Semester Final Solutions

Lecture 10,11,12

Origin42 Final

5. a. What is satellite? What feature of the geostationary orbit sets it apart from the LEO and MEO? How does satellite remain stable in orbit? [5]

Solution: O24

Satellite: Satellite is a specialized antenna in a stable orbit above the earth.

Geostationary satellites:

- Orbit 35,786 km distance from earth surface
- Complete rotation exactly one day, satellite is synchronous to earth rotation
- Fixed antenna positions
- A large footprint (up to 34% of earth surface!)
- Three satellites can cover the whole globe
- Typically used for radio and TV transmission

To keep satellite stable in circular orbit:

$F_g = F_c$, must be maintained.

Here, F_g = Gravitational force, F_c = Centrifugal force

$F_g = m\ g\ (R/r)^2$

$F_c = m\ r\ \omega^2$

So, $F_g = F_c$

$\Rightarrow m\ g\ (R/r)^2 = m\ r\ \omega^2$

=> r = qubic_root (g x R^2 / (2 x pi x f)^2)

The distance r has to be maintained from the satellite to the center of the earth to keep the satellite stable in circular orbit.

5. b. Derive in details the free-space loss (FSL) from Friis' equation and prove that ***FSL = - 20 log(f) + 20 log(d) - 10 log(AtAr) + 169.54 dB***
And all symbols have their usual meaning. [5]

Solution: O24

Friis Free Space eqn:

$$P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$$

$$\Rightarrow \frac{P_t}{P_r} = \frac{(4\pi d)^2}{\lambda^2 G_t G_r}$$

We know,

$$G = \frac{4\pi A_e}{\lambda^2}$$

$$\therefore \frac{P_t}{P_r} = \frac{(4\pi d)^2}{\lambda^2 \cdot \frac{4\pi A_t}{\lambda^2} \cdot \frac{4\pi A_r}{\lambda^2}}$$

$$\Rightarrow \frac{P_t}{P_r} = \frac{\lambda^2 d^2}{A_t A_r}$$

We know,

Free Space Loss, $L_{dB} = 10 \log \left(\frac{P_t}{P_r}\right)$

$$\therefore L_{dB} = 10 \log \frac{\lambda^2 d^2}{A_t A_r}$$

$$= 10 \log(\lambda^2 d^2) + 10 \log(A_t A_r)^{-1}$$

$$= 10 \log\left(\frac{c}{f}\right)^2 + 10 \log d^2 - 10 \log(A_t A_r)$$

$$= 10 \log(3\times10^8)^2 + 10 \log(f)^{-1\times2} + 10 \log(d)^2 - 10 \log(A_t A_r)$$

$$= 169.542\ dB - 20 \log f + 20 \log d - 10 \log(A_t A_r)$$

5. c. A microwave transmitter has an output of 0.1 W at 2 GHz. Assume that this transmitteris used in a microwave communication system where the transmitting and receiving antennas are parabolas, each 1.2 m in diameter. [4]

i. What is the gain in dB of each antenna in decibels?

ii. Taking into account antenna gain, what is the effective radiated power of the transmitted signal?

iii. If the receiving antenna is located 24 km from the transmitting antenna over a free space path, find the available signal power out of the receiving antenna in dB units.

Solution: 024

I. We know, ,Gain of each antenna for parabola,,

G = 7A/λ^2

= 7Af2 /c^2

= (7 × π × (0.6)^2 × (2×10^9)^2]/(3 × 10^8)^2

= 351.85

G_{dB} = 10 log(351.85)

G_{dB} = 25.46 dB

II. Effective radiated power of the transmitted signal is,
0.1 W x 351.85 = 35.185 W

III. Free space loss,

$$\frac{P_t}{P_r} = \frac{(4\pi)^2 (d)^2}{G_r G_t \lambda^2}$$

Using log,

LdB = 20 log (4π) + 20 log (d) + 20 log (f) – 20 log (c) – 10 log(Gr) – 10 log (Gt)

LdB = 21.98 + 87.6 + 186.02 – 169.54 – 25.46 – 25.46 = 75.14 dB

The transmitter power, in dBm is 10 log (0.1 W x 10^3) = 20

The available received signal power is 20 – 75.14 = –55.14 dBm

6. a. Briefly discuss Fresnel zone and calculate the radius of the nth Fresnel zone with appropriate diagram. [4]

Solution: 024

Fresnel Zone: Fresnel zones represent successive regions where secondary waves have a path length from the TX to the RX which are nλ/2 greater in path length than of the LOS path.

