Lecture 3

Quiz-1

Set-A

3. An uniform quadratic B-Spline curve is defined by 6 control points P0 (-1, -2), P1 (0, 2), P2 (1, 2), P3 (3, 5), P4 (8, 0) and P5 (10, 2). Find the midpoint of the last 2 curve segments of the quadratic B-Spline curve. [8]

3. c. Solution:

Given, total control points = n+1 = 6
So, n = 5
Quadratic means k = 2
Total curves = n - k + 1 = 5 - 2 + 1 = 4
Curves are S0, S1, S2, S3.
Last two curves mean S2, S3 and midpoint means t = 0.5.

We know, a uniform quadratic B-spline curve is written as:

$$S_i(t) = \begin{pmatrix} P_i & P_{i+1} & P_{i+2} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

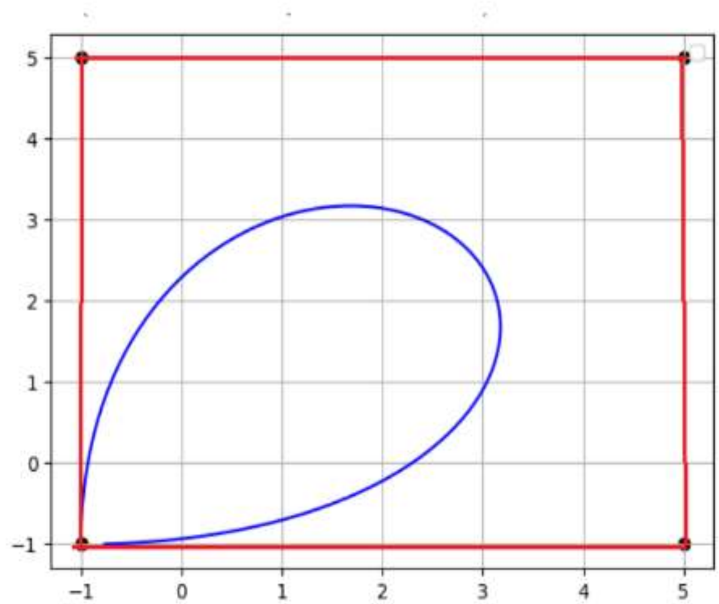
For mid point t = 0.5

$$S_2(0.5) = \begin{bmatrix} P_2 & P_3 & P_4 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} = \begin{bmatrix} 3.375 \\ 4 \end{bmatrix}$$

$$S_3(0.5) = \begin{bmatrix} P_3 & P_4 & P_5 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} = \begin{bmatrix} 7.625 \\ 0.875 \end{bmatrix}$$

Set-B

3. **[8 marks]** A Bezier curve Q is situated inside a square (see the image below) defined by the points (-1, -1), (-1, 5), (5, 5), (5, -1). Find the value of $Q(\frac{1}{3})$ if the curve Q is started and ended on the point (-1, -1).



3. Solution:

n = 5 (5 control points. Since the curve is started and ended on the same point)

$$Q(u) = \sum_{i=0}^{d} B_{i,d}(u)\mathbf{P}_i$$

$$B_{i,d}(u) = \binom{d}{i} u^i (1-u)^{d-i}$$

$$where \binom{d}{i} = \frac{d!}{i!(d-i)!} u^i (1-u)^{d-i}$$

Q(⅓) = (1.370, 3.148)

Solution: 024

Given, total 5 points:

(-1, -1), (-1, 5), (5, 5), (5, -1), (-1, -1)

So, degree = $N-1 = 5-1 = 4$

Using general eqn we get,

$$Q(u) = \sum_{i=0}^{d} B_{i,d}(u)\, P_i$$

$$\Rightarrow Q(u) = \sum_{i=0}^{d} \frac{d!}{i!\,(d-i)!} u^i (1-u)^{d-i} P_i$$

$\therefore$ For $d = 4$,

$$Q(u) = \frac{4!}{0!\,(4-0)!} u^0 (1-u)^{4-0} P_0 + \frac{4!}{1!\,(4-1)!} u^1 (1-u)^{4-1} P_1$$
$$+ \frac{4!}{2!\,(4-2)!} u^2 (1-u)^{4-2} P_2 + \frac{4!}{3!\,(4-3)!} u^3 (1-u)^{4-3} P_3$$
$$+ \frac{4!}{4!\,(4-4)!} u^4 (1-u)^{4-4} P_4$$

