CSE4203: Computer Graphics

Lecture – 8 (Part - B)

Fractal Geometry

Outline

– L-systems

L-systems

- Lindenmayer-systems or L-system is developed by 1968 by biologist Aristid Lindenmayer
- It’s a grammar based technique
  - Represent shape as string of symbol
  - Each symbol has meaning in drawing shape
- Two parts:
  - Grammar for generating strings
  - Rendering algorithm for interpreting strings as shapes

Credit: https://www.wikiwand.com/en/L-system

L-system structure

- L-systems are commonly known as parametric L systems, defined as a tuple

$$\mathbf{G} = (V, \omega, P)$$

- **V (the alphabet)** is a set of symbols containing both elements that can be replaced (variables) and those which cannot be replaced ("constants" or "terminals")
- **ω (start, axiom or initiator)** is a string of symbols from V defining the initial state of the system
- **P is a set of production rules** or productions defining the way variables can be replaced with combinations of constants and other variables.

Credit: https://www.wikiwand.com/en/L-system

Example - 1: algae (1/4)

- L-system for modelling the growth of algae
  variables : **A B**
  constants : **none**
  axiom : **A**
  rules : **(A → AB), (B → A)**

Credit: https://www.wikiwand.com/en/L-system

Example - 1: algae (2/4)

n = 0 : A

n = 1 : AB

n = 2 : ABA

n = 3 : ABAAB

n = 4 : ABAABABA

n = 5 : ABAABABAABAAB

n = 6 : ABAABABAABAABABAABABA

n = 7 : ABAABABAABAABABAABABAABAABABAABAAB

axiom : A
rules : (A → AB), (B → A)

Credit: https://www.wikiwand.com/en/L-system

Example - 1: algae (3/4)

axiom : A
rules : (A → AB), (B → A)

```
n=0:               A           start (axiom/initiator)
                  / \
n=1:             A   B         the initial single A spawned into AB by rule (A → AB), rule (B → A)
couldn't be applied
                /|     \
n=2:           A B      A      former string AB with all rules applied, A spawned into AB again,
former B turned into A
             / | |       | \
n=3:         A B A       A B   note all A's producing a copy of themselves in the first place, then
a B, which turns ...
           / | | | \     | \ \
n=4:       A B A A B     A B A  ... into an A one generation later, starting to spawn/repeat/recurse
then
```

Credit: https://www.wikiwand.com/en/L-system

Example - 1: algae (4/4)

axiom : A
rules : (A → AB), (B → A)

```
n=0:                 A             start (axiom/initiator)
                    / \
n=1:               A   B           the initial single A spawned into AB by rule (A → AB), rule (B → A)
couldn't be applied
                  /|     \
n=2:             A B      A        former string AB with all rules applied, A spawned into AB again,
former B turned into A
               / | |       | \
n=3:           A B A       A B     note all A's producing a copy of themselves in the first place, then
a B, which turns ...
             / | | | \     | \ \
n=4:         A B A A B     A B A   ... into an A one generation later, starting to spawn/repeat/recurse
```

If we count the length of each string, we obtain the Fibonacci sequence of numbers: 1 2 3 5 8 13 21 34 55 89 …

Example - 2: Cantor set (1/6)

variables : **A B**

constants : **none**

axiom : **A**

rules : **(A → ABA), (B → BBB)**

A: Draw a line forward

B: Move forward without drawing

Credit: https://www.wikiwand.com/en/L-system

Example - 2: Cantor set (2/6)

n = 0: A

n = 1: ABA

n = 2: ABABBBABA

n = 3: ABABBBABABBBBBBBBBABABBBABA

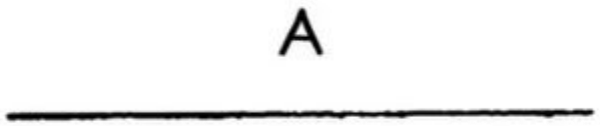
variables : **A B**
constants : **none**
axiom : **A**
rules : **(A → ABA), (B → BBB)**

