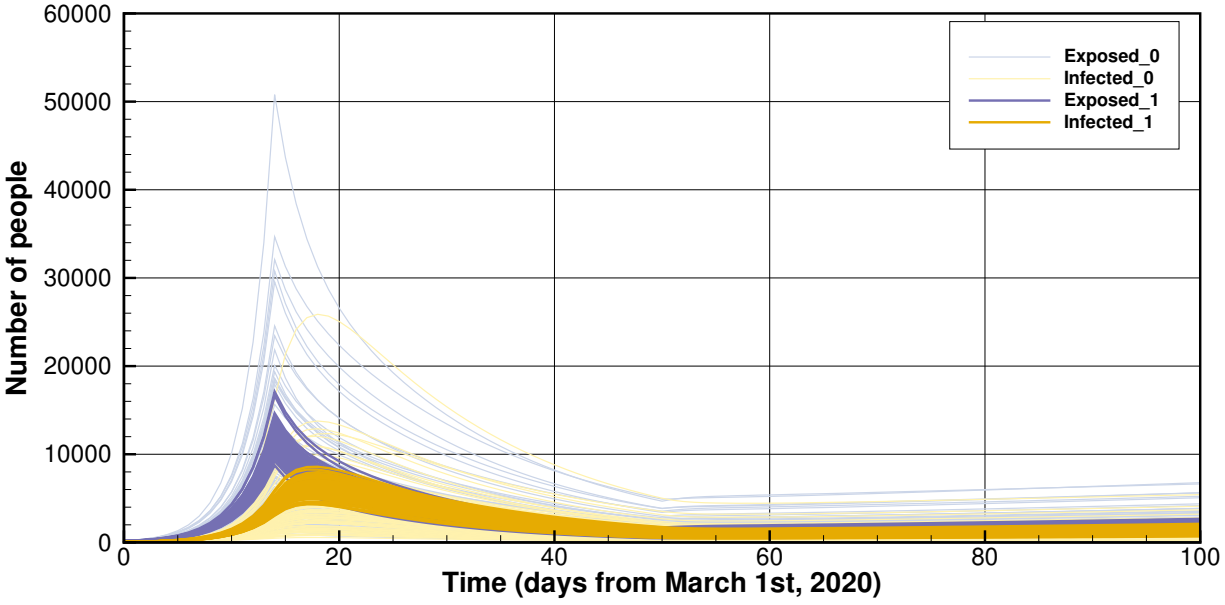
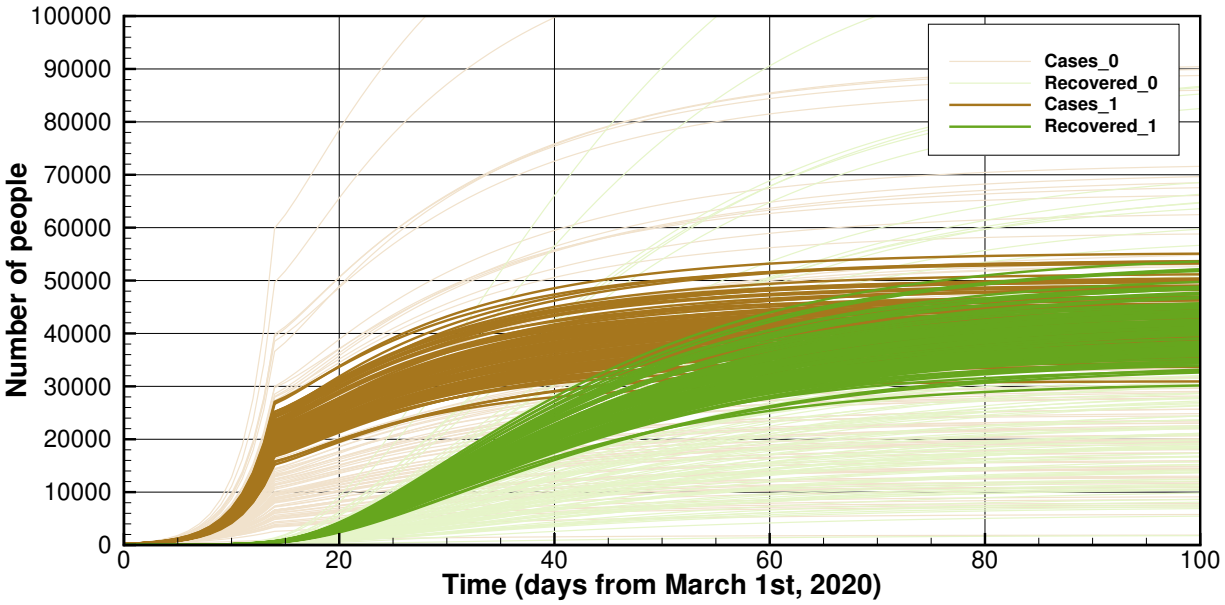
