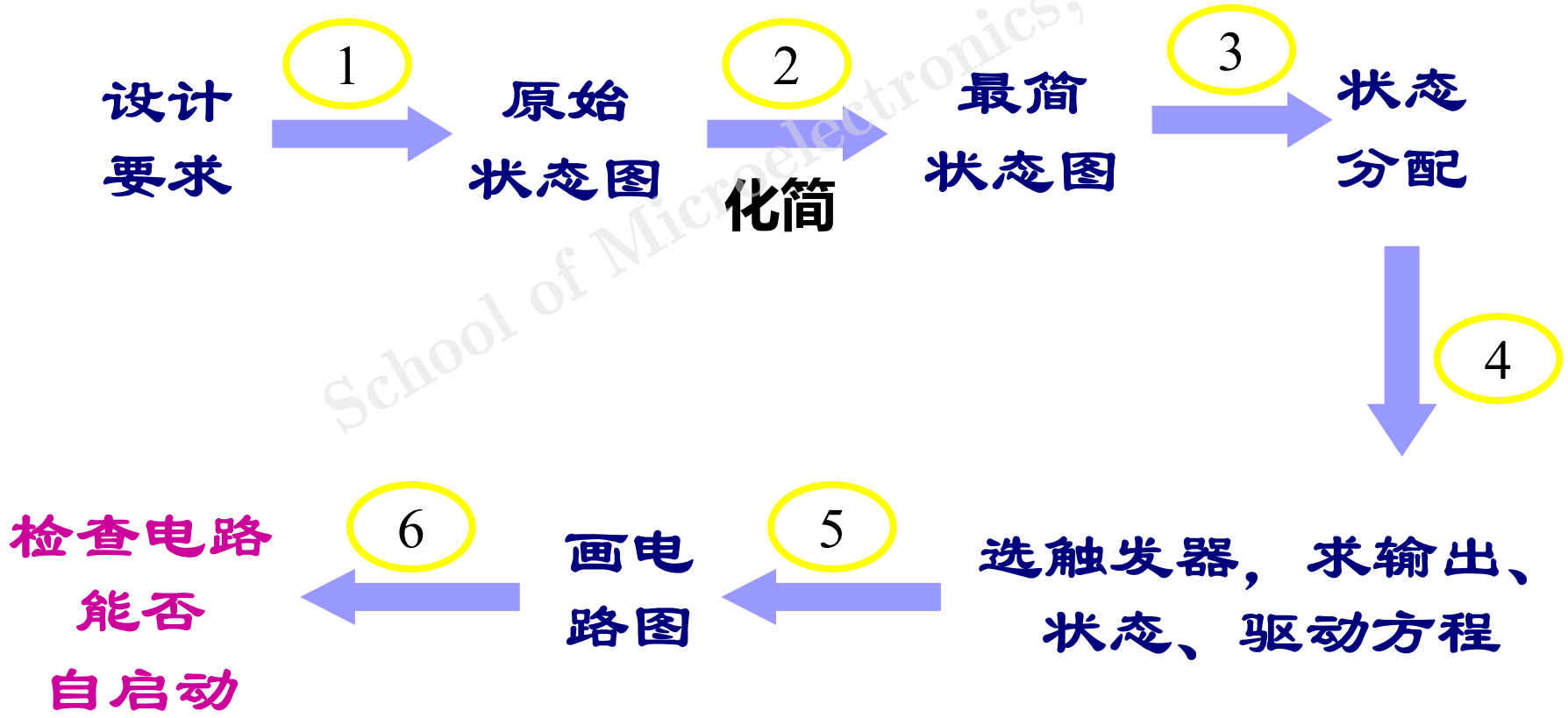
§6.3 同步时序电路设计

Synchronous Sequential Circuit Design

已知 → 功能或状态图

求 → 电路

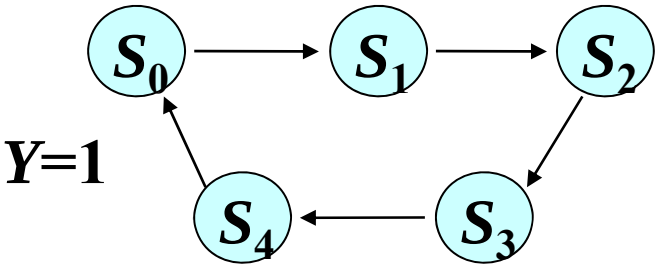


例 1. 设计同步5进制加法计数器（例6.4）

1）确定状态及状态图

M-5 计数器，5 个状态：S_0, S_1, S_2, S_3, S_4

在计数脉冲CLK作用下，5 个状态周期性变换，在 S_4 状态下进位输出 $Y = 1$ 。



2）状态化简

M-5, 5 个状态， 不须再化简

3) 状态分配、编码

$2^{n-1} \leq$ 状态数 $\leq 2^n$　　n：二进制位数　　3位

$S_0 \Rightarrow 000$

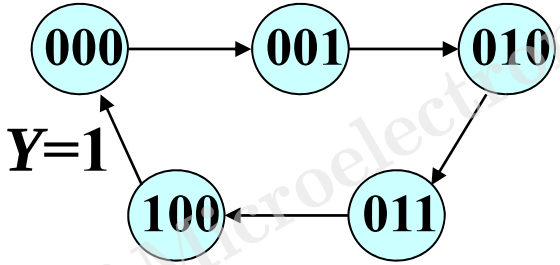
$S_1 \Rightarrow 001$

$S_2 \Rightarrow 010$

$S_3 \Rightarrow 011$

$S_4 \Rightarrow 100$

状态图



状态表

Q_2^n Q_1^n Q_0^n	Q_2^{n+1} Q_1^{n+1} Q_0^{n+1}	Y
0 0 0	0 0 1	0
0 0 1	0 1 0	0
0 1 0	0 1 1	0
0 1 1	1 0 0	0
1 0 0	0 0 0	1

