Some Important Derivations

Chapter-2

2. Propose an alpha compositing formula for blending the colors of three objects C1, C2 and C3. Where C1 is the foreground of C2 and C2 is the foreground of C3 [6]
2. Solution: ***Added from "radia-all-mathcg.pdf"***

<u>Ans:</u> Let,

α = Alpha compositing parameter to blend C_1 and C_2

β = " " " " " C_2 " C_3

We know,

$$C = \alpha C_f + (1-\alpha) C_b$$

Here,
C_f - Foreground color
C_b - background color
C - mixed/final color

$$\therefore C = \alpha C_1 + (1-\alpha) C_2$$

$$= \alpha C_1 + (1-\alpha) \boxed{\{\beta C_2 + (1-\beta) C_3\}}$$

$$= \alpha C_1 + \beta C_2 - \alpha\beta C_2 + (1-\beta) C_3 - \alpha(1-\beta) C_3$$

$$= \alpha C_1 + (1-\alpha)\beta C_2 + (1-\alpha)(1-\beta) C_3$$

$\therefore$ This is the required alpha compositing formula.

Chapter-3

2. Derive the equation of a Bezier curve of degree 4 using de Casteljau's Algorithm. [6]
2. Solution:

de Casteljau's Algorithm:

$P_{i,j} = (1-u)\, P_{i,j-1} + u\, P_{i+1,j-1}$

$P_{0,4} = (1-u)\, P_{0,3} + u\, P_{1,3}$

$= (1-u)\, ((1-u)\, P_{0,2} + u\, P_{1,2})) + u\, ((1-u)\, P_{1,2} + u\, P_{2,2}))$

$= (1-u)\, ((1-u)\, ((1-u)\, P_{0,1} + u\, P_{2,1}) + u\, ((1-u)\, P_{1,1} + u\, P_{2,1}))) + u\, ((1-u)\, ((1-u)\, P_{1,1}$
$u\, P_{2,1}) + u\, ((1-u)\, P_{2,1} + u\, P_{3,1})\,))$

$= (1-u)\, ((1-u)\, ((1-u)\, ((1-u)\, P_{0,0} + u\, P_{1,0}))+ u\, ((1-u)\, P_{2,0} + u\, P_{3,0}))) + u\, ((1-u)\, ((1$
$u)\, P_{1,0} + u\, P_{2,0})) + u\, ((1-u)\, P_{2,0} + u\, P_{3,0}))))) + u\, ((1-u)\, ((1-u)\, ((1-u)\, P_{1,0} + u\, P_{2,0}))$
$u\, ((1-u)\, P_{2,0} + u\, P_{3,0}))) + u\, ((1-u)\, ((1-u)\, P_{2,0} + u\, P_{3,0})) + u\, ((1-u)\, P_{3,0} + u\, P_{4,0})))))$

Chapter-4

1. Show that two successive reflections about either of the principle axis is equivalent to a single rotation about the coordinate origin. [8]

1. a. Solution:

1. Transformation matrix for two successive reflections about either of the principle axis,

   M1 = Ref-Y * Ref-X

   Single rotation matrix

   M2 = R(180)

   You need to show that, M1 == M2

1. Show that, transformation for a reflection about the line y = x, is equivalent to a reflection relative to the x-axis followed by counterclockwise rotations of 90 degree. [8]

1. a. Solution:

1. Transformation matrix for reflection about the line y = x,

   $M1 = R^{-1}(45) * Ref * R(45)$

   Transformation matrix for Reflection relative to the x-axis followed by counterclockwise rotations of 90 degrees,

   M2 = R(90) * Ref-X

   You need to show that, M1 == M2

Solution: 053

M1 = $R^{-1}(45)$ * Ref * R(45)

$$= \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

M2 = R(90) * Ref-X

$$= \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

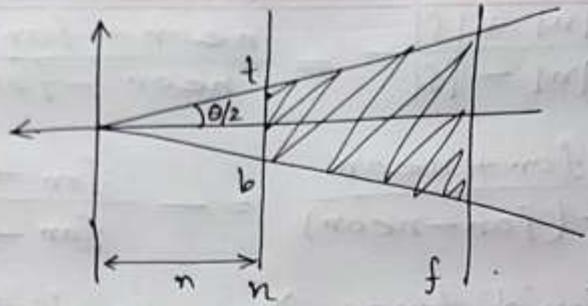
$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

So m1 == m2

Chapter-5

- Show that, the M_{OpenGL} can be written as follows –

$$
\mathbf{M}_{\text{OpenGL}} = \begin{bmatrix} \frac{2|n|}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2|n|}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{|n|+|f|}{|n|-|f|} & \frac{2|f||n|}{|n|-|f|} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{aspect * \tan(\frac{fov}{2})} & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{fov}{2})} & 0 \\ 0 & 0 & -\frac{far + near}{far - near} \\ 0 & 0 & -1 \end{bmatrix}
$$



n: near
f: far
t: top
b: bottom

$$M(1,1) = \frac{2|n|}{r-l} = \frac{1}{\frac{r-l}{2|n|}} = \frac{1}{\frac{r-l}{t-b} \times \frac{t-b}{2|n|}}$$

$$= \frac{1}{\text{aspect} * \frac{\cancel{2}t}{\cancel{2}|n|}} \quad // \; b = -t$$

$$= \frac{1}{\underset{(\text{ratio})}{\text{aspect}} * \tan\left(\frac{fov}{2}\right)} \; ;$$

$l = -r$
$b = -t$
Field-of-view, $fov = \theta$
$\tan(\theta/2) = \frac{t}{|n|} = \tan\left(\frac{fov}{2}\right)$

$$M(1,3) = \frac{r+l}{r-l} = \frac{r-r}{r-l} = 0$$

$$M(2,2) = \frac{2|n|}{t-b} = \frac{1}{\frac{t-b}{2|n|}} = \frac{1}{\frac{2t}{2|n|}} = \frac{1}{\tan\left(\frac{fov}{2}\right)}$$

$$M(2,3) = \frac{t+b}{t-b} = \frac{t-t}{t-b} = 0$$

Correction: M(2,2) er sheshe 1/tan(fov/2) hobe. corrected

$$M(3,3) = \frac{|n| + |f|}{|n| - |f|} = \frac{near + far}{near - far}$$

$$= \frac{far + near}{-(far - near)} = -\frac{far + near}{far - near}$$

// far distance > near distance
so putting far first.

$$M(3,4) = \frac{2|f||n|}{|n| - |f|} = \frac{2 * far * near}{-(far - near)}$$

$$= -\frac{2 * far * near}{far - near}$$