Reformulation-Linearization Technique for Modelling the Scheduling Decisions

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Abstract

The objective of this work is to discuss the use of formulations for the Asymmetric Traveling Salesman Problem for modelling the scheduling decisions in a big bucket approach for the integrated lot sizing and scheduling problem. We apply the Reformulation-Linearization Technique to derive tight constraints and use them to replace the Miller-Tucker and Zemlim type constraints. The new model is studied and compared to two other formulations already used in the literature. The results show that the proposed valid inequalities are useful to obtain tighter dual bounds and thus improve the solution process.

Keywords: Lotscheduling; Asymmetric Traveling Salesman Problem; Reformulation-Linearization Technique; Tight relaxations.

1 Introduction

Lot sizing and the scheduling problems have received a lot of attention in the literature given their importance for the productivity of manufacturing systems. Traditional models have been increasingly refined to integrate the lot sizing decisions with the scheduling decisions in order to competitively satisfy the demand for products within available production capacity [2]. Integrated models have been proposed for several industrial contexts, for example, in the glass container industry [1], animal feed supplements industry [15], soft drink industry [4] - [5] and automobile assembly lines [16]. Two main strategies have been used to model the scheduling decisions. The first one is a small bucket strategy in which each period of the planning horizon is divided into micro-periods. For each micro-period only one item can be produced. This strategy is based on the GLSP model (*General Lotsizing and Scheduling Problem*) [6]. The second strategy is a big bucket one and allows the production of several items in a given period. The decisions associated with lot sizing are based on the Capacitated Lot Sizing Problem (CLSP) (*e.g.* [10]). To obtain the production sequence

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in each period the assignment constraints and the subtour elimination constraints proposed for the Asymmetric Traveling Salesman Problem (ATSP) are used to guarantee that each item produced in a period is sequenced and to eliminate subsequences respectively.

In this work we are interested in studying the quality of three mathematical formulations for the Integrated Lot sizing and Scheduling Problem (ILSP). The first formulation is based on the small bucket strategy and on the GLSP model. The second formulation is based on the big bucket strategy and uses the Miller, Tucker and Zemlin (MTZ) subtour elimination constraints. The attractiveness of the MTZ constraints lies in its compact polynomial representation. However, it is well know that the MTZ constraints produce a weak linear relaxation. Motivated by this, Sherali and Driscoll (2002) [14] develop a tighter representation for ATSP-MTZ using a specialized version of the Reformulation Linearization Technique (RLT). We use the RLT to propose a third formulation for the the (ILSP).

The formulations's quality will be assessed considering the associated dual bounds and their influence on the branch and cut algorithm included in the commercial solver Cplex [8]. The rest of paper is organized as follows. In section 2 we give a brief description of the production process used as a base for the models and present the first two mathematical formulations for the ILSP. In Section 3 we present a new class of valid inequalities derived using the RLT technique and the third formulation for the ILSP. The quality of the three formulations is discussed in Section 4. Final remarks are given in Section 5.

2 Problem Description and Two Classical Formulations

We will consider a one stage, one machine production process in which a set of items is to be produced over a discrete finite planning horizon. The demand of items are known in advance and are given for each period. The machine capacity is taken into account in every period, as well as machine changeover costs and times since the latter values are sequence dependent. To prevent infeasabilities we will allow back orders in each period. It is also assumed that the production periods are independent, that is, the last set up of a given period is not carried over to the following period. The problem is then to define which items to produce and determine their respective lot sizes and production sequence in order to minimize total inventory, backorder and changeover costs.

The GLS1S1M Model - The first formulation, based on the GLSP model, contains the usual balance and capacity constraints. To obtain the production sequence, each planning period (called a macro-period) is divided into a number of micro-periods equal to the maximum number of setups in each period. The micro-period size is flexible and depends on the item lot size. It is a small bucket approach in the sense that only one item can be produced in each micro-period. Given space limitations we do not present the complete model description.

The MTZ1S1M Model - The second formulation uses a big bucket approach and is adapted from the P1S1MTS model proposed by [3]. To obtain the production sequence in each period, assignment and MTZ constraints are used respectively to guarantee that each item produced in a period is sequenced and to eliminate subsequence. To describe the model, let J be the number of items, T the number of periods, and (i, j, t) the index set, $i, j \in \{1, \dots, J\}$; $t \in \{1, \dots, T\}$. We will assume that the changeover time (b_{ij}), the demand (d_{jt}); the machine capacity (c_t), the changeover cost (s_{ij}) and the backorder/inventory costs (g_j/h_j) are known. To represent the problem decisions, let I^+_{jt}/I^-_{jt} be the inventory/backorders for item j at the end of period t, x_{jt} the production quantity, z_{ijt} indicate if there is or not a changeover from item i to item j in period t; and u_{jt} an auxiliary variable.