4) 选择 FF，确定驱动方程、状态方程Q^{n+1}及输出方程

方法 1：先不确定用哪种触发器

由状态表填卡诺图

状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

Q_2^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	0
1	0	1	Φ	Φ

Q_1^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	1	Φ	0
1	1	0	Φ	Φ

Q_0^{n+1}

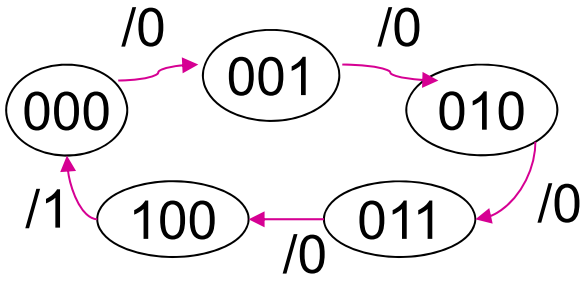
Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	1	1	Φ	0
1	0	0	Φ	Φ

Y

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	1
1	0	0	Φ	Φ

也可直接填卡诺图

直接填卡诺图



5个 有效状态 $5 < 2^3$ **3 个FF**

$Q_2^nQ_1^nQ_0^n$ —/Z→ $Q_2^{n+1}Q_1^{n+1}Q_0^{n+1}$

Y

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	1
1	0	0	Φ	Φ

Q_2^{n+1}

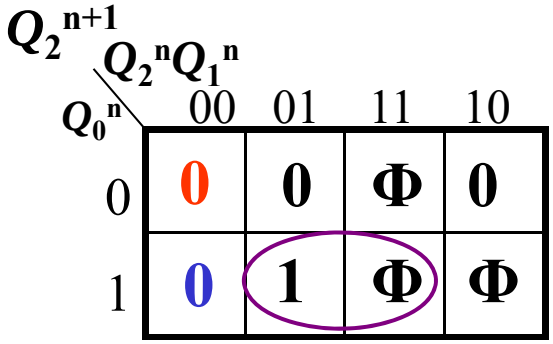
Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	0
1	0	1	Φ	Φ

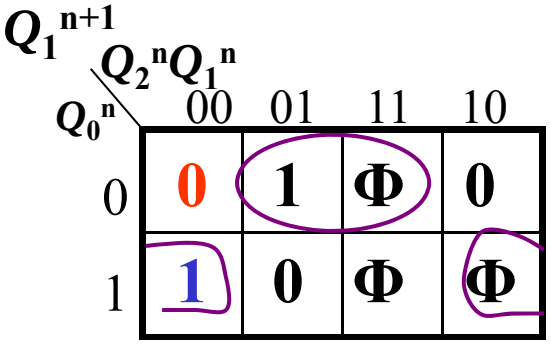
Q_1^{n+1}

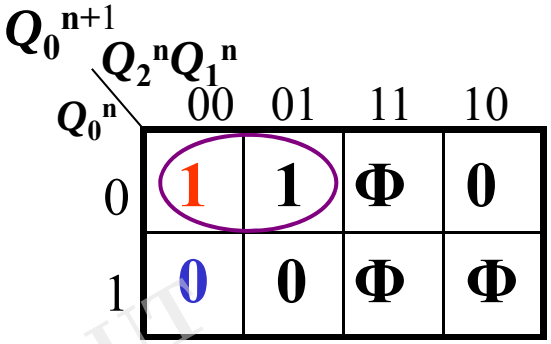
Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	1	Φ	0
1	1	0	Φ	Φ

Q_0^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	1	1	Φ	0
1	0	0	Φ	Φ







$$Q_2^{n+1} = Q_1^n Q_0^n$$

$$= D_2$$

$$D_2 = Q_1^n Q_0^n$$

$$Q_1^{n+1} = Q_0^n \bar{Q}_1^n + \bar{Q}_0^n Q_1^n$$

$$= Q_0^n \oplus Q_1^n$$

$$= T_1 \oplus Q_1^n$$

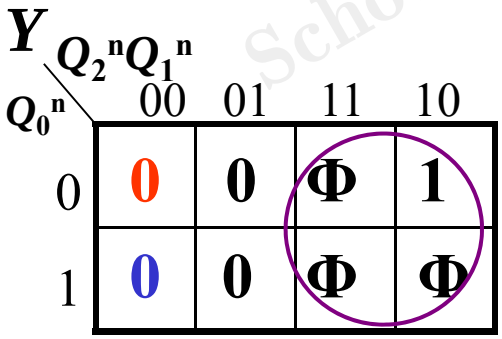
$$T_1 = Q_1^n$$

$$Q_0^{n+1} = \bar{Q}_2^n \bar{Q}_0^n$$

$$= D_0$$

或

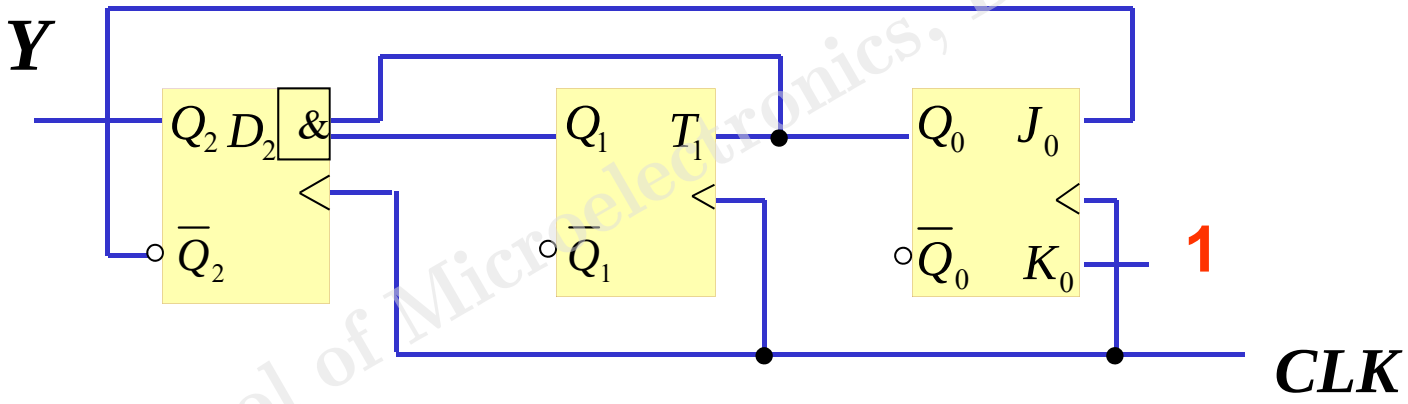
$$\begin{cases} J_0 = \overline{Q_2^n} \\ K_0 = 1 \end{cases}$$



