

# Lecture 6

## Quiz-4

### Set-A

1. Apply the midpoint line drawing algorithm to draw a line from  $(-1, p)$  to  $(4, p - 6)$  and plot the points.

Here,  $p = (-1)^n \times n$   
[n = last 2 digits of your ID]

Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=-x; swap(x, y); plot(-y, x)	(4) x=-x; plot(-x, y)
(5) x=-x; y=-y; plot(-x, -y)	(6) x=-x; y=-y; swap(x, y); plot(-y, -x)	(7) y=-y; swap(x, y); plot(y, -x)	(8) y=-y; plot(x, -y)

- a) [15 marks] Show the values of the decision variables and the points for each step (in a tabular format).  
b) [5 marks] Plot the final points

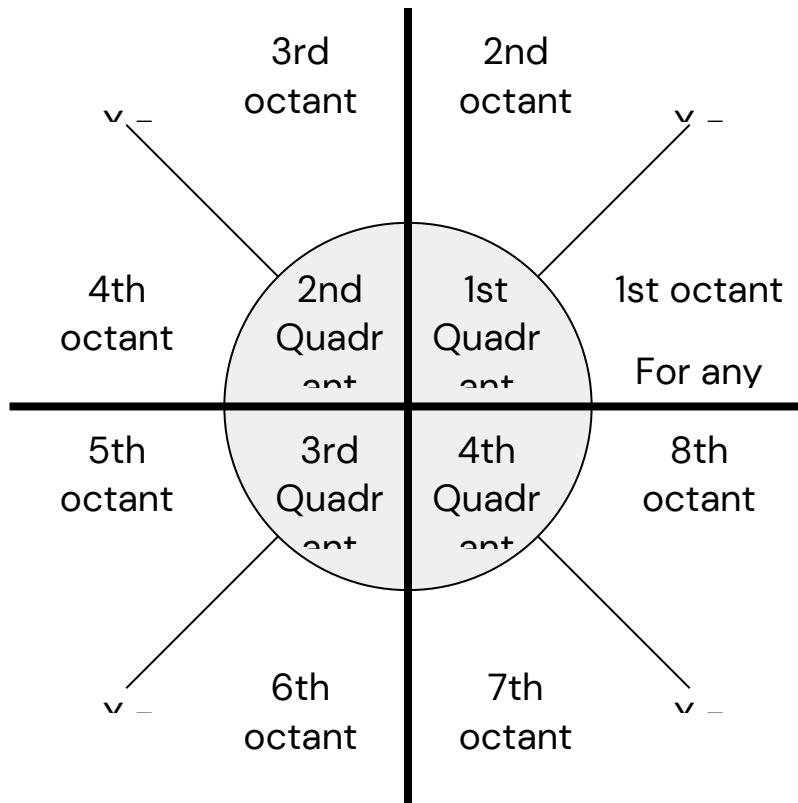
### 1. Solution: 024

(a)

Here,  $p = (-1)^{24} \times 24 = 24$   
So, points are:  $(x_0, y_0) = (-1, 24)$ ,  
 $(x_n, y_n) = (4, 18)$

$m = (y_n - y_0) / (x_n - x_0) = \Delta y / \Delta x = 18 - 24 / 4 - (-1) = -6/5$   
As,  $\Delta x$  value is positive and  $\Delta y$  value is negative, so 4th quadrant.  
For the 4th quadrant we have 7th and 8th octant.  
As,  $|\Delta x| < |\Delta y|$ , so 7th octant.

// Explanation



//

Using given table (for 7th octant),

After negating  $(x_o, y_o) = (-1, -24)$

After negating  $(x_n, y_n) = (4, -18)$

After swapping  $(x_o, y_o) = (-24, -1)$

After swapping  $(x_n, y_n) = (-18, 4)$

**So, we have to go from  $(x_o, y_o) = (-24, -1)$  to  $(x_n, y_n) = (-18, 4)$**

$$dy = y_n - y_o = 4 - (-1) = 5$$

$$dx = x_n - x_o = -18 - (-24) = 6$$

$$d = 2dy - dx = 2 \times 5 - 6 = 4$$

$$\Delta E = 2dy = 2 \times 5 = 10$$

For move to E:  $x = x + 1, y = y$

$$\Delta NE = 2(dy - dx) = 2 \times (5 - 6) = -2$$

For move to NE:  $x = x + 1, y = y + 1$

As  $d=4 > 0$ : move to NE

$$x1 = x_o + 1 = -24 + 1 = -23 \quad y1 = y_o + 1 = -1 + 1 = 0$$

**So,  $(x1, y1) = (-23, 0)$**

$$\text{New } d = d + \Delta NE = 4 + (-2) = 2$$

As  $d=2 > 0$ : move to NE

$$x_2 = x_1 + 1 = -23 + 1 = -22 \quad y_2 = y_1 + 1 = 0 + 1 = 1$$

**So,  $(x_2, y_2) = (-22, 1)$**

$$\text{New } d = d + \Delta NE = 2 + (-2) = 0$$

As  $d=0 \leq 0$ : move to E

$$x_3 = x_2 + 1 = -22 + 1 = -21 \quad y_3 = y_2 = 1$$

**So,  $(x_3, y_3) = (-21, 1)$**

$$\text{New } d = d + \Delta E = 0 + (10) = 10$$

As  $d=10 > 0$ : move to NE

$$x_4 = x_3 + 1 = -21 + 1 = -20 \quad y_4 = y_3 + 1 = 1 + 1 = 2$$

**So,  $(x_4, y_4) = (-20, 2)$**

$$\text{New } d = d + \Delta NE = 10 + (-2) = 8$$

As  $d=8 > 0$ : move to NE

$$x_5 = x_4 + 1 = -20 + 1 = -19 \quad y_5 = y_4 + 1 = 2 + 1 = 3$$

**So,  $(x_5, y_5) = (-19, 3)$**

$$\text{New } d = d + \Delta NE = 8 + (-2) = 6$$

As  $d=6 > 0$ : move to NE

$$x_6 = x_5 + 1 = -19 + 1 = -18 \quad y_6 = y_5 + 1 = 3 + 1 = 4$$

**So,  $(x_6, y_6) = (-18, 4)$**

$$\text{New } d = d + \Delta NE = 6 + (-2) = 4$$

We have found  $(x_n, y_n)$ .

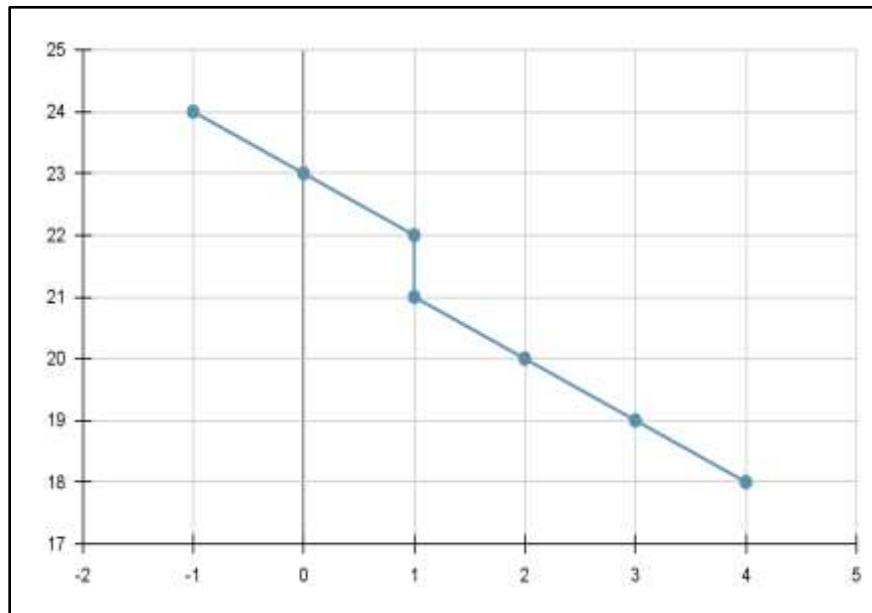
Summary table:

Move	x	y	d
Initial	-24	-1	4
NE	-23	0	2
NE	-22	1	0
E	-21	1	10
NE	-21	2	8
NE	-19	3	6
NE	-18	4	4

(b)

Using values from (a) we get,

$(x, y)$	$\text{plot}(y, -x)$
$(x_0, y_0) = (-24, -1)$	$(-1, 24)$
$(x_1, y_1) = (-23, 0)$	$(0, 23)$
$(x_2, y_2) = (-22, 1)$	$(1, 22)$
$(x_3, y_3) = (-21, 1)$	$(1, 21)$
$(x_4, y_4) = (-20, 2)$	$(2, 20)$
$(x_5, y_5) = (-19, 3)$	$(3, 19)$
$(x_6, y_6) = (-18, 4)$	$(4, 18)$



## Set-B

1. Apply the midpoint algorithm to draw a circle's portions of circumference centered at  $(-2, p)$  on with radius 7.

Here,  $p = (-1)^n \times n$   
[n = last 2 digits of your ID]

If p is even, find the points on the even octants. Otherwise, find the odd octants.

