SEIR with age compartments

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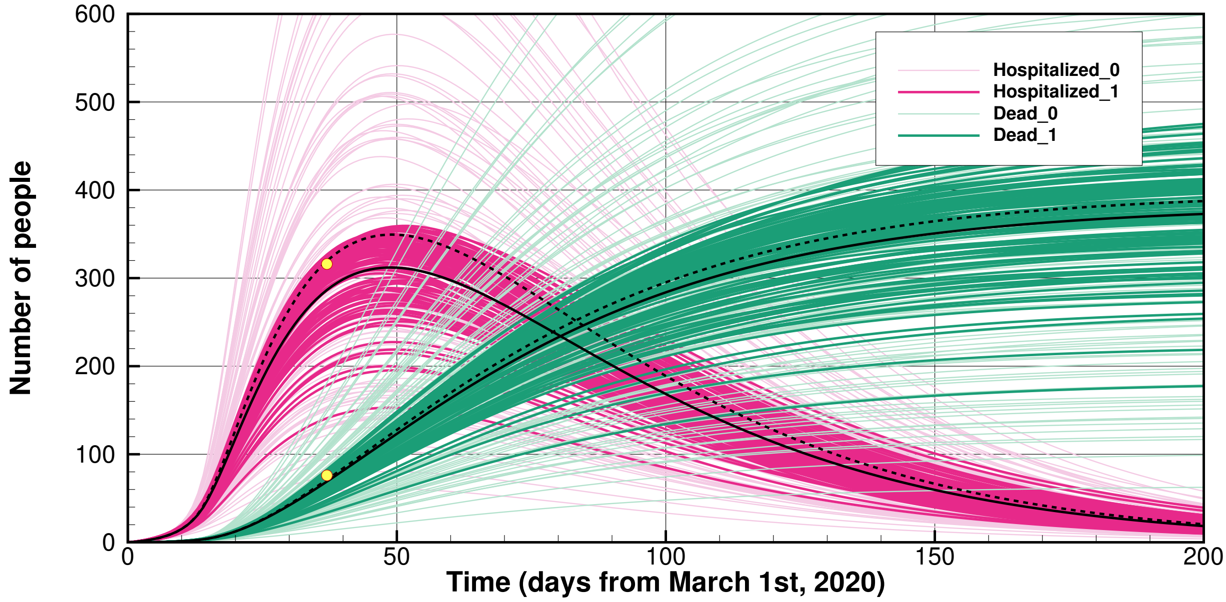
Version compiled April 7, 2020

1 SEIR with age compartments

We have used the model described in Appendix 1 to run a sensitivity simulation to assess the impact of opening children schools and kindergartens after Easter. The age based model considers 11 age groups as defined in Tab. 1. The R matrix used in the simulation after letting children back to kinder gardens and schools is presented in Tab. 2. Note that the P factors and the $\mathbf{R}$ matrix are rather uncertain, and we should gather more data to calibrate these coefficients.

We have run one case where we continue todays measures with $R = 0.8$ as shown in Fig. 1, and then three cases with different $\mathbf{R}$ matrices, which give rise to a stable case Fig. 2, a neutral case Fig. 3, and an unstable case Fig. 4.



