The Options Approach to Valuing Risky Debt

Options Approach to Risky Debt Black/Scholes/Merton

- Consider a firm with equity and one debt issue.
- The debt issue matures at date T and has principal F.
- It is a zero-coupon bond for simplicity.

- Value of the firm is $V(t) = E(t) + D(t)$.
- Value of equity is $E(t)$.
- Value of debt is $D(t)$.

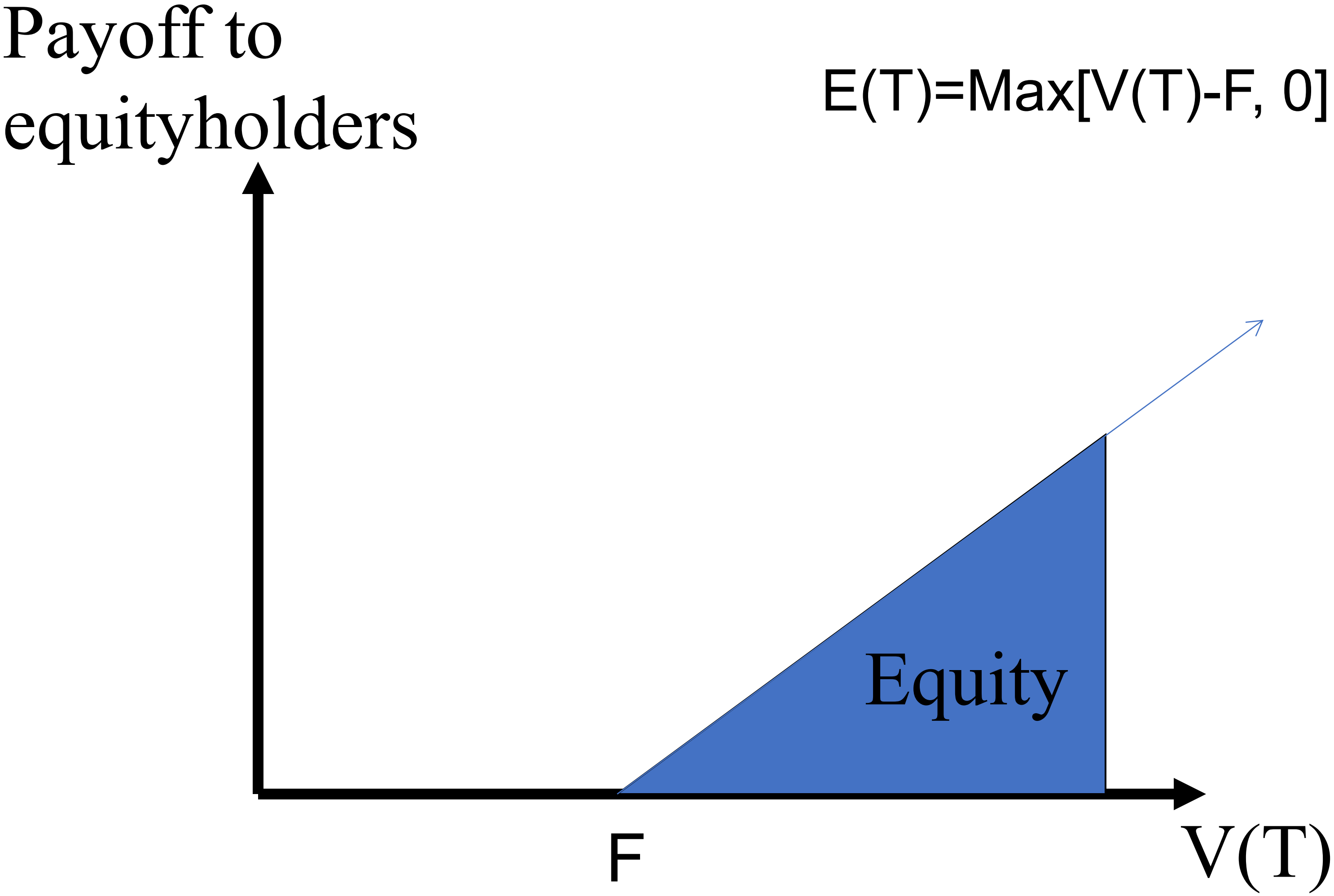
Simplified Firm Balance Sheet

Assets	Liabilities	
Assets $A(t)$	Debt $D(t)$	Face Value F
	Equity $E(t)$	
Value of Firm $V(t)$	Value of Firm $V(t)$	