- a) [15 marks] For each step, show the values of the decision variables and the points (in a tabular format).
- b) [5 marks] Plot the final points.

### 1. Solution: 024

(a)

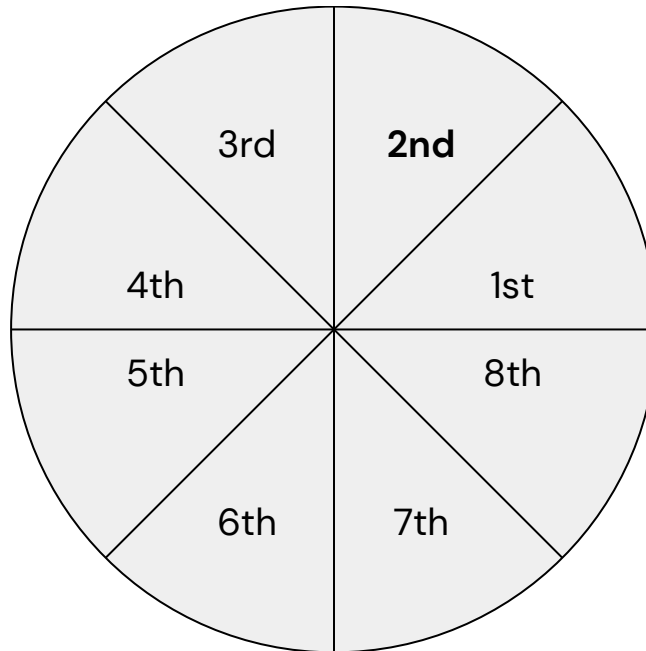
Here,  $p = (-1)^{24} \times 24 = 24$   
So, center  $(-2, 24)$  and radius = 7  
Initial  $x = 0$  and  $y = \text{radius} = 7$   
Initial  $h = 1 - R$

For move to E:  $h = h_{\text{old}} + 2(x_{\text{previous}}) + 3$   
 $x = x_p + 1, y = y_p$

For move to SE:  $h = h_{\text{old}} + 2(x_{\text{previous}}) - 2(y_{\text{previous}}) + 5$   
 $x = x_p + 1, y = y_p - 1$

x	y	2x	2y	h	Move
0	7	0	14	$1 - 7 = -6$	$h < 0$ ; E
1	7	2	14	$-6 + 0 + 3 = -3$	$h < 0$ ; E
2	7	4	14	$-3 + 2 + 3 = 2$	$h \geq 0$ ; SE
3	6	6	12	$2 + 4 - 14 + 5 = -3$	$h < 0$ ; E
4	6	8	12	$-3 + 6 + 3 = 6$	$h \geq 0$ ; SE
5	5	10	10	$6 + 8 - 12 + 5 = 7$	$h \geq 0$ ; SE

As  $p=24$  is even, I have to find points on the even octants.

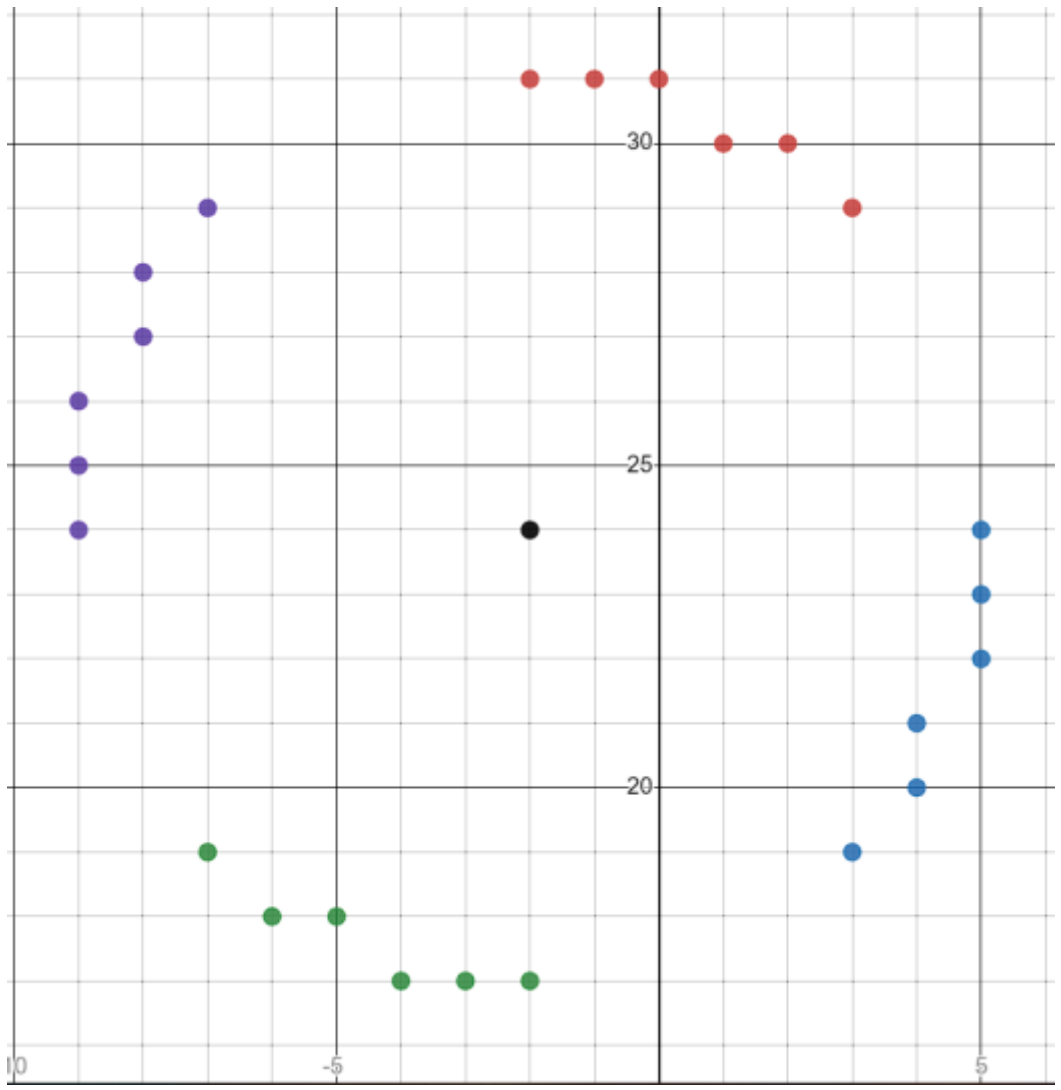


Move	x	y	h	2nd octant	
				(x, y)	(x+xc, y+yc)
Initial	0	7	-6	(0, 7)	(-2, 31)
E	1	7	-3	(1, 7)	(-1, 31)
E	2	7	2	(2, 7)	(0, 31)
SE	3	6	-3	(3, 6)	(1, 30)
E	4	6	6	(4, 6)	(2, 30)
SE	5	5	7	(5, 5)	(3, 29)

