CSE4203: Computer Graphics

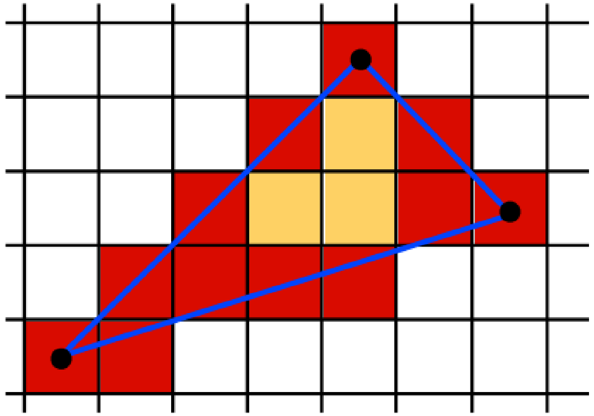
Lecture – 6 (part - C)

Graphics Pipeline

Outline

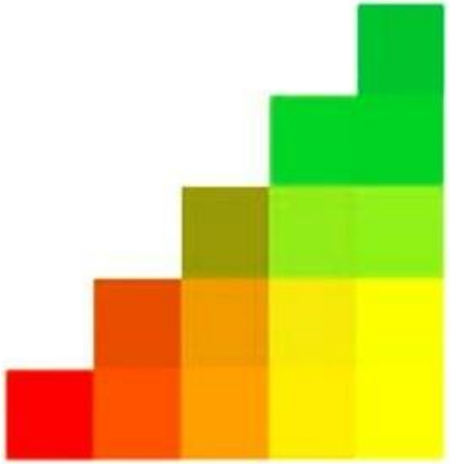
- Barycentric Interpolation
- Rasterizing a triangle

Triangle Rasterization

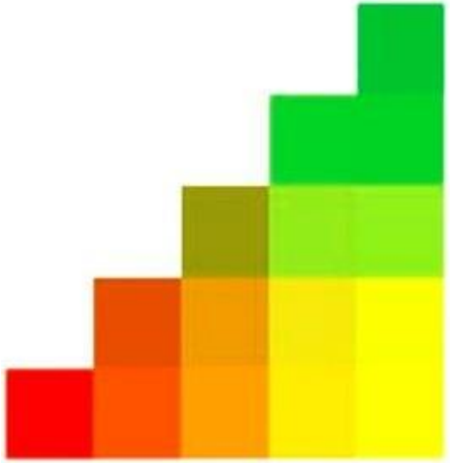


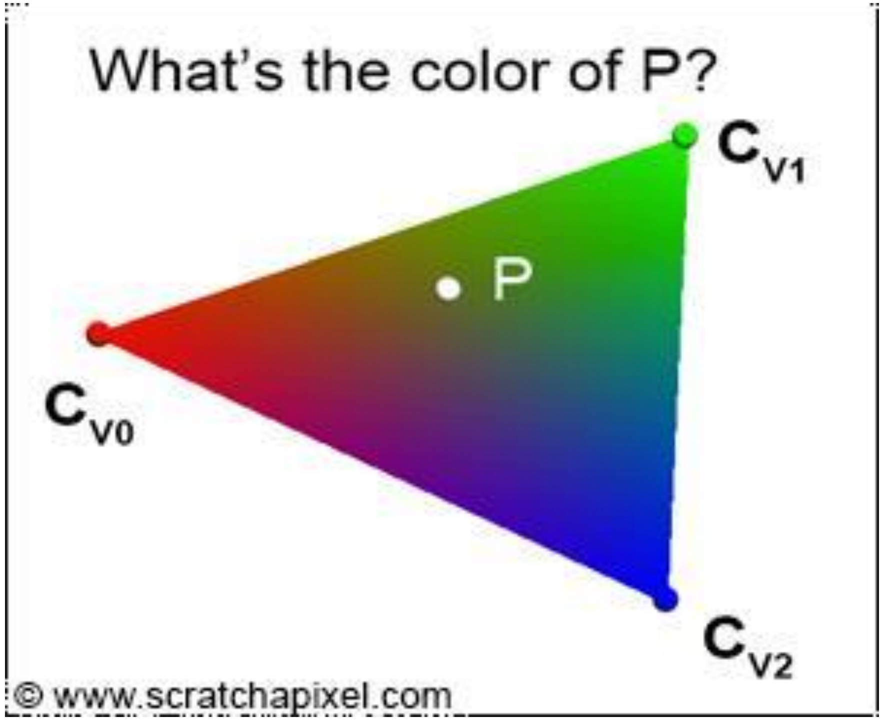
Use Midpoint Algorithm for edges and fill in?

