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Single-stage formulations for synchronised two-stage lot sizing and scheduling in soft drink production

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ABSTRACT

This study deals with industrial processes that produce soft drink bottles in different flavours and sizes, carried out in two synchronised production stages: liquid preparation and bottling. Four single-stage formulations are proposed to solve the synchronised two-stage lot sizing and scheduling problem in soft drink production synchronising the first stage's syrup lots in tanks with the second stage's soft drink lots on bottling lines. The first two formulations are variants of the General Lot Sizing and Scheduling Problem (GLSP) with sequence-dependent setup times and costs, while the other two are based on the Asymmetric Travelling Salesman Problem (ATSP) with different subtour elimination constraints. All models are computationally tested and compared to the original two-stage formulation introduced in Ferreira et al. (2009), using data based on a real-world bottling plant. The results show not only the superiority of the single-stage models if compared to the two-stage formulation, but also the much faster solution times of the ATSP-based models.

1. Introduction

This paper considers a production lot sizing and scheduling problem encountered in a soft drink bottling plant. Although the models developed are based on that plant, its production processes are sufficiently similar to those in many other bottling plants worldwide and even in other industries for the models proposed in this paper to be widely applicable. Integrated lot sizing and scheduling models have been researched in the context of real-word problems (Clark et al., 2011), for example, packaging company production yogurt (Marinelli et al., 2007), foundries (Araujo et al., 2008), electro-fused grains (Luche et al., 2009), glass container industry (Almada-Lobo et al., 2008), animal feed production (Toso et al., 2009), soft drink production (Ferreira et al., 2010), pharmaceutical company (Stadtler, 2011), sand casting operations (Hans and Van de Velde, 2011). Besides the real problem theoretical models have been extensively studied in the last years (Fleischmann, 1990; Haase, 1994; Drexl and Haase, 1995; Fleischmann and Meyr, 1997; Drexl and Kimms, 1997; Kang et al., 1999; Haase and Kimms, 2000; Meyr, 2000, 2002; Gupta and Magnusson, 2005; Fandel and Stammen-Hegene, 2006; Tempelmeier and Buschkühl, 2008; Gicquel et al., 2009; Kaczmarczyk, 2011).

Production lot sizing and scheduling problems can be very difficult depending on the restrictions which have to be met and on the combinatorial structure (classified in general as NP-hard optimization problems, e.g., Meyr, 2002; Bitran and Yanasse, 1982). In general the integrated lot sizing and scheduling problems are based on lot sizing models (Karimi et al., 2003; Toledo and Armentano, 2006; Helber and Sahling, 2010) adapted to incorporate the lot sequences. The sequence of lots involves the determination of when each lot is produced.

Different characteristics have been considered in the lotsizing and scheduling models. For example, the sequence dependent setup times and costs was studied by Fleischmann and Meyr (1997), Haase and Kimms (2000), Meyr (2000), Beraldi et al. (2008), and Kovàcs et al. (2009). The sequence-dependent setup costs and setup times with setup carryover problem was studied in Gupta and Magnusson (2005) and Menezes et al. (2011); Almeder and Almada-Lobo (2011) study the synchronisation in lot sizing and scheduling problems; Supithak et al. (2010) treat lot sizing and scheduling problems with earliness tardiness and setup penalties; Mateus et al. (2010) apply decomposition methods and an iterative approach for the integration of the problems; Stadtler (2011) studies multilevel lot sizing and scheduling

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problems with zero lead times. For reviews in lot sizing and scheduling problems the reader is referred to, e.g., Drexl and Kimms (1997), Koçlar (2005), Zhu and Wilhelm (2006), Jans and Degraeve (2008), and Robinson et al. (2009).

The formulations for lot sizing and scheduling problems can be mainly classified into two groups: small bucket and big bucket models. In small bucket models, such as the DLSP (Discrete Lot Sizing Problem, Fleischmann, 1990; Gicquel et al., 2009), the planning horizon is broken down into relatively small intervals in which at most one item can be produced. The sequence in small bucket formulations is inherent of the model. In situation in which there are many lots (periods), the total number of variables and constraints can increase significantly. In big bucket models, on the other hand, multiple items can be produced in each period. The strategy to incorporate the sequence in the model can be, for example, adding ATSP constraints (Menezes et al., 2011). An advantage is that the total number of variables and constraints is smaller.

An interesting formulation is the GLSP (General Lot Sizing and Scheduling Problem, Fleischmann and Meyr, 1997), in which the planning horizon is broken down into macro-periods and multiple items can be produced in each macro-period. However, to incorporate the sequence, each macro-period is divided into micro-periods in which at most one item can be produced, so its special structure involving subperiods within time periods may be associated with a small bucket framework (Koçlar, 2005). Clark et al. (2010) take a different approach using an asymmetric travelling salesman problem (ATSP) representation for sequencing lots rather than a GLSP-type model, obtaining good results. Although their formulation was inspired by the animal feed production case, the same idea is applicable to soft drinks production.

An important characteristic of soft drink production processes is the synchronisation between its two stages. This is necessary in case the start of production of lots at the second stage (drink bottling) depends on the lots at the first stage (syrup preparation). Toledo et al. (2007, 2009) propose a general model that synchronises the schedules of the soft drink plant's two production stages. Nevertheless, the mathematical model is rather complex, which has motivated the authors to develop approximate methods. An alternative model to represent a synchronised two-stage multi-machine problem is formulated in Ferreira et al. (2009). The authors simplify the overall problem by dedicating bottling lines to tanks.

This paper introduces alternative formulations for the lot sizing and scheduling problem in which the synchronised two-stage problem is formulated as a single-stage model. The first two formulations (models R1 and R2) are based on the single-stage GLSP model with sequence-dependent setup times and costs, while the other two are ATSP-based formulations (models F1 and F2) with different subtour elimination constraints.

In Section 2, we briefly explain the soft drink production process and summarize the synchronised two-stage formulation presented in Ferreira et al. (2009). In Section 3, the single-stage models R1 and R2 are presented, then Section 4 formulates the two models F1 and F2. Section 5 develops the solution procedures to solve the models. In particular, two strategies are detailed for solving model F2, based on the generation of subtour elimination inequalities and patching procedures. The models are computationally tested and analysed in Section 6. Concluding remarks and perspectives for future research are discussed in Section 7.

2. The soft drink production process

The soft drink production process has two main stages: flavour preparation (stage 1) and bottling (stage 2), as shown in Fig. 1. In stage 1, the liquid flavour (concentrated syrup plus some water) is prepared in tanks of varying capacities. Two different flavours cannot be prepared simultaneously in the same tank. For technical reasons a tank must be empty before a new lot of liquid flavour can be prepared in it, even if the flavour does not change. The preparation (cleaning) times and costs depend on the sequence of flavours. A minimum quantity of liquid flavour must be prepared in order to assure homogeneity as the tank propeller has to be completely covered in order to properly mix the necessary ingredients.

In stage 2, the liquid flavours are bottled at the filling lines. A filling line consist of a conveyor belt and machines that wash the bottles, fill them with a combination of liquid flavour and more water (carbonated or non-carbonated) and then seal, label and pack the filled bottles. If a bottle needs to be removed from the conveyor belt, then this is done at the end of the production process, before packaging. There is only one entry point for the bottles in the filling line. Conceptually, we can consider the entire filling line as a single machine processing items characterized by different flavour/bottle-size combinations. Although the syrup preparation is denoted as stage I, the tanks are only freed to start a new syrup preparation once its liquids are completely bottled at stage II. As an example, the *Tank* 1 of Fig. 1 will be available to prepare other syrup once the *Line* 1 finishes the liquid bottling. Obviously, the line can only start the production in case the syrup is ready.