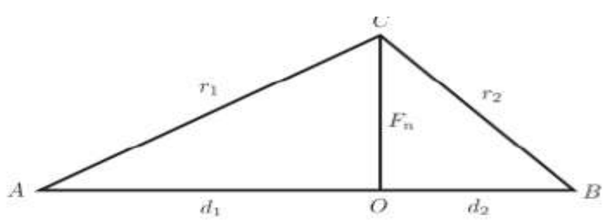


Figure : Fresnel zone radius illustration

The radius of the nth Fresnel zone circle can be found as follows. Consider the triangle below which shows a cross-section of the nth Fresnel zone. Path AB is teh direct path and path ACB is the indirect path. The condition that will locate point C on the nth Fresnel zone is

$$r_1 + r_2 = d_1 + d_2 + n\lambda/2.$$

Hence,

$$\sqrt{d_1^2 + F_n^2} + \sqrt{d_2^2 + F_n^2} = d_1 + d_2 + n\lambda/2,$$

and since $F_n \ll d_1$, $F_n \ll d_2$, we can approximate this as

$$d_1 + \frac{F_n^2}{2d_1} + d_2 + \frac{F_n^2}{2d_2} = d_1 + d_2 + n\lambda/2.$$

Therefore,

$$\frac{F_n^2}{2}\left[\frac{1}{d_1} + \frac{1}{d_2}\right] = F_n^2 \frac{d_1 + d_2}{2d_1 d_2} = n\lambda/2$$

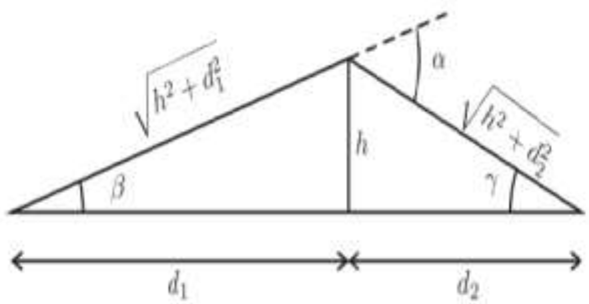
yielding

$$F_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}$$

6. b. Describe the knife edge diffraction model and show that the phase difference between direct signal and diffracted signal: $\phi = 2\pi/\lambda[\frac{h^2((d1+d2)}{2d1d2}]$ [4]

Solution: O24

Knife Edge diffraction model estimates the signal attenuation caused by diffraction of radio waves over hills and buildings.



Path diff:

$$\Delta = \left(\sqrt{d_1^2+h^2} + \sqrt{d_2^2+h^2}\right) - (d_1+d_2)$$

$h << d_1$; $h << d_2$

$$\therefore \Delta = \cancel{d_1} + \frac{h^2}{2d_1} + \cancel{d_2} + \frac{h^2}{2d_2} - \cancel{d_1} - \cancel{d_2}$$

$$= \frac{h^2}{2}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)$$

$$= \frac{h^2(d_1+d_2)}{2d_1d_2}$$

Phase diff:

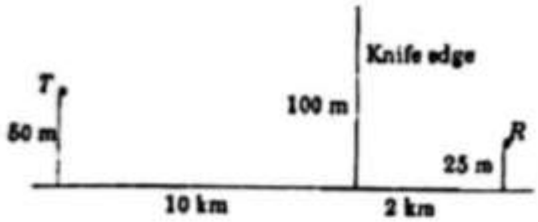
$$\varphi = \omega \tau_d \longrightarrow \text{Time delay} = \frac{\text{Distance}}{\text{Speed}}$$

$$= 2\pi f \cdot \frac{\Delta}{c}$$

$$= 2\pi \cancel{f} \cdot \frac{1}{\cancel{f}\lambda} \cdot \frac{h^2(d_1+d_2)}{2d_1d_2}$$

$$= \frac{2\pi}{\lambda}\left[\frac{h^2(d_1+d_2)}{2\,d_1d_2}\right]$$

6. c. Given the following geometry, determine [6]
(a) the loss due to knife-edge diffraction,
(b) the height of the obstacle required to induce 6 dB diffraction loss. Assume f = 900 MHz.



