Lecture 5

Quiz-3

Set-A

1) Consider a viewport of size **256 x 128** in which pixel coordinates count down from the top of the image, rather than up from the bottom.

   a. **[10 Marks]** Construct the viewport transformation matrix

   b. **[10 Marks]** Transform a 3D line AB from an orthographic view volume to the mentioned viewport. Consider the vertices of the line are A(-3, -4, -3), B(2, 4, -6) and the orthographic view volume has the following setup:

$$l = -6,\ r = 6,\ b = -7,\ t = 7,\ n = -2,\ f = -8$$

1. a. Solution:

"Slide e translate 1,1 & -½ -½ diye korse… :/ *Reply: By default coordinate goes from bottom to top. But here the scenario is different. For a detailed answer you can check Enigma 3(b)*

Solution:

a) Steps to construct the mentioned viewport matrix:

1. Translate by (1, -1)
2. S(nx/2, ny/2)
3. Translate by (-0.5, 0.5)

$$M_{vp} = T(-0.5, 0.5) * S(nx/2, ny/2) * T(1, -1)$$

b) $M = M_{vp} * M_{orth}$

Mvp':

128.0	0	0	127.5
0	64.0	0	63.5
0	0	1	0
0	0	0	1

Morth':

0.17	0	0	0.0
0	0.14	0	0.0
0	0	0.33	1.67
0	0	0	1

A': [63.5, 164.07, 0.67]
B': [170.17, 90.93, 0.33]

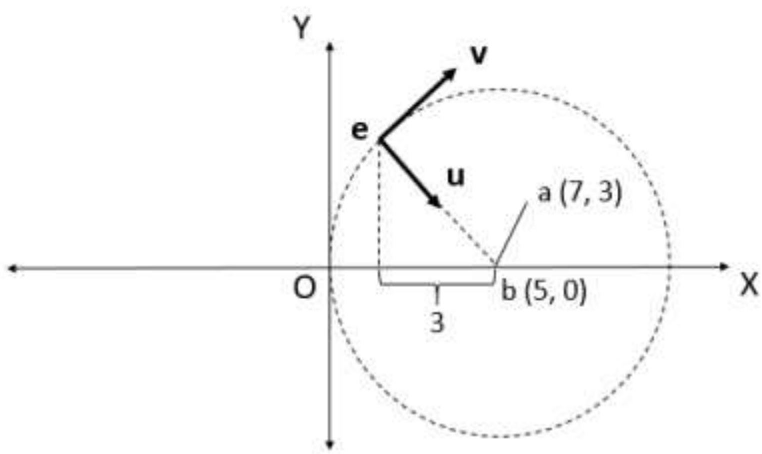
==================

Answer ta ki miltese??

Keu eta solve korle ekhane de

Set-B

1) Here (in the figure), origin O and basis {x,y} construct a 2D canonical coordinate system. Within this, line ab is our model (Pxy). Now, we want to view it from a new 2D camera with eye e and basis {u,v}; which is rotated by θ degrees. Assume that, u is the viewing direction and b is the center of the circle.
    2) **[10 marks]** Determine the basis and eye matrix
    3) **[10 marks]** Calculate Puv.



1. a. Solution:

Solution:

a) Find the camera position (xe, ye) = (2, 4)

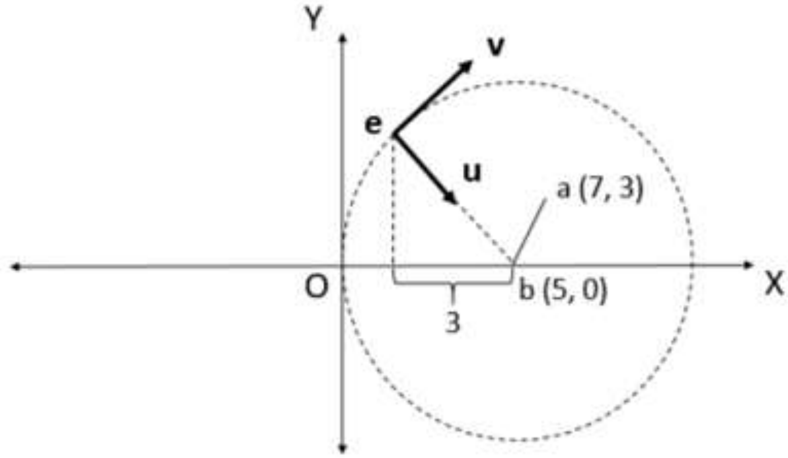
b) Rotate the basis by θ = -53.13 degree to find the new basis

c) Apply the canonical to frame matrix:

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

A': [5.0 0.0 1.0]
B': [3.8 3.4 1.0]

Solution: 024



Here we can see, ob = 5, mb = 3. So, om = 2

Radius ob = eb = 5
For emb triangle, eb^2 = em^2 + mb^2
=> em = 4

So, e position (2, 4)

Again using emb triangle we get, tan(theta) = em/bm = 4/3
=> theta = 53.13

Basis vectors are u = $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, v = $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

So, after rotation basis matrix = Rotate_X(-53.13) x basis_matrix =

$$\begin{pmatrix} \cos\left(-53.13\times\frac{\pi}{180}\right) & -\sin\left(-53.13\times\frac{\pi}{180}\right) & 0 \\ \sin\left(-53.13\times\frac{\pi}{180}\right) & \cos\left(-53.13\times\frac{\pi}{180}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.600001 & 0.799999 \\ -0.799999 & 0.600001 \\ 1 & 1 \end{pmatrix}$$

So, Mcam = basis_matrix x eye_matrix =

$$\begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} 0.600001 & -0.799999 & 0 \\ 0.799999 & 0.600001 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.600001 & -0.799999 & 1.99999 \\ 0.799999 & 0.600001 & -4. \\ 0 & 0 & 1 \end{pmatrix}$$

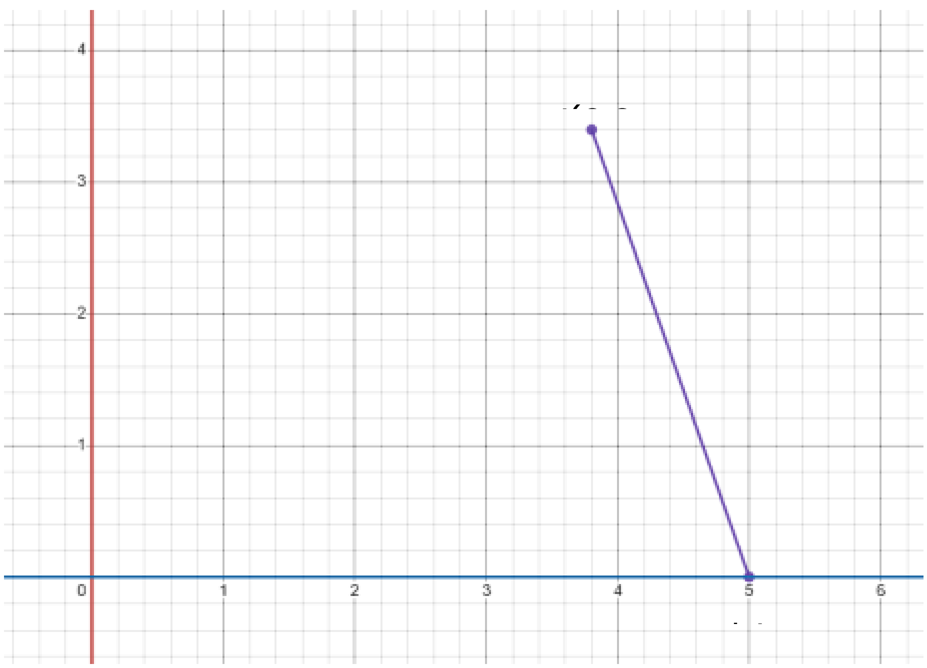
Now, Puv = Mcam x Pxy =

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} 0.600001 & -0.799999 & 1.99999 \\ 0.799999 & 0.600001 & -4 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 7 & 5 \\ 3 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3.8 & 5 \\ 3.4 & 0 \\ 1 & 1 \end{pmatrix}$$

We get, a’ = (3.8, 3.4) and b’(5, 0)



Answer mile nai. Konta thik?

