SUN YEN-SET UNIVERSITY

Geometry

SYSU_TheRavenChaser

doublehh

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目录

Base.h

```
#include <cstdio>
#include <cstring>
#include <cmath>
#include <vector>
#include <utility>
#include <algorithm>
using namespace std;

const double inf = 1e10;
const double eps = 1e-8;
const double pi = acos(-1.0);
const int maxn = 100;

// about output
// %f format for g++, %lf format for c++, about printf

inline int dcmp(double x)
{
    if (fabs(x) < eps) return 0;
    return x < 0 ? -1: 1;
}

inline double adjust(double a)
{
    while (a < 0*pi) a += 2*pi;
    while (a > 2*pi) a -= 2*pi;
    return a;
}
inline double adjust01(double x)
{
    return min(1.0, max(0.0, x));
}
// default: 0-2*pi
inline double normal(double rad, double center=pi)
```

```
{ return rad - 2*pi * floor((rad + pi - center) / (2*pi)); }

inline double rand01() { return rand() / (double)RAND_MAX; }
inline double randeps() { return (rand01() - .5) * eps; }



inline double consinetheorem(double a, double b, double c, double C)
{
	if (C == -1.0)
		return acos((a*a + b*b - c*c) / (2*a*b));
	return -1.0;
}
```

Point.h

```
struct Point
{
	double x, y;
	Point() {}
	Point(double x, double y): x(x), y(y) {}
	void read()
	{ scanf("%lf%lf", &x, &y); }
	void print() const
	{ printf("(%.2lf, %.2lf)\n", x, y); }
	bool operator < (const Point &p) const
	{ return dcmp(x - p.x) < 0 || dcmp(x - p.x) == 0 && dcmp(y - p.y) < 0; }
	bool operator == (const Point &p) const
	{ return !dcmp(x-p.x) && !dcmp(y-p.y); }
};
```

```
typedef Point Vector;
typedef const Point &CP;
typedef const Vector &CV;

Vector operator + (CV A, CV B) { return Vector(A.x+B.x, A.y+B.y); }
Vector operator - (CV A, CV B) { return Vector(A.x-B.x, A.y-B.y); }
Vector operator * (CV A, double p) { return Vector(A.x*p, A.y*p); }
Vector operator / (CV A, double p) { return Vector(A.x/p, A.y/p); }

double Cross(CV A, CV B) { return A.x*B.y-A.y*B.x; }
double Dot(CV A, CV B) { return A.x*B.x+A.y*B.y; }

double Length2(CV v) { return Dot(v, v); }
double Length(CV v) { return sqrt(Dot(v, v)); }

double Angle(CV v) { return atan2(v.y, v.x); }

// 0-pi
double Angle(CV A, CV B) { return acos(Dot(A, B)/Length(A)/Length(B)); }
// -pi ~ pi
// { return fabs(atan2(Cross(A, B), Dot(A, B))); }

Vector Unit(CV A) { return A / Length(A); }

Vector Rotate(CV A, double rad)
{ return Vector(A.x*cos(rad) - A.y*sin(rad), A.x*sin(rad) + A.y*cos(rad)); }

// circumcircle
Point center(CP p1, CP p2, CP p3)
```

```
{
    double d1 = Dot(p2-p1, p3-p1), d2 = Dot(p3-p2, p1-p2), d3 = Dot(p1-p3, p2-p3);
    double c1 = d2 * d3, c2 = d1 * d3, c3 = d1 * d2, c = c1 + c2 + c3;
    if (!dcmp(c)) return p1;
    return (p1 * (c2 + c3) + p2 * (c1 + c3) + p3 * (c1 + c2)) / (2 * c);
}
```