A: Draw a line forward
B: Move forward without drawing

Example - 2: Cantor set (3/6)

n = 0: A

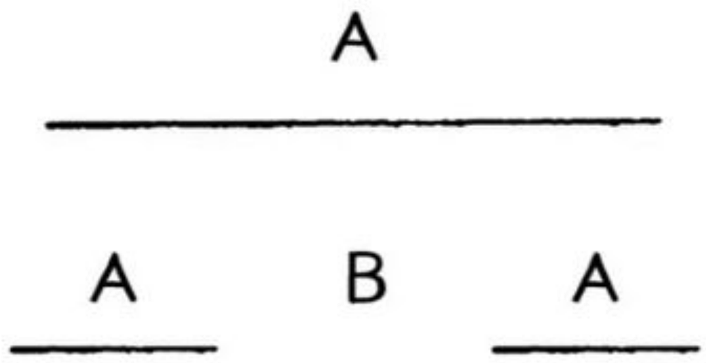
A: Draw a line forward
B: Move forward without drawing



Example - 2: Cantor set (4/6)

n = 1: ABA

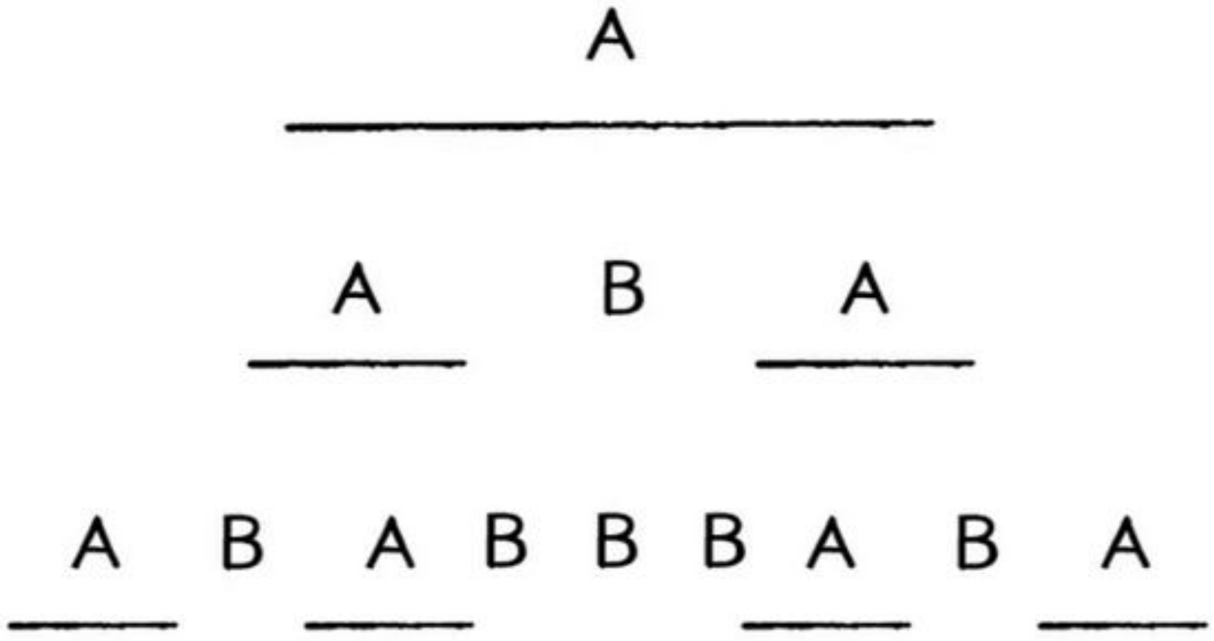
A: Draw a line forward
B: Move forward without drawing



Example - 2: Cantor set (5/6)