$$= (1-u)^4 P_0 + 4u(1-u)^3 P_1 + 6u^2(1-u)^2 P_2$$
$$+ 4u^3(1-u) P_3 + u^4 P_4$$

$$\therefore Q\left(\tfrac{1}{3}\right) = \left(1-\tfrac{1}{3}\right)^4 P_0 + 4\times\tfrac{1}{3}\times\left(1-\tfrac{1}{3}\right)^3 P_1 + 6\times\left(\tfrac{1}{3}\right)^2\times\left(1-\tfrac{1}{3}\right)^2 P_2$$
$$+ 4\times\left(\tfrac{1}{3}\right)^3\times\left(1-\tfrac{1}{3}\right) P_3 + \left(\tfrac{1}{3}\right)^4 P_4$$

$$= \frac{16}{81}P_0 + \frac{32}{81}P_1 + \underset{\frac{8}{27}}{\cancel{\frac{10}{9}}}P_2 + \frac{8}{81}P_3 + \frac{1}{81}P_4$$

Putting point values,

$$\overset{\cancel{16}}{Q}\left(\tfrac{1}{3}\right) = \frac{16}{81}\begin{bmatrix}-1\\-1\end{bmatrix} + \frac{32}{81}\begin{bmatrix}-1\\5\end{bmatrix} + \overset{8/27}{\cancel{\frac{10}{9}}}\begin{bmatrix}5\\5\end{bmatrix} + \frac{8}{81}\begin{bmatrix}5\\-1\end{bmatrix} + \frac{1}{81}\begin{bmatrix}-1\\-1\end{bmatrix}$$

$$= \begin{bmatrix}-16/81\\-16/81\end{bmatrix} + \begin{bmatrix}-32/81\\160/81\end{bmatrix} + \underset{\boxed{\begin{matrix}40/27\\40/27\end{matrix}}}{\cancel{\begin{bmatrix}50/9\\50/9\end{bmatrix}}} + \begin{bmatrix}40/81\\-8/81\end{bmatrix} + \begin{bmatrix}-1/81\\-1/81\end{bmatrix}$$

$$= \begin{bmatrix}-\frac{16}{81} - \frac{32}{81} + \frac{40}{27} + \frac{40}{81} - \frac{1}{81}\\ -\frac{16}{81} + \frac{160}{81} + \frac{40}{27} - \frac{8}{81} - \frac{1}{81}\end{bmatrix}$$

$$= \cancel{\begin{bmatrix}49/9\\65/9\end{bmatrix}} = \begin{bmatrix}37/27\\85/27\end{bmatrix}$$

$$= \cancel{\begin{bmatrix}5.444\\7.222\end{bmatrix}} = \begin{bmatrix}1.370\\3.148\end{bmatrix}$$ Ans:

Set-C

2. Derive the equation of a Bezier curve of degree 4 using de Casteljau's Algorithm. [6]

2. Solution:

de Casteljau's Algorithm:

$P_{i,j} = (1-u)\,P_{i,j-1} + u\,P_{i+1,j-1}$

$P_{0,4} = (1-u)\,P_{0,3} + u\,P_{1,3}$

$= (1-u)\,((1-u)\,P_{0,2} + u\,P_{1,2}) + u\,((1-u)\,P_{1,2} + u\,P_{2,2})$

$= (1-u)\,((1-u)\,((1-u)\,P_{0,1} + u\,P_{2,1}) + u\,((1-u)\,P_{1,1} + u\,P_{2,1}))) + u\,((1-u)\,((1-u)\,P_{1,1} + u\,P_{2,1}) + u\,((1-u)\,P_{2,1} + u\,P_{3,1})\,))$

