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1. **[6 marks]** Explain how a transmissive device works with an example.
2. **[6 marks]** Consider 3 images img1, img2 and img3 (see the image below) overlapping each other where img1 is the foreground of img2 and img2 is the foreground of img3. Additionally, img1 has an alpha mask α_1 and img2 is fully opaque. Find the value of α_1 if we want to get the output image as img4.

130	20	150
50	85	200
230	9	75

img1

15	20	1
200	20	50
110	99	160

img2

23	150	240
1	0	250
22	99	225

img3

38	20	150
200	59	188
176	45	92

img4

3. **[8 marks]** An uniform quadratic B-Spline curve S is defined by 7 control points P_0 (-2, -1), P_1 (-1, 1), P_2 (1, 1), P_3 (3, 4), P_4 (5, 5), P_5 (7, 7) and P_6 (9, 10). Find the point on the curve segments for $S_1(0.4)$, $S_2(0.7)$ and $S_3(1)$.

Solution:

1. See Lecture 2

2. $Img_4 = \alpha_1 img_1 + (1 - \alpha_1) img_2$
 $\alpha_1 = (Img_4 - img_2) / (img_1 - img_2)$ [point-wise subtraction and division]

0.2	0.29	1
0	0.6	0.92
0.55	0.6	0.8

3.

$S_1(0.4) = [[0.8] [1.24]]$
 $S_2(0.7) = [[3.4] [4.11]]$
 $S_3(1) = [[6.] [6.]]$

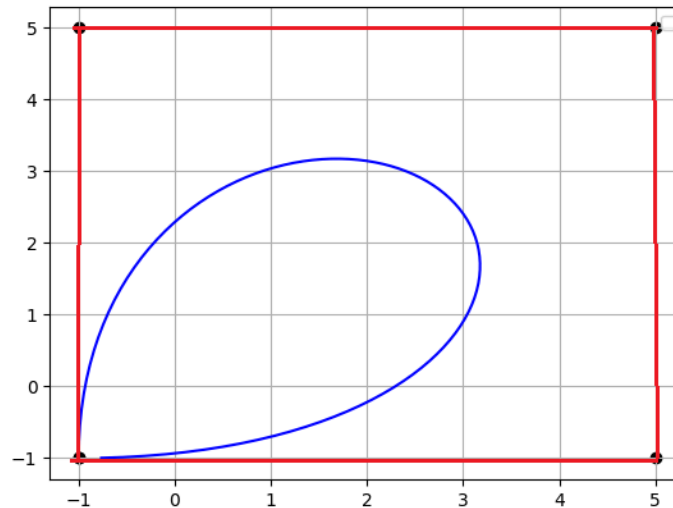
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1. **[6 marks]** Explain why triangles are commonly used as the primary primitive in computer graphics.
2. **[6 marks]** Derive the equation of a Bezier curve of degree 4 using de Casteljau's Algorithm.
3. **[8 marks]** A Bezier curve Q is situated inside a square (see the image below) defined by the points $(-1, -1)$, $(-1, 5)$, $(5, 5)$, $(5, -1)$. Find the value of $Q(\frac{1}{3})$ if the curve Q is started and ended on the point $(-1, -1)$.



Solution:

1. See Lecture 1

2. de Casteljau's Algorithm:

$$P_{i,j} = (1-u) P_{i,j-1} + u P_{i+1,j-1}$$

$$P_{0,4} = (1-u) P_{0,3} + u P_{1,3}$$

$$= \dots\dots\dots$$

$$= (1-u) ((1-u) ((1-u) ((1-u) P_{0,0} + u P_{1,0})) + u ((1-u) P_{2,0} + u P_{3,0})) + u ((1-u) ((1-u) P_{1,0} + u P_{2,0})) + u ((1-u) P_{2,0} + u P_{3,0})) + u ((1-u) ((1-u) P_{1,0} + u P_{2,0})) + u ((1-u) P_{2,0} + u P_{3,0})) + u ((1-u) ((1-u) P_{2,0} + u P_{3,0})) + u ((1-u) P_{3,0} + u P_{4,0}))$$

3. $Q(\frac{1}{3}) = (1.370, 3.148)$

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- [6 marks]** Explain how the level of detail (LoD) rendering optimizes the rendering process.
- [6 marks]** Consider 3 images img1, img2 and img3 (see the image below) overlapping each other where img2 is the foreground of img1 and img1 is the foreground of img3. Additionally, img2 has an alpha mask α_1 given below and img1 is fully transparent. Find the pixel values for the output image.