$$Y = Q_2^n$$

$$D_2 = Q_1^n Q_0^n \qquad T_1 = Q_0^n \qquad \begin{cases} J_0 = \overline{Q}_2^n \\ K_0 = 1 \end{cases} \qquad Y = Q_2^n$$

5）电路



与门可以省略

6)检查是否可以自启动

$$Q_2^{n+1} = Q_1^n Q_0^n$$

$$Q_1^{n+1} = Q_0^n \bar{Q}_1^n + \bar{Q}_0^n Q_1^n$$

$$= Q_0^n \oplus Q_1^n$$

$$Q_0^{n+1} = \bar{Q}_2^n \bar{Q}_0^n$$

状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1
1	0	1	0	1	0	1
1	1	0	0	1	0	1
1	1	1	1	0	0	1

方法 2：确定用哪种触发器

4) 选择 FF　　选 JK-FFs

5) 状态方程Q^{n+1}及控制输入– *J, K*

状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

JK-FF 驱动表

$Q^n \to Q^{n+1}$		J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

Q_2^n	$\Rightarrow$	Q_2^{n+1}	J_2
0		0	0
0		0	0
0		0	0
0		1	1
1		0	X
X		X	X
X		X	X
X		X	X

得到 $2^{\#}$-FF 控制输入 J_2 驱动卡诺图

J_2	$Q_1^nQ_0^n$ 00	01	11	10
Q_2^n 0	0	0	1	0
1	×	×	×	×

状态图

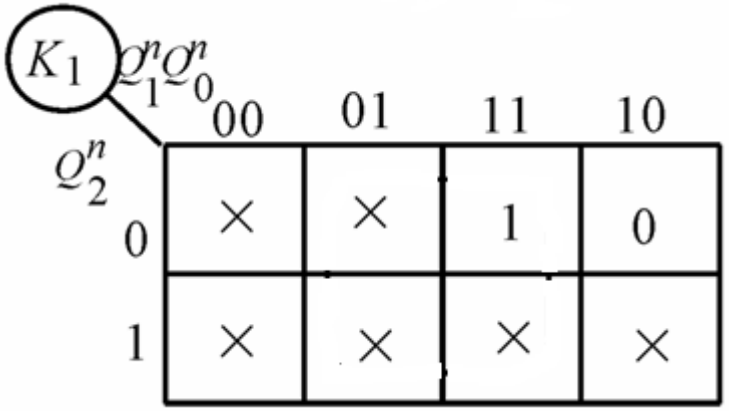
Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

JK-FF 驱动表

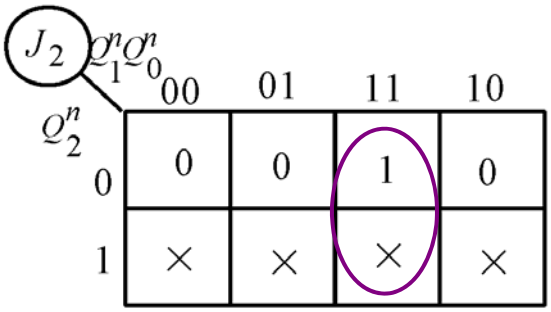
$Q^n \to Q^{n+1}$		J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

Q_1^n	$\Rightarrow$	Q_1^{n+1}	K_1
0		0	X
0		1	X
1		1	0
1		0	1
0		0	X
X		X	X
X		X	X
X		X	X

