Forecast Time Series Data Project 2

Alison Yao (yy2564)

Data Description

This project uses the daily stock price of Netflix from 2002-05-23 to 2022-03-18 (4991 observations) for the time series forecast. The 2022-03-18 data is the latest entry because I obtained the data on April 30th from Yahoo Finance. By the time this report is graded, there may or may not be data for later days, but I am not able to obtain it at the time being. The head of the table is shown below:

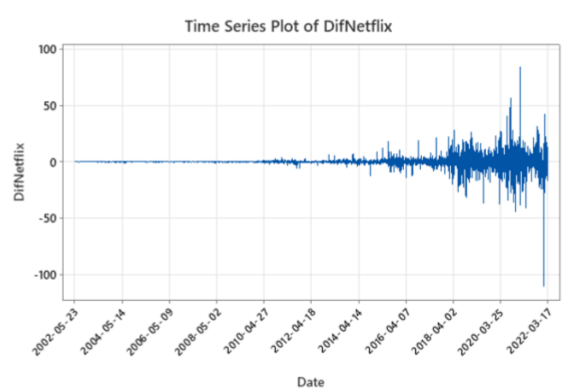
NFLX

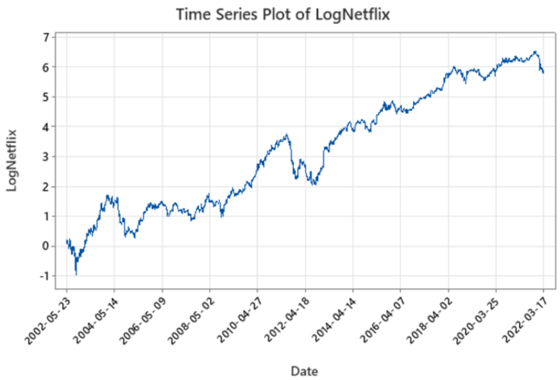
Date	Open	High	Low	Close	Adj Close	Volume
2002-05-23	1.156429	1.242857	1.145714	1.196429	1.196429	104790000
2002-05-24	1.214286	1.225000	1.197143	1.210000	1.210000	11104800
2002-05-28	1.213571	1.232143	1.157143	1.157143	1.157143	6609400
2002-05-29	1.164286	1.164286	1.085714	1.103571	1.103571	6757800
2002-05-30	1.107857	1.107857	1.071429	1.071429	1.071429	10154200

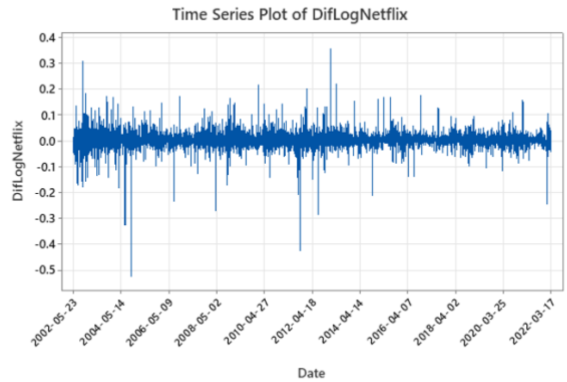
As instructed, I will leave out the last data point, which is the data for 2022-03-18 for the ARIMA-ARCH modeling. Therefore, $n = 4991 - 1 = 4990$ and I will be using the Adj Close data from 2002-05-23 to 2022-03-17.

