MEC2342/AEE2000: Control [2023/24]

Group Design Project

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In this project, you will be exposed to an authentic assessment, in which your theoretical knowledge will be applied to perform modelling, simulation and test of realistic feedback control systems.

As part of your team, you are required to model, analyse, design, and simulate the pitch control system of a helicopter. The aim of the group project is to design a control system using classical control techniques and evaluate the sets of results.

1. Project Scope

In this project you will be exposed to implement control synthesis methods using computer simulation. Starting from a pitch dynamic model, you will get to:

1) Analyse the 1DoF model of the system (plant) and its open-loop response to an impulse and step input. Describe the steady state and dynamic responses of systems in the time domain.
2) Analyse the stability of the control system using a pole-zero map, Rout's Stability criterion and Nyquist stability criterion.
3) Design a controller to derive desired properties in the output response. Determine what kind of controller is necessary: Proportional, Proportional-Integral (PI) or Proportional-Integral Derivative (PID). Describe how the properties of the PID feedback controller impact the output response.
4) Design the controller using the Root-Locus method, tune it through iterations, and propose possible solutions.
5) Evaluate the design solutions against the design's specifications using time-domain and frequency-domain techniques.
6) Evaluate the controller's energy consumption.
7) When inserting a constraint or limit on the control action signal, evaluate the saturation effect.
8) Analyse the multiple-input and multiple-output (MIMO) 2DoF helicopter model and explain the coupling effect when implementing both the pitch 1DoF model system and the given yaw controller system.

You can work in groups of 4 to complete the tasks defined in Section 4.

2. Assessment

The group project contributes to 25% of module grade. Students are required to submit a technical report and all MATLAB files.

There are five sections of tasks in this project. Students are required to refer to the assessment rubrics in appendix 2.

2.1. Submission

Interim Report

You are required to submit a short interim report of Task 1. In the interim report you need to demonstrate that your team is able to get the open-loop response as shown in Figure 3a.

The final report must contain the results of Task 1.

The Interim Report Submission date is in Week 7, Tuesday 18 June 2024 at 23:59.

Full Report

The Full Report Submission date is in Week 10, Wednesday 10 July 2024 at 23:59.

Please submit one PDF format file via the Dropbox. Email will not be accepted as submission.

The Matlab files will be submitted in a zip file format in a second link of the

The system will accept only one submission. In the event that the team requires to re-submit, the team has maximum 24 hours to contact the Module Lead to request for a change of the version. If it is after the deadline, the ML will assess and in case of approval, it will count a late submission.

2.2. Simulation

Complete all tasks defined below and submit the analysis, design, and simulation of the control system. You will be evaluated based on

(i) Correctness of the analysis, design, and implementation in MATLAB Simulink.

(ii) Presentation of the analysis, synthesis and simulation: ensure that all diagrams' models are presented clearly and annotated. When implementing code, ensure that all code is well commented,and all variables are defined. The code must be added in the appendix.

2.3. Technical Report

Based on the tasks defined in Section 4, you are required to produce a technical report which introduces your analysis, design and simulation model. Your report should cover the following:

1) Explain all the steps taken to analyse, synthesise and implement your simulation code. Use control theory to explain your findings.
2) Analysis and Design of the controller system must be solved by hand calculations and all workings reported. Show that you get the same answer with MATLAB simulation
3) Use MATLAB to plot bode diagrams and time response, clearly label each curve and details of the plot (e.g., units).
4) Present your analysis, design steps and simulation results for the different tasks below.

The Page limit is 15 pages for the reports, keep it concise and avoid superfluous information, if more pages are required an appendix can be added, if in doubt check with Module Lead. Some guidelines on how to structure your report is provided in Appendix 1. The reports will be assessed against the criteria provided in Appendix 2.

3. Plant / Dynamic System Model

In Figure 1, the system or plan represents a conventional dual-rotor helicopter. The front rotor that is horizontal to the ground predominantly affects the motion about the pitch axis while the back or tail rotor mainly affects the motion about the yaw axis (about the shaft).

