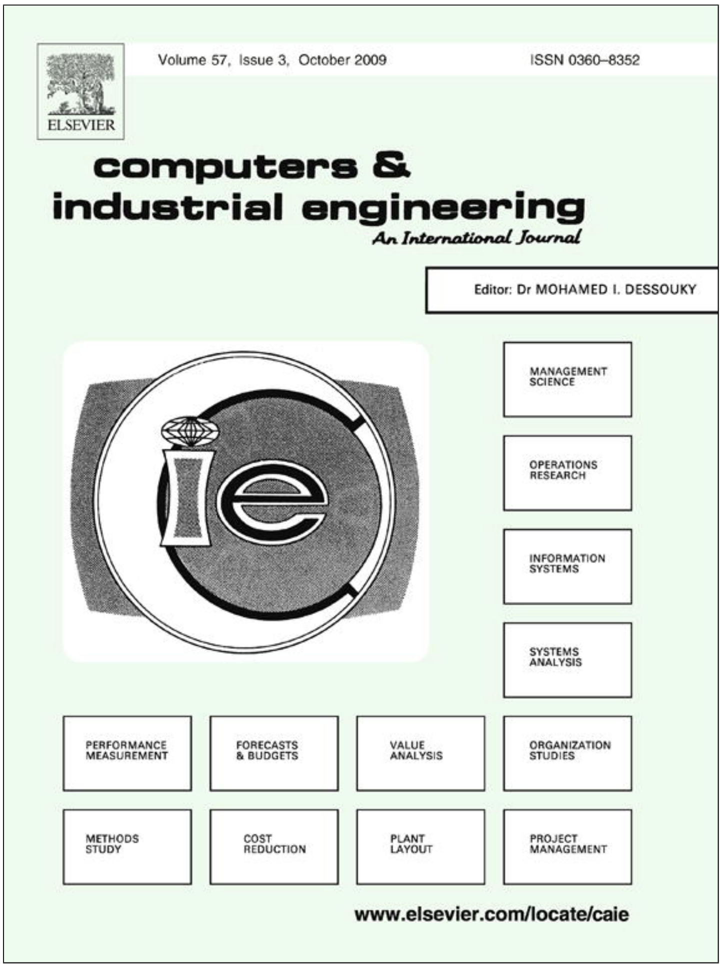
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Lot sizing and sequencing optimisation at an animal-feed plant

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ABSTRACT

This paper studies a challenging case of joint lot sizing and scheduling in a manufacturing plant for animal feed compounds. A key characteristic of this industry is that certain products can perform a production line "cleaning" function if a sufficiently large lot is produced between two products that would otherwise require a cleaning setup. Thus the sequence-dependent setup times do not always obey the triangular inequality. A mixed integer programming model is applied and tested on multiple sets of real data from different seasons. The model takes too long to solve exactly and so alternative formulations and methods are developed to solve the model more quickly, based on two variants of the Relax and Fix heuristic. Test results demonstrate that the formulations are computationally effective and able to take economic advantage of the intermediate cleaning products. The model schedule substantially improves on that practiced at the plant and can be useful for similar companies in the animal-feed industry.

1. Introduction

In a manufacturing system, many products often share valuable capacity which is wasted and not used productively when setting up (changing over) from one product to another. Although automation and process engineering has often reduced the magnitude of setups, a large number of companies still face substantial production setup costs and times within an increasing range of products, with consequent losses of production capacity and missed deadlines if setups are not well managed and controlled. Weak performance in this area generally results in backlogs of unmet demand, customer dissatisfaction and loss of company competitiveness.

While many production lots or batches correspond to specific orders and so have a predetermined size, a product or part may instead feed into many small distinct orders with different deadlines. In such a situation, it makes sense to relate the product or part's lot sizes to its total demand aggregated from the different orders. In other words, the problem becomes one of simultaneous scheduling and sizing of production lots or batches, based on forecasts of product orders and demand, often under limited production capacity (Askin & Standridge, 1993).

This paper investigates such a challenge at Anifeed, a Brazilian animal feed compound company (whose real name has been altered to protect its identity). Two mixed integer programming (MIP) models for joint lot sizing and scheduling with sequence-dependent setup times are applied, taking into account that the setup times, like those in many feed plants, do not always obey the triangular inequality. The first model sequences each period independently of the others. The second model sequences all periods simultaneously, taking into account the linking setup states between periods.

Tests on Anifeed data indicate that in general neither model can be solved optimally within an hour's computing time. In particular, the incumbent solution after an hour is poor for the second model so several alternative formulations and methods are developed to accelerate the solution time, making use of Relax and Fix methods (Wolsey, 1998) on the integer lot sizes or binary setup variables over time.

Computational tests show that the Relax and Fix acceleration is effective while maintaining quality. The solutions show that the second model is able to take advantage of the cleaning function that certain intermediate products can perform if a sufficiently large lot is produced between two products that would otherwise require a cleaning setup. The model's schedules showed a very marked improvement over the schedules implemented at Anifeed. Randomly generated and perturbed data was then used to better evaluate the models and methods through experimental tests.

The rest of this section describes Anifeed's production process and its scheduling context. Section 2 reviews previous research while Section 3 proposes and explains the two optimisation models. Section 4 develops alternative solutions methods which are then tested and analysed in Section 5 and compared to Anifeed's practice in Section 6. Finally Section 7 concludes and points out future directions for research.

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1.1. The Anifeed production process

Anifeed produces about 200 animal feed supplements which can be grouped into approximately 20 product families. Products within the same family do not contaminate each other and have the same production time per batch. All animal feed supplements follow the same basic production route, and make use of the same key resources: silos, dosing machines, pre-mix machines, mixer, and post-mix packaging, as shown in Fig. 1.