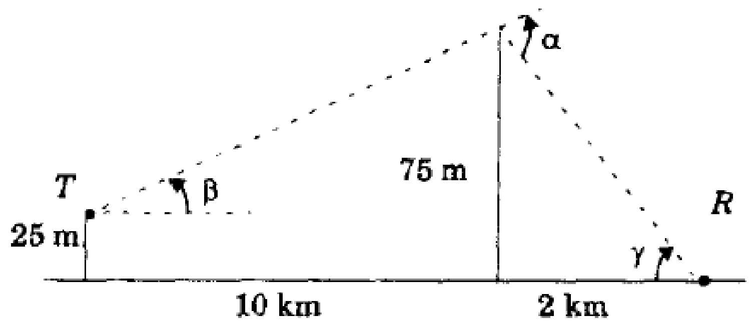
G_d(dB)	v
0	≤ -1
$20 \log(0.5 - 0.62v)$	$[-1, 0]$
$20 \log(0.5\, e^{-0.95v})$	$[0, 1]$
$20 \log(0.4 - (0.1184 - (0.38 - 0.1v)^2)^{1/2})$	$[1, 2.4]$
$20 \log(0.225/v)$	> 2.4

Solution: O24

a.

The wavelength $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3}$ m.

Redraw the geometry by subtracting the height of the smallest structure.



$$\beta = \tan^{-1}\left(\frac{75 - 25}{10000}\right) = 0.2865^o$$

$$\gamma = \tan^{-1}\left(\frac{75}{2000}\right) = 2.15^o$$

and

$$\alpha = \beta + \gamma = 2.434^o = 0.0424 \text{ rad}$$

Formula for v:

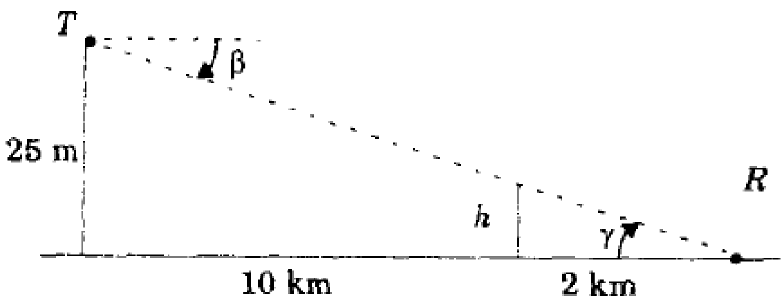
$$v = h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1 + d_2)}}$$

$$v = 0.0424\sqrt{\frac{2 \times 10000 \times 2000}{(1/3) \times (10000 + 2000)}} = 4.24.$$

Using given table, Gd(dB) = 20xlog(0.225 / 4.24) = −25.5
So, knife-edge diffraction loss 25.5 dB

b.

For 6 dB diffraction loss, $\nu = 0$. The obstruction height h may be found using similar triangles ($\beta = -\gamma$) as shown below.



It follows that $\frac{h}{2000} = \frac{25}{12000}$, thus $h = 4.16$ m.

7. a. Discuss in brief that the mobile radio channel can be modeled as a linear time varying channel. [5]

Solution: added by Tamal 122

Impulse Response Model:

Figure 5.2 The mobile radio channel as a function of time and space.

$x(t)$: the transmitted signal.
$y(d,t)$: the received signal at position d.
$h(d,t)$: the channel impulse response at position d.

$$y(d,t) = x(t) \otimes h(d,t)$$
$$= \int_{-\infty}^{\infty} x(\tau) h(d, t-\tau) d\tau$$

For a causal system, $h(d,t) = 0$ for $t < 0$

$$y(d,t) = \int_{-\infty}^{t} x(\tau) h(d, t-\tau) d\tau$$

9

Since the receiver moves along the ground at a constant velocity v, the position of the receiver can by expressed as

$$d = vt$$

$$y(vt, t) = \int_{-\infty}^{t} x(\tau) h(vt, t-\tau) d\tau$$

Since v is a constant, $y(vt, t)$ is just a function of t. Therefore,

$$y(t) = \int_{-\infty}^{t} x(\tau) h(vt, t-\tau) d\tau = x(t) \otimes h(vt, t) = x(t) \otimes h(d, t)$$

τ: the channel multipath delay for a fixed value of t

From equation, it is clear that the mobile radio channel can be modeled as a linear time varying channel, where the channel changes with time and distance. 10