Figure 1. 2DOF helicopter system developed by Quanser.

The tail rotor in helicopters is also known as the anti-torque rotor because it is used to reduce the torque that the main rotor generates about the yaw. Without this, the helicopter would be difficult to stabilize about the yaw axis. Because the rotors on the 2DOF helicopter System are the same size and equidistant from each other, the tail rotor also generates a torque about the pitch axis. As a result, both the front and back/tail rotors generate a torque on each other.

The free-body diagram of the 2DOF helicopter system is illustrated in Figure 2.

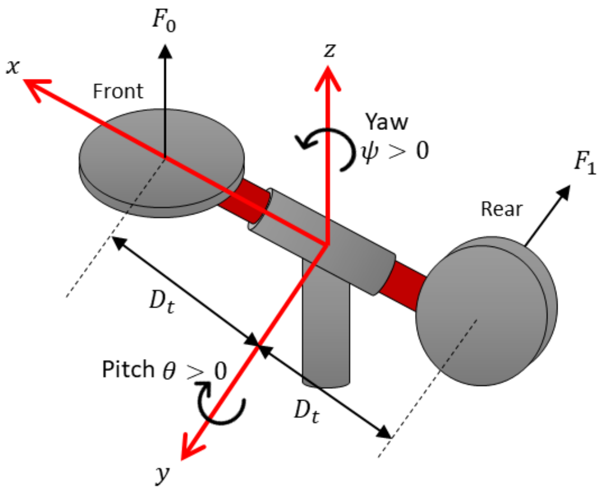

Figure 2: Simple free-body diagram of 2DOF helicopter system.

The following conventions are used for the modelling:

1. The helicopter is horizontal and parallel with the ground when the pitch angle is zero, i.e. θ = 0.

2. The pitch angle increases positively, $\dot{\theta}(t)$ > 0, when the body rotates clockwise (CCW) about the y-axis,i.e., when the front rotor goes up.

3. The yaw angle increases positively, $\dot{\psi}(t)$ > 0, when the body rotates counter-clockwise (CCW) about the Z axis.

4. Pitch increases, $\theta(t)$ > 0, when the front rotor voltage is positive $Vp > 0$.

5. Yaw increases, $\dot{\psi}(t)$ > 0, when the back (or tail) rotor voltage is positive, $Vy > 0$.

When voltage is applied to the pitch motor, Vp , the speed of rotation results in a force, F_0 , that acts normal to the body at a distance D_t from the pitch axis. The rotation of the propeller generates a torque about the pitch rotor motor shaft which is in turn seen about the yaw axis. Thus, rotating the pitch propeller does not only cause motion about the pitch axis but also about the yaw axis. As described earlier, that's why conventional helicopters include a tail, or anti-torque, rotor to compensate for the torque generated about the yaw axis by the large, main rotor. Similarly, the yaw motor causes a force F_1 that acts on the body at a distance D_t from the yaw axis as well as a torque about the pitch axis.

It can be shown that a simple linear model that takes this coupling into account and represents the motions of the 2DOF helicopter system about the horizontal, i.e. when the body is parallel with the ground. The equations of motion (EOM) are:

$$J_p\ddot{\theta}(t) + D_p\dot{\theta}(t) + K_{sp}\theta(t) = \tau_p, \qquad \text{Eq. (1)}$$

$$J_y\ddot{\psi}(t) + D_y\dot{\psi}(t) = \tau_y, \qquad \text{Eq. (2)}$$

where the torques acting on the pitch and yaw axes are

$$\tau_p = K_{pp}D_tV_p + K_{py}D_tV_y \qquad \text{Eq. (3)}$$

$$\tau_y = K_{yp}D_tV_p + K_{yy}D_tV_y \qquad \text{Eq. (4)}$$

Remark about the coupling between the pitch and yaw rotors: The total torque acting on each axis is generated from both rotors. Thus, the total torque acting on the pitch equals τ_p and the total torque on the yaw is τ_y.