The first stage in the production process is to weigh the raw materials based on pre-established formulations. After the operator has specified the feed product and number of batches to be produced, appropriate quantities of the bulkier raw materials are automatically released from the silos into the dosing machines and then held as pre-mix in-process inventory. The bulk materials are transferred to the mixer only after they are all ready in the pre-mixers. Less bulky materials are stored in bags from which they are manually weighed and added directly to the mixer. Mixing occurs in three phases: dry mixing, addition of fluids, final mixing. The mix is then unloaded into the post-mixer and subsequently bagged. The amount of time spent at these operations varies between product families.

A certain number of batches of each product is made before changing to another product. Each batch measures about 2000 l, the capacity of the mixer. The amount produced in the mixer depends on the product density, for example, 2000 kg of basic feeds, 1440 kg of premixes or 2400 kg of mineral salts. Technically, the mixer must be at least half-full to ensure efficient mixing, but economically it makes sense for Anifeed to produce a full batch. However, small batches can be produced in the micro-ingredient mixer, and so small orders are not considered in Anifeed's planning and scheduling of feed supplements.

Further details are available in Toso (2008).

1.2. Production planning and scheduling

Although Anifeed's production process has several stages, logically it can be considered to be single-stage given that the stages are arranged serially, the batch flow is continuous and there is no in-process inventory. The capacity bottleneck is the mixer, so it is at this stage that the whole process can be modelled as a one-machine problem, taking total process times into account.

Product changes are frequent, typically about 30–40 per week. A complicating feature of the animal-feed industry is that some feed families contaminate others if produced in successive batches. As a result the mixer must be often cleaned, consuming potential production time. Products in the same family have negligible changeover times, and identical batch weights and processing times. Thus the amount of mixer cleaning time can be minimised by good sequencing of the production of families.

Furthermore, most of Anifeed's products follow a seasonal pattern of demand, with peaks in certain months. Since employee turnover is high, manpower levels can be adjusted to cope with this seasonality, thus determining basic (pre-overtime) production capacity. The demand in a particular period often exceeds basic capacity, and so overtime is frequently worked to satisfy demand without shortages. When generating production schedules the production manager is particulary concerned to balance overtime and inventory costs, while fulfilling demand without backlogs. The animal feed market is highly competitive, and so delivery delays to clients must be avoided if possible by producing some feed ahead of demand when slack capacity is available.

Thus Anifeed needs to make effective use of production capacity by good lot sizing and sequencing of the production of families. This problem is especially complex for the feed industry due to certain particularities such as highly seasonal demand and sequence-dependent setup times.

The mixer can be completely cleaned during non-productive time between periods (e.g., at weekends), allowing the line to begin and end a period with any product without the initial setup in a period impacting on the total time spent on setups. The setup sequences in consecutive periods are thus delinked and independent of each other. This approach, called *Independent Sequences*, is appropriate in months of slack capacity, and was used not just at Anifeed but also in a second company observed by the authors. It contrasts with a second approach called *Dependent Sequences* which is appropriate in months of tight capacity, when Anifeed's plant is active 24 h a day with no non-productive time between periods. In this case, the cleaning of the mixer at Anifeed is incorporated within the production setup sequencing. Thus the sequences of consecutive periods are linked and dependent, impacting on the total time spent on setups. The Dependent Sequences approach is a more difficult problem as a sequence has

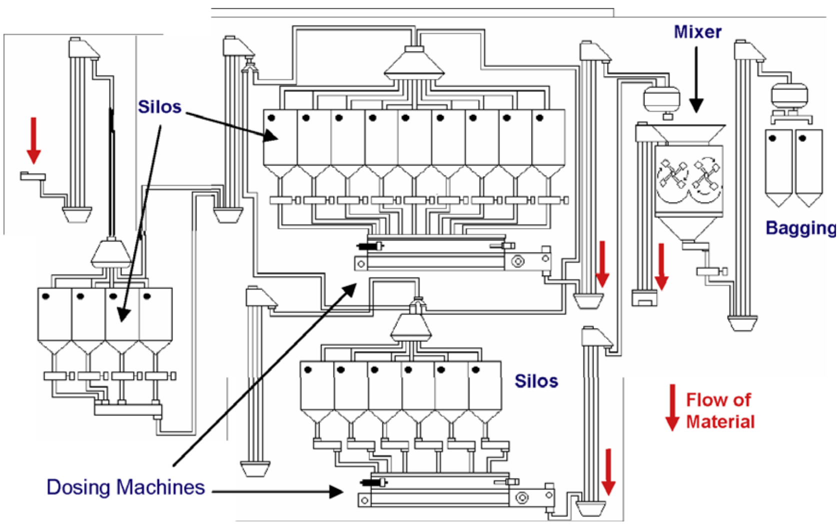

Fig. 1. Production process of Anifeed's food supplements.

to be optimized over multiple periods rather than over a single period as in the case of Independent Sequences.

2. Review of previous research

At a general level, the lot sizing problem consists of determining how much to produce of each family or product in every scheduling period, reconciling demand with capacity either through increases in capacity (such as overtime working) or bringing forward production to slacker periods (Gershwin, 1994; Graves, Rinnoy Kahn, & Zipkin, 1993; Hax & Candea, 1984; Johnson & Montgomery, 1974; Trigeiro, Thomas, & McClain, 1989). The lot sequencing and scheduling problems consists of determining the order in which to produce lots so as minimise setup costs and/or the setup times that consume productive capacity (Allahverdi, Gupta, & Aldowaisan, 1999; Gupta & Magnusson, 2005; Lawler, Lenstra, Rinnoy Kan, & Shmoys, 1993; Potts & Van Wassenhove, 1992).

