An Optimising Model for Supply Chain Management

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ABSTRACT: A linear programming model is developed for the centralized co-ordination of supply chains over many plants and inventory points. Production and supply lead-times are included in the model so that the upstream impact of downstream demand variation can be evaluated and appropriate action taken in time. A simple computational example illustrates well the potential of the model.

KEY WORDS: Supply Chain, Optimization, Linear Program, Inventory Management

1. INTRODUCTION

Supply chain management has become a major concern of industrial enterprises in recent years. Factors such as increasing globalization and the development of information technology have provided opportunities for companies to become more efficient and, at the same time, put competitive pressure on them as their rivals do likewise. With increased internationalization of markets within the EU, Nafta & Mercosul and consolidation of production facilities, many companies today have more extensive supplier and distribution networks than some years ago.

The effective co-ordination of such networks is vital for a company to control its costs and assure its dependability to customers [Andersen Consulting, 1994]. If material flows are not well synchronized, then shortages can occur disrupting production or delaying delivery to end customers. As a result, supply chain planning is increasingly carried out at more centralized levels than previously [Holmes, 1995; Gunn, 1994], making strong use of information support systems and telecommunications technology. The flow of materials is planned and controlled throughout the whole supply chain and over many companies. This is often a considerable challenge and there is a role for a supply chain model of suppliers, producers and transport flows to plan production, inventory and supply over a demand horizon.

This paper proposes a prototype of just such a model. Its benefits include not only less inventory within the supply chain, faster overall lead-times, and swifter responses to demand fluctuations, but also the ability to detect upstream production and supply problems in sufficient time to be able to take corrective action.

2. MODEL SPECIFICATION

The model represents products or groups of products as items that are either processed or stored at stages in the supply chain. A stage is either a plant that produces items or a warehouse that stores items. A plant will take certain incoming items and transform them, through production processes, into other outgoing items. Each plant thus has to have both incoming and outgoing inventories of items. A warehouse has simply an inventory of items, as no transformation processes are involved. Items flow from a plant or warehouse to another plant or warehouse.

A basic model will first be expounded and then developed further through the introduction of production and flow lead-times, and then with the inclusion of capacity limits.

Define the following model indices:

s and *t* are supply chain stages;

i and *j* are item types (or groups of types);

The model's decision variables in which a supply chain planner will be interested are:

x_{isd} is the production or throughput of item *i* at stage *s* on day *d*.

I^{in}_{isd} is the incoming inventory of item *i* at plant stage *s* at the end of day *d*.

I^{out}_{isd} is the outgoing inventory of item *i* at plant stage *s* at the end of day *d*.

I^{W}_{isd} is the outgoing inventory of item *i* at warehouse stage *s* at the end of day *d*.

y_{istd} is the supply of item *i* from stage *s* to stage *t* on day *d*.

To use the model, the supply chain planner must specify which items pass though which stages in the supply chain:

The item-stage pair (*i*,*s*) is in the set *IS* if item *i* is produced or passes through stage *s*.

The item-plant pair (*i*,*s*) is in the set *IP* if item *i* is produced at plant *s*.

The item-warehouse pair (*i*,*s*) is in the set *IW* if item *i* passes through warehouse *s*.

Permitted flows of items must also be specified:

The item-plant pair (*i*,*s*) is in the set *ItoP* if item *i*, is an incoming item to plant *s*.

The item-plant pair (*i*,*s*) is in the set *IfromP* if item *i*, is an outgoing item from plant *s*.

The item-warehouse pair (*i*,*s*) is in the set *ItoW* if item *i*, is an item stored at warehouse *s*.

The item-plant-item triplet (*i*,*s*,*j*) is in the set *IPI* if incoming item *i* to plant *s* is consumed in the production of outgoing item *j*.

The stage-item-stage triplet (s,i,t) is in the set SIS if stage s supplies item i to stage t.

The stage-item-plant triplet (s,i,t) is in the set SIP if stage s supplies item i to plant t.

The stage-item-warehouse triplet (s,i,t) is in the set SIW if stage s supplies item i to warehouse t.