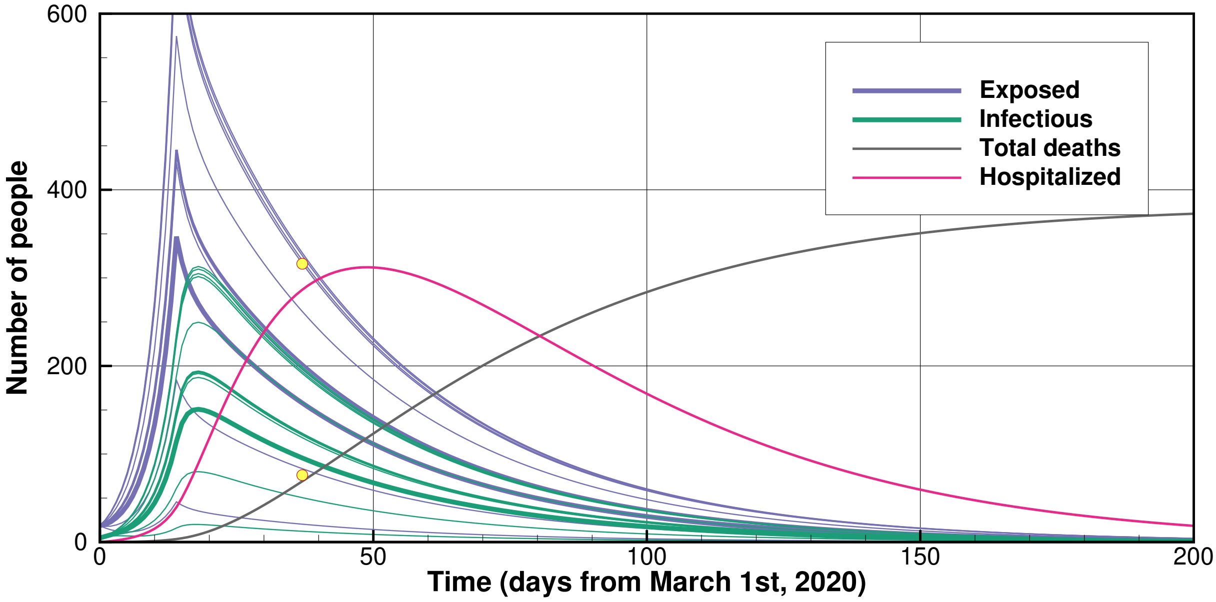


Figure 1: Stable case where the current interventions are continued with $R = 0.8$. The upper plot is an ensemble prediction of Hospitalized and accumulated Deaths. The pale colored lines are the prior realizations while the darker lines are the postrior realizations. The black dashed and solid lines are the enemble means for the prior and the postrior. The lower plot shows the number of exposed and infectious persons in the different age groups. The thickest blue and green lines corresponds to the youngest age group (0–5 years old) while the second thickest lines denote the second age group from (6—12 years old).