The parameters used in the EOMs above are:

- J_p is the total moment of inertia about the pitch axis,
- D_p is the damping about the pitch axis,
- D_y is the damping about the yaw axis,
- K_{sp} is the stiffness about the pitch axis,
- K_{pp} is the thrust force gain acting on the pitch axis from the pitch/front rotor,
- K_{py} is the thrust force gain acting on the pitch from the yaw/rear rotor,

- K_{yy} is the thrust force gain acting on the yaw axis from the yaw/rear rotor,
- K_{yp} is the thrust force gain acting on the yaw axis from the pitch/front rotor,
- V_p is the voltage applied to the pitch rotor, and
- V_y is the voltage applied to the yaw rotor motor.
- D_y is the distance between the Aero 2 pivot and centre of the rotor.

The total moment of inertia and damping about the yaw axis is J_y and D_y, respectively, K_{yy} is the thrust force from the yaw rotor, and K_{yp} is the cross thrust force acting on the yaw axis from the pitch rotor.

The total moment of inertia acting about the pitch and yaw axes are

$$J_p = 23188500.45 \text{ kg-mm}^2$$
$$J_y = 23810415.95 \text{ kg-mm}^2$$

The numerical values of all parameters are in Appendix 3 and also provided in a m-file (MATLAB format).

For this project, the requirement is to design a control system for the pitch angle. Assuming linearity and applying superposition, the pitch control system will focus only on the model of the pitch angle.

$$J_p\ddot{\theta}(t) + D_p\dot{\theta}(t) + \nu K_{sp}\theta(t) = \tau_p, \qquad \text{Eq. (5)}$$

where the torques acting on the pitch and yaw axes are

$$\tau_p = K_{pp}D_tV_p \qquad \text{Eq. (6)}$$

ν is a constant calculated with the equation: ν =(0.05/12)*#Gn+0.9955, where #Gn is your group number.

3.1. Design Specifications

In this project, you will design a feedback controller so that in response to a step command of pitch angle the actual pitch angle overshoots less than 10%, and a steady-state error of less than 1 deg. of angle. Design and evaluate the controller for a step input of 15 degrees of pitch angle.

In summary, the design specifications (design requirements) are the following:

- Overshoot less than 10%.
- Steady-state error less or equal than 1 deg. of pitch angle.
- Peak time: $t_p \leq 2.5$ sec.
- Constraint: Maximum voltage of $|V_y| \leq 24$V and $|V_p| \leq 24$V.

You are required to design a controller using classical control techniques.

Hypothesis 1: The hypothesis is that using control classical control techniques it is possible to meet all the design requirements or specifications.

Hypothesis 2: It is not possible to meet all the specifications. However, it is possible to tune the controllers meet as many as possible specifications at the expense of compromising not meeting some of the specifications. As part of your task 5, tune the inferior controller(s) using heuristic techniques to find the trade-offs. Attempt to improve the performance of the inferior controllers until it is not possible to improve. Present your results using for example spider chart, parallel coordinates or other types of visualisation techniques.

4. Project Tasks

4.1. Task 1 – Modelling and Transfer function representation of the system.

1) Starting from the free body diagram of the 2DOF helicopter system derive the equations of motion (EOM) Eq. 1 and Eq. 2. Assume small angle approximation to derive the EOM. Explain with mathematical evidence the implication of the small angle approximation (e.g., linearity).

2) Derive the transfer functions (i.e., $G_1(s)$, $G_2(s)$, $G_3(s)$, $G_4(s)$) to obtain the following block diagram, see Figure 3.