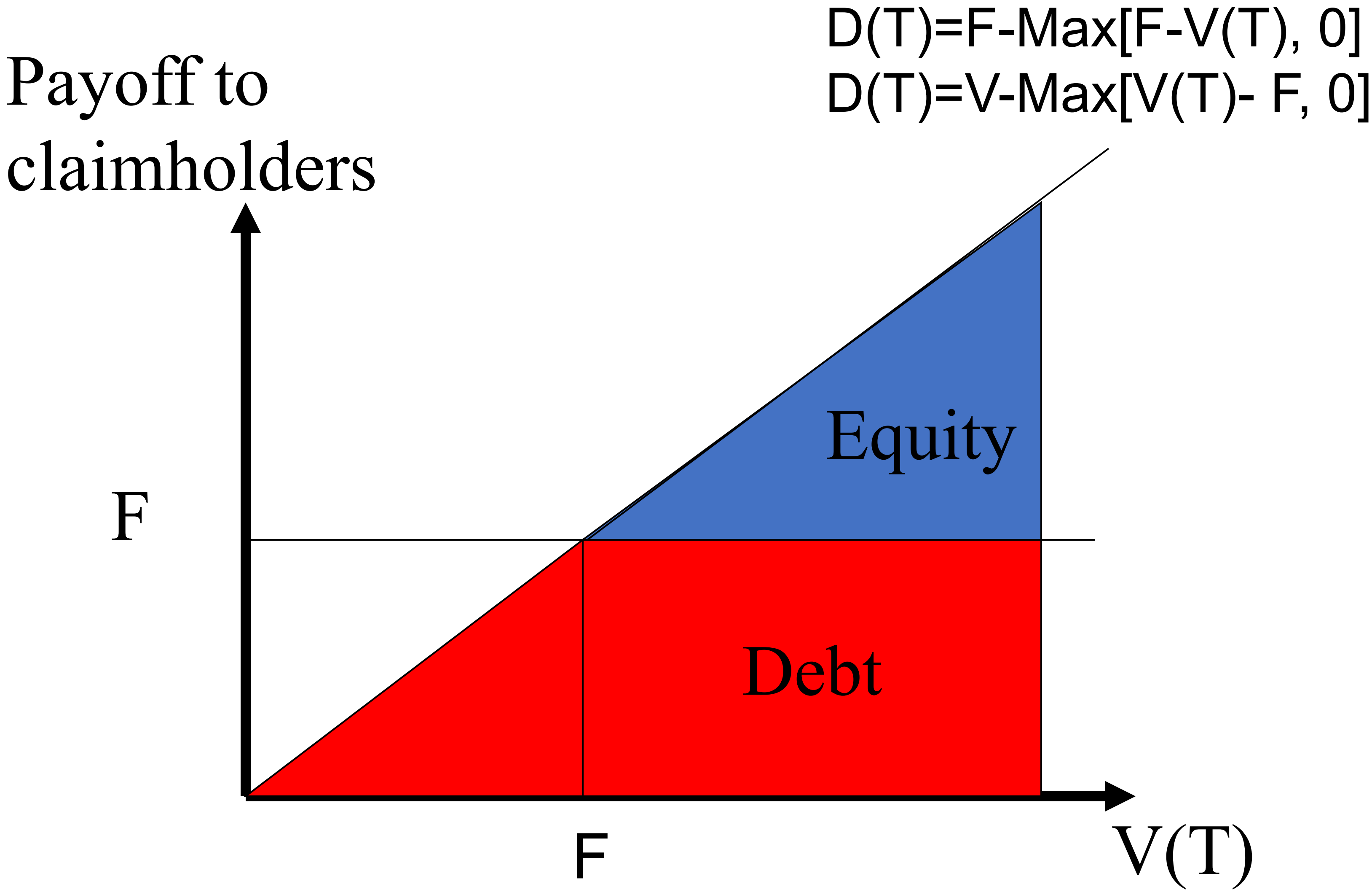
Equity as a Call Option

- Value of the firm at time T is $V(T) = E(T) + D(T)$.
- Value of equity is $E(T)$.
- Value of debt is $D(T)$.
- At maturity T the equity-holders pay the bondholders F if $V(T) > F$, otherwise they pay $V(T)$.
- Then, the value of the equity at maturity is like a call option with exercise price F.

Equity as a Call Option



Debt payoff


D(T)=F-Max[F-V(T), 0]
D(T)=V-Max[V(T)- F, 0]
Payoff to claimholders
Equity
F
Debt
F
V(T)

Payoff of debt

- At date T, the debtholders receive F if V(T) exceeds F and V(T) otherwise.
- They get F - Max[F - V(T), 0]: The payoff of riskless debt minus the payoff of a put on V(T) with exercise price F.
- Equity holders get Max[V(T) - F, 0], the payoff of a call on the firm.