$= (1-u)\,((1-u)\,((1-u)\,((1-u)\,P_{0,0} + u\,P_{1,0})) + u\,((1-u)\,P_{2,0} + u\,P_{3,0}))) + u\,((1-u)\,((1-u)\,P_{1,0} + u\,P_{2,0})) + u\,((1-u)\,P_{2,0} + u\,P_{3,0}))))) + u\,((1-u)\,((1-u)\,((1-u)\,P_{1,0} + u\,P_{2,0})) + u\,((1-u)\,P_{2,0} + u\,P_{3,0}))) + u\,((1-u)\,((1-u)\,P_{2,0} + u\,P_{3,0})) + u\,((1-u)\,P_{3,0} + u\,P_{4,0}))\,))$

3. [8 marks] An uniform quadratic B-Spline curve S is defined by 7 control points P0 (-2, -1), P1 (-1, 1), P2 (1, 1), P3 (3, 4), P4 (5, 5), P5 (7, 7) and P6 (9, 10). Find the point on the curve segments for S1 (0.4), S2 (0.7) and S3 (1).

3. Solution:

$$S_i(0) = \begin{bmatrix} P2 & P_{i-1} & P_{i-2} \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix}$$

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S1(0.4) = [[0.8 ] [1.24]]
S2(0.7) = [[3.4 ] [4.11]]
S3(1) = [[6.] [6.]]
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Set-D

1. State the problems associated with the higher degree Bezier Curve. Explain how this problem can be solved. [6]

1. a. Solution:

- Having a large number of control points requires high polynomials to be evaluated.
- This is expensive to compute.
- It can be solved by B-Spline curves.

3. A Bezier curve Q is defined by five vertices of a pentagon. The vertices of the pentagon are (-3, 3), (3, 3), (5, 0), (0, -2) and (-5, 0). Find the point on the Bezier curve for Q(0.2) and Q(0.8). [8]

3. c. Solution:

$$Q(u) = \sum_{i=0}^{d} B_{i,d}(u)\mathbf{P}_i$$

$$B_{i,d}(u) = \binom{d}{i} u^i (1-u)^{d-i}$$

$$where \ \binom{d}{i} = \frac{d!}{i!(d-i)!} u^i (1-u)^{d-i}$$

Q(0.2) = (0.76, 2.406)
Q(0.8) = (-1.208, -0.737)

What is the value of d? *reply: d means degree. d = control_points - 1. Here, d= 5 - 1 = 4.*

Origin42

2. Lecture -03

State the differences between raster and vector images. [4]

2. a. Solution: solved by- 146 (from chatgpt)

Raster image: convert (an image stored as an outline) into pixels that can be displayed on a screen or printed.

Vector image: store instructions for displaying the image rather than the pixels needed to display it.

Aspect	Raster Image	Vector Image
Basic Representation	Grid of pixels	Geometric shapes and paths
Resolution Dependency	**Resolution-dependent (DPI)**	**Resolution-independent**
Scaling	May result in quality degradation when scaled up or down :**Resizing can result quality degradation**	No loss of quality when scaled
File Size	**Larger file sizes**	**Smaller file size**
Editing Flexibility	Limited flexibility; editing may degrade quality	Highly editable without quality loss
Image Quality	**Suitable for photorealistic images**	**Limited photorealism**; best for line art, logos, and illustrations
Storage Format Examples	**JPEG, PNG, BMP**	**SVG, AI, PDF**
Printing Quality	Quality may vary based on resolution	Consistently high-quality printing
Ideal Use Cases	Photographs, detailed images	Logos, icons, illustrations

6. Lecture -03

(a) Suppose we have a cubic Bézier curve defined by the control points $P_0 = (0, 0)$, $P_1 = (2, 5)$, $P_2 = (5, 5)$, and $P_3 = (8, 0)$. Find the mid-point and end-point of the cubic curve. [7]

6. a. Solution: *by 102*

$P_0(0,0)$, $P_1(2,5)$, $P_2(5,5)$, $P_3(8,0)$ · Find mid point, end point

Control points, $N = 4$; $d = N - 1 = 4-1 = 3$

We know, $B_{i,d}(u) = \frac{d!}{i!(d-i)!} u^i (1-u)^{(d-i)}$

For, $B(\frac{1}{2})$

$$\therefore B_{0,3}(\tfrac{1}{2}) = \frac{3!}{0!\,(3-0)!} (\tfrac{1}{2})^0 (1-\tfrac{1}{2})^{3-0}$$

