Knife-edge Diffraction Model

The electric field due to the diffracted path is

$$E_d = E_o \exp(-j\phi) \quad \text{—— (1)}$$

The difference betn the direct path and the diffracted path called the excess path length (Δ) can be obtained —

$$\Delta = \sqrt{d_1^2 + h^2} + \sqrt{d_2^2 + h^2} - (d_1 + d_2)$$

$$= d_1\sqrt{1 + \frac{h^2}{d_1^2}} + d_2\sqrt{1 + \frac{h^2}{d_2^2}} - d_1 - d_2$$

$$\approx d_1\left(1 + \frac{h^2}{2d_1^2}\right) + d_2\left(1 + \frac{h^2}{2d_2^2}\right) - d_1 - d_2$$

$$\boxed{\sqrt{1+x} = 1 + \frac{x}{2}, \text{ for } x \ll 1}$$

$$\simeq \frac{h^2}{2}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)$$

$$= \frac{h^2(d_1 + d_2)}{2 d_1 d_2}$$

$$\simeq \frac{h^2}{2}\left(\frac{1}{d_1}+\frac{1}{d_2}\right) \qquad \left[\sqrt{1+x} = 1+\frac{x}{2},\ \text{for } x<<1\right]$$

$$= \frac{h^2(d_1+d_2)}{2\,d_1 d_2}$$

The angle $\alpha = \beta + \gamma$, since $d_1, d_2 >> h$

$$\tan\beta = h/d_1 ,\quad \tan\gamma = h/d_2$$

$$\beta = \frac{h}{d_1} ,\quad \gamma = \frac{h}{d_2} \qquad [\tan x \simeq x$$

$$\therefore \alpha = \frac{h}{d_1} + \frac{h}{d_2} \simeq \frac{h(d_1+d_2)}{d_1 d_2}$$

The phase difference is

$$\frac{\Delta}{c} = \frac{\phi}{2\pi f}$$

$$\phi = \omega\, \delta_d = 2\pi f \frac{\Delta}{c} = 2\pi f \frac{\Delta}{\lambda f} = \frac{2\pi}{\lambda}.$$

$$= \frac{2\pi}{\lambda} \cdot \frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right) = \frac{\pi}{2} h^2 \frac{2(d_1+d_2)}{\lambda d_1 d_2}$$

Time delay betn arrival of two components, $\delta_d = \frac{\Delta}{c} = \frac{\phi}{2\pi f}$

The phase difference is usually normalized using Fresnel-Kirchhoff parameter ν, given by

$$\nu = h\sqrt{\frac{2}{\lambda}\left(\frac{d_1+d_2}{d_1 d_2}\right)} = \alpha\sqrt{\frac{2}{\lambda}\left(\frac{d_1 d_2}{d_1+d_2}\right)}$$

— These two parameter describes where the obstacle lies and how sharp it is

From eqn ② ⇒

$$\phi = \frac{\pi}{2}\nu^2$$

From eqn ① ⇒

$$E_d = E_0 \exp\left(-j\frac{\pi}{2}\nu^2\right)$$

Now, we include the effect of all other rays produced by the Huygen's sources. These are produced for all the Huygen's sources above the screen, and hence we sum or integrate from ν to ∞.

$$E_d = E_0 \exp\left(-j\frac{\pi}{2}v^2\right)$$

Now we include the effect of all other rays produced by the Huygen's sources. These are produced for all the Huygen's sources above the screen, and hence we sum or integrate from v to ∞.

$$E_{TOT} = E_0 \frac{1+j}{2} \int_v^\infty \exp\left(-j\frac{\pi}{2}t^2\right)dt$$

or, $$\frac{E_{TOT}}{E_0} = \frac{1+j}{2} \int_v^\infty \exp\left(-j\frac{\pi}{2}t^2\right)dt$$

or, $$\frac{E_{TOT}}{E_0} = F(v)$$ — Complex Fresnal Integral ~~Diffraction Loss (like Free space loss)~~

or, $$\left|\frac{E_{TOT}}{E_0}\right|^2 = |F(v)|^2$$

or, $$G_d(dB) = 20\log|F(v)|$$

$G_d(dB)$ — Diffraction ga

for positive value

for negative value