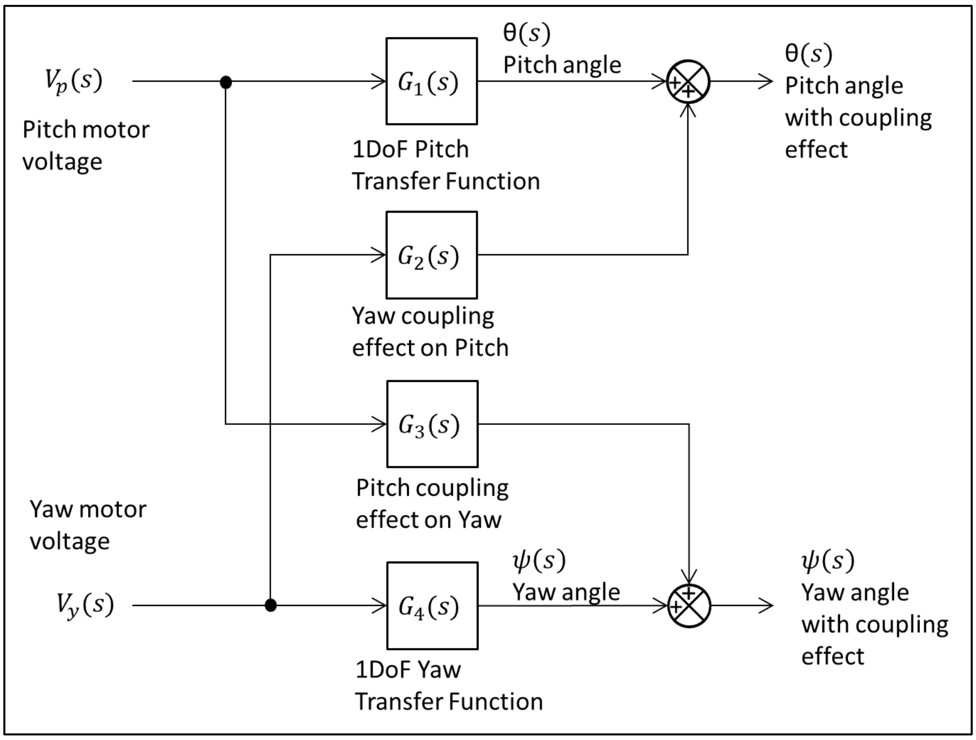

Figure 3a: Block Diagram of the 2DOF helicopter System Model.

1) Analyse the cross-coupling effects by showing the open-loop system response plots for the following three cases:
   - Set the voltage of the yaw motor to zero and apply a step input of 2 volts on the pitch motor.
   - Set the voltage of the pitch motor to zero and apply a step input of 5 volts on the yaw motor.
   - Apply a step input of 5 volts on the yaw motor and 2 volts on the pitch motor.
2) You are required to summarise all these findings in the report.

Download the files TASK1_Model_2DOF_Helicopter.zip

To test that your model is working, input the four transfer function in the Simulink Model FILE3_s_2dof_Model_vis.slx and run the MATLAB file named:

FILE2_Model_2DOF_Helicopter_PlotFigures_Simulation_vis.m

You should get step response as in Figure 3a.

