Small Scale Fading

Small Scale Fading/ Fading

- The rapid fluctuations of received signal strength over short time intervals or travel distances.
- Caused by interference from multiple copies of Tx signal arriving at Rx at slightly **different** times.
- Three most important effects:

1. Rapid changes in signal strengths over small travel distances or short time periods.
2. Random frequency modulation due to varying Doppler shifts on different multipath signals
3. Time dispersion (echoes) caused by multipath propagation delays.

Small scale fading or simply fading. Signal strength = amplitude

2. Changes in the frequency of signals.

3. Multiple signals arriving a different times. When added together at the antenna, signals are spread out in time. This can cause a interference between bits that are received.

Factors Influencing Small-Scale Fading

1) Multipath Propagation

- The presence of reflecting objects and scatterers in the space between transmitter and receiver creates a constantly changing channel environment
- Causes the signal at receiver to fade or distort

2) Speed of Mobile

- Relative motion between base station and mobile causes random frequency modulation due to Doppler shift (f_d)
- Different multipath components may have different frequency shifts.

Relative motion is the calculation of the motion of an object with regard to some other moving object.

Factors Influencing Small-Scale Fading

3) Speed of Surrounding Objects

- If the speed of surrounding objects is greater than mobile, the fading is dominated by those objects
- If the surrounding objects are slower than the mobile, then their effect can be ignored

4) The transmission bandwidth of the signal

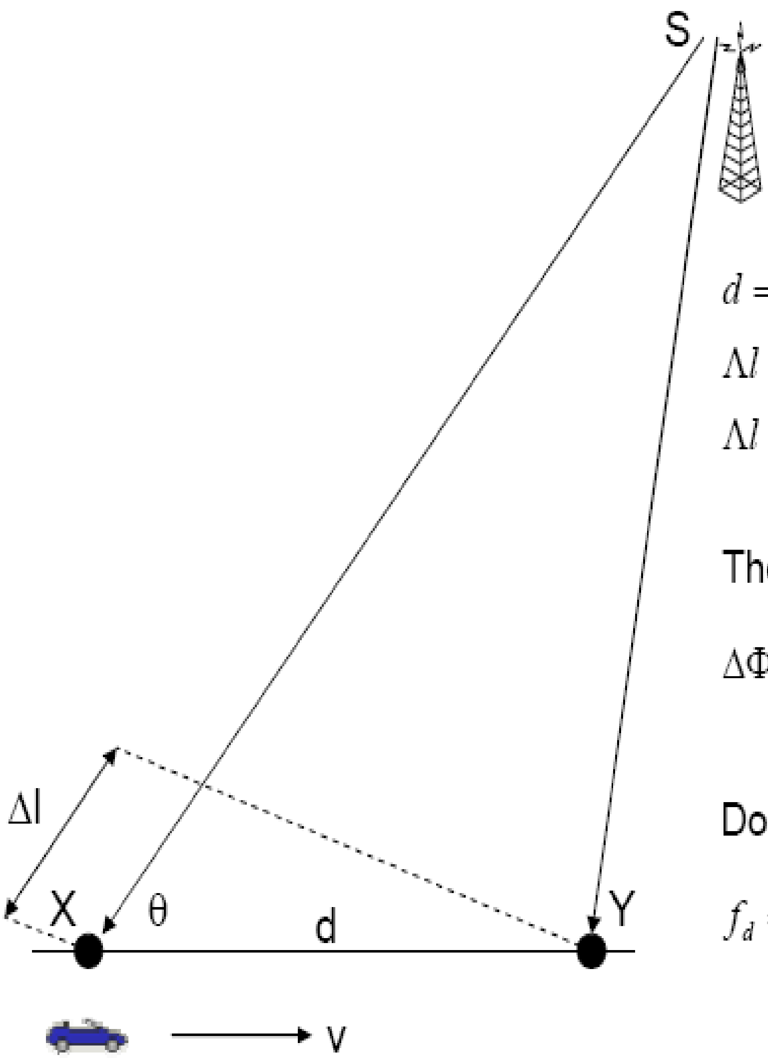
the bandwidth of the channel can be quantified by the coherence bandwidth which is related to the specific multipath structure of the channel. The coherence bandwidth is a measure of the maximum frequency difference for which signals are still strongly correlated in amplitude.