Triangle Rasterization

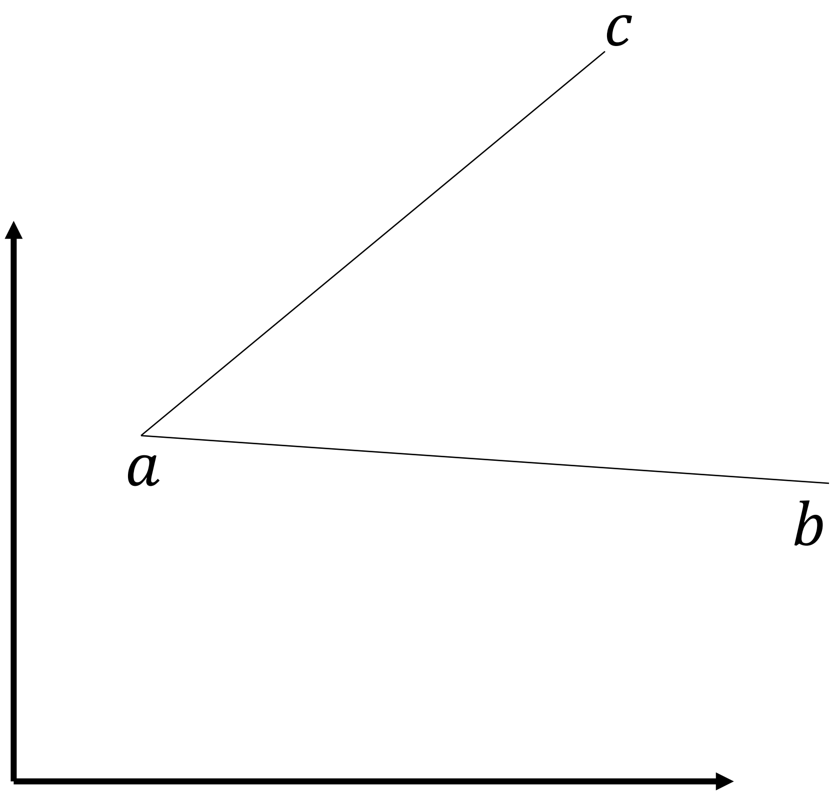


Triangle Rasterization





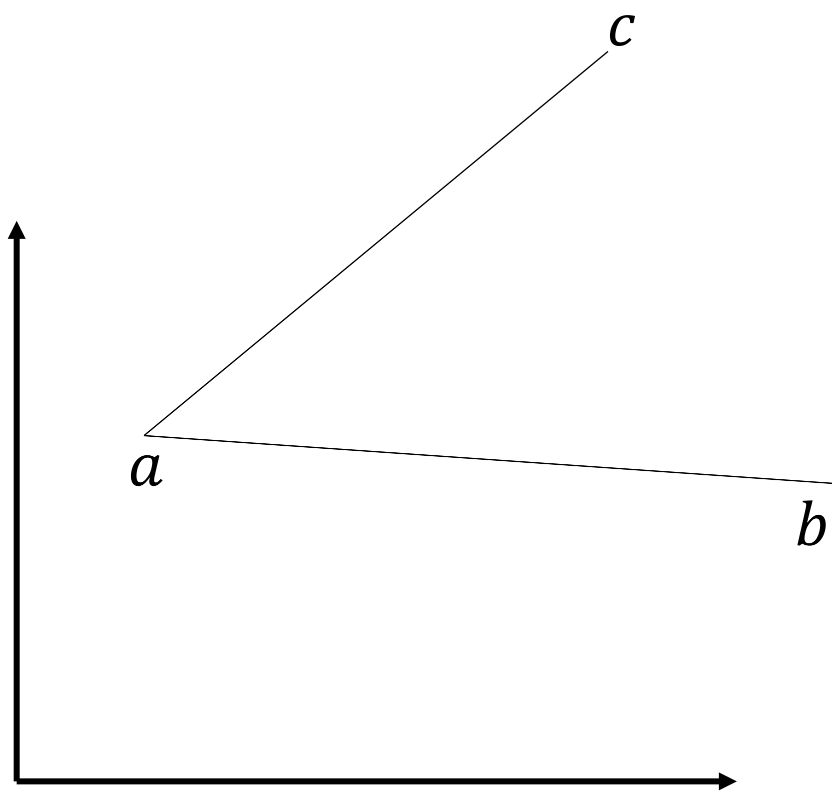
Barycentric Coordinate



$$P(x, y) = \boldsymbol{O} + x\boldsymbol{X} + y\boldsymbol{Y}$$

$$P(\beta, \gamma) = a + \beta\ ? + \gamma\ ?$$

Barycentric Coordinate

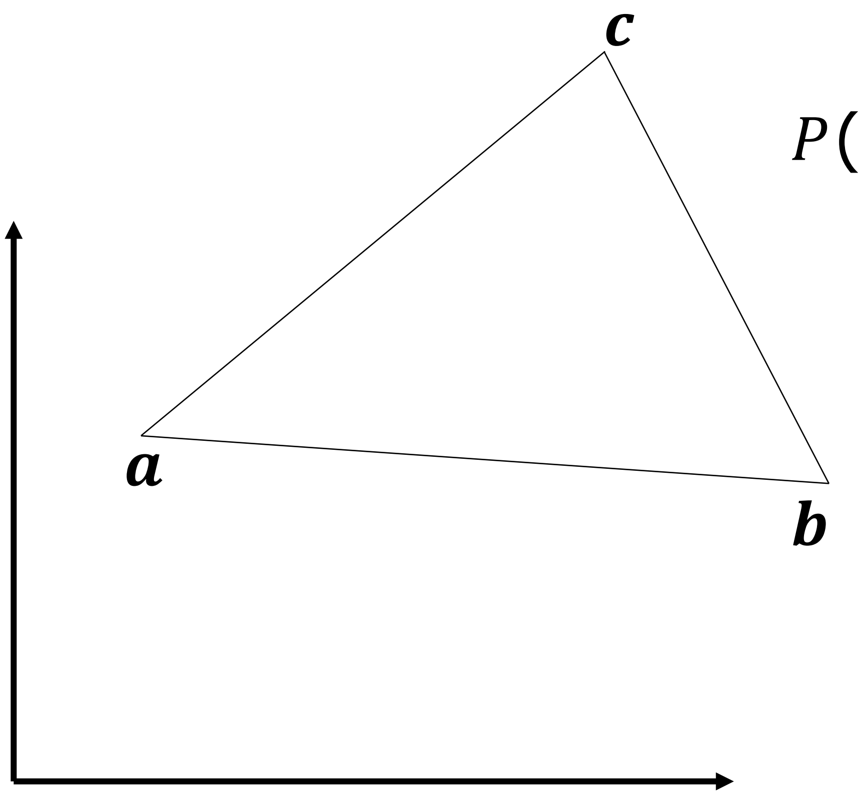


$$P(x, y) = \boldsymbol{O} + x\boldsymbol{X} + y\boldsymbol{Y}$$

$$P(\beta, \gamma) = a + \beta \; ? + \gamma \; ?$$

$$= a + \beta\,(b - a) + \gamma\,(c - a)$$
$$= a + \beta b - \beta a + \gamma c - \gamma a$$
$$= a + \beta b - \beta a + \gamma c - \gamma a$$

Barycentric Coordinate



$$P(x, y) = \boldsymbol{O} + x\boldsymbol{X} + y\boldsymbol{Y}$$

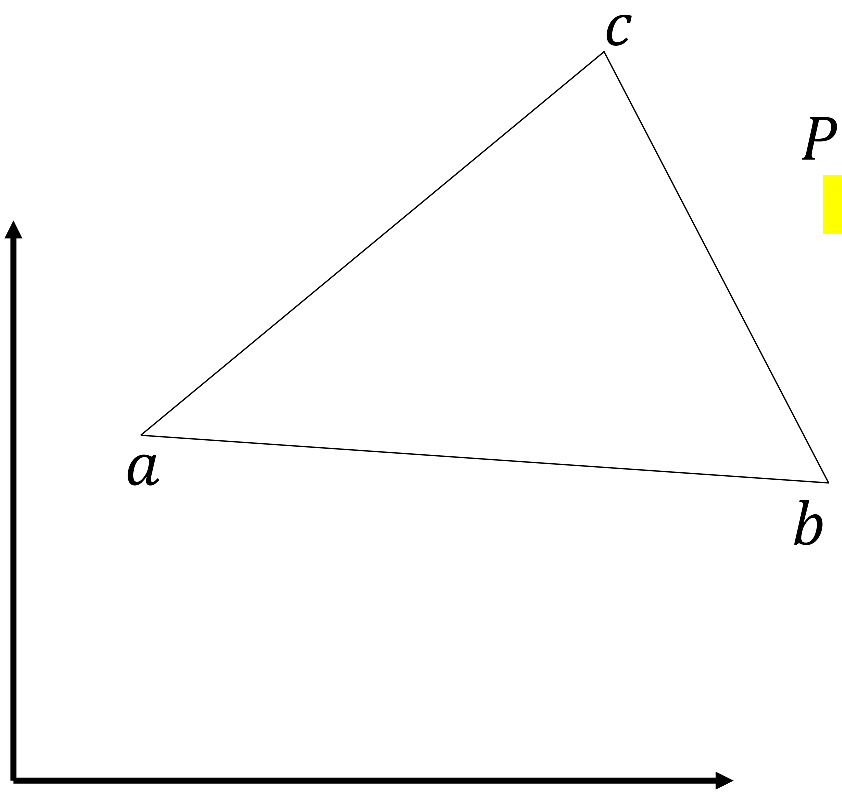
$$P(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) = a + \beta\ ? + \gamma\ ?$$

$$= a + \beta\,(b - a) + \gamma\,(c - a)$$
$$= a + \beta b - \beta a + \gamma c - \gamma a$$
$$= a + \beta b - \beta a + \gamma c - \gamma a$$
$$= a(1 - \beta - \gamma) + b\beta + c\gamma$$
$$= \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{c}$$

$$\boxed{\begin{array}{l} \alpha = (1 - \beta - \gamma) \\ \Rightarrow \alpha + \beta + \gamma = 1 \end{array}}$$