A line can receive a liquid flavour from only one tank at a time, no matter how many tanks are available. However, a tank can

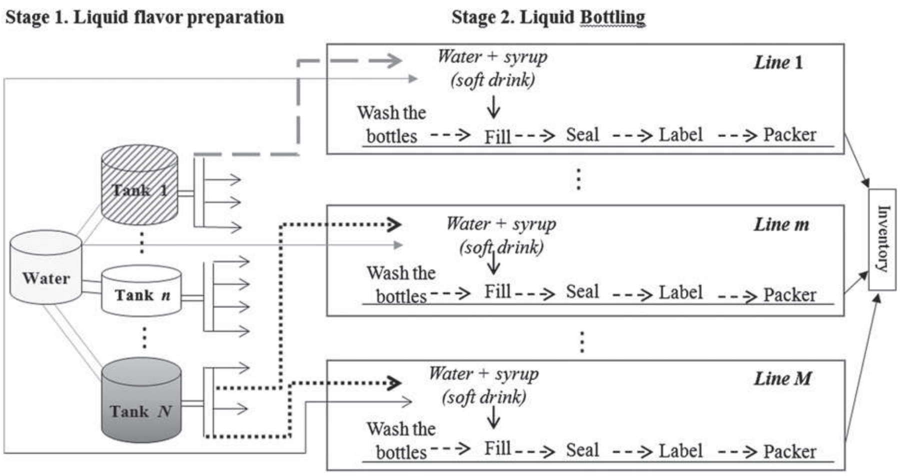

Fig. 1. Soft drink production process.

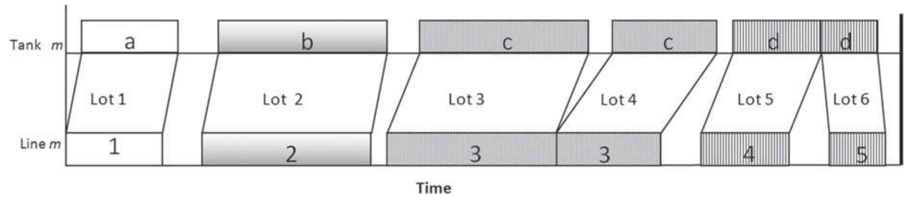

Fig. 2. Batches sequenced but not synchronised.

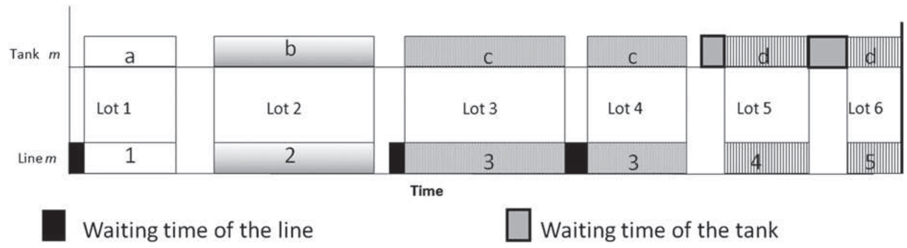

Fig. 3. Sequenced and synchronised batches.

supply a liquid flavour to more than one filling line simultaneously if these are bottling drinks of the same flavour. In Fig. 1, lines *m* and *M* are exclusively connected to *Tank N*. Each changeover from one flavour/bottle-size to another requires cleaning and/or mechanical adjustments, which depend on the sequence of flavour/sizes and can take a long time. The changeover times may vary up to 5 h while the changeover times of the tanks may range between 40 min and 4 h. Therefore, its common in the soft drink industry to have an idle line waiting for the syrup preparation or a tank waiting to supply a lot of prepared syrup due to the changeover of the line.

Because of this fact the relevance of the synchronisation between the two stages in the production scheduling comes into play, as illustrated in Figs. 2 and 3. Both represent the lot sizing and scheduling plans of five different items (1, 2, 3, 4, 5) of four flavours (*a*, *b*, *c*, *d*). The planning horizon is divided into six subperiods of varying lengths, one per lot.

Note that in Fig. 2 the batches of tank liquids and line items are correctly sequenced at both stages, but only the last batch is correctly synchronised. The gaps between consecutive batches of liquids (*a*, *b*, *c*, *d*) and between consecutive lots of items represent the changeover times. Note that item 3 is produced in both subperiods 3 and 4; it uses the same liquid *c* but needs tank replenishment. Observe too that liquid *d* is used in both of the distinct items 4 and 5 which may differ in, say, bottle size.

To achieve synchronisation between tank and line setup times, the product batches have to be delayed by inserting idle times (black rectangles) in the production line in subperiods 1–4, while the liquid batches must also be delayed by inserting idle times (grey rectangles) in subperiods 5 and 6, as shown in Fig. 3. The maximum between changeover times of the tank and of the line defines the respective waiting times to ensure a synchronised plan. The need for synchronisation impacts on the effective utilisation of the capacities of tanks and production lines and may result in delays, or even turn a given problem infeasible. In the practice of production planning and control at a soft drink company, the problem is often solved separately, that is, the lot sizes are decided first and then the lot sequences. Further details about the production of the soft drinks and associated scheduling challenges in practice can be found in Ferreira et al. (2009). An appropriate model often includes extra decision variables to deal with synchronisation, adding complexity to the formulation and solution techniques.

In general, the setups on lines/tanks cannot be carried-over between periods, as syrups are perishable and do not last from one period to the next. We follow this requirement here. Nevertheless, in cases where the company works on a 7-day 24-h schedule, there is no need to perform a setup at the beginning of each day and, therefore, setup carryovers should be considered.

3. The GLSP-based single-stage models (R1 and R2 models)

3.1. Previous work on GLSP models

As noted above, Ferreira et al. (2009) proposed a two-stage lot sizing and scheduling model for the problem detailed in Section 2 that handles synchronisation issues. The first stage deals with the tank-related decisions and constraints, while the bottling-line environment is addressed in the second stage. The authors call their model P2SMM (two stages and multiple machines). The model has the simplifying assumption that a tank is dedicated to a line within a particular planning period. A tank can produce all the flavours needed by the line. In addition to the balancing of inventory, backlogs, production and demand constraints, other requirements take into account capacity, minimum possible lot sizes in tanks, the sequencing of lots at both stages, production-only-if-setup logic, and the synchronisation of delays. The model also recognizes that the capacity bottleneck can shift between stages 1 and 2. The planning horizon is divided into (macro) big-bucket periods of fixed length, each of which is itself subdivided into subperiods of variable length. Note that each subperiod corresponds to a possible setup and production of a certain syrup/item. A subperiod with no production can be of zero length.

In order to solve the model, different solution approaches based on mathematical models and relax-and-fix methods (Wolsey, 1998) were presented in Ferreira et al. (2009). The best strategy found by the authors was the two-step heuristic RA-G21. In the first phase, a simplified model (P1SMM model) composed of the second stage constraints of P2SMM is solved by a relax-and-fix heuristic. Then, the changeover and setup variables for subperiods with positive production amounts are fixed, and P2SMM's resulting sub-MIP is solved. This approach allows for solutions that are better than the real-world schedules from our case study. However, the optimality gaps of these solutions are still considerable. To improve the solutions, this paper first presents two *single-stage* reformulations for the GLSP-based model (Models R1 and R2), and then two ATSP-based models (Models F1 and F2).

