Slide Examples Solutions

Lecture 4,5,6,7,8

Lecture 4

Example- 1

- If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available.

Solution: ***Added from the Lecture Slide***

Total bandwidth = 33 MHz

Channel bandwidth = 25 kHz x 2 simplex channels
= 50 kHz/duplex channel

Total available channels = 33,000/50 = 660 channels

Example- 2

- If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if system uses 4 cell reuse.

Solution: ***Added from the Lecture Slide***

Total bandwidth = 33 MHz
Channel bandwidth = 25 kHz x 2 simplex channels
= 50 kHz/duplex channel

Total available channels, S = 33,000/50 = 660 channels.
Number of cell in a cluster, N = 4.
Number of channel per cell = K
S = KN, K = S/N = 660/4 = 165 channels

Problem 1:

- Assume a system of 32 cells with a cell radius of 1.6 km, a total frequency bandwidth that supports 336 traffic channels, and a reuse factor of N = 7.
  - a) What geographic area is covered by the system?
  - b) How many channels are there per cell?
  - c) What is the capacity of the system?
  - d) Repeat for a cell radius of 0.8 km and 128 cells.

a. Solution: ***by Younus-131***

Area = $\frac{3\sqrt{3}}{2} \times 1.6^2 \times 32$ **= 212.8344 Km^2**

b. Solution:

No. of Traffic Channel per cell = 336 / 7 *[N=7]* **= 48** ***channels***

c. Solution:

Capacity = No. of Channel X No. of Cell = 48 X 32 = 1536

d. Solution:

Area = $\frac{3\sqrt{3}}{2} \times 0.8^2 \times 128$ **= 212.8344 Km^2**
No. of Traffic Channel per cell = 336 / 7 *[N=7]* **= 48** ***channels***
Capacity = No. of Channel X No. of Cell = 48 X 128 = 6144

Problem 2:

- If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular system, which uses two 25 KHz simplex channels to provide full duplex voice and control channels.
  - a) Compute the number of channels available per cell if system uses 4 cell reuse.
  - b) Assume that 1 MHz is dedicated to control channels but that only one control channel is needed per cell. Determine a reasonable distribution of control channels and voice channels in each cell.
  - c) Repeat the (a) and (b) for 7 reuse factor.

Solution:

problem-2

B.W = 33 MHz = 33000 KHz

C.T = 25 KHz x2 simplex channel

= 50 KHz ~~KHz~~ (duplex ")

a) no. of channels available = $\frac{33000}{50}$

= 660 channel

for N = 4

per cell, 4, " " = $\frac{660}{4}$

= 165 channels.

for N = 7

per cell 7, " " = $\frac{660}{7}$

= 95 "

for N = 12,

per cell 12, " " " = $\frac{660}{12}$ "

= 55 "

1 MHz → dedicated to control chan

one control channel needed per cell.

Distribution of control channel &
voice channel

⇒ 1 MHz = 1000 KHz

Available channel = $\frac{1000}{50}$

= 20 control ch

∴ 640 voice channel

20 control "

a) for N = 4

control channel = $\frac{20}{4} = 5$

voice " = $\frac{640}{4} = 160$

b) for N = 7

$C.C = \frac{20}{7} = 2.86$

$V.C = \frac{640}{7} = 91.43$

c) for N = 12

Lecture 5

Example-1

Let the speed of a mobile be v = 35 meters/sec. For n = 4, a cell radius of 500 meters (the distance at which the power is at the threshold), and a 2 second handoff, what Δ is needed?

Solution: ***Added from the Lecture Slide***

Assume the mobile is driving directly away from the BS, so distance d changes by 70 meters in two seconds. Consider the received power at the two times:

$$\text{Pr(minimum useable)} = P_0 - 10n \log d$$
$$\text{Pr(handoff)} = P_0 - 10n \log (d - 70)$$

Taking the difference of the two equations (the 2nd minus the 1st),

$\Delta = 10n \log d - 10n \log (d - 70) = 10n \log (d/d - 70)$

Plugging in that the call is dropped at d = 500 meters, we have

$\Delta = 40 \log 500/430 = 2.6$ dB.

Lecture 6

Solution: ***Added from the Lecture Slide***

Example: 1

You are trying to design a cellular network that will cover an area of at least 2800 km^2. There are K=300 available voice channels. Your design is required to support at least 100 concurrent calls in each cell. If the co-channel cell centre distance is required to be 9 km, how many base stations will you need in this network?

If 100 concurrent voice calls must be supported in each cell, each cell must be allocated 100 voice channels.