$$= \frac{(3\times2)}{1\,(3\times2)} \times 1 \times (\tfrac{1}{2})^3 = \frac{1}{8} = 0.125$$

$$\therefore B_{1,3}(\tfrac{1}{2}) = \frac{3!}{1!\,(3-1)!} \times (\tfrac{1}{2})^1 \times (\tfrac{1}{2})^{3-1}$$

$$= \frac{6}{2} \times \frac{1}{2} \times \frac{1}{4}$$

$$= 0.375$$

$$\therefore B_{2,3}(\tfrac{1}{2}) = \frac{3!}{2!\,(3-2)!} \times (\tfrac{1}{2})^2 \times (\tfrac{1}{2})^{3-2}$$

$$= \frac{6}{2\times1} \times \frac{1}{4} \times \frac{1}{2}$$

$$= 0.375$$

$$B_{3,3}(\tfrac{1}{2}) = \frac{3!}{3!\,(3-3)!} \times (\tfrac{1}{2})^3 \times (\tfrac{1}{2})^{3-3}$$

$$\therefore Q_3\left(\tfrac{1}{2}\right) = 0.125 \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0.375 \times \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 0.375 \times \begin{bmatrix} 5 \\ 5 \end{bmatrix} + 0.125 \times \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.75 \\ 1.875 \end{bmatrix} + \begin{bmatrix} 1.875 \\ 1.875 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3.625 \\ 3.75 \end{bmatrix}$$

For $Q_3(1)$,
$$B_{i,d}(u) = \frac{d!}{i!(d-i)!} \times (u)^i \times (1-u)^{d-i}$$

$$\therefore B_{0,3}(1) = \frac{3!}{0!(3-0)!} \times 1^0 \times (1-1)^{3-0}$$

$$= 1$$

$$\therefore B_{1,3}(1) = \frac{3!}{1!(3-1)!} \times (1)^1 \times \underbrace{(1-1)}_{0}{}^{3-1}$$

$$= \frac{3 \times \cancel{2}}{\cancel{2}} \times 0 = 0$$

$$\therefore B_{2,3}(1) = \frac{3!}{2!\ 1!} (1)^2 \times \underbrace{(1-1)}_{0}{}^{3-2}$$

$$= \frac{3 \times 2}{2} \times 0 = 0$$

$$\therefore B_{3,3}(1) = 1$$

Enigma41

1. Lecture -03

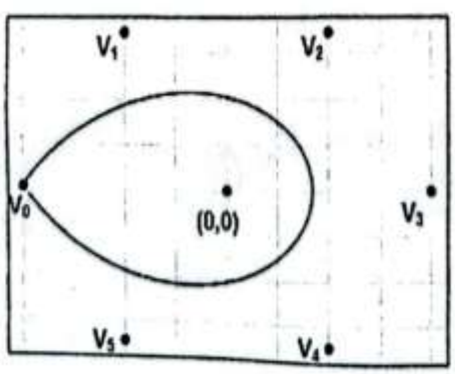
b) State the differences between raster and vector images. [4]

1. b. Solution: 024

Raster Graphics	Vector Graphics
They are composed of pixels.	They are composed of paths.
In Raster Graphics, refresh process is independent of the complexity of the image.	Vector displays flicker when the number of primitives in the image become too large.
Graphic primitives are specified in terms of end points and must be scan converted into corresponding pixels.	Scan conversion is not required.
Raster graphics can draw mathematical curves, polygons and boundaries of curved primitives only by pixel approximation.	Vector graphics draw continuous and smooth lines.
Raster graphics cost less.	Vector graphics cost more as compared to raster graphics.
They occupy more space which depends on image quality.	They occupy less space.
File extensions: .BMP, .TIF, .GIF, .JPG	File Extensions: .SVG, .EPS, .PDF, .AI, .DXF

6. Lecture -03

(b) A 2D Bezier curve Q is situated inside a regular hexagon $V_0V_1V_2V_3V_4V_5$ (see the following figure). The control points are chosen from the vertices of the hexagon. If Q has the same starting and ending point V_0, what is the value of $Q(\frac{1}{5})$? Given that, the vertices V_0 and V_1 are (-1,0) and $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ respectively. Show your calculations. (Hint: a regular hexagon has symmetric property) [7]