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3.2. Model R1

As in Ferreira et al. (2009) our models assume that each filling line has a dedicated tank. A comparison of Figs. 2 and 3 shows that a synchronised schedule is effectively determined by considering the maximum changeover times between stages 1 and 2. This means that the synchronisation between stage 1's syrup lots and stage 2's soft drink lots can be managed just at the filling-line level, considering the changeover time from item i to j as being simply the greater of the line drink setup time and respective tank syrup setup time. Such a single-stage model must still be able to adequately control inter-stage synchronisation. Tank setup times are always positive, even between the same syrup, but line setups between identical items are null. Thus the use of the maximum setups time between these two different kinds of setups must be adapted to properly reflect synchronised setups, particularly when there is no production in a subperiod. If the setups of lines are null in idle periods, then we will see that it is possible to consider the maximum setup times between lines and tanks.

This section presents R1, a single-stage reformulation of Ferreira et al.'s (2009) P2SMM model in which the synchronisation is done via such maximum setups, thus allowing for the modelling of the problem with only one stage, namely the bottling stage. Note that the single-stage formulation is not a simplification of the two-stage model. Both have the same assumptions, model the same type of constraints, deliver the same optimal solution value, and have the same index sets and data. In the following we denote by [M] the set {1, 2,…, M}. [J], [M], [T] refer to the sets of items, machines and periods, respectively. In addition S_t is the set of subperiods in period t, [S] is the set of all subperiods ($|S|=\sum_t |S_t|$) and P_t is the first subperiod of period t. The set λ_j refers to machines that can produce item j, while α_m is the set of items that can be produced on machine (line) m and ϕ_j refers to the syrup used to produce item j. Additional data is given in the following.

Data:

d_{jt}	demand for item j in period t;
h_j	(non-negative) inventory cost for one unit of item j;
g_j	(non-negative) backorder cost for one unit of item j;
b^I_{kl} (c^I_{kl})	changeover time (cost) from liquid flavour k to l (independent of tank m);
b^{II}_{ij} (c^{II}_{ij})	changeover (cost) from item i to j (independent of machine m);
a^{II}_{mj}	time required to produce one unit of item j on machine m;
K^I_{mt}	total capacity of tank m, in litres of liquid;
K^{II}_{mt}	total time capacity of machine m in period t;
r_{jl}	quantity of liquid flavour l necessary for the production of one unit of item j;
q^I_{lm}	minimum production quantity of liquid flavour l in tank m (needed for liquid homogeneity);
I^+_{j0}	initial inventory for item j;
I^-_{j0}	initial backlog for item j;
y_{mj0}	$=1$ if machine m is initially set up for item j; 0 otherwise.

Note that in the particular context of the soft drink process, some parameters such as changeover time and costs are independent of machines and tanks. However, it is straight forward to include the line dependency by simply adding the index m.

The variables of model R1 are

I^+_{jt}	inventory for item j at the end of period t;
I^-_{jt}	backlog for item j at the end of period t;
x_{mjs}	production quantity on machine m of item j in subperiod s;
y_{mjs}	$=1$ if machine m is in setup-state for item j in subperiod s, otherwise 0;
z_{mijs}	1 if there is a changeover on machine m from item i to item j in subperiod s, otherwise 0.

In addition, let

b_{ij}	max $\{b^{II}_{ij}, b^I_{kl}\}$, where syrup $k=\phi_i$ and syrup $l=\phi_j$;
c_{ij}	$c^{II}_{ij}+c^I_{kl}$, where syrup $k=\phi_i$ and syrup $l=\phi_j$;
LB_{mj}	lower bound of each lot of item j on machine m, defined as the minimum lot size of syrup used to produce item j: $LB_{mj}=(q^I_{lm}/r_{jl})$, $l=\phi_j$;
Q_{mj}	maximum quantity of each lot of item j on machine m, derived from the maximum capacity of the tank: $Q_{mj}=(K^I_m)/r_{jl}$, $l=\phi_j$;
UB_{mjs}	$\min\{Q_{mj},(K^{II}_{mt}/a_{mj})\}$, upper bound on the quantity of item j produced on line m in subperiod $s \in S_t$, $t \in [T]$.

Both parameters Q_{mj} and (K^{II}_{mt}/a_{mj}) give an upper bound for variables x_{mjs}, so the maximum lot size of item j on line m in subperiod s is just the minimum of the two quantities. For example, if the maximum lot quantity of item *Cola* is 100,000 units per tank, but the maximum capacity of line m in period t is 80,000 units, then $UB_{m,Cola,s}=80,000$. Although the UB_{mjs} is indexed for subperiod s, the capacity constraints (4) below guarantee that the total capacity of period t is respected. In general, the term K^{II}_{mt}/a_{mj} is bigger than Q_{mj}. Only in the case of a continuous production of a syrup, such as the *Cola* flavour, is Q_{mj} bigger than K^{II}_{mt}/a_{mj}, as K^{II}_{mt} denote the week production rate (in minute) of line m in period t.

In the traditional GLSP model, setup times (and costs) between identical items are null which allows for the setup configuration between identical items in idle (zero-length) subperiods to be maintained at zero setup time and cost. However, our model charges setup times (and costs) in all changeovers between items i and j even if identical ($i=j$). The reason is that setups are always charged in the first stage, even when refilling a tank with the same syrup as previously (see the definitions of b_{ij} and c_{ij} above). An exception occurs when the setup state on a line is maintained for item i just for modelling purposes, and thus it is followed by zero production of i. This exception is handled in our model by defining a new binary variable p_{mjs} in expression (1) below that allows for the non-consideration of setup costs and times of i in both the objective function (2) and capacity constraints (4).

$$p_{mjs}=\begin{cases} 1 & \text{if } y_{mjs}=1 \text{ and } x_{mjs}=0, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Variable p_{mjs} is null ($=0$) for all $j \in \alpha_m$ in subperiod s if machine m is not configured to produce item j, or if machine m is configured for item j and does indeed produce it (i.e., line m is not idle in subperiod s). Conversely, p_{mjs} has value 1 for all $j \in \alpha_m$ in subperiod s if machine m is configured to produce item j, but does not actually produce it (in other words, machine m is idle in subperiod s). Therefore, in subperiods s with production, $p_{mjs}=0$ for all $j \in \alpha_m$ on machine m. In idle subperiods, $p_{mjs}=1$ for the item j allocated to machine m in order to eliminate changeover times and costs. In the single-stage formulation R1, it is not necessary to differentiate stage I from stage II, so from now on, without loss of generality, we skip the superscript II for parameters and variables (e.g., K_{mjt} and x_{mjs}).

The first model, R1, is formulated as follows:

$$\text{Min} \quad \sum_{j \in [J]} \sum_{t \in [T]} (h_j I^+_{jt} + g_j I^-_{jt})$$

$$+\sum_{m\in[M]}\sum_{s\in[S]}\left(\sum_{i,j\in\alpha_m|i\neq j}c_{ij}z_{mijs}+\sum_{j\in\alpha_m}c_{jj}(z_{mjjs}-p_{mjs})\right). \tag{2}$$

