# Lecture 4

## *Quiz-2*

### *Set-A*

**1. Show that, transformation for a reflection about the line y = x, is equivalent to a reflection relative to the x-axis followed by counterclockwise rotations of 90 degree. [8]**

**1. a. Solution:**

1. Transformation matrix for reflection about the line y = x,

$$M1 = R^{-1}(45) * Ref * R(45)$$

Transformation matrix for Reflection relative to the x-axis followed by counterclockwise rotations of 90 degrees,

$$M2 = R(90) * Ref\text{-}X$$

You need to show that, M1 == M2

Solution: 053 (bhul hoise)

Quiz-2 setA

1. $M_1 = R^{-1}(45) * Ref * R(45)$

$$= \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} \sqrt{2}/2 & -\frac{\sqrt{2}}{2} \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \times \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$M_2 = R(90) * Ref\text{-}x$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix}$$

$$= \cancel{1\ 0} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$\therefore M_1 == M_2$

====================== correction:

$$M1 = R^{-1}(45) * Ref * R(45)$$

$$= \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$M2 = R(90) * Ref\text{-}X$$

$$= \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

So m1 == m2

**2. Consider a rectangle with vertices A(1,1), B(6,1), C(6,5) and D(1,5). Apply appropriate transformation to the rectangle to obtain a parallelogram so that point C and D move 10 units to it's right from the original position and the parallelogram is rotated along point A by 45 degree. You must -**

**a. Mention the steps.**

**b. Determine the composite transformation matrix.**

**c. Calculate and plot the final vertices** **[12]**

**2. Solution:** Added by Deb-065

2. Steps:
    a) Translate by (-1, -1)
    b) Shear along x by 2.5
    c) Rotate by 45 degree
    d) Undo step 1

P':

| | | | |
|---|---|---|---|
| 1.0 | 4.54 | 8.78 | 5.24 |
| 1.0 | 4.54 | 14.43 | 10.9 |
| 1.0 | 1.0 | 1.0 | 1.0 |

$$m = T^{-1} R(45)\, S\, T$$

$$m = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} 1 & 2.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 1.06 & -0.767 \\ 0.707 & 2.47 & -2.181 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cdots \end{bmatrix}$$



// ***Explanation: 024***
*For shear factor along X-axis we have to divide the shear value with Δy*
*For shear factor along Y-axis we have to divide the shear value with Δx*

*For this question, C and D move to the right means shear along the X-axis by 10 points. As shear by X-axis we need to calculate Δy. Here, Y_value_difference = Δy = |5 - 1| = 4*
*So, shear value for X-axis, Sh_x = 10/Δy = 10/4 = 2.5*
*//*

## *Set-B*

**1. Show that two successive reflections about either of the principle axis is equivalent to a single rotation about the coordinate origin. [8]**

**1. a. Solution:**

1. Transformation matrix for two successive reflections about either of the principle axis,

   M1 = Ref-Y * Ref-X

   Single rotation matrix

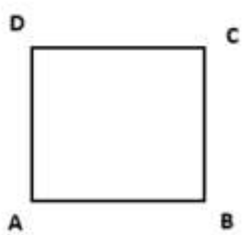
   M2 = R(180)

   You need to show that, M1 == M2

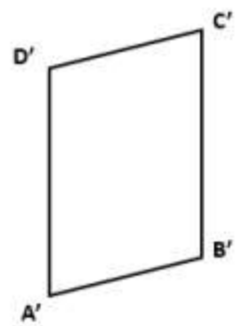
**2.**

**[12 marks]** Consider a square with vertices A(2, 2), B(6, 2), C(6, 6) and D(2, 6). Apply appropriate transformation to the square to obtain a parallelogram so that AD and BC edges are increased by 3 unit and B and C is shifted by 6 units upward. You must -

a. Mention the steps.
b. Determine the composite transformation matrix.
c. Calculate and plot the final vertices



(a) Original Shape



(b) Transformed Shape

**2. Solution:**

2. Steps:
   a) Translate by (-2,-2)
   b) Scale by (1, 1.75)
   c) Shear along y-axis by 1.5
   d) Undo step 1

```
P':
2.0     6.0     6.0     2.0
2.0     8.0     15.0    9.0
1.0     1.0     1.0     1.0
```

// ***Explanation: O24***
*For scaling factor along X-axis we have to divide the new scaled value with $\Delta x$*
*For scaling factor along Y-axis we have to divide the new scaled value with $\Delta y$*