- Depending on the relation between the signal bandwidth and the coherence bandwidth of the channel, the signal is either distorted or faded
- If the signal bandwidth is greater than coherence bandwidth it creates distortion
- If the signal bandwidth is smaller than coherence bandwidth it create small scale fading

Doppler Effect/Shift

- When a transmitter or receiver is moving, the frequency of the received signal changes. It is different than the frequency of transmission. This is called Doppler Effect.
- The change in frequency is called Doppler shift
- The Doppler shift is positive
  - If the mobile is moving towards the direction of arrival of the signal.
- The Doppler shift is negative
  - If the mobile is moving away from the direction of arrival of the signal.

Doppler Shift – Receiver is moving



A mobile receiver is traveling from point X to point Y

$$d = |XY|$$

$$\Delta l = |SX| - |SY| = d\cos\theta$$

$$\Delta l = v\Delta t\cos\theta$$

The phase change in the received signal:

$$\Delta\Phi = \frac{\Delta l}{\lambda}2\pi = \frac{2\pi v\Delta t}{\lambda}\cos\theta$$

Doppler shift (The apparent change in frequency):

$$f_d = \frac{1}{2\pi}\frac{\Delta\Phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$

Apparent = actual

Example – 4.3:

- Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving

(a) directly towards the transmitter,

(b) directly away from the transmitter,

(c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

1 mile = 1609.344m

60 mile = 96560.64m/hr = 26.8224m/sec

1 hr = 60x60 sec

Impulse response of Multipath channel

- The small scale variations of a mobile radio signal can be directly related to the impulse response of mobile radio channel.
- Impulse response contains all information necessary to
  - Simulate the channel
  - Analyze the channel
- A mobile radio channel may be modeled as a linear filter with a time varying impulse response

It is the response(output) from a system (or process) when you put an 'impulse (unit pulse, delta function)' as an input. It is because it fully characterize a system. If you know the impulse response of a system, you can figure out the response (output) of the system for any kind of input without even testing it.

Impulse Response Model:

Figure 5.2 The mobile radio channel as a function of time and space.

$x(t)$: the transmitted signal.

$y(d,t)$: the received signal at position d.

$h(d,t)$: the channel impulse response at position d.

$$y(d,t) = x(t) \otimes h(d,t)$$

$$= \int_{-\infty}^{\infty} x(\tau) h(d, t-\tau) d\tau$$

For a causal system, $h(d,t) = 0$ for $t < 0$

$$y(d,t) = \int_{-\infty}^{t} x(\tau) h(d, t-\tau) d\tau$$

Since the receiver moves along the ground at a constant velocity *v*, the position of the receiver can by expressed as

$$d = vt$$

$$y(vt,t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau$$

Since *v* is a constant, y(*vt*, *t*) is just a function of *t*. Therefore,

$$y(t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau = x(t)\otimes h(vt,t) = x(t)\otimes h(d,t)$$

τ: the channel multipath delay for a fixed value of t

From equation, it is clear that the mobile radio channel can be modeled as a linear time varying channel, where the channel changes with time and distance.