Subject to $$I^+_{j(t-1)}+\sum_{m\in\lambda_j}\sum_{s\in S_t}x_{mjs}+I^-_{jt}=I^+_{jt}+I^-_{j(t-1)}+d_{jt}\quad \forall j\in[J],\ t\in[T]; \tag{3}$$

$$\sum_{j\in\alpha_m}\sum_{s\in S_t}a_{mj}x_{mjs}+\sum_{s\in S_t}\left(\sum_{i,j\in\alpha_m|i\neq j}b_{ij}z_{mijs}+\sum_{j\in\alpha_m}b_{jj}(z_{mjjs}-p_{mjs})\right)\le K_{mt}\quad \forall m\in[M],t\in[T]; \tag{4}$$

$$\sum_{j\in\alpha_m}y_{mjs}=1\quad \forall m\in[M],\ s\in[S]; \tag{5}$$

$$x_{mjs}\le UB_{mjs}(y_{mjs}-p_{mjs})\quad \forall m\in[M],\ j\in\alpha_m,s\in[S]; \tag{6}$$

$$x_{mjs}\ge LB_{mj}(y_{mjs}-p_{mjs})\quad \forall m\in[M],j\in\alpha_m,\ s\in[S]; \tag{7}$$

$$p_{mj(s-1)}\le p_{mjs}\quad \forall m\in[M],\ j\in\alpha_m,t\in[T],\ s\in S_t,\ s\neq P_t; \tag{8}$$

$$\sum_{i\in\alpha_m}z_{mijs}\le y_{mjs}\quad \forall m\in[M],\ j\in\alpha_m,\ s\in[S]; \tag{9}$$

$$z_{mijs}\ge y_{mi(s-1)}+y_{mjs}-1\quad \forall m\in[M],\ i,j\in\alpha_m,\ s\in[S]; \tag{10}$$

$$I^+_{jt},\ I^-_{jt}\ge 0\quad \forall j\in[J],\ t\in[T]; \tag{11}$$

$$x_{mjs},\ z_{mijs}\ge 0,\quad y_{mjs},\ p_{mjs}\in\{0,1\},\quad \forall m\in[M],\ i,j\in\alpha_m,\ s\in[S]. \tag{12}$$

The objective function (2) minimizes the total costs of inventories, backorders and changeovers. Production costs were not added to the objective function as it is assumed that the production unit cost of each item j does not vary by period t, and so the sum of the production costs of all items and periods would be a constant term in the objective function in case all demand is met. On the other hand, the shortage unit costs should be appropriately chosen so that they overcome production costs in case demand is not met. In any optimal solution, the changeover cost in any idle subperiod is between two identical items j. In order to exclude redundant solutions and thus increase solvability, the idle subperiods are forced to be at the end of each period with variables z_{mjjs} and p_{mjs} both taking value 1. Note that the final term $(z_{mjjs}-p_{mjs})$ ensures that the setup cost is charged only if occurs a changeover between identical items j followed by production of j.

Constraints (3) represent the inventory balancing constraints. The total production of item j in a given period t is calculated by summing up production x_{mjs} over all machines $m\in\lambda_j$ where it can be produced and all subperiods of the period $(s\in S_t)$. Constraints (4) represent the machine capacity in each period. The calculation of total changeover times is similar to that for the total costs in the objective function, i.e., they too are not charged in idle subperiods. Constraints (5) ensure that each machine is configured to produce exactly one item per subperiod, allowing the carryover of the setup information.

The explanation of the remaining constraints (6)–(10) is divided into two cases:

1. *Subperiods with production*: Suppose that item j' is produced on machine m in subperiod s, so that the production variable $x_{mj's}$ is positive (>0). By constraints (6), the term $(y_{mj's}-p_{mj's})$ must be 1, and as both variables are binary, $y_{mj's}=1$ and $p_{mj's}=0$. For every $j'\neq j$ the lotsize variables x_{mjs} are zero, so constraints (5) guarantee that the setup variables y_{mjs} are also zero for all $j\neq j'$. Constraints (6) become $0\le UB_{mjs}(0-p_{mjs})$ implying that $p_{mjs}=0$ for all $j\neq j'$. Constraints (6) and (7) define maximum and minimum capacities, respectively.
2. *Idle subperiods*: Differently from subperiods with production, in idle subperiods the lot variables x_{mjs} are zero for all $j\in\alpha_m$. However, constraints (5) force one item $j''\in\alpha_m$ for which the setup variable $y_{mj''s}$ is 1. As $x_{mj''s}=0$, constraints (7) become $0\ge LB_{mj''}(1-p_{mj''s})$, forcing variable $p_{mj''s}$ to equal 1. For all other items $j\neq j''$, we have $x_{mjs}=0$, $y_{mjs}=0$ and constraints (6) guarantee $p_{mjs}=0$ as previously explained.

Constraints (8) force idle subperiods to be placed at the end of a period and be configured for the last item j produced. Constraints (9) guarantee the linkage between changeover and setup variables while constraints (10) take into account the changeovers. Until now variables p_{mjs} were simply related to variables y_{mjs}. It is now necessary to analyse the relation between variables p_{mjs} and changeover variables z_{mjjs}. This is guaranteed by constraints (7)–(10). In idle subperiods, constraints (8) set $p_{mjs}=1$ for a given j', constraints (7) set $y_{mj's}=1$, and constraints (9) and (10) define switchover from j' to j'. Thus $p_{mj's}=1$ forces $z_{mj'j's}$ to be 1, which could be in favour of the gradient of the objective function. Finally, constraints (12) define the variable domain.

3.3. Model R2

It is well known that part of the difficulty of models such as R1 comes from the changeover related constraints (10) that deliver a weak lower bound. These constraints, together with (9), can be replaced by the following sets:

$$y_{mi(s-1)}=\sum_{j\in\alpha_m}z_{mijs}\quad \forall m\in[M],\ i\in\alpha_m,\ s\in[S]; \tag{13}$$

$$\sum_{i\in\alpha_m}z_{mijs}=y_{mjs}\quad \forall m\in[M],\ j\in\alpha_m,\ s\in[S]. \tag{14}$$

Constraints (13) and (14) ensure a balanced network flow of the machine configuration, carrying the setup state on each machine into the next period. Constraints (13) link the setup configuration in a certain period with the flow out of that configuration in the following period. On the other hand, constraints (14) link the flow of the setup configuration on a certain period in time with the necessary flow into that configuration at the beginning of that period. Naturally, (13) and (14) together impose flow in equal to flow out.

This substitution generates fewer constraints as there are $\sum_m|S||\alpha_m|^2+\sum_m|S||\alpha_m|$ constraints of types (10) and (9), and $2\cdot\sum_m|S||\alpha_m|$ of types (14) and (13). Model R2 is formulated as follows:

$$\text{Min}\quad \sum_{j\in[J]}\sum_{t\in[T]}(h_jI^+_{jt}+g_jI^-_{jt})+\sum_{m\in[M]}\sum_{s\in[S]}\left(\sum_{i,j\in\alpha_m|i\neq j}c_{ij}z_{mijs}+\sum_{j\in\alpha_m}c_{jj}(z_{mjjs}-p_{mjs})\right). \tag{15}$$

Subject to constraints (3)–(9) and (12) and (13);

$$I^+_{jt},\ I^-_{jt}\ge 0\quad \forall j\in[J],\ t\in[T]; \tag{16}$$

$$x_{mjs},\ z_{mijs}\ge 0,\ y_{mjs},\ p_{mjs}\in\{0,1\}\quad \forall m\in[M],\ i,j\in\alpha_m,\ s\in[S]. \tag{17}$$

4. The ATSP-based models

Models R1 and R2 are of the small-bucket type. By just allowing at most one setup per subperiod, it is straightforward to accurately incorporate scheduling details and features (such as synchronisation) in such models, but they are well known to deliver weak lower bounds. In this section, we focus on big-bucket formulations to solve our lot sizing and scheduling problem. Here, several production batches may occur per time period. One possibility is to solve an Asymmetric Travelling Salesman Problem (ATSP model) that defines sequences of lots, eliminating disconnected subtour). A challenge of such formulations is how to define a feasible production sequence that excludes disconnected subtours that are not linked to main production sequence).

In the soft drink production process it is necessary to permit the production of several batches of the same item many times in the same time period. Therefore, connected subtours are allowed in a feasible solution, although in an optimal solution there is only one setup from an item i to j, $i \neq j$, in each period, as setups obey the triangular inequality.