ARIMA modeling

1. The time series plots of Netflix stock price (Adj Close), DifNetflix, LogNetflix, and DifLogNetflix are as follows:

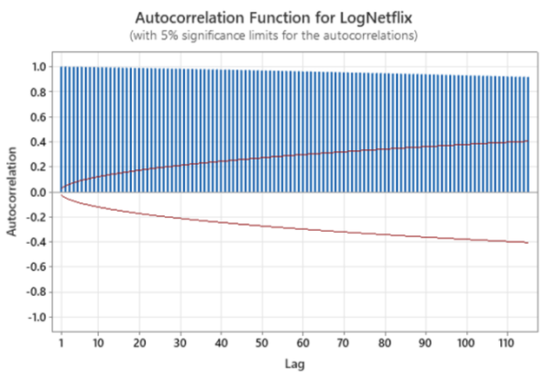


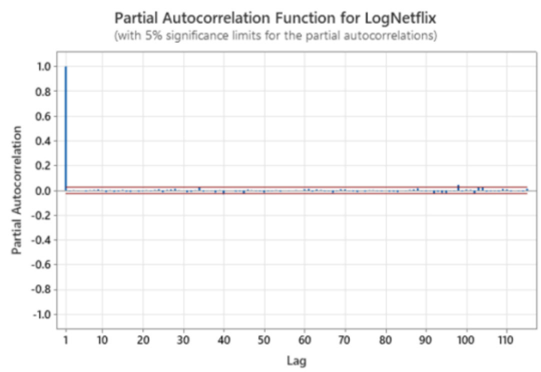




There is strong evidence that Netflix has level-dependent volatility. Netflix stock prices are more volatile in recent years. Taking log can help us mitigate level-dependent volatility and linearize the plot, as the volatility of DifLogNetflix does not seem to depend on a level anymore.

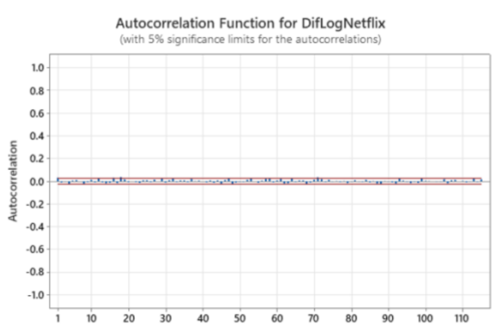
For choosing an ARIMA model, we need to first determine the value of d and see how many times we need to difference the data to make it stationary. The ACF and PACF plot of LogNetflix is as follows:

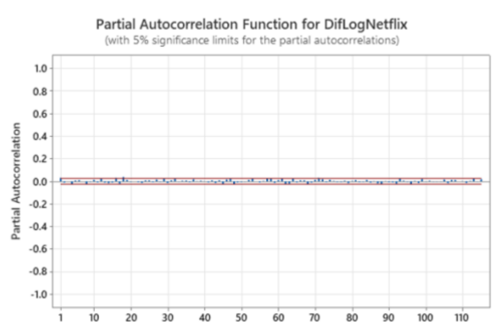




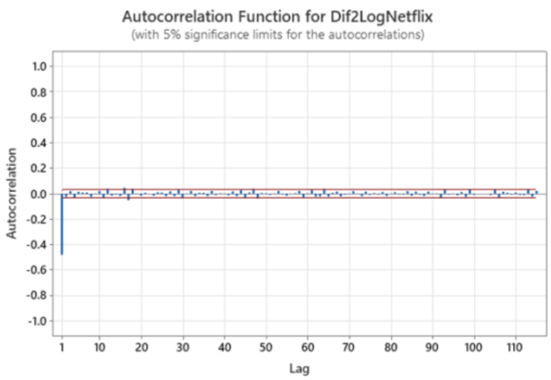
The ACF plot of LogNetflix indicates a hanging behavior and the PACF plot of LogNetflix cuts off after lag 1, so LogNetflix is non-stationary. We need to difference the data.

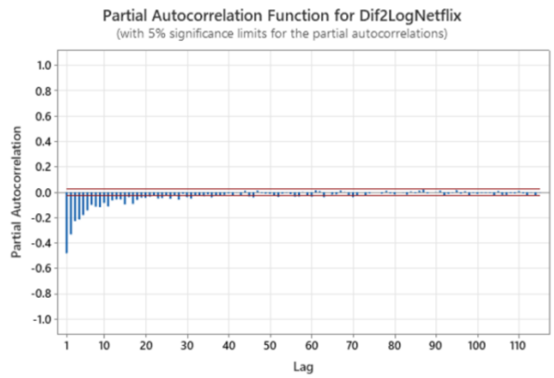
The ACF and PACF plots of DifLogNetflix show that differencing once seems to make the data stationary, as there are no statistically significant lags.





To double-check if d=1, we can difference the data again and look for a sign of over-differencing.





Indeed, there is strong evidence of over-differencing as the PCF of Dif2LogNetflix is statistically significantly negative at lag 1 and close to -0.5. Therefore, we conclude that d=1 in the ARIMA model. The ACF plot of DifLogNetflix suggests an ARIMA(0,1,4) model, while the PACF of DifLogNetflix suggests an ARIMA(4,1,0) model.

