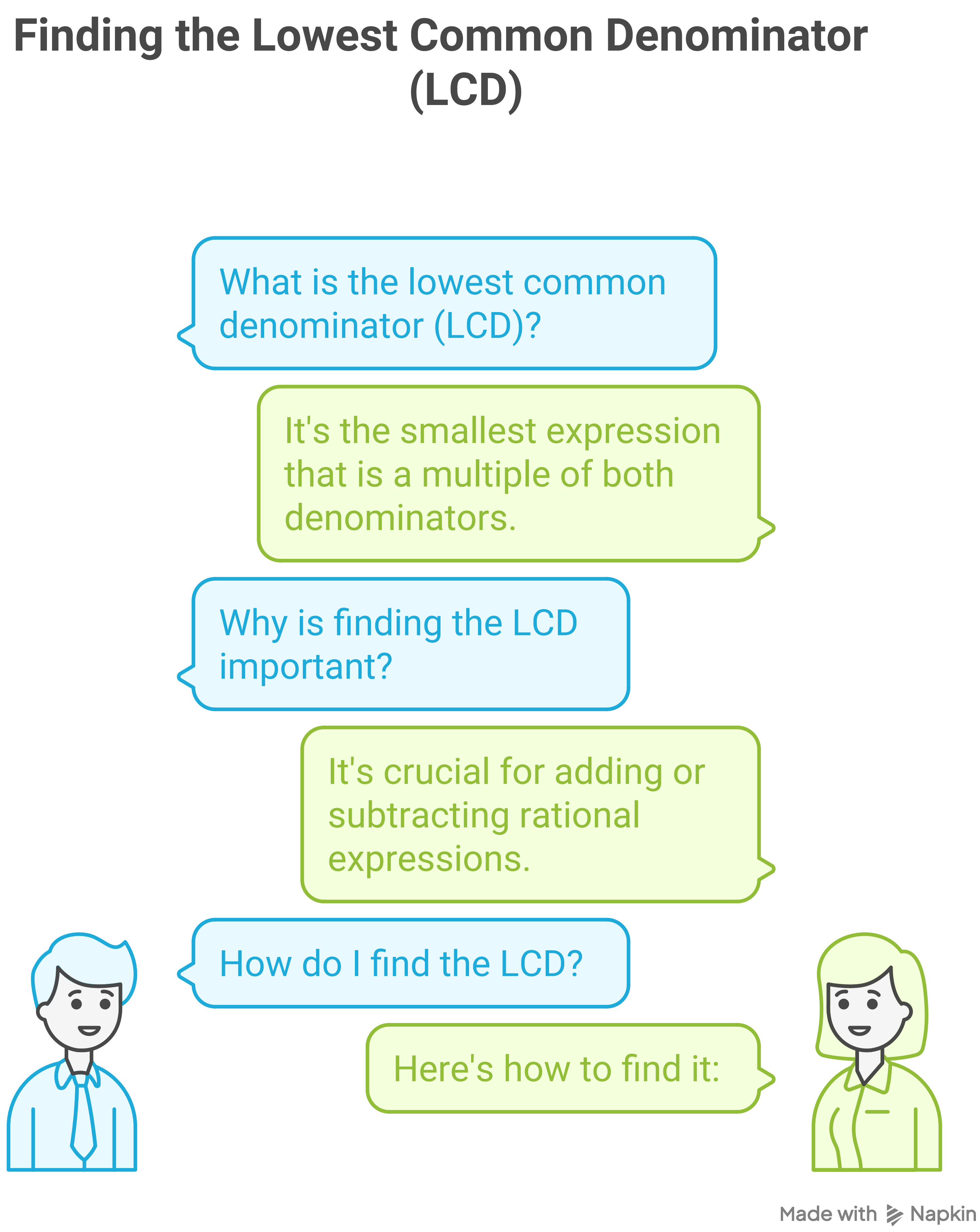
Finding the Lowest Common Denominator and Adding/Subtracting Rational Expressions

This document explains how to determine the lowest common denominator (LCD) for two rational expressions and then how to add or subtract them. We will cover the steps involved with clear examples to illustrate the process.

Determining the Lowest Common Denominator (LCD)

The lowest common denominator (LCD) is the smallest expression that is a multiple of both denominators. Finding the LCD is crucial for adding or subtracting rational expressions. Here's how to find it:



Step 1: Factor Each Denominator Completely

Factor each denominator into its prime factors. This includes factoring out any common factors, factoring quadratic expressions, and so on.

Step 2: Identify All Unique Factors

List all the unique factors that appear in either denominator.

Step 3: Determine the Highest Power of Each Unique Factor

For each unique factor, determine the highest power to which it appears in any of the denominators.