7. b. Describe the Doppler shift effect with appropriate diagram and its impact on the characteristics of the mobile wireless channel. [4]

Solution:

7. c. Calculate the total excess delay, mean delay and RMS delay spread for a channel whose PDP is specified as follows:

Relative delay [μsec]	Average relative power [dB]
0.0	-3.0
0.2	0.0
0.5	-2.0
1.6	-6.0
2.3	-8.0
5.0	-10.0

From the above calculation of PDP, would the channel be regarded as a wideband channel for a binary data system with data rate of 25 Kbits/sec? What is the maximum data rate for the system to be ISI-free?

Solution:
(Toufique 116)

mean excess delay,

$$\bar{\tau} = \frac{10^{\frac{-3.0}{10}} \times 0 + 10^{0} \times 0.2 + 10^{-0.2} \times 0.5 + 10^{-0.6} \times 1.6 + 10^{-0.8} \times 2.3 + 10^{-1} \times 5}{\left(10^{-0.3} + 10^{0} + 10^{-0.2} + 10^{-0.6} + 10^{-0.8} + 10^{-1}\right)}$$

$$= \frac{1.782}{2.642} = 0.6745\ \mu s$$

second moment,

$$\bar{\tau^2} = \frac{10^{-0.3} \times 0 + 10^{0} \times 0.2^2 + 10^{-0.2} \times 0.5^2 + 10^{-0.6} \times 1.6^2 + 10^{-0.8} \times 2.3^2 + 10^{-1} \times 5^2}{\left(10^{-0.3} + 10^{0} + 10^{-0.2} + 10^{-0.6} + 10^{-0.8} + 10^{-1}\right)}$$

$$= \frac{4.179}{2.642} = 1.5818\ \mu s$$

rms delay spread,

$$\sigma_\tau = \sqrt{\bar{\tau^2} - (\bar{\tau})^2}$$

$$= \sqrt{1.5818 - 0.6745^2} = 1.0615\ \mu s$$

$$T_s = \frac{1}{25 \times 10^3} = 40 \times 10^{-6} = 40\ \mu s$$

Since, $T_s >> 10\sigma_\tau$, the channel is narrow-band flat fading, not wideband.

For ISI-free, $T_s > 10\sigma_T$ condition must be fullfilled.

Let $T_s = 10 \times 1.0615$

$= 10.615\ \mu s$

$\therefore$ Maximum data rate is (for ISI free system):

$$\frac{1}{T_s} = \frac{1}{10.615 \times 10^{-6}} = 94206.31 \text{ bits/sec}$$

$$= 94.20 \text{ Kbits/sec}$$

Enigma41 Final

6. a. Explain the two ray ground reflection radio propagation model and prove that received power Pr, is inversely proportional to d^4, where d is the distance between the transmitter and the receiver. [6]

Solution:

6. b. Describe Free space loss (FSL) and prove that [4]

FSL = 32.4 +20 log f (MHz) + 20 log d (km)

Where f is the carrier frequency and d is the distance between antennas.

Solution: ***by Younus - 131***

Rabab **039**

Ekhane 10^3 ar 10^6 multiply hobe, divide na. Naile 32.44 ashe na.

Equation 1=>

$P_t/P_r = ((4\pi * 10^3 * 10^6)/c)^2 * d^2 * f^2$

Friis Free Space Equation:

$$P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$$

$$\Rightarrow \frac{P_t}{P_r} = \frac{(4\pi d)^2}{G_r G_t \lambda^2}$$

$$\Rightarrow \frac{P_t}{P_r} = \frac{(4\pi)^2 d^2 f^2}{G_r G_t c^2}$$

$$= \frac{(4\pi)^2 \left(\frac{d}{1000}\right)^2 \left(\frac{f}{1\times10^6}\right)^2}{G_r G_t c^2} \quad \left[\begin{array}{l} d : m \rightarrow km \\ f : Hz \rightarrow MHz \end{array}\right]$$

$$= \left(\frac{4\pi}{c}\right)^2 \times \left(\frac{d}{1000}\right)^2 \times \left(\frac{f}{1\times10^6}\right)^2 \quad \left[\begin{array}{l} \text{for Isotropic Antenna} \\ G_r, G_t = 1 \end{array}\right]$$

$$\Rightarrow \frac{P_t}{P_r} = \left(\frac{4\pi}{c \times 10^3 \times 10^6}\right)^2 \times d^2 \times f^2 \quad (1)$$