In a barycentric coordinate system: *location of a point is specified by reference to a triangle for points in a plane.*

Barycentric Coordinate



$$P(x, y) = \boldsymbol{O} + x\boldsymbol{X} + y\boldsymbol{Y}$$

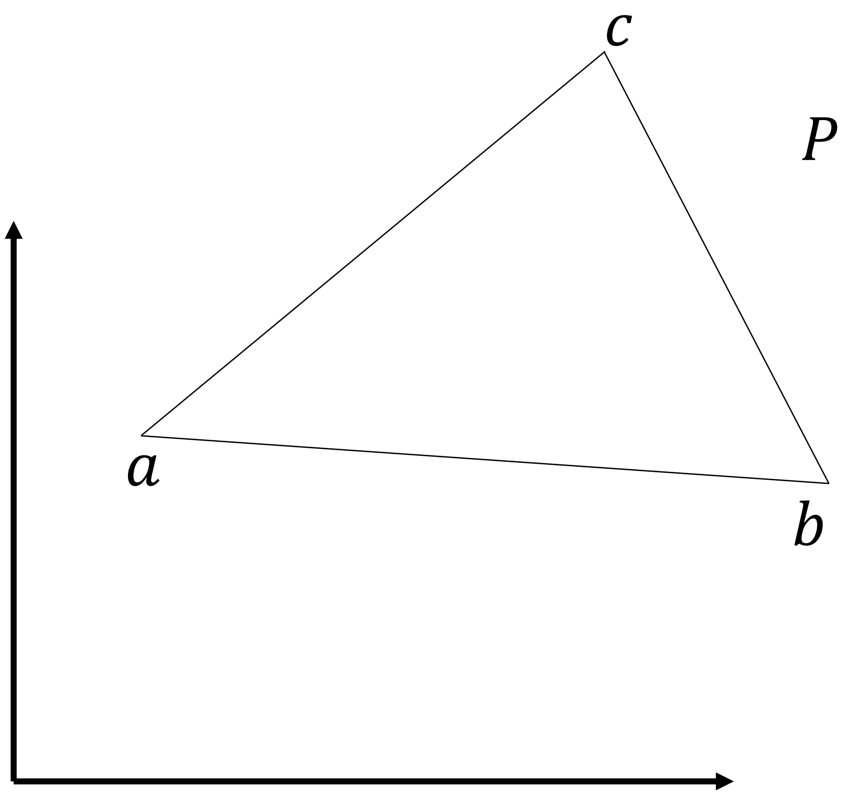
$$P(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) = \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{c}$$

Barycentric

$$\alpha + \beta + \gamma = 1$$

$$0 < \alpha < 1,$$
$$0 < \beta < 1,$$
$$0 < \gamma < 1.$$

Barycentric Coordinate



$$P(x, y) = \boldsymbol{O} + x\boldsymbol{X} + y\boldsymbol{Y}$$

$$P(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) = \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{c}$$

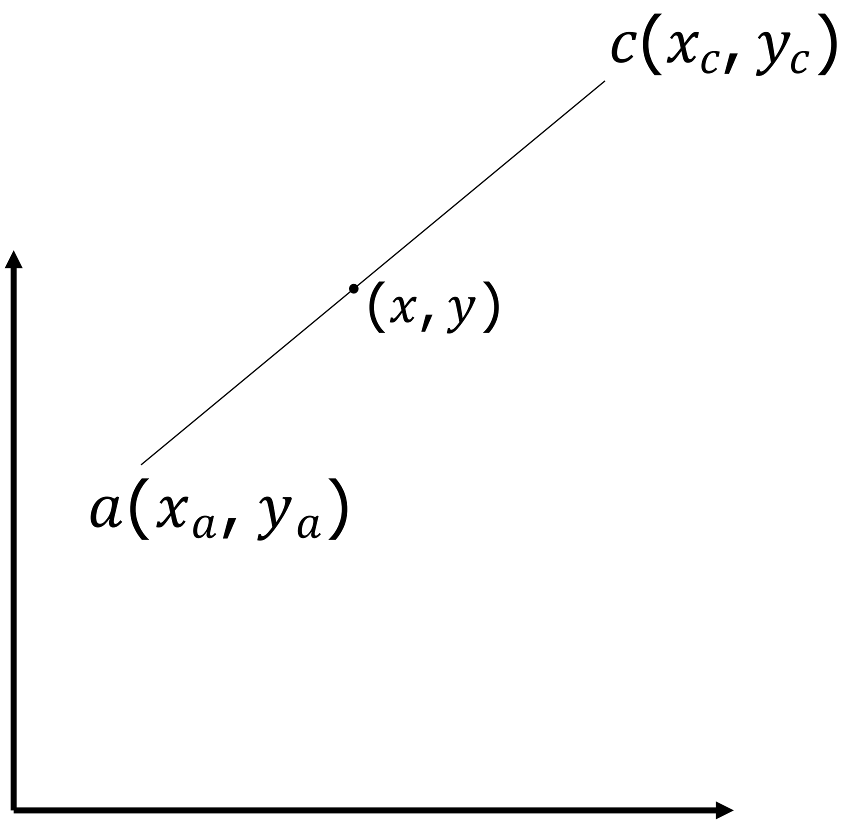
$$\alpha + \beta + \gamma = 1$$

$$0 < \alpha < 1,$$
$$0 < \beta < 1,$$
$$0 < \gamma < 1.$$

Cartesian → Barycentric

$$P(x, y) \to P(\alpha, \beta, \gamma)$$

Cartesian → Barycentric



$$y = mx + b$$

$$\Rightarrow y - mx - b = 0$$

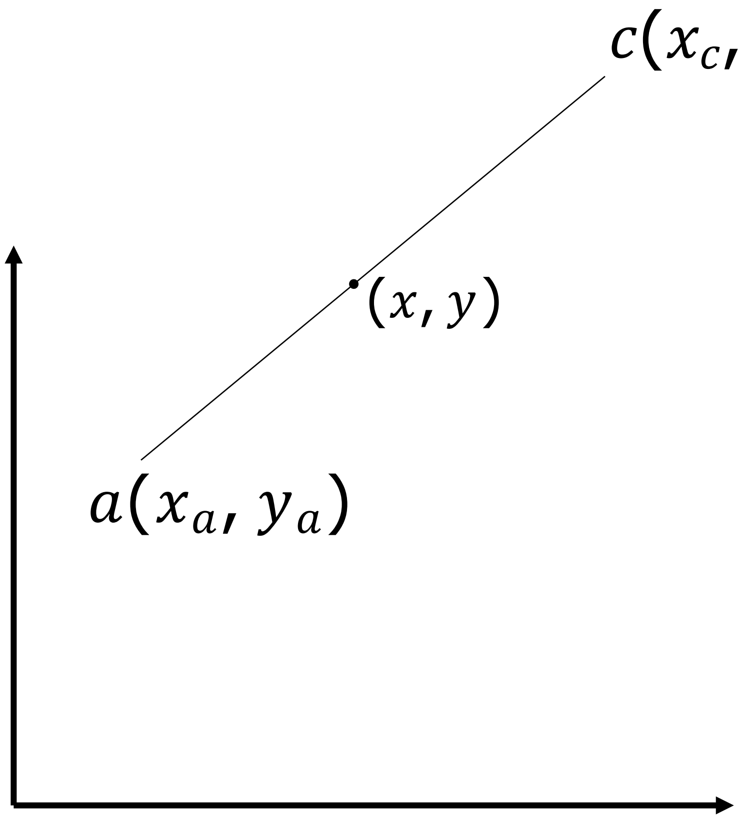
$$\Rightarrow y - \frac{y_c - y_a}{x_c - x_a} x - b = 0$$

