601.445/601.645
Practical Cryptographic Systems

Symmetric Cryptography

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Housekeeping

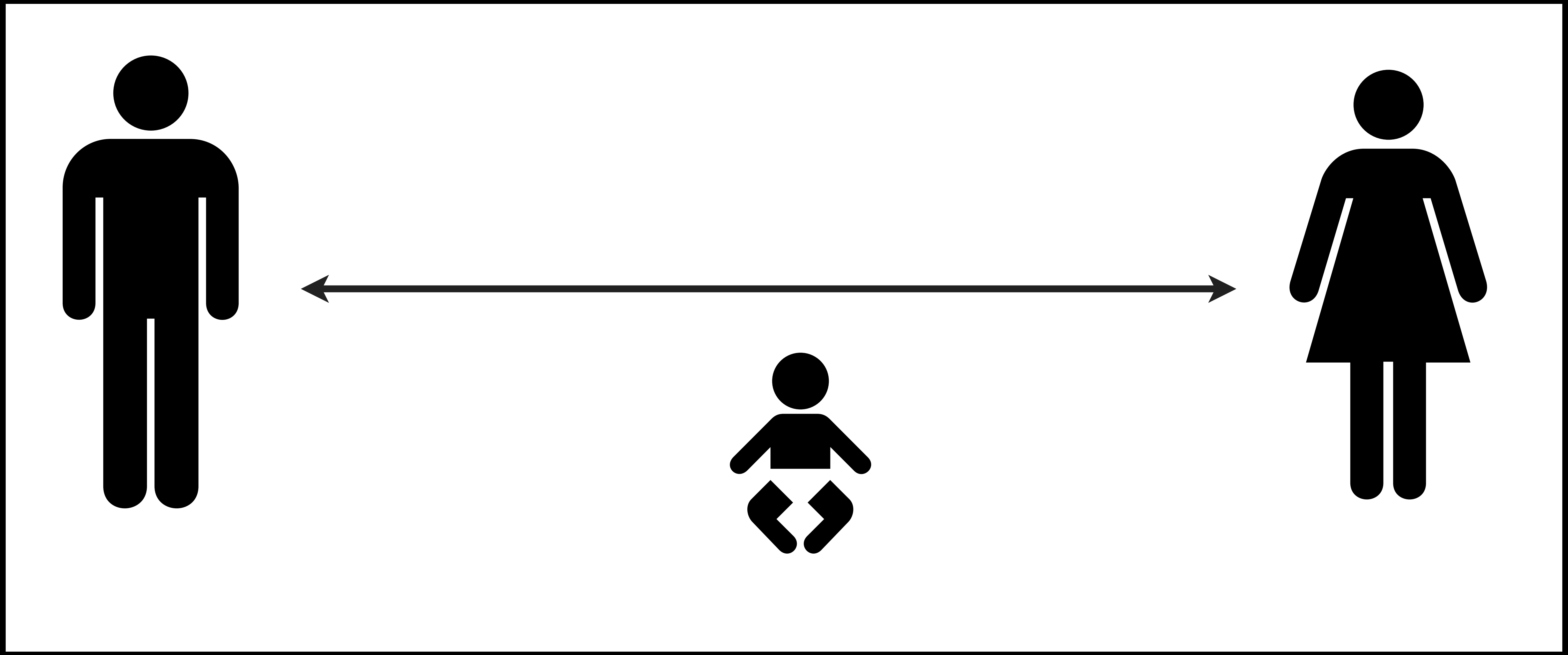
- New (last!) assignment coming Friday
- Will include written and programming portions

News?

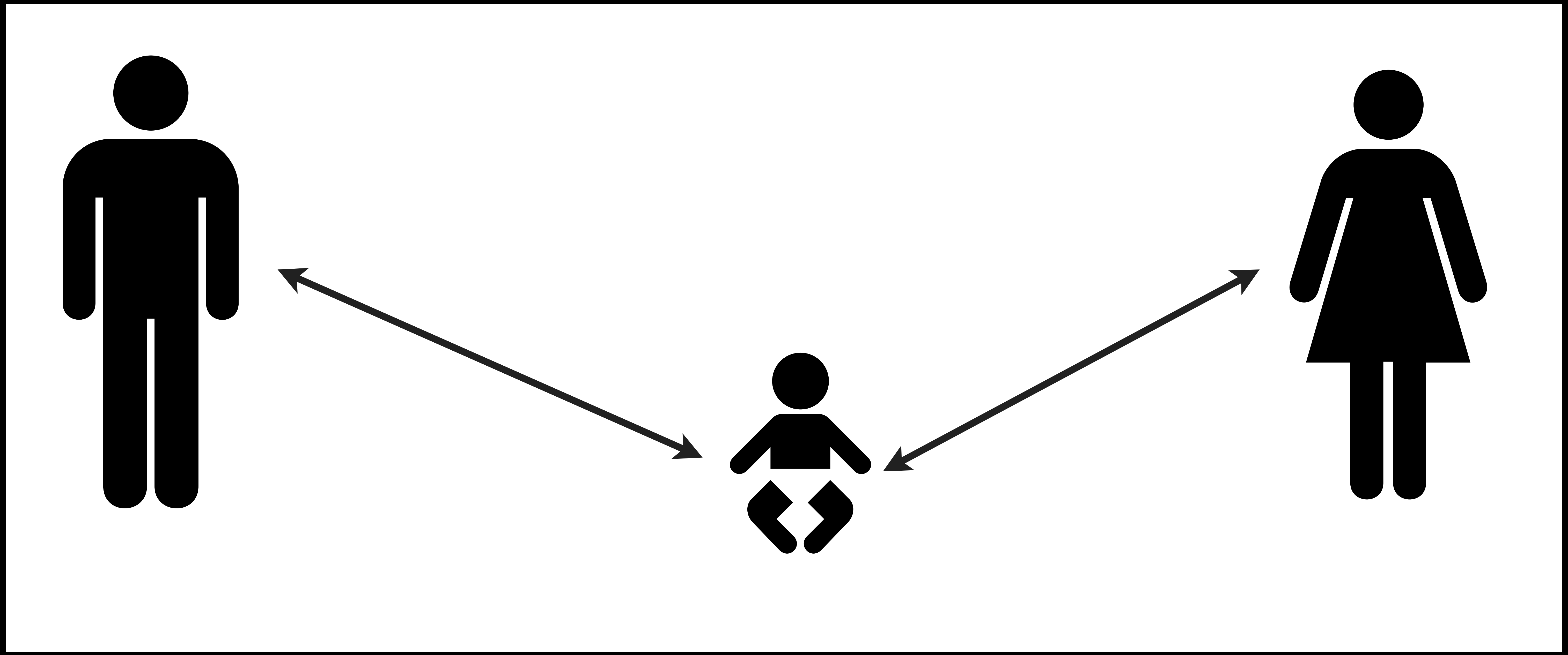
Review

- Last time:
  - Review of GC
- Today:
  - Secret sharing
  - MPC based on secret sharing

