Controle GPC Aplicado a um Robô com Tração Diferencial

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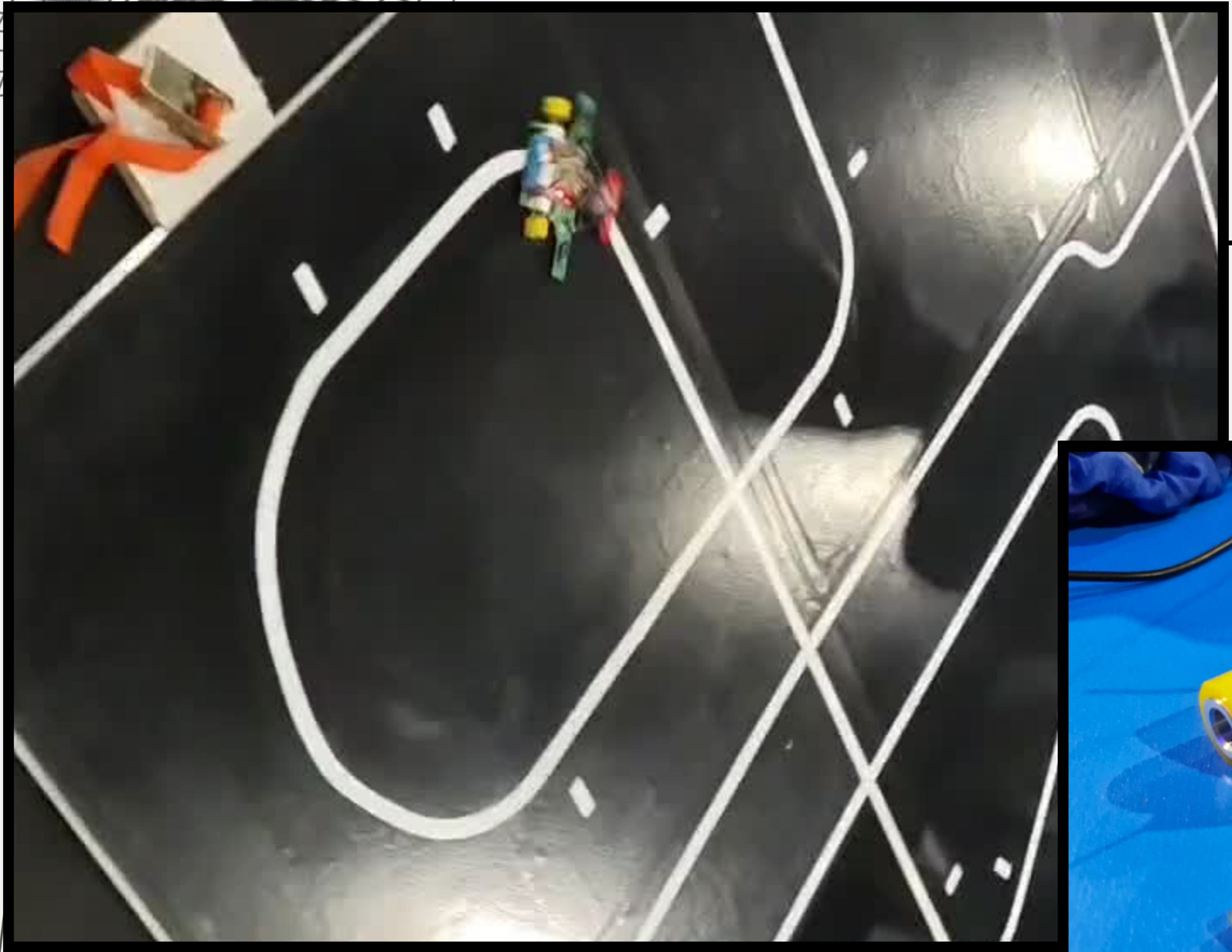
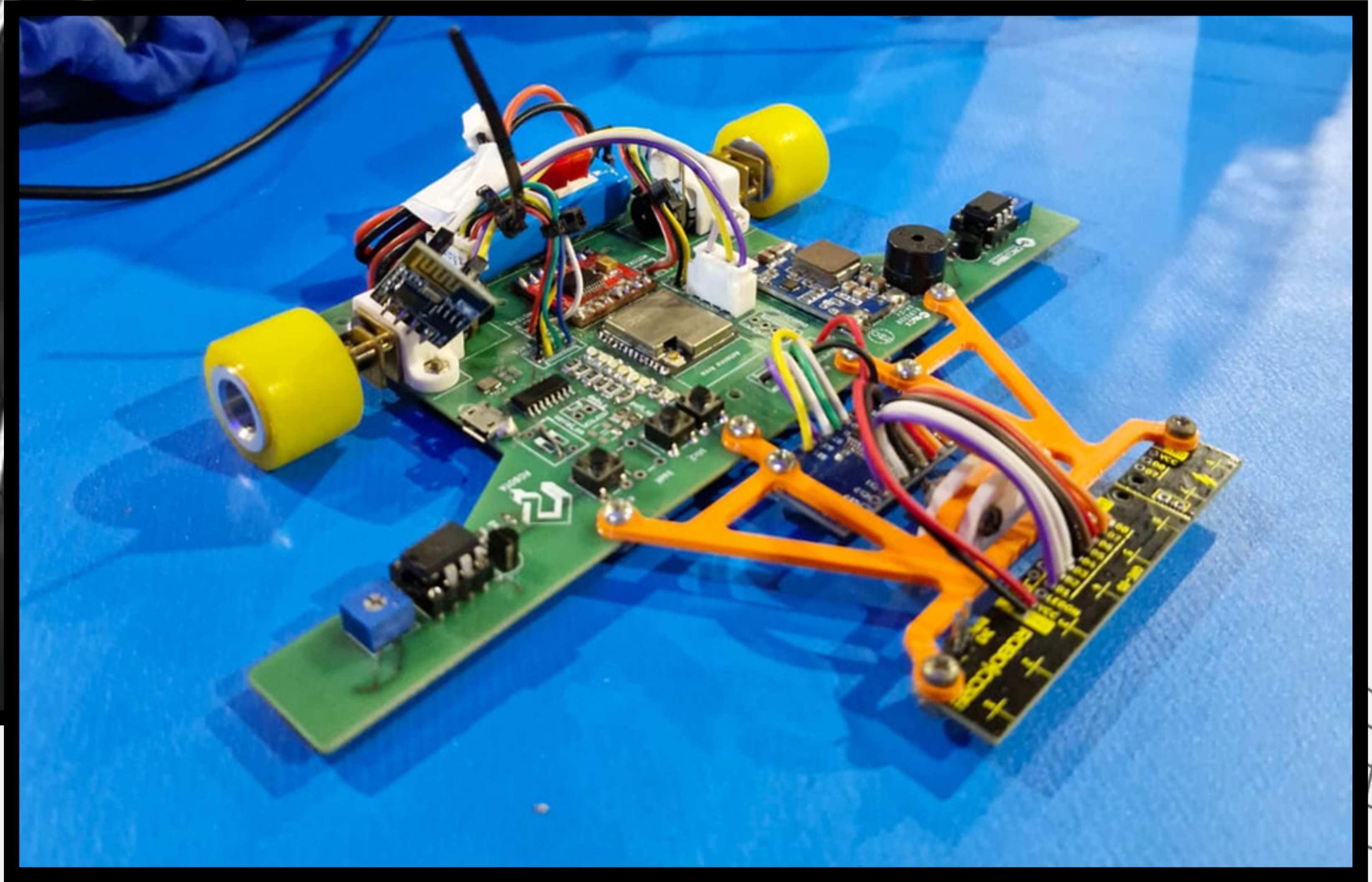
Sumário