Corporate Liabilities as Options

- Equity = Call Option
- Debt = Face Value of Debt – Put Option
- Debt = Value of the Firm – Call Option

Put Call Parity:

- Value of the Firm – Call = Face Value of Debt - Put

Black-Scholes assumptions on firm value

- Now firm value is lognormal; constant vol.; deterministic interest rate; no frictions.
- $E(t) = C(V(t),F,T)$
- $D(t) = Exp[-r(T-t)]*F - P(V(t),F,T)$
- Put-call parity implies also:

  $D(t) = V(t) - C(V(t),F,T)$
- Firm value is simply sum of equity and debt: $V(t) = E(t) + D(t)$.

Equity vs. Assets (Firm Value)

The BS option pricing model enables the value of the firm's equity today, E_0, to be related to the value of its assets today, V_0, and the volatility of its assets, σ_V

$$E_0 = V_0 N(d_1) - F e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/F) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}}; \; d_2 = d_1 - \sigma_V \sqrt{T}$$

Volatility of Equity as a function of the Volatility of the Firm

$$\sigma_E = \frac{\partial E}{\partial V}\frac{V_0}{E_0}\sigma_V = \frac{\%change\ in\ E}{\%change\ in\ V}\sigma_V$$

$$\sigma_E = N(d_1)\frac{V_0}{E_0}\sigma_V$$

This equation together with the option pricing relationship enables V_0 and σ_V to be determined from E_0 and σ_E

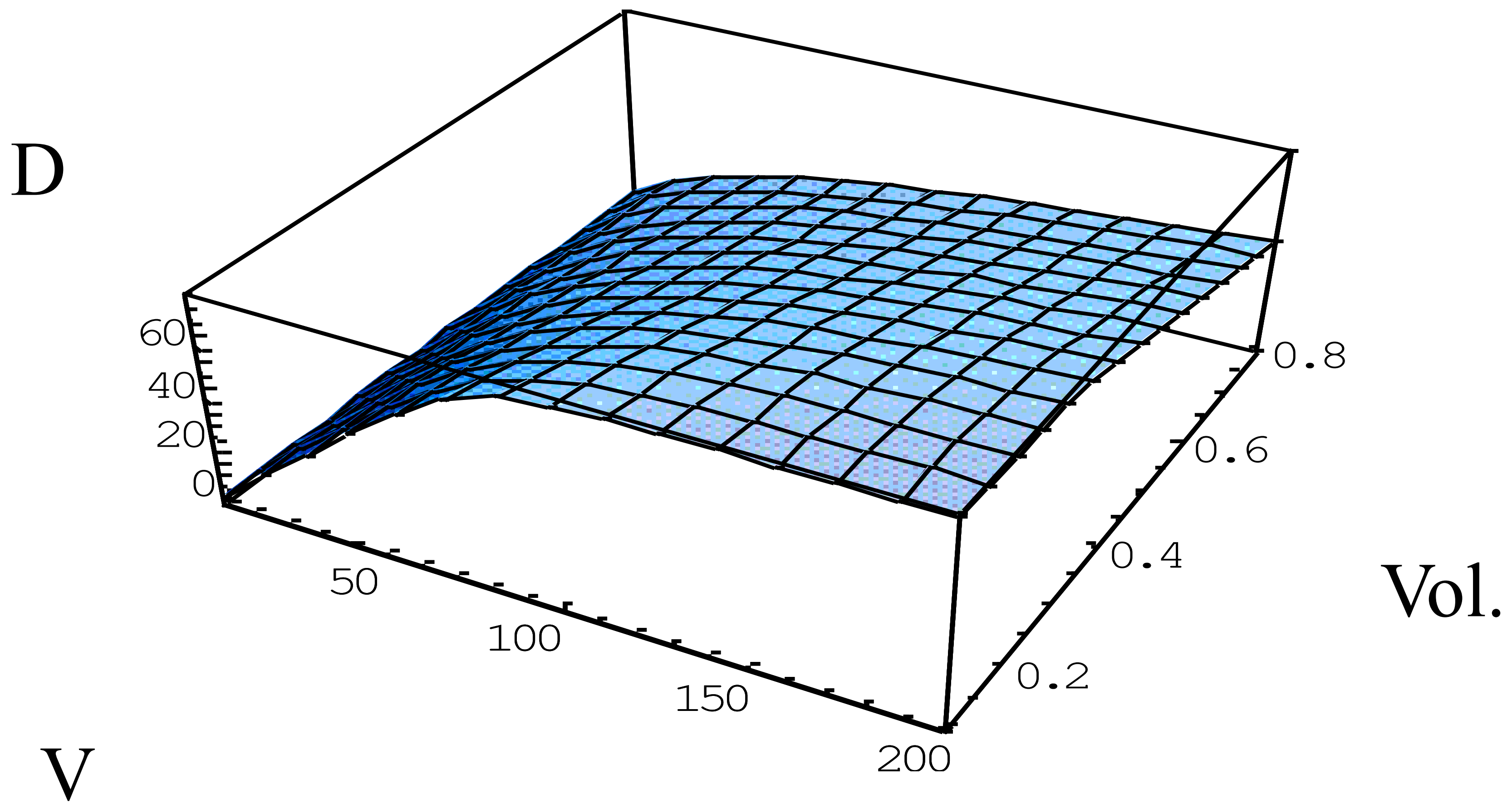
Poll : Should the value of Debt increase with the volatility of V?