As noted in the reviews by Drexl and Kimms (1997), Karimi, Fatemi Ghomia, and Wilson (2003), lot sizing and sequencing decisions have often been separately dealt with by many researchers. This chimes with much industrial practice whereby the sequencing and scheduling of lots is often carried out on the shop floor after the lot sizes have been decided.

However, in the animal-feed industry, and in related industries such as soft drinks (Ferreira, Morabito, & Rangel, 2009; Toledo, Kimms, França, & Morabito, submitted for publication; Toledo, França, Morabito, & Kimms, 2008), dealing with lot sizing independently of sequencing makes it difficult to react flexibly to changes in demand and deliver on-time within available capacity. In line with this, the surveys by Drexl and Kimms (1997), Karimi et al. (2003) show that an increasing number of researchers are jointly considering the two problems of lot sizing and lot-sequencing, and developing a variety of different models and solutions methods, including Haase and Kimms (1999), Salomon, Solomon, Van Wassenhove, and Dumas (1997), Staggemeier and Clark (2001).

Among the more influential papers on joint lot sizing and scheduling, several stand out in terms of modelling innovation and development. Drexl and Haase (1995) presented the *Proportional Lot Sizing and Scheduling Problem* (PLSP), which used "small-bucket" time periods during which at most one setup can occur. In constrast, the "large-bucket" representation of the *Capacitated Lot Sizing and Scheduling Problem* (CLSP) Haase (1996) allows many different products per period, but does not sequence the production lots. Fleischmann and Meyr (1997) formulated the so-called *General Lot Sizing and Scheduling Problem* (GLSP) which models sequence-dependent setup costs on a single machine, allowing multiple setups in each large-bucket time period. Meyr (2000) extended the GLSP to include sequence-dependent setup times, but retained the small-bucket concept, dividing the large planning periods into a predetermined number of micro-periods which contain at most one setup.

Clark and Clark (2000), Clark (2003), Laguna (1999) also present MIP formulations for lot sizing and sequencing, emphasizing different algorithmic aspects. Laguna (1999) developed a tabu search method with short-term memory while the latter two papers used MIP-based heuristics. Araújo, Arenales, and Clark (2007), Luche, Morabito, and Pureza (accepted for publication), Toledo et al. (submitted for publication) present formulations for lot problems in foundries, soft-drinks and electrofused grains production, respectively.

The mathematical model initially presented in this paper results from a simple combination and adaptation of those in Hax and Candea (1984) and the *General Lot Sizing and Scheduling Problem – Setup Times* (GLSP-ST) of Meyr (2000) who considers the loss of capacity resulting from sequence-dependent setup times. Differently from the GLSP-ST, it includes non-triangular setups times and also overtime.

3. Modelling approach

This paper now applies two MIP models to decide family lot sizes and sequences in each planning period, the first for Independent Sequences (used in seasons of lesser demand), the second for Dependent Sequences (used in seasons of greater demand). The aim is to minimise overtime and excess inventory, while satisfying demand and keeping within available capacity.

The unit of production is a single batch, whose weight size for a product depends on its density. Anifeed's sales department aggregates sales orders to result in integer family demand quantities in each period.

3.1. Dependent Sequences model

To formulate the model, the following indices are used:

i : *Product family,* $i = 1, \ldots, N$
t : *Time period,* $t = 1, \ldots, T$
s : *Subperiod,* $s = 1, \ldots, S$

where N is the number of families, T is the number of periods in the planning horizon, and S is the total number of sub-periods over the planning horizon.

A period t is split into a fixed number S_t of subperiods s of flexible duration (and can even be of zero length). Just one lot (or none) can be produced in each subperiod, so that S_t is the maximum number of lots that can be produced in period t. Thus, if $S_t = N$, then all families can be produced in period t (but do not need to be).

The subperiods do not overlap and the length of each is a decision variable. Subperiods can be viewed as a device to model family changeovers within a period. The length of a subperiod is the sum of the setup time of the family produced within it and the actual production time of its single lot. If no lot is produced, then this duration can be zero in which case the machine's setup state is conserved. In other words, if family i_{s-1} is produced in subperiod $s-1$ and subperiod s is inactive, then no setup time is needed at the start of subperiod $s+1$ to resume production of family i_{s-1}.

If the same family is produced in consecutive subperiods, then the family lot size is the sum of the production quantities in these subperiods. Thus a lot can be produced over multiple consecutive subperiods and periods. The subperiod decisions determine the number, size and sequence of the family production lots.

The input data required by the model are:

C_t Available capacity time in each period t.
p_i Time needed to produce one batch of each product family i.
lm_i Minimum lot size of family i (integer number of batches).
h_i Cost in monetary units (m.u.) of holding one week's inventory of family i.
co_t Unit cost of overtime for week t.
st_{ji} Setup times needed to changeover from product family j to family i
d_{it} Forecasts of demand for family i at the end of week t in the planning horizon.
I_{i0} Inventory of family i at the start of the planning horizon.
x_{i0} indicates (=1) if production is already set up to produce family i at the start of the first period (otherwise =0).
u_t Upper limit on the number of overtime hours permitted in period t.

The variables, i.e., decisions output by the model, are:

I_{it} Inventory of product family i at the end of period t.
q_{is} Number of batches of family i produced in subperiod s (integer variable).
x_{is} indicates (=1) if production is to be set up for product family i in subperiod s (otherwise =0).
y_{jis} indicates (=1) if production is to be changed over from product family j to family i in subperiod s (otherwise =0).
O_t Number of overtime hours needed in period t.