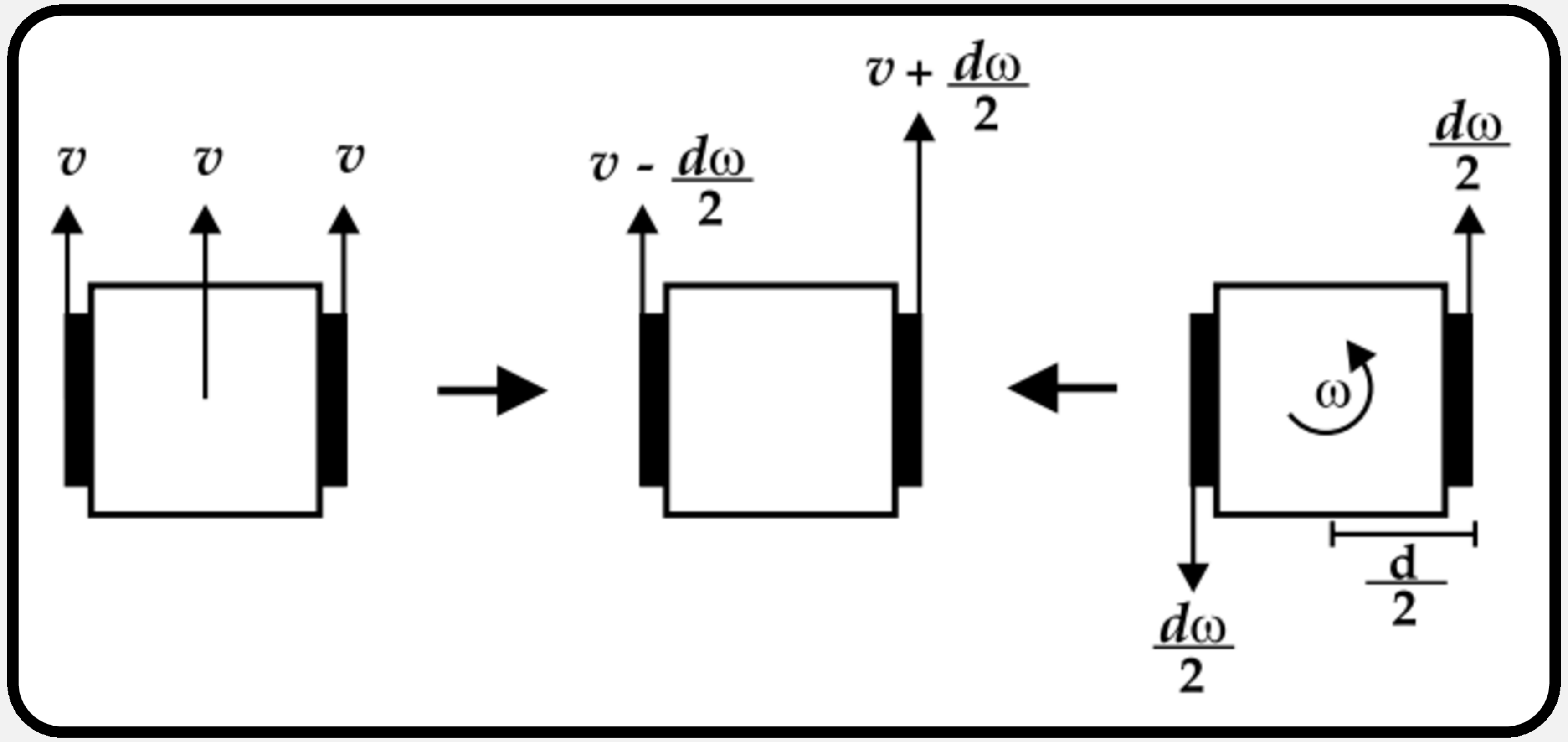
Introdução

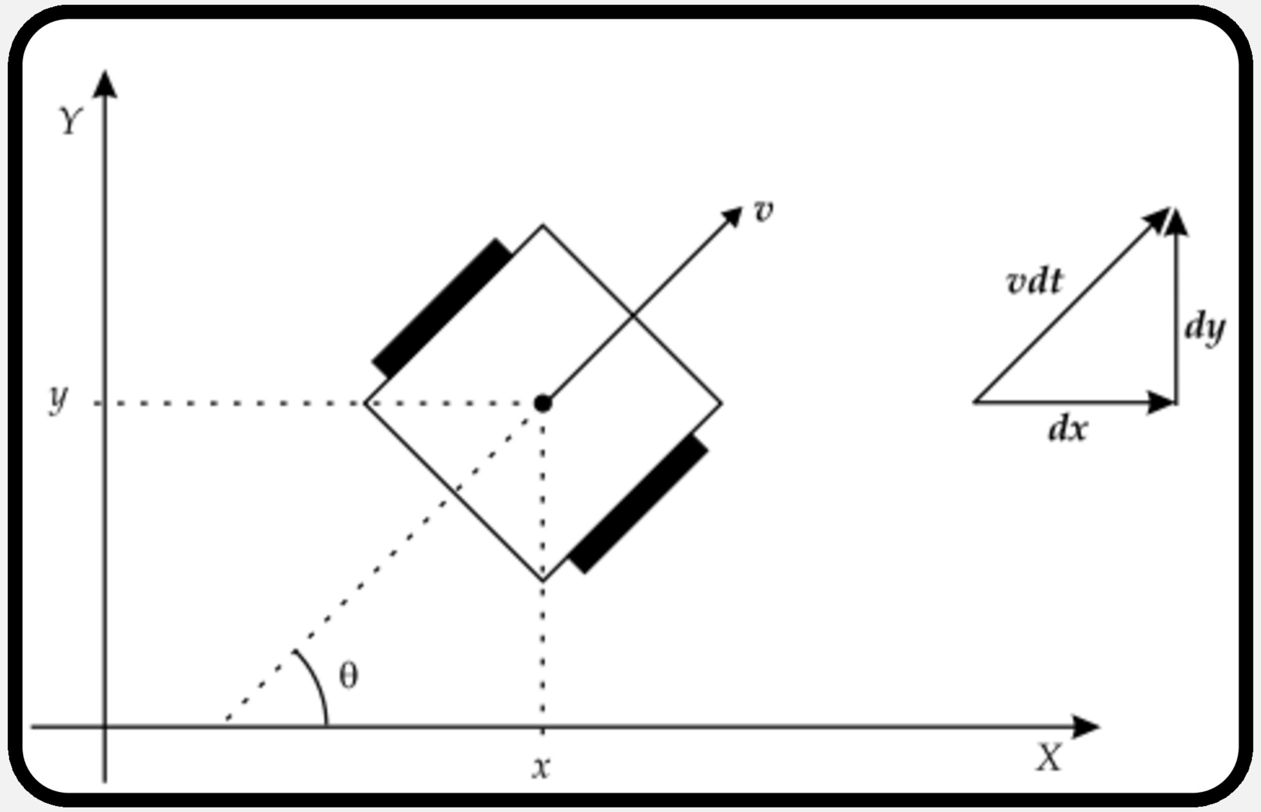




Modelagem Cinemática

Relação entre as velocidades





$$v_e = v - \frac{d}{2}\omega$$

$$v_d = v + \frac{d}{2}\omega$$

$$\omega = \omega_d \frac{r_d}{d} - \omega_e \frac{r_e}{d}$$

$$v = \omega_d \frac{r_d}{2} + \omega_e \frac{r_e}{2}$$

$$\begin{cases} \dot{x} = v\cos(\theta) \\ \dot{y} = v\sin(\theta) \\ \dot{\theta} = \omega \end{cases}$$

Modelagem Dinâmica

Modelagem dos Atuadores

Circuito de armaduta
Balanço de torque

$$u = L\frac{di}{dt} + Ri + K_\omega \omega$$

$$t_r = K_t i = J\frac{d\omega}{dt} + \beta\omega$$

Eliminando a corrente

$$LJ\frac{d^2\omega}{dt^2} + (L\beta + RJ)\frac{d\omega}{dt} + (R\beta + K_\omega K_t)\omega = K_t u$$