- Consider a pulse with a small width at the input of the channel

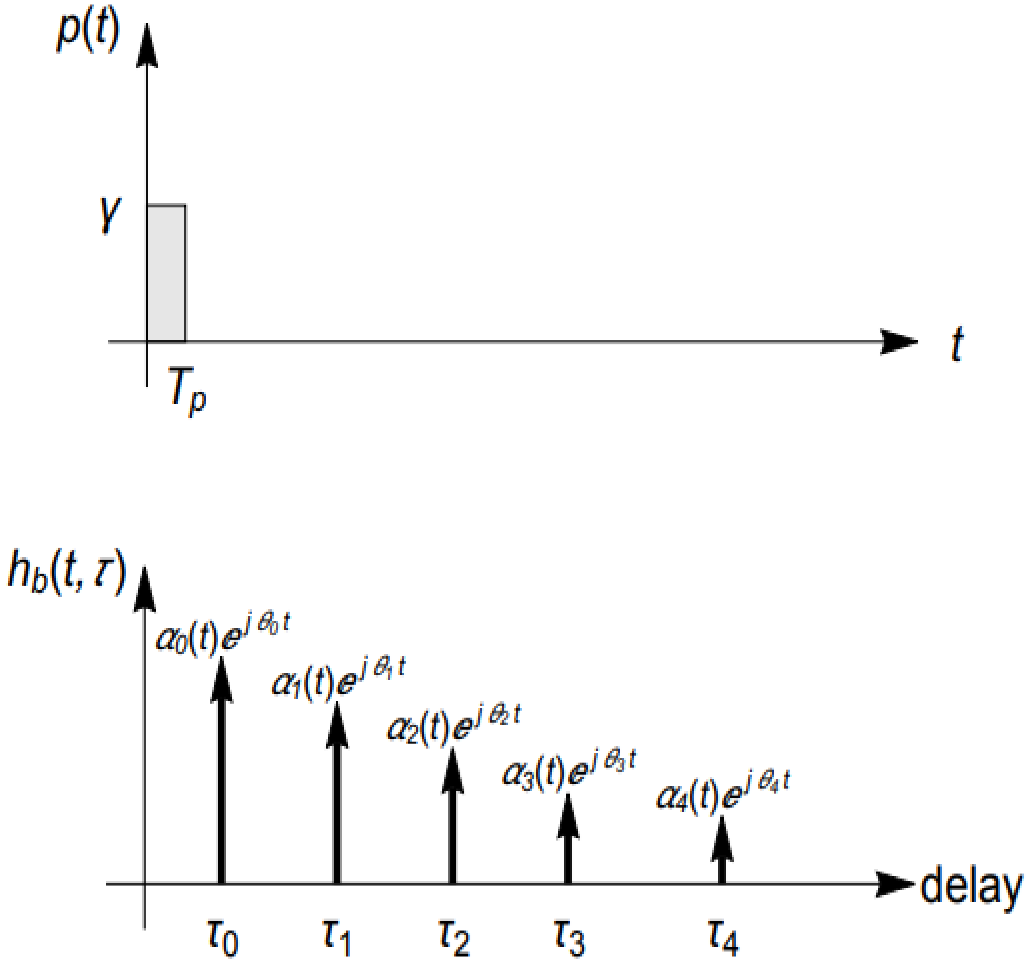


Fig. . Signal power representation in the time domain (*top*) and multipath introduced in the wireless channel (*bottom*).

- After convolution, the channel output is

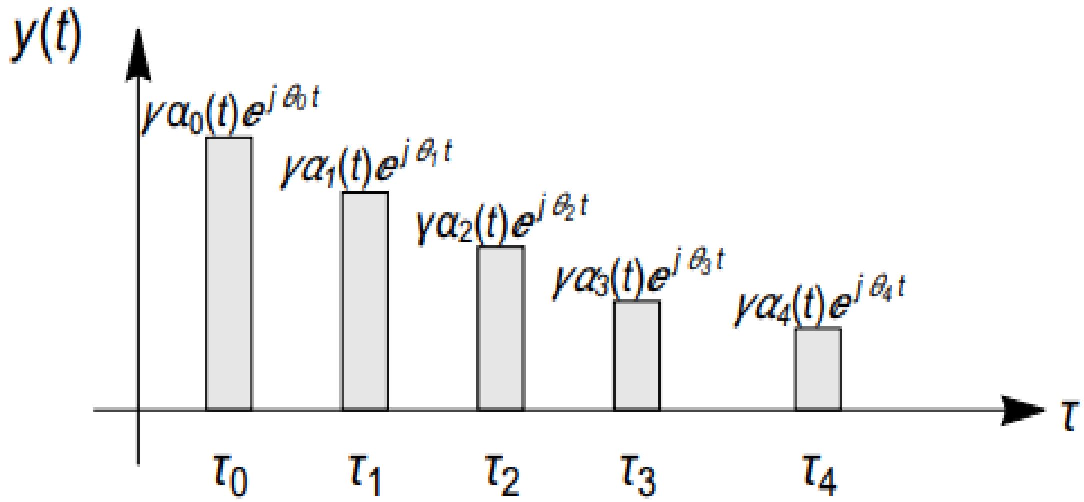


Fig. Received multipath components at the MS.