Sir A point er value B point e likhe felse naki ? :3

Set-C

1) Suppose we have a viewport of size **256 x 512** which is obtained by rotating the viewport 45 degree and keeping the (0,0) pixel to its original position.

   a) **[10 Marks]** Construct the viewport transformation matrix

   b) **[10 Marks]** Transform a 3D line AB from an orthographic view volume to the mentioned viewport. Consider the vertices of the line are A(10, -5, 3), B(12, 6, 2) and the orthographic view volume has the following setup:

$$l = -5, r = 5, b = -8, t = 8, n = -4, f = -8$$

1. a. Solution:

Solution:

a) You need to rotate the normal viewport matrix by 45 degree to find the new viewport matrix

$$M_{vp\_new} = R(45) * M_{vp}$$

b) $A' = M_{vp\_new} * M_{orth} * A$

$B' = M_{vp\_new} * M_{orth} * B$

```
A': [204.48, 340.09, 4.5]
B': [-9.09, 626.36, 4.0]
```

// Rabab 039 (values slightly differ), **thik ase, same value ashe**

$$M_{vp} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{256}{2} & 0 & 0 & \frac{256-1}{2} \\ 0 & \frac{512}{2} & 0 & \frac{512-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 90.5 & -181 & 0 & -90.5 \\ 90.5 & 181 & 0 & 270.82 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\therefore A'B' = M_{vp} \times M_{orth} \times AB$$

$$= M_{vp} \times \begin{bmatrix} 2/10 & 0 & 0 & 0 \\ 0 & 2/16 & 0 & 0 \\ 0 & 0 & 2/4 & 12/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 10 & 12 \\ -5 & 6 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 18.101 & -22.62 & 0 & -90.5 \\ 18.101 & 22.62 & 0 & 270.8 \\ 0 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 10 & 12 \\ -5 & 6 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

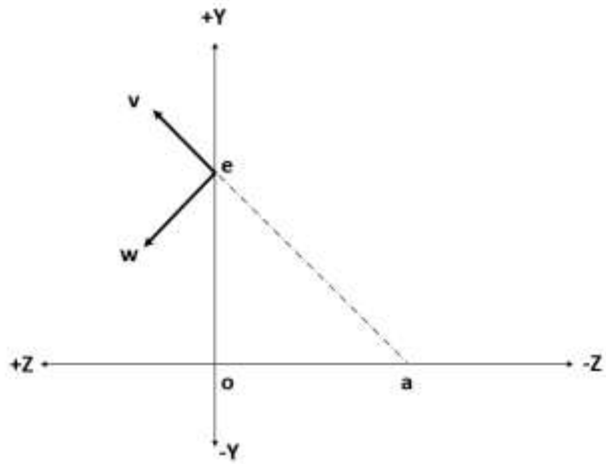
$$= \begin{bmatrix} 203.61 & -9.008 \\ 338.73 & 623.75 \\ 4.5 & 4 \\ 1 & 1 \end{bmatrix}$$

Set-D

1. Here (in the figure), origin **O** and basis vectors **{z, y}** construct a 2D canonical coordinate system where –z is the viewing direction and y is the up vector. Consider a camera coordinate with origin **e** and basis **{w, v}**. Here **e** is located on the y-axis and edge **oe** and **oa** of the triangle Δoea have a length of 3 and $\sqrt{3}$ unit respectively. The goal is to point the camera viewing direction at point **a** and capture it

   a) **[10 Marks]** Determine the basis and eye matrix

   b) **[10 Marks]** Determine the position of point **a** w.r.t the camera coordinate.



1. a. Solution:

Solution:

a) Find the camera position (xe, ye) = (0, 3)

b) Rotate the basis by 60 degree to find the new basis

c) Apply the canonical to frame matrix on point A($-\sqrt{3}$, 0)

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

A_{uv} : [-3.46 -0.0 1.0]

// Rabab 039 #hoini :p

Quiz 3, set-D

$a = (-\sqrt{3}, 0)$, $e = (0, 3)$ [$-\sqrt{3}$ because it is on $-z$ plane]

Basis vectors $\{w, v\}$: $w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Now, $\tan\theta = \frac{oe}{oa} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore \theta = \tan^{-1}\sqrt{3} = 60°$

$\therefore$ Basis matrix, $M_{Basis} = \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} .5 & -.86 \\ .86 & .5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} .5 & -.86 & 0 \\ .86 & .5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore P_{uv} = M_{Basis}^{-1} \times M_{eye}^{-1} \times P_{xy}$

$$\Rightarrow \begin{bmatrix} .5 & .86 & 0 \\ -.86 & .5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -\sqrt{3} \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3.46 \\ 0 \\ 1 \end{bmatrix}$$

Origin42

2. Lecture -05

b) Explain the problems associated with it if homogeneous coordinates were not used in matrix transformation. [2]

2. b. Solution: 024

In general, Translation process is a summation process. But other transformation processes are matrix multiplication. So, if homogeneous coordinates were not used, the translation process could not be combined with other transformations.