Considerando que L é desprezível

$$RJ\frac{d\omega}{dt} + (R\beta + K_\omega K_t)\omega = K_t u$$

$$t_r = \rho K_t u - \beta\omega - J\frac{d\omega}{dt}$$

Modelagem Dinâmica

Esforços Sobre a Estrutura

Força e torque sobre a estruta

$$f = f_d + f_e = \frac{t_d}{r_d} + \frac{t_e}{r_e}$$

$$t = t_d \frac{d}{2r_d} - t_e \frac{d}{2r_e}$$

Utilizando a dinâmica dos motores

$$\begin{bmatrix} t_d \\ t_e \end{bmatrix} = \begin{bmatrix} \rho_d K_{td} & 0 \\ 0 & \rho_e K_{te} \end{bmatrix} \begin{bmatrix} u_d \\ u_e \end{bmatrix} - \begin{bmatrix} \beta_d & 0 \\ 0 & \beta_e \end{bmatrix} \begin{bmatrix} \omega_d \\ \omega_e \end{bmatrix} - \begin{bmatrix} J_d & 0 \\ 0 & J_e \end{bmatrix} \begin{bmatrix} \dot{\omega}_d \\ \dot{\omega}_e \end{bmatrix}$$

Substituindo pela velocidade linear e angular

$$\begin{bmatrix} \omega_d \\ \omega_e \end{bmatrix} = \begin{bmatrix} \frac{1}{r_d} & \frac{d}{2r_d} \\ \frac{1}{r_e} & -\frac{d}{2r_e} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$\mathbf{t} = \mathbf{K}_{mot}\mathbf{u} - \mathbf{B}_{mot}\,{}^{\omega}\mathbf{T}_{\mathbf{v}}\mathbf{v} - \mathbf{J}_{mot}\,{}^{\omega}\mathbf{T}_{\mathbf{v}}\dot{\mathbf{v}}$$

$$\mathbf{t} = \begin{bmatrix} t_d \\ t_e \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} u_d \\ u_e \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} v \\ \omega \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} f \\ t \end{bmatrix} \quad {}^{\omega}\mathbf{T}_{\mathbf{v}} = \begin{bmatrix} \frac{1}{r_d} & \frac{d}{2r_d} \\ \frac{1}{r_e} & -\frac{d}{2r_e} \end{bmatrix}$$

$$\mathbf{K}_{mot} = \begin{bmatrix} \rho_d K_{td} & 0 \\ 0 & \rho_e K_{te} \end{bmatrix} \quad \mathbf{B}_{mot} = \begin{bmatrix} \beta_d & 0 \\ 0 & \beta_e \end{bmatrix} \quad \mathbf{J}_{mot} = \begin{bmatrix} J_d & 0 \\ 0 & J_e \end{bmatrix} \quad {}^{\mathbf{f}}\mathbf{T}_{\mathbf{t}} = \begin{bmatrix} \frac{1}{r_d} & \frac{1}{r_e} \\ \frac{d}{2r_d} & -\frac{d}{2r_e} \end{bmatrix}$$

$$\mathbf{f} = {}^{\mathbf{f}}\mathbf{T}_{\mathbf{t}}\mathbf{t}$$

Modelagem Dinâmica

Dinâmica Total do Robô

$$m\dot{v} + \beta_l v = \frac{t_d}{r_d} + \frac{t_e}{r_e}$$

$$J\dot{\omega} + \beta_\theta \omega = t_d \frac{d}{2r_d} - t_e \frac{d}{2r_e}$$

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \beta_l & 0 \\ 0 & \beta_\theta \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{r_d} & \frac{1}{r_e} \\ \frac{d}{2r_d} & -\frac{d}{2r_e} \end{bmatrix} \begin{bmatrix} t_d \\ t_e \end{bmatrix}$$

deixando v em função de u

$$(\mathbf{M}_{robo} + {}^{\mathrm{f}}\mathbf{T}_{\mathrm{t}} \mathbf{J}_{mot} {}^{\omega}\mathbf{T}_{\mathbf{v}})\dot{\mathbf{v}} + (\mathbf{B}_{robo} + {}^{\mathrm{f}}\mathbf{T}_{\mathrm{t}} \mathbf{B}_{mot} {}^{\omega}\mathbf{T}_{\mathbf{v}})\mathbf{v} = {}^{\mathrm{f}}\mathbf{T}_{\mathrm{t}} \mathbf{K}_{mot}\mathbf{u}$$