This necessitates the frequency re-use factor, N, to be 300/100=3.

The distance between co-channel cell centers D is related to R and N via the formula:

$$\frac{D}{R} = \sqrt{3 \cdot N}$$

D = 9 km, then, R = 3 km

cell area is $\frac{3 \cdot \sqrt{3}}{2} \cdot R^2 = 23.38$

2800/23.38 = 120 base stations are required

Example-2:

- If a SIR of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is (a) n = 4, (b) n = 3? Assume that there are six co-channel cells in the first tier, and all of them are at the same distance from the mobile. Use suitable approximation.

Solution: ***Added from the Lecture Slide***

(a) $n = 4$

First, let us consider a seven-cell reuse pattern.

Using Equation (3.4), the co-channel reuse ratio $D/R = 4.583$.

Using Equation (3.9), the signal-to-noise interference ratio is given by

$$S/I = (1/6) \times (4.583)^4 = 75.3 = 18.66 \text{ dB}$$

Since this is greater than the minimum required S/I, $N = 7$ can be used.

(b) $n = 3$

First, let us consider a seven-cell reuse pattern.

Using Equation (3.9), the signal-to-interference ratio is given by

$$S/I = (1/6) \times (4.583)^3 = 16.04 = 12.05 \text{ dB}$$

Since this is less than the minimum required S/I, we need to use a larger N.

Using Equation (3.3), the next possible value of N is 12, ($i = j = 2$).

The corresponding co-channel ratio is given by Equation (3.4) as

$$D/R = 6.0$$

Using Equation (3.3), the signal-to-interference ratio is given by

$$S/I = (1/6) \times (6)^3 = 36 = 15.56 \text{ dB}$$

Since this is greater than the minimum required S/I, $N = 12$ is used.

- Example-3:

- Suppose the subscriber is at a distance of 1000 m from the BS and another mobile which is using an adjacent channel is unfortunately at a distance of only 100 m from the base station.

Solution:

S/I = $(1000/100)^{-4}$ = 0.0001

S/I in dB = 10log(0.0001) = -40 dB

$$\frac{S}{I} = -40\text{dB}$$

Problem 1:

Consider two different cellular systems that share the following characteristics. The frequency bands are 825 to 845 MHz for mobile unit transmission and 870 to 890 MHz for base station transmission. A duplex circuit consists of one 30-kHz channel in each direction. The systems are distinguished by the reuse factor, which is 4, and 19, respectively.

Suppose that in each of the systems, the cluster of cells (4, 19) is duplicated 16 times. Find the number of simultaneous communications that can be supported by each system.

Find the number of simultaneous communications that can be supported by a single cell in each system.

What is the area covered, in cells, by each system?

Suppose the cell size is the same in all two systems and a fixed area of 100 cells is covered by each system. Find the number of simultaneous communications that can be supported by each system.

Solution: added by Deb 065

Channel = mobile unit transmission + Base unit tran.

= Reverse + forward

$= ((845-825) + (890-870))$ MHz

$= (20+20) = 40$ MHz (Total Bandwidth)

Duplex channel $= (30 \times 2)$ kHz $= 60$ kHz

$\therefore$ Num of channel $= \frac{40 \times 10^6}{60 \times 10^3}$ Hz ≈ 667 channel

Given reuse factor $N = 4, 19$

(a) Cluster is repeated 16 times. Hence,

Total channel in one system = 667 × 16
(simultaneous call) = 10672

(b) for $N = 4$, $S = kN$
$\Rightarrow \frac{667}{4} = k \Rightarrow 167$

for $N = 19$
$k = \frac{667}{19} = 35$

(c) Total cell is 1st system $(4 \times 16) = 64$
2nd system $(19 \times 16) = 304$

(d) For 1st system $167 \times 100 = 16700$
2nd system $35 \times 100 = 3500$

Lecture 7

Erlang--Example

- If a group of 100 users made 30 calls in one hour, and each call had an average call duration (holding time) of 5 minutes, then the number of Erlangs is worked out as follows:
  - Minutes of traffic in the hour = number of calls x duration
  - Minutes of traffic in the hour = 30 x 5 = 150
  - Hours of traffic in the hour = 150 / 60 = 2.5
  - Traffic Intensity= 2.5 Erlangs

BCC System Example-1

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a BCC system? (a) 5, (b) 10,(c)=20. Assumed that each user generates 0.1 Erlangs of traffic.