The plant-item-stage triplet (s,i,t) is in the set PIS if plant s produces item i for consumption at stage t.

The warehouse-item-stage triplet (s,i,t) is in the set WIS if warehouse s supplies item i for consumption at next stage t.

The following parameters must also be defined:

T is the number of days in the supply chain planning horizon

h_{is}^{in} is the unit daily cost of incoming inventory of item i at plant s

h_{is}^{out} is the unit daily cost of outgoing inventory of item i at plant s

h_{is}^{W} is the unit daily cost of inventory of item type i at warehouse s

q_{ijs} is the quantity of item i consumed by the production of one unit of item j at plant s

The basic data input is the demand for items at various stages over the planning horizon:

d_{isd} is the independent demand, firm or forecast, for item i at stage s on day d

The objective is to satisfy this demand over the planning horizon with minimum total supply chain inventory. Suppose for the time being that there is no production or supply lead time. Then an initial model is:

Model M_0^U :

$$\min \quad \sum_{(i,s)\in ItoP} \sum_{d=1}^{T} h_{is}^{\text{in}} I_{isd}^{\text{in}} + \sum_{(i,s)\in IfromP} \sum_{d=1}^{T} h_{is}^{\text{out}} I_{isd}^{\text{out}} + \sum_{(i,s)\in IW} \sum_{d=1}^{T} h_{is}^{\text{W}} I_{isd}^{\text{W}} \tag{1}$$

such that

$$I_{isd}^{\text{in}} = I_{is,d-1}^{\text{in}} - \sum_{(i,s,j)\in IPI} q_{ijs} x_{jsd} + \sum_{(t,i,s)\in SIP} y_{itsd} \qquad \text{for all } (i,s)\in ItoP, \text{ and } d=1,..,T \tag{2}$$

$$I_{isd}^{\text{out}} = I_{is,d-1}^{\text{out}} + x_{isd} - d_{isd} - \sum_{(s,i,t)\in PIS} y_{istd} \qquad \text{for all } (i,s)\in IfromP \text{ and } d=1,..,T \tag{3}$$

$$I_{isd}^{\mathrm{W}} = I_{is,d-1}^{\mathrm{W}} + \sum_{(t,i,s)\in SIW} y_{itsd} - d_{isd} - \sum_{(s,i,t)\in WIS} y_{istd} \qquad \text{for all } (I,S)\in IW \text{ and } d=1,..,T \qquad (4)$$

$$x_{isd} \geq 0\,; I_{isd}^{\mathrm{in}} \geq 0\,; I_{isd}^{\mathrm{out}} \geq 0 \qquad \text{for all } (i,s)\in IP \text{ and } d=1,..,T \qquad (5)$$

$$I_{isd}^{\mathrm{W}} \geq 0 \qquad \text{for all } (i,s)\in IW \text{ and } d=1,..,T \qquad (6)$$

$$y_{istd} \geq 0 \qquad \text{for all } (s,i,t)\in SIS \text{ and } d=1,..,T \qquad (7)$$

The key constraints in the model are (2) to (4) which balance the number of items entering inventory with the number leaving plants inward, plants outward and warehouses respectively. Initial inventory levels must be specified, namely I_{is0}^{in} and I_{is0}^{out} for all $(i,s)\in IP$ and I_{is0}^{W} for all $(i,s)\in IW$. Note that WIP and in-transit inventory are not included in the model.

Assuming no initial inventory, the optimal solution to this model is zero inventory since there are no capacity limits on production or supply. However, before adding capacity constraints, let us add production and supply lead-time to the model in the form of the following parameters:

$Lp(i,s)$ is the lead-time for item i at plant s to be produced and available in its outward inventory.

$Ls(s,i,t)$ is the lead time for the supply of item i from stage s to stage t and be available in its inward stock.