Line.h

```
struct Line
{
    Point p;
    Vector v;
    Line() {}
    Line(Point A, Point B): p(A), v(B-A) {}
    Point point(double t) const
    { return p + v*t; }
    Point A() const
    { return p; }
    Point B() const
    { return p+v; }
    void print() const
    {
        p.print();
        (p+v).print();
    }
};
typedef const Line &CL;
typedef Line Segment;
typedef const Segment &CS;

Point GetLineIntersection(CL L1, CL L2)
{
    Vector u = L1.p-L2.p;
    double t = Cross(L2.v, u) / Cross(L1.v, L2.v);
    return L1.p+L1.v*t;
}

double DistanceToLine(CP P, CL L)
{
    Vector v1 = L.v, v2 = P-L.p;
    return fabs(Cross(v1, v2)) / Length(v1);
}

double DistanceToSegment(CP P, CP A, CP B)
{
    if (A == B) return Length(P - A);
    Vector v1 = B-A, v2 = P-A, v3 = P-B;
    if (dcmp(Dot(v1, v2)) < 0) return Length(v2);
    if (dcmp(Dot(v1, v3)) > 0) return Length(v3);
    return fabs(Cross(v1, v2)) / Length(v1);
}

double UntouchedSegSegDistance(CP a, CP b, CP c, CP d)
{ return min(min(DistanceToSegment(a, c, d), DistanceToSegment(b, c, d)),
min(DistanceToSegment(c, a, b), DistanceToSegment(d, a, b))); }

Point GetProjection(CP P, CL L)
```

```
{ return L.p + L.v*(Dot(L.v, P-L.p) / Dot(L.v, L.v)); }

bool SegmentProperIntersection(CP a1, CP a2, CP b1, CP b2)
{
	double c1 = Cross(a2-a1, b1-a1), c2 = Cross(a2-a1, b2-a1),
		c3 = Cross(b2-b1, a1-b1), c4 = Cross(b2-b1, a2-b1);
	return dcmp(c1) * dcmp(c2) < 0 && dcmp(c3) * dcmp(c4) < 0;
}

bool OnSegment(CP p, CP a1, CP a2)
{ return dcmp(Cross(a1-p, a2-p)) == 0 && dcmp(Dot(a1-p, a2-p)) < 0; }
```

Triangle.h

```
double Area(double a, double b, double c)
{
	double p = (a+b+c)/2;
	return sqrt(p*(p-a)*(p-b)*(p-c));
}

double ExcircleRadium(double a, double b, double c)
{ return Area(a, b, c) * 2/ (-a + b + c); }
double InCircleRadium(double a, double b, double c)
{ return Area(a, b, c) * 2/ (a + b + c); }
```

Polygon.h

```
typedef vector<Point> Polygon;

bool PointInPolygon(Point p, Polygon poly)
{
	int wn = 0;
	int n = poly.size();
	for (int i = 0; i < n; i++)
	{
		Point C = poly[i], D = poly[(i+1)%n];
		if (OnSegment(p, C, D)) return true;
		int k = dcmp(Cross(D-C, p-C));
		int d1 = dcmp(C.y-p.y);
		int d2 = dcmp(D.y-p.y);
		if (k > 0 && d1 <= 0 && d2 > 0) wn++;
		if (k < 0 && d2 <= 0 && d1 > 0) wn--;
	}
	return wn;
}

double PolygonArea(Point poly[], int n)
{
	double area = 0;
	poly[n] = poly[0];
	for (int i = 0; i < n; i++)
		area += Cross(poly[i], poly[i+1]) / 2;
	return fabs(area);
}

// Andrew Convex Hull algorithm
int ConvexHull(Point p[], int n)
{
```

```
    static Point q[maxn];

    // important
    sort(p, p+n);
    // n = unique(p, p+n) - p;
    int m = 0;
    for (int i = 0; i < n; i++)
    {
        while (m > 1 && Cross(q[m-1]-q[m-2], p[i]-q[m-2]) <= 0) m--;
        q[m++] = p[i];
    }
    int k = m;
    for (int i = n-2; i >= 0; i--)
    {
        while (m > k && Cross(q[m-1]-q[m-2], p[i]-q[m-2]) <= 0) m--;
        q[m++] = p[i];
    }
    if (n > 1) m--;
    for (int i = 0; i < m; i++)
        p[i] = q[i];
    return m;
}

struct line
{
    Point p;
    Vector v;
    double ang;
    line() {}
    line(CP A, CP B): p(A), v(B-A) { ang = atan2(v.y, v.x); }
    bool operator < (const line &L) const
    { return ang < L.ang; }
};
typedef const line &Cl;

bool OnLeft(Cl L, CP p)
{ return Cross(L.v, p-L.p) > 0; }

Point GetIntersection(Cl a, Cl b)
{
    Vector u = a.p-b.p;
    double t = Cross(b.v, u) / Cross(a.v, b.v);
    return a.p + a.v * t;
}

int HalfplaneIntersection(line L[], int n, Point poly[])
{
    sort(L, L+n);

    int first, last;
    static Point p[maxn];
    static line q[maxn];

    q[first=last=0] = L[0];

    for (int i = 1; i < n; i++)
    {
        while (first < last && !OnLeft(L[i], p[last-1])) last--;
```

```
            while (first < last && !OnLeft(L[i], p[first])) first++;
            q[++last] = L[i];
            // Select inter
            if (!dcmp(Cross(q[last].v, q[last-1].v)))
            {
                last--;
                if (OnLeft(q[last], L[i].p)) q[last] = L[i];
            }
            if (first < last) p[last-1] = GetIntersection(q[last-1], q[last]);
        }
        while (first < last && !OnLeft(q[first], p[last-1])) last--;
        if (last-first <= 1) return 0;
        p[last] = GetIntersection(q[last], q[first]);

        int m = 0;
        for (int i = first; i <= last; i++)
            poly[m++] = p[i];
        return m;
}

// Euler Formula
// V + F - E = 2(2D-plane) V -> # of vertex, F -> # of region, E -> # of edge
// V + F - E = X(P) X(P) -> euler topological invariant (2 - 2*h)
```