4th octant		6th octant		8th octant	
(-y, x)	(-y+xc, x+yc)	(-x, -y)	(-x+xc, -y+yc)	(y, -x)	(y+xc, -x+yc)
(-7, 0)	(-9, 24)	(-0, -7)	(-2, 17)	(7, -0)	(5, 24)
(-7, 1)	(-9, 25)	(-1, -7)	(-3, 17)	(7, -1)	(5, 23)
(-7, 2)	(-9, 26)	(-2, -7)	(-4, 17)	(7, -2)	(5, 22)

$(-6, 3)$	$(-8, 27)$	$(-3, -6)$	$(-5, 18)$	$(6, -3)$	$(4, 21)$
$(-6, 4)$	$(-8, 28)$	$(-4, -6)$	$(-6, 18)$	$(6, -4)$	$(4, 20)$
$(-5, 5)$	$(-7, 29)$	$(-5, -5)$	$(-7, 19)$	$(5, -5)$	$(3, 19)$

(b)



## Set-C

1. Apply the midpoint line drawing algorithm to draw a line from  $(p, -3)$  to  $(p + 7, -6)$  and plot the points.

Here,  $p = (-1)^n \times n$   
 $[n = \text{last 2 digits of your ID}]$

Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) $x = -x$ ; swap(x, y); plot(-y, x)	(4) $x = -x$ ; plot(-x, y)
(5) $x = -x$ ; $y = -y$ ; plot(-x, -y)	(6) $x = -x$ ; $y = -y$ ; swap(x, y); plot(-y, -x)	(7) $y = -y$ ; swap(x, y); plot(y, -x)	(8) $y = -y$ ; plot(x, -y)

- a) [15 marks] Show the values of the decision variables and the points for each step (in a tabular format).  
 b) [5 marks] Plot the final points

### 1. Solution: 035

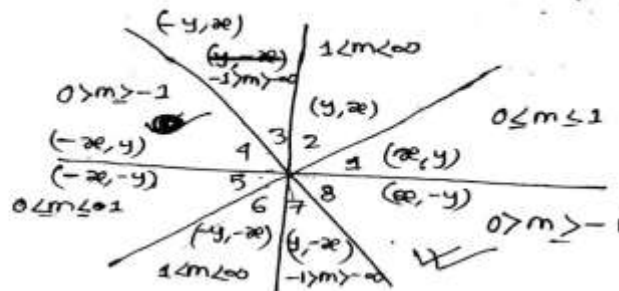
Quiz #1 Set #C

①  $P = (-1)^n * n = (-1)^{35} * 35 = -35$

$(x_0, y_0) = (p, -3) = (-35, -3)$

$(x_f, y_f) = (p+7, -6) = (-35+7, -6) = (-28, -6)$

$m = \frac{dy}{dx} = \frac{-6+3}{-28+35} = \frac{-3}{7} = -0.428$ ,  $\Delta x = +ve$ ,  $\Delta y = -ve$   $\therefore$  8th octant



Modification for 8th octant :

- ① ~~y~~ Negating:  $y = -y$

$\therefore (x_0, y_0) = (-35, 3)$

$(x_f, y_f) = (-28, 6)$

②

So, the line should be drawn from  $(-35, 3)$  to  $(-28, 6)$



Now,

$$dy = 6 - 3 = 3$$

$$\therefore 2dy = 2 \times 3 = 6$$

$$\therefore \Delta E = 2dy = 6$$

$$d\alpha = -28 + 35 = 7$$

$$\therefore \Delta NE = 2(dy - d\alpha)$$

$$= 2(3 - 7)$$

$$= -8$$

$$d_{init} = 2dy - d\alpha$$

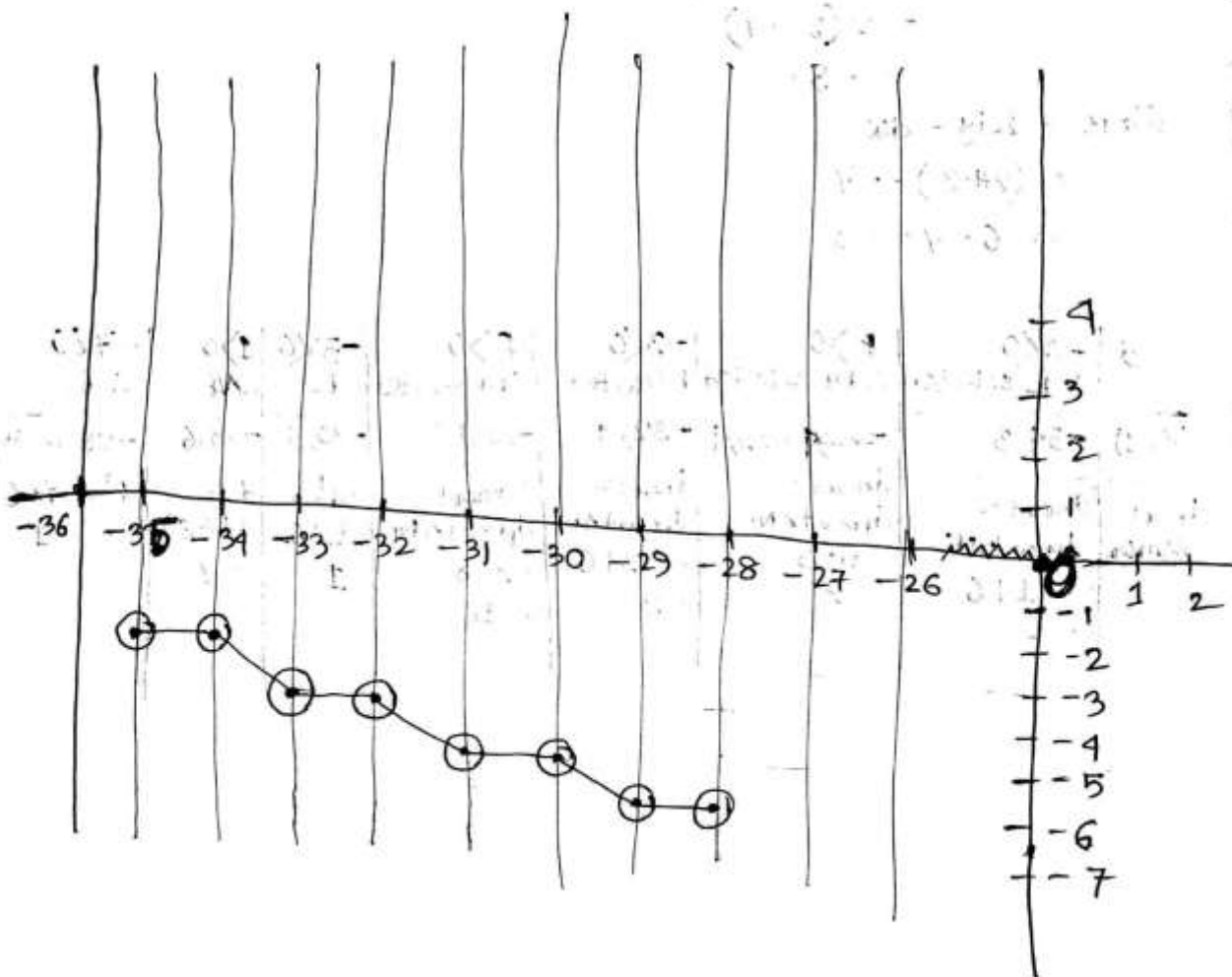
$$= (2 \times 3) - 7$$

$$= 6 - 7 = -1$$

$d$	$-1 < 0$ $\therefore E$ selected	$5 > 0$ $\therefore NE$ selected	$-3 < 0$ $E$ selected	$3 > 0$ $\therefore NE$ selected	$-5 < 0$ $\therefore E$	$1 > 0$ $\therefore NE$	$-7 < 0$ $\therefore E$
$(\alpha, y)$	$-34, 3$	<del><math>-32, 4</math></del>	$-32, 4$	$-31, 5$	$-30, 5$	$-29, 6$	$-28, 6 = \alpha_f, y_f$
$d_{next}$ Calculation	$d_{next} =$ $d_{prev} + \Delta E$ $= -1 + 6$ $= 5$	$d_{next} =$ $d_{prev} + \Delta NE$ $= 5 - 8$ $= -3$	$d_{next} =$ $d_{prev} + \Delta E$ $= -3 + 6$ $= 3$	$d_{next} =$ $d_{prev} + \Delta NE$ $= 3 - 8$ $= -5$	$d' =$ $-5 + 6$ $= 1$	$d' =$ $1 + (-8)$ $= -7$	$d' =$ <del><math>-7 + 6</math></del> $= -1$

b) Plotting:  $(x, -y)$

$(x, y)$	$(-35, 3)$	$(-34, 3)$	$(-33, 4)$	$(-32, 4)$	$(-31, 5)$	$(-30, 5)$	$(-29, 6)$	$(-28, 6)$
$(x, -y)$	$(-35, -3)$	$(-34, -3)$	$(-33, -4)$	$(-32, -4)$	$(-31, -5)$	$(-30, -5)$	$(-29, -6)$	$(-28, -6)$