2. Now, we need to select the p and q in the ARIMA model. Let's pick p,q = 0,1,2 arbitrarily and compare AICc. We know N = n – d = 4990 – 1 = 4989.

Without Constant					With Constant				
p	d	q	SS	AICc	p	d	q	SS	AICc
0	1	0	6.42032	-33202.40248	0	1	0	6.41372	-33205.53214
0	1	1	6.41394	-33205.36101	0	1	1	6.40772	-33208.19910
0	1	2	6.41325	-33203.89534	0	1	2	6.40691	-33206.82659
1	1	0	6.41406	-33205.26767	1	1	0	6.40785	-33208.09788
1	1	1	6.41215	-33204.75113	1	1	1	6.40578	-33207.70658
1	1	2	6.41264	-33202.36668	1	1	2	6.40593	-33205.58575
2	1	0	6.41330	-33203.85644	2	1	0	6.40696	-33206.78765
2	1	1	6.41264	-33202.36668	2	1	1	6.40593	-33205.58575
2	1	2	6.41205	-33200.82171	2	1	2	6.40558	-33203.85352

ARIMA(0,1,1) with constant gives us the smallest AICc value, so we choose ARIMA(0,1,1) with constant.

The Minitab output for ARIMA(0,1,1) with constant is:

Final Estimates of Parameters

Type	Coef	SE Coef	T-Value	P-Value
MA 1	-0.0309	0.0142	-2.19	0.029
Constant	0.001151	0.000523	2.20	0.028

Differencing: 1 regular difference

Number of observations: Original series 4990, after differencing 4989

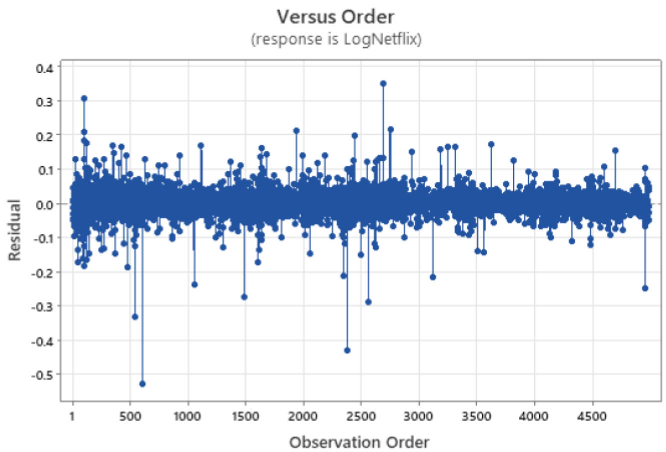
Both the MA1 coefficient and the constant are statistically significant, with p-values less than 0.05. Therefore, if we denote $\{x_t\}$ as the time series of Netflix, $\{y_t\}$ as LogNetflix and $\{z_t\}$ as DifLogNetflix. The best estimate of the MA1 coefficient is -0.0309 and that of the constant is 0.001151. Therefore, the fitted model is $z_t = \epsilon_t + 0.0309\epsilon_{t-1} + 0.001151$ where $z_t = y_t - y_{t-1} = log\, x_t - log\, x_{t-1}$.

The one step ahead forecast and 95% forecast interval are as follows:

Forecasts from period 4990

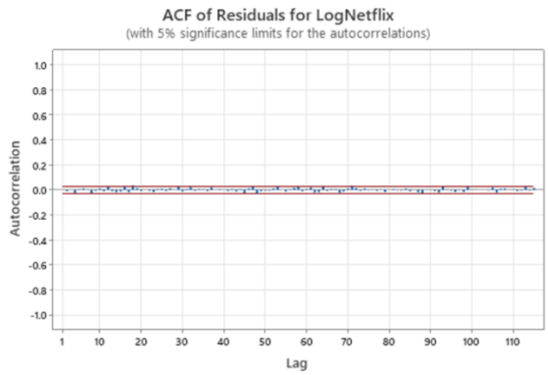
		95% Limits		
Period	Forecast	Lower	Upper	Actual
4991	5.91954	5.84927	5.98981	