- A: Yes
- B: No
- C: Depends
- D: Can’t tell

Poll : Should the value of Debt increase with V?

- A: Yes
- B: No
- C: Depends
- D: Can’t tell

Debt value, firm value and vol. (F=100,t=5,r=0.05)



Debt value determinants

- The value of the debt falls with volatility since the value of the equity (a call option of the value of the assets) increases with volatility.
- The debt value increases with firm value since the value of the put decreases with firm value.

Example

- A company's equity is \$3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the face value of debt is \$10 million and time to debt maturity is 1 year
- Solving the two equations yields V_0=12.40 and σ_v=21.23%

$$E_0 = V_0 N(d_1) - Fe^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/F) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}}; \; d_2 = d_1 - \sigma_V \sqrt{T}$$

$$\sigma_E = \frac{\partial E}{\partial V} \frac{V_0}{E_0} \sigma_V = \frac{\%change\ in\ E}{\%change\ in\ V} \sigma_V$$

$$\sigma_E = N(d_1) \frac{V_0}{E_0} \sigma_V$$

Example continued

- The market value of the debt is V_0 - E_0 = 12.4 – 3 = 9.40
- The present value of the promised payment if risk free is 10*exp(-0.05) = 9.51
- The PV of the expected loss is about 1.2% (.11/9.51)