Convert the equation i into dB =>

$$FSL = \{20\log 4 + 20\log 3.1416 - 20\log(3\times10^8)$$
$$- 20\times(-3)\log 10 - 20\times(-6)\log 10\}$$
$$+ 20\log d + 20\log f$$
$$FSL = 32.44 + 20\log f\ (\mathrm{MHz}) + 20\log d\ (\mathrm{km})$$

6. c. Compute the diffraction loss and the Fresnel zone within which the tip of the obstruction lies where the effective height (h) of obstruction screen is positive and h = 25m. Assume frequency is 900 MHz, distance from transmitter to obstacle is 1 km and from obstacle to receiver is 1 km. The approximation solution for diffraction loss due to the presence of knife edge is given below: [4]

$G_d(dB)$	υ
0	≤ -1
$20\log(0.5-0.62\upsilon)$	$[-1,0]$
$20\log(0.5\, e^{-0.95\upsilon})$	$[0,1]$
$20\log(0.4-(0.1184-(0.38-0.1\upsilon)^2)^{1/2})$	$[1, 2.4]$
$20\log(0.225/\upsilon)$	> 2.4

Solution:

Check Lecture 11 example 3.7

7. a. Write short note on following terms: [4]

i) Coherence Bandwidth

ii) Doppler Shift

Solution: by Rabab 039

(i) Coherence Bandwidth (B_c) is a statistical measure of frequency range where channel amplitude is flat. The amplitudes of different frequency components are correlated here. If they are 90% correlated, $B_c = 1/50\sigma_t$. If 50% correlated, $B_c = 1/5\sigma_t$.

(ii) When a transmitter or receiver is moving, the frequency of the received signal changes. This change in frequency is called Doppler Shift. If the mobile is moving towards the direction of arrival of the signal, doppler shift, f_d is positive, and if moving away, it is negative. If moving towards the signal at an angle θ, $f_d = v\cos\theta/\lambda$.

7. b. Describe different types of small scale fading based on time delay spread and Doppler spread. [5]

Solution:

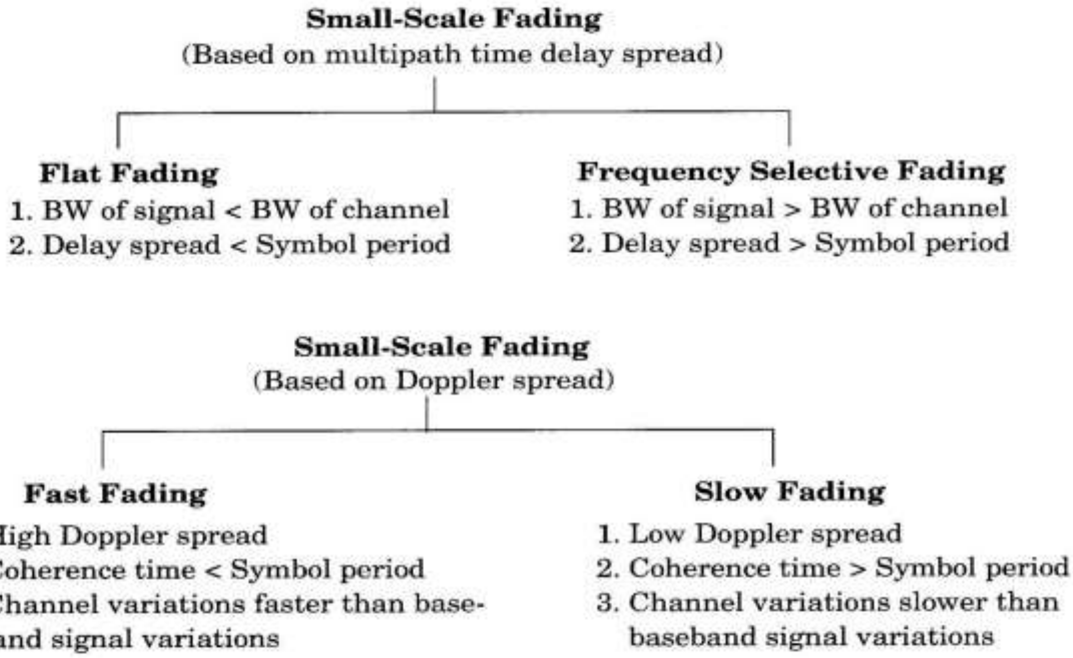


Figure 5.11 Types of small-scale fading.