This means that constraints (2) to (4) become

$$I_{isd}^{\mathrm{in}} = I_{is,d-1}^{\mathrm{in}} - \sum_{(i,s,j)\in IPI} q_{ijs} x_{jsd} + \sum_{(t,i,s)\in SIP} y_{its,d-Ls(t,i,s)} \quad \text{for all } (i,s)\in ItoP, \text{ and } d=1,..,T \qquad (8)$$

$$I_{isd}^{\mathrm{out}} = I_{is,d-1}^{\mathrm{out}} + x_{is,d-Lp(i,s)} - d_{isd} - \sum_{(s,i,t)\in PIS} y_{istd} \qquad \text{for all } (i,s)\in IfromP \text{ and } d=1,..,T \qquad (9)$$

$$I_{isd}^{\mathrm{W}} = I_{is,d-1}^{\mathrm{W}} + \sum_{(t,i,s)\in SIW} y_{its,d-Ls(t,i,s)} - d_{isd} - \sum_{(s,i,t)\in WIS} y_{istd} \qquad \text{for all } (I,S)\in IW \text{ and } d=1,..,T \qquad (10)$$

We must also specify the following recent values of production and supply which are needed to fully specify the above constraints, namely:

$$x_{isd} = x_{isd}^{past} \text{ for all } (i,s)\in IP \text{ and } d=1\text{-}Lx(i,s),..,0 \qquad (11)$$

$$y_{stid} = y_{stid}^{past} \text{ for all } (s,i,t)\in SIS \text{ and } d=1\text{-}Ly(s,i,t),..,0 \qquad (12)$$

Thus the uncapacitated lead-time model M_L^U becomes:

Model M_L^U:

$$\min \quad \sum_{(i,s)\in ItoP} \sum_{d=1}^{T} h_{is}^{\text{in}} I_{isd}^{\text{in}} + \sum_{(i,s)\in IfromP} \sum_{d=1}^{T} h_{is}^{\text{out}} I_{isd}^{\text{out}} + \sum_{(i,s)\in IW} \sum_{d=1}^{T} h_{is}^{\text{W}} I_{isd}^{\text{W}} \tag{13}$$

such that

$$I_{isd}^{\text{in}} = I_{is,d-1}^{\text{in}} - \sum_{(i,s,j)\in IPI} q_{ijs} x_{jsd} + \sum_{(t,i,s)\in SIP} y_{its,d-Ls(t,i,s)} \quad \text{for all } (i,s)\in ItoP, \text{ and } d=1,..,T \tag{14}$$

$$I_{isd}^{\text{out}} = I_{is,d-1}^{\text{out}} + x_{is,d-Lp(i,s)} - d_{isd} - \sum_{(s,i,t)\in PIS} y_{istd} \quad \text{for all } (i,s)\in IfromP \text{ and } d=1,..,T \tag{15}$$

$$I_{isd}^{\text{W}} = I_{is,d-1}^{\text{W}} + \sum_{(t,i,s)\in SIW} y_{its,d-Ls(t,i,s)} - d_{isd} - \sum_{(s,i,t)\in WIS} y_{istd} \quad \text{for all } (I,S)\in IW \text{ and } d=1,..,T \tag{16}$$

$$x_{isd} = x_{isd}^{past} \text{ for all } (i,s)\in IP \text{ and } d=1\text{-}Lx(i,s),..,0 \tag{17}$$

$$y_{stid} = y_{stid}^{past} \text{ for all } (s,i,t)\in SIS \text{ and } d=1\text{-}Ly(\mathrm{s},i,t),..,0 \tag{18}$$

$$x_{isd} \geq 0;\ I_{isd}^{\text{in}} \geq 0;\ I_{isd}^{\text{out}} \geq 0 \quad \text{for all } (i,s)\in IP \text{ and } d=1,..,T \tag{19}$$

$$I_{isd}^{W} \geq 0 \quad \text{for all } (i,s)\in IW \text{ and } d=1,..,T \tag{20}$$

$$y_{istd} \geq 0 \quad \text{for all } (s,i,t)\in SIS \text{ and } d=1,..,T \tag{21}$$

The model now incorporates lead times, but there are still no limits on production and supply. Suppose that:

x_{isd}^{max} is the maximum possible production of item i at stage s on day d

y_{sitd}^{max} is the maximum possible flow of item i from stage s to stage t on day d

Then we may include the following constraints in the model:

$$x_{isd}^{\text{max}} \leq x_{isd} \quad \text{for all } (i,s)\in IP \text{ and } d=1,..,T \tag{22}$$

$$y_{sitd}^{\text{max}} \leq y_{sitd} \quad \text{for all } (s,i,t)\in SIS \text{ and } d=1,..,T \tag{23}$$

thus defining the capacitated lead-time model M_L^C.