Algorithm.h

```
// p1 and p2 are both counterclockwise
// need min((p1, n1, p2, n2), (p2, n2, p1, n1))
// Max very similary
double MinDistcanceBetween2Polygon(Point p1[], int n1, Point p2[], int n2)
{
    double ans = inf;
    int i = 0, j = 0;
    for (int k = 1; k < n1; k++) if (p1[k] < p1[i]) i = k;
    for (int k = 1; k < n2; k++) if (p2[j] < p2[k]) j = k;
    for (int t = 0; t < n1; t++)
    {
        for (;;)
        {
            // Area
            int diff = dcmp(Cross(p1[i+1]-p1[i], p2[j+1]-p2[j]));
            if (diff <= 0)
            {
                if (!diff) ans = min(ans, UntouchedSegSegDistance(p1[i], p1[i+1],
p2[j], p2[j+1]));
                else ans = min(ans, DistanceToSegment(p2[j], p1[i], p1[i+1]));
                break;
            }
            j = (j+1) % n2;
        }
        i = (i+1) % n1;
    }
    return ans;
}

double shadow_length(double alpha, Point a, Point b)
{
    double dx = a.x - b.x;
```

```
    double dy = a.y - b.y;
    return fabs(dx * cos(alpha) + dy * sin(alpha));
}

void SmallestEnclosingRectangle(Point p[], int n, double &area, double &peri)
{
    area = peri = inf;
    Point *q[4] = {NULL, NULL, NULL, NULL};
    for (int i = 0; i < n; i++)
    {
        Point *t = p+i;
        if (!q[0] || t->y < q[0]->y || t->y == q[0]->y && t->x < q[0]->y) q[0] = t;
        if (!q[1] || t->x > q[1]->x || t->x == q[1]->x && t->y < q[1]->y) q[1] = t;
        if (!q[2] || t->y > q[2]->y || t->y == q[2]->y && t->x > q[2]->x) q[2] = t;
        if (!q[3] || t->x < q[3]->x || t->x == q[3]->x && t->y > q[3]->y) q[3] = t;
    }

    double alpha = 0;
    for (int k = 0; k < n+5; k++)
    {
        int j = -1;
        double Min = inf;
        for (int i = 0; i < 4; i++)
        {
            double tmp = adjust(Angle(q[i][1] - q[i][0]) - (alpha + i*pi/2));
            if (tmp < Min)
            {
                j = i;
                Min = tmp;
            }
        }
        if (++q[j] == p + n) q[j] = p + 0;
        alpha = adjust(alpha + Min);

        double a = shadow_length(alpha + pi / 2, *q[0], *q[2]);
        double b = shadow_length(alpha, *q[1], *q[3]);
        area = min(area, a*b);
        peri = min(peri, 2*(a+b));

        if (dcmp(alpha - pi / 2) > 0) break;
    }
}

const int maxp = 1e4;
struct Edge
{
    int from, to;
    double ang;
    Edge(int from, int to, double ang):
        from(from), to(to), ang(ang) {}
};

struct PSLG
{
    int n, m;
    Point p[maxp];
    vector<int> G[maxp];
    vector<Edge> edges;
```

```
int prev[maxp<<1];
bool vis[maxp<<1];

double getAngle(int from, int to)
{ return Angle(p[to] - p[from]); }

void init(int n, Point inter[])
{
	this->n = n;
	for (int i = 0; i < n; i++)
	{
		G[i].clear();
		p[i] = inter[i];
	}
	edges.clear();
}

void addEdge(int from, int to)
{
	edges.push_back(Edge(from, to, getAngle(from, to)));
	edges.push_back(Edge(to, from, getAngle(to, from)));
	m = edges.size();
	G[from].push_back(m-2);
	G[to].push_back(m-1);
}

// PSLG build the graph and output is on faces
// all face are counter-clockwise
void build(vector<Polygon> &faces)
{
	// clear the faces
	faces.clear();

	// calculate prev
	for (int u = 0; u < n; u++)
	{
		int sz = G[u].size();
		for (int i = 0; i < sz; i++) for (int j = i + 1; j < sz; j++)
			if (edges[G[u][i]].ang > edges[G[u][j]].ang) swap(G[u][i], G[u][j]);
		for (int i = 0; i < sz; i++)
			prev[G[u][(i+1)%sz]] = G[u][i];
	}

	// find plane region
	memset(vis, false, sizeof(vis));
	// infinite border
	for (int u = 0; u < n; u++)
	{
		if (G[u].size() == 1)
		{
			int e = G[u][0];
			if (!vis[e])
			{
				Polygon poly;
				for (;;)
				{
					int from = edges[e].from;
					if (from != u && G[from].size() == 1) break;
```

```
                        vis[e] = true;
                        poly.push_back(p[from]);
                        e = prev[e^1];
                    }
                    int from = edges[e].from;
                    poly.push_back(p[from]);
                    faces.push_back(poly);
                }
            }
        }
        // finite region
        for (int u = 0; u < n; u++)
        {
            for (int i = 0; i < G[u].size(); i++)
            {
                int e = G[u][i];
                if (!vis[e])
                {
                    Polygon poly;
                    for (;;)
                    {
                        vis[e] = true;
                        // left[e] = face_cnt;
                        int from = edges[e].from;
                        poly.push_back(p[from]);
                        e = prev[e^1];
                        if (e == G[u][i]) break;
                    }
                    faces.push_back(poly);
                }
            }
        }
    }
} solver;
```