Power Delay Profile

- The *power delay profile* $P_h(\tau)$ indicates how the channel power is distributed along the delay τ

- The power delay profile (PDP) gives the strength of a signal received through a multipath channel as a function of the time delay

- For small scale channel modelling, the power delay profile of the channel is found by taking the average of $|h_b(\tau, t)|^2$ over a local area

$$P_h(\tau) = \langle |h_b(\tau, t)|^2 \rangle$$

Time Dispersion Parameters

- The time dispersion parameters that can be determined from a power delay profile are
  - mean excess delay
  - rms delay spread
  - excess delay spread

- To quantify “how spread-out” the arriving signals are, we use time dispersion parameters

Dispersion = spreading

Time Dispersion Parameters

The mean excess delay

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

The rms delay spread

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

The maximum excess delay

$$\tau_{\max} = \tau_X - \tau_0$$

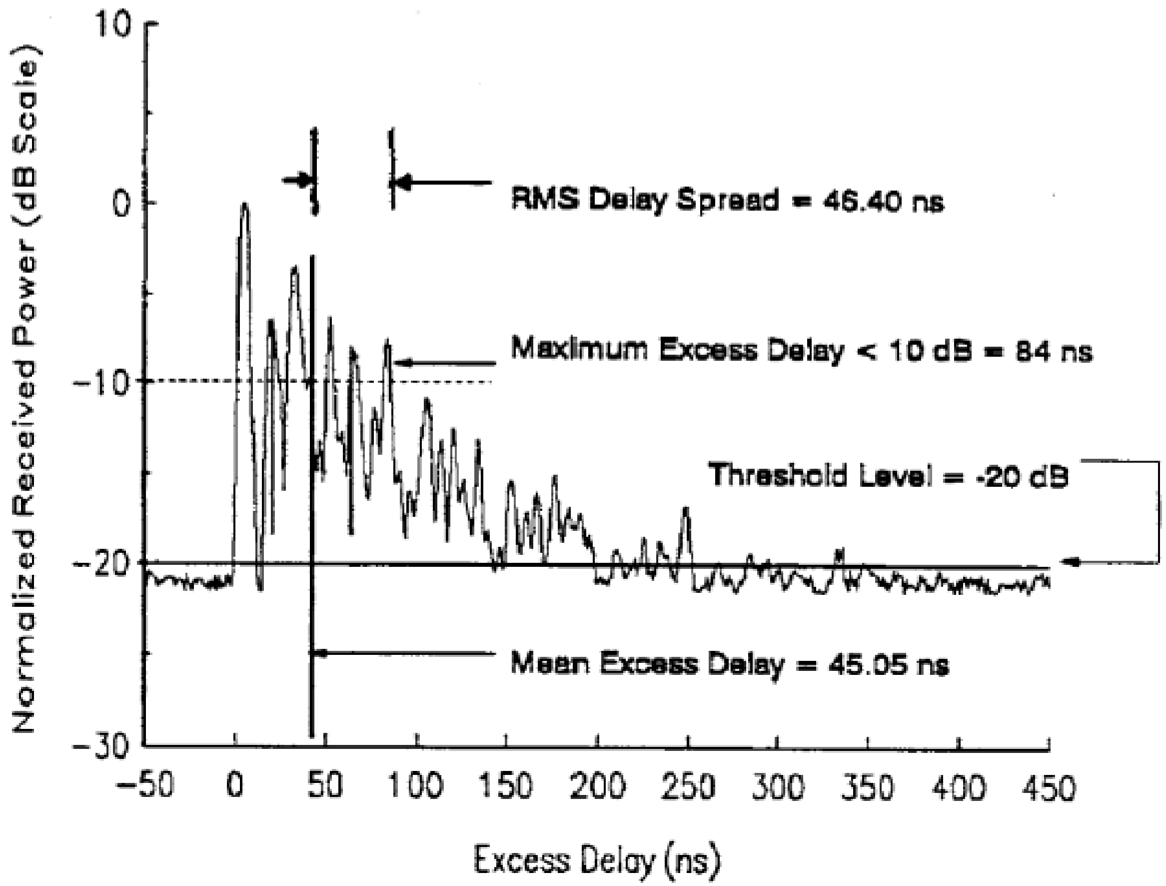
τ_0 : the first arrival

τ_X : the max. delay at which a multipath component is within X dB

Dispersion = spreading

the second moment of the mean excess delay

Time Dispersion Parameters: Max Excess Delay(X dB)



Example 5.5

Calculate the mean excess delay, rms delay spread, and the maximum excess delay (10 dB) for the multipath profile given in the figure below. Estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer?