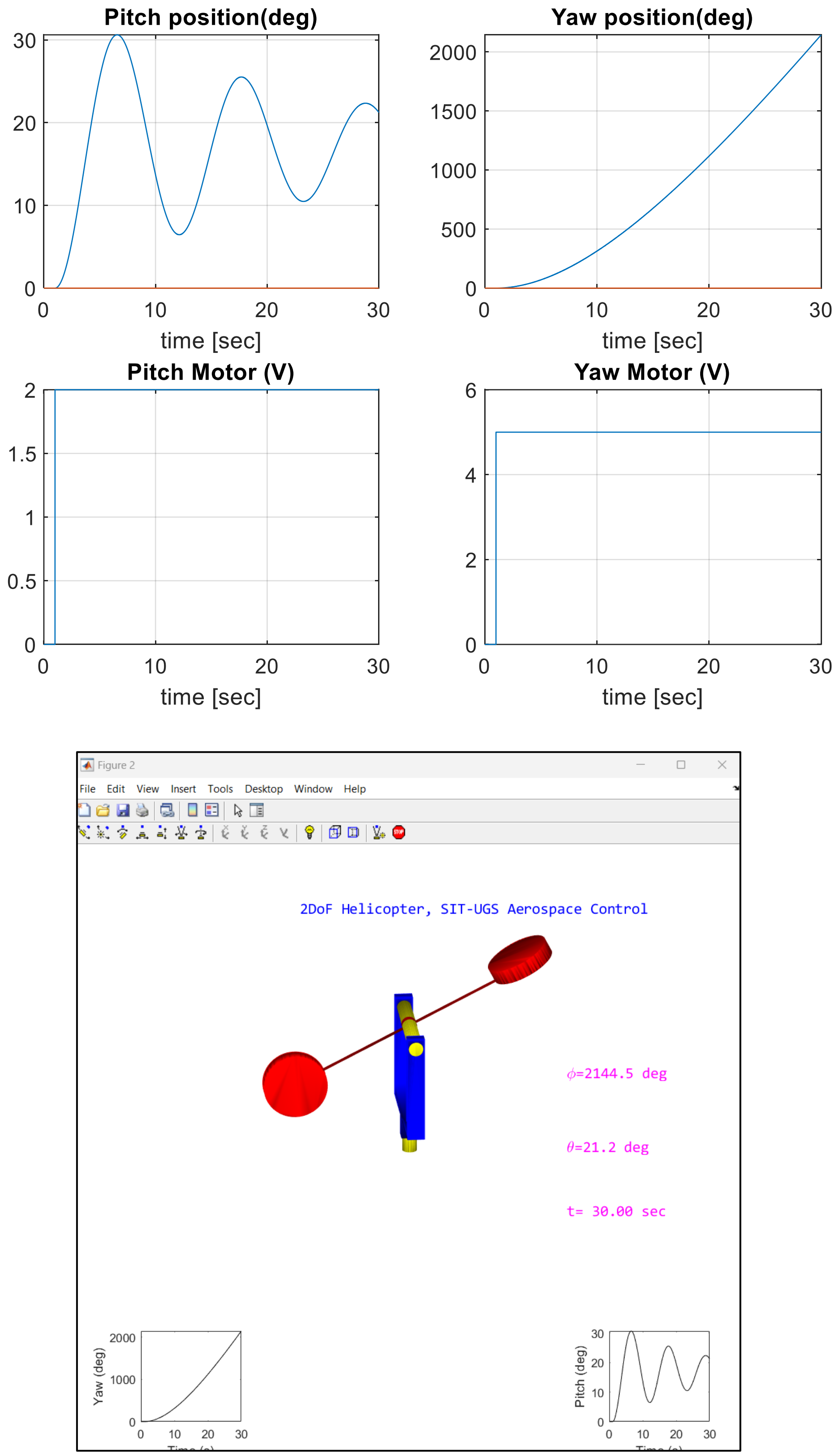

Figure 3a: Simulation Results of the 2DOF helicopter System Model.

4.2. Task 2 – Transfer function and State-space representation.

Using the models from task 1, analyse the open-loop system response of the pitch angle:

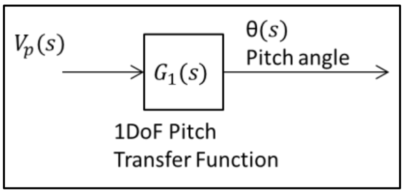

Figure 4: Transfer Function of the Pitch Angle Model.

3) Show how the open-loop system response to a step input of 2 volts.
4) Would the open-loop response satisfy the design specifications?
5) Explain how to analyse the stability of the open-loop system.
6) Show how the close-loop system or feedback system performs to a step input of 15 degrees (11 degrees), use a proportional controller set to 1 (unit gain).
7) Would the closed-loop response satisfy the design criteria? Explain your findings.
8) Explain how to analyse the stability of the closed-loop system or feedback system. It is recommended to use Routh's stability criterion.
9) Create MATLAB scripts to simulate and show the step response of your control system and you are required to summarise all these findings in the report.