A MIP formulation of the problem is:

$$\text{Minimise} \quad \sum_i \sum_t h_{it} I_{it} + \sum_t co_t O_t \tag{1}$$

$$\text{such that} \quad I_{it} = I_{i,t-1} + \sum_{s \in S_t} q_{is} - d_{it} \quad \forall i,t \tag{2}$$

$$\sum_i \sum_{s \in S_t} p_i q_{is} + \sum_j \sum_i \sum_{s \in S_t} st_{ji} y_{jis} \leqslant C_t + O_t \quad \forall t \tag{3}$$

$$p_i q_{is} \leqslant (C_t + u_t) x_{is} \quad \forall i,t,s \in S_t \tag{4}$$

$$\sum_i x_{is} = 1 \quad \forall s \tag{5}$$

$$y_{ijs} \geqslant x_{i,s-1} + x_{js} - 1 \quad \forall i,j,s \tag{6}$$

$$q_{is} \geqslant lm_i (x_{is} - x_{i,s-1}) \quad \forall i,s \tag{7}$$

$$0 \leqslant O_t \leqslant u_t \quad \forall t \tag{8}$$

$$I_{it} \geqslant 0 \quad \forall i,t \tag{9}$$

$$q_{is} \geqslant 0 \text{ and integer} \quad \forall i,s \tag{10}$$

$$x_{is} \in \{0,1\} \quad \forall i,s \tag{11}$$

$$0 \leqslant y_{ijs} \leqslant 1 \quad \forall i,j,s \tag{12}$$

where the symbol $\forall$ means 'for all'.

The objective function (1) minimizes the costs of inventory and overtime, two criteria of major importance to the company. The costs of shortages or backlogs are not included in (1) as the company is always able to avoid these through the use of overtime.

Differently to Hax and Candea (1984), Meyr (2000), the objective function (1) does not consider setup costs. The reason is that setups essentially consume only labour and time, neither of which incurs an immediate direct cost, apart from overtime, as basic manpower levels are fixed over the planning horizon. Thus setup times are not directly penalized in the objective function (1), but instead indirectly though their use of overtime. When basic capacity is tight, the minimisation of overtime in the objective function will prevent superfluous setups via constraints (3). However, when capacity is slack, superfluous setups can occur. This situation is avoided by including term (13) below in the objective function:

$$\alpha \sum_j \sum_i \sum_s st_{ji} y_{jis} \tag{13}$$

where α is sufficiently small to make the value of term (13) minute compared to the other terms in the objective function (1). The inclusion of the setup times st_{ji} in (13) ensures that only the cleaning setups are minimised.

Constraints (2) balance inventory, production and demand over consecutive periods. The capacity constraints (3) take into account the setup times as well as actual production times, and the possibility of a limited amount of overtime. Note that if family i is produced in subperiod s, then the duration of this subperiod is the setup time $st_{ji} y_{jis}$ needed to changeover from the previous family j plus the production time $p_i q_{is}$, as illustrated in Fig. 2.

Constraints (4) ensure that production of a family can occur in a subperiod only if the line is set up accordingly. Constraints (5) restricts setups in any subperiod to just one family. Constraints (6) relate setup states to setup changeovers such that, if production changes from family i in subperiod $s-1$ to family j in subperiod s, then the binary changeover variable y_{ijs} must have value 1, but 0 in all other situations, given that term (13) has been added to the objective function (1).

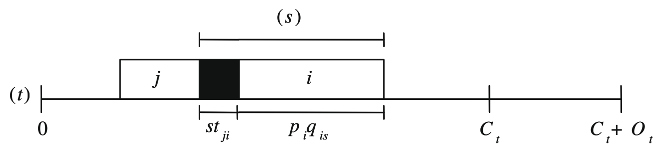

Fig. 2. Setup and production times within capacity.

In constraints (7), the minimum lot size is enforced if family i was not produced in the previous subperiod. A minimum lot size is needed as setup times do not always satisfy the triangular inequality. To better understand why, consider a family i whose production contaminates that of family k unless a thorough cleaning occurs as part of the substantial setup time st_{ik}. However, in the animal-feed industry, such cleaning can sometimes occur during the production of an intermediate family j whose setup time st_{ij} from i and setup time st_{jk} to j are both shortened to the extent that $st_{ik} > st_{ij} + st_{jk}$. Thus the triangular inequality $st_{ik} \leqslant st_{ij} + st_{jk}$ does not hold in this case. Without constraints (7) to impose sufficient production of j to allow proper cleaning of i's contaminants, an optimal schedule could setup from i to k via zero production of j rather than directly.

Constraints (8) impose limits on overtime working, and constraints (9) prohibit backlogs of demand. Constraints (10) require the production to be a whole number of batches while constraints (11) ensure the setup state variables x_{is} to be binary.

The changeover y_{jis} variables will turn out to be binary in an optimal solution without having to impose this in constraints (12), as demonstrated by the following *reductio ad absurdum* argument: if there were an optimal solution with fractional y_{jis} values, then constraints (3) and (6) imply that it is always possible to find a better solution by reducing the value of y_{jis} to be 0 when $0 < y_{jis} < 1$ and to remain at 1 when $y_{jis} = 1$. However, the absence or presence of constraints (12) does make a difference when the model is used within a heuristic procedure to obtain near-optimal solution, as shown in Section 5 below.

Model (1)–(12) has $N(NS + 2S + T) + T + 1$ variables, of which NS are binary, and NS are integer. If the variables y_{jis} were modelled as binary, then there would be $NS(N + 1)$ binary variables, i.e., $N + 1$ times as many.