Constraints (22) and (23) are simple by-item limits. Other more sophisticated capacity constraints can be represented, for example, limits on the total tonnage that a supply link can handle each day.

3. COMPUTATIONAL EXAMPLE

Let us now illustrate the use of the model with a simple example. An upstream plant s1 supplies a component c1 to a downstream plant f1, taking a day to do so. A second upstream plant s2 supplies another component c2 to both f1 and to a second downstream plant f2. Plant f1 uses one unit each of c1 and c2 to produce a unit of product p1. Plant f1 also uses just one unit of c2 to produce a unit of product p2. Similarly, plant f2 also uses one unit of c2 to produce a unit of a third product p3. Plant f1 supplies p1 and p2 to a distribution warehouse w1 and just p2 to a second warehouse w2. Plant f2 simply supplies p3 to w2. The sets for this system are shown on Appendix 1.

The plants all have a production lead-time $Lp(i,s)$ of 1 day and a maximum capacity of 500 items a day for each product. The supply lines can handle 500 units of each permitted product per day and have a lead-time of 1 day except for Ls(s2,c2,f1) = 2 = Ls(f1,p2,w2). The only independent demand for products p1 to p3 is via warehouses w1 and w2. The unit daily holding cost of all items is 1. The demand forecast horizon is 8 days.

Suppose the demand for p1 and p2 at w2 and for p2 and p3 at w2 is each 100 units/day. By some simple manual calculations, it is clear that the steady state for the supply chain is:

Production of :	at :	is :
c1	s1	300
c2	s2	300
p1	f1	100
p2	f1	200
p3	f2	100

Supply from :	of :	to :	is :
s1	c1	f1	300
s2	c2	f1	200
s2	c2	f2	100
f1	p1	w1	100
f1	p2	w1	100
f1	p2	w2	100
f2	p3	w2	100

Allocating these values to the past production and supply on day 0 and to y(s2,c2,f1,−1) and y(f1,p2,w2,−1), the model should reproduce the steady state with zero inventory and, indeed, it does. The results are shown in Appendix 2. Not that because of the cumulative lead-times, the production of p1 at f1 and of p3 at f2 is only specified up to day 6, and of p2 at f1 up to day 5. Similarly, the production of c1 at s1 tapers off at day 4 and is zero at day 5. The longest cumulative lead-time path is of c2 from s2 to f1 and then of p2 from f1 to w2. Thus the production of c2 at s2 begins to taper off even earlier at day 3 and is zero at day 5.

Note that this steady state production is possible within the available capacity. Suppose the demand for all products was doubled on days 7 and 8. This will cause the production of c1 and c2 to hit the limit of production capacity at s1 and s2 and of the supply of c1 from s1 to f1. The solution is obviously to bring forward production and build up inventory to meet demand. This is possible and, in fact, the only inventory the system needs to hold is 100 unit of p1 at w2 at the end of day 6. The production plan is shown in Appendix 3. Note that the maximum limit of 500 was reached for c1 in plant s1 on days 2 and 3 and for c2 in s1 on day 2, i.e., quite early on ! Thus a simple doubling of downstream demand near the end of the planning horizon causes at-capacity production upstream much nearer in time. This should serve as a warning that we are already near the overall limits of the system.

Now suppose demand was also doubled on day 6. The model cannot find a feasible solution, showing that the situation has now become inviable - it is not possible to meet this extra demand within the production and supply capacity limits and the specified lead-times.

This example shows the value of the model in flagging in advance inviable future demand. The supply chain planner can thus take appropriate action in time, particularly at upstream facilities.