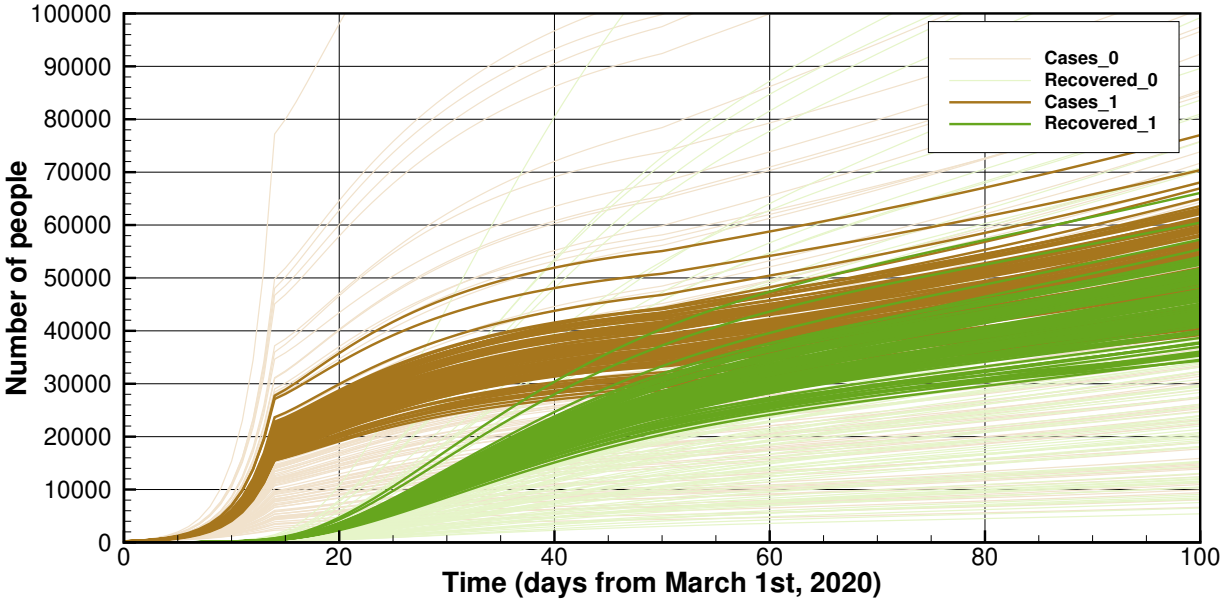