3.2. Independent Sequences model

To model the independent sequencing of setups between periods, the number S of subperiods s is divided by the number T of periods t. The following variables are then redefined including a third index t:

q_{ist} Number of batches of family i produced in subperiod s of period t (integer variable).
x_{ist} indicates (=1) if production is to be set up for product family i in subperiod s of period t (otherwise =0).
y_{jist} indicates (=1) if production is to be changed over from product family j to family i in subperiod s of period t (otherwise =0).

The first value of s is 0 in each period t, being a "phantom" subperiod representing the clean state at the start of the period. Each period t has S subperiods that are sequenced independently of each other.

A MIP formulation of the problem is:

$$\text{Minimise} \quad \sum_i \sum_t h_{it} I_{it} + \sum_t co_t O_t + \alpha \sum_j \sum_i \sum_s st_{ji} y_{jis} \tag{14}$$

$$\text{such that} \quad I_{it} = I_{i,t-1} + \sum_s q_{ist} - d_{it} \quad \forall i, t \tag{15}$$

$$\sum_i \sum_s p_i q_{ist} + \sum_j \sum_i \sum_s st_{ji} y_{jist} \leqslant C_t + O_t \quad \forall t \tag{16}$$

$$p_i q_{ist} \leqslant (C_t + u_t) x_{ist} \quad \forall i, s > 0, t \tag{17}$$

$$\sum_i x_{ist} = 1 \quad \forall s > 0, t \tag{18}$$

$$y_{ijst} \geqslant x_{i,s-1,t} + x_{jst} - 1 \quad \forall i, j, s > 0, t \tag{19}$$

$$q_{ist} \geqslant lm_i (x_{ist} - x_{i,s-1,t}) \quad \forall i, s > 0, t \tag{20}$$

$$0 \leqslant O_t \leqslant u_t \quad \forall t \tag{21}$$

$$I_{it} \geqslant 0 \quad \forall i, t \tag{22}$$

$$q_{ist} \geqslant 0 \text{ and integer} \quad \forall i, s, t \tag{23}$$

$$x_{ist} \in \{0, 1\} \quad \forall i, s, t \tag{24}$$

$$x_{i0t} = 0 \quad \forall i, t \tag{25}$$

$$0 \leqslant y_{ijst} \leqslant 1 \quad \forall i, j, s, t \tag{26}$$

Note that constraints (25) permit a zero setup time from the clean state at the start of each period.

4. Solution methods

The models of Section 3 were implemented in the AMPL mathematical programming language (Fourer, Gay, & Kernighan, 2003). The MIPs were solved using Cplex 9 (Ilog, 2004). The tests were run on a 1.5 GHz Sun V208 Dual Opteron processor with 4 Gb of RAM.

The test data from Anifeed was modified to maintain confidentiality but also retain proportionality among the product families. It initially comprised two 4-week months of production, corresponding to the Brazilian rainy and dry seasons. Compared to the dry season, demand in the rainy season is spread over a larger number of product families, albeit each with smaller demand. Demand varied substantially between families – a few had zero or very small demand over the four weeks while just 4 or 5 families accounted for the greater part of all demand. Demand also varied over time within each family and, in total, would require overtime if a lot-for-lot production policy were followed.

Additional Anifeed data was also collected over 7 consecutive months, permitting a wider comparison of the models' results with Anifeed's practice.

Family changeover times st_{ji} were either 100 min or zero (rounded down from negligible near-zero times). The family production times p_i and inventory holding costs h_i ranged between 0.2 and 0.6 h per batch and 102 and 922 monetary units per period, respectively (the exact data set is available from the authors). The capacity C_t, overtime limit u_t and overtime costs were constant and equal to 64, 16 h and 859.2 monetary units per hour, respectively. Tables with the complete experimental data can be found in Toso (2008).

The Dependent Sequences model (1)–(12) had 40,661 variables (of which 1764 were integer and 1764 binary) and 40,749 constraints. Initial computational tests showed that there was little improvement in the incumbent solution after several hours of CPU time, with a large duality gap in relation to the Cplex lower bound. As the Dependent Sequences model is the more used by Anifeed, several ways of improving its solution and computing time were investigated as follows.

The following constraints were proposed by Fleischmann and Meyr (1997) as valid inequalities for a GLSP-ST model in order to reduce the search space:

$$\sum_{j,i,j\neq i} y_{ji,s-1} \geqslant \sum_{j,i,j\neq i} y_{jis} \quad \forall t, s = f_t + 2, \ldots, l_t \tag{27}$$

$$q_{is} \leqslant \frac{C_t + u_t}{p_i} \left(2 - \sum_j y_{ji,s-1} - y_{iis} \right) \quad \forall i, t, s = f_t + 1, \ldots, l_t \tag{28}$$

where f_t and l_t are respectively the first and last subperiods of period t. Within a period, constraints (27) ensure that setups which involve a change of family (i.e., $\sum_{j,i,j\neq i} y_{ji,s-1} = 1$) are carried out before those which do not (i.e., $\sum_{j,i,j\neq i} y_{jis} = 0$). Constraints (28) prevent the production of the same family in two consecutive subperiods within a period. However the computational results for the company data show that the incumbent solution after 1 h did not improve.

Close examination of the incumbent solution revealed that some of its y_{jis} changeover variables had value 1 when the corresponding production quantity q_{is} was 0. The formulation permits this and it occurred particularly for the zero changeover times. In an attempt to flush out these misleading phantom setups and speed up convergence to an optimal solution, the y_{jis} variables were penalized in the objective function, but to no effect. We also added the redundant constraints $\sum_{j,i} y_{jis} = 1 \forall s$ (given that $st_{ii} = 0 \ \forall i$), but again to no effect.