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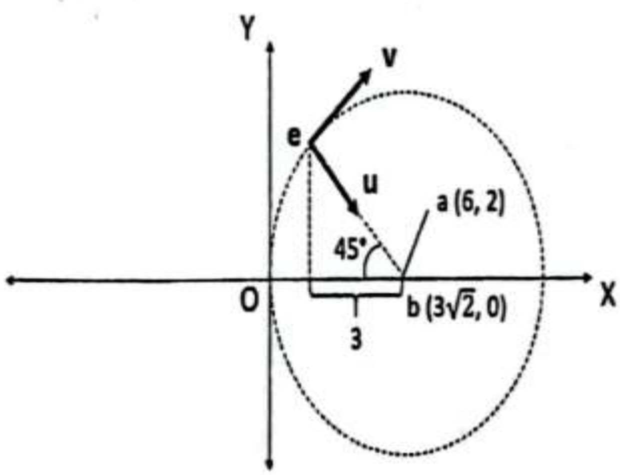
Homogeneous coordinates are especially important in 3D graphics, where transformations involve both rotations and translations in three dimensions. Without homogeneous coordinates, the representation and manipulation of 3D transformations would be significantly more complex and error-prone.

perspective projection, are essential for creating realistic 3D scenes. Homogeneous coordinates are crucial for representing and efficiently applying projective transformations. Without them, handling perspective projection would be extremely complex and inefficient

2. Lecture - 05

c) Here (in the figure), origin O and basis {x,y} construct a 2D canonical coordinate system. Within this, line ab is our model (Pxy). Now, we want to view it from a new 2D camera with eye e and basis {u,v}; which is rotated by −45 degrees around b. Determine the position of a and b w.r.t camera coordinate. [8]

Assume that, u is the viewing direction and b is the center of the circle.



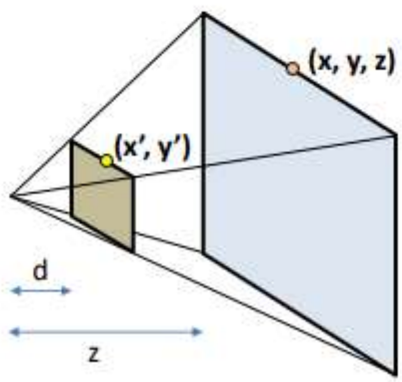
2. c. Solution:

4.

(b) Derive 2D perspective projection matrix. [4]

4. b. Solution: 45

Perspective Projection (8/17)



For 2D:

$$y' = dy/z$$
$$x' = dx/z$$

$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} dx \\ dy \\ z \end{bmatrix} \sim \begin{bmatrix} dx/z \\ dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

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7. **Lecture -05***

(a) Transform a 3D line AB from an orthographic view volume to the viewport of size 256 x 128. Consider the vertices of the line are A(-2, -4, -1), B(1, 5, -5) and the orthographic view volume has the following setup: [7]

l = - 6, r = 6, b = - 7, t = 7, n = - 2, f = - 8

7. a. Solution: 024

Given,

nx = 256, ny = 128

l = −6, r = 6, t = 7, b = −7, n = −2, f = −8

For Orthographic -> Canonical -> Viewport:

M = Mvp x Morth =

$$\begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{x} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \frac{256}{2} & 0 & 0 & \frac{256-1}{2} \\ 0 & \frac{128}{2} & 0 & \frac{128-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \frac{2}{6--6} & 0 & 0 & -\frac{6-6}{6--6} \\ 0 & \frac{2}{7--7} & 0 & -\frac{7-7}{7--7} \\ 0 & 0 & \frac{2}{-2--8} & -\frac{-2-8}{-2--8} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{64}{3} & 0 & 0 & \frac{255}{2} \\ 0 & \frac{64}{7} & 0 & \frac{127}{2} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Also given, A(-2, -4, -1) and B(1, 5, -5).
We know,

$$\begin{bmatrix} x_{\text{pixel}} \\ y_{\text{pixel}} \\ z_{\text{canonical}} \\ 1 \end{bmatrix} = (\mathbf{M}_{\text{vp}} \mathbf{M}_{\text{orth}}) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