### **Set-D**

2. Apply the midpoint algorithm to draw a circle's portions of circumference centered at (p, -2) on with radius 8.

$$\text{Here, } p = (-1)^n \times n$$

[n = last 2 digits of your ID]

If p is even, find the points on the even octants. Otherwise, find the odd octants.

- c) **[15 marks]** For each step, show the values of the decision variables and the points (in a tabular format).
- d) **[5 marks]** Plot the final points.

**2. Solution: 035**

# # Quiz #4 set #D

$$P = (-1)^n * n = (-1)^{35} * 35 = -35$$

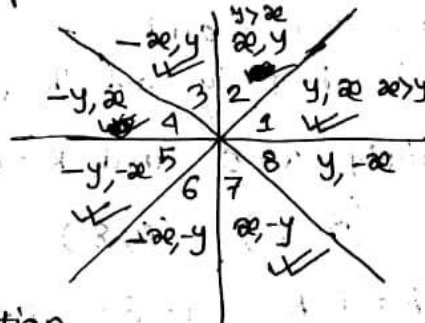
$$\text{Centre} = (P, -2) = (-35, -2)$$

$$\text{Radius} = R = 8$$

$P = -35 \Rightarrow \text{odd}$ ; so we have to find the points on the odd octants.

$$\text{Initially, } (x, y) = (0, R) = (0, 8)$$

$$\therefore h = 1 - R = 1 - 8 = -7$$



Calculations for 2nd octant:

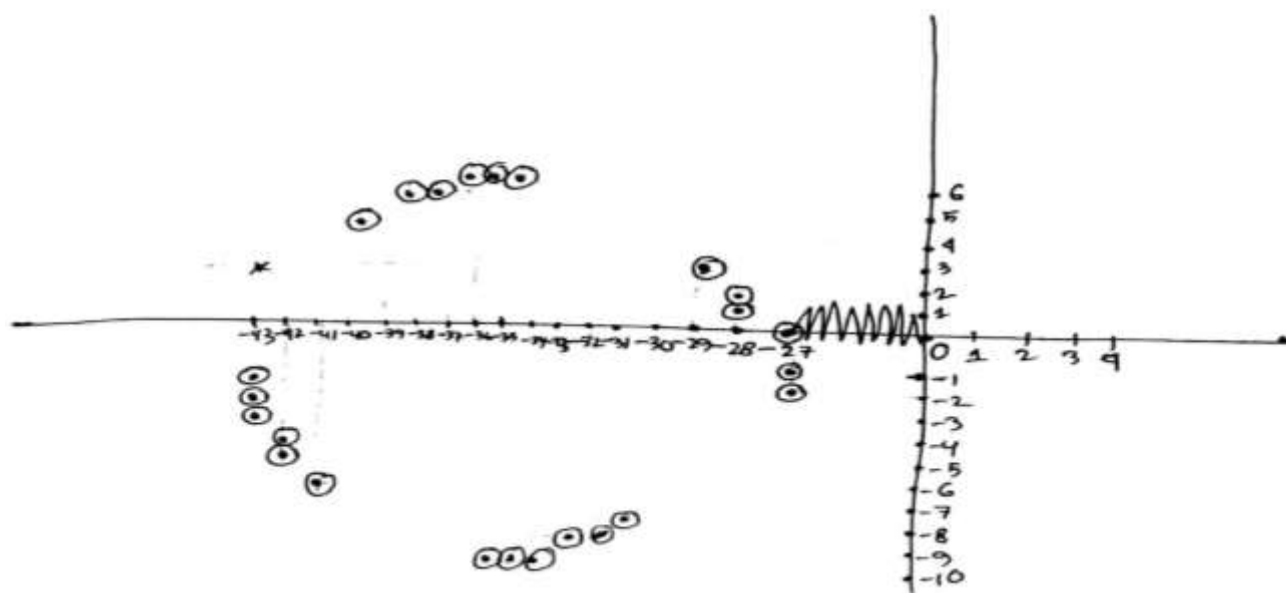
$x, y$	$x', y'$	$h$	$h_{\text{next}}$ Calculation
0, 8	-35, 6	$-7 < 0$ $\therefore E \checkmark$	$h_{\text{next}} = h_{\text{prev}} + \Delta E$ $= h_{\text{prev}} + (2x + 3) = -7 + (2*0 + 3) = -4$
1, 8	-34, 6	$-4 < 0$ $\therefore E \checkmark$	$h_{\text{next}} = -4 + (2*1 + 3) = 1$
2, 8	-33, 6	$1 > 0$ $\therefore SE \checkmark$	$h_{\text{next}} = h_{\text{prev}} + \Delta SE$ $= h_{\text{prev}} + (2x - 2y + 5) = 1 + (2*2 - (2*8) + 5) = -6$
3, 7	-32, 5	$-6 < 0$ $\therefore E \checkmark$	$h_{\text{next}} = -6 + (2*3 + 3) = 3$
4, 7	-31, 5	$3 > 0$ $\therefore SE \checkmark$	$h_{\text{next}} = 3 + (2*4 - (2*7) + 5) = 2$
5, 6	-30, 4	$2 > 0$ $\therefore SE \checkmark$	$h_{\text{next}} = 2 + (2*5 - (2*6) + 5) = 5$
6, 5	-29, 3	$5 > 0$ $\therefore SE \checkmark$	$h_{\text{next}} = 5 + (2*6 - (2*5) + 5) = 12$
7, 4		$12 > 0$ $\therefore SE \checkmark$	$h_{\text{next}} = 12 + (2*7 - (2*4) + 5) = 23$

no need of calculation

as  $y < x$

2nd octant		1st octant		3rd octant		5th octant		7th octant	
$(x, y)$	$(x', y')$	$(y, x)$	$(y', x')$	$(-x, y)$	$(-x', y')$	$(-y, -x)$	$(-y', -x')$	$(-x, -y)$	$(-x', -y')$
0, 8	-35, 6	8, 0	-27, -2	0, 8	-35, 6	-8, 0	-43, -2	0, -8	-35, -10
1, 8	-34, 6	8, 1	-27, -1	-1, 8	-36, 6	-8, -1	-43, -3	1, -8	-34, -10
2, 8	-33, 6	8, 2	-27, 0	-2, 8	-37, 6	-8, -2	-43, -4	2, -8	-33, -10
3, 7	-32, 5	7, 3	-28, 1	-3, 7	-38, 5	-7, -3	-42, -5	3, -7	-32, -9
4, 7	-31, 5	7, 4	-28, 2	-4, 7	-39, 5	-7, -4	-42, -6	4, -7	-31, -9
5, 6	-30, 4	6, 5	-29, 3	-5, 6	-40, 4	-6, -5	-41, -7	5, -6	-30, -8

(b) Plotting:



## Origin42

### 3. Lecture - 06

- b) Write down the algorithm to create a half circle given the radius and the center using Bresenham's Circle drawing algorithm. [6]