Credit spreads

$$\textit{Risky rate} = R$$

$$\exp(RT) = \frac{F}{D}$$

$$R = \frac{1}{T}\ln\frac{F}{D} = \ln\frac{10}{9.4} = \ln 1.0638 = 0.0612$$

$$\textit{Credit spread} = R - r = 6.12 - 5.00 = 1.12\%$$

Subordinated debt

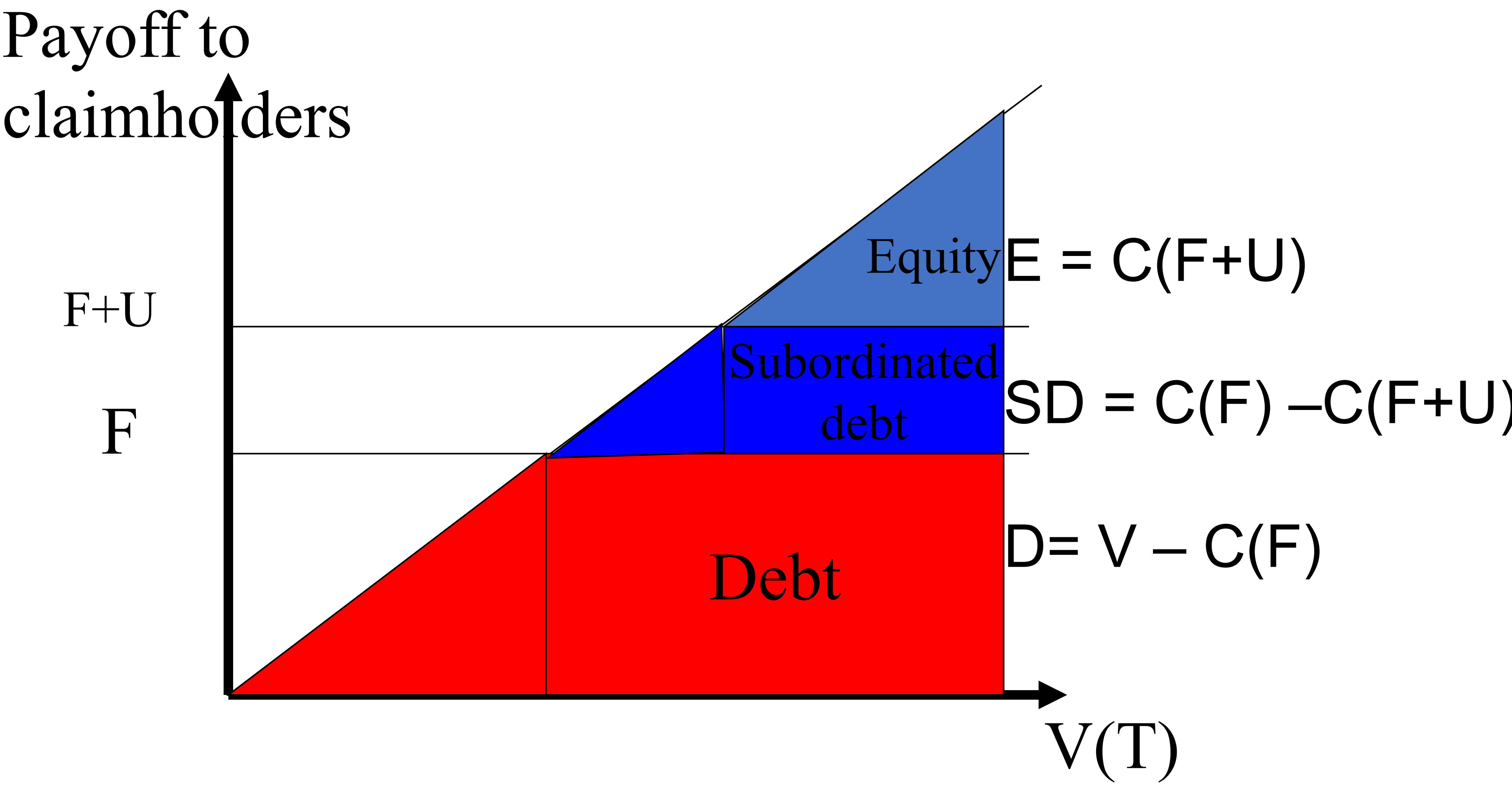
- Let the firm have subordinated debt with face value of U.
- The subordinated debt holders receive U if firm value exceeds U + F.
- If firm value is less than F, they receive nothing.
- If firm value is less than U + F but more than F, they get what is left after paying the senior debtholders.

Simplified Firm Balance Sheet

Assets	Liabilities	Promised at Maturity
	Senior Debt D(t)	F
Assets A(t)	Subordinated Debt SD(t)	U
	Equity E(t)	
Value of Firm V(t)	Value of Firm V(t)	

Subordinated debt payoff

V=E+SD+D



Probability of default and expected loss

- If Merton's model applies, the probability of default and the expected loss can be computed.
- It is the probability that firm value will not exceed the debt face value at T.
- Properties of the log-normal distribution are well known.
- The distribution of firm value depends on true expected return, μ.

Why is it hard to use Merton's model?

- Firms are much more complicated
  - Multiple types of debt
  - Debt with different maturities
  - Coupon-paying debt
- Many firms to which banks lend do not have traded equity
- Recovery is difficult to estimate even if equity is traded
- Number of credits in portfolios makes it impractical to use an approach that requires careful attention to details of firms' situations

Solution

- Use the spirit of Merton's model, but devise practical shortcuts.
- KMV approach.

The Implementation of Merton’s Model by Moody’s KMV

- Choose time horizon
- Calculate cumulative obligations to the time horizon. This is termed by KMV the “default point”. We denote it by F
- Use Merton’s model to calculate the “distance to default”
- Use historical data to develop a one-to-one mapping between the distance to default and the real-world probability of default.