Point3.h

```
struct Point3
{
    double x, y, z;
    Point3(double x=0, double y=0, double z=0):
        x(x), y(y), z(z) {}
    void read()
    { scanf("%lf%lf%lf", &x, &y, &z); }
    void print()
    { printf("(%.2lf, %.2lf, %.2lf)\n", x, y, z); }
};
typedef Point3 Vector3;
typedef const Point3 &CP3;
typedef const Vector3 &CV3;

typedef Point3 Triangle[3];
typedef Point3 Tetrahedron[4];

bool operator == (CV3 A, CV3 B) { return !dcmp(A.x-B.x) && !dcmp(A.y-B.y)
&& !dcmp(A.z-B.z); }
Vector3 operator + (CV3 A, CV3 B) { return Vector3(A.x+B.x, A.y+B.y, A.z+B.z); }
Vector3 operator - (CV3 A, CV3 B) { return Vector3(A.x-B.x, A.y-B.y, A.z-B.z); }
```

```
Vector3 operator * (CV3 A, double b) { return Vector3(A.x*b, A.y*b, A.z*b); }
Vector3 operator / (CV3 A, double b) { return Vector3(A.x/b, A.y/b, A.z/b); }

double Dot(CV3 A, CV3 B) { return A.x*B.x + A.y*B.y + A.z*B.z; }
Vector3 Cross(CV3 A, CV3 B) { return Vector3(A.y*B.z - A.z*B.y, A.z*B.x - A.x*B.z,
A.x*B.y - A.y*B.x); }
double Length(CV3 A) { return sqrt(Dot(A, A)); }

double torad(double deg) { return deg/180 * pi; }

double Area2(CP3 A, CP3 B, CP3 C) { return Length(Cross(B-A, C-A)); }

double Volume6(CP3 A, CP3 B, CP3 C, CP3 D) { return Dot(D-A, Cross(B-A, C-A)); }

// lat == latitude, lng == longitude
// north latitude > 0, east longitude > 0
Vector3 get_coord(double R, double lat, double lng)
{
	lat = torad(lat);
	lng = torad(lng);
	return Point3(R*cos(lat)*sin(lng), R*cos(lat)*cos(lng), R*sin(lat));
}

// R is ball radium, o is (0, 0, 0)
double DistanceOnBall(CV3 A, CV3 B, double R)
{
	double d = Length(A-B);
	double alpha = 2*asin(d/2./R);
	return alpha * R;
}

// (jingdu, weidu)
// (lag, lat)
// A(alpha1, belta1) B(alpha2, belta2)
// double DistanceOnBall(double alpha1, double alpha2, double belta1, double belta2)
// { return R *
acos(cos(belta1)*cos(belta2)*cos(alpha1-alpha2)+sin(belta1)*sin(belta2)); }

double DistanceToPlane(CP3 p, CP3 p0, CV3 n)
{ return fabs(Dot(p-p0, n)) / Length(n); }

// p's projection in plane (p0, n)
Point3 GetPlaneProjection(CP3 p, CP3 p0, CV3 n)
{
	double d = Dot(p-p0, n) / Length(n);
	return p - n * (d / Length(n));
}

// P must in plane(tri)
bool PointInTri(CP3 P, Triangle tri)
{
	double area1 = Area2(P, tri[0], tri[1]);
	double area2 = Area2(P, tri[1], tri[2]);
	double area3 = Area2(P, tri[2], tri[0]);
	return !dcmp(area1 + area2 + area3 - Area2(tri[0], tri[1], tri[2]));
}
```