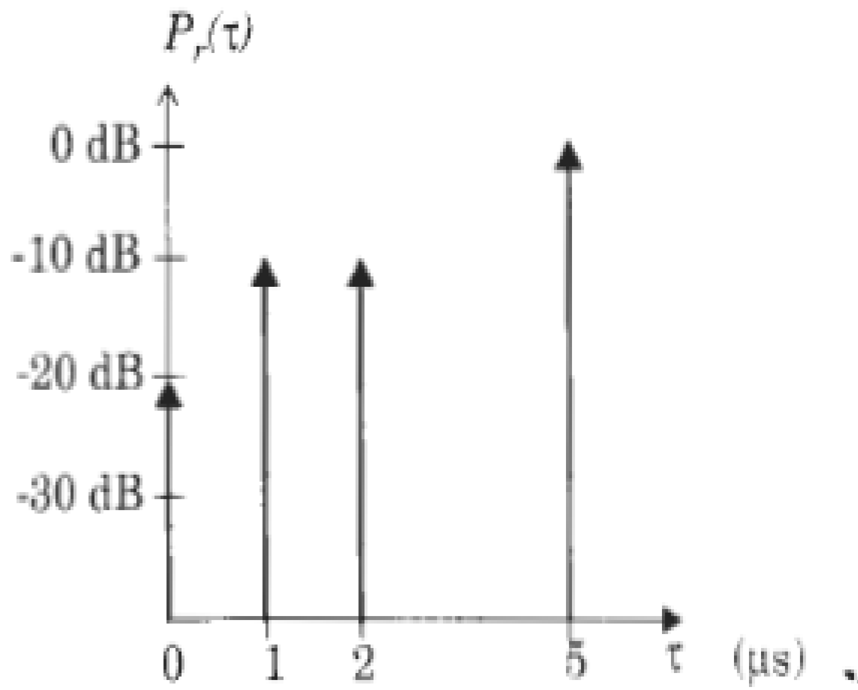


Figure E5.5

Solution

Using the definition of maximum excess delay (10 dB), it can be seen that $\tau_{10\,dB}$ is 5 μs. The rms delay spread for the given multipath profile can be obtained using Equations (5.35)–(5.37). The delays of each profile are measured relative to the first detectable signal. The mean excess delay for the given profile is

$$\bar{\tau} = \frac{(1)(5) + (0.1)(1) + (0.1)(2) + (0.01)(0)}{[0.01 + 0.1 + 0.1 + 1]} = 4.38\ \mu s$$

The second moment for the given power delay profile can be calculated as

$$\overline{\tau^2} = \frac{(1)(5)^2 + (0.1)(1)^2 + (0.1)(2)^2 + (0.01)(0)}{1.21} = 21.07\ \mu s^2$$

Therefore the rms delay spread is $\sigma_\tau = \sqrt{21.07 - (4.38)^2} = 1.37\ \mu s$

The coherence bandwidth is found from Equation (5.39) to be

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\mu s)} = 146\ \text{kHz}$$

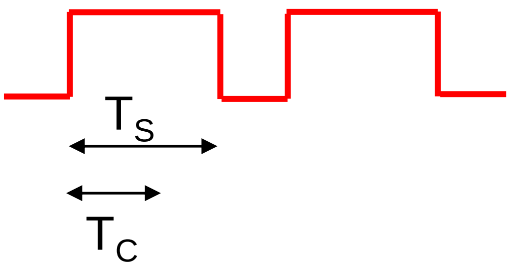
Since B_c is greater than 30 kHz, AMPS will work without an equalizer. However, GSM requires 200 kHz bandwidth which exceeds B_c, thus an equalizer would be needed for this channel.

Coherence Bandwidth (B_c)

- B_c : statistical measure of frequency range where MRC response is **flat**
  - MRC response is **flat** = passes all frequencies with ≈ equal gain & linear phase
  - Amplitudes of different frequency components are correlated
  - If two sinusoids have frequency separation greater than B_c, they are affected quite differently by the channel
  - Estimates coherence BW (B_c)
    - 0.9 correlation $\rightarrow B_c \approx 1/50\sigma_\tau$ (signals are above 90% correlated with each other)
    - 0.5 correlation $\rightarrow B_c \approx 1/5\sigma_\tau$ (signals are above 50% correlated with each other)

Coherence Time (T_c)

- Coherence time is the time duration over which the channel impulse response is essentially invariant.
- If the symbol period of the baseband signal is greater the coherence time, than the signal will distort.
- Coherence time is also defined as: $$T_C \approx \frac{9}{16\pi f_m}$$



where f_m is the maximum Doppler spread given by

$$f_m = \frac{v}{\lambda}$$

Example

Assuming the speed of a vehicle to be equal to 60 mph (88 ft/s), carrier frequency, $f_c = 860$ MHz, and rms delay spread $\tau_d = 2\ \mu s$, calculate coherence time and coherence bandwidth. At a coded symbol rate of 19.2 Kbps what kind of symbol distortion will be experienced? What type of fading will be experienced by the channel?