We also tested including the constraints (29) to avoid a changeover to a family ($y_{jis} > 0$) when it is not then produced ($x_{is} = 0$), as allowed without setup cost by the objective function. In other words, variable y is free to be positive when it does not need to be and could just as well be zero. The aim of including constraints (29) was to reduce the search space without loss of generality in an attempt to improve the solution quality within a fixed amount of computing time.

$$\sum_j y_{jis} \leqslant x_{is} \quad \forall i, s \tag{29}$$

Again the tests for the company data showed that it was largely ineffective in obtaining a better solution within a hour's CPU time.

In order to accelerate the solution time, three variants of the Relax and Fix heuristic were developed and tested. The Relax and Fix method (Wolsey, 1998) solves a series of partially relaxed MIPs, each with a number of integer variables that is small enough to be quickly solved by Cplex's branch and cut default algorithm. As the series progresses, each set of integer variables is permanently fixed at their solution values, and the relaxed variables are reduced in number, eventually disappearing. The procedure is broadly similar to a depth-first identification of an initial integer solution for a MIP model in a branch and bound search. Its big advantage is its speed.

The three variants of the Relax and Fix (RF) heuristic were:

1. *Method RF on the lot sizes*q_{is}: Relax and Fix on the q_{is} variables: Relax only integrality of the q_{is} variables, solve model (1)–(12), then fix the values of the x_{is} and y_{jis} variables and resolving with the q_{is} variables constrained to be integer. The rationale is that the relaxed solution will be near the optimal solution, but there is a possibility that the procedure might not reach a feasible solution.
2. *Method RF forwards over the periods* t: Maintaining the q_{is} variables as integer, relax the x_{is} and y_{jis} variables in periods 2 onwards, solve (1)–(12), permanently fix the period 1 solution values, restore integrality constraints to the x_{is} and y_{jis} variables in period 2 but relax them for periods 3 onwards, solve (1)–(12) again, permanently fix the period 2 solution values, restore integrality constraints to the x_{is} and y_{jis} variables in period 3 but relax them for periods 4 onwards, solve (1)–(12) again, and so on until period T.
3. *Method RF backwards over the periods* t: As immediately above, but backwards in time over periods $t = T, T-1, \ldots, 1$.

The above variants of the Relax and Fix heuristic are tested in the next section for the Dependent Sequences model.

5. Computational tests and results

The purpose of the tests was to gain insight rather than to carry out conclusive experimentation. In addition, the use of real operational four-week data from Anifeed enabled a comparison with the company's own scheduling. It also provided a basis for further research.

All the tests were limited to 1 h of CPU time, with most solutions being the incumbent of truncated branch and cut searches. (Pochet & Wolsey, 2006).

Table 1 shows the results of the computational tests. The first column identifies the test instance, followed by the solution value for the Independent Sequences GLSP model and its computing time in seconds. The remaining columns shows the solution value for the Dependent Sequences model, firstly solved exactly with just constraints (1)–(12), then with the RF methods. In the case of RF over the periods, the optimality gap is not shown as it changes each time the model is executed with a different group of relaxed variables.

Note from Table 1 that for the Independent Sequences, an optimal solution is obtained in month 5 only (and in just 28 s). Over all months, the mean gap between the solution and lower bound is only 2.43%. The overall mean solution time is 21 min (1275 s), well within the 1-h limit preset for the branch and cut search.

Preliminary tests showed that the first step in applying RF on lot sizes (solving the model with relaxed lot sizes) generally required substantial computational effort, while the second step (forcing integer lot sizes) was solved in a few seconds. As a result, the 1-h time limit was applied only to the first step. Note from Table 1 that method RF on lot sizes obtained either better solutions than truncated branch and cut (in months *Rainy* and 7) or the same, but always in less computing time, in fact in about a fifth of the time on average.

Initial tests also showed that applying RF Forward over the periods acted too much like a greedy heuristic, without aggregating lots or forming sufficient inter-period stocks, resulting at times in infeasible solutions for instances known to be feasible. This method was thus abandoned in favour of applying RF Backwards over the periods, which did not exhibit so greedy a behaviour.

Observe in Table 1 that Method RF (Backwards) over the periods performed better than truncated branch and cut in two months (*Rainy* and 7), worse in month 3 and was about 30% faster on average. All three methods in Table 1 met the demand without backlogs in all instances.

5.1. Comparing the Independent Sequences and Dependent Sequences models

The Independent Sequences model has a lower (or equal) optimal value than the Dependent Sequences model, since the setup for the first lot in a period is a cleansing one and uses zero production capacity, as explained in Section 3.2. Thus more time is available for production than in the Dependent Sequences model (and so less overtime is required, if at all). In practice, Anifeed only carries out inter-period setups in months of slack demand. However, an increasing concern with hygiene means that the company is considering an obligatory weekly cleaning, i.e., either interrupting production during the working week (and thus losing capacity) or using inter-period setups (with consequent extra costs).

To assess the costs, the following three model scenarios were compared using the Anifeed data:

1. The Independent Sequences model with no reduction in capacity.
2. The Dependent Sequences model where cleaning is carried out between periods, i.e., at the weekend.
3. The Independent Sequences model with cleaning carried out within productive time, i.e., with a capacity reduction equivalent to one setup in each period, i.e., losing 1.67 h per period.

Fig. 3 shows the main differences between the three scenarios in terms of mean capacity use per period, total overtime over all periods, and total number of setups over all periods. The means were calculated over all the real instances.

Observe from Fig. 3 that using productive time for weekly cleaning in the Independent Sequences model (scenario 3) significantly impacts on overtime (82 h) and consequent capacity usage (117% of available non-overtime capacity) while these two measures vary relatively little between scenarios 1 (48 h and 99%) and 2 (39 h and 100%).