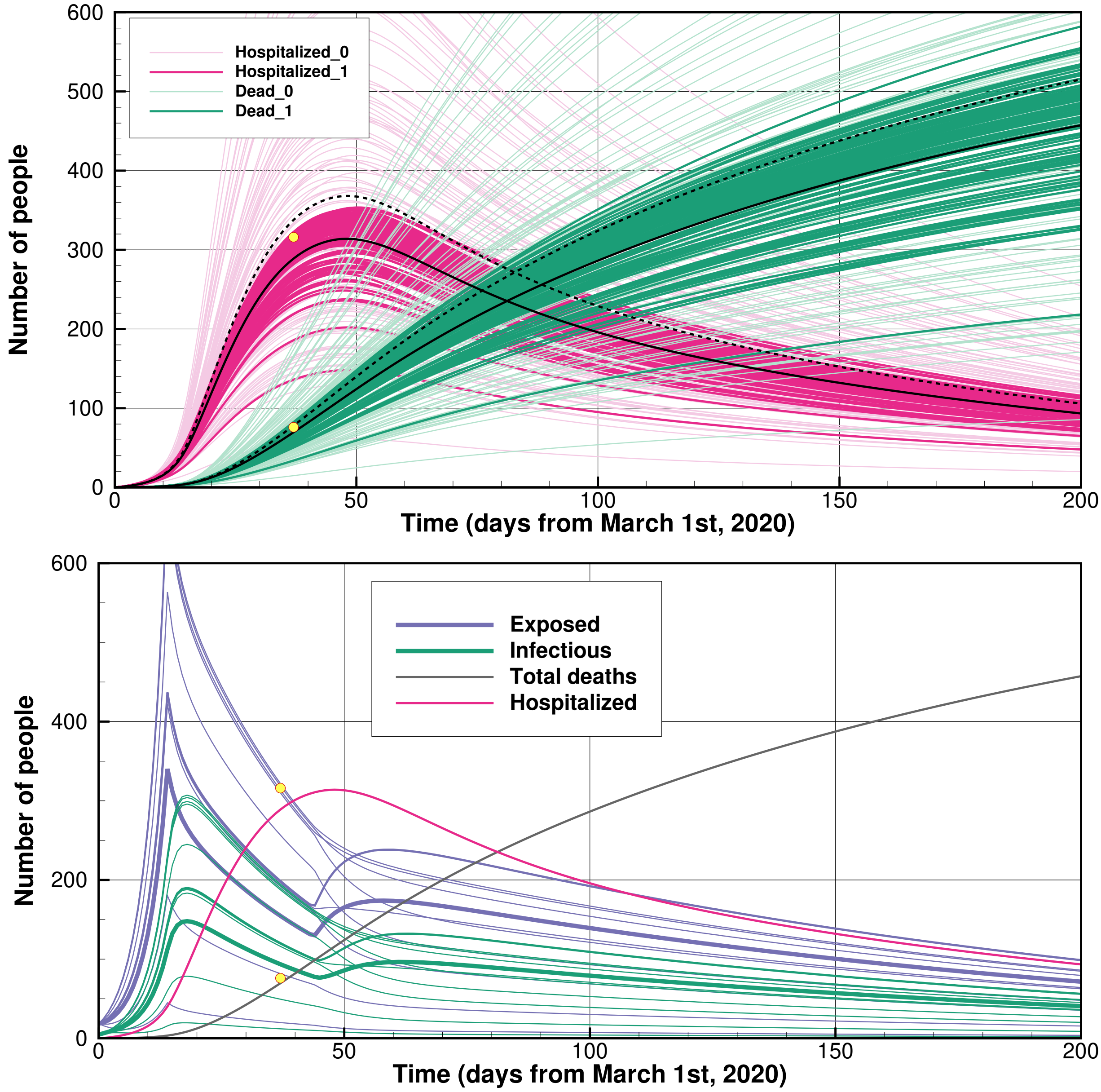


Figure 2: Stable case where we open kinder gardens and schools for children aged 6–12 years old after Easter. The **R** matrix from Tab. 2 is used after Easter but with all 0.8 numbers set to 0.7.

