

Practical Cryptographic Systems

**Symmetric Cryptography II &
Asymmetric Cryptography**

Instructor: Matthew Green

Housekeeping

- **A2 (part 1) due tonight**
- **A2 (part 2) out now**
- **New reading: attacks on RSA paper**
 - **Dan Boneh**
- **Late day policy update (A2 and beyond):**
 - **3 total late days to be used at discretion**
 - **Please note these on your assignment!**
 - **25% per day late after that**

Housekeeping

- **Projects**
 - **I will put up a tentative list on Github and we'll talk Weds about this**

News

Review

- **Last time:**
 - **Padding oracles**
 - **Introduction to algebraic groups**
 - **Diffie-Hellman (MITM)**

Hash Functions

Asymmetric Crypto

- So far we've discussed symmetric crypto
 - Requires both parties to share a key
 - Key distribution is a hard problem!



Key Agreement

- Establish a shared key in the presence of a passive adversary



D-H Protocol

Malcolm Williamson in 72

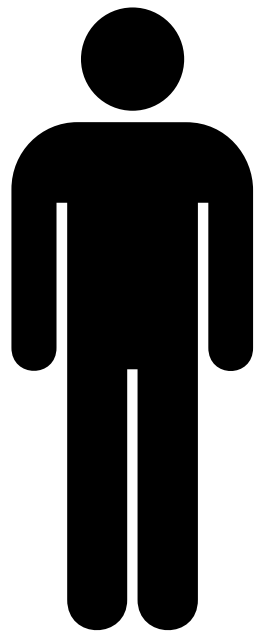
Diffie-Hellman in 76



$$b \in \mathbb{Z}_q$$

$$p, q : p = 2q + 1$$

$$a \in \mathbb{Z}_q$$



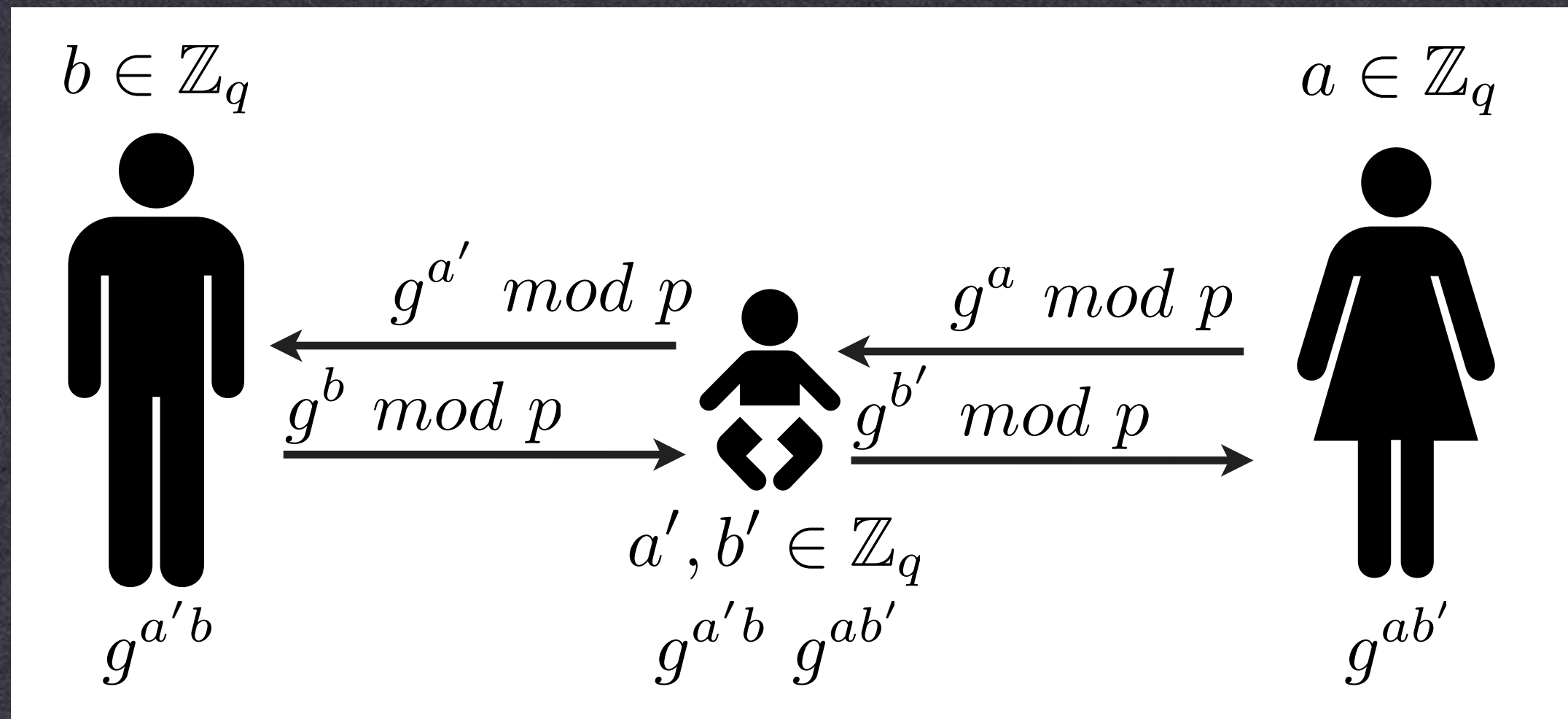
$$g^{ab}$$



$$g^{ab}$$

Man in the Middle

- Assume an active adversary:

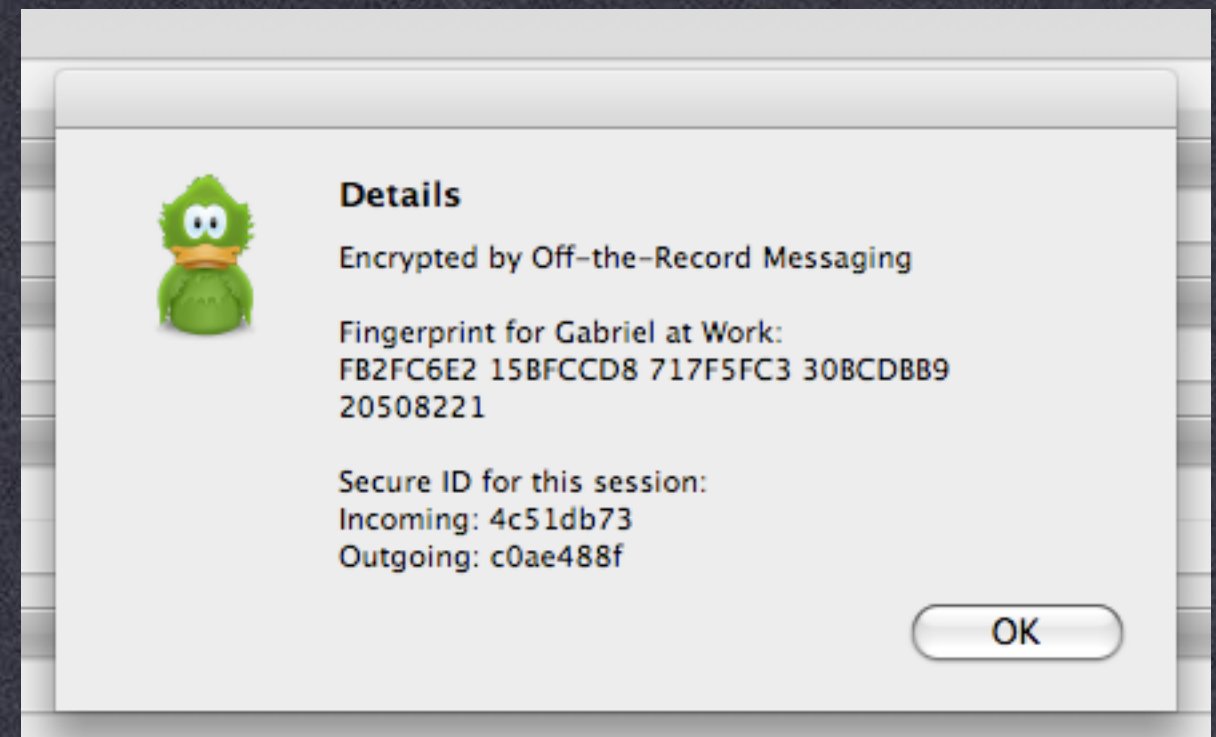


Man in the Middle

- **Caused by lack of authentication**
 - **D-H lets us establish a shared key with anyone...
but that's the problem...**
- **Solution: Authenticate the remote party**