Furthermore it is permitted to not produce an item. These production flexibilities differentiate soft drink production planning from the animal-feed problem as defined in Toso et al. (2009), in which the items can be produced more than once in a period. Such features of the problem differentiate it from the traditional asymmetric travelling salesman problem in which each item (city) is produced (visited) exactly once in a period.

In this study, we present two models based on the capacitated lot sizing model with subtour elimination constraints. Both models deliver the sizing and sequencing of lots, but it is necessary to exclude any disconnected subtours. Two different sets of inequalities are tested to eliminate these subtours through two models, F1 and F2. In the first formulation F1, the number of inequalities are polynomial so the inequalities are included *a priori* in the model. In the second formulation F2, the exponential number of constraints forces the model to be solved iteratively, and subtour elimination cuts are dynamically generated and added into it, as discussed in Section 5.

4.1. The Base Model

Models F1 and F2, which have different subtour elimination constraints, build on the same Base Model. Like model P2SMM, they assume that the tanks are dedicated to bottling lines and setups are not carried from one period to the next. The parameters are the same as in models R1 and R2, but parameter UB_{mj} is the maximum lot of syrup prepared per item, that is, it is similar to UB_{mjs} in R1 and R2.

Consider the following variables:

I_{jt}^{+} inventory for item j at the end of period t;
I_{jt}^{-} backorder for item j at the end of period t;
x_{mjt} production quantity on machine m of item j in period t;
$\eta_{mjt}=1$ if machine m is configured to item j at the beginning of period t, 0 otherwise;
ξ_{mjt} number of times the machine m is configured to item j in period t;
$z_{mijt}=1$ if there is changeover on machine m from item i to j, for all i, $j \in \alpha_m$ in period t, 0 otherwise;
$z_{mjjt}=0$ for all $j \in \alpha_m$.

Note that another way to view variables η_{mjt} is that they define the last configuration of machine m in period $t-1$. Thus this set of variables does not imply that machine m is producing item j in period t; rather, they simply keep track of the configuration of the machine from period $t-1$ to period t. To better understand the model, note that the term $(\xi_{mjt}-\eta_{mjt})$ returns the total number of setups (and lots) performed to item j on machine m in period t. Moreover, as the sum $\sum_{i,i\neq j} z_{mijt}$ represents the total number of changeovers to item j (the flow into node j), the term $\xi_{mjt}-\eta_{mjt}-\sum_{i,i\neq j} z_{mijt}$ accounts for the total number of changeovers from item j to j.

The Base Model reads

$$\text{Min} \sum_{j\in[J]}\sum_{t\in[T]}(h_j I_{jt}^{+}+g_j I_{jt}^{-})+\sum_{m\in[M]}\sum_{t\in[T]}\sum_{j\in\alpha_m} c_{jj}\left(\xi_{mjt}-\eta_{mjt}-\sum_{i\in\alpha_m|i\neq j} z_{mijt}\right) + \sum_{m\in[M]}\sum_{t\in[T]}\sum_{i,j\in\alpha_m|i\neq j} c_{ij} z_{mijt} \tag{18}$$

$$\text{subject to} \quad I_{j(t-1)}^{+}+\sum_{m\in[M]}\sum_{s\in S_t} x_{mjt}+I_{jt}^{-}=I_{jt}^{+}+I_{j(t-1)}^{-}+d_{jt} \quad \forall j\in[J],\ t\in[T]; \tag{19}$$

$$\sum_{j\in\alpha_m} a_{mj}x_{mjt}+\sum_{i,j\in\alpha_m|i\neq j} b_{ij}z_{mijt} + \sum_{j\in\alpha_m} b_{jj}\left(\xi_{mjt}-\eta_{mjt}-\sum_{i\in\alpha_m|i\neq j} z_{mijt}\right)\le K_{mt} \quad \forall m\in[M],\ t\in[T]; \tag{20}$$

$$\sum_{j\in\alpha_m}\eta_{mjt}=1 \quad \forall m\in[M],\ t\in[T]; \tag{21}$$

$$\eta_{mjt}+\sum_{i\in\alpha_m} z_{mijt}=\sum_{i\in\alpha_m} z_{mjit}+\eta_{mj(t+1)} \quad \forall m\in[M],\ j\in\alpha_m,\ t\in[T]; \tag{22}$$

$$\xi_{mjt}\le |S_t|\left(\sum_{i\in\alpha_m} z_{mijt}+\eta_{mjt}\right) \quad \forall m\in[M],\ j\in\alpha_m,\ t\in[T]; \tag{23}$$

$$x_{mjt}\le UB_{mj}(\xi_{mjt}-\eta_{mjt}) \quad \forall m\in[M],\ j\in\alpha_m,\ t\in[T]; \tag{24}$$

$$x_{mjt}\ge LB_{mj}(\xi_{mjt}-\eta_{mjt}) \quad \forall m\in[M],\ j\in\alpha_m,\ t\in[T]; \tag{25}$$

$$\sum_{i\in\alpha_m} z_{mijt}\le \xi_{mjt} \quad \forall m\in[M],\ j\in\alpha_m,\ t\in[T]; \tag{26}$$

$$I_{jt}^{+},I_{jt}^{-}\ge 0 \quad \forall j\in[J],\ t\in[T]; \tag{27}$$

$$x_{mjt}\ge 0;\ \eta_{mjt},z_{mijt}\in\{0,1\};\ \xi_{mjt}\in Z^{+}, \quad \forall m\in[M],\ i,j\in\alpha_m,\ t\in[T]; \tag{28}$$

The objective function (18) minimizes the total inventory, backorder and changeover costs. The term $\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{i\in\alpha_m}\sum_{j\in\alpha_m|i\neq j} c_{ij}z_{mijt}$ takes into account the changeover costs, c_{ij}, between different items. The term $(\xi_{mjt}-\eta_{mjt}-\sum_{i\in\alpha_m|i\neq j} z_{mijt})$ represents the number of changeovers from item j to j as explained before. Constraints (19) balance inventory, backorders and production with demand. Constraints (20) reflect the finite capacity where setup times are interpreted in the same way as setup costs before. Each machine must be configured for exactly one item at the beginning of each period, as enforced by constraints (21). Constraints (22) represent the setup flow balance equations that carry the setup configuration into the next period. Note that the Base Model is not accurate in its current form as it allows for solutions with cycles that are disjoint, also known as disconnected subtours.

Constraints (23) define that, if machine m is not configured to produce item j at the beginning of period t and there are no changeovers into j, then variable $\xi_{mjt}=0$ and consequently $x_{mjt}=0$. The maximum and minimum lots of item j in period t on machine m are defined by constraints (24) and (25), respectively. As explained above, $\xi_{mjt}-\eta_{mjt}$ corresponds to the number of lots produced. Constraints (26) guarantee that ξ_{mjt} is positive when z_{mijt} is positive for at least one i in α_m. Finally, constraints (28) define the domain of the decision variables. It is necessary to add valid inequalities into the Base Model to eliminate disconnected subtours.