The complete description of the MTZ1S1M model is given by (1) - (9). The lot sizing decisions are restricted by constraints (2)-(4) which model the flow conservation of each item in each time period, the machine capacity, and guarantee that there is production only if the machine is prepared.

Constraints (5)-(8) model the order in which the items will be produced in a given period t. In each period the machine is initially set-up for a ghost item i_0. The changeover costs associated with the ghost item are zero and do not interfere in the total solution cost.

$$\text{Min } \; \text{Z} = \sum_{j \in J} \sum_{t \in T} (h_j I^+_{jt} + g_j I^-_{jt}) + \sum_{t \in T} \sum_{i \in J} \sum_{\substack{j \in J \\ j \neq i}} s_{ij} z_{ijt} \tag{1}$$

subject to:

$$I^+_{j(t-1)} + I^-_{jt} + x_{jt} - I^+_{jt} - I^-_{j(t-1)} = d_{jt}, \qquad \forall j, \forall t \tag{2}$$

$$\sum_{j \in J} p_j x_{jt} + \sum_{i \in J} \sum_{j \in J; j \neq i} b_{ij} z_{ijt} \leq C_t, \qquad \forall t \tag{3}$$

$$x_{jt} \leq \frac{C_t}{p_j} \Big(\sum_{i = i_0 i \neq j}^{J} z_{ijt} \Big), \qquad \forall j, \forall t \tag{4}$$

$$\sum_{j \in J} z_{i_0 jt} \geq \sum_{i = i_0}^{J} z_{ikt}, \qquad \forall k \in J; \; k \neq i, \; \forall t \tag{5}$$

$$\sum_{i = i_0 i \neq k}^{J} z_{ikt} = \sum_{j = i_0 j \neq k}^{J} z_{kjt}, \qquad \forall k \in J, \; \forall t \tag{6}$$

$$\sum_{j = i_0; j \neq i}^{J} z_{ijt} \leq 1, \qquad \forall i = i_0, 1, \ldots, J, \; \forall t \tag{7}$$

$$u_{jt} \geq u_{it} + 1 - (J)(1 - z_{ijt}); \qquad \forall i, \forall j; i \neq j; \forall t \tag{8}$$

$$x_{jt}, u_j \geq 0, z_{ijt} = 0/1, \qquad \forall i, j; \forall t. \tag{9}$$

3 Application of RLT to tighten MTZ1S1M

Consider a nonlinear restatement of constraints (8) from the MTZ1S1M formulation:

$$u_{jt} z_{ijt} \leq (u_{it} + 1) z_{ijt} \qquad \forall i, j, t \tag{10}$$

$$u_{jt} z_{i_0 jt} \geq z_{i_0 jt} \qquad \forall j, t \tag{11}$$

$$u_{jt} z_{j i_0 t} \leq (J) z_{j i_0 t} \qquad \forall j, t. \tag{12}$$

and the following lower and upper bounds for the variable u_{jt}:

$$1 \leq u_{jt} \leq J \qquad \forall j, t \tag{13}$$

The formulation given by (1)-(7), (9) and (10)-(13) will be called MTZ1S1M2. To apply the RLT to MTZS1M2, we will first reformulate it by generating additional implied constraints (Reformulation

Phase). The reformulation phase is done in two steps. The Linearization Phase is then applied to the reformatted MTZ1S1M2 by using a variable substitution for each distinct nonlinear term.

The Reformulation Phase

Step1 - Multiply a inequality by a lower bound. Take constraints (7) and restate as (14), since $u_{kt} \geq 0$.

$$u_{it}\left[\sum_{\substack{j=i_0\\ j\neq i}}^{J} z_{ijt} - 1\right] \leq 0 \qquad \forall i,\ t. \tag{14}$$

Consider now constraints (6) and (7). Together they imply (15), since $u_{kt} \geq 0$.

$$u_{kt}\left[\sum_{\substack{i=i_0\\ i\neq k}}^{J} z_{ikt} - 1\right] \leq 0 \qquad \forall k,\ t. \tag{15}$$

Step2 - Multiply a given inequality by a lower and an upper bound of the u_{ijt} variables. By (7) and (6) we have that $z_{ijt} + z_{jit} \leq 1\ \forall\ i,\ j,\ t;\ i \neq j$. This is true if we desconsider the ghost item i_0. Then we can use the variable u_{jt} lower bound defined in (13) to obtain the valid inequalities (16).

$$(u_{jt} - 1)(1 - z_{ijt} - z_{jit}) \geq 0 \qquad \forall\ i \geq 1; \forall j \geq 1;\ i \neq\ j. \tag{16}$$

Using the u_{jt} upper bound stated in (13), we obtain the valid inequalities (17).