$$\Rightarrow (x_c - x_a)y - (y_c - y_a)x - (x_c - x_a)b = 0$$

$$\Rightarrow (x_c - x_a)y - (y_c - y_a)x - C = 0$$

$$\Rightarrow (y_a - y_c)x + (x_c - x_a)y - C = 0$$

Cartesian → Barycentric

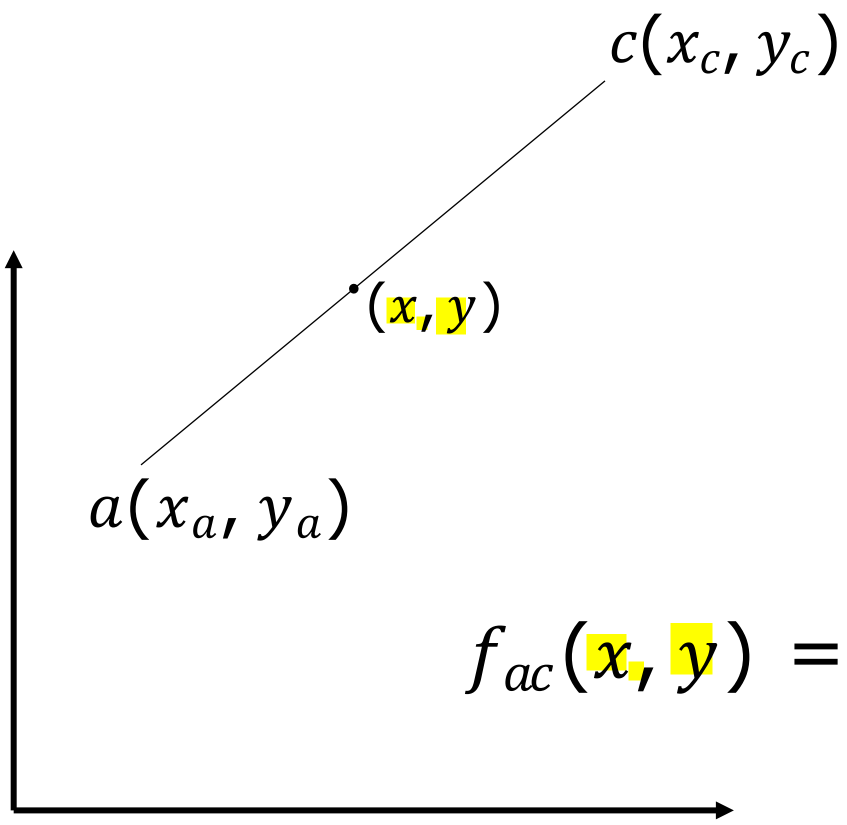


$$C = (y_a - y_c)x + (x_c - x_a)y$$

$$= (y_a - y_c)\mathbf{x_a} + (x_c - x_a)\mathbf{y_a}$$

$$= x_c y_a - x_a y_c$$

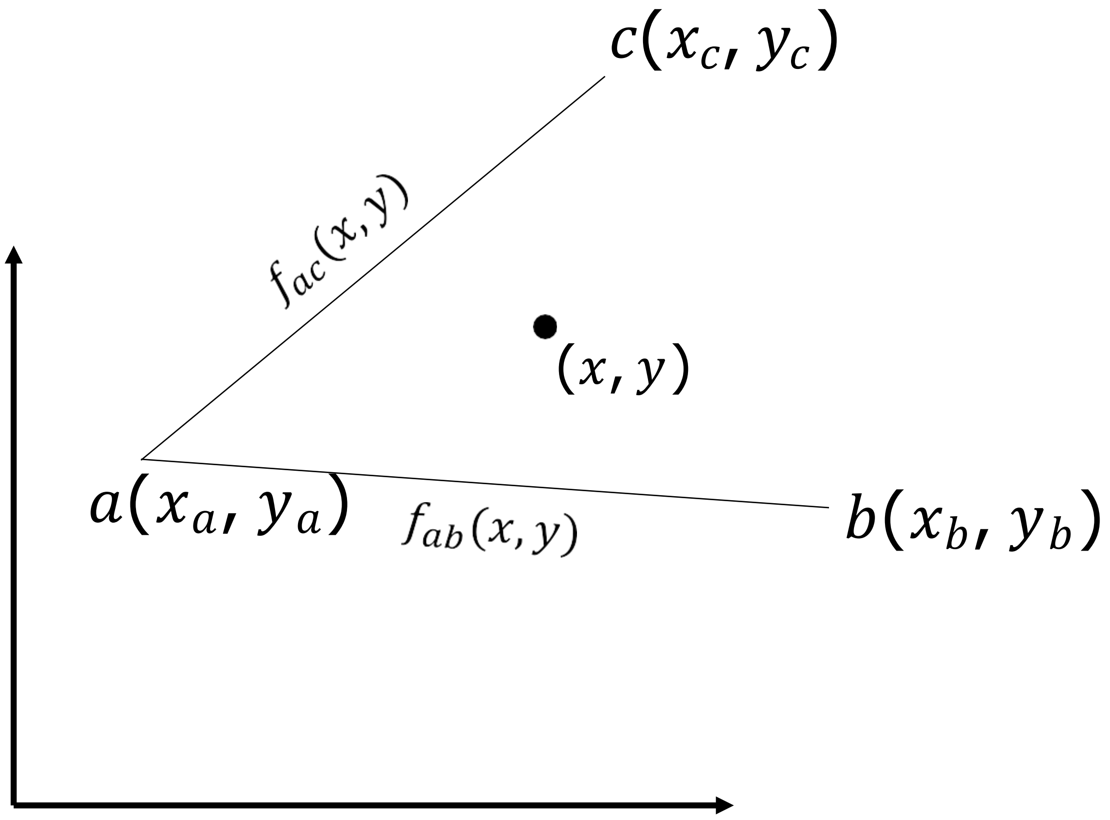
Cartesian → Barycentric



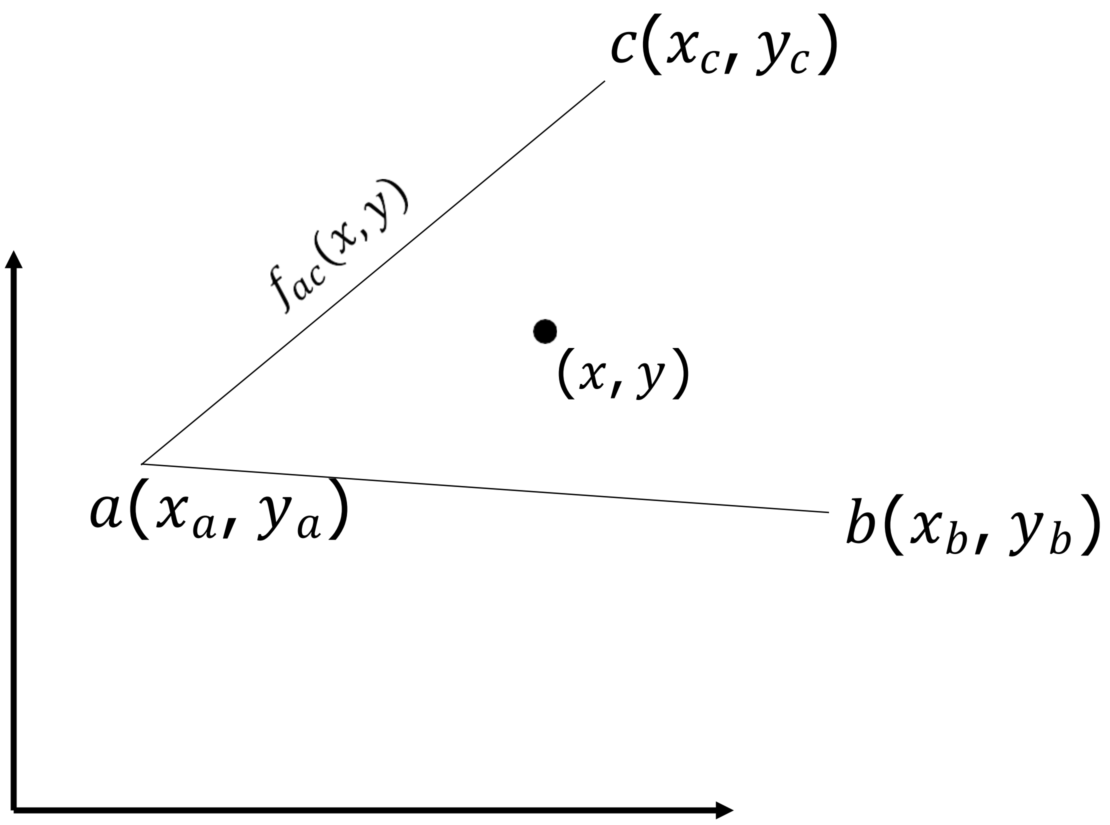
$$
\begin{aligned}
C &= (y_a - y_c)x + (x_c - x_a)y \\
&= (y_a - y_c)x_a + (x_c - x_a)y_a \\
&= x_c y_a - x_a y_c
\end{aligned}
$$