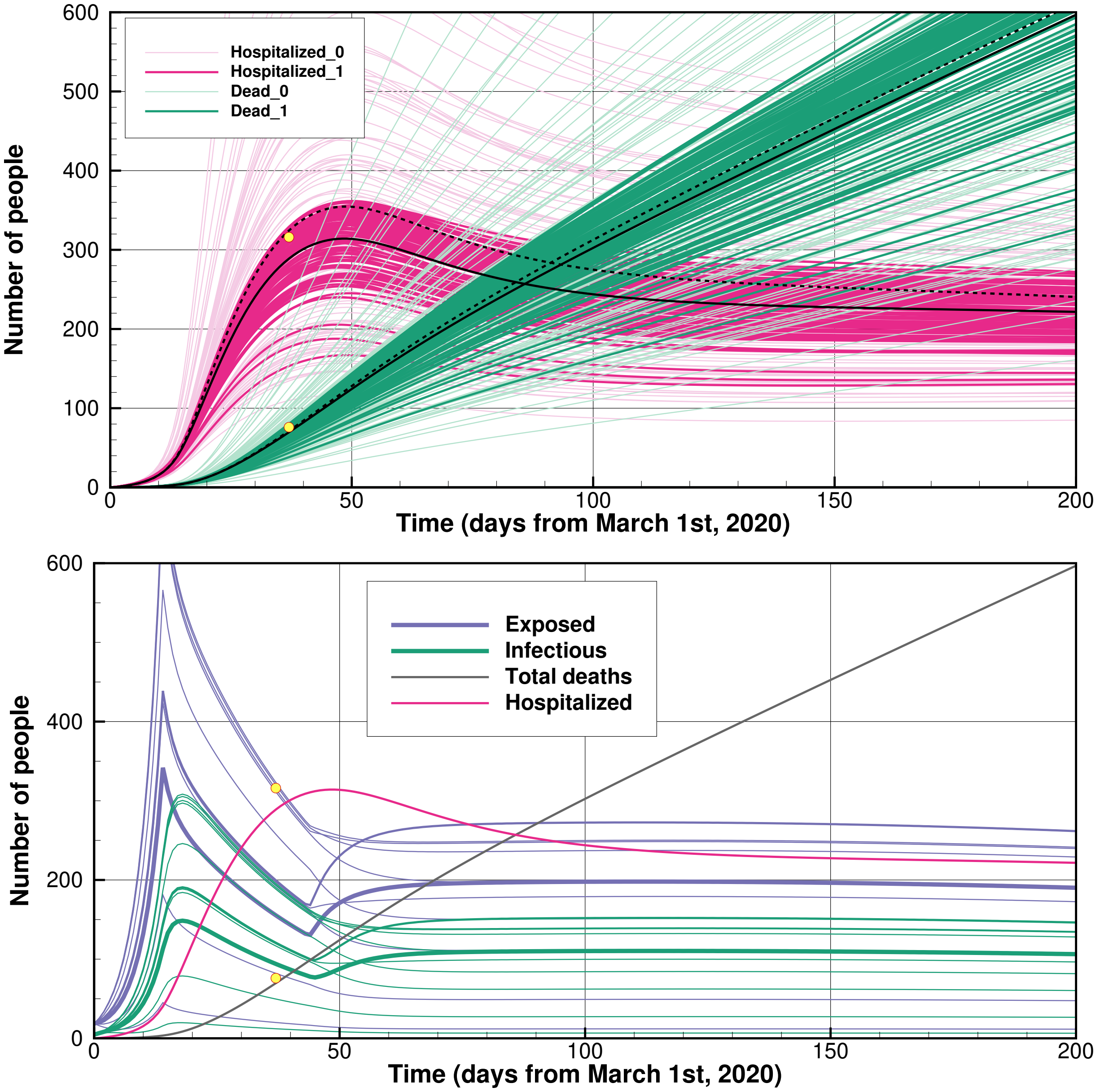


Figure 3: Neutral case where we open kinder gardens and schools for children aged 6–12 years old after Easter. The **R** matrix from Tab. 2 is used after Easter but with all 0.8 numbers are retained at 0.8.