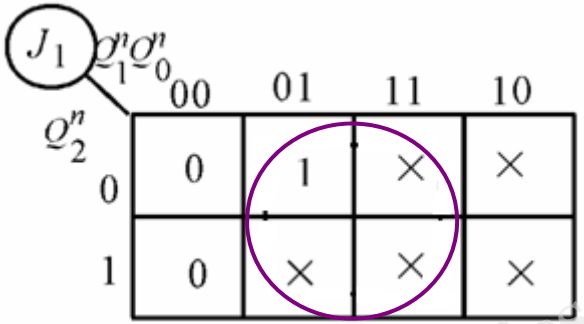
得到 $1^{\#}$-FF 控制输入 K_1 驱动卡诺图



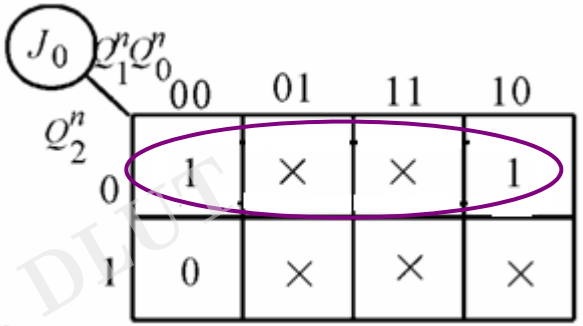
得到各个触发器控制输入驱动卡诺图及控制输入



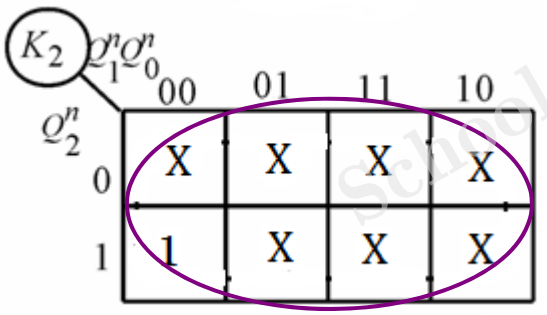
$$J_2 = Q_1^n Q_0^n$$



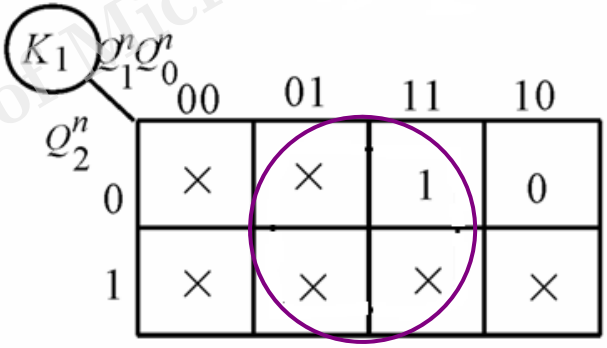
$$J_1 = Q_0^n$$



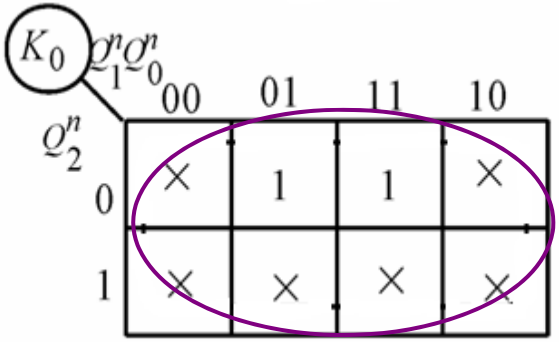
$$J_0 = \overline{Q_2^n}$$



$$K_2 = 1$$

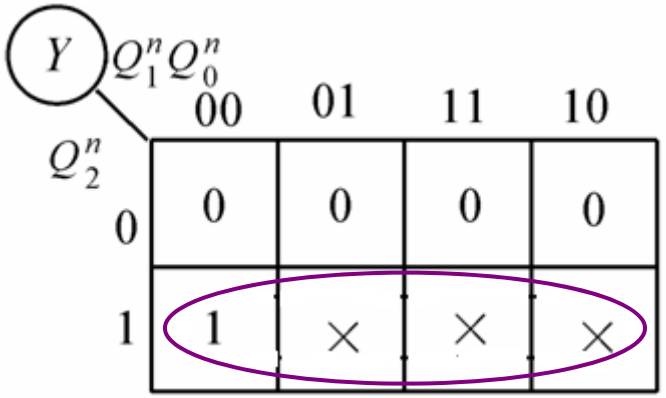


$$K_1 = Q_0^n$$



$$K_0 = 1$$

输出卡诺图



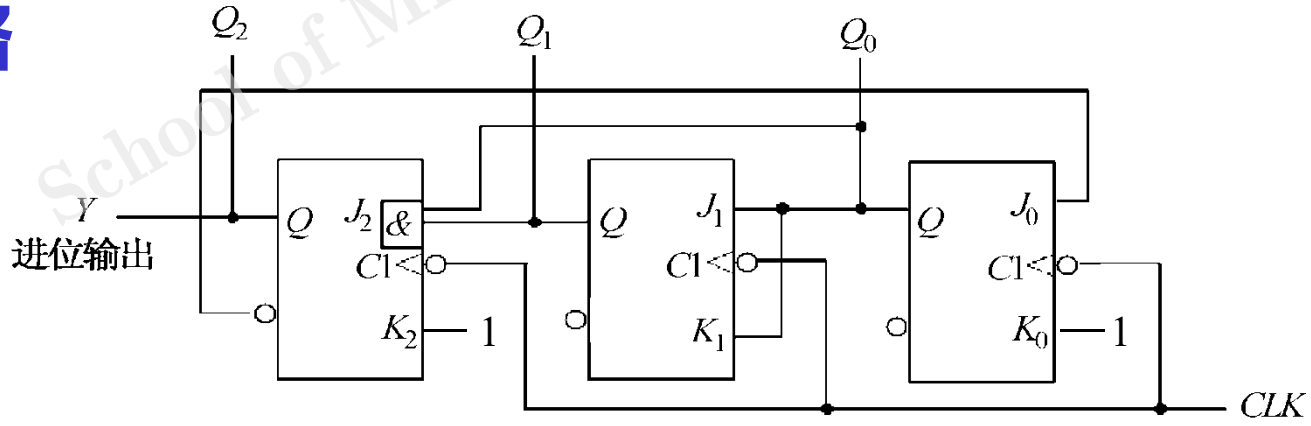
状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

$$Y = Q_2^n$$

$$\begin{cases} J_2 = Q_1^nQ_0^n \\ K_2 = 1 \end{cases} \qquad \begin{cases} J_1 = Q_0^n \\ K_1 = Q_0^n \end{cases} \qquad \begin{cases} J_0 = \overline{Q_2^n} \\ K_0 = 1 \end{cases}$$