*For this question, AD and BC edge increases by 3 units means Y-axis gets scaled up. As scale by Y-axis we need to calculate $\Delta y$. Here, Y_value_difference = $\Delta Y = |6 - 2| = 4$*
*Here after scaling the Y-axis value will be 4+3 = 7.*
*So, scaling factor for Y-axis, Sc_y = $7/\Delta y = 7/4 = 1.75$*
—
*For shear factor along X-axis we have to divide the shear value with $\Delta y$*
*For shear factor along Y-axis we have to divide the shear value with $\Delta x$*

*For this question, B and C shifted to the up means shear along the Y-axis by 6 points. As shear by Y-axis we need to calculate $\Delta x$. Here, X_value_difference = $\Delta x = |6 - 2| = 4$*
*So, shear value for Y-axis, Sh_y = $6/\Delta x = 6/4 = 1.5$*
//

## *Set-C*

1. **[8 marks]** Show that the composite transformation of two rotations $R(\theta_1)$ and $R(\theta_2)$ can be obtained with a single rotation of $R(\theta_1 + \theta_2)$.

**1. a. Solution:**

1. Transformation matrix of two rotations $R(\theta_1)$ and $R(\theta_2)$

$$M1 = R(\theta_1) * R(\theta_2)$$

Single rotation of $R(\theta_1 + \theta_2)$

$$M2 = R(\theta_1 + \theta_2)$$

You need to show that, M1 == M2

**2.**

2. **[12 marks]** Consider a rectangle with vertices A(10, 6), B(16, 6), C(16, 10) and D(10, 10). Reflect the rectangle along the line y - 3x - 4 = 0 using 2D transformation. You must -
   a. Mention the steps.
   b. Determine the composite transformation matrix.
   c. Calculate and plot the final vertices

**2. Solution:**

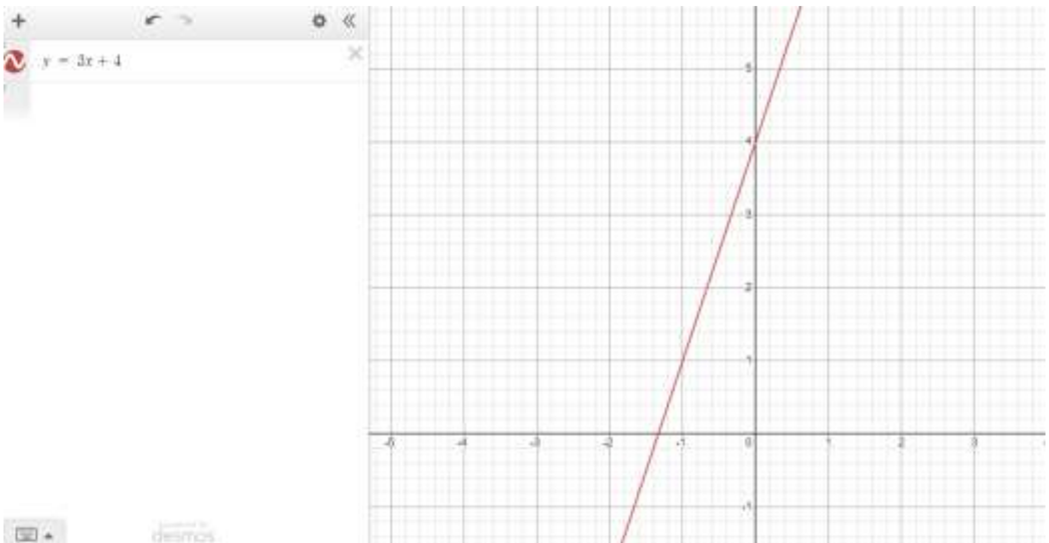
2. Steps:
    a) Translate by (0,-4)
    b) Rotate by 18.5 degree
    c) Reflect along y-axis
    d) Undo step 2
    e) Undo step 1