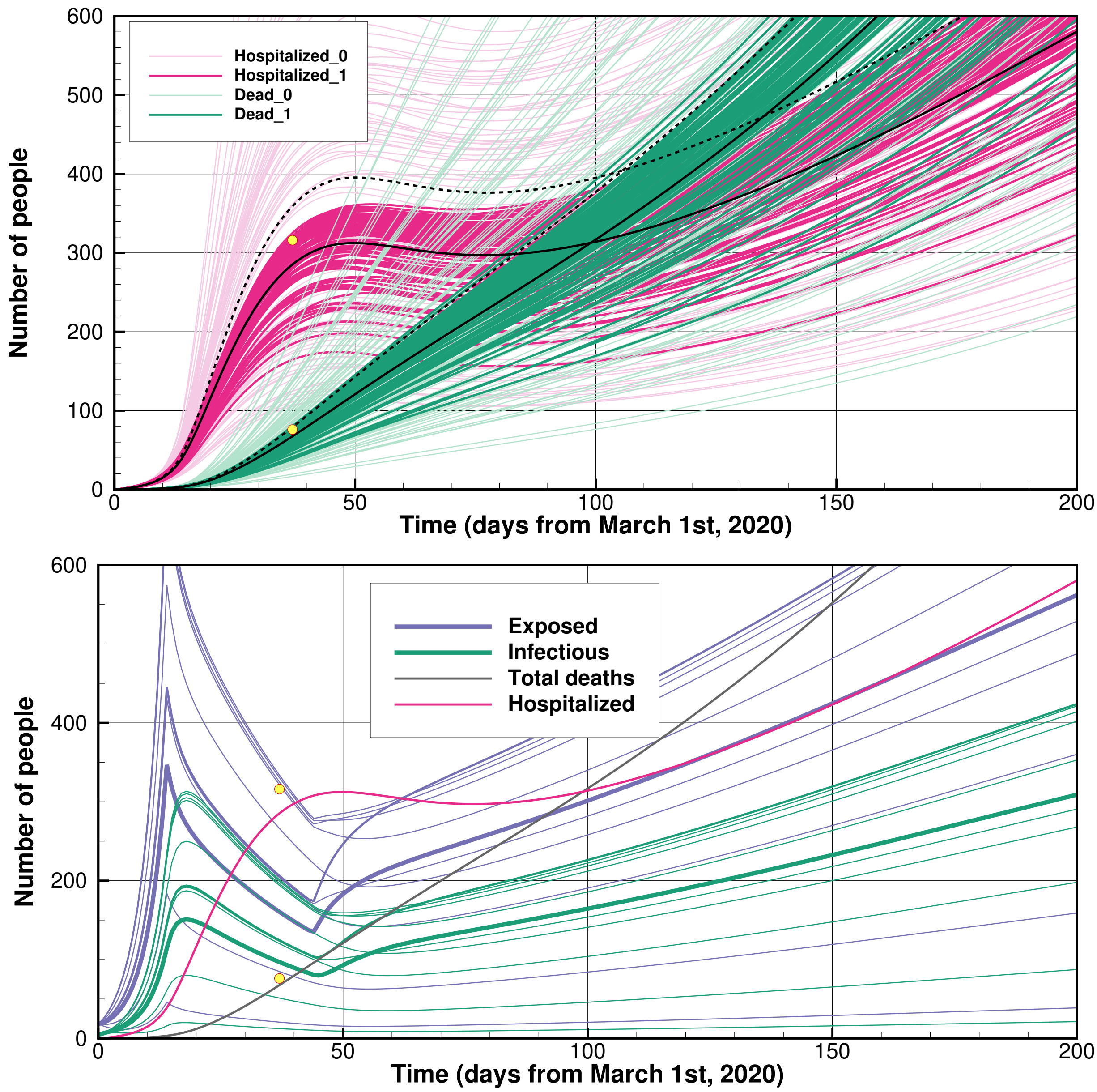


Figure 4: Unstable case where we open kinder gardens and schools for children aged 6–12 years old after Easter. The **R** matrix from Tab. 2 is used after Easter but with all 0.8 numbers set to 0.9.

2 Model description

We have added age compartments to the standard SEIR model. The susceptible, exposed and infected populations is modeled per age group, while after they get sick we gather them in common groups.

$$\begin{cases} \mathbf{S}_1 \to & \mathbf{E}_1 \to \mathbf{I}_1 \\ & \vdots \\ \mathbf{S}_i \to & \mathbf{E}_i \to \mathbf{I}_i \quad \to \begin{cases} \mathbf{Q}_\mathrm{m} & \to \mathbf{R}_\mathrm{m} \\ \mathbf{Q}_\mathrm{s} \to \mathbf{Q}_\mathrm{h} & \to \mathbf{R}_\mathrm{s} \\ \mathbf{Q}_\mathrm{f} & \to \mathbf{D} \end{cases} \\ & \vdots \\ \mathbf{S}_n \to & \mathbf{E}_n \to \mathbf{I}_n \end{cases} \tag{1}$$

The model equations are as follows:

$$\frac{\partial \mathbf{S}_i}{\partial t} = -\frac{1}{\tau_\mathrm{inf}} \left(\sum_{j=1}^{n} R_{ij}(t)\, \mathbf{I}_j \right) \mathbf{S}_i \tag{2}$$

$$\frac{\partial \mathbf{E}_i}{\partial t} = \frac{1}{\tau_\mathrm{inf}} \left(\sum_{j=1}^{n} R_{ij}(t)\, \mathbf{I}_j \right) \mathbf{S}_i - \frac{1}{\tau_\mathrm{inc}} \mathbf{E}_i \tag{3}$$

$$\frac{\partial \mathbf{I}_i}{\partial t} = \frac{1}{\tau_\mathrm{inc}} \mathbf{E}_i - \frac{1}{\tau_\mathrm{inf}} \mathbf{I}_i \tag{4}$$

$$\frac{\partial \mathbf{Q}_\mathrm{m}}{\partial t} = \sum_{i=1}^{n} \frac{p_\mathrm{m}^i}{\tau_\mathrm{inf}} \mathbf{I}_i - (1/\tau_\mathrm{recm})\, \mathbf{Q}_\mathrm{m} \tag{5}$$

$$\frac{\partial \mathbf{Q}_\mathrm{s}}{\partial t} = \sum_{i=1}^{n} \frac{p_\mathrm{s}^i}{\tau_\mathrm{inf}} \mathbf{I}_i - (1/\tau_\mathrm{hosp})\, \mathbf{Q}_\mathrm{s} \tag{6}$$

$$\frac{\partial \mathbf{Q}_\mathrm{f}}{\partial t} = \sum_{i=1}^{n} \frac{p_\mathrm{f}^i}{\tau_\mathrm{inf}} \mathbf{I}_i - (1/\tau_\mathrm{death})\, \mathbf{Q}_\mathrm{f} \tag{7}$$

$$\frac{\partial \mathbf{Q}_\mathrm{h}}{\partial t} = (1/\tau_\mathrm{hosp})\, \mathbf{Q}_\mathrm{s} - (1/\tau_\mathrm{recs})\, \mathbf{Q}_\mathrm{h} \tag{8}$$

$$\frac{\partial \mathbf{R}_\mathrm{m}}{\partial t} = (1/\tau_\mathrm{recm})\, \mathbf{Q}_\mathrm{m} \tag{9}$$

$$\frac{\partial \mathbf{R}_\mathrm{s}}{\partial t} = (1/\tau_\mathrm{recs})\, \mathbf{Q}_\mathrm{h} \tag{10}$$

$$\frac{\partial \mathbf{D}}{\partial t} = (1/\tau_\mathrm{death})\, \mathbf{Q}_\mathrm{f} \tag{11}$$