Step 4: Construct the LCD

The LCD is the product of each unique factor raised to its highest power.

Example 1:

Find the LCD of $\frac{1}{x^2 - 4}$ and $\frac{1}{x^2 + 4x + 4}$.

1. **Factor the denominators:**

> * $x^2 - 4 = (x - 2)(x + 2)$

> * $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$

2. **Identify unique factors:**

> * $(x - 2)$

> * $(x + 2)$

3. **Determine the highest power of each unique factor:**

> * $(x - 2)$ appears to the power of 1.

> * $(x + 2)$ appears to the power of 2.

4. **Construct the LCD:**

> * LCD = $(x - 2)(x + 2)^2$

Example 2:

Find the LCD of $\frac{1}{6x}$ and $\frac{1}{4x^2}$.

1. **Factor the denominators:**

> * $6x = 2 \cdot 3 \cdot x$

> * $4x^2 = 2^2 \cdot x^2$

2. **Identify unique factors:**

> * 2

> * 3

> * x

3. **Determine the highest power of each unique factor:**

> * 2 appears to the power of 2.

> * 3 appears to the power of 1.

> * x appears to the power of 2.

4. **Construct the LCD:**

> *   LCD = $2^2 \cdot 3 \cdot x^2 = 12x^2$

Adding and Subtracting Rational Expressions

Once you have the LCD, you can add or subtract rational expressions. Here's how:

Step 1: Find the LCD

Determine the lowest common denominator of the rational expressions.

Step 2: Rewrite Each Fraction with the LCD

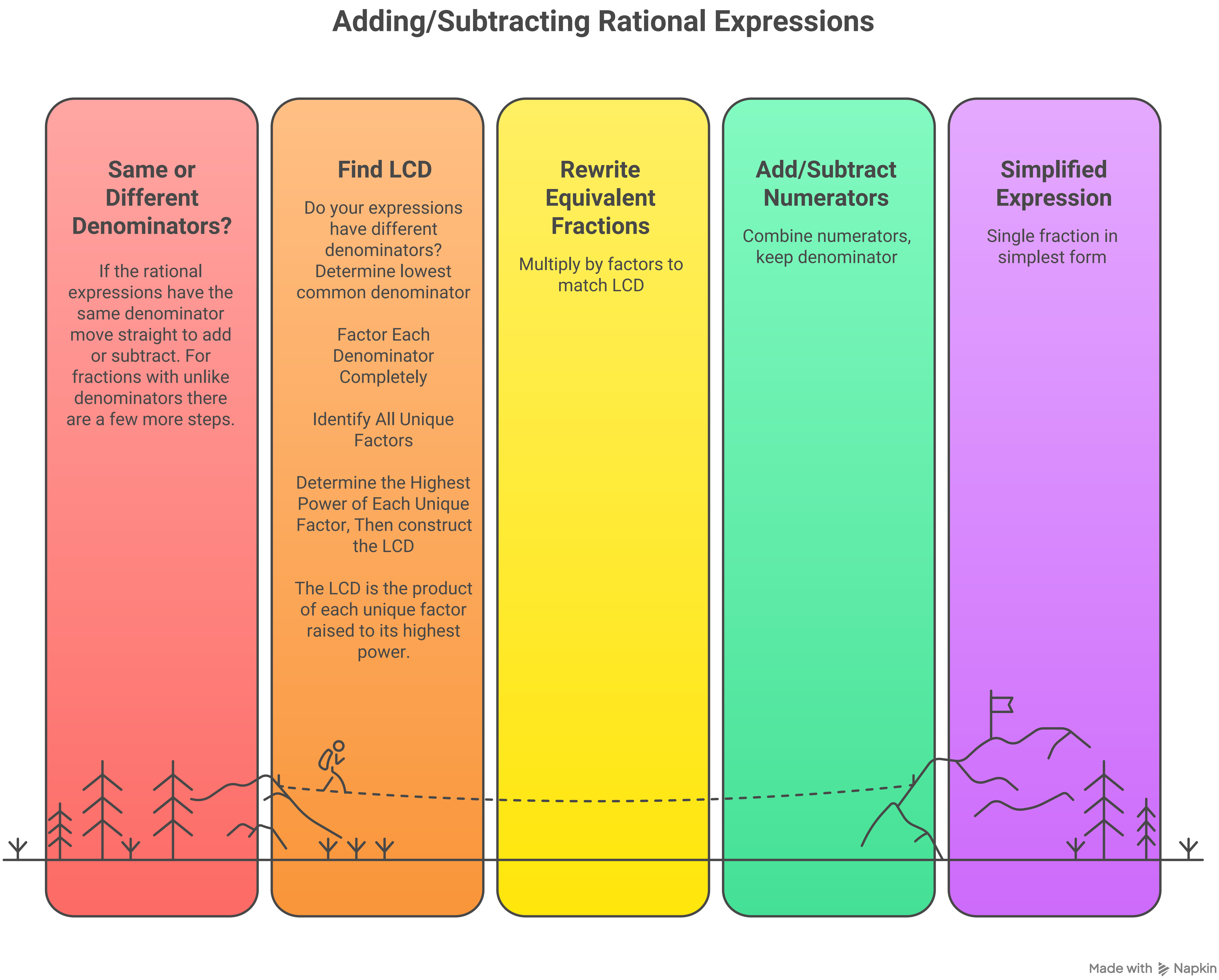
Multiply the numerator and denominator of each fraction by the factors needed to make the denominator equal to the LCD. This is equivalent to multiplying each fraction by a form of 1, so the value of the fraction does not change.