6) 电路



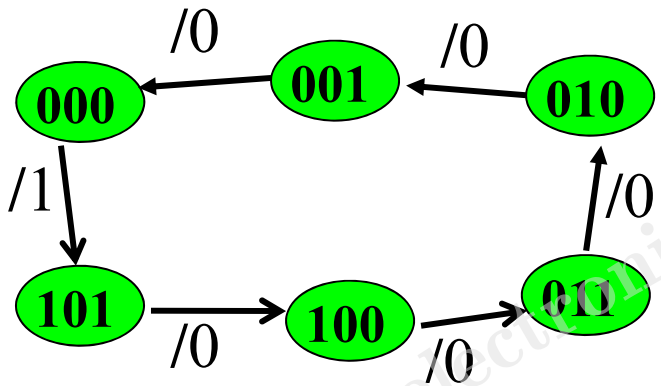
7)检查是否可以自启动

例 2. 设计 M-6 减法计数器

6 个状态　　直接用3位数编码　　$Q_3Q_2Q_1$

借位输出 Z

/ Z



Z

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	0

Q_3^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	1

Q_2^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	0	0	Φ	1
1	0	1	Φ	0

Q_1^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	1	Φ	1
1	0	0	Φ	0

Q_3^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	1

$$Q_3^{n+1} = \overline{Q_3}\ \overline{Q_2}\ \overline{Q_1} + Q_3Q_1$$

$$D_3 = \overline{Q_3}\ \overline{Q_2}\ \overline{Q_1} + Q_3Q_1$$

Q_2^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	0	0	Φ	1
1	0	1	Φ	0

$$Q_2^{n+1} = Q_2Q_1 + Q_3\overline{Q_1}$$

$$D_2 = Q_2Q_1 + Q_3\overline{Q_1}$$

Q_1^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	1	Φ	1
1	0	0	Φ	0

$$Q_1^{n+1} = \overline{Q_1}$$

$$D_1 = \overline{Q_1}$$

Z

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	0

$$Z = \overline{Q_3}\ \overline{Q_2}\ \overline{Q_1}$$

自启动及电路图略

例 3. (例6.5)

设计一个串行数据检测器。该检测器有一个输入端X。电路的功能是对输入信号进行检测。当连续输入三个1（以及三个以上1）时，该电路输出$Y=1$，否则输出$Y=0$。

1）根据设计要求，设定状态，画出状态转换图。

S_0—初始状态或没有收到1时的状态；

S_1—收到一个1后的状态；

S_2—连续收到两个1后的状态；

S_3—连续收到三个1（以及三个以上1）后的状态。

X=1，收到一个 “1”

S_0—初始状态或没有收到1时的状态；

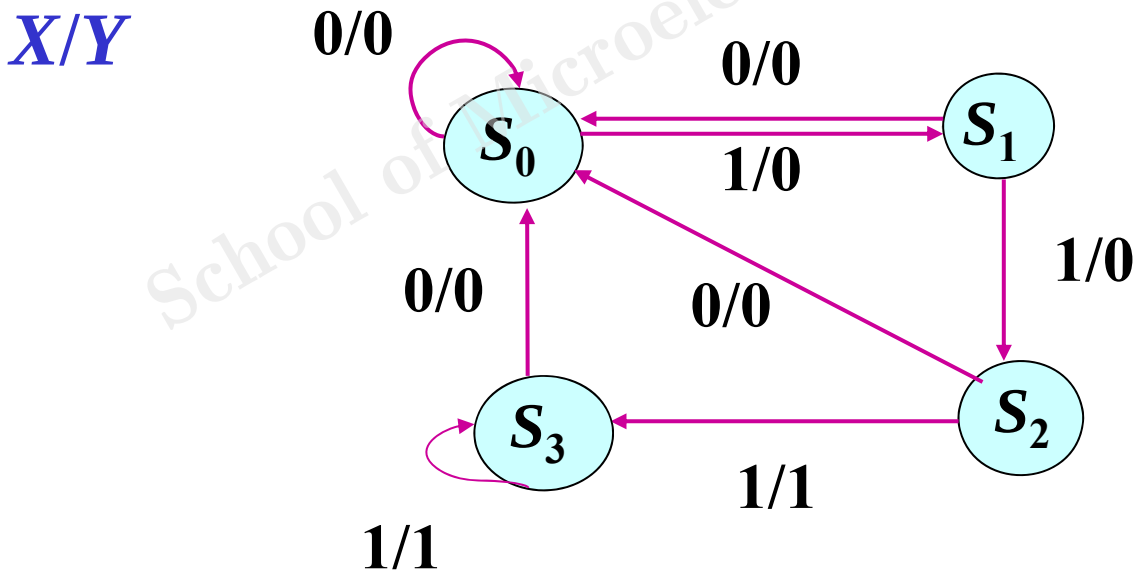
S_1—收到一个1后的状态；

S_2—连续收到两个1后的状态；

S_3—连续收到三个1（以及三个以上1）后的状态。

X=1, 收到一个 “1”