#### 3. b. Solution:

##### Algorithm

```
void MidpointCircle(int radius)
{
    int x = 0;
    int y = radius;
    int d = 1 - radius;
    CirclePoints(x, y);
    while (y > x)
    {
        if (d < 0) /* Select E */
            d = d + 2 * x + 3;
        else
        {
            /* Select SE */
            d = d + 2 * (x - y) + 5;
            y = y - 1;
        }
        x = x + 1;
        CirclePoints(x, y);
    }
}
```

```
CirclePoints(x, y)
    Plotpoint(x, y);
    Plotpoint(x, -y);
    Plotpoint(-x, y);
    Plotpoint(-x, -y);
    Plotpoint(y, x);
    Plotpoint(y, -x);
    Plotpoint(-y, x);
    Plotpoint(-y, -x);
end
```

### 4. Lecture -06

- (a) Consider a line with a start and end point of (0, 0) and (-1, -2) respectively. Apply the necessary transformation to increase the size of the line by 100% and find the final vertices after the transformation. Also, determine the coordinates of each pixel along the transformed line segment using the midpoint line drawing algorithm. [10]

Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=-x; swap(x, y); plot(-y, x)	(4) x=-x; plot(-x, y)
(5) x=-x; y=-y; plot(-x, -y)	(6) x=-x; y=-y; swap(x, y); plot(-y, -x)	(7) y=-y; swap(x, y); plot(y, -x)	(8) y=-y; plot(x, -y)

4. a. Solution: Rabab 039

#Origin42 4(a)

$$\Delta x = (-1-0) = -1, \Delta y = (-2-0) = -2$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{-1} = 2$$

<sup>and  $\Delta x, \Delta y$  are</sup>  
 $1 < m < \infty$ . It's in the 6th octant.

$$\therefore (x_0, y_0) = (0, 0) \text{ and } (x_s, y_s) = (2, 1)$$

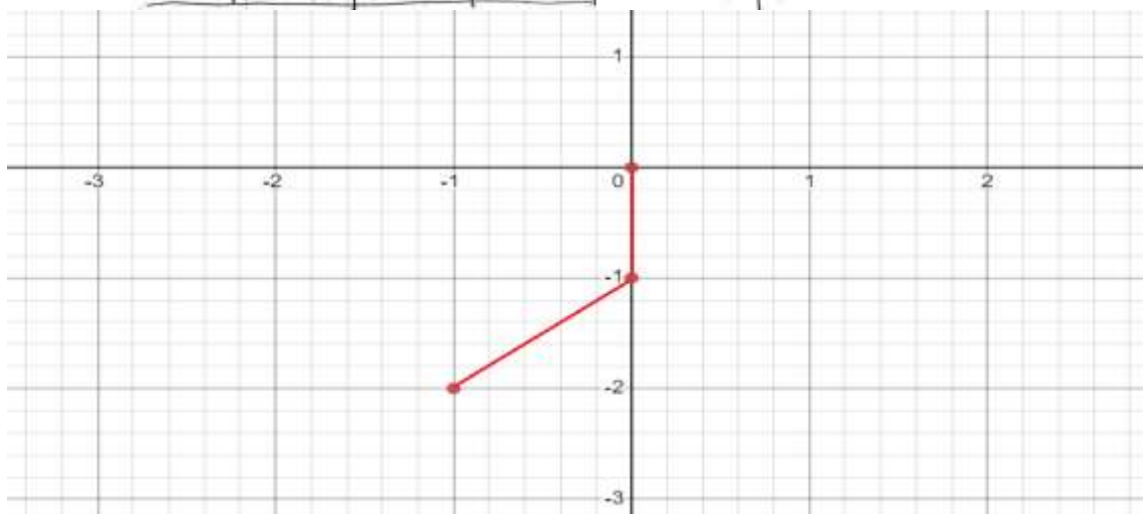
$$\text{Now, } dy = (1-0) = 1, dx = (2-0) = 2$$

$$\therefore d_{init} = 2dy - dx = (2 \times 1) - 2 = 0$$

$$\therefore \Delta E = 2dy = 2 \times 1 = 2$$

$$\therefore \Delta NE = 2(dy - dx) = 2(1-2) = -2$$

$x, y$	0, 0	1, 0	2, 1
$d$	$0 \leq 0$ $\therefore E$	$2 > 0$ $\therefore NE$	$0 \leq 0$ $\therefore E$
$d_{next}$	$0+2=2$	$2-2=0$	-
$-y, -x$	0, 0	0, 1	-1, -2





### 5. Lecture -06

- (a) Consider a triangle with vertices A(1, 1), B(5, 1), and C(3, 3) and color values of red(1, 0, 0), green(0, 0.9, 0), and blue(0, 0, 0.8) at each vertex of the triangle. Find the color of the point P(3, 2) inside the triangle using the concept of barycentric interpolation.

5. a. Solution: 024

Barycentric Interpolation.

Given: A(1, 1), B(5, 1), C(3, 3) ; P(3, 2)

$$\beta = \frac{f_{AC}(x, y)}{f_{AC}(x_B, y_B)}$$

$$= \frac{(y_A - y_C)x + (x_C - x_A)y + x_A y_C - x_C y_A}{(y_A - y_C)x_B + (x_C - x_A)y_B + x_A y_C - x_C y_A}$$

$$= \frac{(1-3)3 + (3-1)2 + 1 \times 3 - 3 \times 1}{(1-3)5 + (3-1)1 + 1 \times 3 - 3 \times 1}$$

$$= 0.25$$

$$\gamma = \frac{f_{AB}(x, y)}{f_{AB}(x_C, y_C)}$$

$$= \frac{(y_A - y_B)x + (x_B - x_A)y + x_A y_B - x_B y_A}{(y_A - y_B)x_C + (x_B - x_A)y_C + x_A y_B - x_B y_A}$$

$$= \frac{(1-1)3 + (5-1)2 + 1 \times 1 - 5 \times 1}{(1-1)3 + (5-1)3 + 1 \times 1 - 5 \times 1}$$

$$= 0.5$$

$$\therefore \alpha + \beta + \gamma = 1 \Rightarrow \alpha = 1 - 0.25 - 0.5 = 0.25$$



$$\text{So, for } P(3, 2) \Rightarrow P(0.25, 0.25, 0.50) \\ P(x, y) \Rightarrow P(\alpha, \beta, \gamma)$$

For color,

Given, red (1, 0, 0), green (0, 0.9, 0),  
blue (0, 0, 0.8)

we know,

$$C = \alpha C_0 + \beta C_1 + \gamma C_2$$

$$\Rightarrow C = 0.25 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0.25 \begin{bmatrix} 0 \\ 0.9 \\ 0 \end{bmatrix} + 0.50 \begin{bmatrix} 0 \\ 0 \\ 0.8 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.225 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \text{red} \\ \text{green} \\ \text{blue} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.225 \\ 0.4 \end{bmatrix} \text{ Ans!}$$

## Enigma41

### 4. Lecture -06

- (a) Apply the midpoint algorithm to draw a circle's portions of circumference centered at  $(-5, -1)$  on the 5<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> octant with radius 6. Plot the obtained points. For each step, show the values of the decision variables and the points (in a tabular format). [9]

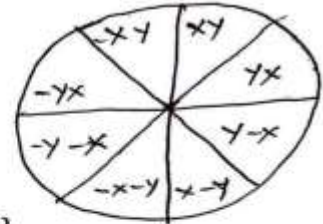
#### 4. a. Solution: Rabab 039

# Enigma41 4(a)

$$h = 1 - R = 1 - 6 = -5$$

For 2nd octant,

#	x, y	h	h <sub>next</sub>
0	0, 6	$-5 < 0$ $\therefore E$	$-5 + 2x + 3$ $= -5 + (2 \times 0) + 3 = -2$
1	1, 6	$-2 < 0$ $\therefore E$	$-2 + (2 \times 1) + 3 = 3$
2	2, 6	$3 > 0$ $\therefore SE$	$3 + (2 \times 2) + 5 = 10$ $- (2 \times 6)$
3	3, 5	$10 > 0 \therefore SE$	$10 + (2 \times 3) + (2 \times 5) + 5 = 28$
4	4, 4		