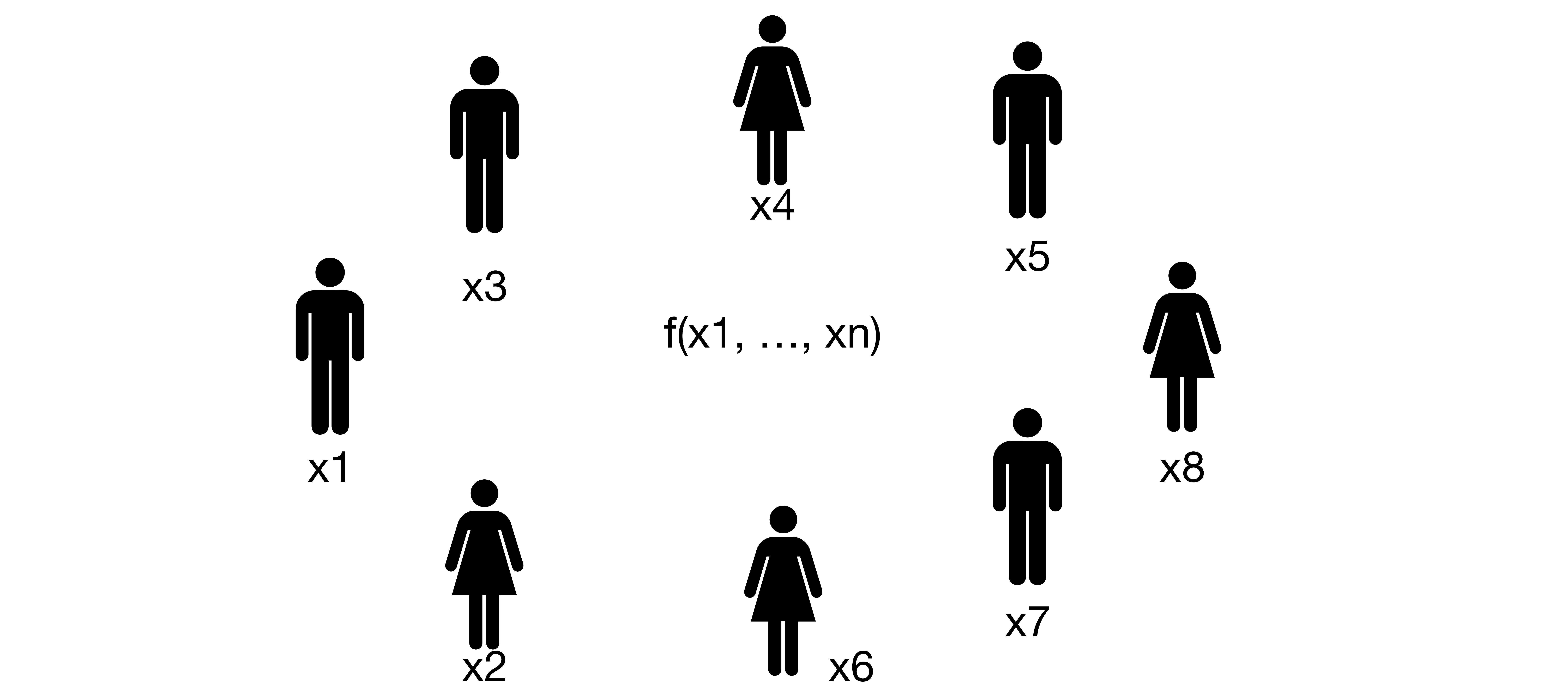
Communication Model



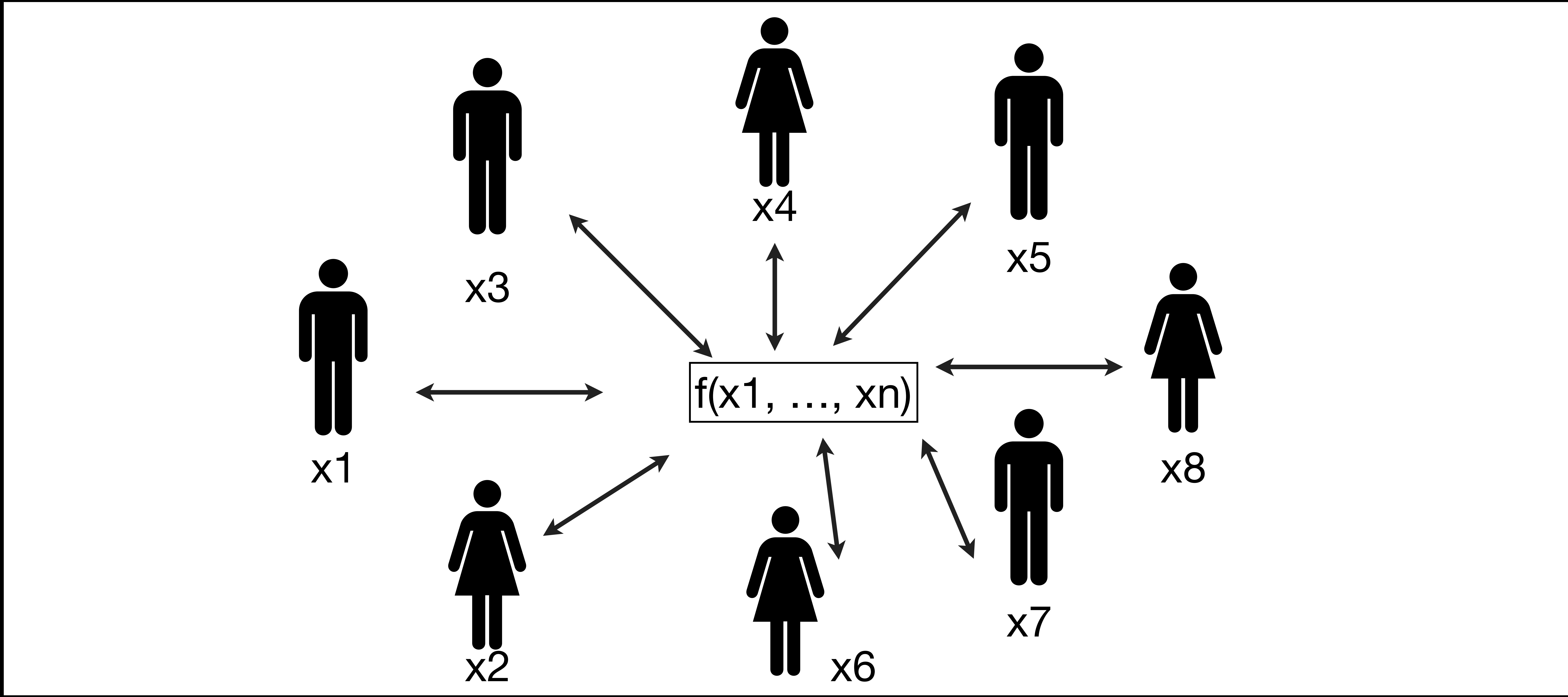