输入三个1（以及三个以上1）时，输出$Y = 1$



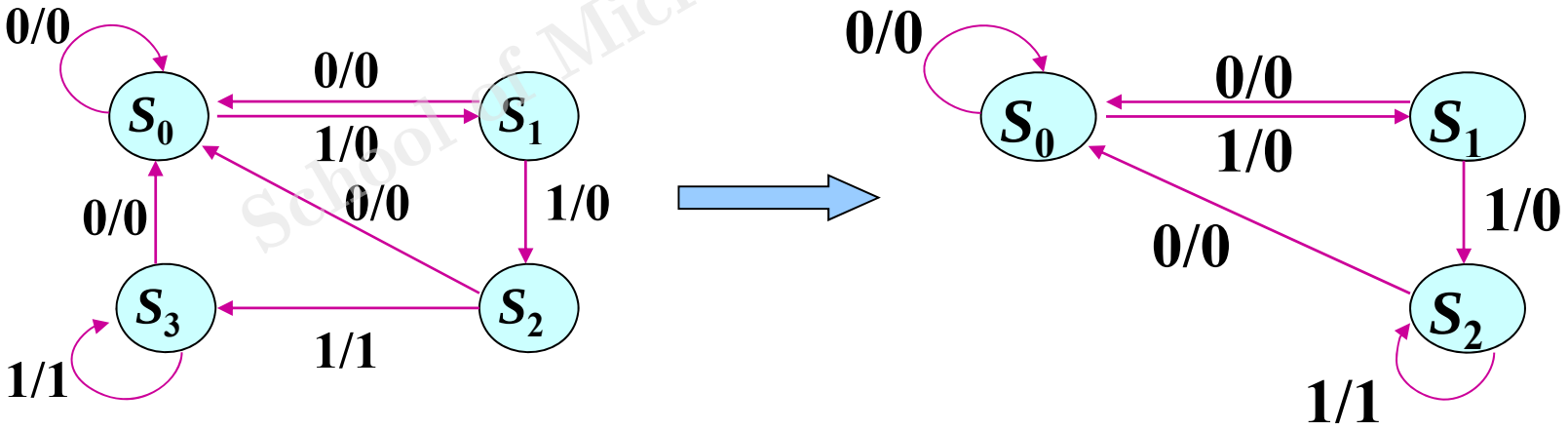
2）状态化简

状态化简： 合并等效状态

等效状态:

在相同的输入条件下，输出相同、次态也相同的状态

S_2 和 S_3 是等效状态，将S_2 和 S_3 合并为S_2



3）状态分配、编码

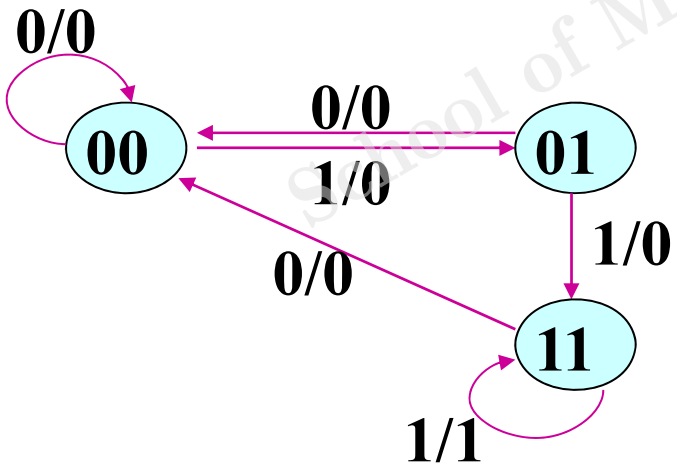
Set $S_0 = 00$

$S_1 = 01$

$S_2 = 11$

编码可以不连续

编码后的状态图



状态表

X	Q_1^n	Q_0^n	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	Φ	Φ	Φ
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	Φ	Φ	Φ
1	1	1	1	1	1

4）选触发器及控制输入

X	Q_1^n	Q_0^n	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	Φ	Φ	Φ
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	Φ	Φ	Φ
1	1	1	1	1	1