4.2. Model F1

Sections 4.2 and 4.3 present two different types of valid inequalities that will give rise to the two different formulations, F1 and F2. For the first approach, the Muller–Tucker–Zemlin (MTZ) subtour (Sherali and Driscoll, 2002) elimination constraints are tested. Almada-Lobo et al. (2007) adapted these constraints, also presented in Nemhauser and Wolsey (1988), for setup carryovers. New continuous variables v_{mit} are necessary to schedule production lots on machine m in period t. The larger the value of v_{mit}, the later item i is scheduled in period t, assuming that each machine is set up for only one item at a time. Constraints (29) eliminate the disconnected subtours

$$v_{mit}+1 \le v_{mjt}+|\alpha_m|(1+\eta_{mit}-z_{mijt}) \quad \forall t\in[T],\ m\in[M],\ i\in\alpha_m,\ j\in\alpha_m\backslash\{i\}. \tag{29}$$

For example, consider machine m in period t, the set $\alpha_m=\{1,2,3,4\}$ and a disconnected subtour $ST=\{2,3,4\}$ with $z_{23}=1$, $z_{34}=1$, $z_{42}=1$, $\eta_{m2t}=\eta_{m3t}=\eta_{m4t}=0$. The following constraints are generated: $V_2\le V_3-1$, $V_3\le V_4-1$ and $V_4\le V_2-1$, resulting in $V_2\le V_2-3$ which is infeasible. Note that each item is set up only once in the period and that solutions in which items are produced more than once (connected subtours) are also eliminated. Almada-Lobo et al. (2007) apply a similar modelling device.

Model F1 is defined as follows:

$$\text{Min} \sum_{j\in[J]}\sum_{t\in[T]}(h_jI^+_{jt}+g_jI^-_{jt})+\sum_{m\in[M]}\sum_{t\in[T]}\sum_{j\in\alpha_m}c_{jj}\left(\xi_{mjt}-\eta_{mjt}-\sum_{i\in\alpha_m|i\neq j}z_{mijt}\right)+\sum_{m\in[M]}\sum_{t\in[T]}\sum_{i,j\in\alpha_m|i\neq j}c_{ij}z_{mijt}. \tag{30}$$

Subject to constraints (19)–(26) and (29);

$$I^+_{jt},I^-_{jt}\ge 0 \quad \forall j\in[J],\ t\in[T]; \tag{31}$$

$$v_{mjt},\ x_{mjt}\ge 0;\quad \eta_{mjt},\ z_{mijt}\in\{0,1\};\quad \xi_{mjt}\in Z^+ \quad \forall m\in[M],\ i,j\in\alpha_m,\ t\in[T]; \tag{32}$$

4.3. Model F2

The following set of valid inequalities from Almada-Lobo et al. (2007) are also effective to eliminate disconnected subtours.

$$\sum_{j\in\alpha_m} z_{mijt}\le 1 \quad \forall m\in[M],\ t\in[T],\ i\in\alpha_m; \tag{33}$$

$$\sum_{i\in\alpha_m} z_{mijt}\le 1 \quad \forall m\in[M],\ t\in[T],\ j\in\alpha_m; \tag{34}$$

$$\sum_{i\in C}\sum_{j\in\alpha_m\backslash C} z_{mijt}+\sum_{j\in C}\eta_{mj(t+1)}\ge\sum_{i\in\alpha_m}z_{mikt} \quad \forall m\in[M],\ t\in[T],\ k\in C,\ C\subseteq\alpha_m, \tag{35}$$

where C is the node set of a subtour. Constraints (33) and (34) ensure that each item is produced at most once per period (i.e., the flow in and flow out of each item equal to zero or one). Constraints (35) eliminate disconnected subtours. For all disconnected subtours, the following holds:

$$\sum_{i\in C}\sum_{j\in\alpha_m\backslash C} z_{mijt}=0 \quad\text{and}\quad \sum_{j\in C}\eta_{mj(t+1)}=0 \quad \forall m\in[M],\ t\in[T], \tag{36}$$

so that $0\ge\sum_{i\in[J]}z_{mikt}$ but $z_{mikt}=1$ for $i\in C$ and $k\in C$. Hence, a solution with a disconnected subtour does not satisfy constraint (35). Clearly we need to identify the subtours to generate (35), as there are an exponential number of them. Almada-Lobo et al. (2007) show that a formulation with this type of valid inequalities is at least as strong as a formulation with the MTZ-type cuts.

Model F2 is defined by

$$\text{Min} \sum_{j\in[J]}\sum_{t\in[T]}(h_jI^+_{jt}+g_jI^-_{jt})+\sum_{m\in[M]}\sum_{t\in[T]}\sum_{j\in\alpha_m}c_{jj}\times\left(\xi_{mjt}-\eta_{mjt}-\sum_{i\in\alpha_m|i\neq j}z_{mijt}\right)+\sum_{m\in[M]}\sum_{t\in[T]}\sum_{i,j\in\alpha_m|i\neq j}c_{ij}z_{mijt}. \tag{37}$$

Subject to constraints (19)–(28); (33)–(35);

$$I^+_{jt},I^-_{jt}\ge 0 \quad \forall j\in[J],\ t\in[T]; \tag{38}$$

$$x_{mjt}\ge 0;\ \eta_{mjt},z_{mijt}\in\{0,1\};\ \xi_{mjt}\in Z^+\ \forall m\in[M],\ i,j\in\alpha_m,\ t\in[T]. \tag{39}$$

Because of the exponential number of constraints (35), two different strategies to solve model F2 are introduced in the next section, namely SF2 and PS (patching).

5. Strategies to solve the models

Models R1, R2 and F1 have a polynomial number of constraints, so it is not difficult to apply an optimization solver to them. On the other hand, model F2 has an exponential number of constraints which can be dynamically added in different ways. In this section, we discuss two strategies (SF2, PS) to solve F2. The R1, R2 and F1 models are solved by the exact branch and cut method implemented in CPLEX version 11.0 (ILOG, 2008) with default parameters.

5.1. Strategy F2—SF2

In this study the inequalities of family (35) are generated and introduced in a dynamic way. The pseudo-code of the algorithm to solve model F2 is given below

Algorithm 1. Pseudo-code of strategy SF2.

while *run time < total-time-limit* **do**
  Step 1. Solve the Base Model together with constraints (33)and(34);
  Step 2. Check the status of the solution of Step 1;
  if *no feasible solution found* **then**
    if *run time > total-time-limit with no feasible solution* **then**/* no feasible solution found for original problem */
      Stop.
    end
    if *run time < total-time-limit with no feasible solution* **then**
      Solve the Base Model together with constraints (33)and(34), until total-time-limit or a feasible solution;
      Go to Step 2;
    end
  end
  else
    Go to Step 3;
  end
  Step 3. **for** $i\leftarrow 1$ **to** M **do**
    Detect the subtours;
    if *There is no subtour* **then**
      if *Optimal solution* **then** Stop;
      else Use the current solution as the initial solution Go to Step 1;
    end
  end
  Step 4. Generate and insert constraints (35) into Base Model; Go to Step 1;
end

In Step 1 of Algorithm 1 we solve the Base Model. However, after a few iterations including valid inequalities, the Base Model gets more difficult and it might not be possible to solve it to optimality in reasonable time, then in Step 2 the execution time is limited. Nevertheless, it is necessary to guarantee that an integer solution is found (Step 3). Note that the F2 strategy can become an exact method if the total execution time and execution of Step 1 are long enough to include all the valid inequalities. In general, the optimal solution of the Base Model, which has subtours, is a lower bound to the original problem, while a feasible solution with no subtours is an upper bound.

5.2. Patching strategy—PS to solve F2

As discussed above, to solve the asymmetric travelling salesman problem it is necessary to detect and eliminate subtours. In many cases a huge number of subtour elimination constraints are needed and it is possible that the modified Base Model does not provide an integer feasible solution in a reasonable amount of time. Accordingly, we have recourse to heuristics that construct a feasible solution which yields an upper bound on the quality of optimal solution.