n = 2: ABABBBABA

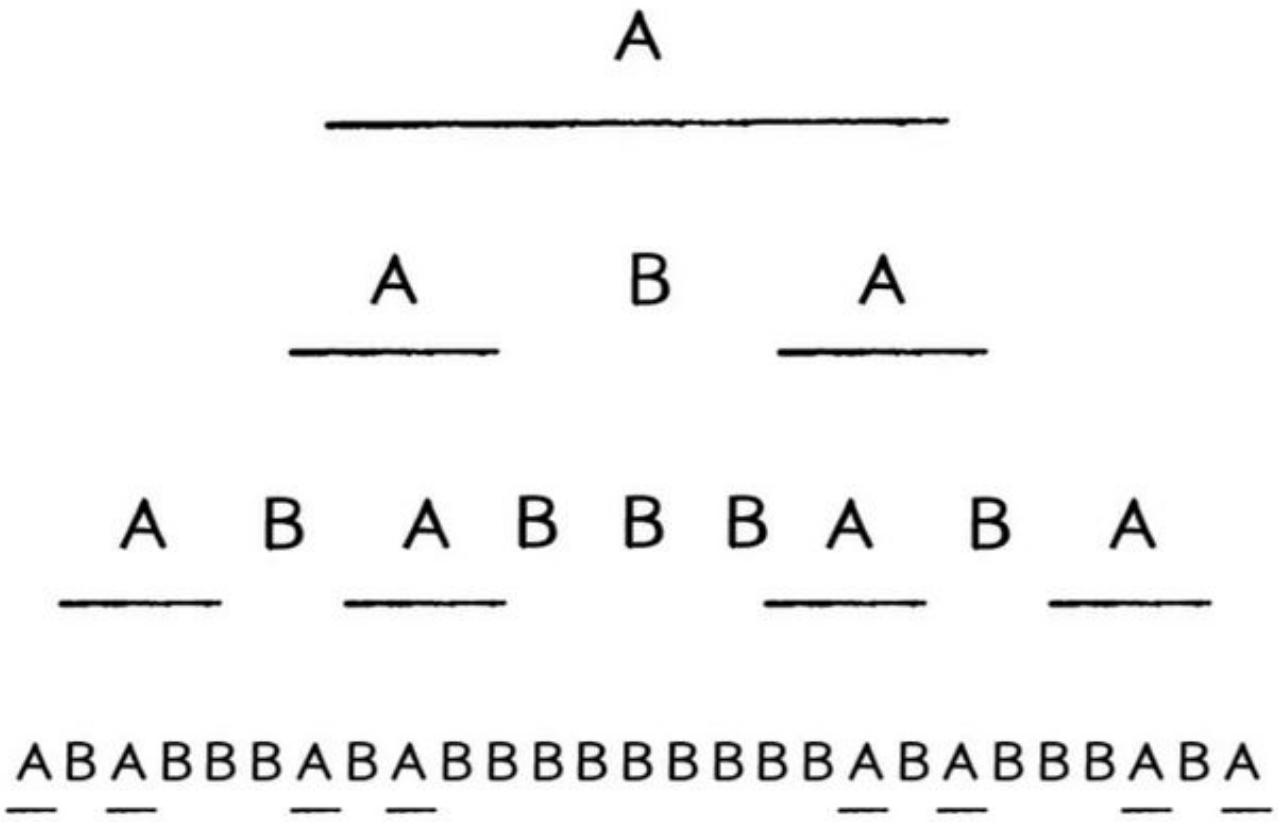
A: Draw a line forward
B: Move forward without drawing



Example - 2: Cantor set (6/6)

n = 3: ABABBBABABBBBBBBBBABABBBABA

A: Draw a line forward
B: Move forward without drawing

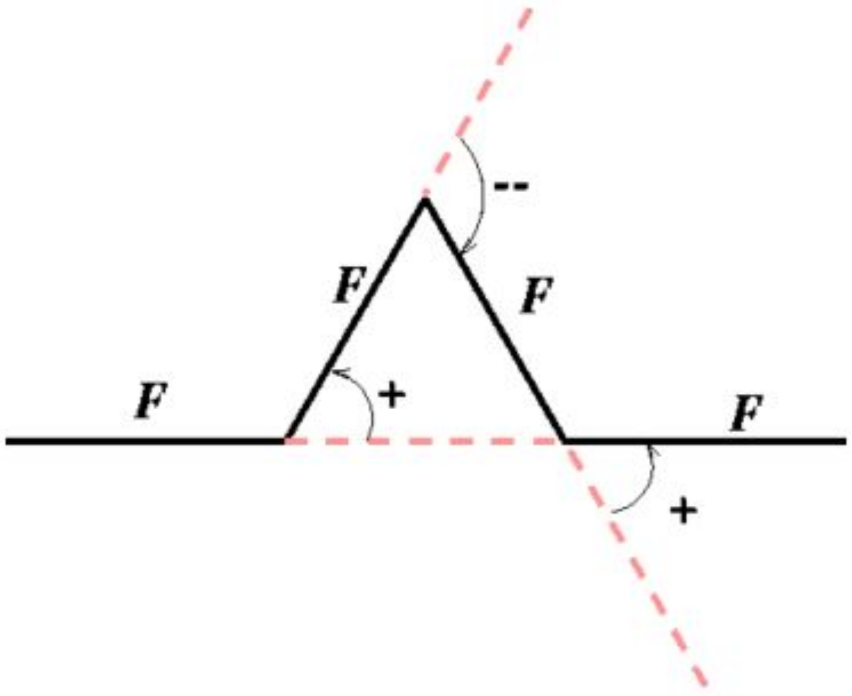


Credit:https://medium.com/@hhtun21/l-systems-draw-your-first-fractals-139ed0bfcac2