Communication Model



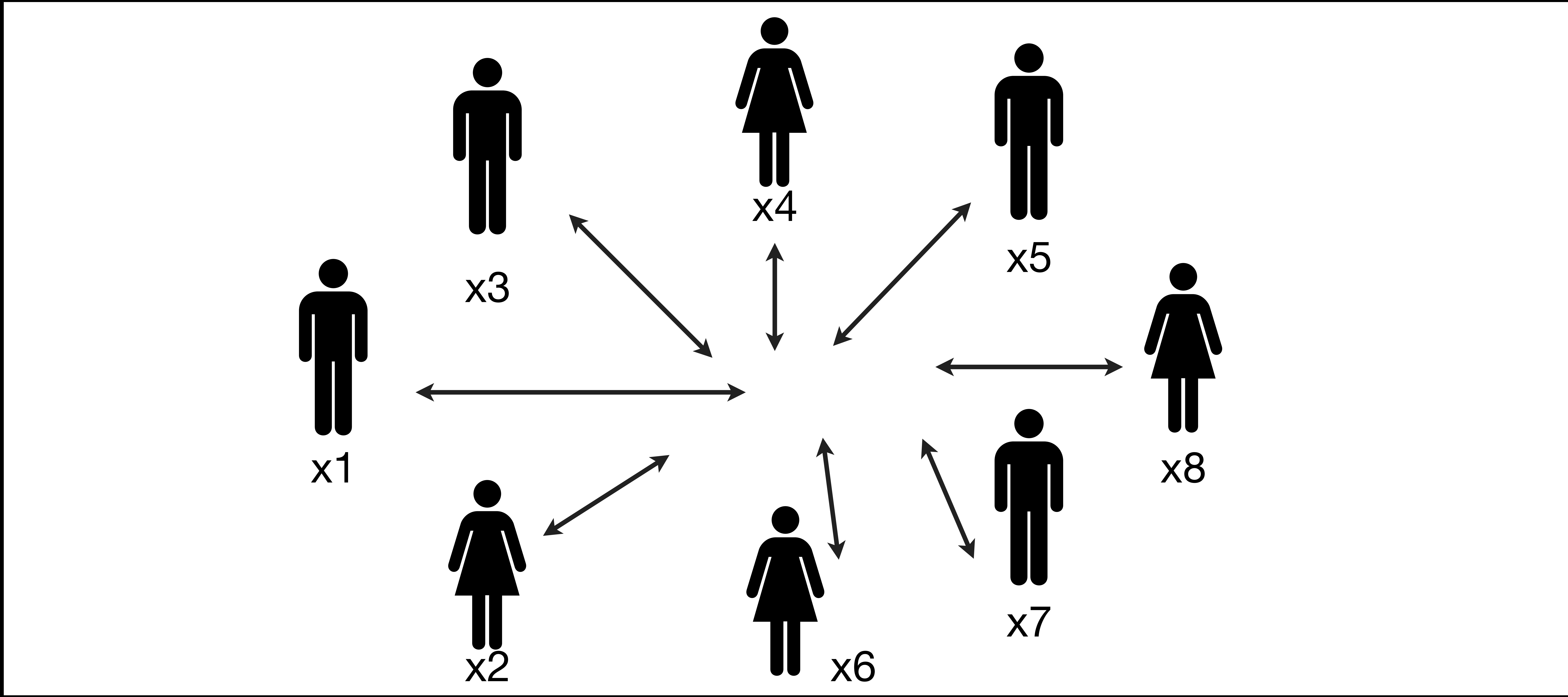
Computation model



Computation model (ideal)