KMV approach

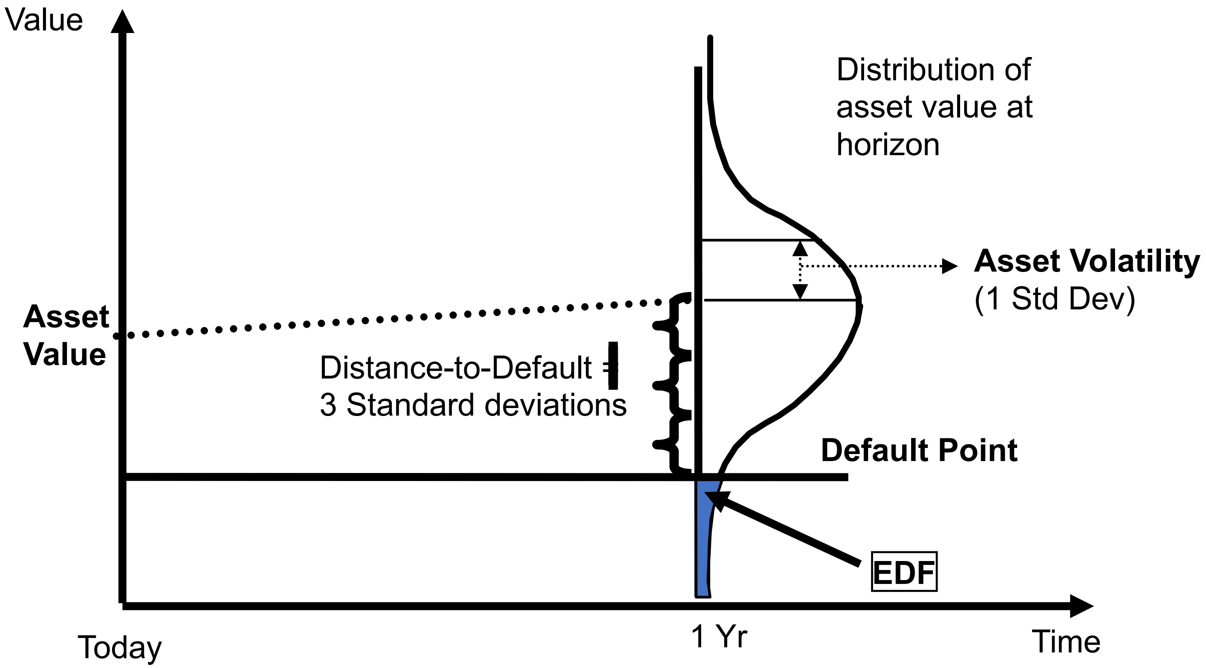
- Uses Merton model, but assumes that capital structure is more complex.
- Reflects current situation of firm through stock price.
- Key concepts: DD (distance to default) and EDF (expected default frequency).

Default Point (DPT) and Distance to default (DD)

- To compute probability of default, KMV first computes the default point and the distance to default for the firm.

- STD, short-term debt; LTD, long-term debt.
- DPT = default point = STD +0.5LTD
- DD = [V(t+1) - DPT]/Vol(V)

Distance to Default



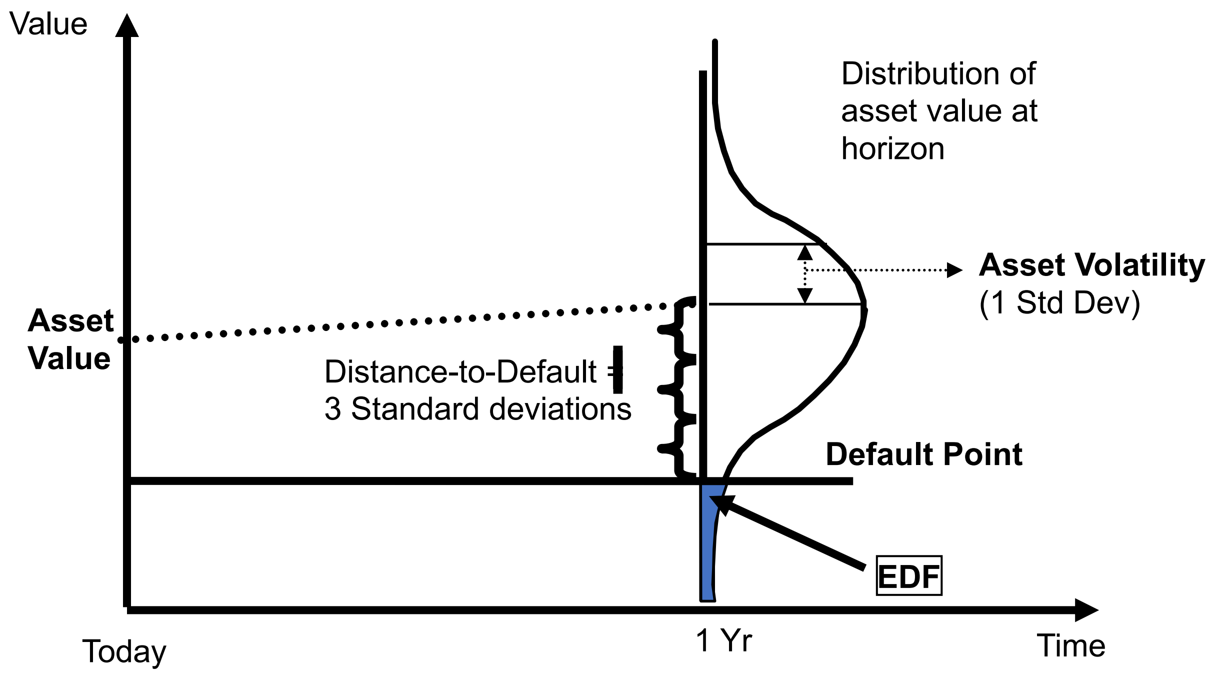
Expected default frequency (EDF)

- EDF is the probability of default.
- One could use Merton's results to compute expected default frequency.
- That's not what KMV does.
- KMV uses a proprietary historical database that gives historical default frequency for DD values.