Example - 3: Koch Snowflake (1/5)

variables : **F**
constants : **+, -**
axiom : **F**
rules : **F -> F+F--F+F**
Angle: 60 degrees

F: forward one unit
+ : turn left 60 degrees
- : turn right 60 degrees



Example - 3: Koch Snowflake (2/5)

n = 0: F

n = 1: F+F--F+F

n = 2: F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F

variables : **F, +, -**
constants : **none**
axiom : **F**
rules : **F -> F+F--F+F**
Angle: 60 degrees

F: forward one unit
+ : turn left 60 degrees
- : turn right 60 degrees

Example - 3: Koch Snowflake (3/5)

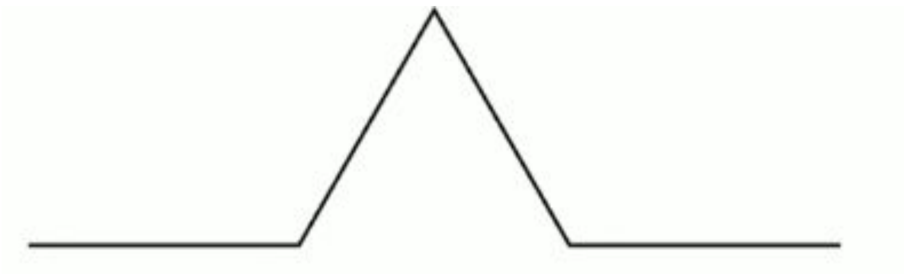
n = 0: F

F: forward one unit
+ : turn left 60 degrees
- : turn right 60 degrees

Example - 3: Koch Snowflake (4/5)

n = 1: F+F--F+F

F: forward one unit
+ : turn left 60 degrees
- : turn right 60 degrees

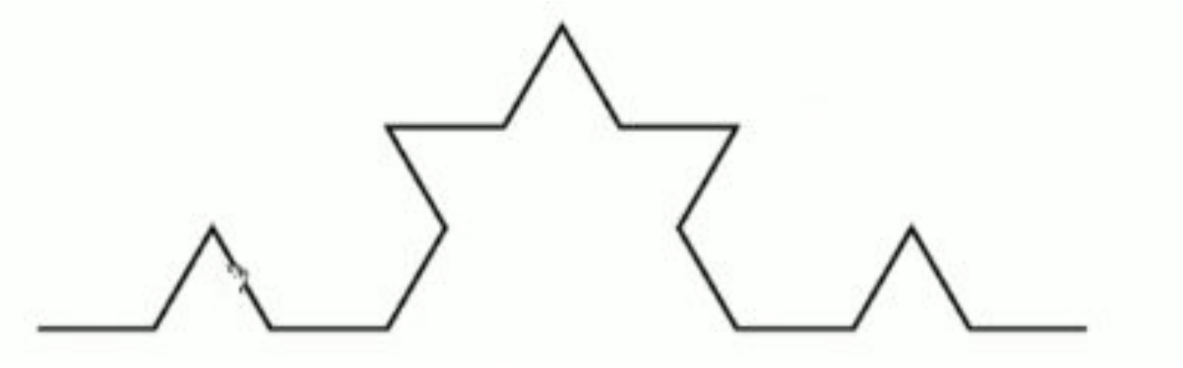


Credit: https://www.wikiwand.com/en/L-system

Example - 3: Koch Snowflake (5/5)