Q_1^{n+1} / XQ_1^n / Q_0^n

Q_0^n \ XQ_1^n	00	01	11	10
0	0	Φ	Φ	0
1	0	0	1	1

$$Q_1^{n+1} = XQ_0^n = D_1 \qquad D_1 = XQ_0^n$$

Q_0^{n+1} / XQ_1^n / Q_0^n

Q_0^n \ XQ_1^n	00	01	11	10
0	0	Φ	Φ	1
1	0	0	1	1

$$Q_0^{n+1} = X = D_0 \qquad D_0 = X$$

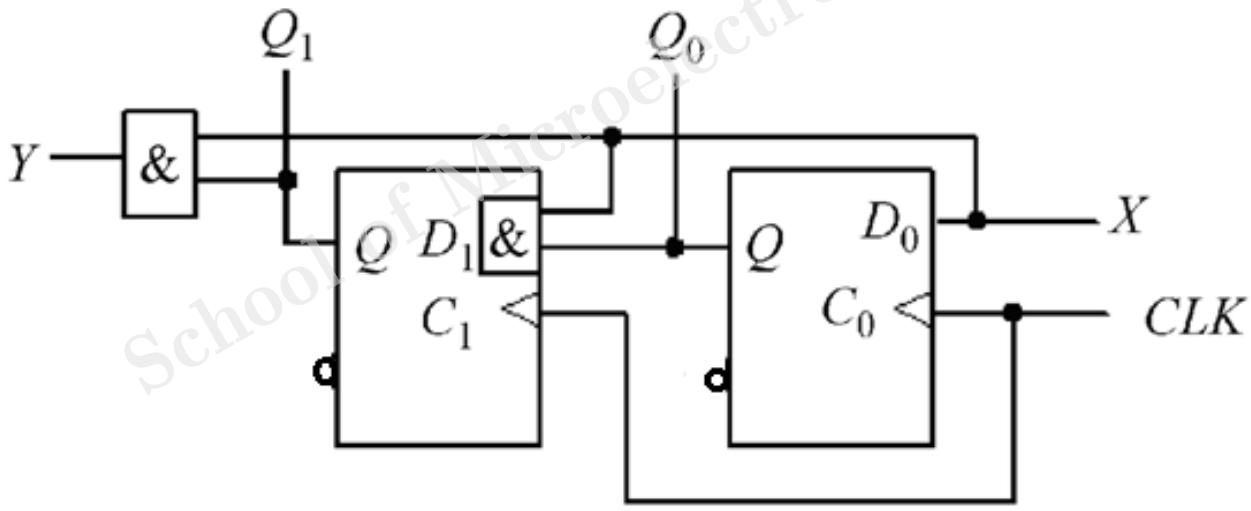
Y / XQ_1^n / Q_0^n

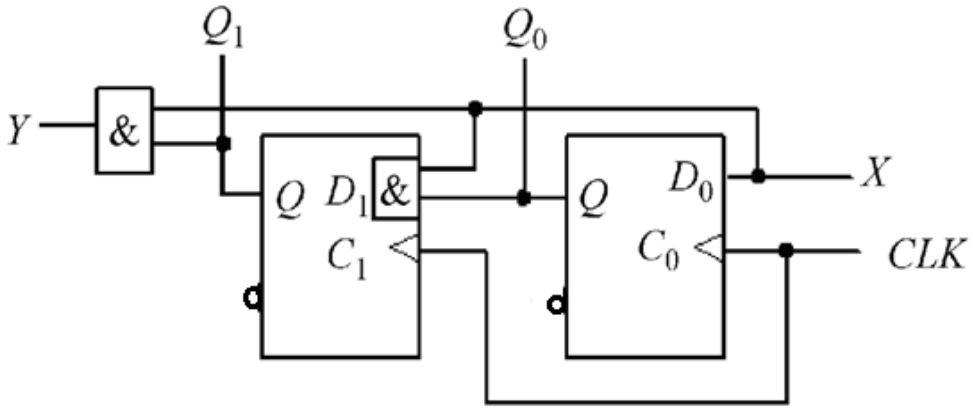
Q_0^n \ XQ_1^n	00	01	11	10
0	0	Φ	Φ	0
1	0	0	1	0

$$Y = XQ_1^n$$

5) 电路

2 D-FFs $D_1 = XQ_0^n$ $D_0 = X$ $Y = XQ_1^n$





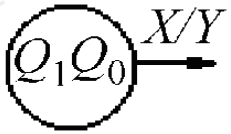
$$Q_1^{n+1} = XQ_0^n$$

$$Q_0^{n+1} = X$$

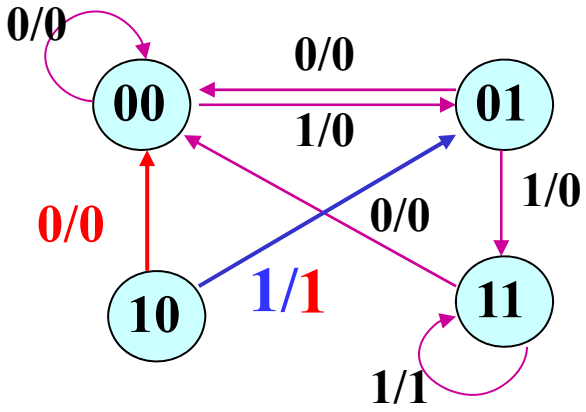
$$Y = XQ_1^n$$

6）自启动

从电路的状态图分析



可以自启动



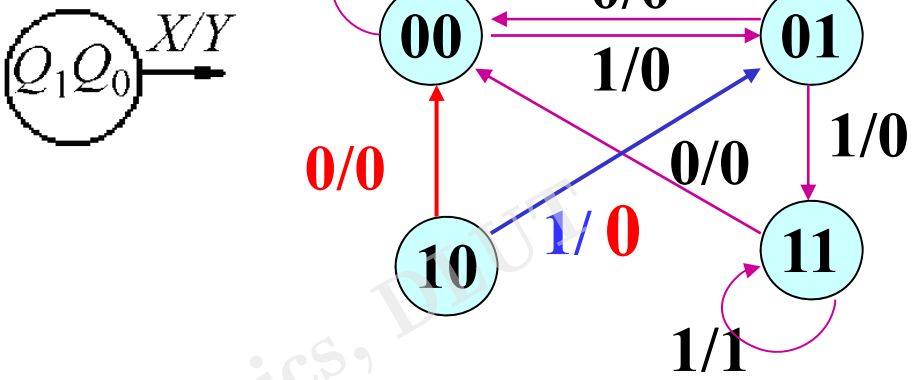
但其功能错误，

输出应设置为0，才符合题意

检测 连续输入三个及以上个1时，电路输出$Y = 1$。

自启动

让110对应的输出为0



状态表

X	Q_1^n	Q_0^n	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	1	1	1	1

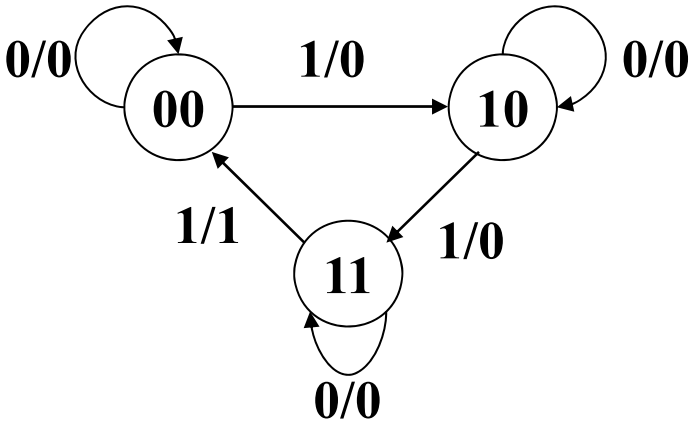
Y / XQ_1^n	00	01	11	10
Q_0^n = 0	0	0	0	0
Q_0^n = 1	0	0	1	0

$$Y = XQ_1^nQ_0^n$$

既实现自启动，也符号题意。

可以在最初设计时就考虑自启动问题。

例 4. 按照下面状态图设计电路 (例6.6)



确定状态及状态表

状态数 $\xrightarrow{\text{确定}}$ FF 个数

n FFs $\rightarrow 2^n$ 状态

$2^{n-1} \leq$ 状态数 $\leq 2^n \rightarrow$ n FFs

$3 < 2^2$ 需要 2 个 FF

状态表 (根据状态图)

X/Z

X	Q_2^n	Q_1^n	Q_2^{n+1}	Q_1^{n+1}	Z
0	0	0	0	0	0
0	0	1	ϕ	ϕ	ϕ
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	1	0	0
1	0	1	ϕ	ϕ	ϕ
1	1	0	1	1	0
1	1	1	0	0	1