30	21	140
27	78	200
222	25	224

img1

50	22	152
55	85	20
230	19	100

img2

150	20	1
90	25	70
112	99	165

img3

0.2	0.39	1
0	0.5	0.82
0.45	0.5	0.7

 α_1

- [8 marks]** An uniform quadratic B-Spline curve is defined by 6 control points P_0 (-1, -2), P_1 (0, 2), P_2 (1, 2), P_3 (3, 5), P_4 (8, 0) and P_5 (10, 2). Find the midpoint of the last 2 curve segments of the quadratic B-Spline curve.

Solution:

1. See Lecture 1

2. Pixel values of img4: $\text{img4} = \alpha * \text{img2} + (1 - \alpha) * \text{img3}$

```
[[130.    20.78 152.   ]
 [ 90.    55.    29.   ]
 [165.1   59.   119.5 ]]
```

3. For mid point $t = 0.5$

$$S_2(0.5) = [P_2 \ P_3 \ P_4] \frac{1}{2} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} = \begin{bmatrix} 3.375 \\ 4 \end{bmatrix}$$

$$S_3(0.5) = [P_3 \ P_4 \ P_5] \frac{1}{2} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} = \begin{bmatrix} 7.625 \\ 0.875 \end{bmatrix}$$

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1. **[6 marks]** State the differences between raster and vector images.
2. **[6 marks]** Propose an alpha compositing formula for blending the colors of four objects C_1 , C_2 , C_3 and C_4 . Where C_1 is the foreground of C_2 , C_2 is the foreground of C_3 and C_3 is the foreground of C_4 .
3. **[8 marks]** A Bezier curve Q is defined by five vertices of a pentagon. The vertices of the pentagon are $(-3, 3)$, $(3, 3)$, $(5, 0)$, $(0, -2)$ and $(-5, 0)$. Find the point on the Bezier curve for $Q(0.2)$ and $Q(0.8)$.

Solution:

1. See Lecture 2
2. $C = \alpha_1 C_1 + (1 - \alpha_1) C'' = \alpha_1 C_1 + (1 - \alpha_1) [\alpha_2 C_2 + (1 - \alpha_2) (\alpha_3 C_3 + (1 - \alpha_3) C_4)]$
3. $Q(0.2) = (0.76, 2.406)$ $Q(0.8) = (-1.208, -0.737)$

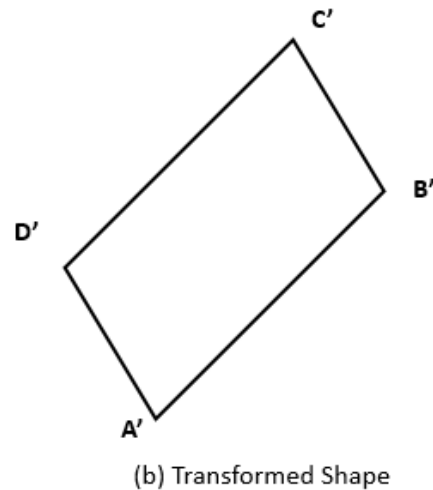
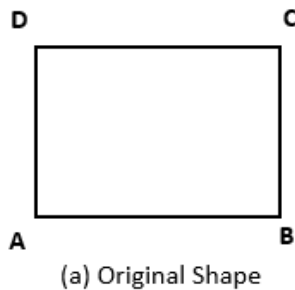
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1. **[8 marks]** Show that the transformation matrix for the reflection about the line $y = -x$ is equivalent to a reflection relative to the y -axis followed by a counter-clockwise rotation of 90 degrees.
2. **[12 marks]** Consider a rectangle with vertices A(1,1), B(6,1), C(6,5) and D(1,5). Apply appropriate transformation to the rectangle to obtain a parallelogram so that point C and D move 20 units to its right from the original position and the parallelogram is rotated along point A by 30 degree. You must -
 - a. Mention the steps.
 - b. Determine the composite transformation matrix.
 - c. Calculate and plot the final vertices



Solution:

1. Show that, $R(-\theta) * \text{Ref-y} * R(\theta) = R(90) * \text{Ref-y}$
2. A'(1,1) B'(5.33, 3.5) C'(20.65, 16.96) D'(16.32, 14.46)

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1. **[8 marks]** Show that two successive reflections about either of the principle axis is equivalent to a single rotation about the coordinate origin.
2. **[12 marks]** Consider a square OACB with vertices O(10, 2), A(15, 2), C(15, 6) and B(10, 6). Reflect the square along a line $x = -3$ using 2D transformation. Determine the composite transformation matrix and find the final vertices.