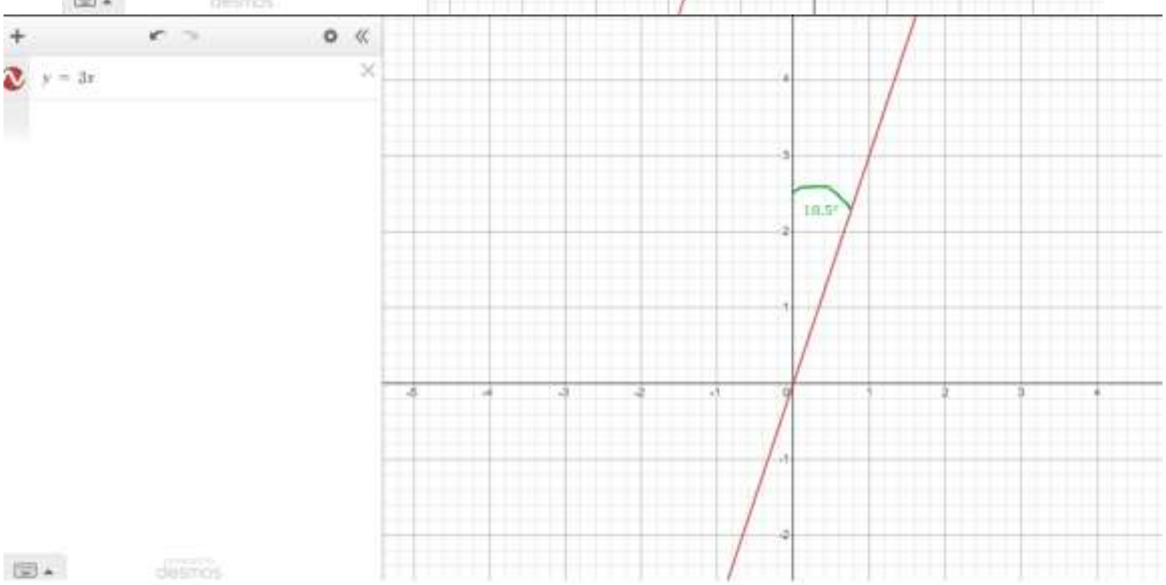
| | | | |
|---|---|---|---|
| -6.78 | -11.57 | -9.17 | -4.38 |
| 11.62 | 15.23 | 18.42 | 14.81 |
| 1.0 | 1.0 | 1.0 | 1.0 |

**//Explanation: Rabab 039**

$y = 3x + 4$

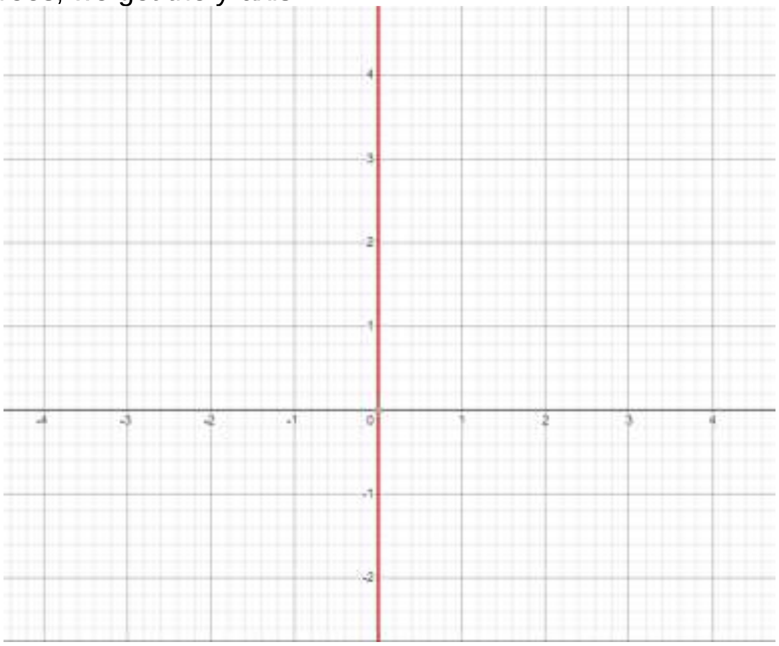
Here, the line is 4 units above the origin (on the y-intercept). So, by translating by (0,-4), we get $y = 3x$




Here, slope, $m = \tan\theta = y/x = 3$. So, angle, $\theta = \tan^{-1}(y/x) = \tan^{-1}(3) \approx 71.5$

So, rotation angle = 90 - 71.5 = 18.5
By rotating 18.5 degrees, we get the y-axis



Now, we reflect the rectangle along the y-axis, and then, we rotate by -18.5 degrees.
Finally, we translate it by (0,4) back to the original position.

## *Set-D*

1. **[8 marks]** Show that reflection about the y-axis is equivalent to a rotation about the y-axis in three-dimensional space.