$$(J - u_{jt})(1 - z_{ijt} - z_{jit}) \geq 0 \qquad \forall\ i \geq 1; \forall j \geq 1; i \neq\ j. \tag{17}$$

The Linearization Phase

Consider the following set of variables:

$$\lambda_{ijt} = u_{it} z_{ijt} \qquad \forall\ i,\ j, t;\ i \neq j; \tag{18}$$
$$y_{ijt} = u_{jt} z_{ijt} \qquad \forall\ i,\ j, t;\ i \neq j; \tag{19}$$
$$y_{ijt} = \lambda_{ijt} + z_{ijt} \qquad \forall\ i,\ j, t;\ i \neq j. \tag{20}$$

By expanding constraints (14) and using (12) and (18) we obtain a new set of linear valid inequalities:

$$\sum_{j=1}^{J} \lambda_{ij} + (J) z_{ii_0t} \leq u_{it}, \qquad \forall i,\ t. \tag{21}$$

With a similar argument and using constraints (11) and (18) the linearization of (15) is given by (22).

$$\sum_{i=1}^{J} \lambda_{ikt} + z_{i_0kt} \leq u_{kt}, \qquad \forall k,\ t. \tag{22}$$

Expanding the nonlinear valid inequalities (16) and using the variable redefinitions given by (18) and (19) we obtain the linear valid inequalities (23).

$$\lambda_{ijt} + \lambda_{jit} \leq u_{jt} - (1 - z_{jit}), \qquad \forall\ i \geq 1; \forall j \geq 1; i \neq\ j \tag{23}$$

With a similar argument and using the variable redefinitions given by (18)-(20) we obtain from (17) the linear valid inequalities (24).

$$u_{it} + (J-1)z_{ijt} - (J)(1 - z_{jit}) \leq \lambda_{ijt} + \lambda_{jit}, \qquad \forall\ i \geq 1; \forall j \geq 1; i \neq\ j \tag{24}$$

The SD1S1M model is then defined using the same constraints as model MTZ1S1M, but replacing constraints (8) by the tighter constraints (21) - (24). Using polyhedral theory it is possible to show that the polyhedron associated with the MTZ1S1M formulation contains the polyhedron associated with the SD1S1M formulation. Therefore we may expect that the latter provides a stronger LP relaxation bound. A preliminary version of this model was discussed in [13] and [11].

4 Computational Results

The three formulations presented in Sections 2 and 3 (GLS1S1M, MTZ1S1M and SD1S1M) have been codified in the AMPL syntax [7] and solved by the Cplex optimizer [8]. The computational study was divided in two phases. In the first phase the three models were compared using an illustrative example. In the second phase only models based on the big bucket approach were compared using instances adapted from [9]. All the runs were executed on a computer Intel Core i7-2600 CPU, 3,4 GHz, 16 GB RAM.

4.1 Illustrative Example

Consider $J = 15$ and $T = 5$, the number of items and periods respectively. The total number of variables were 20580, 1610, and 2895 for the GLS1S1M, MTZ1S1M and SD1S1M instances respectively; and the total number of constraints were 17220, 1465, 2665 respectively. The maximum execution time was defined as three minutes.

Optimality was not proved for the GLS1S1M instance and the gap was still 99.99% after one hour. Less than five seconds were necessary to prove optimality for the two other model instances. The best linear relaxation bound in the root node was obtained from the SD1S1M model (13074.44 against 12954.41 for the MTZM1S1M instance). Also, the total number of nodes were smaller for the SD1S1M instance than for the MTZ1S1M instance (1015 against 2733).

4.2 Results

To given an idea of the new model we present results from instances taken from [9]. The data set is related to single machine problems with different capacities. The maximum execution time was defined as one hour (3600 seconds). The instances are grouped using the quadruplet J, T, $\%C_t$, s_{ijt} (representing: number of products, number of periods, average capacity utilization per period and cost of setup per time unit). This gave a total of 240 instances, being 10 different instances of each the 24 groups created by combining different values of the parameters: $J \in \{15, 25\}$, $T \in \{5, 10, 15\}$, $\%C_t \in \{0.6, 0.8\}$ and $s_{ijt} \in \{50, 100\}$.

In the Figure 1, the horizontal axis shows each group and the vertical axis real numbers. The blue line presents the average values of the difference between the linear relaxation of the models MTZ1S1M and SD1S1M $((LR_{SD1S1M} - LR_{MTZ1S1M}))$. The average and the standard deviation of the dual bounds values lie in the interval [0; 130] and [0; 170] respectively. In the SD1S1M model always provides better bounds than the MTZ1S1M. It is possible to observe that as the set up cost increases the average difference between the dual bound decrease. For the instances with $J = 25$, $T = 5$ and $\%C_t = 0.8$ the dual bounds are the same.