Line3.h

```
struct Line3
{
	Point3 p;
	Vector3 v;
	Line3(CP3 A, CP3 B): p(A), v(B-A) {}
	Point3 A() { return p; }
	Point3 B() { return p + v; }
	Point3 point(double t) { return p + v * t; }
};
typedef Line3 Segment3;

Point3 GetLineProjection(CP3 P, Segment3 seg)
{ return seg.p + seg.v * (Dot(seg.v, P-seg.p) / Dot(seg.v, seg.v)); }

double DistanceToLine(Point3 P, Segment3 seg)
{ return Length(Cross(seg.v, P - seg.p)) / Length(seg.v); }

Point3 LinePlaneIntersection(Line3 L, CP3 p0, CV3 n)
{
	double t = (Dot(n, p0-L.p) / Dot(n, L.v));
	return L.p + L.v*t;
}

// the distance between two line in different planes
// return s in p1 + s * v1(the intersection)
bool LineDistance3D(Line3 L1, Line3 L2, double &s)
{
	double b = Dot(L1.v, L1.v) * Dot(L2.v, L2.v) - Dot(L1.v, L2.v) * Dot(L1.v, L2.v);
	if (!dcmp(b)) return false; // parallel or coincide
	double a = Dot(L1.v, L2.v) * Dot(L2.v, L1.p-L2.p) - Dot(L2.v, L2.v) * Dot(L1.v,
L1.p-L2.p);
	s = a / b;
	return true;
}

double PointToSegment3D(CP3 p, Segment3 seg)
{
	Vector3 v1 = seg.v, v2 = p - seg.A(), v3 = p - seg.B();
	if (dcmp(Dot(v1, v2)) <= 0) return Length(v2);
	if (dcmp(Dot(v1, v3)) >= 0) return Length(v3);
	return Length(Cross(v1, v2)) / Length(v1);
}

double SegmentDistance3D(Segment3 seg1, Segment3 seg2)
{
	double s, t;
	if (LineDistance3D(seg1, seg2, s) && dcmp(s) >= 0 && dcmp(s-1) <= 0
			&& LineDistance3D(seg2, seg1, t) && dcmp(t) >= 0 && dcmp(t-1) <= 0)
		return Length(seg1.point(s) - seg2.point(t));
	return min(min(PointToSegment3D(seg1.A(), seg2), PointToSegment3D(seg1.B(),
```

```
seg2)),
                min(PointToSegment3D(seg2.A(), seg1), PointToSegment3D(seg2.B(),
seg1)));
}

double SegmentTriDistance3D(Segment3 seg, Triangle tri)
{
    Vector3 n = Cross(tri[1]-tri[0], tri[2]-tri[0]);
    Vector3 proj1 = GetPlaneProjection(seg.A(), tri[0], n);
    Vector3 proj2 = GetPlaneProjection(seg.B(), tri[0], n);

    double ans = inf;
    if (PointInTri(proj1, tri))
        ans = min(ans, Length(seg.A()-proj1));
    if (PointInTri(proj2, tri))
        ans = min(ans, Length(seg.B()-proj2));

    for (int i = 0; i < 3; i++)
        ans = min(ans, SegmentDistance3D(seg, Segment3(tri[i], tri[(i+1)%3])));
    return ans;
}

double TriDistance3D(Triangle tri1, Triangle tri2)
{
    double ans = inf;
    for (int i = 0; i < 3; i++)
    {
        ans = min(ans, SegmentTriDistance3D(Segment3(tri1[i], tri1[(i+1)%3]), tri2));
        ans = min(ans, SegmentTriDistance3D(Segment3(tri2[i], tri2[(i+1)%3]), tri1));
    }
    return ans;
}

double TetrahedronDistance(Tetrahedron tet[2])
{
    static Triangle tri[2][4];
    for (int i = 0; i < 2; i++)
    {
        tri[i][0][0] = tet[i][0], tri[i][0][1] = tet[i][1], tri[i][0][2] = tet[i][2];
        tri[i][1][0] = tet[i][0], tri[i][1][1] = tet[i][1], tri[i][1][2] = tet[i][3];
        tri[i][2][0] = tet[i][0], tri[i][2][1] = tet[i][2], tri[i][2][2] = tet[i][3];
        tri[i][3][0] = tet[i][1], tri[i][3][1] = tet[i][2], tri[i][3][2] = tet[i][3];
    }

    double ans = inf;
    for (int i = 0; i < 4; i++) for (int j = 0; j < 4; j++)
        ans = min(ans, TriDistance3D(tri[0][i], tri[1][j]));
    return ans;
}
```

Ball.h

```
struct Ball
{
    Point3 o;
    double r;
    Ball(Point3 o, double r): o(o), r(r) {}
};
```

```
bool SegmentBallIntersection(Segment3 seg, Ball ball, double &t1, double &t2)
{
	double dist = DistanceToLine(ball.o, seg);

	if (dcmp(dist - ball.r) >= 0)
		return false;

	double t = Dot(seg.v, ball.o-seg.p) / Dot(seg.v, seg.v);
	double dt = sqrt(ball.r*ball.r - dist*dist) / Length(seg.v);
	t1 = adjust01(t-dt);
	t2 = adjust01(t+dt);

	if (!dcmp(t1 - t2))
		return false;

	if (dcmp(t1 - t2) > 0)
		swap(t1, t2);

	return true;
}
```