3 Some model parameters

Age group	1	2	3	4	5	6	7	8	9	10	11
Age range	0–5	6–12	13–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–105
Population	351159	451246	446344	711752	730547	723663	703830	582495	435834	185480	45230
P–mild	1.0000	1.0000	1.0000	1.0000	1.0000	0.9640	0.9185	0.9210	0.8900	0.9070	0.9120
P–severe	0.0000	0.0000	0.0000	0.0000	0.0000	0.0360	0.0720	0.0600	0.0720	0.0360	0.0120
P–fatal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0095	0.0190	0.0380	0.0570	0.0760

Table 1: The population numbers are obtained from SSB and are accurate. The total Norwegan population is 5367580. The P numbers indicate the fraction of sick people in an age group ending up with mild symptons, severe symptons (hospitalized), and fatal infection (hospitalized and then dead). With the current numbers, the average case fatality rate is 0.0090 and the average percentage of severe (hospitalized) cases is 0.0280.

Age group	1	2	3	4	5	6	7	8	9	10	11
1	**3.80**	2.00	2.00	1.50	1.50	1.10	0.80	0.80	0.80	0.80	0.80
2	2.00	**3.80**	2.00	1.50	1.50	1.50	0.80	0.80	0.80	0.80	0.80
3	2.00	2.00	**1.00**	1.00	0.90	0.80	0.80	0.80	0.80	0.80	0.80
4	1.50	1.50	1.00	**0.80**	0.80	0.80	0.80	0.80	0.80	0.80	0.80
5	1.50	1.50	0.90	0.80	**0.80**	0.80	0.80	0.80	0.80	0.80	0.80
6	1.10	1.50	0.80	0.80	0.80	**0.80**	0.80	0.80	0.80	0.80	0.80
7	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80	0.80	0.80	0.80
8	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80	0.80	0.80
9	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80	0.80
10	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**	0.80
11	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	**0.80**

Table 2: The R matrix allows for using different transmission factors in between different age groups. This matrix was used after opening up children schools and kinder gardens. On the diagonal the value gives the transmission of desease within the same age group. The off-diagonal terms are the transmissions between age groups. Here it is assumed that open kinder gardens and schools leads to "normal" transmission within these groups $R = 3.8$. We also assume that there are increased transmission between parent groups and children.

3.1 Model paramters

$\tau_{\text{2death}} = 32.0$	$P(1)$	Days to death
$N = 5000000.0$	$P(2)$	Initial population
$I_0 = 50.0$	$P(3)$	Initial infectious (19 cases 1st march)
$R_0 = 2.2$	$P(4)$	Basic Reproduction Number
$\tau_{\text{inc}} = 5.2$	$P(5)$	Incubation period (Tinc)
$\tau_{\text{inf}} = 2.9$	$P(6)$	Duration patient is infectious (Tinf)
$\tau_{\text{recm}} = 14.0 - \tau_{\text{inf}}$	$P(7)$	Recovery time mild cases (11.1)
$\tau_{\text{recs}} = 31.5 - \tau_{\text{inf}}$	$P(8)$	Recovery time severe cases Length of hospital stay
$\tau_{\text{hosp}} = 5.0$	$P(9)$	Time to hospitalization.
$p_{\text{f}} = 0.02$	$P(10)$	Case fatality rate
$p_{\text{s}} = 0.2$	$P(11)$	Hospitalization rate % for severe cases
$R(t) = 0.8$	$P(12)$	Basic Reproduction Number during intervention
$\tau_{\text{intervention}} = 30.0$	$P(13)$	Interventions start here (15th march)
$\tau_{\text{death}} = \tau_{\text{2death}} - \tau_{\text{inf}}$		

3.2 Diagnostic variables

Number of hospitelized	$N(\mathbf{Q}_\mathrm{f} + \mathbf{Q}_\mathrm{h})$
Number of recovered	$N(\mathbf{R}_\mathrm{m} + \mathbf{R}_\mathrm{s})$
Number of deaths	$N\mathbf{D}$
Number of exposed	$N \sum \mathbf{E}_i$
Number of infectious	$N \sum \mathbf{I}_i$
Number of susceptible	$N \sum \mathbf{S}_i$