Now, for other octants, and shifting by  $(-5, -1)$ ,

5 <sup>th</sup>		6 <sup>th</sup>		7 <sup>th</sup>		8 <sup>th</sup>	
$-y, -x$	$-y', -x'$	$-x, -y$	$-x', -y'$	$x, -y$	$x', -y'$	$y, -x$	$y', -x'$
-6, 0	-11, -1	0, -6	-5, -7	0, -6	-5, -7	6, 0	1, -1
-6, -1	-11, -2	-1, -6	-6, -7	1, -6	-4, -7	6, -1	1, -2
-6, -2	-11, -3	-2, -6	-7, -7	2, -6	-3, -7	6, -2	1, -3
-5, -3	-10, -4	-3, -5	-8, -6	3, -5	-2, -6	5, -3	0, -4
-4, -4	-9, -5	-4, -4	-9, -5	4, -4	-1, -5	4, -4	-1, -5

1

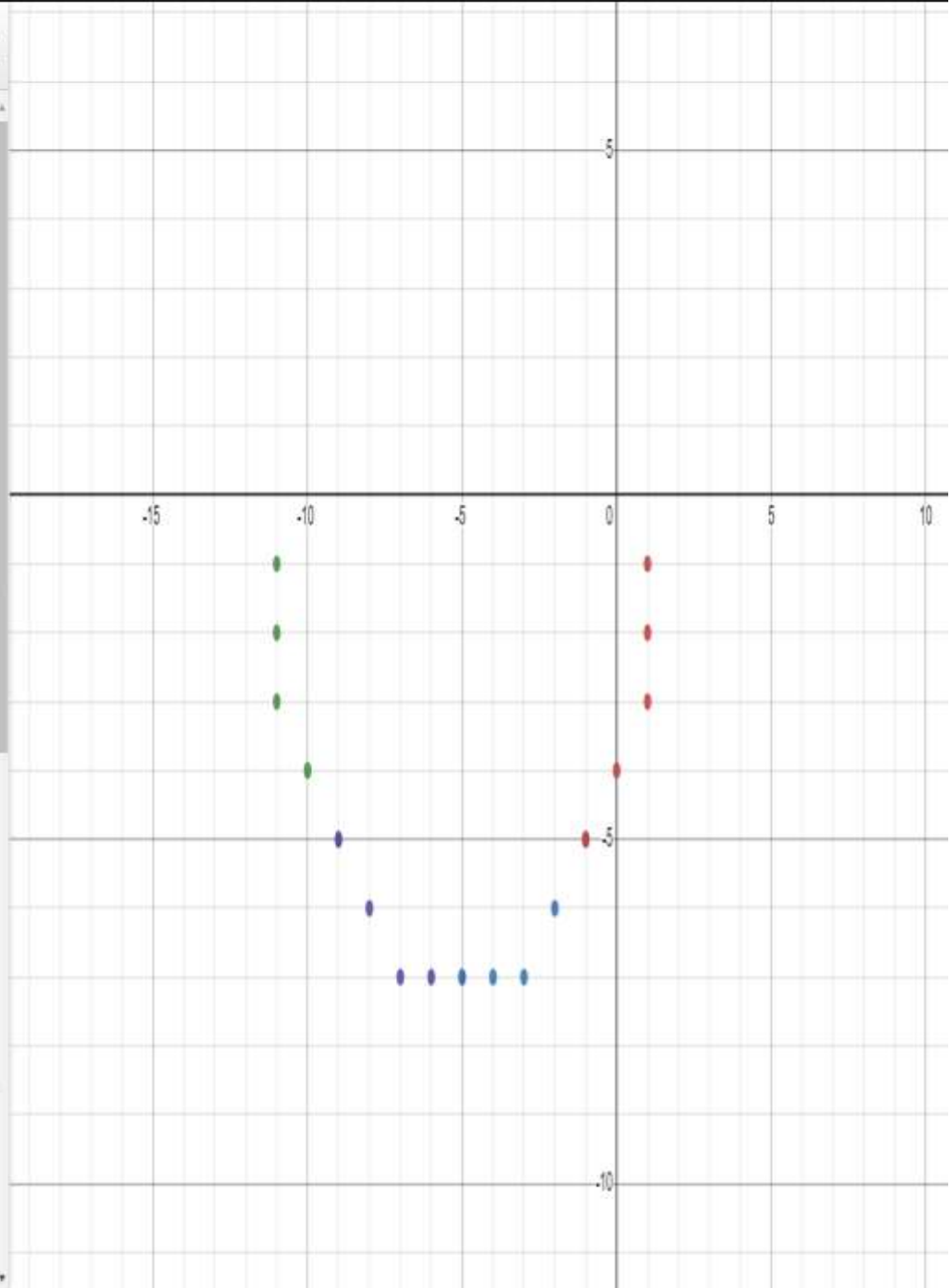
$x_4$	$y_4$
-11	-1
-11	-2
-11	-3
-10	-4
-9	-5

2

$x_1$	$y_1$
-5	-7
-6	-7
-7	-7
-8	-6
-9	-5

3

$x_2$	$y_2$
-5	-7
-4	-7



## 5. Lecture -06

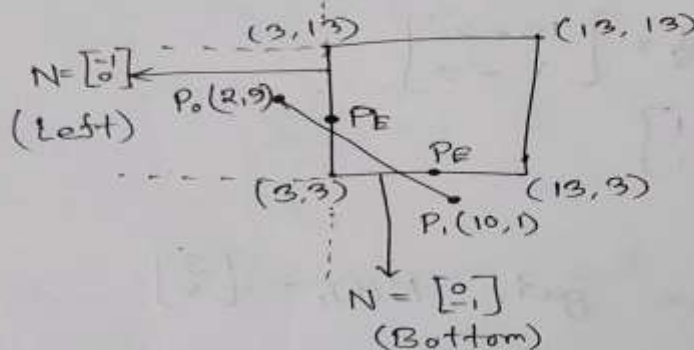
### Question 5. [Marks: 14]

- (a) Consider a clipping rectangle defined by the vertices (3,3), (13,3), (13,13) and (3,13). Also, consider a line which has starting and ending points of (10,1) and (2,9) respectively. Find the line-edge intersecting points with respect to all four edges of the clipping rectangle using the Cyrus-Beck clipping algorithm and determine the true clipping points. Show the steps and calculations for your solution. [8]

5. a. Solution: 024, Correction: 018

Start point =  $P_0(10, 1)$  and end point =  $P_1(2, 9)$

### Cyrus-Beck Line Algorithm



For Left:

$$N_F = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$P_E = \begin{bmatrix} 2 \\ 9 \end{bmatrix} \text{ ; (Let) and } P_0 = \begin{bmatrix} 10 \\ 1 \end{bmatrix} \text{ ; } P_1 = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

we know,

$$\begin{aligned} t_1 &= \frac{N \cdot [P_0 - P_E]}{-N \cdot [P_1 - P_0]} \\ &= \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 - 10 \\ 9 - 1 \end{bmatrix}}{-\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 - 10 \\ 9 - 1 \end{bmatrix}} \\ &= \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -8 \\ 8 \end{bmatrix}}{-\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -8 \\ 8 \end{bmatrix}} = \frac{1}{8} \end{aligned}$$

Again we know,

$$P(t) = P_0 + t(P_1 - P_0)$$

$$\begin{aligned}\Rightarrow \begin{bmatrix} x_t \\ y_t \end{bmatrix} &= \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \frac{1}{8} \times \begin{bmatrix} 10-2 \\ 1-9 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ 8 \end{bmatrix}\end{aligned}$$

So, for left we get,  $P(t)_1 = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

For bottom:

$$N_b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$P_E = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \text{ (let) and } P_0 = \begin{bmatrix} 2 \\ 9 \end{bmatrix}, P_1 = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$

We know,

$$\begin{aligned}t_2 &= \frac{N \cdot [P_0 - P_E]}{-N \cdot [P_1 - P_0]} \\ &= \frac{\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2-7 \\ 9-3 \end{bmatrix}}{-\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 10-2 \\ 1-9 \end{bmatrix}} \\ &= \frac{\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 6 \end{bmatrix}}{-\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -8 \end{bmatrix}} = \frac{-6}{-8} = \frac{3}{4}\end{aligned}$$

$$D = P_1 - P_0 \quad N \cdot D < 0 \rightarrow P_E \cdot N \cdot D > 0 \rightarrow P$$



$$\text{So, } P(t) = P_0 + t(P_1 - P_0)$$

$$\Rightarrow \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \frac{3}{4} \begin{bmatrix} 10-2 \\ 1-9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

So, for bottom we get  $P(t)_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

$\therefore$  Line-edge intersecting points are  $P(t)_1 = (3, 8)$  and  $P(t)_2 = (8, 3)$ .