4.3. Task 3 – Root Locus Approach:

Based on the design specification design a controller using classical control techniques:

10) Design a controller to derive desired properties in the output response. Determine what kind of controller is necessary: Proportional, Proportional-Integral (PI) or Proportional-Integral Derivative (PID). Describe how the properties of the chosen feedback controller impact the output response.
11) You are required to provide details of the steps taken to design the controller. Use the lecture slides provided.
12) Design the controller using the Root-Locus method, tune it through iterations, and propose possible solutions.
13) Use MATLAB commands to create plots and using the plots explain how the controller is ''shaped''.
14) Explain the characteristics of your controller and how the controller was designed to meet the design specifications.
15) Show how the close-loop system or feedback system performs to a step input of 15 degrees of pitch angle.
16) Would the closed-loop response satisfy the design criteria? If not tune the controller and report your design iterations.
17) Create a Simulink model to simulate and show the step response of your control system.

You are required to summarise all these findings in the report.

18) What are the closed loop stability margins? Show both the Nyquist plot and the Bode plot to respond to the question. You may use MATLAB plots to provide your answers.

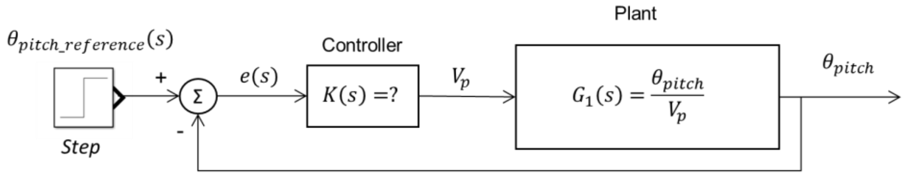

Figure 5: Closed-Loop Control System of the Pitch Angle System.

4.4. Task 4 – Evaluation of Control System

19) Improve the initial or inferior controller by tuning the controller using heuristic techniques.

20) Create a table to report all the iterations by means of comparing against the design specifications.

21) Illustrate the evolution of design iterations using the step response (time domain analysis).

22) Identify trade-offs between design specifications by means of comparing all the design iterations. Explain your findings. Use parallel coordinates or any other appropriate visualisation techniques to compare multiple objectives or specifications, show objectives or specifications are conflicting.

23) Prove or disprove hypothesis 1 and 2 with evidence of your results.

24) You are required to summarise all these findings in the report.

4.5. Task 5 – Evaluation of Energy Consumption & performance in the 2DOF helicopter control system.

25) Implement both the pitch 1DoF model system and the given yaw controller system to control the full 2DOF helicopter, see Figure 6. Note that the controller $K_2(s)$ will be given and the controller $K_1(s)$ will be the one calculated in the previous tasks.

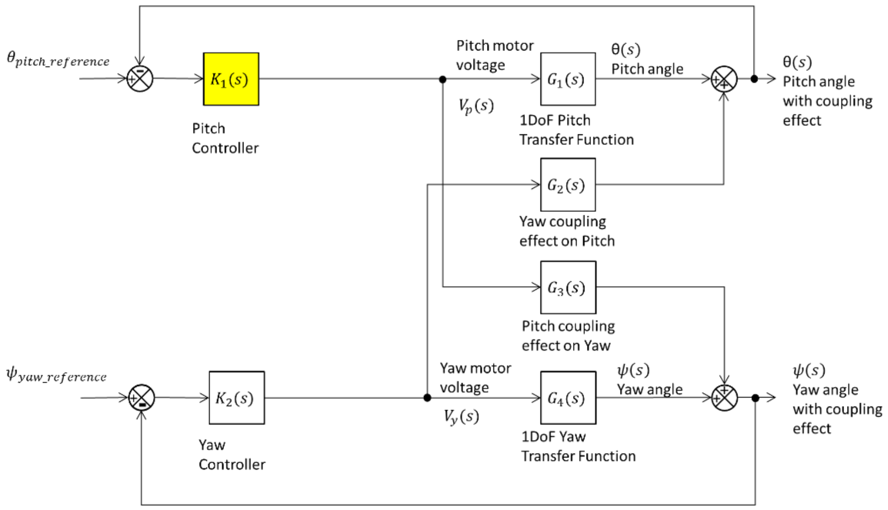

Figure 6: 2DOF helicopter control system

26) Evaluate the controller's notional cost of energy consumption by computing the following equation:

$$Notional\ Cost\ of\ Energy\ Consumption, J_C = \int_0^{t=30sec} \left(V_p^2(t) + V_y^2(t)\right) dt$$