*// here signs between matrices are **x** (not dot) and multiplications are **cross product.***

A' = MA

$$\begin{pmatrix} \frac{64}{3} & 0 & 0 & \frac{255}{2} \\ 0 & \frac{64}{7} & 0 & \frac{127}{2} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} -2 \\ -4 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{509}{6} \\ \frac{377}{14} \\ \frac{4}{3} \\ 1 \end{pmatrix}$$

B′ = MB

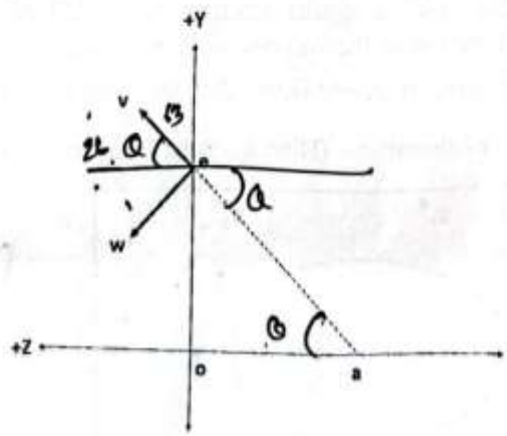
$$\begin{pmatrix} \frac{64}{3} & 0 & 0 & \frac{255}{2} \\ 0 & \frac{64}{7} & 0 & \frac{127}{2} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ -5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{893}{6} \\ \frac{1529}{14} \\ 0 \\ 1 \end{pmatrix}$$

Enigma41

3. Lecture -05

a) Origin **O** and basis vectors {z, y} construct a 2D canonical coordinate system where $-z$ is the viewing direction and y is the up vector. Consider a frame coordinate with origin **e** and basis {**w**, **v**}. Here **e** is located on the y-axis and edge **oe** and **oa** of the triangle *oea* has a length of 1 and 2 unit respectively. Determine the position of the point **a** w.r.t the frame coordinate. **[8]**



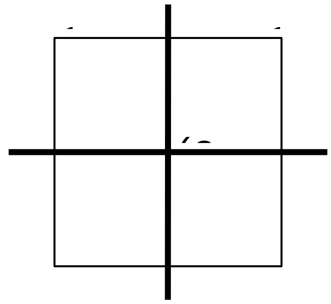
3. a. Solution:

3. Lecture -05

b) Construct the viewport matrix required for a system in which pixel coordinates count down from the top of the image, rather than up from the bottom. **[6]**

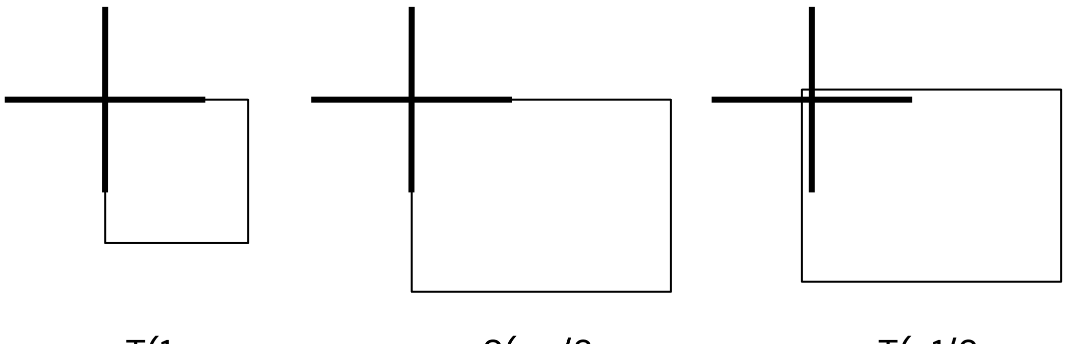
3. b. Solution: O24

This is a general unit image rectangle. We have to convert it to screen pixels.