For true clipping mask points:

$$N_1 \cdot D = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -8 \end{bmatrix} = -8 < 0 \rightarrow \text{Potentially entering}$$

$$N_6 \cdot D = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -8 \end{bmatrix} = 8 > 0 \rightarrow \text{Potentially leaving}$$

$$\therefore$$

$P_e$	$P_l$
$t_1 = 0.125$	$t_2 = 0.75$

As  $\max(P_e) < \min(P_l)$  ; So we can say

that  $t_1 = 0.125$  and  $t_2 = 0.75$  are true intersecting points.

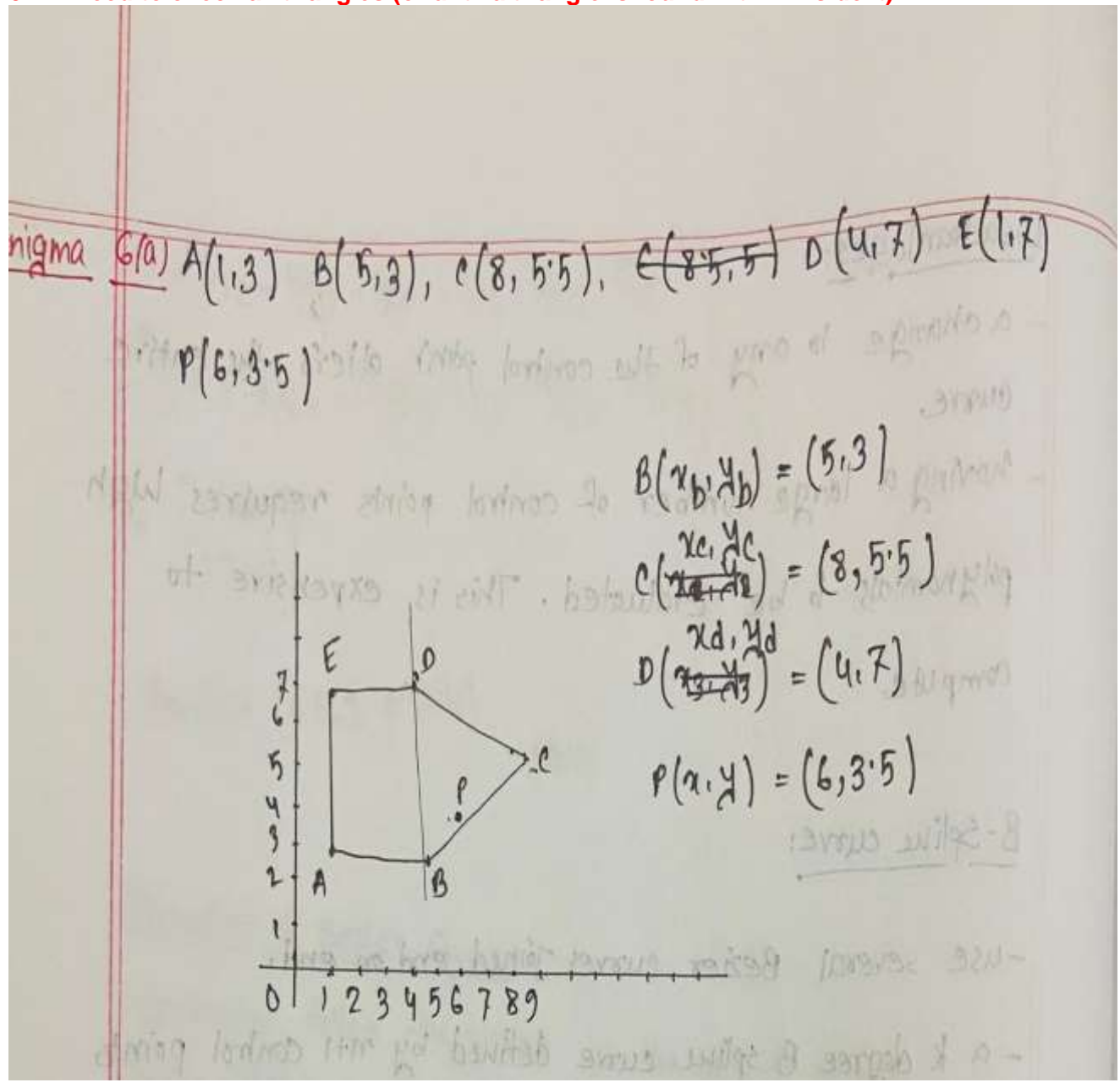
## 6. Lecture -06

- (a) Consider a pentagon ABCDE with vertices A(1,3), B(5,3), C(8,5.5), D(4,7) and E(1,7). Using the concept of barycentric coordinate, determine if a point P(6, 3.5) is inside the pentagon or not. Describe your approach and show your calculations.

[7]

6. a. Solution: 109

021 - Need to check all triangles (or until a triangle is found with P inside it)



$$\begin{aligned}
 \beta &= \frac{f_{cd}(x, y)}{f_{cd}(x_b, y_b)} \\
 &= \frac{(y_c - y_d)x + (x_d - x_c)y + x_c y_d - x_d y_c}{(y_c - y_d)x_b + (x_d - x_c)y_b + x_c y_d - x_d y_c} \\
 &= \frac{(5.5 - 7) \times 6 + (4 - 8) \times 3.5 + (8 \times 7) - (4 \times 5.5)}{(5.5 - 7) \times 5 + (4 - 8) \times 3 + (6 \times 5.5) - 22} \\
 &= \frac{-9 - 14 + 56 - 22}{-7.5 - 12 + 56 - 22} = 0.759
 \end{aligned}$$

$$\begin{aligned}
 \gamma &= \frac{f_{bc}(x, y)}{f_{bc}(x_d, y_d)} \\
 &= \frac{(y_b - y_c)x + (x_c - x_b)y + x_b y_c - x_c y_b}{(y_b - y_c)x_d + (x_c - x_b)y_d + x_b y_c - x_c y_b} \\
 &= \frac{(3 - 5.5) \times 6 + (8 - 5) \times 3.5 + (5 \times 5.5) - (8 \times 3)}{(3 - 5.5) \times 6.4 + (8 - 5) \times 7 + (5 \times 5.5) - (8 \times 3)} \\
 &= \frac{-15 + 10.5 + 27.5 - 24}{-10 + 21 + 27.5 - 24} = 0.276
 \end{aligned}$$

$$\begin{aligned}
 \alpha &= 1 - \beta - \gamma \\
 &= (1 - 0.759 - 0.276) = -0.035
 \end{aligned}$$

Not inside.



## Recursive40

3. Apply the midpoint algorithm to draw a line from (2, 1) to (-8, -6) and plot the obtained points. [11]  
1. Show step-wise values of the decision variables and the points (in a tabular format).

1. a. Solution: ch6

### 3. lecture-6

- a) Suppose we have a 2D quad  $OABC$  with the vertices  $O(0,0)$ ,  $A(1, 0.5)$ ,  $B(2, 1.5)$  and  $C(0.75, 3)$ . Using the concept of barycentric coordinate, determine if a point  $P(1.5, 2.5)$  is inside the quad. Describe your approach and show your calculations.