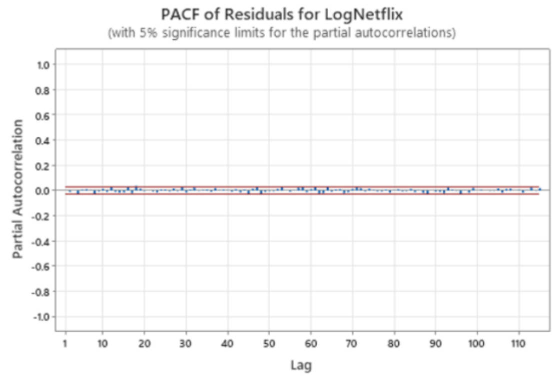
3. The time series plot of the residuals is:



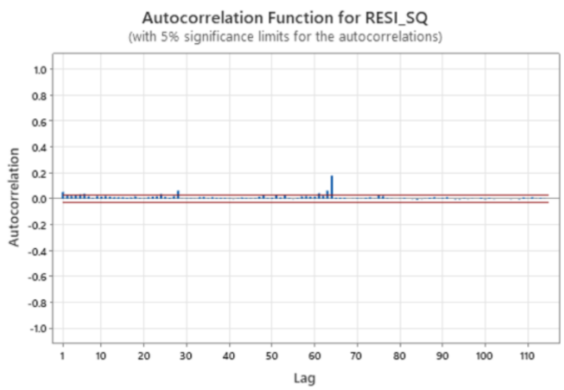
The residuals look relatively random, with a few with absolute values over 0.3.

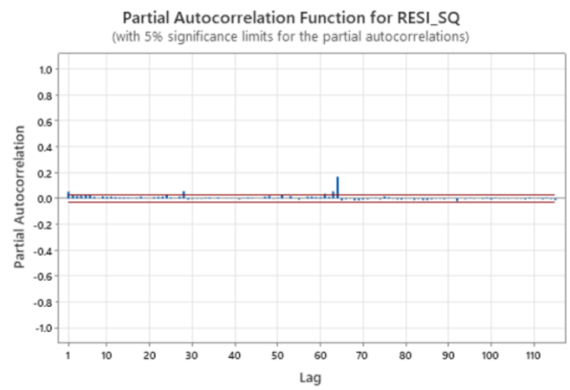
The ACF and PACF plots of the residuals are as follows:





There aren't many statistically significant lags in the residuals, meaning that the residuals look uncorrelated. The ACF and PACF of squared residuals are:





However, the squared residuals have multiple lags that are statistically significant, meaning that the residuals are not independent. Also, we can see from the time series plot of the residuals that the variance is not constant. There exists evidence of conditional heteroscedasticity.

ARCH/GARCH modeling

4. To select the ARCH(q) model with q ranging from 0 to 10, we can calculate the AICc. We know N = n – d = 4990 – 1 = 4989.

q	logLik	AICc	q	logLik	AICc
0	9528.019	-19054.03719791458	6	9745.581	-19477.13951455531
1	9629.999	-19255.99559326113	7	9745.042	-19474.05508433735
2	9642.285	-19278.56518555667	8	9701.958	-19385.879848162283
3	9649.818	-19291.627974317817	9	9739.789	-19459.533805544397
4	9662.440	-19314.867959060808	10	9729.909	-19437.764955997587
5	9746.715	-19481.413139301487			

We can also consider the GARCH(1,1) model. We have logLike = 9799.593 and the corresponding AICc is -19593.18118555667. GARCH(1,1) yields the smallest AICc, so we choose GARCH(1,1).

```
> print(summary(model))

Call:
garch(x = res, order = c(1, 1), trace = F)

Model:
GARCH(1,1)

Residuals:
      Min        1Q    Median        3Q       Max
-12.15300  -0.45908  -0.02503   0.46956   9.20783

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 5.875e-06   3.106e-07    18.92   <2e-16 ***
a1 1.425e-02   5.761e-04    24.73   <2e-16 ***
b1 9.814e-01   6.882e-04  1426.09   <2e-16 ***
```

```
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 79436, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.61832, df = 1, p-value = 0.4317

> print(logLik(model))
'log Lik.' 9799.593 (df=3)
```

The p-values are not correct because they are for a 2-tailed test. We should look at half of the presented p-values, but they are so small that half of any of these p-values is strongly statistically significant. Therefore, a0, a1, and b1 are all statistically significant. However, the Jarque Bera test has p-value less than 0.05, which indicates that the conditional distribution of GARCH residuals is not normally distributed. The Ljung-Box test has a p-value greater than 0.05, which means that the GARCH residuals are not correlated. But overall, the model is inadequate.