Preventing MITM

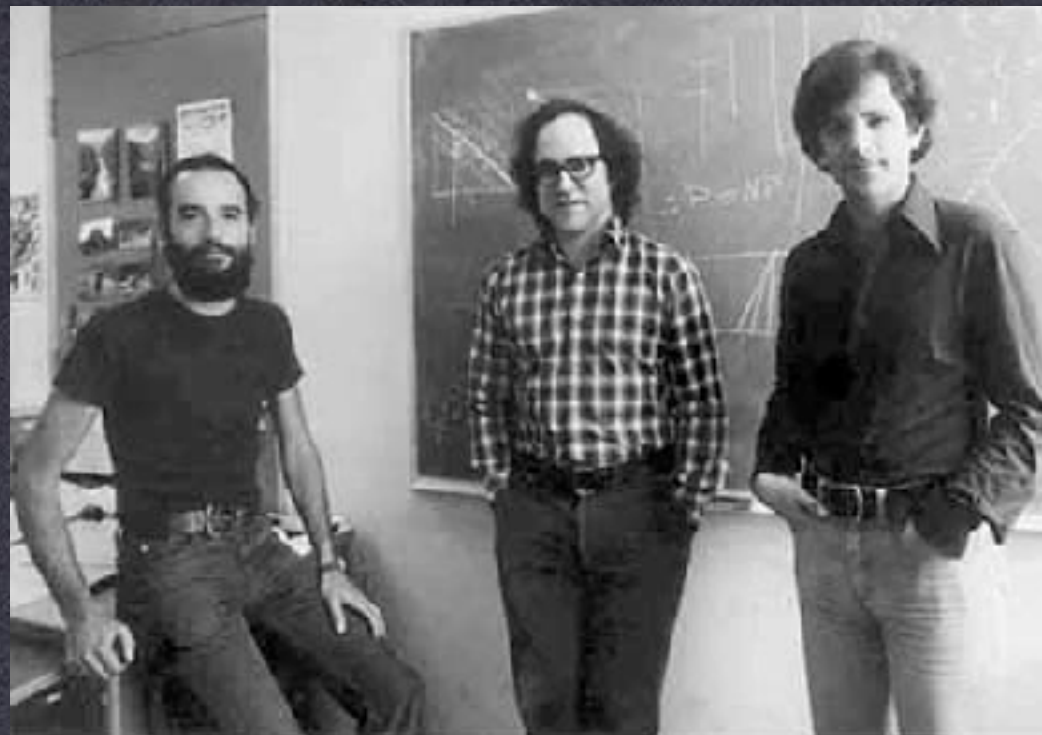
- **Verify key via separate channel**
- **Password-based authentication**
- **Authentication via PKI**



