Homework 1 – ME 890 Fundamentals of Modern Control Theory

Reference: Lectures on Dynamic Systems and Control: https://tinyurl.com/DSCLec

Q1: Exercise 1.2 in Reference.

Q2: Exercise 1.3 in Reference.

Q3: Exercise 1.5 in Reference.

Q4: Exercise 1.6 in Reference.

Q5: Prove that $\|x\| = \max_i |x_i|$ is a norm for $x \in R^{n\times 1}$. In the literature, this is called ∞-norm.

Q6: Prove the Cauchy-Schwarz Inequality on Page 6 of Reference.

Q7: Define the following inner product and norm for $R^{3\times 1}$:

$$\langle x, y\rangle = x^\top Q y, \quad \|x\| = \sqrt{\langle x, x\rangle},$$

for $x, y \in R^{3\times 1}$, where Q is a symmetric positive definite matrix. Consider $M \subset R^{3\times 1}$ spanned by $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}4\\5\\6\end{bmatrix}$, and $y = \begin{bmatrix}7\\8\\12\end{bmatrix}$. Use the projection theorem to find $\hat{m}$ as the minimizing solution to

$$\min_{m\in M} \|y - m\|,$$

when $Q = \begin{bmatrix}1 & 0.5 & 0.2\\0.5 & 2 & 0.5\\0.2 & 0.5 & 3\end{bmatrix}$ and $Q = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$, respectively.

Q8: Define the following inner product and norm for C, the vector space of continuous functions over $[0,1]$:

$$\langle f(x), g(x)\rangle = \int_0^1 f(x)g(x)dx, \qquad \|f(x)\| = \sqrt{\langle f(x), f(x)\rangle},$$

where both $f(x)$ and $g(x)$ are continuous over $[0,1]$. Now consider the vector space of polynomials of order less than or equal to 3, denoted as $P \subset C$, based on the basis elements $\{1, x, x^2, x^3\}$. Use the projection theorem to find out the best polynomial within P to approximate $h(x) = e^x$ by solving

$$\hat{h}(x) = \arg \min_{p(x)\in P} \|h(x) - p(x)\|.$$

Verify your result by comparing the plots of $h(x)$ and $\hat{h}(x)$.