Given, pixel coordinates count from top to down.

So, transformations are as follows:



So viewport matrix, Mvp = T(-1/2, 1/2) x S(nx/2, ny/2) x T(1, -1)

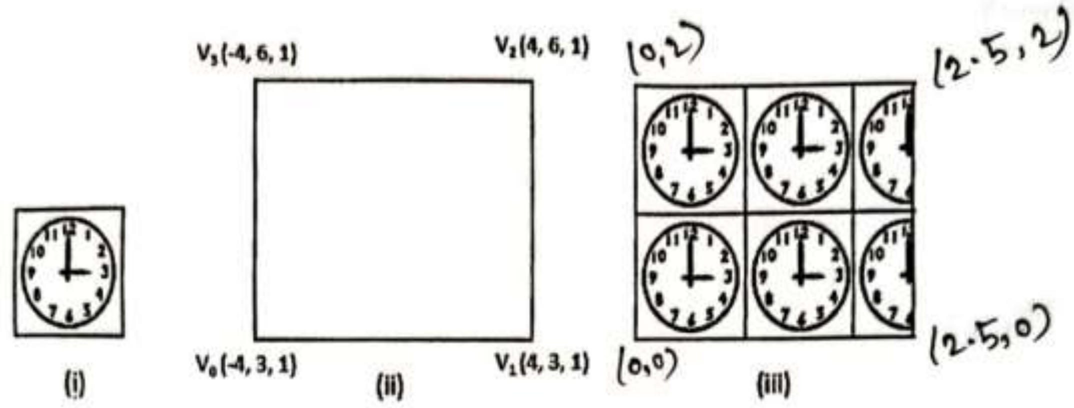
$$\begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{nx}{2} & 0 & 0 \\ 0 & \frac{ny}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{nx}{2} & 0 & \frac{nx}{2} \\ 0 & \frac{ny}{2} & -\frac{ny}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{nx}{2} & 0 & \frac{nx}{2} - \frac{1}{2} \\ 0 & \frac{ny}{2} & \frac{1}{2} - \frac{ny}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

4.

(b) In the following figure, (i) is a texture, (ii) is a rectangular face $V_0V_1V_2V_3$ to be mapped with the texture, and (iii) is the output after texture mapping. List the texture coordinates for corresponding xyz-coordinates to perform texture lookup. (assume any data if necessary) [5]



4. b. Solution:

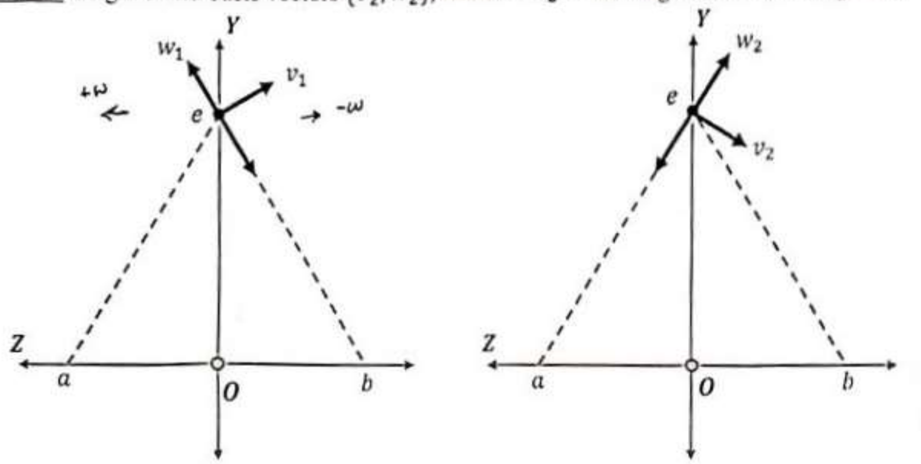
5. **Lecture -05**

(c) Differentiate between orthographic and oblique projections. [3]

5. c. Solution:

6.	(a) Consider a 2D canonical camera coordinate system with origin O and basis vectors $\{y, z\}$; where $-z$ is the viewing direction and y is the up vector. Also consider 2 frame coordinate system inside the canonical (see the following figures), which are – [11]

- Frame-1: Origin e and basis vectors $\{v_1, w_1\}$; where $-w_1$ = viewing direction, and v_1 = up
- Frame-2: Origin e and basis vectors $\{v_2, w_2\}$; where $-w_2$ = viewing direction, and v_2 = up



Frame-1 *Frame-2*

Here e is located on y axis and is a vertex of an equilateral Δeab, where each edge has a length of *one* unit for both the frames. Determine the positions of the O w.r.t *Frame-1* and *Frame-2*.