Algorithm3.h

```
struct Face
{
	int v[3];
	Face(int a, int b, int c)
	{ v[0] = a, v[1] = b, v[2] = c; }
	Vector3 normal(Point3 p[]) const
	{ return Cross(p[v[1]]-p[v[0]], p[v[2]]-p[v[0]]); }
	bool cansee(Point3 p[], int i) const
	{ return Dot(p[i]-p[v[0]], normal(p)) > 0; }
};

Point3 add_noise(CP3 p)
{ return Point3(p.x + randeps(), p.y + randeps(), p.z + randeps()); }

// increment method
vector<Face> CH3D(Point3 q[], int n)
{
	static Point3 p[maxn];
	for (int i = 0; i < n; i++)
		p[i] = add_noise(q[i]);
	static bool vis[maxn][maxn];
	vector<Face> cur;
	cur.push_back(Face(0, 1, 2));
	cur.push_back(Face(2, 1, 0));
	for (int i = 3; i < n; i++)
	{
		vector<Face> nex;
		for (int j = 0; j < cur.size(); j++)
		{
			Face &f = cur[j];
			bool flag = f.cansee(p, i);
			if (!flag) nex.push_back(f);
			for (int k = 0; k < 3; k++) vis[f.v[k]][f.v[(k+1)%3]] = flag;
		}
```

```
            for (int j = 0; j < cur.size(); j++)
            {
                for (int k = 0; k < 3; k++)
                {
                    int a = cur[j].v[k], b = cur[j].v[(k+1)%3];
                    if (vis[a][b] != vis[b][a] && vis[a][b])
                        nex.push_back(Face(a, b, i));
                }
            }
            cur = nex;
        }
        return cur;
}
```