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{B}\mathbf{v} = \mathbf{K}\mathbf{u}$$

$$\mathbf{M} = \mathbf{M}_{robo} + {}^{\mathrm{f}}\mathbf{T}_{\mathrm{t}} \mathbf{J}_{mot} {}^{\omega}\mathbf{T}_{\mathbf{v}} \qquad \mathbf{B} = \mathbf{B}_{robo} + {}^{\mathrm{f}}\mathbf{T}_{\mathrm{t}} \mathbf{B}_{mot} {}^{\omega}\mathbf{T}_{\mathbf{v}} \qquad \mathbf{K} = {}^{\mathrm{f}}\mathbf{T}_{\mathrm{t}} \mathbf{K}_{mot}$$

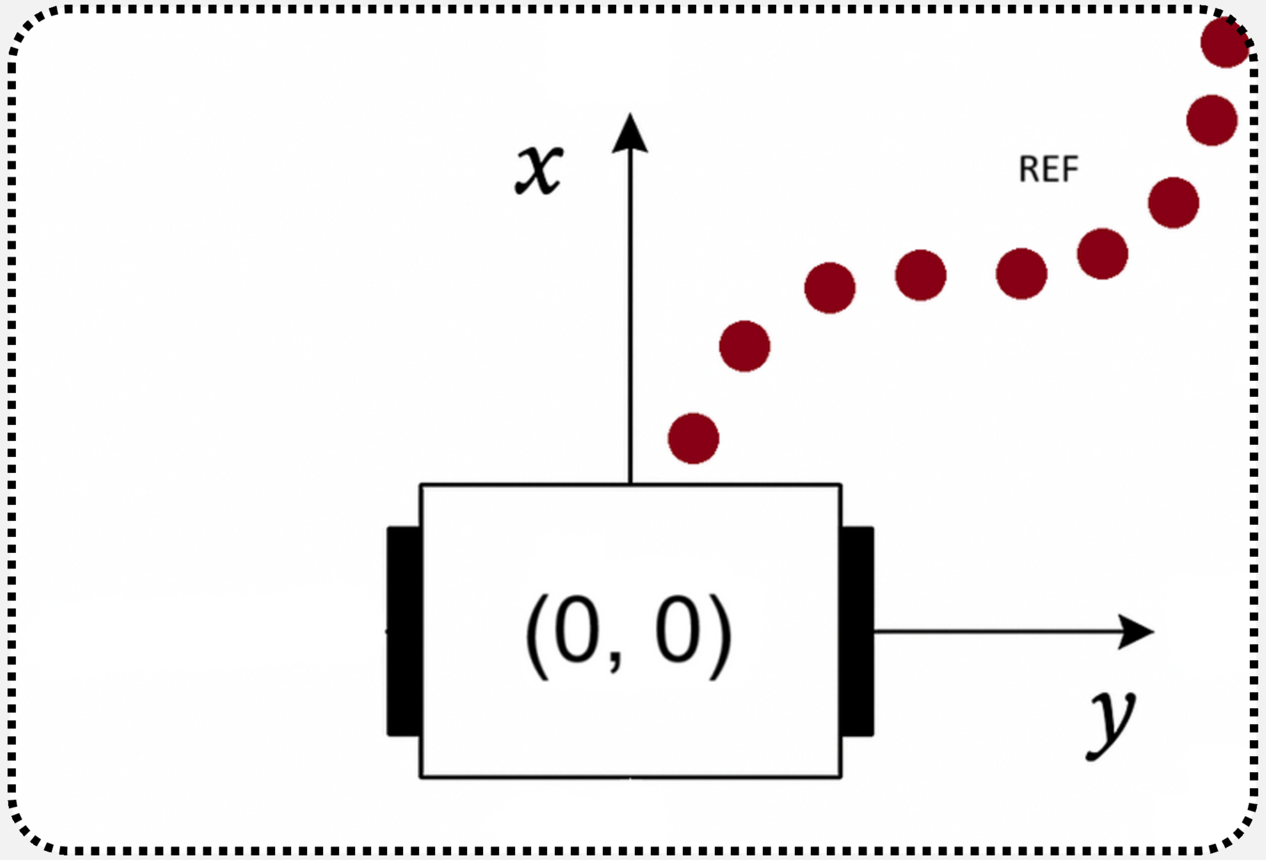
Modelagem Dinâmica

Resultado Final

$$\begin{cases} \dot{\mathbf{v}} = -\mathbf{M}^{-1}\mathbf{B}\mathbf{v} + \mathbf{M}^{-1}\mathbf{K}\mathbf{u} \\ \dot{\mathbf{q}} = {}^{v}\mathbf{T}_{(\mathbf{x},\mathbf{y})}\mathbf{v} \end{cases}$$