Solution: ***Added from the Lecture Slide***

- Given C=5, GOS=0.005, Au=0.1,
- From graph/Table using C=5 and GOS=0.005,A=1.13
- Total Number of users U=A/Au=1.13/0.1=11 users

BCC System Example-2

- Assuming that each user in a system generates a traffic intensity of 0.2 Erlangs, how many users can be supported for 0.1% probability of blocking in an Erlang B system for a number of trunked channels equal to 60.
- Solution :

System is an Erlang B

$A_u = 0.2$ Erlangs

Pr [Blocking] = 0.001

$C = 60$ *Channels*

From the Erlang B figure, we see that

$A \approx 40$ *Erlangs*

Therefore U=A/Au=40/0.02=2000users.

Solution: ***Added from the Lecture Slide***

System is an Erlang B

$A_u = 0.2$ Erlangs

Pr [Blocking] = 0.001

$C = 60$ *Channels*

From the Erlang B figure, we see that

$A \approx 40$ *Erlangs*

Therefore $U = A/Au = 40/0.02 = 2000$ *users.*

Problem 1

An urban area has a population of two million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of three minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider..

Solution: **by Rabab 039 (correct if necessary)** ~~**incomplete**~~
Completed by Hussain 060 [from *Lecture 4 (ext), Example 3.5*]

Solution

System A

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 19$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 19$, from the Erlang B chart, the total carried traffic, A, is obtained as 12 Erlangs.

Therefore, the number of users that can be supported per cell is

$$U = A/A_u = 12/0.1 = 120$$

Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to $120 \times 394 = 47280$

System B

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 57$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, A, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 45/0.1 = 450$

Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44{,}100$

System C

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 100$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, A, is obtained as 88 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 88/0.1 = 880$

Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43{,}120$

Therefore, total number of cellular subscribers that can be supported by these three systems are $47{,}280 + 44{,}100 + 43{,}120 = 134{,}500$ users.

Since there are two million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to

$47{,}280/2{,}000{,}000 = 2.36\%$

Similarly, market penetration of System B is equal to

$44{,}100/2{,}000{,}000 = 2.205\%$

and the market penetration of System C is equal to

$43{,}120/2{,}000{,}000 = 2.156\%$

The market penetration of the three systems combined is equal to

$134{,}500/2{,}000{,}000 = 6.725\%$

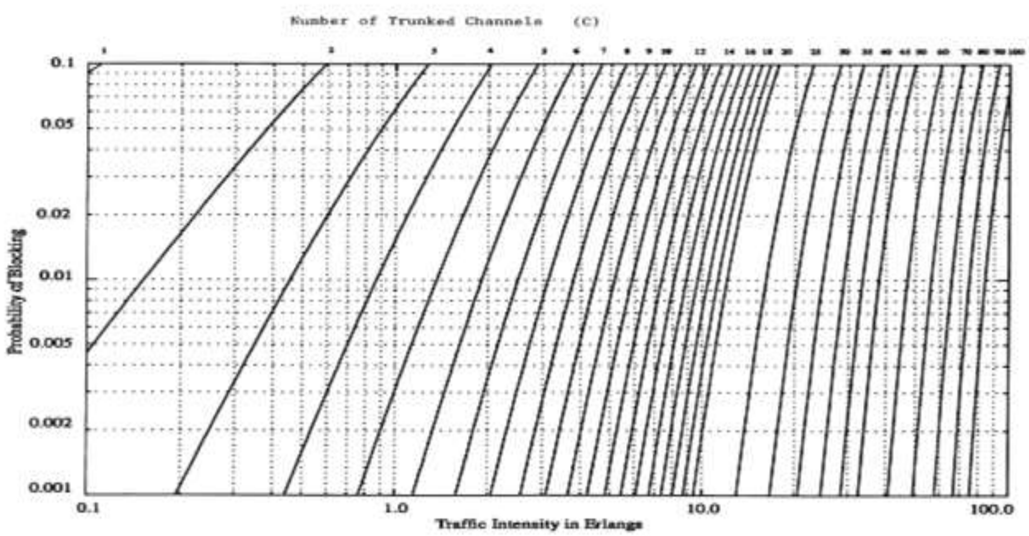


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

Problem 2

A certain city has an area of 1,300 square miles and is covered by a cellular system using a seven-cell reuse pattern. Each cell has a radius of four miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per unique channel (where it is understood that channels are reused), and (g) the theoretical maximum number of users that could be served at one time by the system.

Solution: **By Bappy (067)**

(a) Given:

Total coverage area = 1300 miles

Cell radius = 4 miles

The area of a cell (hexagon) can be shown to be $2.5981R^2$, thus each cell covers

$$2.5981 \times (4)^2 = 41.57 \text{ sq mi.}$$

Hence, the total number of cells are $N_c = 1300/41.57 = 31$ cells.