4. CONCLUSION

As supply chain management becomes more crucial to a company's competitiveness, it will tend to be more centrally co-ordinated. There is a strong need for an overall model such as that developed in this paper. The challenge now is to test its usefulness in the field. Clearly, the model will need adapting to the particular circumstances of the supply chain in hand. In some cases, the day-long planning periods will have to be tighter, measured in hours.

The model would have great value implemented as a planning and control module of an integrated computerized supply chain management system, with fast or real-time access to information about demand, capacity, inventory and production.

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Appendix 1

Sets for the example

Set `IS = (c1,s1) (p1,f1) (p3,f2) (p2,w1) (p3,w2) (c2,s2) (p2,f1) (p1,w1) (p2,w2)`

Set `ItoP = (c1,f1) (c2,f1) (c2,f2)`

Set `IfromP = (c1,s1) (c2,s2) (p1,f1) (p2,f1) (p3,f2)`

Set `IW = (p1,w1) (p2,w1) (p2,w2) (p3,w2)`

Set `IPI = (c1,f1,p1) (c1,f1,p2) (c2,f1,p2) (c2,f2,p3)`

Set `SIS = (s1,c1,f1) (s2,c2,f2) (f1,p2,w1) (f2,p3,w2) (s2,c2,f1) (f1,p1,w1)  f1,p2,w2)`

Set `SIP = (s1,c1,f1) (s2,c2,f1) (s2,c2,f2)`

Set `SIW = (f1,p1,w1) (f1,p2,w1) (f1,p2,w2) (f2,p3,w2)`

Set `PIS = (s1,c1,f1) (s2,c2,f2) (f1,p2,w1) (f2,p1,w2) (s2,c2,f1) (f1,p1,w1) (f1,p2,w2) (f2,p3,w2)`

Set `WIS` was empty;

Appendix 2
Steady state production

Production at plant `f1` is

Day	p1	p2
0	100	200
1	100	200
2	100	200
3	100	200
4	100	200
5	100	200
6	100	100
7	0	0
8	0	0

Production at plant `f2` is

Day	p3
0	100
1	100
2	100
3	100
4	100
5	100
6	100
7	0
8	0

Production at plant `s1` is

```
Day c1
0   100
1   100
2   100
3   100
4   100
5   100
6   100
7     0
8     0
```

Production at plant `s2` is

```
Day c2
0   300
1   300
2   300
3   200
4   100
5     0
6     0
7     0
8     0
```

Appendix 3
Production when demand is doubled on days 6 and 7

Production at plant `f1` is

```
Day  p1    p2
0   100   200
1   100   200
2   100   200
3   100   200
4   200   300
5   100   400
6   200   200
7     0     0
8     0     0
```

Production at plant `f2` is

```
Day  p3
0   100
1   100
2   100
3   100
4   100
5   200
6   200
7     0
8     0
```

Production at plant `s1` is

```
Day c1
0   300
```

1	300
2	500
3	500
4	400
5	0
6	0
7	0
8	0

Production at plant `s2` is

Day	c2
0	300
1	400
2	500
3	400
4	200
5	0
6	0
7	0
8	0

The supply from `f1` to `w1` was

Day	p1	p2
0	100	100
1	100	100
2	100	100
3	100	100
4	100	100
5	200	100
6	100	200
7	200	200
8	0	0

The supply from `f1` to `w2` was

Day	p2
-1	100
0	100
1	100
2	100
3	100
4	100
5	200
6	200
7	0
8	0

The supply from `f2` to `w2` was

Day	p3
0	100
1	100
2	100
3	100
4	100
5	100
6	200
7	200

```
8      0
```

The supply from `s1` to `f1` was

```
Day  c1
0   300
1   300
2   300
3   500
4   500
5   400
6     0
7     0
8     0
```

The supply from `s2` to `f1` was

```
Day   c2
-1   200
0    200
1    200
2    300
3    400
4    200
5      0
6      0
7      0
8      0
```

The supply from `s2` to `f2` was

```
Day c2
0   100
1   100
2   100
3   100
4   200
5   200
6     0
7     0
8     0
```

-o0o-