Exposed_0
Infected_0
Exposed_1
Infected_1
Number of people
Time (days from March 1st, 2020)
0
10000
20000
30000
40000
50000
60000
0
20
40
60
80
100


Exposed_0
Infected_0
Exposed_1
Infected_1
Number of people
Time (days from March 1st, 2020)
0
10000
20000
30000
40000
50000
60000
0
20
40
60
80
100


Number of people
Time (days from March 1st, 2020)
Cases_0
Recovered_0
Cases_1
Recovered_1
0
10000
20000
30000
40000
50000
60000
70000
80000
90000
100000
0
20
40
60
80
100


Number of people
Time (days from March 1st, 2020)
Cases_0
Recovered_0
Cases_1
Recovered_1
0
10000
20000
30000
40000
50000
60000
70000
80000
90000
100000
0
20
40
60
80
100

1 SEIR model with agecompartments

$$\begin{cases} \mathbf{S}_1 \rightarrow & \mathbf{E}_1 \rightarrow \mathbf{I}_1 & & \\ & \vdots & & \\ \mathbf{S}_i \rightarrow & \mathbf{E}_i \rightarrow \mathbf{I}_i & \rightarrow \begin{cases} \mathbf{Q}_\mathrm{m} & \rightarrow \mathbf{R}_\mathrm{m} \\ \mathbf{Q}_\mathrm{s} \rightarrow \mathbf{H}_\mathrm{s} & \rightarrow \mathbf{R}_\mathrm{s} \\ \mathbf{Q}_\mathrm{f} \rightarrow \mathbf{H}_\mathrm{f} & \rightarrow \mathbf{D} \end{cases} \\ & \vdots & & \\ \mathbf{S}_n \rightarrow & \mathbf{E}_n \rightarrow \mathbf{I}_n & & \end{cases} \tag{1}$$

The model equations are as follows:

$$\frac{\partial \mathbf{S}_i}{\partial t} = -\frac{1}{\tau_\mathrm{inf}} \left(\sum_{j=1}^{n} R_{ij}(t)\, \mathbf{I}_j \right) \mathbf{S}_i \tag{2}$$

$$\frac{\partial \mathbf{E}_i}{\partial t} = \frac{1}{\tau_\mathrm{inf}} \left(\sum_{j=1}^{n} R_{ij}(t)\, \mathbf{I}_j \right) \mathbf{S}_i - \frac{1}{\tau_\mathrm{inc}} \mathbf{E}_i \tag{3}$$

$$\frac{\partial \mathbf{I}_i}{\partial t} = \frac{1}{\tau_\mathrm{inc}} \mathbf{E}_i - \frac{1}{\tau_\mathrm{inf}} \mathbf{I}_i \tag{4}$$

$$\frac{\partial \mathbf{Q}_\mathrm{m}}{\partial t} = \sum_{i=1}^{n} \frac{p_\mathrm{m}^i}{\tau_\mathrm{inf}} \mathbf{I}_i - (1/\tau_\mathrm{recm})\, \mathbf{Q}_\mathrm{m} \tag{5}$$

$$\frac{\partial \mathbf{Q}_\mathrm{s}}{\partial t} = \sum_{i=1}^{n} \frac{p_\mathrm{s}^i}{\tau_\mathrm{inf}} \mathbf{I}_i - (1/\tau_\mathrm{hosp})\, \mathbf{Q}_\mathrm{s} \tag{6}$$

$$\frac{\partial \mathbf{Q}_\mathrm{f}}{\partial t} = \sum_{i=1}^{n} \frac{p_\mathrm{f}^i}{\tau_\mathrm{inf}} \mathbf{I}_i - (1/\tau_\mathrm{hosp})\, \mathbf{Q}_\mathrm{f} \tag{7}$$

$$\frac{\partial \mathbf{H}_\mathrm{s}}{\partial t} = (1/\tau_\mathrm{hosp})\, \mathbf{Q}_\mathrm{s} - (1/\tau_\mathrm{recs})\, \mathbf{H}_\mathrm{s} \tag{8}$$

$$\frac{\partial \mathbf{H}_\mathrm{f}}{\partial t} = (1/\tau_\mathrm{hosp})\, \mathbf{Q}_\mathrm{f} - (1/\tau_\mathrm{death})\, \mathbf{H}_\mathrm{f} \tag{9}$$

$$\frac{\partial \mathbf{R}_\mathrm{m}}{\partial t} = (1/\tau_\mathrm{recm})\, \mathbf{Q}_\mathrm{m} \tag{10}$$

$$\frac{\partial \mathbf{R}_\mathrm{s}}{\partial t} = (1/\tau_\mathrm{recs})\, \mathbf{H}_\mathrm{s} \tag{11}$$

$$\frac{\partial \mathbf{D}}{\partial t} = (1/\tau_\mathrm{death})\, \mathbf{H}_\mathrm{f} \tag{12}$$