$${}^{v}\mathbf{T}_{(\mathbf{x},\mathbf{y})} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad \dot{\mathbf{q}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

Linearização



$$\begin{cases} x \approx v_0\cos(\theta_0) + \cos(\theta_0)\Delta v - v_0\sin(\theta_0)\Delta\theta \\ y \approx v_0\sin(\theta_0) + \sin(\theta_0)\Delta v + v_0\cos(\theta_0)\Delta\theta \end{cases}$$

$$\begin{cases} \dot{x} \approx v \\ \dot{y} \approx v_0\theta \\ \dot{\theta} = \omega \end{cases} \quad \Rightarrow \quad \begin{cases} x \approx s \\ y \approx s_0\theta \end{cases}$$

Controle GPC

Função de transferência v e w para as entradas ue e ud

$$\mathbf{H}_{dv}(z^{-1}) = \frac{b_{dv}z^{-1}}{1-a_d z^{-1}} \qquad \mathbf{H}_{d\omega}(z^{-1}) = \frac{b_{d\omega}z^{-1}}{1-a_d z^{-1}}$$

$$\mathbf{H}_{ev}(z^{-1}) = \frac{b_{ev}z^{-1}}{1-a_e z^{-1}} \qquad \mathbf{H}_{e\omega}(z^{-1}) = \frac{b_{e\omega}z^{-1}}{1-a_e z^{-1}}$$

Função de transferência s e theta para as entradas delta ue e delta ud

$$\mathbf{H}_{ds}(z^{-1}) = \frac{b_{dv}z^{-1}}{1-a_d z^{-1}} \left(\frac{T}{1-z^{-1}}\right)^2 \qquad \mathbf{H}_{d\theta}(z^{-1}) = \frac{b_{d\theta}z^{-1}}{1-a_d z^{-1}} \left(\frac{T}{1-z^{-1}}\right)^2$$

$$\mathbf{H}_{es}(z^{-1}) = \frac{b_{ev}z^{-1}}{1-a_e z^{-1}} \left(\frac{T}{1-z^{-1}}\right)^2 \qquad \mathbf{H}_{e\theta}(z^{-1}) = \frac{b_{e\theta}z^{-1}}{1-a_e z^{-1}} \left(\frac{T}{1-z^{-1}}\right)^2$$

$$\hat{\mathbf{y}} = \mathbf{G}\Delta\mathbf{u} + \mathbf{F}(z^{-1})\mathbf{y}[k]$$

$$J_u = \boldsymbol{\delta}(\hat{\mathbf{y}} - \mathbf{w})^2 + \boldsymbol{\lambda}\Delta\mathbf{u}^2$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{ds} & \mathbf{G}_{es} \\ \mathbf{G}_{d\theta} & \mathbf{G}_{e\theta} \end{bmatrix} \qquad \Delta\mathbf{u} = \begin{bmatrix} \Delta\mathbf{u_d} \\ \Delta\mathbf{u_e} \end{bmatrix} \qquad \mathbf{F}(z^{-1}) = {}^{\omega}\mathbf{T_v}^{-1} \begin{bmatrix} F_{\omega d} & 0 \\ 0 & F_{\omega e} \end{bmatrix}$$

Simulação

Referências

[1] K. Ogata. *Modern Control Engineering.* Prentice Hall, 3rd edition, 1997.

[2] F. C. Vieira. Controle dinâmico de robôs móveis com acionamento diferencial. Dissertação de mestrado, Universidade Federal do Rio Grande do Norte, Natal–RN, Brasil, Feb. 2005. Disponível em PDF.

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