Create a MATLAB script to compute the $Notional\ Cost\ of\ Energy\ Consumption, J_C$. Check the Matlab function "trapz", in matlab enter ''>help trapz''. This function is used to integrate time domain signals.

27) When inserting a constraint or limit on the control action signal, evaluate the saturation effect. Evaluate once again the notional cost of energy consumption.

The response will be evaluated with a series of step inputs and changes. The Simulink file is available in the xSiTe page.

28) Analyse the multiple-input and multiple-output (MIMO) 2DoF helicopter model (See Figure 7) and explain the coupling effect when implementing both the pitch 1DoF model system and the given yaw controller system.

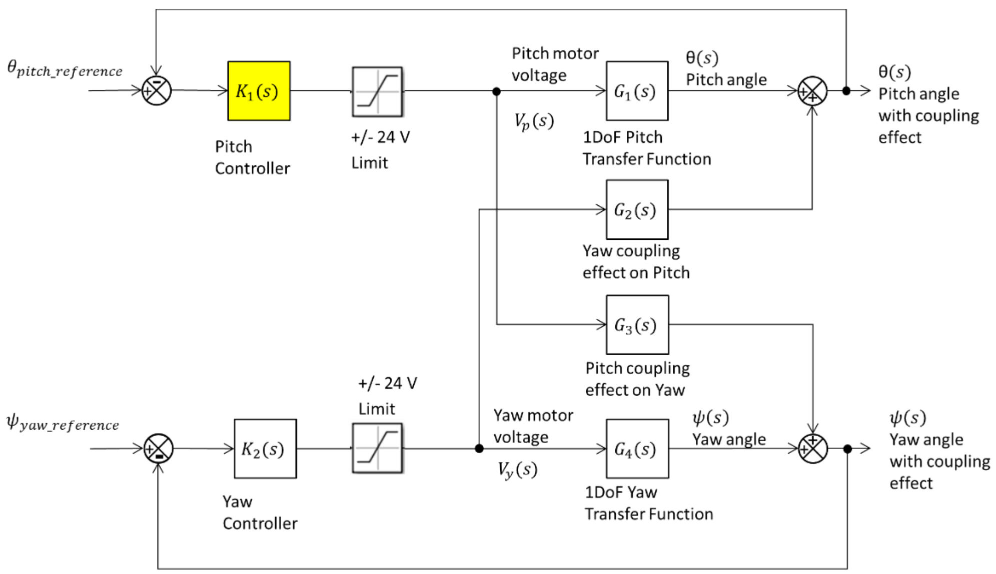

Figure 7: 2DOF helicopter control system with limit on the control action.

Appendix 1

Guidelines for the structure of the report

Report Technical Content:

Informative and persuasive:

Topic of discussion is relevant to the assignment brief. Content is adequately researched. The report covers:

- **Introduction** – establishes the purpose of the report; convincingly identifies the problem that motivates the project; establishes the context in which the problem occurs; identifies a suitable solution; clearly defines key terms
- **Body** – clearly and accurately describes the solution in keeping with the reader's needs; anticipates and addresses challenges to the solution's development and implementation; includes clearly labelled and reader-centred graphical information where appropriate
- **Conclusion** – emphasizes the benefits of adopting the solution, both to the reader and other stakeholders

Report Format & Organisation:

Front matter contains

- Title page – proper Title of the proposal, Module Name, Students' names
- Table of Contents
- Informative Summary