Step 3: Add or Subtract the Numerators

Once all fractions have the same denominator, add or subtract the numerators. Keep the common denominator.

Step 4: Simplify the Result

Simplify the resulting rational expression by combining like terms in the numerator and factoring both the numerator and denominator to see if any factors can be cancelled.



```
*   The LCD is $(x + 1)(x - 1)$.
```

2. **Rewrite each fraction with the LCD:**

```
*   $\frac{1}{x + 1} = \frac{1(x - 1)}{(x + 1)(x - 1)} = \frac{x - 1}{(x + 1)(x - 1)}$
```

```
*   $\frac{2}{x - 1} = \frac{2(x + 1)}{(x - 1)(x + 1)} = \frac{2x + 2}{(x + 1)(x - 1)}$
```

3. **Add the numerators:**

> * $\frac{x - 1}{(x + 1)(x - 1)} + \frac{2x + 2}{(x + 1)(x - 1)} = \frac{(x - 1) + (2x + 2)}{(x + 1)(x - 1)} = \frac{3x + 1}{(x + 1)(x - 1)}$

4. **Simplify the result:**

> *   The numerator $3x + 1$ cannot be factored further. The denominator can be written as $x^2 - 1$.

> *   The simplified expression is $\frac{3x + 1}{x^2 - 1}$.

Example 2: Subtracting Rational Expressions

Subtract $\frac{3}{x} - \frac{1}{x + 2}$.

1. **Find the LCD:**

> *   The denominators are x and $(x + 2)$.

> *   The LCD is $x(x + 2)$.

2. **Rewrite each fraction with the LCD:**

> * $\frac{3}{x} = \frac{3(x + 2)}{x(x + 2)} = \frac{3x + 6}{x(x + 2)}$

> * $\frac{1}{x + 2} = \frac{1(x)}{(x + 2)(x)} = \frac{x}{x(x + 2)}$

3. **Subtract the numerators:**

> * $\frac{3x + 6}{x(x + 2)} - \frac{x}{x(x + 2)} = \frac{(3x + 6) - x}{x(x + 2)} = \frac{2x + 6}{x(x + 2)}$

4. **Simplify the result:**

> *   Factor the numerator: $2x + 6 = 2(x + 3)$

> *   The expression becomes $\frac{2(x + 3)}{x(x + 2)}$. There are no common factors to cancel.

> *   The simplified expression is $\frac{2(x + 3)}{x(x + 2)}$ or $\frac{2x+6}{x^2+2x}$.

Example 3: Adding with Factoring Required

Add $\frac{x}{x^2 - 9} + \frac{1}{x + 3}$.

1. **Find the LCD:**

```
*   Factor the first denominator: $x^2 - 9 = (x - 3)(x + 3)$
```

```
*   The denominators are $(x - 3)(x + 3)$ and $(x + 3)$.
```

```
*   The LCD is $(x - 3)(x + 3)$.
```

2. **Rewrite each fraction with the LCD:**

```
*   $\frac{x}{(x - 3)(x + 3)}$ is already in the correct form.
```

```
*   $\frac{1}{x + 3} = \frac{1(x - 3)}{(x + 3)(x - 3)} = \frac{x - 3}{(x - 3)(x
+ 3)}$
```

3. **Add the numerators:**

```
*   $\frac{x}{(x - 3)(x + 3)} + \frac{x - 3}{(x - 3)(x + 3)} = \frac{x + (x -
3)}{(x - 3)(x + 3)} = \frac{2x - 3}{(x - 3)(x + 3)}$
```

4. **Simplify the result:**

```
*   The numerator $2x - 3$ cannot be factored further.
```

```
*   The simplified expression is $\frac{2x - 3}{(x - 3)(x + 3)}$ or $\frac{2x-3}
{x^2-9}$.
```

By following these steps, you can confidently find the lowest common denominator and add or subtract rational expressions. Remember to always factor completely and simplify your final answer.