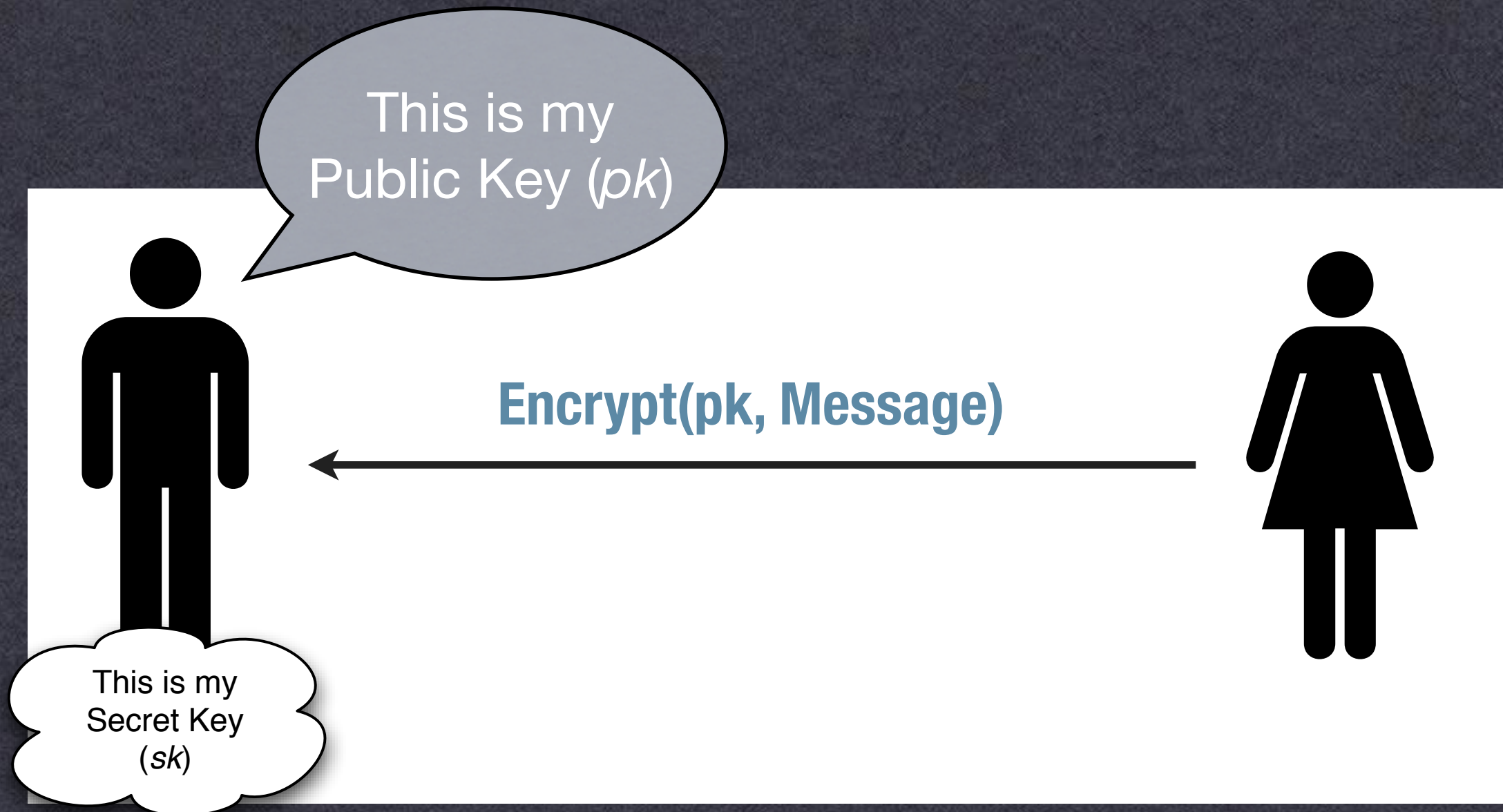
Public Key Encryption

- What if our recipient is offline?
 - Key agreement protocols are interactive
 - e.g., want to send an email

Ellis in 72, Cocks a few months later



Public Key Encryption



RSA Cryptosystem

Key Generation

Choose large primes: p, q

$$N = p \cdot q$$

$$\phi(N) = (p - 1)(q - 1)$$

Choose:

$$e : \gcd(e, \phi(N)) = 1$$

$$d : ed \bmod \phi(N) = 1$$

Output:

$$pk = (e, N)$$

$$sk = d$$

Encryption

$$c = m^e \bmod N$$

Decryption

$$m = c^d \bmod N$$

“Textbook RSA”

- In practice, we don't use Textbook RSA
 - Fully deterministic (not semantically secure)
 - Malleable

$$c' = c \cdot x^e \bmod N$$

$$c'^d = (m^e \cdot x^e)^d = m \cdot x \bmod N$$

- Might be partially invertible
- Coppersmith's attack: recover part of plaintext (when m and e are small)