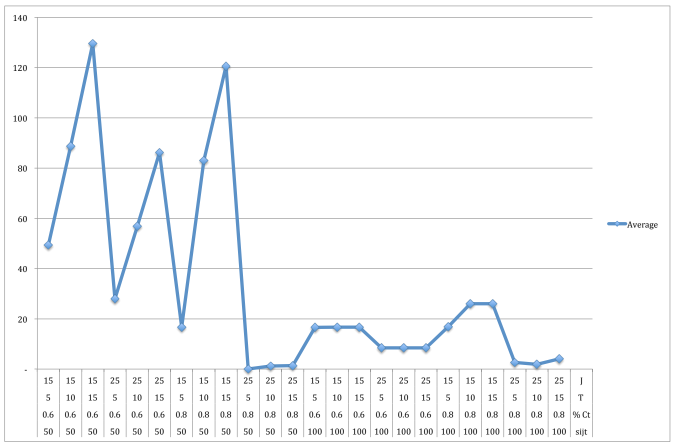

Figure 1: Average Values of the $(LR_{SD1S1M} - LR_{MTZ1S1M})$

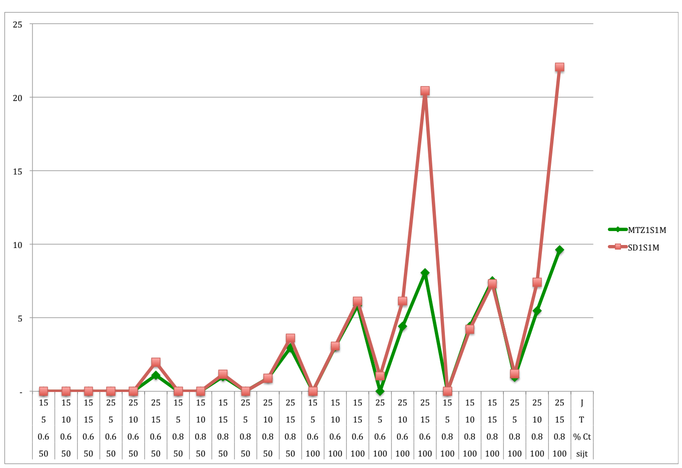

Figure 2: Average Gap (%) found in 60 minutes

Optimality was proven for the SD1S1M model in ten groups (Gap=0 in Figure 2). For the other 14 groups for which the CPLEX did not find the optimal value, the instances have $T = 15$ in half of them. Two higher gaps are for the instances with $J = 25$, $T = 15$ and $s_{ijt} = 100$. In terms of CPU time (Figure 3) the models have the same behavior.

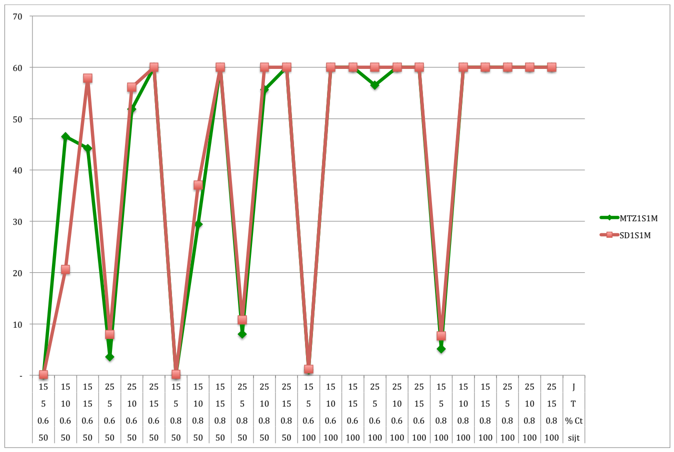

Figure 3: Average CPU Times in minutes

5 Conclusions

The objective of this paper was to present a new and tighter formulation for the ILSP. The MTZ constraints presented an interesting challenge in this vein. Considering the ATSP constraints, much research has shown (e.g. [12]) that models based on them consistently yield the weakest formulation among of all those available. However, for the ILSP, the MTZ1S1M and the SD1S1M formulations provided better computational behavior than the GLS1S1M as shown by the illustrative example results given in Section 4.1. The new formulation provides better dual bounds. The computational study in 4.2 showed that the SD1S1M formulation outperforms the MTZ1S1M formulation in terms of dual bound values, however it presents some computational difficulties for the groups of instances with high number of items and periods, and high values of set up costs. This might be due to the time necessary to solve the associated LR. Using polyhedral theory it is possible to show that the polyhedron associated with the MTZ1S1M formulation contains the polyhedron associated with the SD1S1M formulation. It would be interesting to study how other polynomial formulations for the ATSP behave when applied to the ILSP.

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