V = 88 ft/s

Tc = 9/16pifm = 9/16x3.14x77 = 2.3 ms

Fm = v/lemda = 88/1.142 = 77 Hz

Lemda = c / f =9.8 x 10^8/ 860 x 10^6 = 1.1442 ft

Ts = 10^6/19200 = 5.2 micro s

Doppler Spread (B_D)

- Measure of spectral broadening caused by motion
- We know how to compute Doppler shift: f_d
- Doppler spread, B_D, is defined as the maximum Doppler shift: $f_m = v/\lambda$
- If the baseband signal bandwidth is much greater than B_D then effect of Doppler spread is negligible at the receiver.

Types of small-scale fading

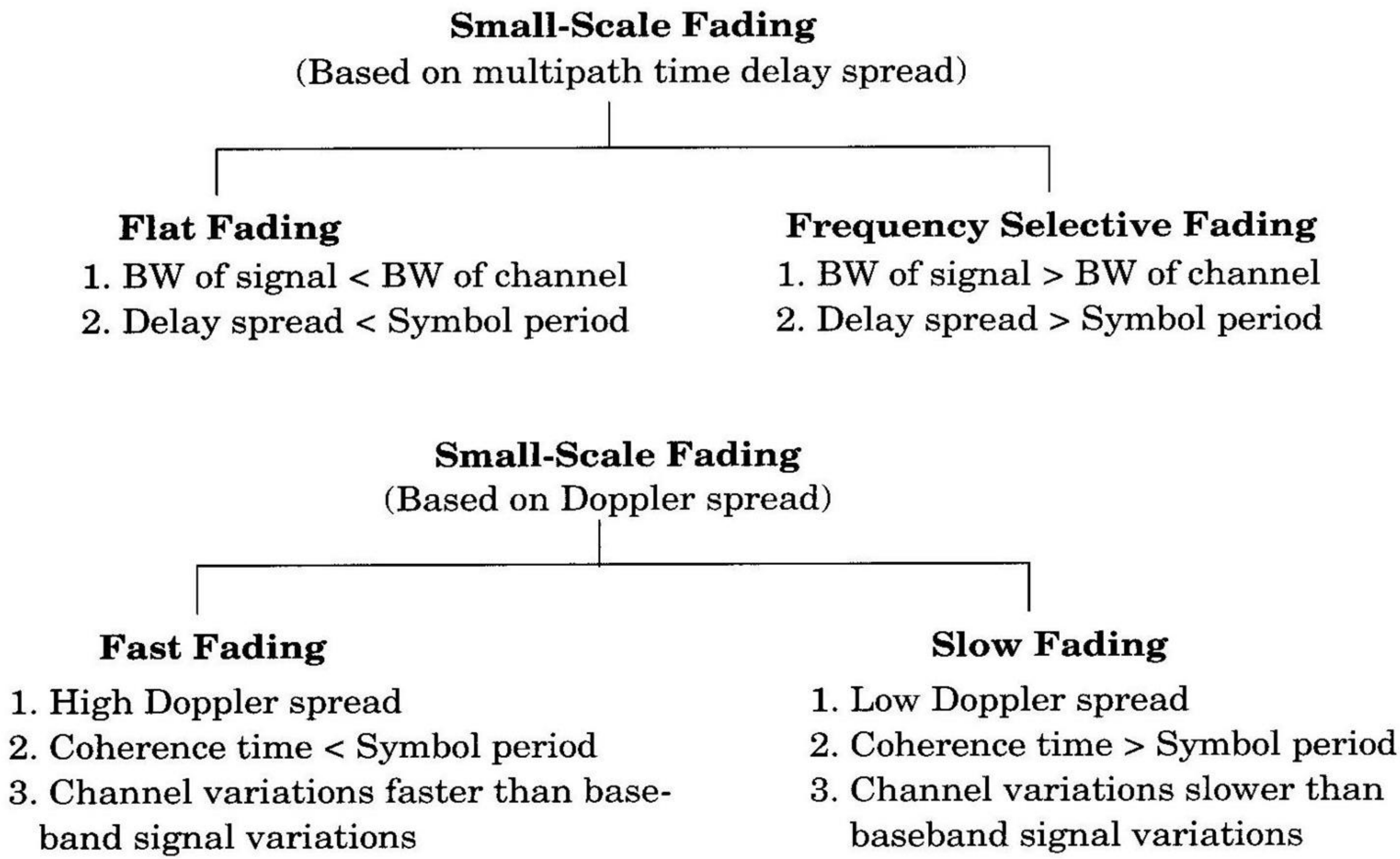


Figure 5.11 Types of small-scale fading.