To write the complete form of the chosen GARCH(1,1) model, we have $\omega = 0.000005875, \alpha = 0.01425 \; and \; \beta = 0.9814$. Therefore, the complete form is $\epsilon_t|\psi_{t-1} \sim N(0, h_t) \; where \; h_t = 0.000005875 + 0.01425\epsilon_{t-1}^2 + 0.9814h_{t-1}$.

The unconditional variance is $Var(\epsilon_t) = \frac{\omega}{1-(\alpha+\beta)} = \frac{0.000005875}{1-(0.01425+0.9814)} = 0.00135$.

5. We know from the residual data that $\epsilon_{4990} = 0.035765$. We also know that $h_{4990} = 0.00138539642318183$. Therefore,

$$h_{4991} = 0.000005875 + 0.01425 \times 0.035765^2 + 0.9814 \times 0.00138539642318183 = 0.001383731$$

From the ARIMA(0,1,1) model, we know that $f_{4990,1} = 5.91954$. The 95% forecast interval of the ARIMA-GARCH model is therefore:

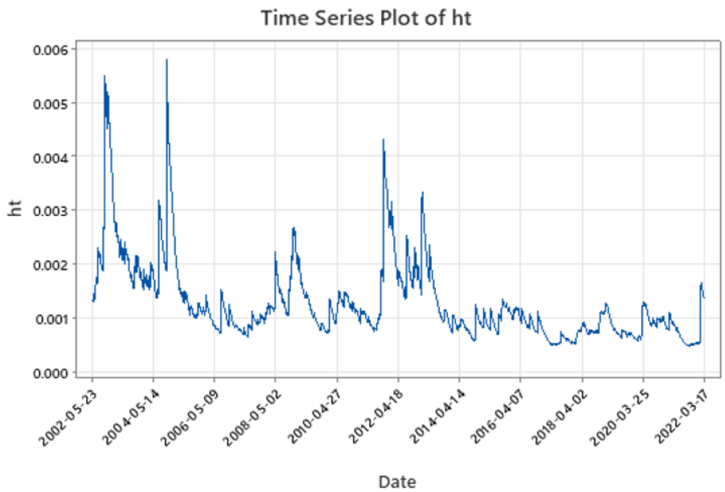
$$f_{4990,1} \pm z_{0.05/2}\sqrt{h_{4991}} = \left(5.91954 - 1.96 \times \sqrt{0.001383731}\right) = (5.846631, 5.992449)$$

Compared to the ARIMA(0,1,1) model, which has a 95% forecast interval of (5.84927, 5.98981), the ARIMA-GARCH model has a wider forecast interval.

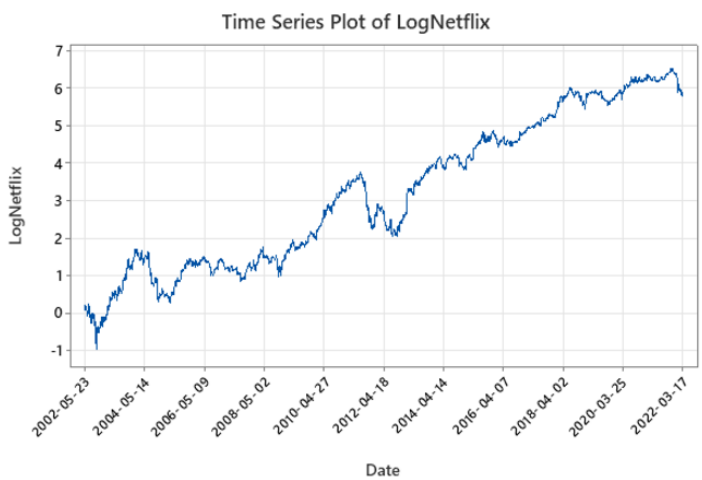
The 5th percentile of the conditional distribution of the next period's log exchange rate is:

$$5.91954 - 1.645 \times \sqrt{0.001383731} = 5.858348$$

6. The time series plot of ht is:

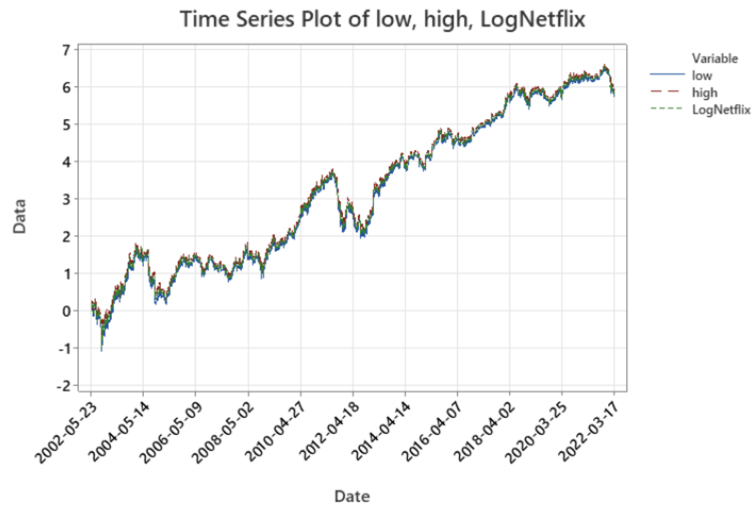


The ht from 2002/5/23 to 2014/4/14 is more volatile than later ht. Bursts of high volatility reside between 2002/5/23 to 2014/4/14, with the highest volatility residing between 2022/5/23 to 2005/5/9.



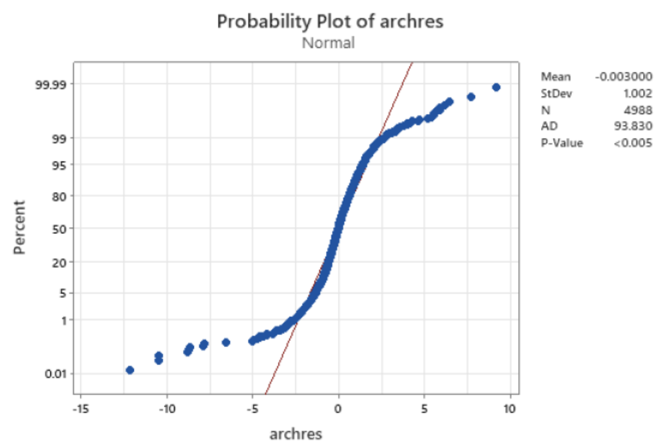
Compared with the time series plot of LogNetflix, it seems that these bursts of high volatility do agree with those found from the examination of the time series plot of the log exchange rates themselves.

7. The time series plot of logNetflix and one-step ahead 95% forecast intervals is as follows:



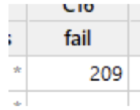
According to the plot, LogNetflix is mostly between low and high, so the forecast interval is quite accurate. However, since the ARIMA-GARCH parameters are estimated from the entire data set, not just the observations up to the time at which the forecast is to be constructed, the performance may be somewhat better here than in an actual forecasting context, so the practical usefulness is somewhat questionable for x_1 to x_{4990}. However, x_{4991} was not utilized in building the ARIMA-GARCH model, so the forecast for x_{4991} is more practically useful.

8.



The test tells us that the p-value is smaller than 0.005, so archres is not normally distributed. The s-shaped plot is an indicator that a heavier tail, which means that the model does not seem to have adequately described the leptokurtosis in the data.

9. There are 209 failures in prediction intervals, which is 209/4988 = 4.19% of all intervals.



fail
209

Performance Check

For LogNetflix, the 95% forecast interval of the ARIMA model is (5.84927, 5.98981), which corresponds to an interval of (347.89, 399.339) for Netflix. $x_{4991} = 380.6$ is within the forecast interval.

The 95% forecast interval of the ARIMA-GARCH model is (5.846631, 5.992449), which corresponds to an interval of (346.0665, 400.39398) for Netflix. $x_{4991} = 380.6$ is within the forecast interval.

The ARIMA interval and ARIMA-GARCH interval are very similar. Both forecast intervals seem neither too wide nor too narrow. The ARIMA-GARCH interval is slightly wider to adapt to the recent higher volatility because ARIMA-GARCH is adaptive, but ARIMA is not.