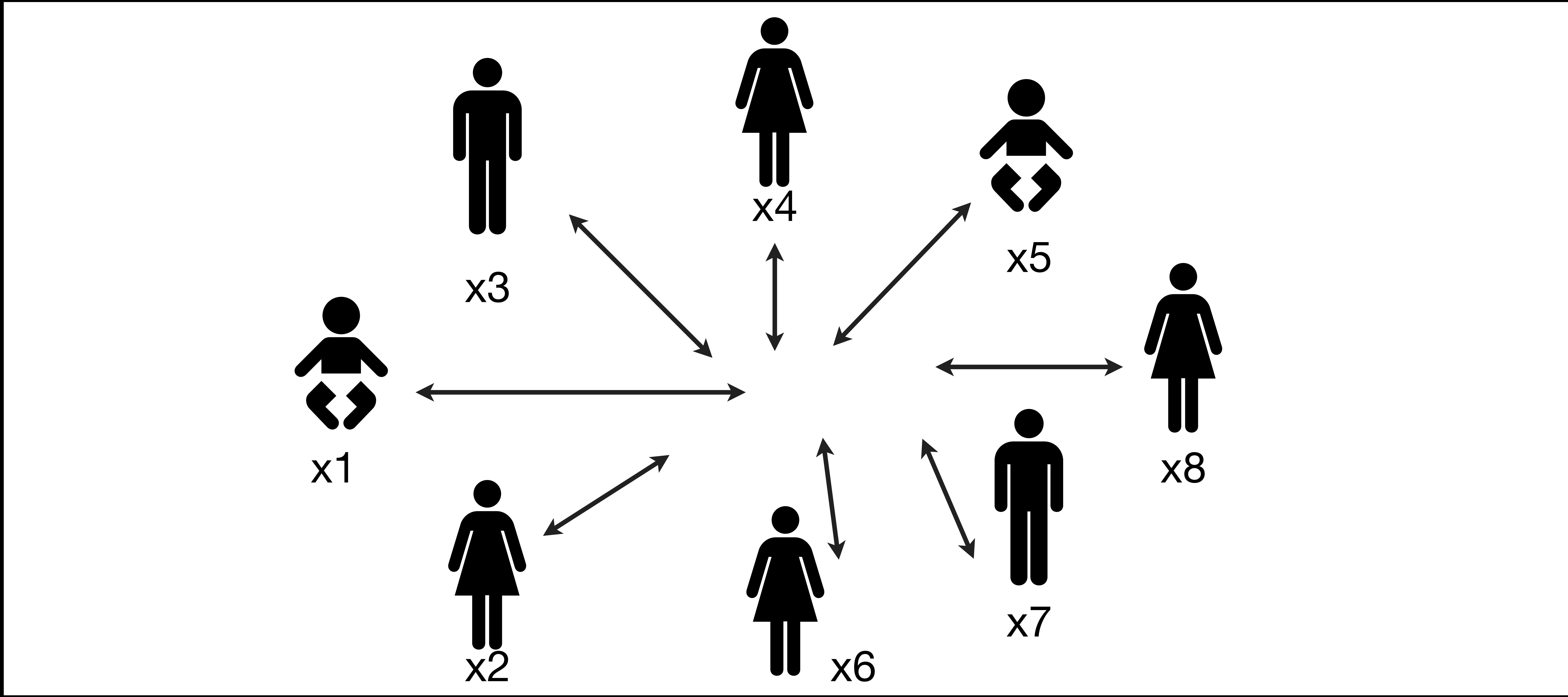
Computation model (MPC)



MPC

- Three basic properties we want to achieve
  - Correctness: the output of the function is actually what it should be
  - Privacy: nobody learns anything about <u>honest parties'</u> inputs (other than what they would learn from the function output)
  - Guaranteed output delivery (no dishonest party can prevent honest parties from getting outputs)

Adversarial model



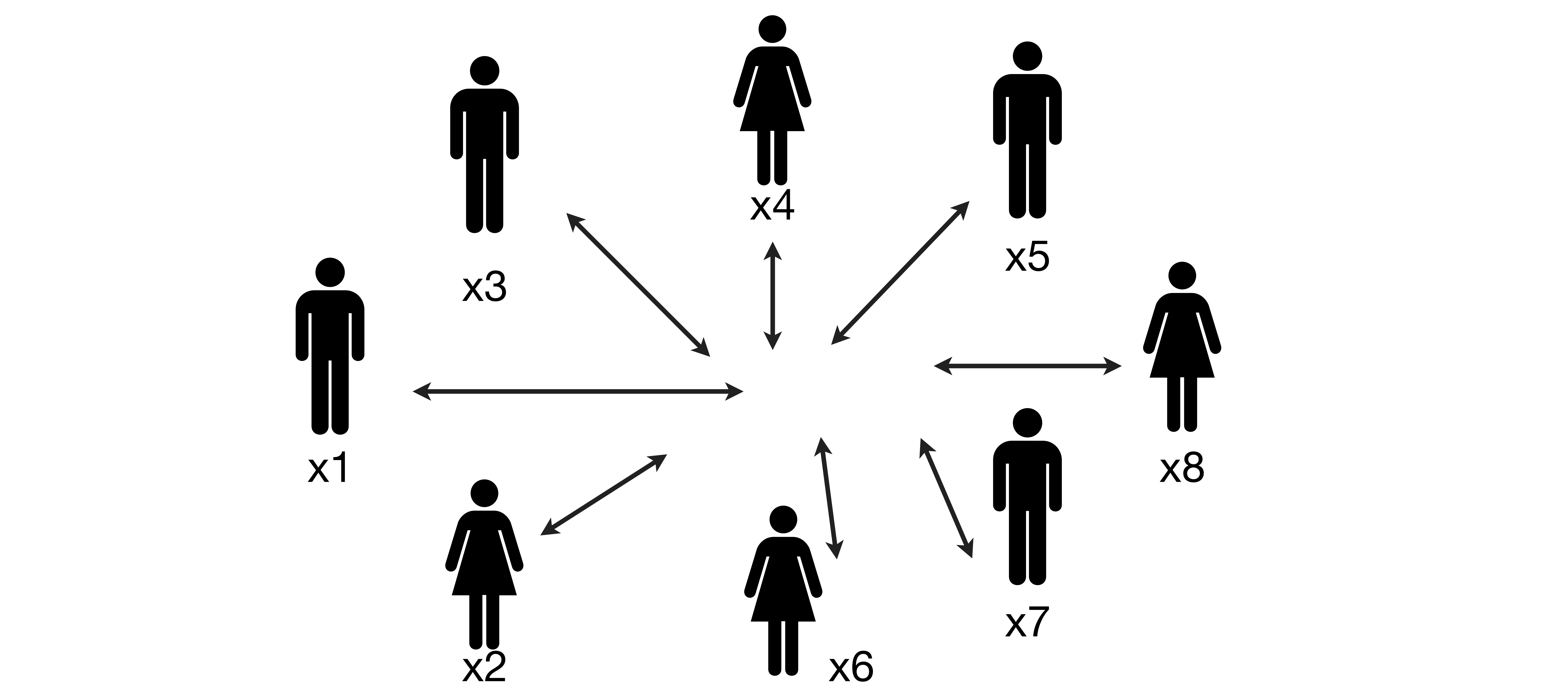
MPC

- We must consider an adversarial model
  - If all parties are totally malicious (and colluding) then there can be no security!
  - So we will assume some parties are “honest” and some are “corrupted”
- What do corrupted parties do?
  - Semi-honest (“honest but curious)”) model: they obey the protocol as written, but also try to learn information about the honest parties’ input <u>passively</u>
  - Malicious model: adversaries can do/send anything they want

Semi-honest model?

