2-Ray Ground Reflection Model

(1) Determine Total Received E-field (in V/m) E_{TOT}

↳ We represent EM wave in Electric field (E-field) and Magnetic field (H-field). For geometric calculation we take E-field.

Here, E_{TOT} = Total received E-field

E_{LOS} = Direct LOS component

E_g = Ground reflected component

h_t = Height of the Transmitter (T_x)

h_r = Height of the Receiver (R_x)

d = Distance between T_x and R_x

let E_0 = Free space E-field (V/m) at distance d_0, Propagating Free space E-field al distance $d > d_0$ is given by,

$$E(d,t) = \frac{E_0 d_0}{d} \cos\left(\omega\left(t - \frac{d}{c}\right)\right) \quad \text{—} (1)$$

$$\left[\text{Propagation delay} = \frac{\text{distance}}{\text{speed}} = \frac{d}{c}\right]$$

↳ With the help of eqn (1),

E-field for LOS relative to E_0 given by,

$$E_{LOS}(d', t) = \frac{E_0 d_0}{d'} \cos\left(\omega\left(t - \frac{d'}{c}\right)\right) \quad \text{—} (2)$$

[d' = Distance of LOS wave]

E-field for reflected wave relative to E_0 given by,

$$E_g(d'', t) = \frac{E_0 d_0}{d''} \cos\left(\omega\left(t - \frac{d''}{c}\right)\right) \quad \text{—} (3)$$

[d'' = Distance of reflected wave]

↳ The electric field $E_{TOT}(d,t)$ can be expressed as the sum of equations 2 and 3.

$$E_{TOT}(d,t) = \frac{E_0 d_0}{d'} \cos\left(\omega\left(t - \frac{d'}{c}\right)\right) + (-1)\frac{E_0 d_0}{d''} \cos\left(\omega\left(t - \frac{d''}{c}\right)\right)$$

[(-1) = Reflection co-efficient]

↳ Evaluate E-field when reflected path arrives at receiver at $t = \frac{d''}{c}$,

$$E_{TOT}\left(d, t = \frac{d''}{c}\right) = \frac{E_0 d_0}{d'} \cos\left(\omega\left(\frac{d'' - d'}{c}\right)\right) - \frac{E_0 d_0}{d''} \cos 0^\circ$$

$$= \frac{E_0 d_0}{d'} \cos \theta_\Delta - \frac{E_0 d_0}{d''}$$

↳ If d becomes large, then $d'' = d' = d$.

$$E_{TOT}\left(d, t = \frac{d''}{c}\right) = \frac{E_0 d_0}{d} \cos \theta_\Delta - \frac{E_0 d_0}{d}$$

$$= \frac{E_0 d_0}{d}\left(\cos \theta_\Delta - 1\right)$$

↳ Determine exact E-field for 2-ray ground reflection model at d using phasor diagram,

$$E_{TOT}(d) = \sqrt{\left(\frac{E_0 d_0}{d}\right)^2 (\cos\theta_\Delta - 1)^2 + \left(\frac{E_0 d_0}{d}\right)^2 \sin^2\theta_\Delta}$$

$$= \frac{E_0 d_0}{d} \sqrt{2 - 2\cos\theta_\Delta}$$

$$= 2\frac{E_0 d_0}{d} \sin\left(\frac{\theta_\Delta}{2}\right)$$ [using Trigonometric identities]

$$= 2\frac{E_0 d_0}{d}\left(\frac{\theta_\Delta}{2}\right)$$ [using $\sin\left(\frac{\theta_\Delta}{2}\right) = \frac{\theta_\Delta}{2}$] — 4

(2) Compute Path difference, Phase difference and Time delay

↳ compute path difference using the method of images,

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$= d\sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2} - d\sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2}$$

[From Tylor series, $\sqrt{1+x} = 1 + \frac{x}{2}$; for $x \ll 1$]

$$= d\left(1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right) - d\left(1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right)$$

$$= \frac{1}{2d}\left((h_t + h_r)^2 - (h_t - h_r)^2\right)$$

$$= \frac{2 h_t h_r}{d}$$

↳ Compute phase difference,

$$\theta_\Delta = \omega \tau_d = 2\pi f \frac{\Delta}{c} = 2\pi f \frac{\Delta}{\lambda f} = \frac{2\pi\Delta}{\lambda}$$

↳ Compute Time delay,

$$\tau_d = \frac{\Delta}{c} = \frac{\theta_\Delta \lambda}{2\pi \cdot f\lambda} = \frac{\theta_\Delta}{2\pi f}$$

(3) Determine Received Power

↳ From eq^n (4) →

$$E_{TOT}(d) = 2\frac{E_0 d_0}{d}\left(\frac{\theta_\Delta}{2}\right) = \frac{2E_0 d_0}{d} \times \frac{2\pi\, 2 h_t h_r}{2\pi d}$$

$$= \frac{4\pi E_0 d_0 h_t h_r}{\lambda d^2}$$

$$|E_{TOT}(d)|^2 = \left|\frac{4\pi E_0 d_0 h_t h_r}{\lambda d^2}\right|^2 \quad [P \propto E^2]$$

$$P_r(d) = \frac{(4\pi)^2 E_0^2 d_0^2 h_t^2 h_r^2}{\lambda^2 d^4} = \frac{(4\pi)^2 P_0 d_0^2 h_t^2 h_r^2}{\lambda^2 d^4} \quad [P_0 \propto E_0^2] \quad (5)$$

↳ Free Space Loss for LOS,

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{G_t G_r \lambda^2} \Rightarrow P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$$

Here, $P_0 = P_r$; $P_0 = P_t G_t G_r \dfrac{\lambda^2}{(4\pi d_0)^2}$

↳ From eqn (5) →

$$P_r = P_t G_t G_r \frac{\cancel{\lambda^2}}{\cancel{(4\pi d_0)^2}} \cdot \frac{\cancel{(4\pi d_0)^2} h_t^2 h_r^2}{\cancel{\lambda^2} d^4}$$

$$= P_t G_t G_r \frac{h_t^2 h_r^2}{d^4} \quad \text{(Proved)}$$

↳ Free space loss for 2-ray model,

$$\frac{P_t}{P_r} = \frac{d^4}{G_t G_r h_t^2 h_r^2} \quad \text{(Proved)}$$

↳ Free Space loss/Path loss for 2-ray model in dB,

$$L_{dB} = 40 \log d - 10 \log(G_t G_r) - 20 \log(h_t h_r) \quad \text{(Proved)}$$