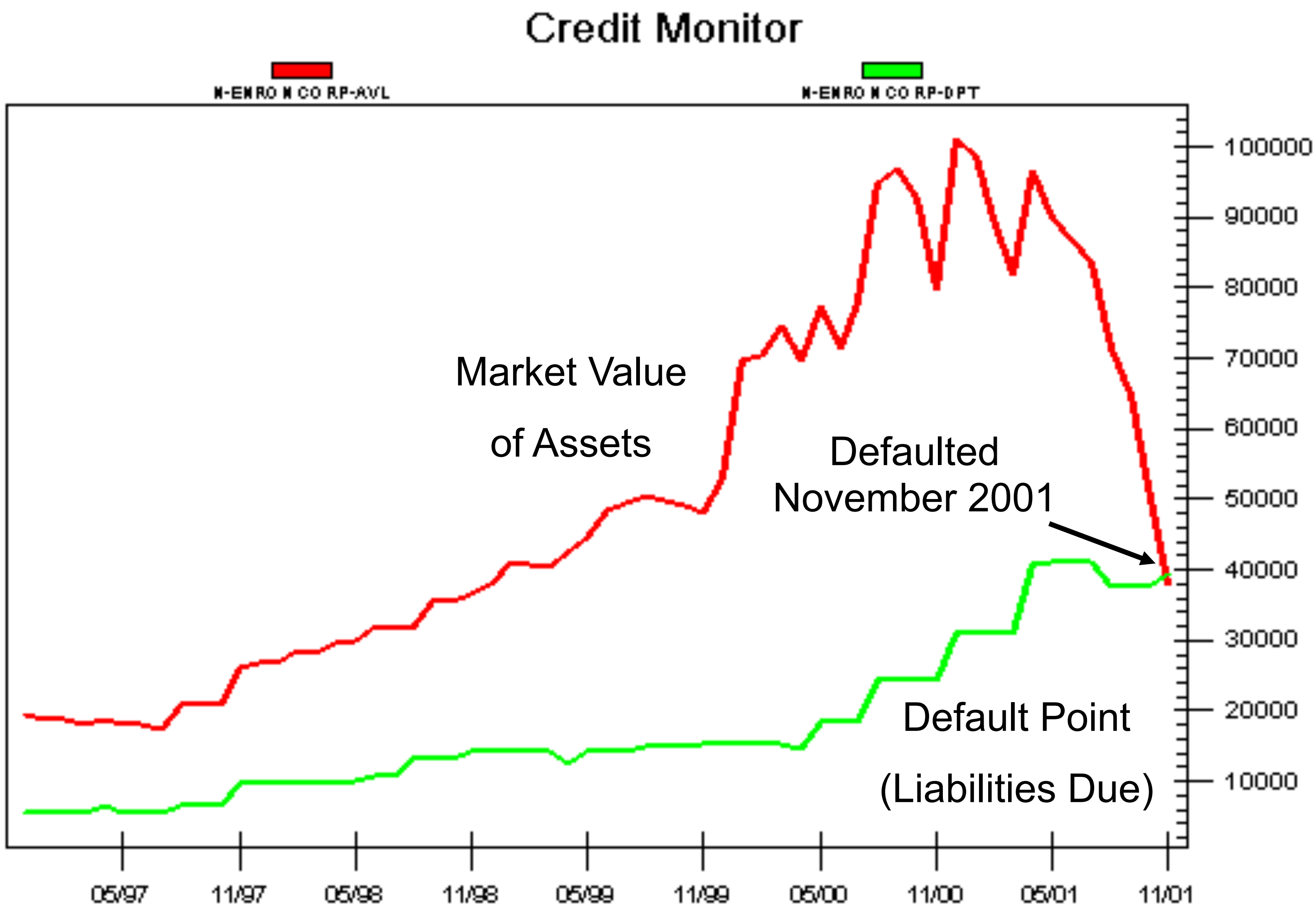
Probability of default

- Say a firm has DD = 4.
- KMV then uses a large database of firms to find the percentage of firms with DD = 4 that defaulted within one year.
- If it finds 40 bp, then the probability of default of the firm is 0.4%.