Note also that scenario 3's 2 setups within productive times do not include the 4 setups implicitly carried out within the reduced capacity at the start of each of the 4 weeks, ie, there are really 6 setups. Similarly there are really 7 setups for the Independent Sequences model with no reduction in capacity (scenario 1). These compare with the 15 setups for the Dependent model (scenario 2).

These comparisons are not straightforward as they do not include, for example, the likely additional costs of weekly cleaning. However they do show, using Anifeed's actual data, that weekly cleaning within productive time does greatly increase direct costs such as overtime, and almost certainly some indirect opportunity costs resulting from unproductively using part of the installed capacity.

Table 1
Results from the *Independent Sequences* and *Dependent Sequences* models.

Month	Independent Sequences			Dependent Sequences							
	Branch and cut			Branch and cut			Relax and Fix on lot sizes			Relax and Fix on time	
	Value	Time	Gap (%)	Value	Time	Gap (%)	Value	Time	Gap (%)*	Value	Time
Rainy	3028	2860	3.28	3519	360	16.58	3453	273	13.9	3445	415
Dry	15425	779	0.18	16616	157	7.32	16616	132	7.29	16616	511
1	4029	706	1.08	4312	339	7.58	4312	132	6.69	4312	115
2	7655	2531	4.69	8176	438	10.76	8176	258	10.5	8176	65
3	3207	968	0.99	4328	1049	10.85	4328	240	7.69	4589	1243
4	3378	234	5.31	3378	2854	5.47	3378	183	0.1	3378	2150
5	13056	28	0	13510	775	3.36	13510	117	3.12	13510	341
6	9611	790	0.23	10019	345	4.3	10019	128	4.23	10019	227
7	9088	2579	6.09	11597	2998	15.87	10726	484	13.03	10851	1506
Mean	7609	1275	2.43	8384	1035	9.12	8280	216	7.39	8322	730

* At 1st MIP (2nd MIP is always solved optimally).

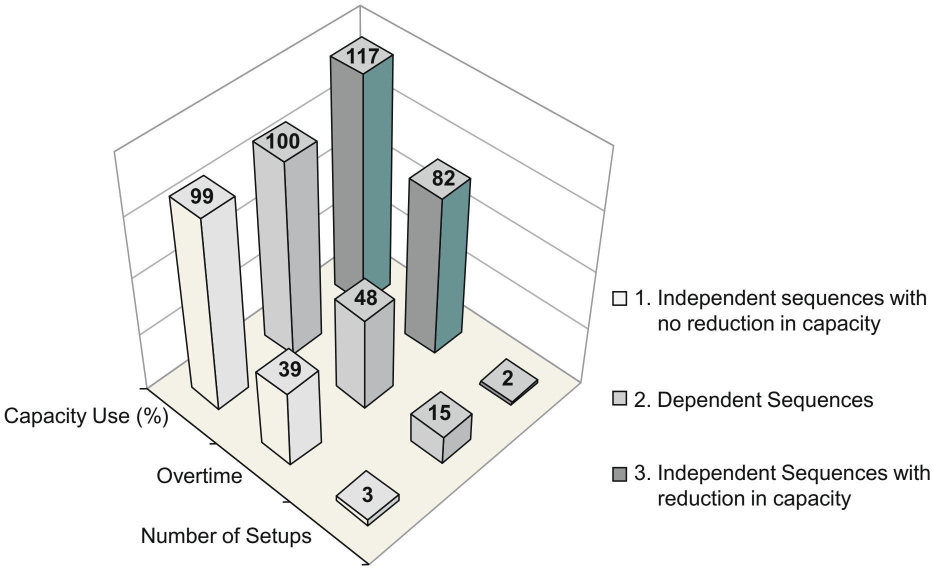

Fig. 3. Comparing the Independent Sequences and Dependent Sequences models.

5.2. Detailed production schedule

Table 2 illustrates the production schedule for the *Rainy* month generated by RF over the periods. For each of the 4 periods, it shows the lot sizes and sequences, total production time, total setup time, capacity used, and overtime required.

Note from Table 2 that period 1's production not only meets demand but also has some left over for stock. Only one setup of 1.67 h was required. This plus the 59.8 h of production time constitutes 96.0% of the 64 h of available capacity. However, period 2's demand is met by calling on the stocks produced in period 1 as well as production in the period itself, using 99.95% of available capacity. Again, some of period 2's production creates stocks for future consumption with only one setup needed. Although no setup is needed in period 3, production consumes 100% of available capacity. Period 4's demand is met by having no setups that would eat into capacity and by calling upon stocks produced in previous periods. No overtime was required over the four periods, so the only costs are due to inventory.

Table 2 shows that some products perform a cleaning function, thus avoiding extra setups. For example, in period 2, the sequence fam14 → fam17 → fam8 does not exhibit triangular setup times as fam14 → fam8 would require a setup whereas neither fam14 → fam17 nor fam17 → fam8 do.

6. Comparison with Anifeed's own planning

Anifeed follows a *chase strategy* (Nahmias, 1995) of producing only the demand forecast for the forthcoming week, making substantial use of overtime. Sequencing of production is not planned, but rather carried out on the shop floor. For example, production records for the same rainy season month in hand show that there were 4 non-trivial setups in period 1, 3 setups in period 2, 4 in

Table 2
Production schedule for the *Rainy* month generated by RF over the periods.