Flat Fading

- Occurs when symbol period of the transmitted signal is much larger than the Delay Spread of the channel
  - Bandwidth of the applied signal is narrow.
  - If $B_s << B_c$, and $T_s >> \sigma_\tau \Rightarrow$ Flat fading
- Flat fading channels are known as amplitude varying channels or narrow-band channels.
- A rule of thumb that channel is assumed to be flat fading if $T_s > 10\sigma_\tau$
- All frequencies undergone the same amount of fading

Frequency Selective Fading

- Occurs when channel multipath delay spread is greater than the symbol period.
  - If $B_s >> B_c$, and $T_s << \sigma_\tau \Rightarrow$ Frequency selective fading
  - Channel induces Intersymbol Interference (ISI)
- Frequency selective fading channels are known as wideband channels since the BW of the signal is wider than the BW of the channel impulse response.
- A rule of thumb that channel is assumed to be frequency selective fading if $T_s <= 10\sigma_\tau$
- Not all frequencies undergone the same amount of fading.

Example (1)

- Calculate the total excess delay, mean delay and RMS delay spread for a channel whose PDP is specified as follows:

Relative delay [μsec]	Average relative power [dB]
0.0	-3.0
0.2	0.0
0.5	-2.0
1.6	-6.0
2.3	-8.0
5.0	-10.0

- Total excess delay is simply the difference between the shortest and longest delays: 5 μsec
- The mean delay is found by multiplying the powers by the delays and summing:

$$\tau_o = \frac{1}{P_T} \sum_{i=1}^{n} P_i \, \tau_i = \frac{1}{P_T} \left(P_1\tau_1 + P_2\tau_2 + P_3\tau_3 + P_4\tau_4 + P_5\tau_5 + P_6\tau_6 \right)$$

τ_o= 1/2.64 × {(0.5×0) + (1×0.2) + (0.63×0.5) + (0.25×1.6) + (0.16×2.3) + (0.10×5)}

= 0.678 μsec

Example (2)

- To find the RMS delays spread, the average power is normalized as follows:

Average relative power [dB]	Average relative power [W]	Average relative power (normalised)
-3.0	0.50	0.19
0.0	1.00	0.38
-2.0	0.63	0.24
-6.0	0.25	0.09
-8.0	0.16	0.06
-10.0	0.10	0.04

- The RMS delay spread can be found as:

$$\tau_{RMS} = \sqrt{\frac{1}{P_T}\sum_{i=1}^{n} P_i \, \tau_i^{\,2} - \tau_o^{\,2}}$$

$\tau^2_{RMS} = (0.19\times 0^2) + (0.38\times 0.2^2) + (0.24\times 0.5^2) + (0.09\times 1.6^2) + (0.06\times 2.3^2) + (0.04\times 5^2) - (0.678^2)$

$= 1.163$ (μsec^2)

This is still correct because $\tau_i^{\,2}$ above is expressed in squared of micro-second .(μsec^2). By performing square-root onto (μsec^2) gives us back (μsec) in unit. (think about it!)

Hence, $\tau_{RMS} = 1.08$ μsec

Example

- From the previous calculation of PDP, would the channel be regarded as a wideband channel for a binary data system with data rate of 25 Kbits/sec?

If the data rate is 25×10^3 bits/sec, then the symbol period T_S is:

$$T_S = \frac{1}{25\times10^3} = 40\times10^{-6}\,\text{sec} = 40\mu\,\text{sec}$$

It is found previously that $\tau_{RMS} = 1.08$ µsec, so rule of thumb $\Longrightarrow$ $10\times\tau_{RMS} \approx 10$ µsec

Hence, the channel can regards as narrowband flat fading since $T_S \gg 10\tau_{RMS}$

- What is the maximum data rate for the system to be ISI-free? (For ISI-free, $T_S > 10\tau_{RMS}$ condition must be fulfilled)

Since $10\tau_{RMS}$ is about 10 µsec, then the minimum symbol period T_S must not be less than 10 µsec. Therefore, we can write that $T_S = 10$ µsec

Hence, the maximum data rate is: $1/T_S = 1/(10\times10^{-6}) = 100$Kbits/sec, for ISI-free system

Example

- Can the channel considered as narrowband when the data rate in the previous example is increased to 200Kbits/sec?