n = 2: F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F

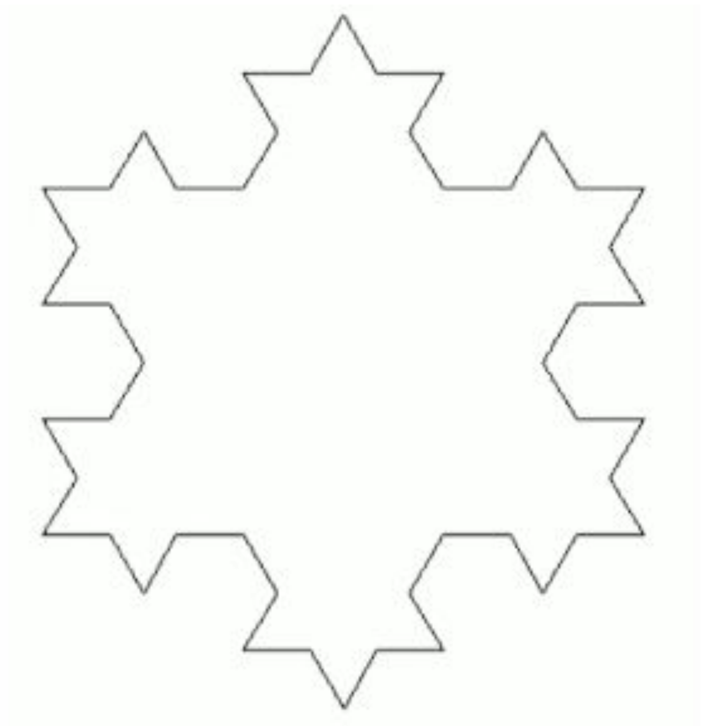
F: forward one unit
+ : turn left 60 degrees
- : turn right 60 degrees



Credit: https://www.wikiwand.com/en/L-system

Example - 3: Koch Snowflake (5/5)

For all 3 sides of triangle



Example - 4: Fractal (binary) tree (1/10)

variables : **0 1**
constants : **[ ]**
axiom : **0**
rules : **(1 → 11), (0 → 1[0]0)**
Angle: 45 degrees

0: draw a line segment ending in a leaf
1: draw a line segment
[: push position and angle, turn left 45 degrees
]: pop position and angle, turn right 45 degrees

Credit: https://www.wikiwand.com/en/L-system

Example - 4: Fractal (binary) tree (2/10)

n = 0: 0

n = 1: 1[0]0

n = 2: 11[1[0]0]1[0]0

n = 3: 1111[11[1[0]0]1[0]0]11[1[0]0]1[0]0

variables : **0 1**
constants : **[ ]**
axiom : **0**
rules : **(1 → 11), (0 → 1[0]0)**
Angle: 45 degrees

0: draw a line segment ending in a leaf
1: draw a line segment
[: push position and angle, turn left 45 degrees
]: pop position and angle, turn right 45 degrees

Credit: https://www.wikiwand.com/en/L-system

Example - 4: Fractal (binary) tree (3/10)

n = 0: 0

0: draw a line segment ending in a leaf

Example - 4: Fractal (binary) tree (4/10)

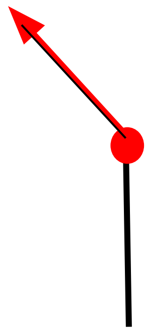
n = 1: 1[0]0

1: draw a line segment

Example - 4: Fractal (binary) tree (5/10)

n = 1: 1[0]0

[: push position and angle, turn left 45 degrees

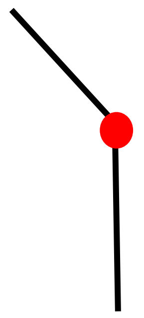


Credit: https://www.wikiwand.com/en/L-system

Example - 4: Fractal (binary) tree (6/10)

n = 1: 1[0]0

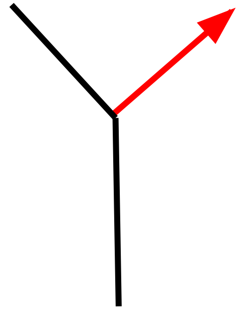
0: draw a line segment ending in a leaf



Example - 4: Fractal (binary) tree (7/10)

n = 1: 1[0]0

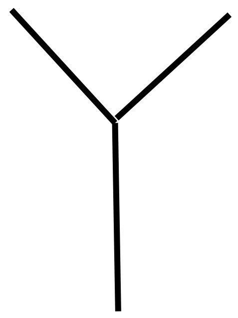
] : pop position and angle, turn right 45 degrees



Example - 4: Fractal (binary) tree (8/10)