6. a. Solution:

7.

b) Consider a 3D line AB that needs to be transformed from an orthographic view volume to a viewport with 64×64 resolution. Vertices of the line are $A(-1, -3, -6)$ and $B(2, 4, -7)$. The orthographic view volume has the following setup: [6]

$$l = -5, \quad r = 5, \quad b = -5, \quad t = 5, \quad n = -3, \quad f = -10$$

Determine the matrix M to transform the vertices of the line to viewport. Determine the transformed vertices.

7. b. Solution: ch5

Recursive40

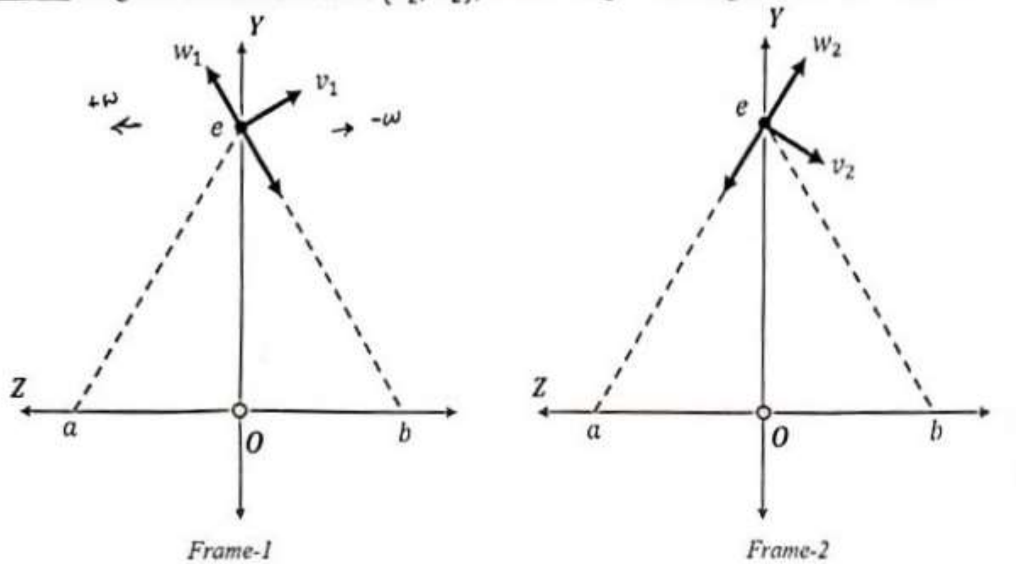
5. **b)** With an example, explain the shading computation for a simple *ray-tracing* algorithm. [4]

5. b. Solution:

6.

(a) Consider a 2D canonical camera coordinate system with origin O and basis vectors $\{y, z\}$; where $-z$ is the viewing direction and y is the up vector. Also consider 2 frame coordinate system inside the canonical (see the following figures), which are – [11]

- Frame-1: Origin e and basis vectors $\{v_1, w_1\}$; where $-w_1$ = viewing direction, and v_1 = up
- Frame-2: Origin e and basis vectors $\{v_2, w_2\}$; where $-w_2$ = viewing direction, and v_2 = up



Here e is located on y axis and is a vertex of an equilateral Δeab. where each edge has a length of *one* unit for both the frames. Determine the positions of the $\boldsymbol{O}$ w.r.t *Frame-1* and *Frame-2*.

6. a. Solution: ch5

7.

b) Consider a 3D line AB that needs to be transformed from an orthographic view volume to a viewport with 64×64 resolution. Vertices of the line are $A(-1, -3, -6)$ and $B(2, 4, -7)$. The orthographic view volume has the following setup: [6]

$$l = -5, \quad r = 5, \quad b = -5, \quad t = 5, \quad n = -3, \quad f = -10$$

Determine the matrix M to transform the vertices of the line to viewport. Determine the transformed vertices.

7. b. Solution: ch5