Body contains

- Introduction – provides a background with some review of literature, defines key terms, establishes the problem clearly, establishes the purpose of the proposal
- Research Methods – states the research methods and any limitations
- Findings & Analyses – states the findings and their analyses clearly, explains discusses the various options/ possible solutions clearly
- Conclusions & Recommendations – based on the conclusions drawn, recommend 1 solution which the proposal accurately describes in keeping with the reader's needs; anticipates and addresses challenges to the solution's development and implementation
- Graphics – includes any tables, pictures, charts or graphs clearly labelled and reader-centred graphical information where appropriate
- Short conclusion – emphasizes the benefits of adopting the solution, both to the reader and other stakeholders

End matter

- full list of references (sources cited in the report) in correct APA or IEEE style;
- properly labelled appendices to supply additional information where appropriate

Headings and sub-headings

- appropriately chosen to identify different sections and sub-sections of the report in a suitable sequence

Report Language:

- topic sentences – clearly stated and appropriately positioned
- paragraphs – unified in their content and coherent in their development
- syntax – effectively uses various sentence types to achieve intended purpose or meaning; shows a full range of simple, compound and complex structures
- grammar – uses accurately
- vocabulary, word usage & idiomatic expressions – appropriately uses in context
- fluency & cohesion – appropriately uses transitions/connections to present ideas fluently and cohesively
- punctuation/spelling – uses accurately
- style – presents ideas appropriately for the intended audience, i.e. a corporate decision maker who may not be a technical expert

Appendix 2

Assessment Rubrics – Report

A, A+, A-	B+, B, B-	C+, C	D+, D	F	Grade Awarded
Excellent (10-9)	**Very Good (8 -7)**	**Good (6 - 5)**	**Developing (4-3)**	**Poor (2-0)**	
WRITING (7.5%)					
Exceptionally clear, precise and concise English. Excellent spelling & grammar, few typos.	*Clear and well written, easy to understand, and mostly free of errors.*	*Most of the text is clear and easily understood. There are some issues with grammar and spelling.*	*The text can be understood but some elements are not entirely clear. A sizeable volume of errors is noticeable.*	*The volume and nature of the grammatical errors, combined with poor writing makes this report difficult to read.*	
ORGANISATION & STRUCTURE (5%)					
Structure is entirely correct with all sections correctly placed. Reading contents gives clear overview.	*A well organised report with all sections logically placed enhancing understanding of work*	*A report which is sufficiently well organised to make reading report easy.*	*There may be some issues with the structure, but these don't detract from overall quality.*	*Serious flaws in structure which makes it difficult to read and understand the report.*	
PRESENTATION AND FIGURES (2.5%)					
Professional standard of presentation. All illustrations are well formatted and presented.	*A clear and consistent presentation style making it easy to read. Most of the figures are clear and well presented.*	*There are some minor flaws in the presentation and the clarity of the figures, but overall a well presented report.*	*A number of basic errors present – inconsistent use of styles, margins etc. Figures are satisfactory.*	*Unacceptable presentation: untidy and inconsistent use of styles. Figures are messy and unclear.*	
TECHNICAL CONTENT AND QUALITY OF ANALYSIS (85%)– Introduction, Simulation, Production of Detailed Analysis and Design of Task 1 to 6, Conclusion					
Exemplary range of references indicating comprehensive background reading. Well informed and authoritative discussion of a significantly complex technical problem. Excellent breadth and depth of knowledge.	*An appropriate range of relevant references suggesting substantial background reading. Clear and reasoned arguments indicating a very good grasp of a difficult technical problem.*	*Sufficient references to indicate a good level of background reading. Arguments presented are of a reasonable technical level, and have been well considered and clearly stated.*	*Perhaps just enough references to suggest some background reading was undertaken. Too many www references. The arguments presented are of reasonable technical depth and show a satisfactory understanding.*	*Only a few references and majority are irrelevant. Little evidence of background reading. Very little evidence of critical discussion of technical work or results. Superficial understanding of problem.*	

Appendix 3 2DoF Helicopter Parameters