**1. a. Solution:**

1. Matrix for reflection about the y-axis

M1 = Ref-Y

rotation about the y-axis by 180 in three-dimensional space.

M2 = R(180)

You need to show that, M1 == M2

# 3D REFLECTION

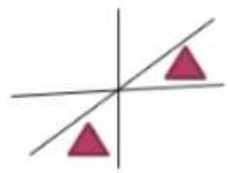
- The matrix for reflection about y-axis:-

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

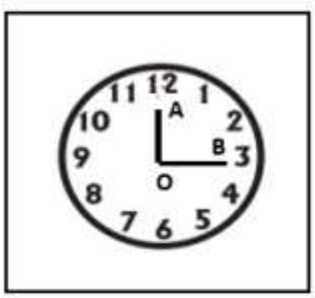
- Reflection about z-axis:-

**x'=-x  y'=-y  z'=z**

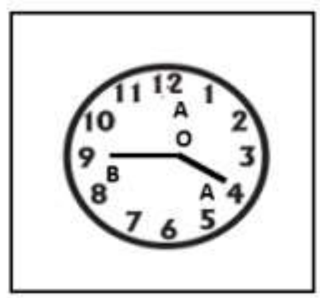
$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$



2. **[12 marks]** Suppose, a clock has an hour and a minute hand denoted by OA and OB respectively. Initially, the clock shows a time of 12:15 pm. Apply appropriate transformations so that the time of the clock gets changed to 4:45 pm. Initial vertices of the hour and minute hands are O(8, 8), A(8, 14) and B(16, 8). You must -
   a. Mention the steps.
   b. Determine the composite transformation matrix.
   c. Calculate and plot the final vertices




(a) Initial State




(b) Transformed State

**2. Solution:**

2. Each hour have 360/12 = 30 degree of angle in between.We need to add four hours. 30*4 = 120 degree clockwise rotation.

   Each minute have 360/60 = 6 degree of angle in between. need to add (45-15) = 30 minutes. 30*6 = 180 degree clockwise rotation.

| | |
|---|---|
| Steps for OA:<br>a) Translate by (-8,-8)<br>b) Rotate by -120 degree<br>c) Translate by (8, 8)<br><br>A' = [13.2, 5, 1] | Steps for OB:<br>a) Translate by (-8,-8)<br>b) Rotate by -180 degree<br>c) Translate by (8, 8)<br><br>B' = [0, 8, 1] |

## *Origin42*

**3.** **Lecture - 04**

a) Consider a rectangle with vertices A(1, 1), B(6, 1), C(6, 5) and D(1, 5). Apply appropriate transformation to the rectangle to obtain a parallelogram in such a way that point C and D move 4 units to its right from the original position and point A and B remain unchanged. [8]

**3. a. Solution: O24**

Steps:

1. Translate by (-1, -1)
2. Shear along X-axis by 1
3. Translate by (1, 1)

*// Explanation:*
*For shear factor along X-axis we have to divide the shear value with Δy*
*For shear factor along Y-axis we have to divide the shear value with Δx*

*For this question, C and D move to the right means shear along the X-axis by 4 points. As shear by X-axis we need to calculate Δy. Here, Y_value_difference = Δy = |5 - 1| = 4*
*So, shear value for X-axis, Sh_x = 4/Δy = 4/4 = 1*
*//*

M = T(1, 1) x Shear_X(1) x T(-1, -1) =

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

*// here signs between matrices are **x** (not dot) and multiplications are **cross product.***

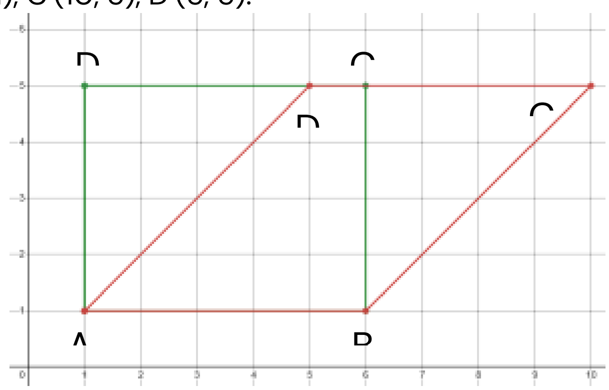
So, M x V =

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 6 & 10 & 5 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

So, points are A(1, 1), B(6, 1), C′(10, 5), D′(5, 5).