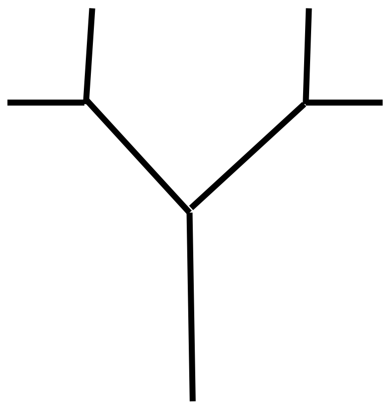
n = 1: 1[0]0

0: draw a line segment ending in a leaf



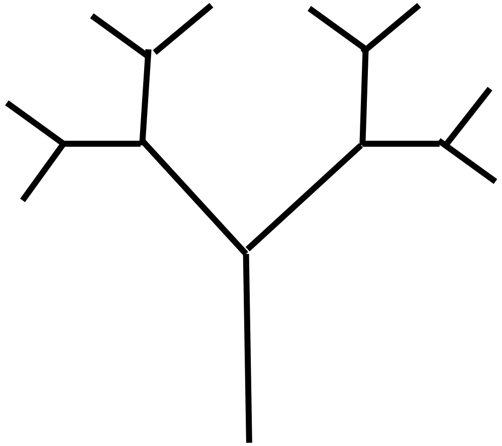
Example - 4: Fractal (binary) tree (9/10)

n = 2: 11[1[0]0]1[0]0



Example - 4: Fractal (binary) tree (10/10)

