Homework 1 – ME 890 Fundamentals of Modern Control Theory

Q1: A discrete-time linear system can be expressed by the following transfer function:

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) = \frac{b_1 q^{-1} + \cdots + b_n q^{-m}}{1 + a_1 q^{-1} + \cdots + a_n q^{-n}}u(t),$$

where q^{-1} is the unit backward shift operator (please refer to the z-transform). With the above, an often-used model in system identification is the autoregressive exogenous input (ARX) modelmodel:

$$A(q^{-1})y(t) = B(q^{-1})u(t) + e(t),$$

where $e(t)$ represents an error or noise.

1) Show that the ARX model can be rewritten in the following linear regression form:

$$y(t) = \varphi^{\top}(t)\theta + e(t),$$

where

$$\varphi(t) = [-y(t-1) \quad \cdots \quad -y(t-n) \quad u(t-1) \quad \cdots \quad u(t-m)]^{\top},$$

$$\theta = [a_1 \quad \cdots \quad a_n \quad b_1 \quad \cdots \quad b_n]^{\top}.$$

2) Identify the parameter vector θ using the batch least squares and recursive least squares for the dryer based on the dataset "[96-006] Data of a laboratory setup acting like a hair dryer" at https://homes.esat.kuleuven.be/~smc/daisy/daisydata.html.

   (Please feel free to play with other datasets)