(b) The total number of channels per cell (C)

= allocated spectrum / (channel width × frequency reuse factor)

= 40, 000, 000/(60, 000 × 7) = 95 channels/cell

(c) Given:

$C = 95$, and $GOS = 0.02$

From the Erlang B chart, we have

traffic intensity per cell $A = 84$ Erlangs/cell

(d) Maximum carried traffic = number of cells × traffic intensity per cell

$= 31 \times 84 = 2604$ Erlangs.

(e) Given traffic per user = 0.03 Erlangs

Total number of users = Total traffic / traffic per user

= 2604 / 0.03 = 86,800 users.

(f) Number of mobiles per channel = number of users/number of channels

= 86,800 / 666 = 130 mobiles/channel.

(g) The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)

$= C \times N_C = 95 \times 31 = 2945$ users, which is 3.4% of the customer base.

Problem 3

A hexagonal cell within a four-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) How many users per square kilometer will this system support?

(b) What is the probability that a delayed call will have to wait for more than 10 s?

(c) What is the probability that a call will be delayed for more than 10 seconds?

Solution: **By Bappy (067)**

Given,

Cell radius, $R = 1.387$ km

Area covered per cell is $2.598 \times (1.387)^2 = 5$ sq km

Number of cells per cluster = 4

Total number of channels = 60

Therefore, number of channels per cell = 60 / 4 = 15 channels.

(a) From Erlang C chart, for 5% probability of delay with $C = 15$, traffic i sity = 9.0 Erlangs.

Therefore, number of users = total traffic intensity / traffic per user

$= 9.0/0.029 = 310$ users

$= 310$ users$/5$ sq km $= 62$ users/sq km

(b) Given $\lambda = 1$, holding time

$H = A_u/\lambda = 0.029$ hour $= 104.4$ seconds.

The probability that a delayed call will have to wait for more than 10 s

$$Pr[delay > t|delay] = \exp(-(C-A)t/H)$$

$$= \exp(-(15-9.0)10/104.4) = 56.29\ \%$$

(c) Given $Pr[delay > 0] = 5\% = 0.05$

Probability that a call is delayed more than 10 seconds,

$$Pr[delay > 10] = Pr[delay > 0]Pr[delay > t|delay]$$

$$= 0.05 \times 0.5629 = 2.81\ \%$$

Problem 4

a. Consider a 7-cell system covering an area of 3100 km2. The traffic in the seven cells is as follows:

Cell number	1	2	3	4	5	6	7
Traffic (Erlangs)	30.8	66.7	48.6	33.2	38.2	37.8	32.6

Each user generates an average of 0.03 Erlangs of traffic per hour, with a mean holding time of 120 s. The system consists of a total of 395 channels and is designed for a grade of service of 0.01.

a. Determine the number of subscribers in each cell.
b. Determine the number of calls per hour per subscriber.
c. Determine the number of calls per hour in each cell.
d. Determine the number of channels required in each cell. *Hint:* You will need to extrapolate using Table 10.3.
e. Determine the total number of subscribers.
f. Determine the average number of subscribers per channel.
g. Determine the subscriber density per km2•
h. Determine the total traffic (total Erlangs).
i. Determine the Erlangs per km2.
j. What is the radius of a cell?

Solution:

Lecture 8

Example-1

Consider Figure; Assume each base station uses 60 channels, regardless of cell size. If each original cell has a radius of 1 Km and each microcell has a radius of 0.5 Km, find the number of channels contained in a 3 Km by 3 Km square centered around *A* under the following conditions:

a) without the use of microcells;
b) when the lettered microcells as shown in Figure 3 are used, also calculate increase in capacity; and
c) if all the original base stations are replaced by microcells. Assume cells on the edge of the square to be contained within the square.