**2. Lecture -04**

**b)** What are the properties of affine transformation? Mention an example of non-affine transformation operation. **[4]**

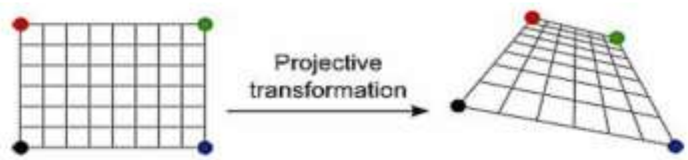
**2. b. Solution:**

Affine:

# Affine transformation (1/2)

- Maps points to points, lines to lines, planes to planes.
- Preserves the ratio of lengths of parallel line segments.
- Sets of parallel lines remain parallel.
- Does not necessarily preserve angles between lines or distances between points.


Non-Affine:




**5. Lecture -04**

Consider a square OACB with vertices O(3, 2), A(3, 6), C(7, 6) and B(7, 2). Reflect the square along a line x = -1 using 2D transformation. Determine the composite transformation matrix and find the final vertices. [7]

**5. b. Solution: 024**

Steps:

1. Translate by (1, 0) // to align x=-1 line with Y-axis we have to move it to right by 1 point
2. Reflect by Y-axis // given x=-1 is parallel to Y-axis. So, reflect by Y-axis
3. Translate by (-1, 0) // undo step 1

M = T(-1, 0) x Reflect_Y x T(1, 0) =

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

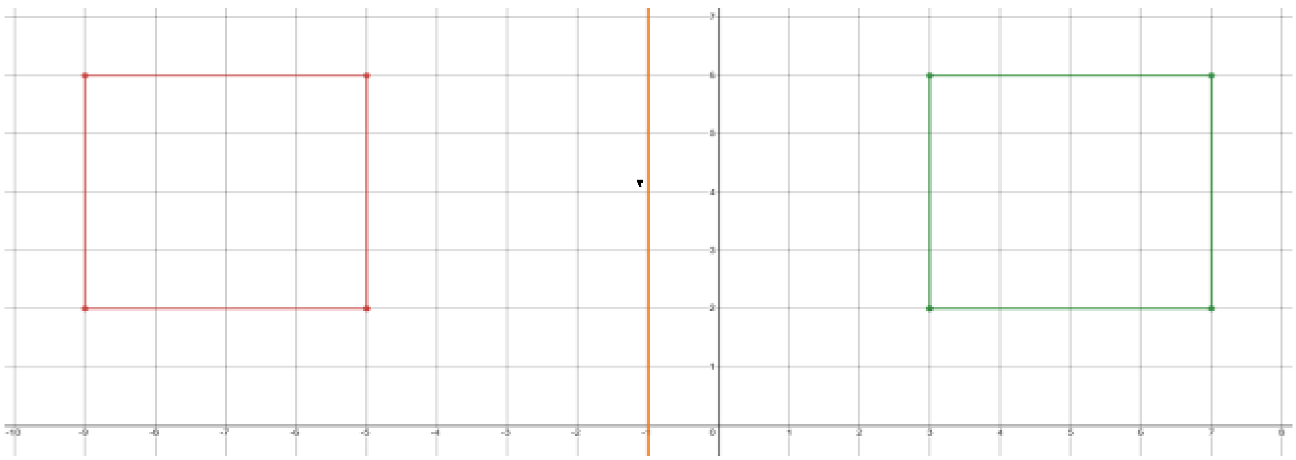
*// here signs between matrices are **x** (not dot) and multiplications are **cross product.***

So, M x V =

$$\begin{pmatrix} -1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 3 & 3 & 7 & 7 \\ 2 & 6 & 6 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -5 & -5 & -9 & -9 \\ 2 & 6 & 6 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

So, points are A′(−5, 2), B′(−5, 6), C′(−9, 6), D′(−9, 2).