2 Some model parameters

Age group	1	2	3	4	5	6	7	8	9	10	11
Age range	0–5	6–12	13–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–105
Population	351159	451246	446344	711752	730547	723663	703830	582495	435834	185480	45230
P–mild	1.0000	1.0000	1.0000	1.0000	1.0000	0.9640	0.9185	0.9210	0.8900	0.9070	0.9120
P–severe	0.0000	0.0000	0.0000	0.0000	0.0000	0.0360	0.0720	0.0600	0.0720	0.0360	0.0120
P–fatal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0095	0.0190	0.0380	0.0570	0.0760

Table 1: The population numbers are obtained from SSB and are accurate. The total Norwegan population is 5367580. The P numbers indicate the fraction of sick people in an age group ending up with mild symptons, severe symptons (hospitalized), and fatal infection (hospitalized and then dead). **These numbers are currently defined in m_pfactor.F90 but will later be read from a file.**

Age group	1	2	3	4	5	6	7	8	9	10	11
1	**3.80**	2.00	2.00	1.50	1.50	1.10	0.80	0.80	0.80	0.80	0.80
2	2.00	**3.80**	2.00	1.50	1.50	1.50	0.80	0.80	0.80	0.80	0.80
3	2.00	2.00	**1.00**	1.00	0.90	0.80	0.80	0.80	0.80	0.80	0.80
4	1.50	1.50	1.00	**0.80**	0.80	0.80	0.80	0.80	0.80	0.80	0.80
5	1.50	1.50	0.90	0.80	**0.80**	0.80	0.80	0.80	0.80	0.80	0.80
6	1.10	1.50	0.80	0.80	0.80	**0.80**	0.80	0.80	0.80	0.80	0.80
7	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80	0.80	0.80	0.80
8	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80	0.80	0.80
9	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80	0.80
10	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80
11	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**

Table 2: The R matrix allows for using different transmission factors in between different age groups. This matrix was used after opening up children schools and kinder gardens. On the diagonal the value gives the transmission of desease within the same age group. The off-diagonal terms are the transmissions between age groups. Here it is assumed that open kinder gardens and schools leads to "normal" transmission within these groups $R = 3.8$. We also assume that there are increased transmission between parent groups and children. **These numbers are currently defined in m_Rmatrix.F90 but will later be read from a file.**

2.1 Model paramters

$I_0 = 50.0$	Initial infectious
$R_0 = 5.0$	Basic Reproduction Number
$\tau_{\text{inf}} = 2.9$	Infections time
$\tau_{\text{inc}} = 5.2$	Incubation period
$\tau_{\text{recm}} = 11.1$	Recovery time mild cases (11.1)
$\tau_{\text{recs}} = 15.0$	Recovery time severe cases Length of hospital stay
$\tau_{\text{hosp}} = 5.0$	Time to hospitalization.
$\tau_{\text{death}} = 15.0$	Days to death
$p_{\text{f}} = 0.006$	Case fatality rate
$p_{\text{s}} = 0.012$	Hospitalization rate % for severe cases
$R(t) = 0.63$	Basic Reproduction Number during intervention

2.2 Diagnostic variables

Number of hospitelized	$N(\mathbf{H}_\mathrm{s} + \mathbf{H}_\mathrm{f})$
Number of recovered	$N(\mathbf{R}_\mathrm{m} + \mathbf{R}_\mathrm{s})$
Number of deaths	$N\mathbf{D}$
Number of exposed	$N \sum \mathbf{E}_i$
Number of infectious	$N \sum \mathbf{I}_i$
Number of susceptible	$N \sum \mathbf{S}_i$