7. c. A local spatial average of a power delay profile is shown in Figure 7(c).

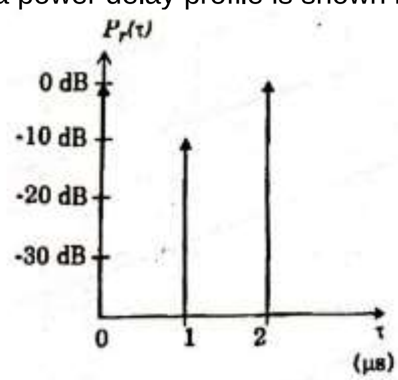


Figure: 7(c)

i) Determine the rms delay spread and mean excess delay for the channel.
ii) Determine the maximum excess delay (20 dB).
iii) If a mobile traveling at 30 km/hr receives a signal through the channel, determine the coherence time of the channel.

Solution: by Rabab 039 (correct if necessary)

(i)

* Enigma 7(b)

(i) At $\tau_0 = 0$, $a_k^2 = 0 \text{ dB} = 10^{\frac{0}{10}} = 1$

at $\tau_1 = 1$, $a_k^2 = -10 \text{ dB} = 10^{-\frac{10}{10}} = 0.1$

at $\tau_2 = 2$, $a_k^2 = -20 \text{ dB} = 10^{-\frac{20}{10}} = 0.01$

$$\therefore \bar{\tau} = \frac{(0\times 1) + (1\times 0.1) + (2\times 0.01)}{(1 + 0.1 + 0.01)} = 0.108 \ \mu s$$

$\Rightarrow$ Mean excess dela

$$\therefore \overline{\tau^2} = \frac{(0^2\times 1) + (1^2\times 0.1) + (2^2\times 0.01)}{(1 + 0.1 + 0.01)} = 0.126 \ \mu s$$

$\therefore$ rms delay spread, $\sigma_\tau = \sqrt{0.126 - (0.108)^2}$

$= 0.338 \ \mu s$

(ii) From the figure we can see that,

First 20dB arrival τ_0 = 0 μs; and, last 20dB arrival τ_{20} = 2 μs

So, τ_{max} (20dB) = 2 - 0 = 2 μs

(iii)

Coherence time, $T_c = 9/(16\pi f_m)$, where $f_m = v/\lambda$, $\lambda = c/f$

Here,

v = 30 kmph = 8.33 m/s

c = 3x10^8

f = f

$T_c = 9/(16\pi f_m)$ = …

Recursive40 Final

5. a. Why do we use cell sectoring? Explain with an example that cell sectoring decreases trunking efficiency. [5]

Solution: 039

Cell sectoring is the technique of using directional antennas for reducing the number of cells per cluster by reducing co-channel reuse ratio and keeping cell radius constant, thus increasing frequency reuse. We use it to overcome the problem of cell splitting where cell radius is

decreased and thereby increases CCI.
Check Origin 42 3.b

5. b. Describe free space loss (FSL) and prove that

FSL= - 20 log(f) + 20 log(d) - 10 log(AtAr) + 169.54 dB

Solution: 039
If no other sources of attenuation or impairment are assumed, a transmitted signal attenuates over distance because the signal is being spread over a larger and larger area.This form of attenuation is known as free space loss. It can be expressed in terms of the ratio of the radiated power Pt to the power Pr, received by the antenna.

$$\frac{P_t}{P_r} = \frac{(4\pi)^2 (d)^2}{G_r G_t \lambda^2} = \frac{(\lambda d)^2}{A_r A_t} = \frac{(cd)^2}{f^2 A_r A_t}$$

$$L_{dB} = 20\log(\lambda) + 20\log(d) - 10\log(A_t A_r)$$
$$= -20\log(f) + 20\log(d) - 10\log(A_t A_r) + 169.54\text{dB}$$