Circle.h

```
struct Circle
{
    Point o;
    double r;
    Circle() {}
    Circle(Point o, double r): o(o), r(r) {}
    bool operator < (const Circle &C) const
    { return o < C.o || o == C.o && dcmp(r - C.r) < 0; }
    bool operator == (const Circle &C) const
    { return o == C.o && dcmp(r - C.r) == 0; }
    Point point(double a) const
    { return Point(o.x + cos(a)*r, o.y + sin(a)*r); }
    void read()
    {
        o.read();
        scanf("%lf", &r);
    }
    void print() const
    {
        o.print();
        printf("r = %.2lf\n", r);
    }
};
typedef const Circle &CC;

int GetLineCircleIntersection(Line L, CC C, double &t1, double &t2, vector<Point> &sol)
{
    double a = L.v.x, b = L.p.x - C.o.x, c = L.v.y, d = L.p.y - C.o.y;
    double e = a*a + c*c, f = 2*(a*b + c*d), g = b*b + d*d - C.r*C.r;
    double delta = f*f - 4*e*g;
    if (dcmp(delta) < 0) return 0;
    if (!dcmp(delta))
    {
        t1 = t2 = -f / (2*e); sol.push_back(L.point(t1));
        return 1;
    }

    t1 = (-f - sqrt(delta)) / (2 * e); sol.push_back(L.point(t1));
    t2 = (-f + sqrt(delta)) / (2 * e); sol.push_back(L.point(t2));
    return 2;
}
```

```
int GetCircleSegmentIntersection(CC C, Line L, double &t1, double &t2)
{
	double a = L.v.x, b = L.p.x - C.o.x, c = L.v.y, d = L.p.y - C.o.y;
	double e = a*a + c*c, f = 2*(a*b + c*d), g = b*b + d*d - C.r*C.r;
	double delta = f*f - 4*e*g;
	if (delta < 0) return 0;
	int tot = 0;
	if (!dcmp(delta))
	{
		t1 = t2 = -f / (2*e);

		if (dcmp(t1) >= 0 && dcmp(t1-1) <= 0) tot++;
		// else t1 = 0, t2 = 1, tot++;
		return tot;
	}

	t1 = (-f - sqrt(delta)) / (2 * e);
	if (dcmp(t1) > 0 && dcmp(t1-1) < 0) tot++;
	// else t1 = 0, tot++;

	t2 = (-f + sqrt(delta)) / (2 * e);
	if (dcmp(t2) > 0 && dcmp(t2-1) < 0) tot++;
	// else t2 = 1, tot++;

	return tot;
}

bool PointInCircle(CP p, const Circle &c)
{ return dcmp(Length(p-c.o) - c.r) < 0; }

bool CircleInCircle(const Circle &c1, const Circle &c2)
{ return dcmp((Length(c1.o-c2.o) + c1.r) - c2.r) <= 0; }

bool GetCircleCircleIntersection(const Circle &c1, const Circle &c2, double &a1, double
&a2)
{
	double d = Length(c1.o - c2.o);
	// external
	if (dcmp(c1.r+c2.r - d) < 0) return false;
	// coincide or internal
	if (!dcmp(d) || dcmp(fabs(c1.r - c2.r) - d) > 0) return false;

	double alpha = Angle(c2.o - c1.o), beta = consinetheorem(c1.r, d, c2.r, -1);
	a1 = adjust(alpha - beta), a2 = adjust(alpha + beta);

	return true;
}

typedef pair<double, int> Rad;
bool CirclesIntersect(vector<Circle> &circle, double &area, Point &P)
{
	bool ret = false;
	area = 0;

	sort(circle.begin(), circle.end());
	circle.erase(unique(circle.begin(), circle.end()), circle.end());
```

```
    for (int i = 0; i < circle.size(); i++)
    {
        vector<Rad> a;
        a.push_back(Rad(0, 0));
        a.push_back(Rad(2*pi, 0));
        int k = 0;
        for (int j = 0; j < circle.size(); j++) if (i != j)
        {
            double a1, a2;
            if (CircleInCircle(circle[i], circle[j])) { k++; continue; }
            if (!GetCircleCircleIntersection(circle[i], circle[j], a1, a2)) continue;
            a.push_back(Rad(a1, 1));
            a.push_back(Rad(a2, -1));
            if (a1 > a2) k++;
        }
        sort(a.begin(), a.end());

        for (int j = 0; j+1 < a.size(); j++)
        {
            k += a[j].second;
            if (k + 1 == circle.size())
            {
                ret = true;
                double angle = a[j+1].first - a[j].first;
                area += .5 * circle[i].r*circle[i].r * (angle - sin(angle));
                area += .5 * Cross(circle[i].point(a[j].first),
circle[i].point(a[j+1].first));
            }
        }
    }

    return ret;
}

void MinimalCoverCircle(vector<Point> points, Point &o, double &r)
{
    random_shuffle(points.begin(), points.end());
    o = points[0]; r = 0;
    for (int i = 1; i < points.size(); ++i)
    {
        if (dcmp(Length(points[i] - o) - r) <= 0) continue;
        o = points[i]; r = 0;
        for (int j = 0; j < i; ++j)
        {
            if (dcmp(Length(points[j] - o) - r) <= 0) continue;
            o = (points[i] + points[j]) * .5; r = Length(points[j] - o);
            for (int k = 0; k < j; ++k)
            {
                if (dcmp(Length(points[k] - o) - r) <= 0) continue;
                o = center(points[i], points[j], points[k]); r = Length(points[k] -
o);
            }
        }
    }
}

double CircleIntersectTriangle(Point p1, Point p2, CC C)
{
```

```
        p1 = p1 - C.o;
        p2 = p2 - C.o;
        if (dcmp(0.25 * Dot(p1+p2, p1+p2) - C.r * C.r) < 0)
                return Cross(p1, p2) / 2;
        else
        {
                double ang = adjust(atan2(p2.y, p2.x) - atan2(p1.y, p1.x));
                return C.r * C.r * ang / 2;
        }
}

double CircleIntersectPolygon(Point poly[], int n, CC C)
{
        static Point p[3*maxn];

        int m = 0;
        for (int i = 0; i < n; i++)
        {
                vector<Point> sol;
                double t1, t2;
                p[m++] = poly[i];
                if (GetCircleSegmentIntersection(C, Line(poly[i], poly[(i+1)%n]), t1, t2))
                {
                        p[m++] = Line(poly[i], poly[(i+1)%n]).point(t1);
                        p[m++] = Line(poly[i], poly[(i+1)%n]).point(t2);
                }
        }

        double area = 0;
        for (int i = 0; i < m; i++)