RSA Padding

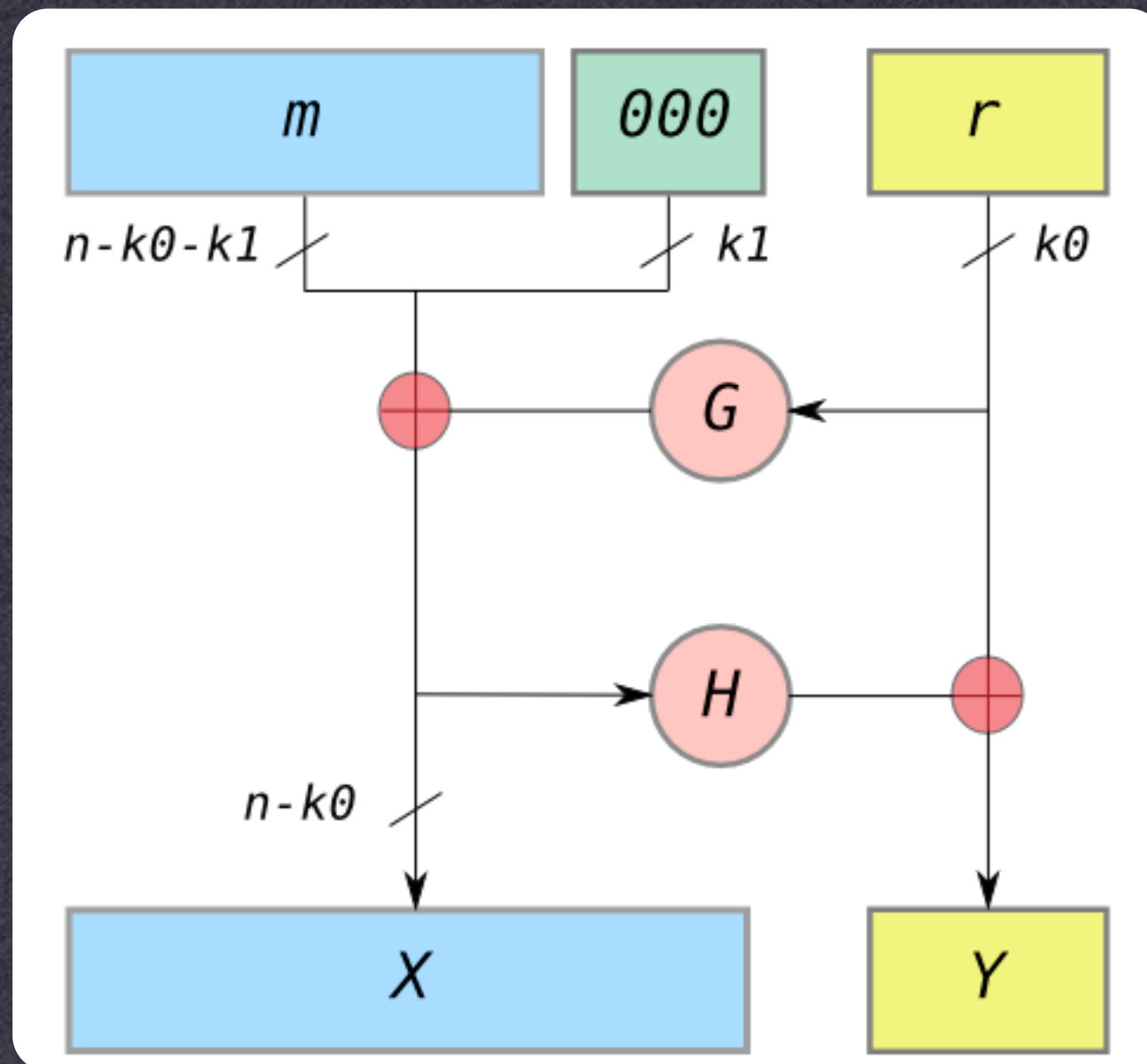
- **Early solution (RSA PKCS #1 v1.5):**
 - Add “padding” to the message before encryption
 - Includes randomness
 - Defined structure to mitigate malleability
 - PKCS #1 v1.5 badly broken (Bleichenbacher)



~ 1024 bits (128 bytes)

RSA Padding

- Better solution (RSA-OAEP):
 - G and H are hash functions



Efficiency

$m^e \bmod N$
 $e = 65,537$

$m^d \bmod N$

	Cycles/Byte
AES (128 bit key)	18
DES (56 bit key)	51
RSA (1024 bit key) <u>Encryption</u>	1,016
RSA (1024 bit key) <u>Decryption</u>	21,719

Hybrid Encryption

- **Mixed Approach**

- **Use PK encryption to encrypt a symmetric key**
- **Use (fast) symmetric encryption on data**

$$k \xleftarrow{\$} \{0, 1\}^k$$

$$C_k \leftarrow \text{RSA.Encrypt}_{pk}(k)$$

$$C_m \leftarrow \text{AES.Encrypt}_k(\text{message})$$

“Key encapsulation”



“Data encapsulation”