- Why do we think there is a semi-honest model?
- Does this make any sense?
- Can we come up with obviously broken protocols in this model?

Secure channels



Secure channels

- For simplicity we will often assume that the parties can communicate with each other securely
  - Ie, the adversaries are the corrupted parties, not eavesdroppers on the wire
  - In practice: how do we achieve this?

Types of MPC

- Honest majority: we assume that if there are N participants, then strictly <u>more than N/2 of the participants are honest</u>
- Dishonest majority: more than N/2 may be corrupted, all the way up to N-1
- What does this mean for 2PC?

Applications of MPC

- Key splitting: break a single encryption key into multiple pieces, use MPC to compute decryption/signatures etc.
- Evaluating secret data: compute functions over data that nobody wants to reveal, e.g.,:
  - Sealed-bid auctions: nobody learns non-winning bids
  - Statistical calculations: e.g., compute salary ranges
  - Machine learning: train ML models on large amounts of private data (often this uses a technique called “differential privacy”)

Secret sharing

- Problem:
  - Take a given secret *s* and break it into *N* different pieces (“shares”)
  - Want to recover the original secret from any *M* of the shares, M <= N
  - What is the security goal?

Secret sharing

- Two algorithms:
  - Share(N, M, s): outputs (t1, …., tN)
  - Recover(N, M, t_i1, …, t_iM): outputs s’

Secret sharing

- Correctness?
- Security definition:
  - (Informal) Given any subset of M-1 shares, no adversary learns any information about *s* (other than its size)
  - Alternative definition: Given a set of M-1 shares of *s*, and a set of M-1 shares of some *random value s’,* no adversary can tell the difference
  - (E.g., there is no detectable difference between the shares of s and a random element of the same length.)

How do we build secret sharing

- Let’s try to build 2-out-of-2 secret sharing
- We have a bitstring *s,* and wish to compute t1, t2 such that:
  - Neither t1 or t2 (by itself) reveals anything about *s* (other than length)
  - Given both t1, t2 we can recover *s*

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  - Neither t1 or t2 (by itself) reveals anything about *s* (other than length)
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- Solution (share algorithm):
  - Pick a random string t1 such that |t1| = |s|
  - Set t2 = s XOR t1

How do we build secret sharing

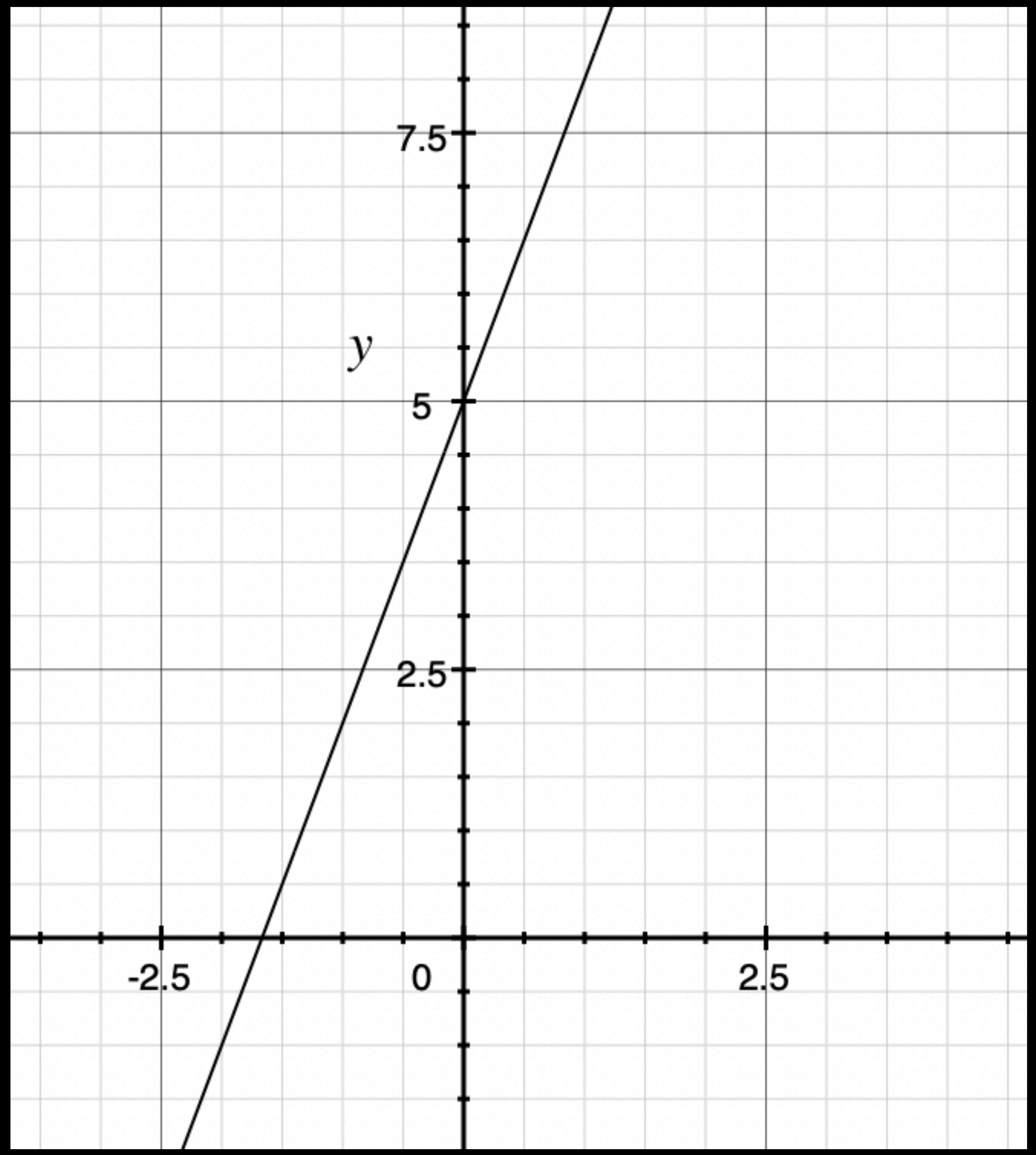
- Let’s try to build 2-out-of-3 secret sharing using the same technique

General secret sharing

- What are the downsides of the XOR approach?
- Can we build a more efficient, general-purpose approach?