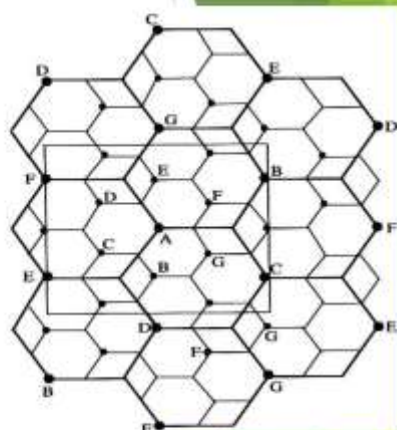


Figure 3: Figure for Example

Solution: ***Added from the Lecture Slide***

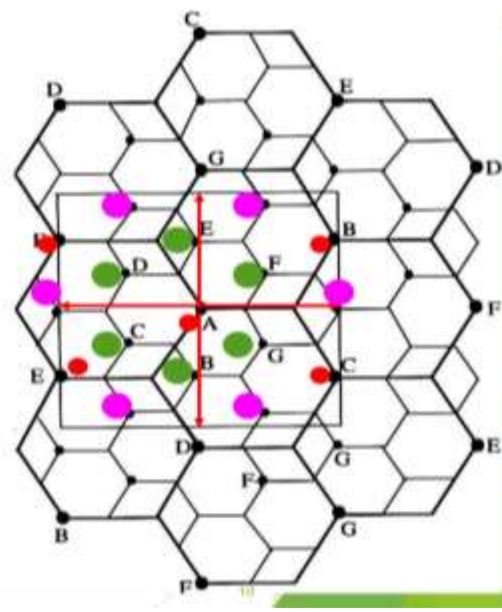
(a) 5x60 = 300 channels

(b) 5+6 = 11

11 x 60 = 660 channels

(c) 5 + 12 = 17 channels

17 x 60 = 1020 channels



Problem 1

- A cellular service provider decides to use a digital TDMA scheme which can tolerate a signal to interference ration 15 dB in the worst case. Find the optimal value of N for
  a) Omni directional antenna
  b) 120 degree sectoring and
  c) 60 degree sectoring.
  d) Should sectoring be used?
  e) If so, which case (120 degree or 60 degree) should be used? (Assuming a path loss factor n = 4 and consider trunking efficiency.)

Solution: 039 (from chap 8 Lect23(01-09-21).mp4)

(a)
Given:

Path loss exponent (n)=4

Tolerable signal to interference ratio, $\frac{S}{I} = 15\ dB$

$$\frac{S}{I} = 31.623$$

Assuming 6 interferers from the first tire of co-channel cells,

$$\frac{S}{I} = \frac{\left(D/R\right)^n}{i_0} = \frac{\left(\sqrt{3N}\right)^n}{i_0}$$

$$31.623 = \frac{\left(\sqrt{3N}\right)^4}{6}$$

$N = 4.592$

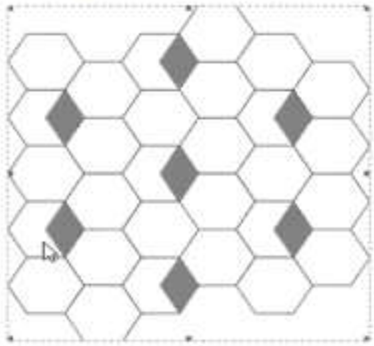
Since we have to choose higher possible value to satisfy the S/I requirement,

N=7

If we calculate S/I from N=7, we get **18.66 dB** (Calculate!), which is better than the requirement.

(b)
120° Sectoring

Let us consider first that N=4, (i=2,j=0) .If we see the layout of the cells with N=4, it seems as below.



It is clear from the diagram that with 120° sectoring and N=4,there are 2 interferers in the first tier of co-channel cells.

Taking $i_0=2$ in the expression

$$\frac{S}{I}=\frac{\left(\sqrt{3N}\right)^n}{i_0}$$

$$\frac{S}{I}=\frac{\left(\sqrt{3\times 4}\right)^4}{2}=72$$

$$\frac{S}{I}=18.57\ dB$$

With 120° sectoring, the S/I obtained is better than required. So N=4 can be used.

We have to again check for N=3

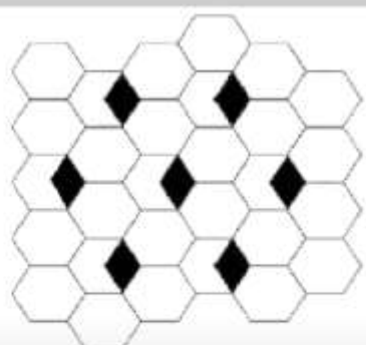
Let us consider that N=3, (i=1,j=1) .If we see the layout of the cells with N=3, it seems as below.

Taking $i_0=3$ in the expression

$$\frac{S}{I}=\frac{\left(\sqrt{3N}\right)^n}{i_0}$$

$$\frac{S}{I}=\frac{\left(\sqrt{3\times 3}\right)^4}{3}=27$$

$$\frac{S}{I}=14.314\ dB$$



Since it is lower than the required value, N=3 cannot be used.

For rest, check: [chap 8]