$$f_{ac}(x, y) = (y_a - y_c)x + (x_c - x_a)y - x_c y_a + x_a y_c = 0$$

Cartesian → Barycentric

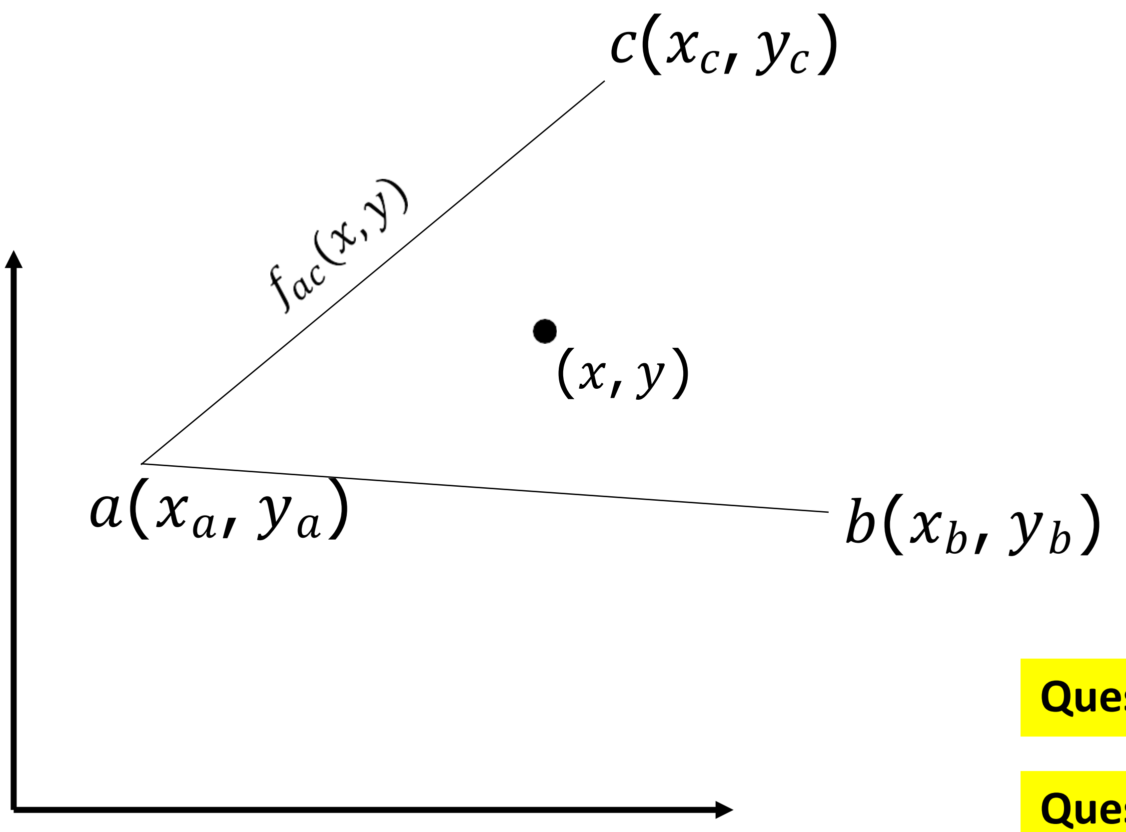


Cartesian → Barycentric



$$\beta = \frac{f_{ac}(x, y)}{f_{ac}(x_b, y_b)}$$

Cartesian → Barycentric



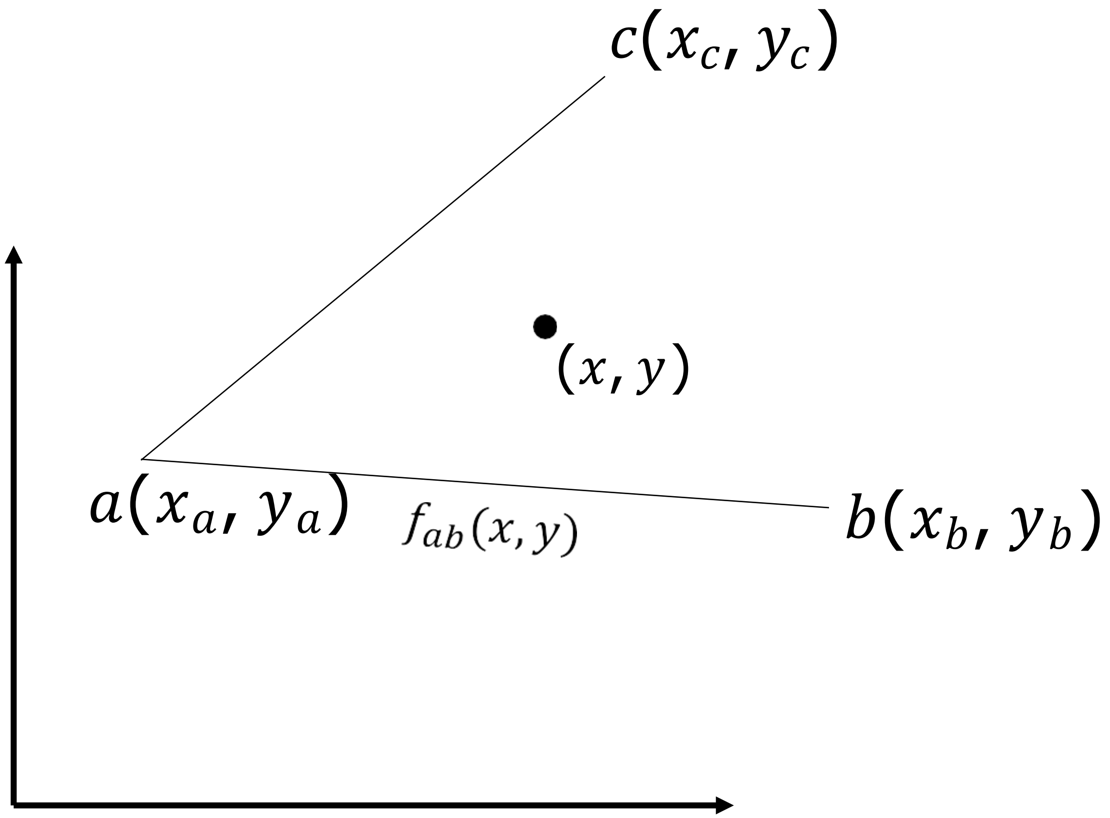
$$\beta = \frac{f_{ac}(x,y)}{f_{ac}(x_b,y_b)}$$

$$= \frac{(y_a - y_c)x + (x_c - x_a)y + x_a y_c - x_c y_a}{(y_a - y_c)x_b + (x_c - x_a)y_b + x_a y_c - x_c y_a},$$

Question – 1: In which case β becomes 1?