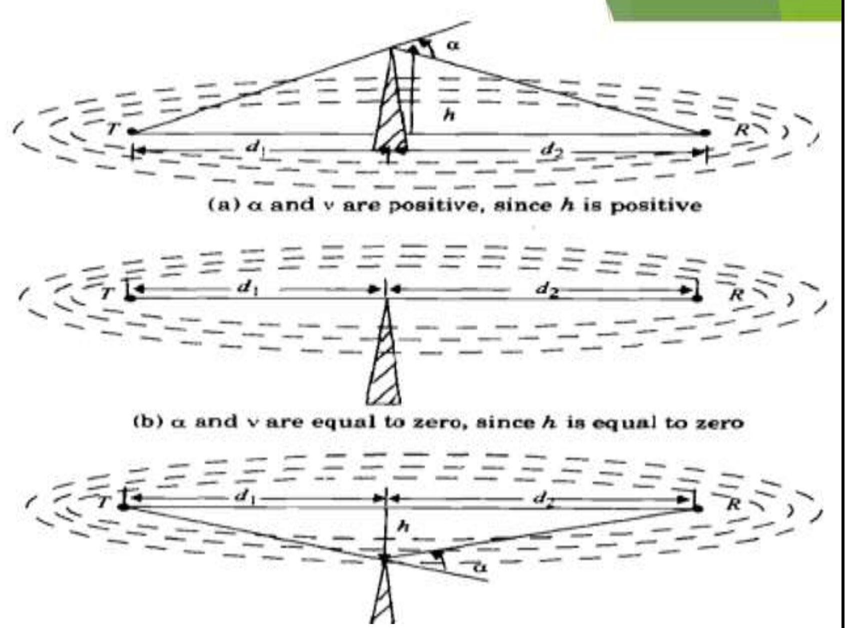
5. c. Determine the isotropic free space loss at 4 GHz for the shortest path to a synchronous satellite from earth (35,863 km). What is the power received at the satellite antenna? (Assume antenna gain of both the satellite and ground-based antennas are 44 dB and 48 dB, respectively. a transmit power of 250 W at the earth station.)

Solution: given above (Lecture 10, example 3)

6. a. Briefly describe the three cases of *Fresnel zone* with an appropriate diagram. [5]

Solution:

Three Cases:



6. b. For the knife edge diffraction model, show that the phase difference between direct signal and diffracted signal $\emptyset = 2\pi/\lambda[\frac{h^2((d1+d2)}{2d1d2}].$ [5]

Solution:
Check Origin42 6.b

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6. c. If the received power at reference distance do = 1km is equal to 1 Watt, find the received powers at distance of 5 km from the same transmitter with transmitting power of 1 Watt for the path loss model i) Free space and ii) Two ray ground reflection. Assume that, f = 1800 MHz, ht = 40 m, hr = 3 m, Gt = Gr = 1 Watt. [4]

Solution:

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7. a. What is power delay profile? Define the three time dispersion parameters that can be determined from a power delay profile. [5]

Solution:
The Power Delay Profile $P_h(\tau)$ indicates how channel power is distributed along time delay τ.

Time Dispersion Parameters

The mean excess delay

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

The rms delay spread

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

The maximum excess delay

$\tau_{max} = \tau_X - \tau_0$
τ_0 : the first arrival
τ_X : the max. delay at which a multipath component is within X dB

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7. b. Show that the mobile radio channel can be modeled as a linear time varying channel. [5]

Solution:

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7. c. Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving
(i) directly towards the transmitter,
(ii) directly away from the transmitter, and
(iii) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

Solution: 039

Therefore, wavelength $\lambda = c/f_c = \dfrac{3 \times 10^8}{1850 \times 10^6} = 0.162\,\text{m}$

Vehicle speed v = 60 mph = 26.82 m/s

(a) The vehicle is moving directly towards the transmitter.
The Doppler shift in this case is positive and the received frequency is given by equation (4.2)

$$f = f_c + f_d = 1850 \times 10^6 + \frac{26.82}{0.162} = 1850.000165\ \text{MHz}$$

(b) The vehicle is moving directly away from the transmitter.
The Doppler shift in this case is negative and hence the received frequency is given by

$$f = f_c - f_d = 1850 \times 10^6 - \frac{26.82}{0.162} = 1849.999834\ \text{MHz}$$

(c) The vehicle is moving perpendicular to the angle of arrival of the transmitted signal.
In this case, $\theta = 90°$, $\cos\theta = 0$, and there is no Doppler shift.
The received signal frequency is the same as the transmitted frequency of 1850 MHz.