One of the heuristics (among many others) is based on the Karp–Steele patching (KSP) heuristic (Karp, 1979). In Clark et al. (2010) such a heuristic is applied to solve an animal-feed lot sizing and scheduling problem. A model is proposed and different subtour elimination strategies are applied. A strategy was tested in which all of a solution's subtours are patched to generate a feasible solution for the original problem. In models that are hard to solve, this strategy can help by finding a hopefully better feasible solution at each iteration of the procedure.

The Patching strategy can be seen as strategy SF2 with the addition of two steps, one for patching subtours, and another to update the upper bound. The algorithms in pseudo-code is provided below

Algorithm 2. Pseudo-code of Patching Strategy (PS).

```
while run time < total-time-limit do
    Steps 1−3 of Algorithm 1;
    Step 4. Patch the subtours to the main sequence, solve the
    resulting model and update the upper bound;
    Step 5. Generate and insert constraints (35)
    into Base Model; Go to Step 1;
end
```

6. Computational testing

In order to compare the performance of models R1, R2, F1 and F2 (the last solved by strategies SF2 and PS), we ran experiments on three sets of instances, based on real world data of a soft drink company. The first set (instances E1–E28) contains relatively small instances for which provably optimal solutions are obtained by the models and strategies. The second set (instances I1–I15) is composed of realistic instances used in Ferreira et al. (2009), for which no proven optimal solutions are known. The tests in the second set of instances revealed that the changeover costs are much higher than inventory and backorder costs, which could make the scheduling (sequence) decisions more important than the lot sizing decisions. In the third set of instances (instances I1b–I15b), in order to represent situations in which the sequence decisions are less important, the changeover costs were disconsidered (i.e., their unit costs were nulled in the models).

Model F1 and strategies SF2 and PS were also compared with model P2SMM and heuristic RA-G21 of Ferreira et al. (2009) (see Sections 1 and 3). The solutions in Ferreira et al. (2009) were compared to the company solutions of the case study. When defining the lot sizes and schedule, the soft drink plant requires that the product inventories in a given macro-period must be sufficient to cover the product demands in the next period. To have a fair comparison among the solutions, the following set of constraints (40) were added to models R1, R2, F1 and F2:

$$I^+_{jt} \geq d_{j(t+1)} \quad \forall t \in [T],\, j \in [J], \tag{40}$$

where $d_{j(T+1)}$ is the forecasted demand for item j in period $T+1$, i.e., the first period beyond the planning horizon.

In models R1 and R2, which subdivide the periods to consider the scheduling, the total number of subperiods (lots) is previously defined. So as to compare appropriately R1 and R2 (small bucket models) with F1 and F2 (big bucket models), the following constraints were included in models F1 and F2, in order to limit the total number of production lots

$$\sum_{j \in \alpha_m} (\xi_{mjt} - \eta_{mjt}) \leq |S_t| \quad \forall t \in [T],\, m \in [M]. \tag{41}$$

As mentioned in Section 5, the optimization solver CPLEX version 11.0 was used with default parameters to solve models R1, R2, F1, as well as the Base Model in strategies SF2 and PS. The AMPL modelling language (Fourer et al., 2003) was used to generate the model instances and to code the strategies SF2 and PS. The runs were executed on a 3.0 GHz Intel Pentium 4 processor with 2.0 GB RAM. The total execution time was limited to 4 h, and each instance of Base Model F2 was limited to 15 min.

6.1. Small instances

In this section, we assess the computational performance of the different strategies to solve relatively small-sized instances up to optimality. Each instance encompasses two lines, four items, two syrups and three periods, each with six sub-periods, totalling 18 subperiods. As previously mentioned, these instances are based on real-world data. However, they consider a smaller number of items and machines than those typically found in practice. The tests addressed 28 instances (E1–E28) grouped in four categories. Instances E8–E14 are similar to instances E1–E7, but with reduced capacity; instances E15–E21 are similar to instances E8–E14, but with syrup changeover costs; instances E22–E28 are similar to instances E1–E7, but with syrup changeover costs.

Table 1 shows the total execution times to optimality for each instance. The fastest times are shown in bold. The RA-G21 heuristic was not included as it did not provide optimal solutions and, for some instances with tight capacities, not even feasible solutions. Infeasible solutions are due to the relax-and-fix strategy that fixed the sequences of production in lines and was unable to find any feasible solution when the tank constraints were included (for synchronisation). Note that F1 is by far the fastest to solve 26 out of the 28 instances. For instance, the execution time of E17 is similar to the total execution time consumed by the PS strategy, while for instance E25 the run time of PS is shorter. Note that PS and SF2 are the second and third best strategies in the tests, which suggests that big-bucket models (and their respective strategies) are more efficient than small-bucket models for the current lot-scheduling problem.

In relation to the linear relaxation, the computational tests show that for instances E15–E28 the models R2, F1 and F2 obtain the same values that are on average 2.77% better than those of model R1. For instances E1–E14 all models present the same values.

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Table 1
Run times in seconds to obtain the optimal solution of instances E1–E28.

Instance	Optimal Sol.	SP2SMM	R1	R2	F1	SF2	PS
E1	*257.7*	1692.6	207.2	128.4	**7.5**	290.0	456.0
E2	*264.3*	2565.0	199.9	98.1	**9.0**	1201.0	2007.0
E3	*275.3*	492.6	117.3	116.5	**8.2**	208.0	266.0
E4	*278.2*	1294.3	150.3	137.8	**14.4**	1392.0	1542.0
E5	*260.9*	4309.9	126.1	74.8	**7.6**	73.0	70.0
E6	*271.0*	4695.7	109.4	43.2	**3.0**	137.0	255.0
E7	*215.3*	3361.4	60.7	68.8	**4.1**	128.0	86.0
E8	*257.7*	146.5	430.3	111.7	**1.0**	57.0	56.0
E9	*264.3*	571.4	137.5	76.3	**7.0**	71.0	181.0
E10	*345.4*	91.5	157.2	68.0	**5.0**	49.0	48.0
E11	*336.0*	741.5	423.4	167.6	**3.0**	106.0	105.0
E12	*272.5*	218.5	250.9	70.6	**4.0**	12.0	5.0
E13	*354.4*	575.5	82.7	58.3	**2.0**	28.0	28.0
E14	*215.3*	72.5	67.9	34.1	**2.0**	32.0	47.0
E15	*1028.1*	7.0	61.0	21.0	**1.0**	6.0	5.0
E16	*347.7*	16.0	88.0	24.0	**3.0**	13.0	9.0
E17	*720.4*	47.0	88.0	21.0	**7.0**	14.0	**7.0**
E18	*770.9*	6.0	151.0	26.0	**3.0**	7.0	11.0
E19	*658.2*	47.0	49.0	19.0	**2.0**	7.0	6.0
E20	*719.7*	12.0	60.0	14.0	**1.0**	10.0	11.0
E21	*910.6*	12.0	53.0	12.0	**1.0**	14.0	13.0
E22	*267.7*	1442.0	439.0	63.0	**1.0**	22.0	17.0
E23	*274.3*	964.0	428.0	50.0	**6.0**	97.0	51.0
E24	*285.3*	96.0	260.0	82.0	**4.0**	12.0	8.0
E25	*288.2*	157.0	518.0	889.0	13.0	21.0	**11.0**
E26	*270.9*	75.0	153.0	53.0	**2.0**	17.0	21.0
E27	*280.9*	171.0	83.0	24.0	**6.0**	36.0	7.0
E28	*225.3*	38.0	89.0	21.0	**2.0**	8.0	8.0
Average		854.2	180.0	91.9	**4.6**	145.3	190.6

Table 2
Total costs (without considering production costs) obtained from real instances.