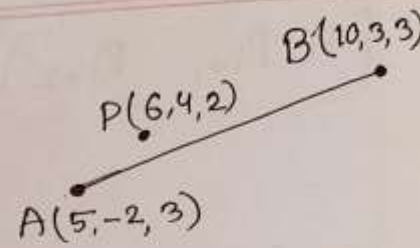


## Enigma41

**2.** **Lecture -04**

a) AB is a line and P is a point in 3D space; where the points A, B and P are (5,-2,3), (10,3,3) and (6,4,2) respectively. We want to rotate P along AB by -90°. Determine the composite transformation matrix to do the task and calculate the rotated point P'. **[10]**

**2. a. Solution: 024, Sohom bolse "Thikase"**




Steps

1. Translate by $(-5, 2, -3)$
2. Rotate along Z
3. Rotate along X
4. Rotate along Y
5. Rotate along X
6. Rotate along Z
7. Translate by $(5, -2, 3)$

Step-1

$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~Ue =~~

Step-2

$$u_e = \frac{B-A}{|B-A|} = C_x, C_y, C_z$$

$$C_x = \frac{10-5}{\sqrt{(10-5)^2 + (3+2)^2 + (3-3)^2}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$C_y = \frac{3-(-2)}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$C_z = \frac{3-3}{5\sqrt{2}} = 0$$

Step 5 (undo 3)

$$R\_x(\beta^{-1}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 6 (undo 2)

$$R\_z(\alpha^{-1}) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 7 (undo 1)

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ M = Step 7 matrix X ..... X Step 1 matrix

=

**Beta value ta negative hobe cause clockwise rotation**

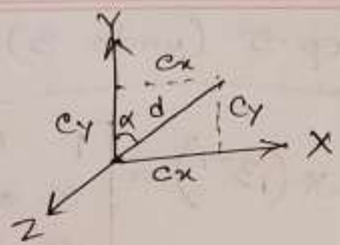
***Calculation incomplete***

**2.** **Lecture -04**

$$d = \sqrt{Cx^2 + Cy^2} = 1$$

$$\cos\alpha = \frac{Cy}{d} = \frac{1}{\sqrt{2}}$$

$$\sin\alpha = \frac{Cx}{d} = \frac{1}{\sqrt{2}}$$

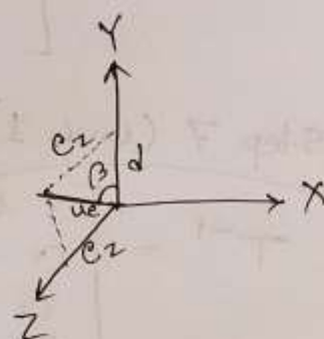



$$R\_z(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

<u>Step 3</u>

$$\cos\beta = \frac{d}{Ue} = \frac{1}{1} = 1$$

$$\sin\beta = \frac{Cz}{Ue} = 0$$




$$R\_x(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>Step 4</u> //it was asked to do

$$R\_Y = \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**b)** What are the properties of affine transformation? Mention an example of non-affine transformation operation. **[4]**

**2. b. Solution:** ***same as Origin 2(b)***

**7. Lecture -04**

**(a)** Consider a rectangle with vertices A(1,1), B(6,1), C(6,5) and D(1,5). Reflect the rectangle along the line $y = \frac{1}{\sqrt{3}}x - 3$ using 2D transformation. Determine the composite transformation matrix and find the final vertices. **[8]**

**7. a. Solution: almost** same as set-c (2b)

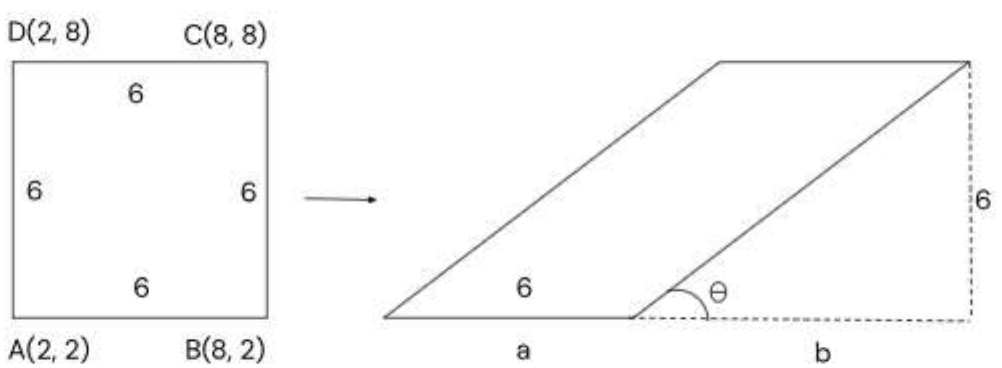
## Recursive40

**2.** a) $AB$ is a line and $P$ is a point in 3D space; where the points $A, B$ and $P$ are $(1,1,1)$, $(3,3,3)$ and $(2,2,4)$ respectively. We want to rotate a point $P$ with respect to $AB$ by $90°$. Determine the composite transformation matrix to perform the task. [12]