Solution:

1. Transformation matrix for two successive reflections about either of the principle axis,

$$M1 = \text{Ref-Y} * \text{Ref-X}$$

Single rotation matrix

$$M2 = R(180)$$

You need to show that, $M1 == M2$

2. O'(-16, 2)
A'(-21, 2)
C(-21, 6)
B'(-16, 6)

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1. **[8 marks]** Show that the composite transformation of two rotations $R(\theta_1)$ and $R(\theta_2)$ can be obtained with a single rotation of $R(\theta_1 + \theta_2)$.
2. **[12 marks]** Consider a rectangle with vertices A(10, 12), B(16, 12), C(16, 16) and D(10, 16). Reflect the rectangle along the line $y - 6x + 2 = 0$ using 2D transformation. You must -
 - a. Mention the steps.
 - b. Determine the composite transformation matrix.
 - c. Calculate and plot the final vertices

Solution:

1. Transformation matrix of two rotations $R(\theta_1)$ and $R(\theta_2)$

$$M1 = R(\theta_1) * R(\theta_2)$$

Single rotation of $R(\theta_1 + \theta_2)$

$$M2 = R(\theta_1 + \theta_2)$$

You need to show that, $M1 == M2$

2. A'(-9.46, 15.25)
B'(-15.13, 17.19)
C'(-13.83, 20.98)
D'(-8.16, 19.03)

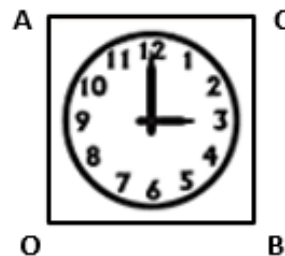
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1. **[8 marks]** Show that reflection about the y-axis is equivalent to a rotation about the y-axis in three-dimensional space.
2. **[12 marks]** Stretch the clock OACB (shown in the figure) by 150% along one of its diagonals so that 10:00 through 4:00 move to the northeast, and 9:00 through 5:00 move to the southwest. The four vertices of the clock are O(2,2), A(2,6), C(6,6), and B(6,2). Perform all the transformations and find the final vertices.



Solution:

1. Matrix for reflection about the y-axis

$$M1 = \text{Ref-Y}$$

rotation about the y-axis by 180 in three-dimensional space.

$$M2 = R(180)$$

You need to show that, $M1 == M2$

2. O'(2, 2)
A' (5, 9)
C' (12, 12)
B' (9, 5)

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1. Consider a viewport of size **256 x 128** in which the origin coordinate is in the center of the viewport and pixel coordinates can have negative values.

a. **[10 Marks]** Construct the viewport matrix

b. **[10 Marks]** Transform a 3D line AB from an orthographic view volume to the mentioned viewport.

Consider the vertices of the line are A(-2, -3, -4), B(2, 4, -6) and the orthographic view volume has the following setup:

$$l = -6, r = 6, b = -7, t = 7, n = -2, f = -8$$

Solution:

Mvp' : 128.0 0 0 0 0 64.0 0 0 0 0 1 0 0 0 0 1	A' : -42.67 -27.43 0.33 1.0
Morth' : 0.17 0 0 0.0 0 0.14 0 0.0 0 0 0.33 1.67 0 0 0 1	B' : 42.67 36.57 -0.33 1.0