Inst.	RA-G21	R1	R2	F1	SF2	PS
I1	306,834	966,958	333,276	191,968	*	**188,799**
I2	321,811	1,522,085	378,614	210,997	*	**198,811**
I3	290,841	1,009,121	440,035	**189,872**	*	191,434
I4	317,599	4,298,926	415,388	195,432	*	**177,874**
I5	379,529	4,066,081	517,836	279,525	*	**269,424**
I6	526,473	1,363,424	770,111	**352,464**	*	352,975
I7	509,464	1,833,328	435,889	266,198	*	**251,274**
I8	509,668	2,648,909	425,340	243,319	*	**236,749**
I9	412,237	4,692,963	364,571	**215,631**	*	245,796
I10	429,868	704,501	301,130	**185,736**	*	206,912
I11	289,170	696,055	316,921	**191,831**	*	195,105
I12	491,725	5,383,103	370,827	**200,519**	*	238,320
I13	369,540	667,375	371,917	**210,624**	*	317,830
I14	449,511	1,809,175	596,365	247,827	*	**233,950**
I15	446,194	5,835,839	556,393	297,638	*	**288,639**
Average	403,364	2,499,856	439,641	**231,972**	*	239,593

*: no feasible solution was found within the time-limit.

Table 3
Average of inventory, backorder and changeover costs.

Strategy	Inventory	Backorder	Changeover	Total
Average RA-G21	**12,256**	53,675	337,433	403,364
Average F1	13,102	**22,596**	**196,274**	**231,972**
Average PS	13,389	26,011	200,192	239,593

6.2. *Real-world instances*

Fifteen real-world instances analyzed in Ferreira et al. (2009) were also tackled here. Each instance encompasses 18 flavours which are bottled on two machines, the first producing 23 items and the second producing only 10 out of those 23 items. Machine 1 is available for four days (i.e., 5760 min per week), and machine 2 for six days (i.e., 8640 min per week). The single tank can handle up to 25 changeovers per week. Instance 1 reflects the plant's actual parameters. Instances 2–5 were generated by modifying instance 1's data as follows: in instance 2 the inventory costs were doubled, in instance 3 the backorder cost was doubled, in instance 4 the total demand of each item was randomly redistributed among the three periods (weeks), and in instance 5 the machine capacities were reduced by 25%. Instances I6–I15 have the same costs of instances I1 but different weekly demands. The planning horizon is 3 weeks, each divided into 25 subperiods (a total of 75 subperiods).

Like model P2SMM, models R1, R2, F1 and F2 are also difficult to solve when applied to these large-scale problem instances. Table 2 presents the total costs (without considering production costs) obtained for each instance, as well as the solutions obtained with strategy RA-G21. The smallest cost for each instance is shown in bold. None of the models was solved to optimality within the time limit of four hours (14,400 s) and all the values in the table correspond to the best solutions found within this time frame. Production costs, by far the largest component of total costs, are not considered in these values (as they do not vary by period and all demand should be met), but only inventory, shortage and setup costs.

During the Patching Strategy an average of 845 subtour elimination inequalities were included (maximum 1776, minimum 198). Note in Table 2 that the big-bucket model F1 and strategy PS provide the best solutions for all instances (F1 is the best in seven instances and PS in other eight instances). However, F1 presents the lowest average cost of the 15 instances. The results indicate that, when benchmarking against PS, the performance of SF2, which did not provide any integer feasible solutions, shows that patching helps the strategy, providing integer solutions besides the upper bounds.

The solutions of the models and strategies improved the best solutions reported in Ferreira et al. (2009). In particular, for instance I1, the PS strategy was able to reduce the company solution value of 422,720 monetary units (without considering production costs) by 55%.

Table 3 shows the average of each component cost of the objective function, namely inventory, backorder and changeover costs of the final solutions obtained by RA-G21, F1 and PS. The smallest costs are in bold. Note that F1 and PS reduced significantly the average of backorder and changeover costs compared to RA-G21. Observe that F1 and PS are more flexible than RA-G21. In the RA-G21 heuristic, different lot-scheduling models have to be solved and binary variables are hard-fixed during the procedure, which can turn the last sub-models of the heuristic less flexible, compromising the quality of the final solution.

6.3. *Instances without changeover*

A third set of instances (I1b–I15b) were solved to analyse scenarios in which the sequencing decisions are less important than the lot sizing decisions. Therefore, the changeover costs of instances I1–I15 were reduced to zero, while the inventory and backorder costs were left unchanged. When changeover costs are disregarded, instance I3b is similar to instance I1b, so it was not addressed. Table 4 presents the total costs (without production and changeover costs) and the respective solution execution time (in s) obtained for each instance by R2, F1 and PS. Strategy RA-G21 did not provide feasible solutions for some instances, and R1 yielded solutions worse than those obtained by R2, so their results are not reported in Table 4.

Table 4
Costs and execution times of strategies when changeover costs are not considered.

Inst.	Cost			Time (s)		
	R2	**F1**	**PS**	**R2**	**F1**	**PS**
I1b	10,843	10,820[a]	10,821	14,400	**551**	14,400
I2b	21,786	21,641[a]	21,641[a]	14,400	381	**177**
I4b	14,528	14,352	14,352[a]	14,400	14,400	**850**
I5b	829,444	47,717	**33,240**	14,400	14,400	14,400
I6b	113,508	43,199[a]	43,199	14,400	**13,187**	14,400
I7b	104,394	20,928[a]	20,928	14,400	**908**	14,400
I8b	20,894	10,479	10,479[a]	14,400	14,400	**947**
I9b	9525	9525[a]	9525[a]	14,400	**982**	1361
I10b	9687	8933[a]	8933[a]	14,400	**53**	233
I11b	14,074	9176[a]	9176[a]	14,400	2628	**1103**
I12b	10,883	9143	9143[a]	14,400	14,400	**1072**
I13b	22,755	8539	8539	14,400	14,400	14,400
I14b	20,964	10,311	**10,231**	14,400	14,400	14,400
I15b	82,640	17,204[a]	17,204	14,400	**12,211**	14,400
Average	91,852	15,310	16,244	14,400	8379	7610

[a] Provably optimal solutions.

Table 4 shows that R2 did not obtain any provably optimal solution. For some instances such as I5b–I7b, the differences of the solution quality when compared to those of F1 and PS are significant. The performance of strategies F1 and PS was very close—F1 was faster in seven instances and PS in other five instances, and both reported the same solution for 13 out of 14 instances. Instance I5b deserves a closer look as PS clearly beats F1. Even though there are no changeover costs in the objective function, the setup times play an important role because of the tight capacity feature of the instances. Therefore, sequencing decisions get more important, which has favoured PS over F1, and F1 over R2.

7. Conclusions

This paper proposes single-stage strategies (the R1, R2 and F1 models, and the SF2 and PS strategies to solve model F2) to solve a two-stage lot sizing and scheduling problem with sequence-dependent setup times and costs at each stage and with synchronisation between the two stages. The solutions were compared to those in Ferreira et al. (2009). The R1 and R2 models are based on small-bucket formulations, which divide each period into subperiods, thus increasing the total number of variables and constraints. In contrast, F1, SF2 and PS rely on the big-bucket structure of an ATSP-formulation. Computational tests show that F1 and PS provide the best solutions of all the instances analyzed.

The results also indicate that in the soft drink production context, strategies based on ATSP have a better performance than the ones based on small-bucket formulations.

Future research could explore alternative solution methods to incorporate subtour elimination constraints and patching within the linear programmes at nodes in a single branch-and-cut tree search (Kang et al., 1999), instead of solving the relaxed problem without subtour-elimination constraints to integer optimality and then prohibiting the specific subtours that arise. It would also be interesting to test different methods to solve the sub-MIPS present in the PS strategy.

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