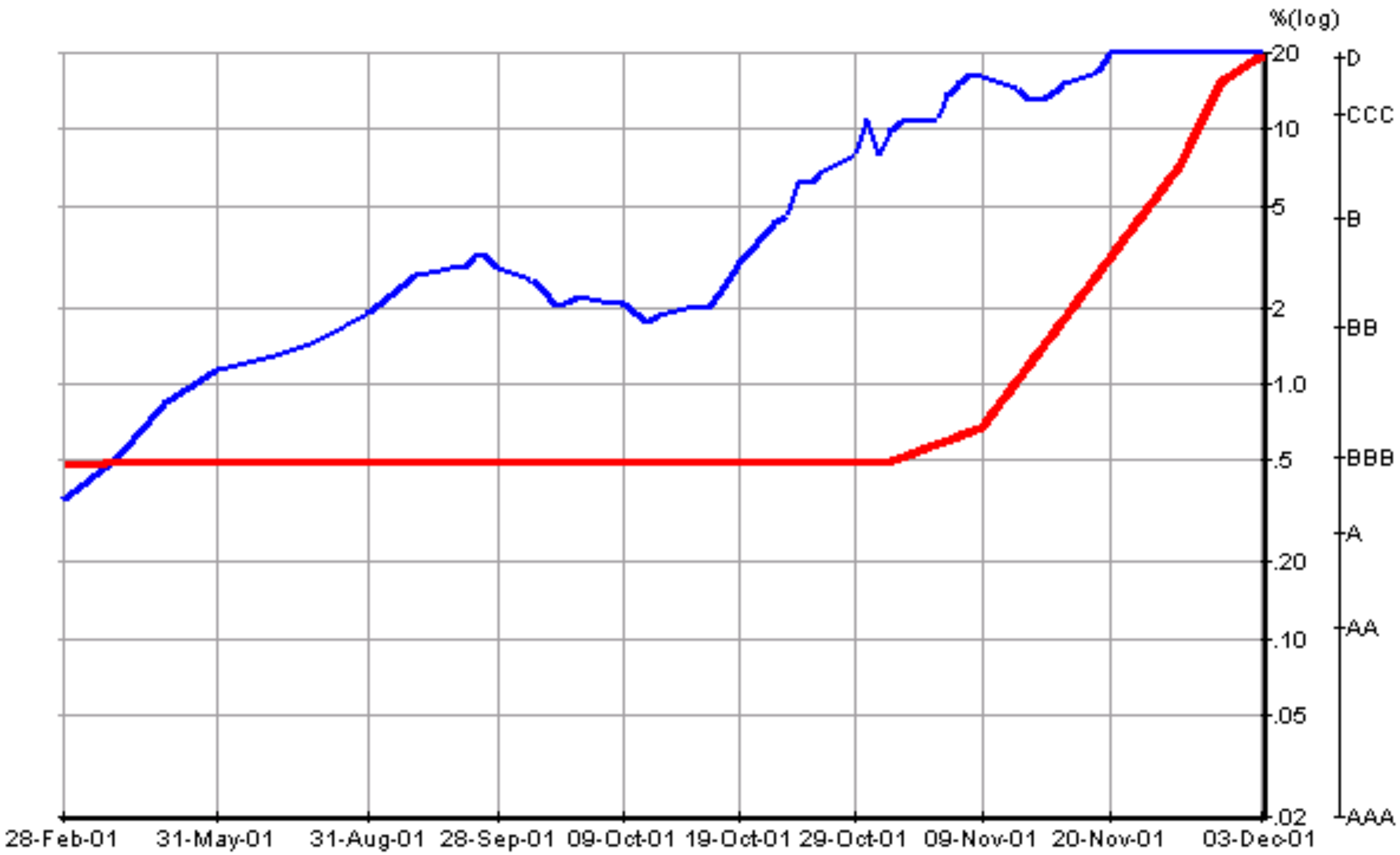
Putting It All Together



Enron


Credit Monitor
N-ENRON CORP-AVL
N-ENRON CORP-DPT
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
05/97
11/97
05/98
11/98
05/99
11/99
05/00
11/00
05/01
11/01
Market Value
of Assets
Defaulted
November 2001
Default Point
(Liabilities Due)

Ratings agencies and credit risk (KMV EDF): Enron



An alternative approach

- In the approach presented so far, we used a process for the value of the firm (structural models).
- Alternatively, we could use a process for default (reduced form models).
- If the firm defaults, then the debtholder receives a fraction of his claim.
- This recovery fraction can be random or deterministic
- This approach leads to a formula that is similar to the formulas for fixed income claims.