If the data rate is 200×10^3 bits/sec, then the symbol period T_S is:

$$T_S = \frac{1}{200 \times 10^3} = 5 \times 10^{-6}\,\text{sec} = 5\mu\,\text{sec}$$

We know from previously that $10 \times \tau_{\text{RMS}} \approx 10$ μsec which is greater than T_S

Therefore, the channel can no longer considered as narrowband since $T_S < 10\tau_{\text{RMS}}$.
Hence, a wideband frequency selective fading channel.

Fast Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics is **larger** than the Rate of change of the transmitted signal
  - The channel changes during a symbol period.
  - The channel changes because of receiver motion.
  - Coherence time of the channel is smaller than

    the symbol period of the transmitter signal
  - It causes frequency dispersion due to Doppler spread and leads to distortion.

Occurs when:

$B_S < B_D$

and

$T_S > T_C$

Slow Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics is **much smaller** than the rate of change of the transmitted signal

Occurs when:
$B_S >> B_D$
and
$T_S << T_C$

B_S: Bandwidth of the signal
B_D: Doppler Spread
T_S: Symbol Period
T_C: Coherence Bandwidth

Summary of Channel Classification

1) Frequency Flat (no delay-spread) & Time-Flat (slow fading) Channel $B_S << B_C$ or $T_S >> \tau_{rms}$ $T_S << T_C$ or $B_S >> B_D$ *(Doubly –flat fading channel)*	2) Frequency Flat (no delay-spread) & Time-Selective (fast fading) Channel $B_S << B_C$ or $T_S >> \tau_{rms}$ $T_S > T_C$ or $B_S < B_D$ *(Fast-varying flat fading channel)*
3) Frequency Selective (Delay-spread) & Time-Flat (slow fading) Channel $B_S > B_C$ or $T_S < \tau_{rms}$ $T_S << T_C$ or $B_S >> B_D$ *(Slow-varying frequency selective channel)*	4) Frequency Selective (Delay-spread) & Time-Selective (fast fading) Channel $B_S > B_C$ or $T_S < \tau_{rms}$ $T_S > T_C$ or $B_S < B_D$ *(Doubly dispersive fading channel)*

B_S = Bandwidth of the baseband signal
B_C = Coherence bandwidth of the channel
B_D = Doppler spread of the channel

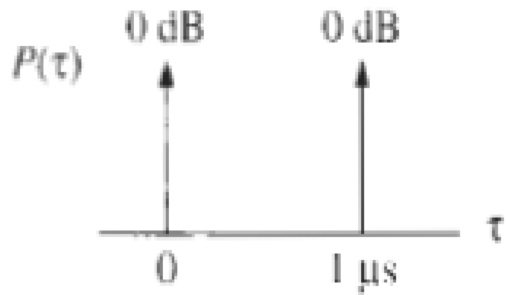
T_S = Symbol duration of the baseband signal
T_C = Coherence time of the channel
τ_{rms} = *RMS delay spread of the channel*

Example

Example 5.4

Compute the RMS delay spread for the following power delay profile:

(a)



(b) If BPSK modulation is used, what is the maximum bit rate that can be sent through the channel without needing an equalizer?

Example

Solution

(a) $$\bar{\tau} = \frac{(1)(0)+(1)(1)}{1+1} = \frac{1}{2} = 0.5\mu s$$

$$\overline{\tau^2} = \frac{(1)(0)^2+(1)(1)^2}{1+1} = \frac{1}{2} = 0.5\mu s^2$$

$$\sigma_\tau = \sqrt{\overline{\tau^2}-(\bar{\tau})^2} = \sqrt{0.5-(0.5)^2} = \sqrt{0.25} = 0.5\mu s$$

(b) $$\frac{\sigma_\tau}{T_s} \le 0.1$$

$$T_s \ge \frac{\sigma_\tau}{0.1}$$

$$T_s \ge \frac{0.5\mu s}{0.1}$$

$$T_s \ge 5\mu s$$

$$R_s = \frac{1}{T_s} = 0.2\times 10^6 \text{sps} = 200\text{ksps}$$

$$R_b = 200\text{kbps}$$