Sequence	Period $t = 1$		Period $t = 2$		Period $t = 3$		Period $t = 4$	
	Product	Lot size	Product	Lot size	Product	Lot size	Product	Lot size
1	fam17	10	fam20	5	fam21	65	fam2	1
2	fam5	25	Setup		fam3	9	fam17	3
3	fam3	9	fam10	57	fam11	6	fam21	14
4	fam11	2	fam3	16	fam14	20	fam14	19
5	fam19	2	fam14	15	fam4	1	fam3	25
6	Setup		fam17	3	fam5	2	fam13	1
7	fam17	10	fam8	29	fam13	1	fam8	52
8	fam9	40	fam7	16	fam8	32	fam9	50
9	fam15	2	fam12	1	fam12	1	fam7	11
10	fam16	1	fam9	34	fam17	3	fam10	79
11	fam8	29	fam5	15	fam9	32	fam5	5
12	fam12	2	fam13	1	fam10	65	fam11	5
13	fam10	58	fam11	6	fam7	12	fam19	4
14	fam2	2	fam20	5	fam2	9		
15	fam21	38	fam21	49				
16	fam14	12						
17	fam20	4						
Production time	59.8		62.3		64		64	
Setup time	1.67		1.67		0		0	
Capacity used	61.47		63.97		64		64	
Overtime	0		0		0		0	

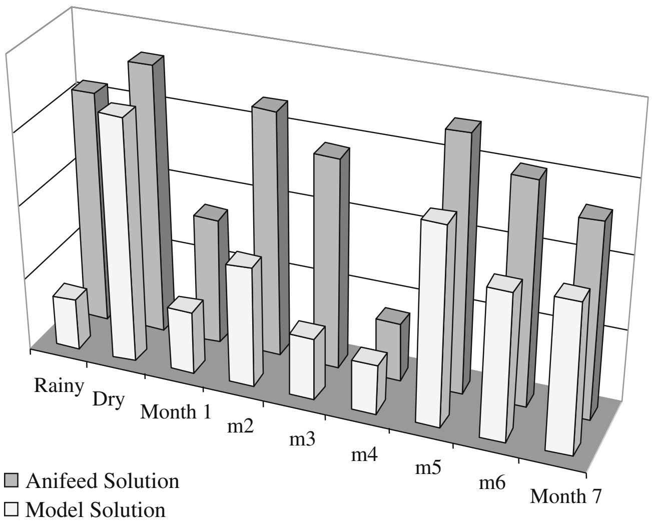

Fig. 4. Model solutions compared to Anifeed solutions.

period 3, and 2 in period 4. This required total production and setup times of 64.18, 61.41, 67.98 and 78.25 h, respectively, over the 4 periods, giving a value of 15,809 for the objective function (1) compared to the Dependent Sequence model's best value of 3445 m.u. in Table 1, a striking difference.

During the dry season, characterized by a smaller variety of products, but each with much larger demand, the Dependent Sequence model solutions are more uniform and generally faster. The planning is also simpler for Anifeed, and the advantage of the model was much smaller than for the rainy season.

Tests over the same 7 months as Table 1 showed that the Dependent Sequence model reduced costs considerably compared to the best possible solution that Anifeed could achieve using its chase strategy. Illustrating these results, Fig. 4 shows how the objective function value was substantially improved, due to the release of capacity as a result of better sequencing and thus the anticipation of production in slacker periods.

Even without improved sequencing, the gap between the Anifeed and model results could have been less if Anifeed had not followed its chase strategy and instead had brought production forward from periods 3 and 4 to take advantage of the underuse of capacity in period 2. However, the company is wary of over-producing to demand forecasts that will invariably change – indeed this is the main reason that the differences between its practice and the model results are so large. This suggests that the model could be profitably used by Anifeed to explore and quantify the costs of its caution in relation to demand forecasts, for example, via scenario analysis. It also indicates that Anifeed could adopt a rolling horizon strategy of implementing only the immediate period's production schedule and then to reschedule the following period's production with updated forecasts of demand, as in Clark and Clark (2000).

7. Conclusions and future research

This paper applied two mixed integer programming models for joint lot sizing and scheduling, motivated by an animal-feed plant where the sequence-dependent setup times were non-triangular. One model treats each period as having an independent sequence of lots, suitable for when the production line is cleaned between periods. A second model addresses the case when there is no time for cleaning between periods so that the periods' sequences are linked and thus dependent on each other.

Tested on data from the Anifeed plant, the models take too long to solve exactly. Several alternative formulations and methods were explored to solve the Dependent Sequences model more quickly so as to make it viable for operational use. The method that seems most promising is Relax and Fix on the integer lot sizes q_{is}.

The results confirm that the models are able to take advantage of the "cleaning" function that certain intermediate product families can perform if a sufficiently large lot is produced between two families that would otherwise require a non-trivial setup. The model solution is a clear improvement on that practiced at the feed plant, but further testing of the RF on q_{is} and other methods to properly compare them with Anifeed's practice, particularly on the basis of rolling horizon usage.

A continuing challenge is to develop both exact and approximate solution approaches that are much faster and yet obtain near-optimal solutions. To this end, the authors are considering the following research directions: (i) further development of relax-and-fix based heuristics for lot sizing and sequencing (Clark, 2003), including their application to overlapping periods (Almada-Lobo, Klabjan, Oliveira, & Carravilla, 2007); (ii) lot sequencing methods based on the Asymmetric Travelling Salesman problem (ATSP) methods that very efficiently solve a series of Assignment Problems with sub-tour elimination constraints (Lawler, Lenstra, Rinnoy Kan, & Shmoys, 1985); and (iii) alternative optimisation approaches based on modern metaheuristics such as memetic algorithms (Hart, Krasnogor, & Smith, 2005; Krasnogor & Smith, 2005; Toledo et al., 2008).

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