n = 3: 1111[11[1[0]0]1[0]0]11[1[0]0]1[0]0



Credit: https://www.wikiwand.com/en/L-system

Example - 5: Fractal Plant

variables : **X F**

constants : **+ - [ ]**

axiom : **X**

rules : **(X → F+[[X]-X]-F[-FX]+X), (F → FF)**

Angle: 25 degrees

F: draw forward

X: do Nothing

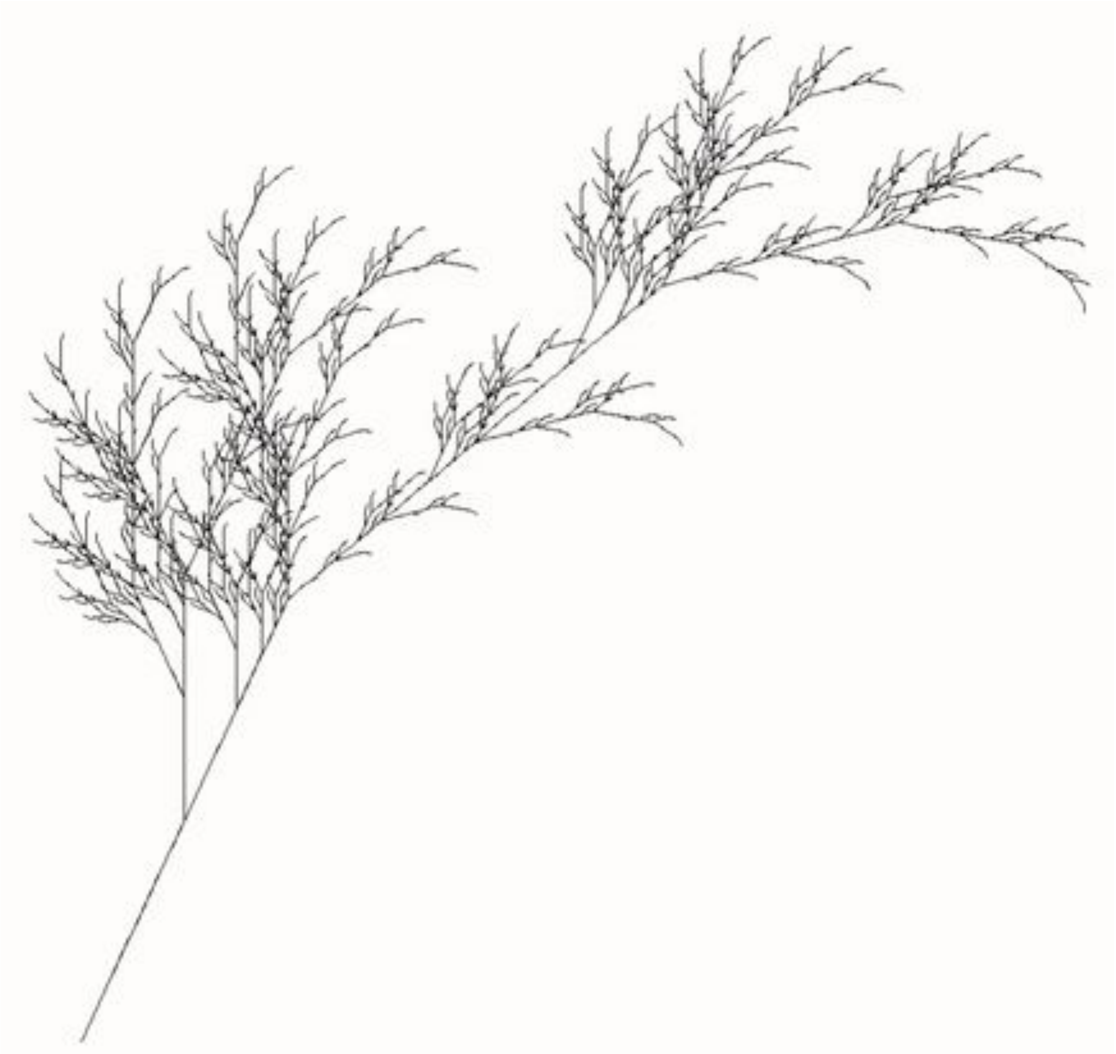
+ : turn left 25 degrees

- : turn right 25 degrees

[: push state

] : pop state

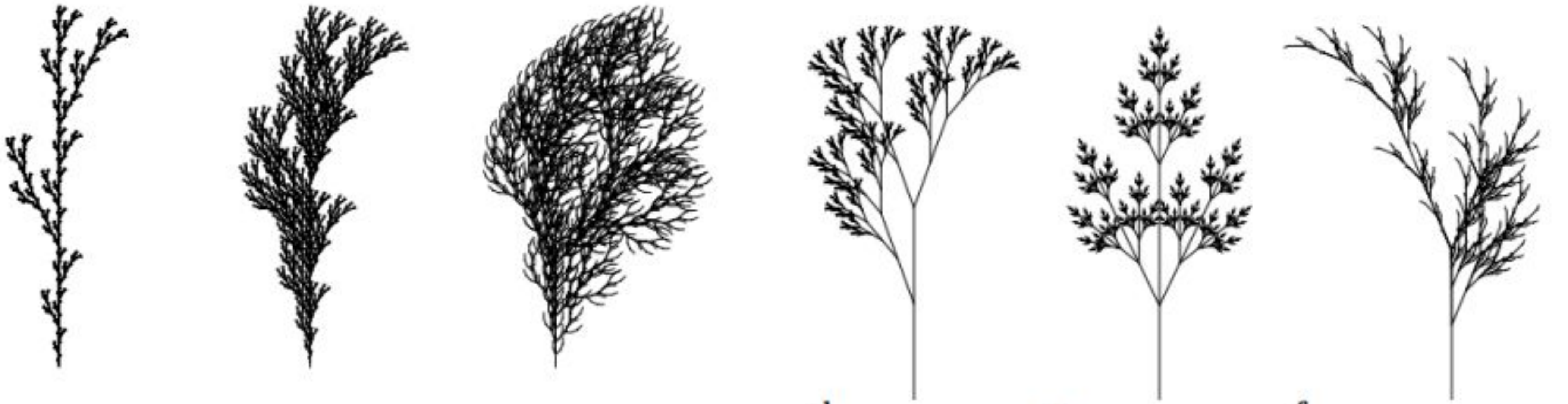
Example - 5: Fractal Plant



Fractal plant for n = 6 (GIF available here)

Credit: https://www.wikiwand.com/en/L-system

Example - 5: Fractal Plant



a
n=5, δ=25.7°
F
F →F[+F]F[-F]F

b
n=5, δ=20°
F
F →F[+F]F[-F][F]

c
n=4, δ=22.5°
F
F →FF-[-F+F+F]+
[+F-F-F]

d
n=7, δ=20°
X
X →F[+X]F[-X]+X
F →FF

e
n=7, δ=25.7°
X
X →F[+X][-X]FX
F →FF

f
n=5, δ=22.5°
X
X →F-[[X]+X]+F[+FX]-X
F →FF

Practice Problems

Apply the concept of L-systems for third iteration (n = 3) to draw -

- Sierpinski Triangle
- Koch curve
- Dragon curve

Further Reading

- https://www.wikiwand.com/en/L-system
- http://paulbourke.net/fractals/lsys/