Question – 2: What will happen when *(x,y)* lies on *f_{ab}(x,y)*

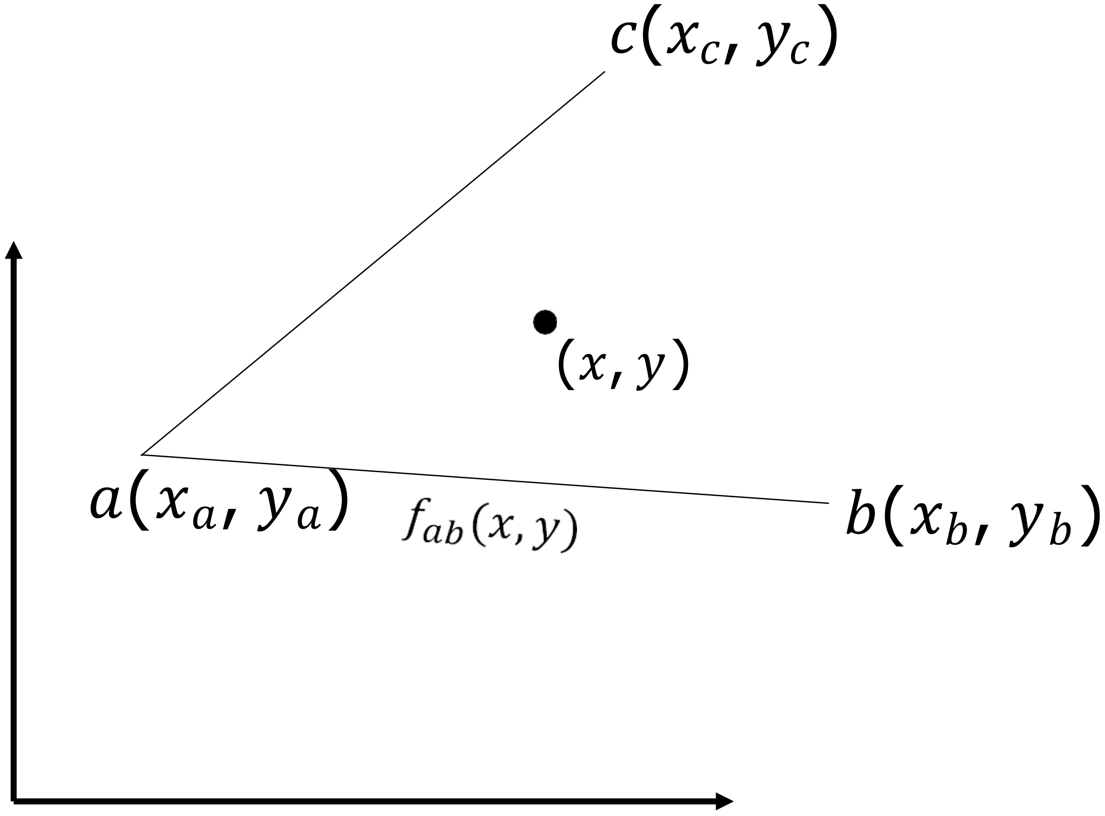
Cartesian → Barycentric



$$\beta = \frac{f_{ac}(x, y)}{f_{ac}(x_b, y_b)}$$

$$\gamma = \frac{f_{ab}(x, y)}{f_{ab}(x_c, y_c)}$$

Cartesian → Barycentric

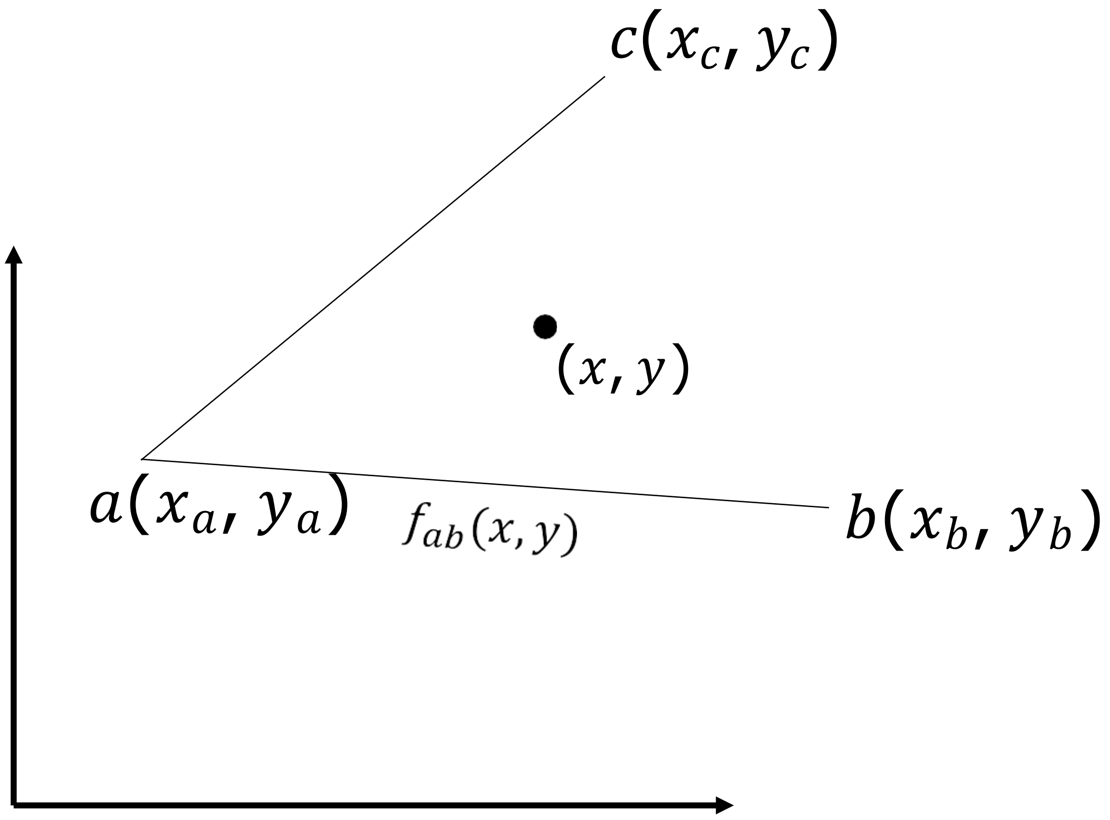


$$\beta = \frac{f_{ac}(x, y)}{f_{ac}(x_b, y_b)}$$

$$\gamma = \frac{f_{ab}(x, y)}{f_{ab}(x_c, y_c)}$$

$$\alpha = 1 - \beta - \gamma$$

Cartesian → Barycentric



$P(x, y) \rightarrow P(\alpha, \beta, \gamma)$

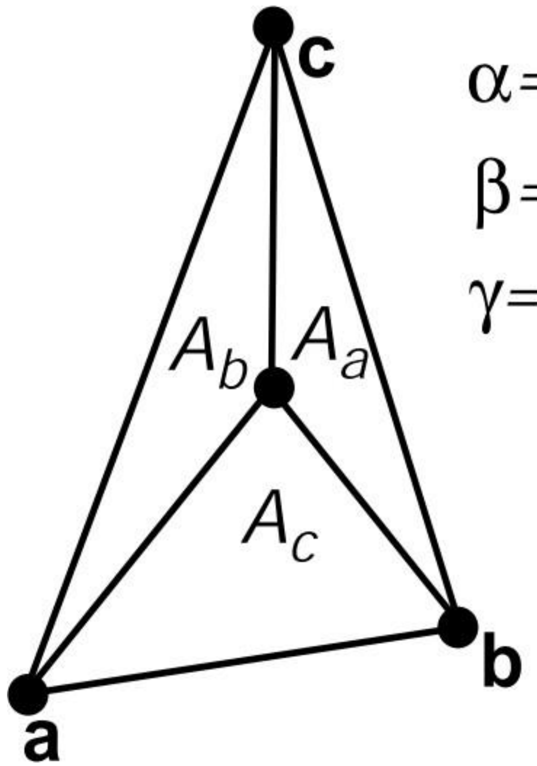
$$\beta = \frac{f_{ac}(x, y)}{f_{ac}(x_b, y_b)}$$

$$\gamma = \frac{f_{ab}(x, y)}{f_{ab}(x_c, y_c)}$$

$$\alpha = 1 - \beta - \gamma$$

Barycentric Coordinate

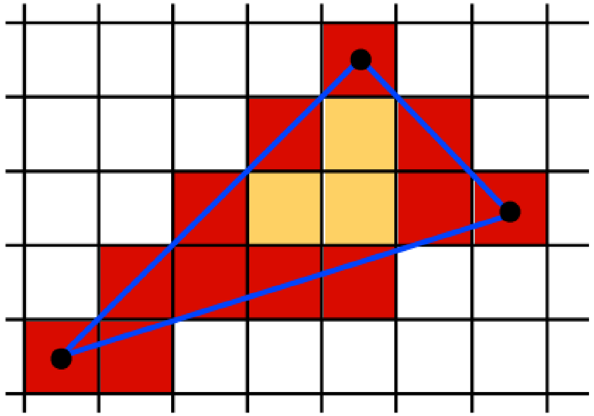
<u>Another approach:</u>



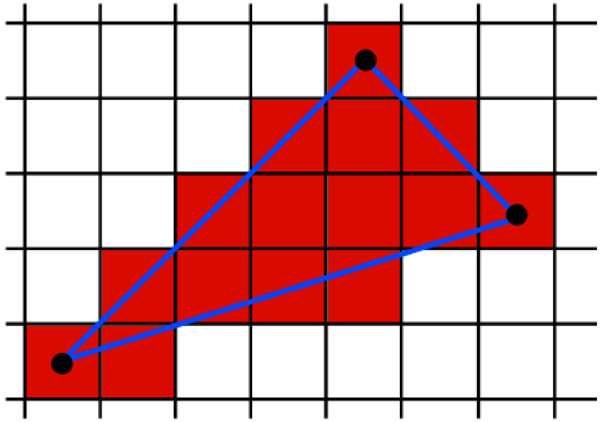
$$\alpha = A_a / A$$
$$\beta = A_b / A$$
$$\gamma = A_c / A$$

Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (1/7)



Use Midpoint Algorithm for edges and fill in?



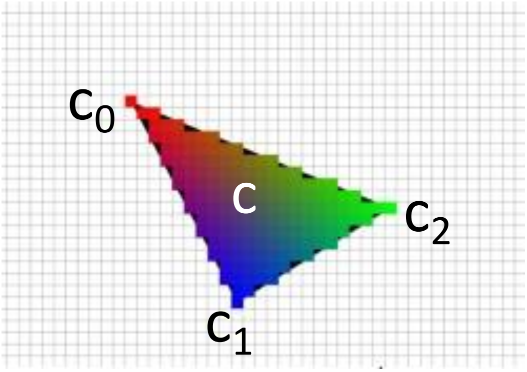
Use an approach based on barycentric coordinates

Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (2/7)

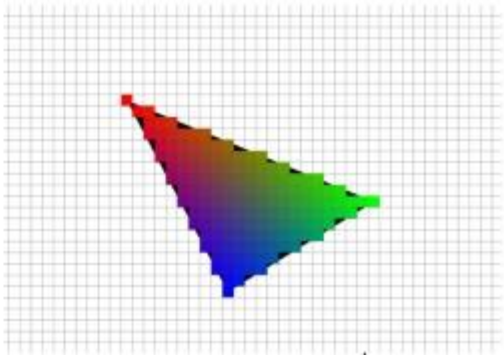
- If the vertices have colors c_0, c_1, and c_2, the color at a point in the triangle with *Barycentric coordinates* (α, β, γ) is:

$$\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$$



  - This type of interpolation of color is known in graphics as ***Gouraud interpolation***

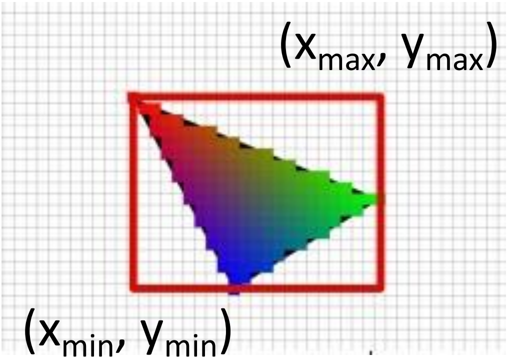
Triangle Rasterization (3/7)



for all x **do**
  for all y **do**
    compute (α, β, γ) for (x, y)
    if $(\alpha \in [0,1]$ and $\beta \in [0,1]$ and $\gamma \in [0,1])$ **then**
      $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$
      drawpixel (x, y) with color $\mathbf{c}$

Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (4/7)



for $y = y_{\min}$ to $y_{\max}$ **do**
  for $x = x_{\min}$ to $x_{\max}$ **do**

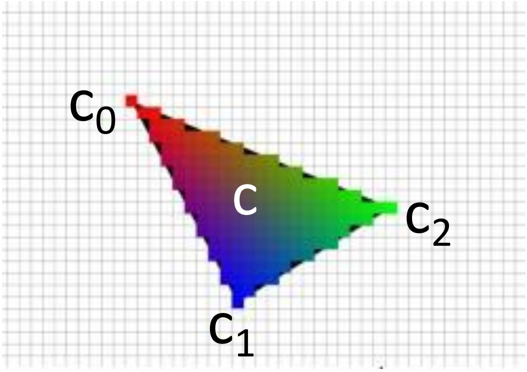
    compute (α, β, γ) for (x, y)

    if $(\alpha > 0$ and $\beta > 0$ and $\gamma > 0)$ **then**
      $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$
      drawpixel (x, y) with color $\mathbf{c}$

Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (5/7)

for $y = y_{\text{min}}$ to y_{max} **do**
  for $x = x_{\text{min}}$ to x_{max} **do**
    $\alpha = f_{12}(x, y)/f_{12}(x_0, y_0)$
    $\beta = f_{20}(x, y)/f_{20}(x_1, y_1)$
    $\gamma = f_{01}(x, y)/f_{01}(x_2, y_2)$
    if $(\alpha > 0$ and $\beta > 0$ and $\gamma > 0)$ **then**
      $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$
      drawpixel (x, y) with color $\mathbf{c}$

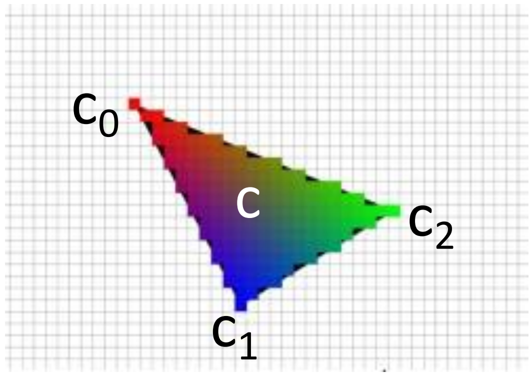


Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (6/7)

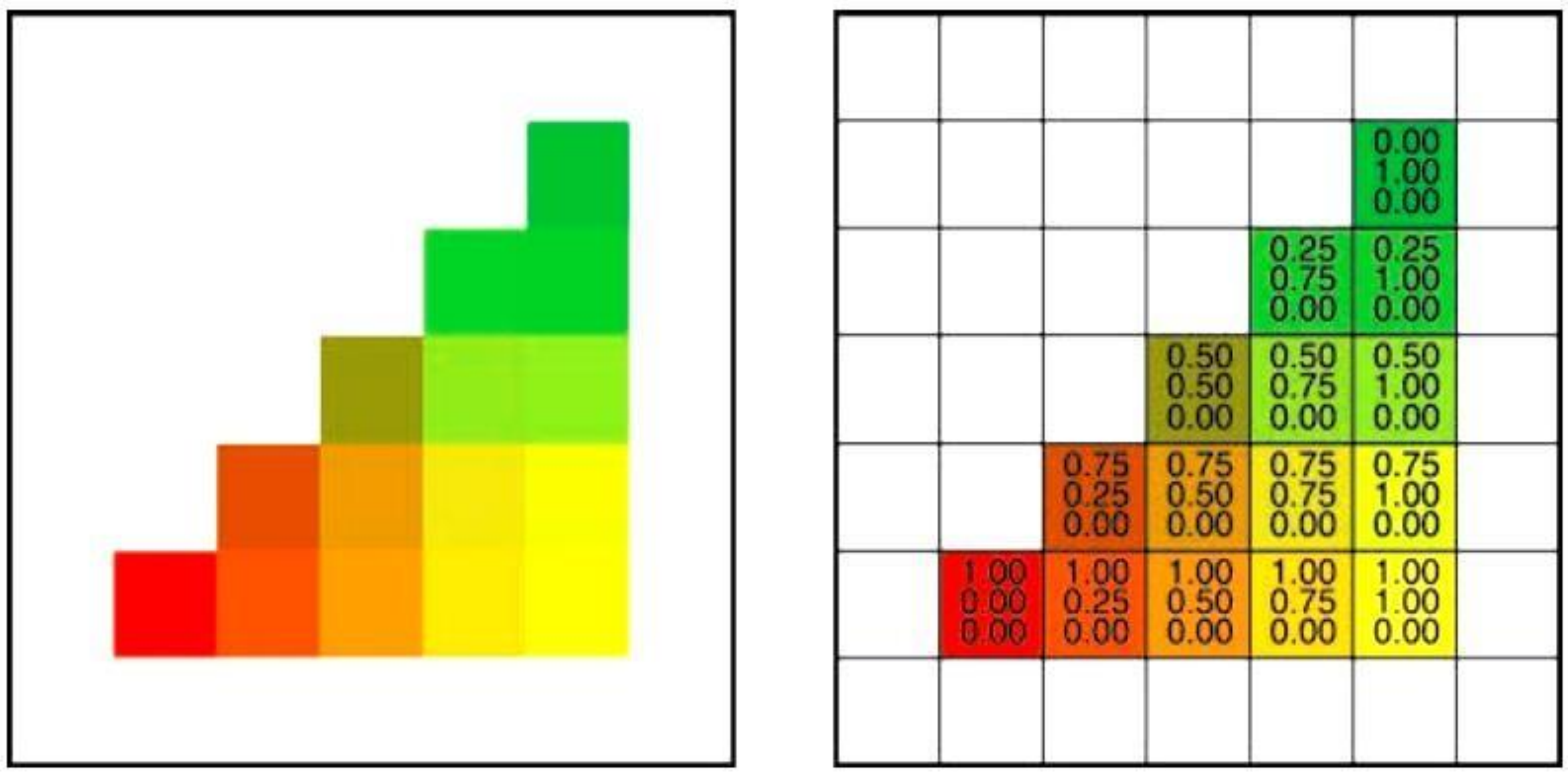
$$f_{01}(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0,$$
$$f_{12}(x,y) = (y_1 - y_2)x + (x_2 - x_1)y + x_1y_2 - x_2y_1,$$
$$f_{20}(x,y) = (y_2 - y_0)x + (x_0 - x_2)y + x_2y_0 - x_0y_2.$$

for $y = y_{\min}$ to $y_{\max}$ **do**
  for $x = x_{\min}$ to $x_{\max}$ **do**
    $\alpha = f_{12}(x,y)/f_{12}(x_0,y_0)$
    $\beta = f_{20}(x,y)/f_{20}(x_1,y_1)$
    $\gamma = f_{01}(x,y)/f_{01}(x_2,y_2)$
    if $(\alpha > 0$ and $\beta > 0$ and $\gamma > 0)$ **then**
      $\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$
      drawpixel (x,y) with color **c**



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (7/7)



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Practice Problem

- Take three vertices of a triangle, choose two points, P and Q , such that they stay inside and outside the triangle respectively.
  - Apply barycentric interpolation and verify that P lies inside and Q lies outside the triangle.

Further Reading

- Fundamentals of Computer Graphics, 4th Edition - Chapter 8