Annexe B

Développements limités usuels en zéro

Les développements limités suivants sont valables à l'origine et pour tout entier naturel n :

— $\mathrm{e}^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots + \dfrac{x^n}{n!} + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\sinh x = x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \ldots + \dfrac{x^{2n+1}}{(2n+1)!} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n+1}\big)$

— $\cosh x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \ldots + \dfrac{x^{2n}}{(2n)!} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n}\big)$

— $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \ldots + (-1)^n\,\dfrac{x^{2n+1}}{(2n+1)!} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n+1}\big)$

— $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \ldots + (-1)^n\,\dfrac{x^{2n}}{(2n)!} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n}\big)$

— $(1+x)^\alpha = 1 + \alpha\, x + \dfrac{\alpha\,(\alpha-1)}{2!}\,x^2 + \ldots + \dfrac{\alpha\,(\alpha-1)\,\ldots\,(\alpha-n+1)}{n!}\,x^n + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\ln(1-x) = -x - \dfrac{x^2}{2} - \dfrac{x^3}{3} - \dfrac{x^4}{4} - \ldots - \dfrac{x^n}{n} + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots + (-1)^n\,x^n + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots + (-1)^{n-1}\,\dfrac{x^n}{n} + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\sqrt{1+x} = 1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \ldots + (-1)^{n-1}\dfrac{1\cdot 3\cdot \ldots \cdot (2n-3)}{2\cdot 4\cdot \ldots \cdot (2n)}\,x^n + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\dfrac{1}{\sqrt{1+x}} = 1 - \dfrac{x}{2} + \dfrac{3}{8}x^2 - \ldots + (-1)^n\dfrac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot \ldots \cdot (2n)}\,x^n + \underset{x\to 0}{\mathrm{o}}(x^n)$

— $\arctan x = x - \dfrac{x^3}{3} + \ldots + (-1)^n\,\dfrac{x^{2n+1}}{2n+1} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n+1}\big)$

— $\operatorname{argtanh} x = x + \dfrac{x^3}{3} + \ldots + \dfrac{x^{2n+1}}{2n+1} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n+1}\big)$

— $\arcsin x = x + \dfrac{1}{2}\cdot\dfrac{x^3}{3} + \ldots + \dfrac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot \ldots \cdot (2n)}\cdot\dfrac{x^{2n+1}}{2n+1} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n+1}\big)$

— $\operatorname{argsinh} x = x - \dfrac{1}{2}\cdot\dfrac{x^3}{3} + \ldots + (-1)^n\,\dfrac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot \ldots \cdot (2n)}\cdot\dfrac{x^{2n+1}}{2n+1} + \underset{x\to 0}{\mathrm{o}}\big(x^{2n+1}\big)$

— $\tan x = x + \dfrac{x^3}{3} + \dfrac{2}{15}\,x^5 + \dfrac{17}{315}\,x^7 + \underset{x\to 0}{\mathrm{o}}\big(x^7\big)$

— $\tanh x = x - \dfrac{x^3}{3} + \dfrac{2}{15}\,x^5 - \dfrac{17}{315}\,x^7 + \underset{x\to 0}{\mathrm{o}}\big(x^7\big)$