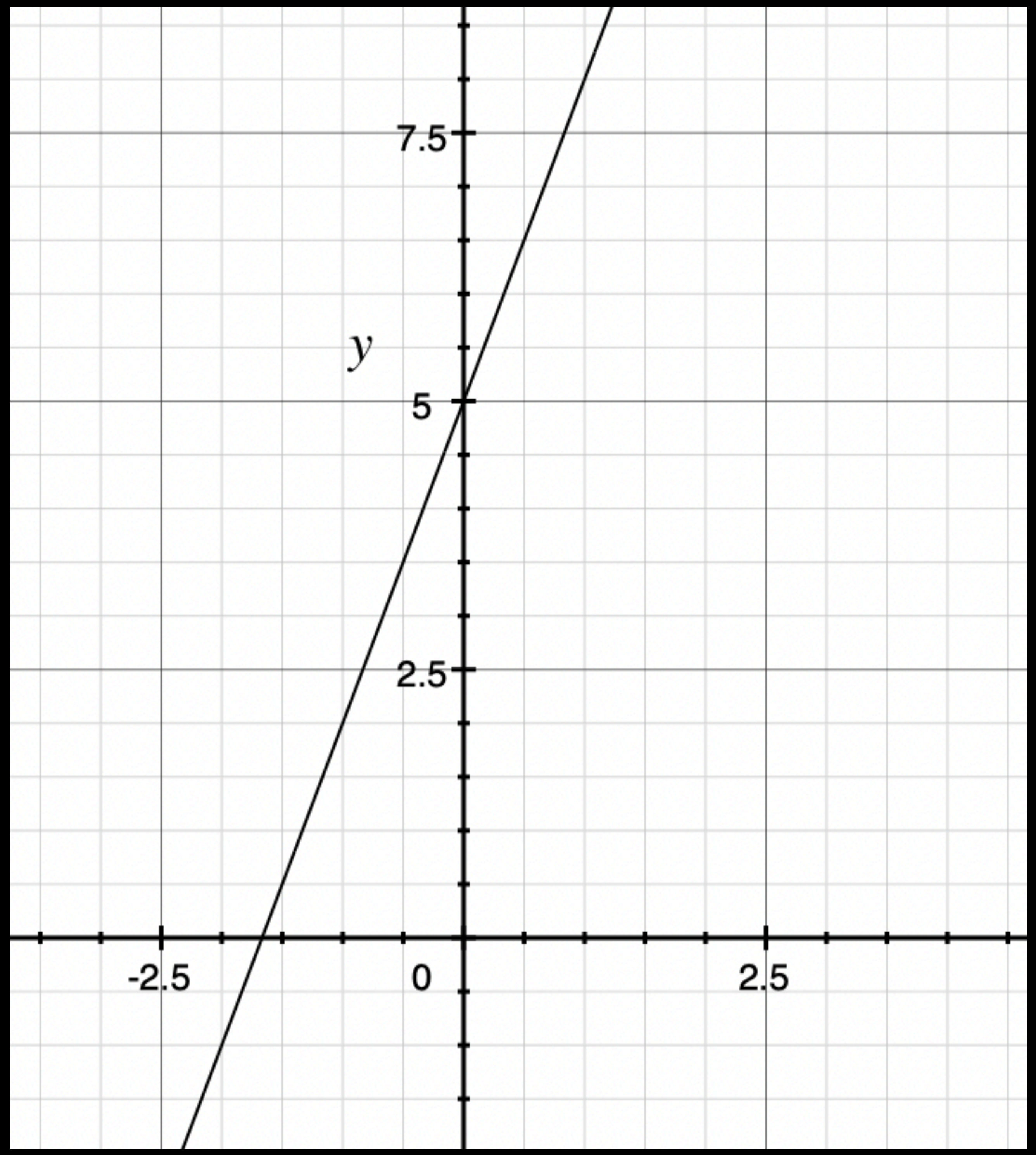
(Simple) linear secret sharing

- Let y=*mx*+*b* be the equation of a line
- Imagine I give you a point (x, y) for x != 0
- What can we learn about *b*?