**2. a. Solution:** ch4

**3.** (b) Assume, $ABCD$ is a 2D rectangle and the vertices are $A(2,2)$, $B(8,2)$, $C(8,8)$, and $D(2,8)$. Apply shear to obtain $A'B'C'D'$ such that $A'D'$ and $B'C'$ both create 30 degree with $X$-axis after the transformation. Design the steps to perform the task and determine the composite transformation matrix. Plot $A'B'C'D'$. [8]

**3. b. Solution: 024 (shear-x er khetre angle with Y axis consider korte hoy i think, as per book, Please Check)** *reply: book page no please? as said "30 degree with X-axis" so I did that.*




Here, theta = 30 degree (given)

tan(30) = 6 / b => b = 10.392

So, to shear by X-axis for 10.392 point, Shear factor = 10.392 / 6 = 1.732

Steps for transformation:

1. Translate by (-2, -2)
2. Shear along X-axis by 1.732
3. Translate by (2, 2)

M = T(2, 2) x Shear_X(1.732) x T(-2, -2) =

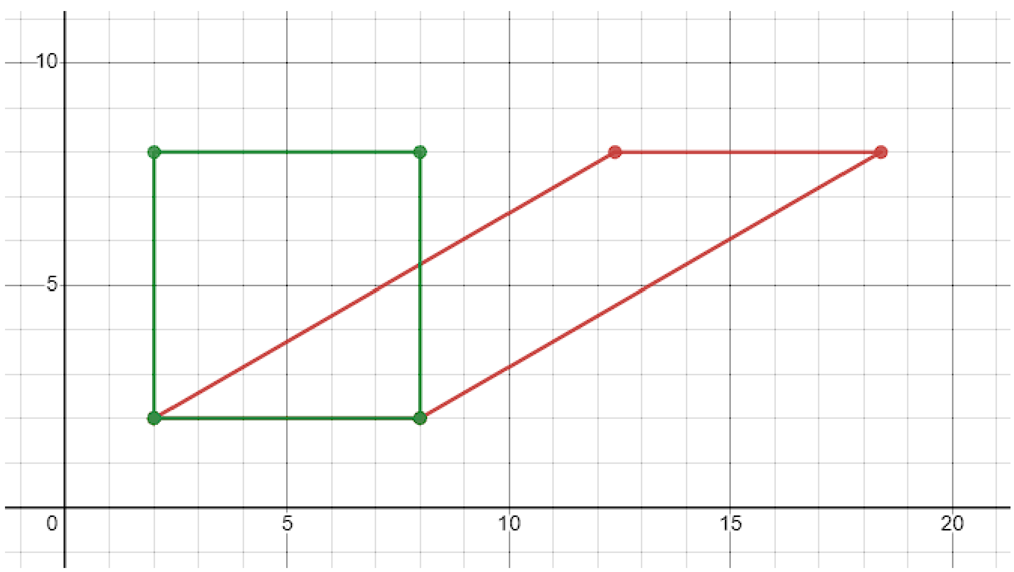
$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 1.732 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So, M x V =

$$\begin{pmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 2 & 8 & 8 & 2 \\ 2 & 2 & 8 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2. & 8 & 18.392 & 12.392 \\ 2 & 2 & 8 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$



**4.**

(b) Suppose we want to reflect a 2D point $P(4, 5)$ against a line that goes through $(-1, -3)$ and $(3, 2)$. Determine the composite transformation to perform this task. What is the final position of $P$? **[6]**

**4. b. Solution: 024, Correction: 019, 076**

For (-1, -3) and (3, 2) line equation:

=> (x-x1) / (x1-x2) = (y-y1) / (y1-y2)

=> (x+1) / (-1-3) = (y+3) / (-3-2)

=> 5x - 4y - 7 = 0

=> y = (5/4)x + (-7/4)     // y=mx+c

Steps for transformation:

1. Translate by (0, +7/4)     // as the equation cuts Y-axis through (-7/4)
2. Rotate by 38.66  // m = tan(theta) = 5/4 => theta = 51.34 with X-axis. So with Y: (90-51.34)
3. Reflect by Y-axis     // after rotation the line became parallel to Y-axis
4. Rotate by -38.66     // undo step 2
5. Translate by (0, -7/4)     // undo step 1

M =

M x V =

***Calculations incomplete***