Key Strength

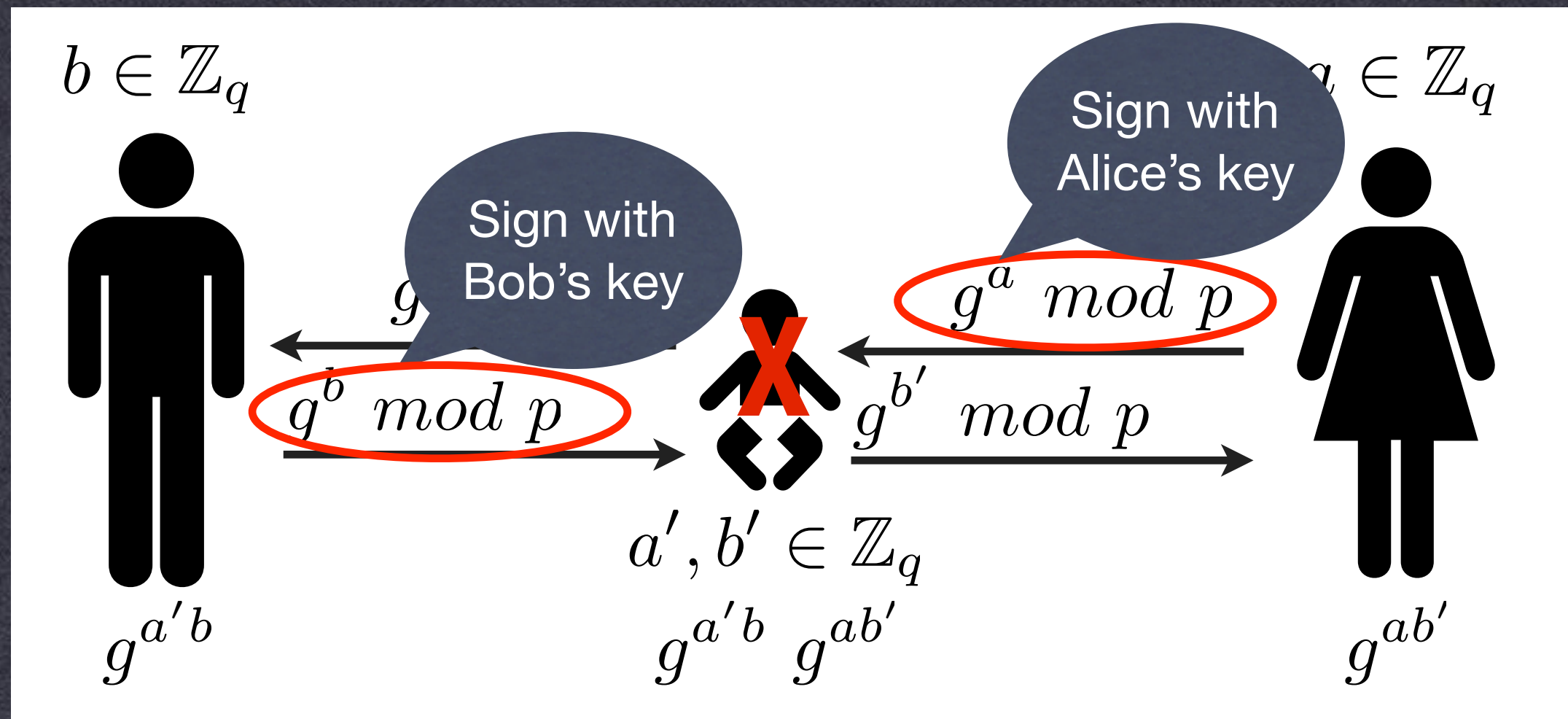
Level	Protection	Symmetric	Asymmetric	Discrete Logarithm Key Group		Elliptic Curve	Hash
1	Attacks in "real-time" by individuals <i>Only acceptable for authentication tag size</i>	32	-	-	-	-	-
2	Very short-term protection against small organizations <i>Should not be used for confidentiality in new systems</i>	64	816	128	816	128	128
3	Short-term protection against medium organizations, medium-term protection against small organizations	72	1008	144	1008	144	144
4	Very short-term protection against agencies, long-term protection against small organizations <i>Smallest general-purpose level, Use of 2-key 3DES restricted to 2^{40} plaintext/ciphertexts, protection from 2009 to 2011</i>	80	1248	160	1248	160	160
5	Legacy standard level <i>Use of 2-key 3DES restricted to 10^6 plaintext/ciphertexts, protection from 2009 to 2018</i>	96	1776	192	1776	192	192
6	Medium-term protection <i>Use of 3-key 3DES, protection from 2009 to 2028</i>	112	2432	224	2432	224	224
7	Long-term protection <i>Generic application-independent recommendation, protection from 2009 to 2038</i>	128	3248	256	3248	256	256
8	"Foreseeable future" <i>Good protection against quantum computers</i>	256	15424	512	15424	512	512

Digital Signatures

- **Similar to MACs, with public keys**
 - **Secret key used to sign data**
 - **Public key can verify signature**
 - **Advantages over MACs?**

Preventing MitM

- Assume an active adversary:



PKI & Certificates

- **How do I know to trust your public key?**
 - **Put it into a file with some other info, and get someone else to sign it!**



Next Time

- **Protocols & Implementation**
- **Reading!**
- **A2 coming up this week**