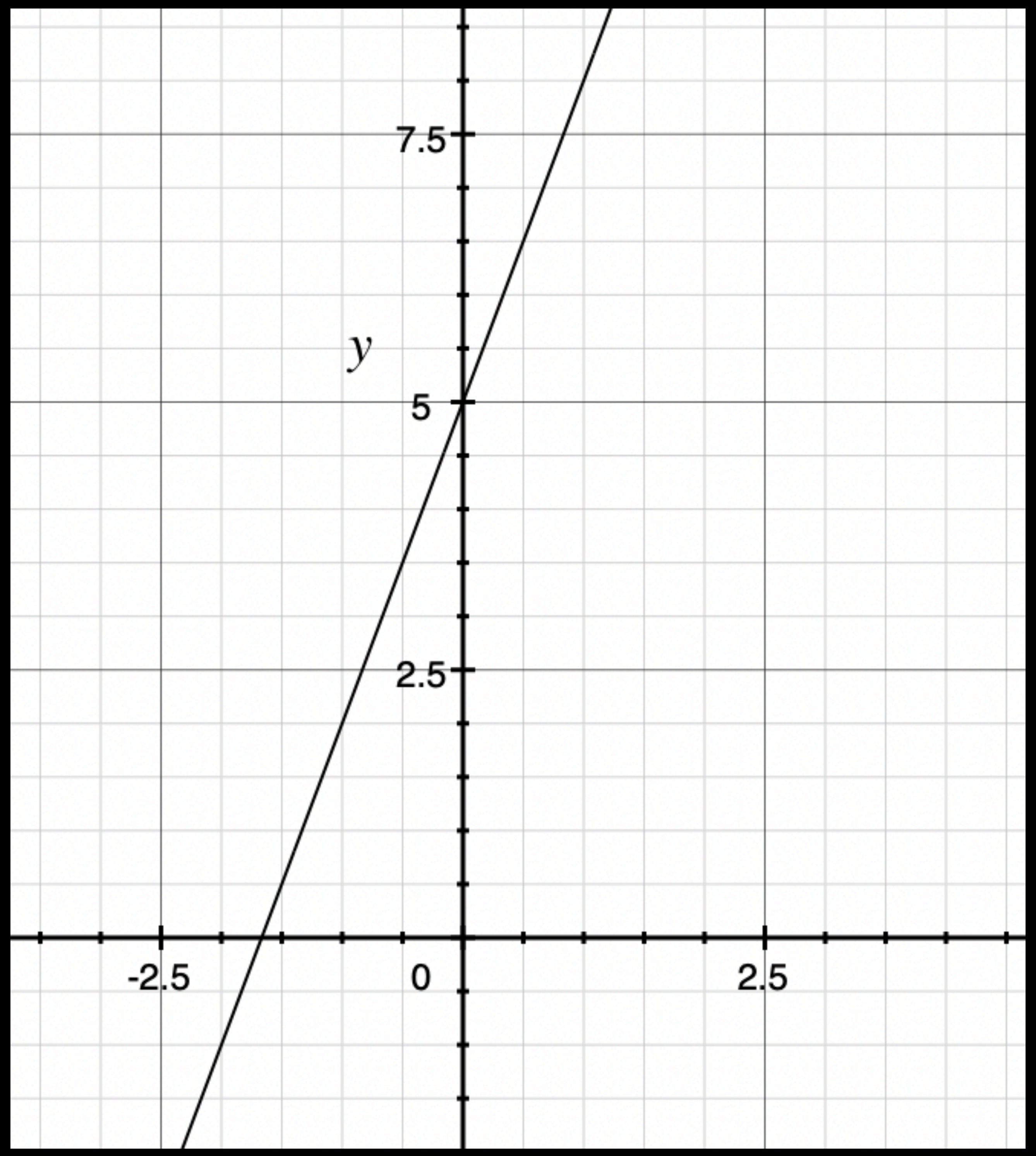
(Simple) linear secret sharing

- Let y=*mx*+*b* be the equation of a line
- Imagine I give you a point (x, y) for x != 0
- What can we learn about *b*?
  - For every b, (x, y) there exists a line that passes through (0, b)



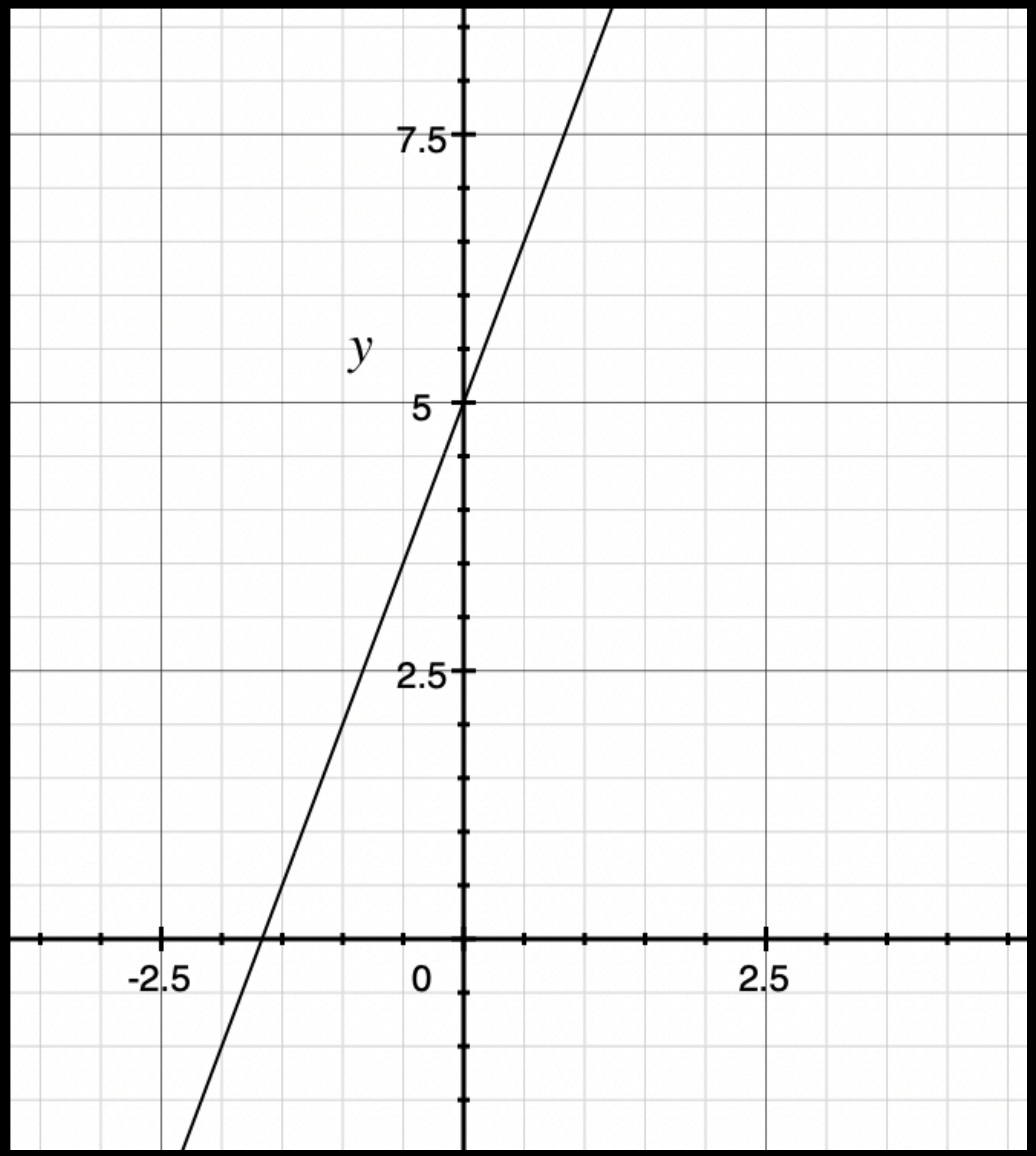
(Simple) linear secret sharing

- Let y=*mx*+*b* be the equation of a line
- Imagine I give you two distinct points (x2, y2), (x1, y1)
- What can we learn about *b*?