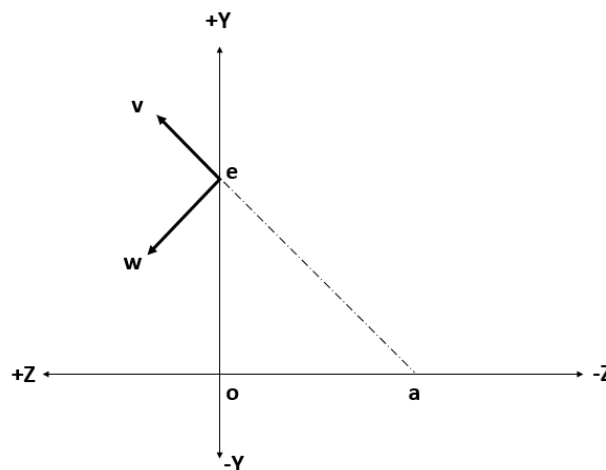
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1. Here (in the figure), origin **O** and basis vectors **{z, y}** construct a 2D canonical coordinate system where $-z$ is the viewing direction and y is the up vector. Consider a camera coordinate with origin **e** and basis **{w, v}**. Here **e** is located on the y -axis and edge **oe** and **oa** of the triangle Δoea have a length of 5 and 3 unit respectively. The goal is to point the camera viewing direction at point **a** and capture it.
- a) **[10 Marks]** Determine the basis and eye matrix
- b) **[10 Marks]** Determine the position of point **a** w.r.t the camera coordinate.



Solution: $P_{wv} = (-5.83, 0)$

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1. Consider a viewport of size **1024 x 512** in which pixel coordinates count down from the top of the image, rather than up from the bottom.
 - a. **[10 Marks]** Construct the viewport transformation matrix
 - b. **[10 Marks]** Transform a 3D line AB from an orthographic view volume to the mentioned viewport. Consider the vertices of the line are A(-2, 5, -4), B(6, 4, -5) and the orthographic view volume has the following setup:

$$l = -7, r = 7, b = -6, t = 6, n = -2, f = -8$$

Solution:

Mvp': 512.0 0 0 511.5 0 256.0 0 -256.5 0 0 1 0 0 0 0 1	A': 365.21 -43.17 0.33 1.0
Morth': 0.14 0 0 0.0 0 0.17 0 0.0 0 0 0.33 1.67 0 0 0 1	B': 950.36 -85.83 0.0 1.0

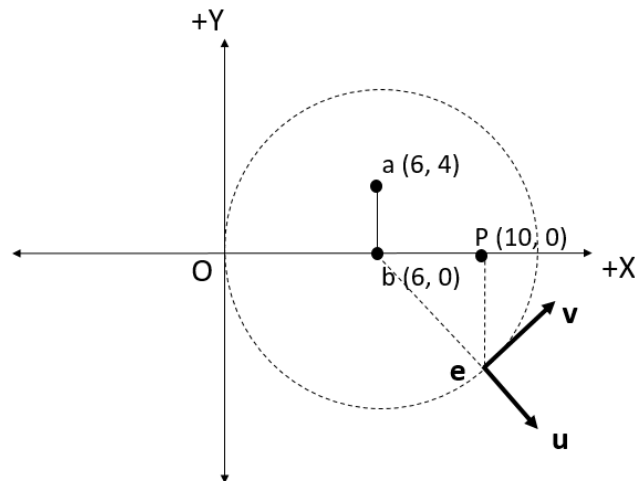
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1. Here (in the figure), origin O and basis $\{x,y\}$ construct a 2D canonical coordinate system. Within this, line ab is our model (P_{xy}). Now, we want to view it from a new 2D camera with eye e and basis $\{u,v\}$; which is rotated by θ degrees from its' default orientation. Assume that, u is the viewing direction and b is the center of the circle.
- [10 Marks]** Determine the basis and eye matrix
 - [10 Marks]** Determine the position of the point a and b w.r.t the camera coordinate.



Solution: $a_{uv} = (-9.03, 2.68)$, $b_{uv} = (-6.03, 0)$

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1. Apply the midpoint line drawing algorithm to draw a line from (-1, 9) to (-5, 15) and plot the points.

Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=-x; swap(x, y); plot(-y, x)	(4) x=-x; plot(-x, y)
(5) x=-x; y=-y; plot(-x, -y)	(6) x=-x; y=-y; swap(x, y); plot(-y, -x)	(7) y=-y; swap(x, y); plot(y, -x)	(8) y=-y; plot(x, -y)

- a) **[15 marks]** Show the values of the decision variables and the points for each step (in a tabular format).
- b) **[5 marks]** Plot the final points

Try Yourself

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1. Apply the midpoint algorithm to draw a circle's portions of circumference centered at (5, 8) on with radius 7.
 - a) **[15 marks]** For each step, show the values of the decision variables and the points (in a tabular format).
 - b) **[5 marks]** Plot the final points.

Try Yourself