3. a. Solution: Rabab 039

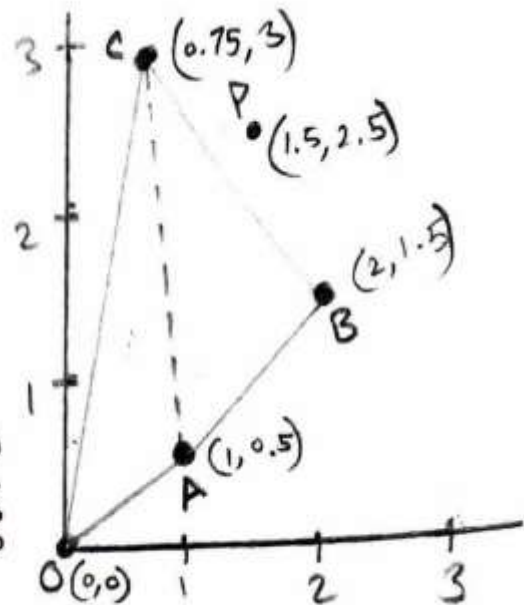
021 - Need to check all triangles (or until a triangle is found with P inside it)

# Recursive40, 3(a)

$$a = \frac{f_{bc}(x, y)}{f_{bc}(x_a, y_a)}$$

$$= \frac{(y_b - y_c)x + (x_c - x_b)y + x_b y_c - x_c y_b}{(y_b - y_c)x_a + (x_c - x_b)y_a + x_b y_c - x_c y_b}$$

$$= \frac{(1.5 - 3)1.5 + (.75 - 2)2.5 + (2 \times 3) - (.75 \times 1.5)}{(1.5 - 3)1 + (.75 - 2)0.5 + (2 \times 3) - (.75 \times 1.5)} = -0.181$$



**Shurutei khela shesh. There's  
a negative value so, outside**

#### 4. . lecture-6

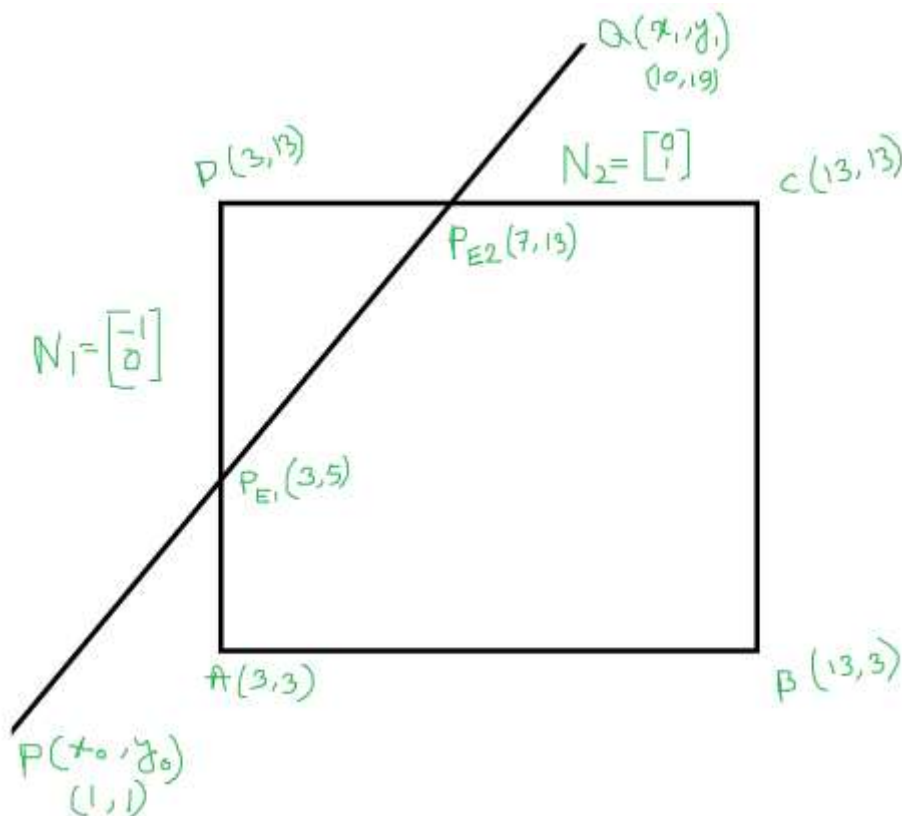
- a) Apply the midpoint algorithm to draw a circle's portions of circumference centered at (2, 0) on the 3<sup>rd</sup>, 4<sup>th</sup>, 5<sup>th</sup> and 6<sup>th</sup> octant with radius 7. Plot the obtained points. For each step, show values of the decision variables and the points (in a tabular format). [8]

#### 4. a. Solution:

#### 7. lecture-6

- Consider a clipping rectangle which has width and height of 10 units. Its lower left corner is located at (3, 3). Also consider a line which has a starting point at (1, 1), length = 20 units, and slope = 2. Perform the line-edge intersecting points with respect to *all four edges* of the clipping rectangle using Cyrus-Beck algorithm and determine the true clipping points. Show your steps and calculations for your solution (assume any data if necessary). [8]

#### 7. a. Solution: Younus-131



Given,  $m = 2$

$$\Rightarrow \frac{y_1 - y_0}{x_1 - x_0} = 2$$

$$\Rightarrow \frac{y_1 - 1}{x_1 - 1} = 2$$

$$\Rightarrow 2x_1 - 2 - y_1 + 1 = 0$$

$$\Rightarrow y_1 = 2x_1 - 1 \quad \text{--- (1)}$$

$\therefore$  Length of PQ = 20

$$\Rightarrow \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} = 20$$

$$\Rightarrow (1 - x_1)^2 + (1 - y_1)^2 = 400$$

$$\Rightarrow 1^2 - 2x_1 + x_1^2 + 1^2 - 2y_1 + y_1^2 = 400$$

$$\Rightarrow 1 - 2x_1 + x_1^2 + 1 - 2(2x_1 - 1) + (2x_1 - 1)^2 = 400$$

$$\Rightarrow 5x_1^2 - 10x_1 - 395 = 0 \quad [\text{from (1)}]$$

$$x_1 = 9.94 \quad \& \quad x_1 = -7.94$$

$$\approx 10 \quad \& \quad \approx -8$$

$$\text{From (1)} \Rightarrow y_1 = 2x_1 - 1$$

$$\therefore y_1 = 19 \quad \& \quad -17$$

$(-8, -17)$  is invalid because PQ line intersects the rectangle. So  $Q = (10, 19)$ .

The PQ line intersects the AD line at  $P_{E1}$  point.

Eq<sup>n</sup> of AD line is  $x = 3$  [parallel line of y axis]

$$\text{From (1)} \Rightarrow y_1 = 2 \times 3 - 1 = 5$$

$$\therefore P_{E1} = (3, 5)$$

PQ line intersects the CD line at  $P_{E2}$  point.

Eq<sup>n</sup> of CD line is  $y = 13$  [parallel line of x axis]

$$\text{From (1)} \Rightarrow 13 = 2x - 1$$

$$\Rightarrow x = 7$$

$$\therefore P_{E2} = (7, 13)$$

For left half plane:

$$t_1 = \frac{N_1 [P_0 - P_{E1}]}{N_1 [P_1 - P_0]} = \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}}{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}} \\ = \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -2 \\ -4 \end{bmatrix}}{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} 9 \\ 18 \end{bmatrix}} = \frac{2}{9} = 0.222$$

For top half plane:

$$t_2 = \frac{N_2 [P_0 - P_{E2}]}{-N_2 [P_1 - P_0]} = \frac{\begin{bmatrix} 0 \\ 1 \end{bmatrix} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 13 \end{bmatrix} \right\}}{-\begin{bmatrix} 0 \\ 1 \end{bmatrix} \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}} \\ = 0.666$$

Intersection points are:

$$P(t_1) = P_0 + t_1 (P_1 - P_0)$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.222 \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 2.98 \\ 4.99 \end{bmatrix} \approx \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$P(t_2) = P_0 + t_2 (P_1 - P_0)$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.66 \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 6.94 \\ 12.88 \end{bmatrix} \approx \begin{bmatrix} 7 \\ 13 \end{bmatrix} \quad \text{Answer}$$