                area +=    CircleIntersectTriangle(p[i], p[i+1], C);
        return fabs(area);
}

// Circles Union in n*m plant
double CirclesUnion(Circle C[maxn], int tot, int n, int m)
{
        static Point border[4];
        static Circle tC[maxn];

        border[0] = Point(0, 0);
        border[1] = Point(n, 0);
        border[2] = Point(n, m);
        border[3] = Point(0, m);

        int ttot = 0;
        for (int i = 0; i < tot; i++)
        {
                bool flag = true;
                for (int j = 0; j < tot; j++) if (i != j)
                {
                        double d = Length(C[i].o - C[j].o);
                        if (!dcmp(d))
                        {
                                if (dcmp(C[i].r - C[j].r) < 0 || dcmp(C[i].r - C[j].r) == 0 && i < j)
                                        flag = false;
                        }
                        else if (dcmp(d + C[i].r - C[j].r) <= 0)
```

```
                flag = false;
        }
        if (flag)
                tC[ttot++] = C[i];
}
copy(tC, tC+ttot, C);
tot = ttot;

double area = 0;
for (int i = 0; i < tot; i++)
{
        vector<Rad> vec;
        vec.push_back(Rad(0, 0));
        vec.push_back(Rad(2*pi, 0));
        int k = 0;
        for (int j = 0; j < tot; j++) if (i != j)
        {
                double a1, a2;
                if (GetCircleCircleIntersection(C[i], C[j], a1, a2))
                {
                        vec.push_back(Rad(a1, 1));
                        vec.push_back(Rad(a2, -1));
                        if (a1 > a2)
                                k++;
                }
        }
        for (int j = 0; j < 4; j++)
        {
                double t1, t2;
                Line L = Line(border[j], border[(j+1)%4]);
                if (GetCircleSegmentIntersection(C[i], L, t1, t2))
                {
                        double a1 = adjust(Angle(L.point(t1) - C[i].o)),
                                        a2 = adjust(Angle(L.point(t2) - C[i].o));
                        vec.push_back(Rad(a1, 1));
                        vec.push_back(Rad(a2, -1));
                        if (a1 > a2)
                                k++;
                }
        }
        sort(vec.begin(), vec.end());

        for (int j = 0; j+1 < vec.size(); j++)
        {
                k += vec[j].second;
                if (!k)
                {
                        double a = vec[j+1].first - vec[j].first;
                        area += C[i].r * C[i].r * (a - sin(a)) / 2;
                        area += Cross(C[i].point(vec[j].first), C[i].point(vec[j+1].first)) / 2;
                }
        }
}
for (int i = 0; i < 4; i++)
{
        Line L = Line(border[i], border[(i+1)%4]);
        vector<Rad> vec;
        vec.push_back(Rad(0, 0));
```

```
            vec.push_back(Rad(1, 0));
            for (int j = 0; j < tot; j++)
            {
                double t1, t2;
                if (GetCircleSegmentIntersection(C[j], L, t1, t2))
                {
                    vec.push_back(Rad(t1, 1));
                    vec.push_back(Rad(t2, -1));
                }
            }
            sort(vec.begin(), vec.end());

            int k = 0;
            for (int j = 0; j+1 < vec.size(); j++)
            {
                k += vec[j].second;
                if (k)
                    area += Cross(L.point(vec[j].first), L.point(vec[j+1].first)) / 2;
            }
        }
        return area;
}

int GetTangents(Circle A, Circle B, Point *a, Point *b)
{
        int cnt = 0;
        if (A.r < B.r) swap(A, B), swap(a, b);
        double d2 = Length2(A.o-B.o);
        double rdiff = A.r - B.r;
        double rsum = A.r + B.r;

        if (dcmp(d2 - rdiff * rdiff) < 0) return 0; // contain

        // B.o - A.o !!!!
        double base = Angle(B.o - A.o);
        if (dcmp(d2) == 0 && dcmp(rdiff) == 0) return -1; // coincide
        if (dcmp(d2 - rdiff * rdiff) == 0) // inscibe
        {
            a[cnt] = A.point(base);
            b[cnt] = B.point(base);
            cnt++;
            return 1;
        }

        // two external common tangent
        double ang = acos(rdiff / sqrt(d2));
        a[cnt] = A.point(base + ang);
        b[cnt] = B.point(base + ang);
        cnt++;
        a[cnt] = A.point(base - ang);
        b[cnt] = B.point(base - ang);
        cnt++;

        if (dcmp(d2 - rsum * rsum) == 0) // one internal common tangent
        {
            a[cnt] = A.point(base);
            b[cnt] = B.point(pi + base);
            cnt++;
```

```
        }
        else if (dcmp(d2 - rsum * rsum) > 0) // two
        {
                ang = acos(rsum / sqrt(d2));
                a[cnt] = A.point(base + ang);
                b[cnt] = B.point(pi + base + ang);
                cnt++;
                a[cnt] = A.point(base - ang);
                b[cnt] = B.point(pi + base - ang);
                cnt++;
        }
        return cnt;
}

int GetTangents(const Circle &C, CP p, vector<double> &rad)
{
        Vector u = p - C.o;
        double base = Angle(u);
        double dist = Length(u);

        if (dcmp(dist-C.r) < 0) return 0; // in circle
        if (dcmp(dist-C.r) == 0)
        {
                rad.push_back(base);
                return 1;
        }
        double ang = acos(C.r / dist);
        rad.push_back(base - ang);
        rad.push_back(base + ang);
                return 2;
}
```