6. b. Solution: Rabab 039
We know, rectangular hexagons have symmetric properties. From the figure,
V0 = (-1,0)
V1 = (-½, √3/2)
V3 is at the opposite x-axis end of V0. So, V3 = (1,0)
V5 is at the opposite y-axis end of V1. So, V5 = (-½, -√3/2)
V2 is at the opposite side of the origin from V1. So,V2 = (½, √3/2)
V4 is at the opposite y-axis end of V2. So, V4 = (½, -√3/2)

So, the number of control points, N = 7 (As V0 is both the starting and ending point)
So, dimensions, d = N - 1 = 7 - 1 = 6

Pro tip: Too tired to calculate the coefficients? Use Pascal's triangle! (jodi time na thake)

d = 0	1
d = 1	1 1
d = 2	1 2 1
d = 3	1 3 3 1
d = 4	1 4 6 4 1
d = 5	1 5 10 10 5 1
d = 6	1 6 15 20 15 6 1

Using Bezier curve equation,
$Q(u) = (1-u)^6V0 + 6(1-u)^5uV1 + 15(1-u)^4u^2V2 + 20(1-u)^3u^3V3 + 15(1-u)^2u^4V4 + 6(1-u)u^5V5 + u^6V0$
So,
$Q(0.2) = (0.8)^6V0 + 6(0.8)^5(0.2)V1 + 15(0.8)^4(0.2)^2V2 + 20(0.8)^3(0.2)^3V3 + 15(0.8)^2(0.2)^4V4 + 6(0.8)(0.2)^5V5 + (0.2)^6V0$
Calculate the rest…

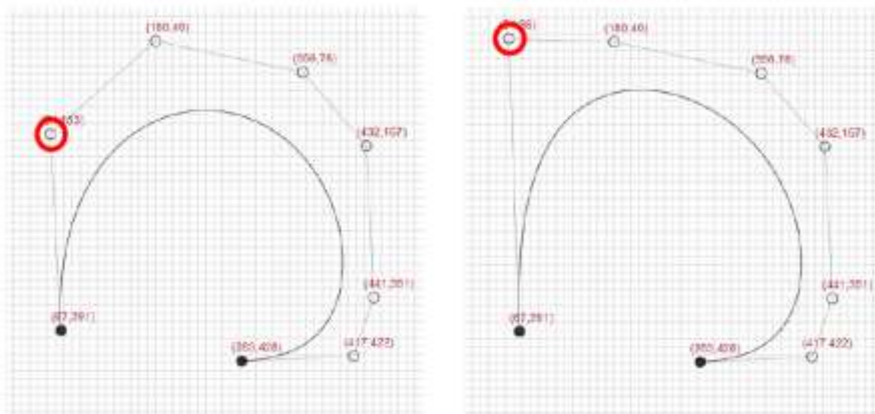
Recursive40

2. Lecture - 03
b) Discuss the limitations of Bezier curve. [2]

2. b. Solution: Rabab 039

Disadvantages

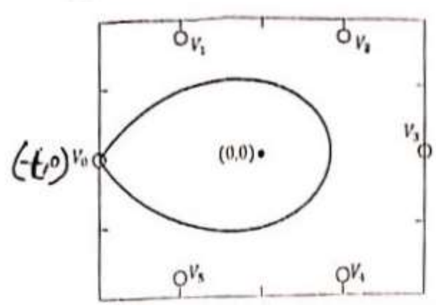
- A change to any of the control point alters the entire curve.
- Having a large number of control points requires high polynomials to be evaluated. This is expensive to compute.



Credit: CPSC 589/689 Course Notes, University of Calgary, Faramarz Samavati

5. **lecture-3**

(a) A 2D Bezier curve Q is situated inside a regular hexagon $V_0V_1V_2V_3V_4V_5$ (see the following figu
The control points are chosen from the vertices of the hexagon. If Q has the same starting and ending point V_0, what is the Euclidean distance between $Q\left(\frac{1}{2}\right)$ and $Q\left(\frac{1}{6}\right)$? Given that, the vertic
V_0 and V_1 are $(-1,0)$ and $(-1,\frac{\sqrt{3}}{2})$ respectively. Show your calculations.



5. a. Solution:

Same as Enigma41 6(b)