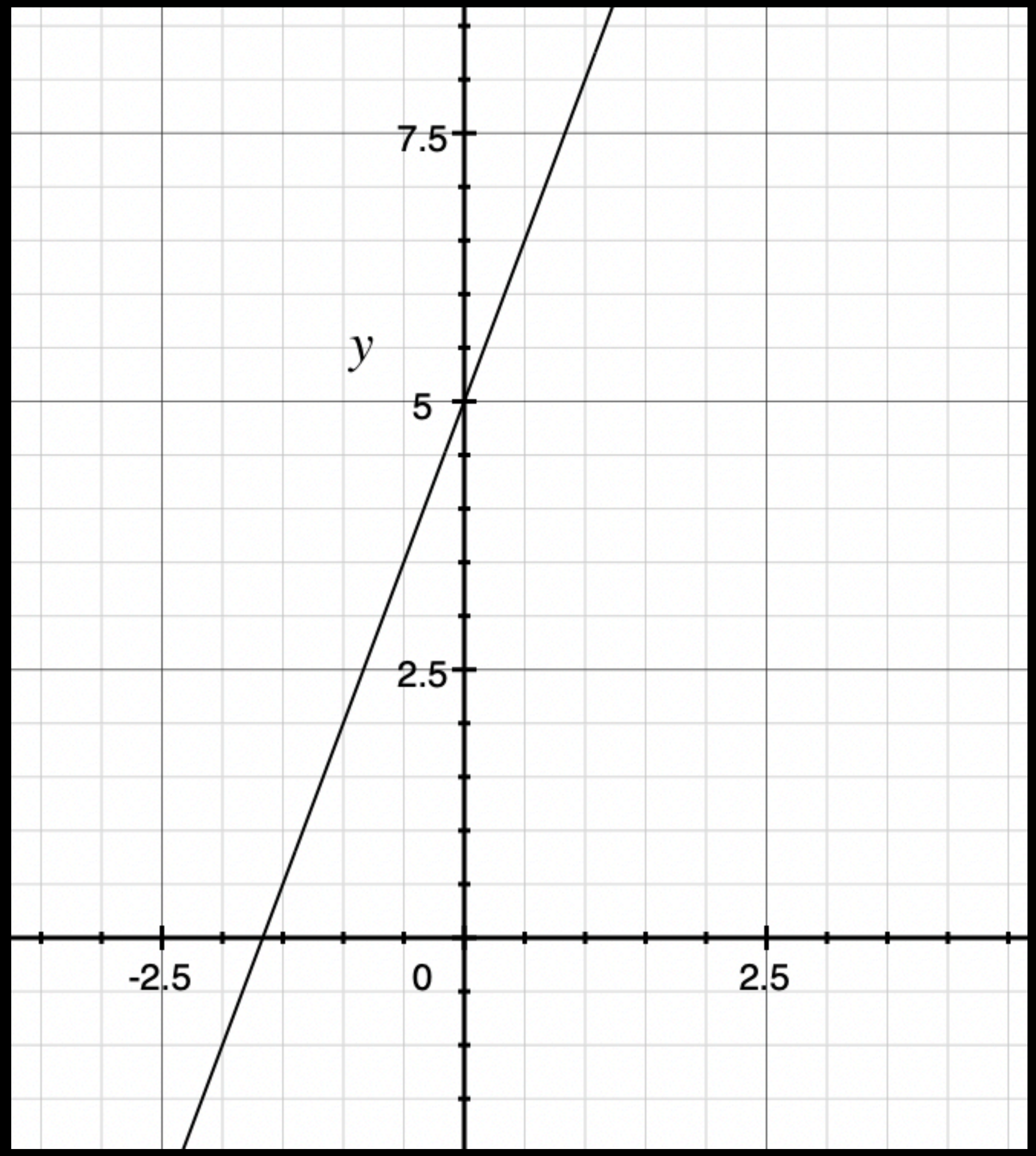
(Simple) linear secret sharing

- Further optimization:
  instead of computing over the real numbers,
  let's compute over the field Zp
- Let y=*mx+b mod p* be the equation of a line
- Same questions



(Simple) linear secret sharing

- This allows us to compute a 2-of-N secret sharing
- Fix some Zp (for largish p)
- Pick a line with constant term (y-intercept) set to *s* and a random coefficient (slope) *m*
- For x=1 to N, output shares:
  - t_i = (x, mx+s) mod p
- Recovery is just linear interpolation



Can we generalize this to M>2?

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- Shamir’s observations:
  - Any degree-(M-1) polynomial can be uniquely interpolated given M distinct points (using Lagrangian interpolation)
  - Given only M-1 points (or fewer) the polynomial is not constrained

Can we generalize this to M>2?

- Share(M, N, s):
  - Fix Zp
  - Sample coefficients (a1, …, a_{M-1}), and set P(x) to the polynomial defined by these coefficients, with constant term s
  - Compute shares: (1, P(1)), (2, P(2)), …, (N, P(N))

Other nice facts about secret sharing

- Polynomials can be added easily
  - Given two (random) polynomials F(), G() with constant terms s1, s2
  - The sum of F() + G() has constant term s1+s2
  - Similarly, adding together a vector of secret shares for secrets s1, s2 (respectively) will produce a set of shares for (s1 + s2)

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  - Similarly, adding together a vector of secret shares for secrets s1, s2 (respectively) will produce a set of shares for (s1 + s2)
  - Better yet, if F() and G() are random polynomials, then their sum will also be a random polynomial

Can we multiply secret shares?

- Not quite as elegantly
  - If we multiply two polynomials of degree d, we get a polynomial of degree 2d. Also it's not random anymore.
  - This also prevents us from just multiplying shares
  - However, there are *interactive* protocols for multiplying secret shares, then reducing the degree of the resulting polynomial