选择 FF (K-map, 圈 1)

Q_2^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10
0	0	1	1	1
1	ϕ	1	0	ϕ

2# FF

$$Q_2^{n+1} = \overline{X}Q_2^n + X\overline{Q_1}^n$$

选择 JK-FF

$$Q_2^{n+1} = J_2\overline{Q}_2^n + \overline{K}_2Q_2^n$$

找到 $J_2 = ?$ $K_2 = ?$

不能按上面方法圈，必须圈成 $Q_2^{n+1} = \_\_\_\ \overline{Q}_2^n + \_\_\_\ Q_2^n$

Q_2^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10
0	0	1	1	1
1	ϕ	1	0	ϕ

$\overline{Q}_2^n$ ← Q_2^n → $\overline{Q}_2^n$

$$Q_2^{n+1} = X\overline{Q}_2^n + (\overline{X} + \overline{Q}_1^n)Q_2^n$$

$$= X\overline{Q}_2^n + \overline{XQ_1}^n Q_2^n$$

$$\therefore \begin{cases} J_2 = X \\ K_2 = XQ_1^n \end{cases}$$

能找到系数（控制变量）时尽量化简；找不到系数时，牺牲化简也要找到系数。

X	Q_2^n	Q_1^n	Q_2^{n+1}	Q_1^{n+1}	Z
0	0	0	0	0	0
0	0	1	ϕ	ϕ	ϕ
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	1	0	0
1	0	1	ϕ	ϕ	ϕ
1	1	0	1	1	0
1	1	1	0	0	1

1# FF

Q_1^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10
0	0	0	1	0
1	ϕ	1	0	ϕ

$0 \rightarrow \overline{Q_1}^n$

$1 \rightarrow Q_1^n$

JK-FF

$$Q_1^{n+1} = J_1\overline{Q_1}^n + \overline{K}_1 Q_1^n$$

$$= XQ_2^n\overline{Q_1}^n + \overline{X}Q_1^n$$

$$\therefore \begin{cases} J_1 = XQ_2^n \\ K_1 = X \end{cases}$$

输出 Z

Z

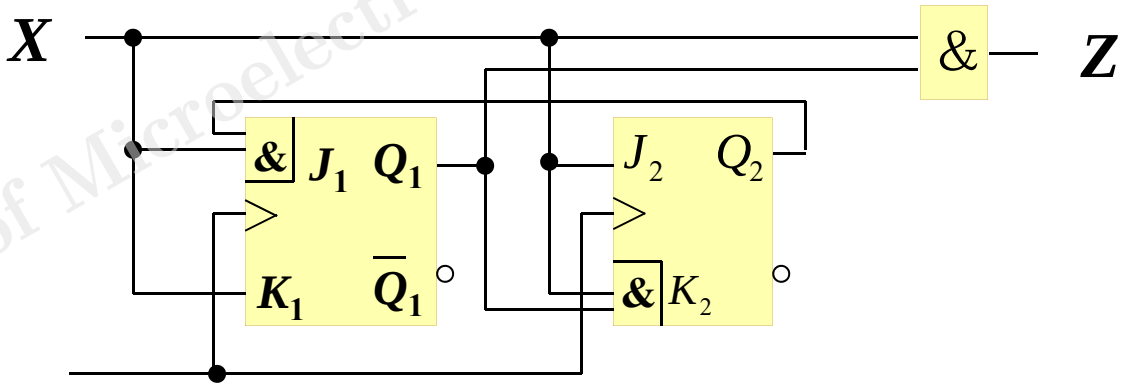
Q_1^n \ XQ_2^n	00	01	11	10
0	0	0	0	0
1	Φ	0	1	Φ

$$Z = XQ_1^n$$

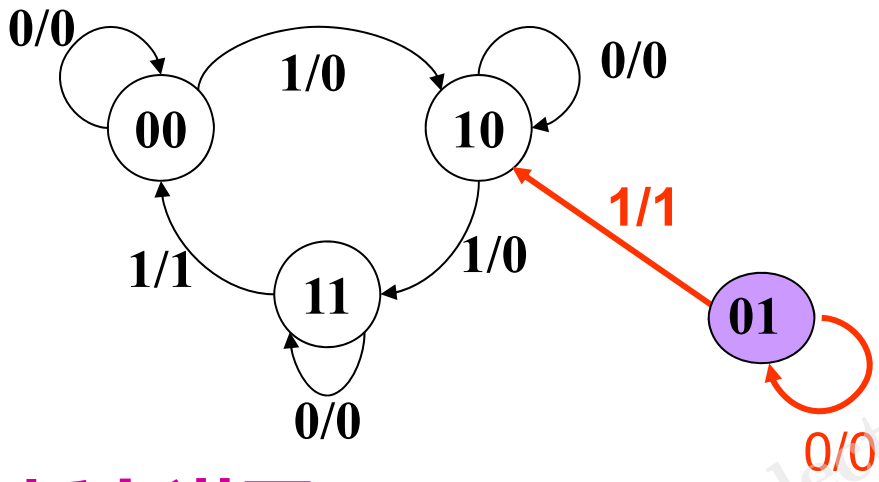
电路

$$\begin{cases} J_2 = X \\ K_2 = XQ_1^n \end{cases}$$

$$\begin{cases} J_1 = XQ_2^n \\ K_1 = X \end{cases}$$



讨论：01 状态



Q_1^n \ XQ_2^n	00	01	11	10
0	0	1	1	1
1	0	1	0	1

Q_2^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10
0	0	0	1	0
1	1	1	0	0

Q_1^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10
0	0	0	0	0
1	0	0	1	1

Z

分析卡诺图 K-map

$XQ_2^nQ_1^n = 001$, (Z=0)

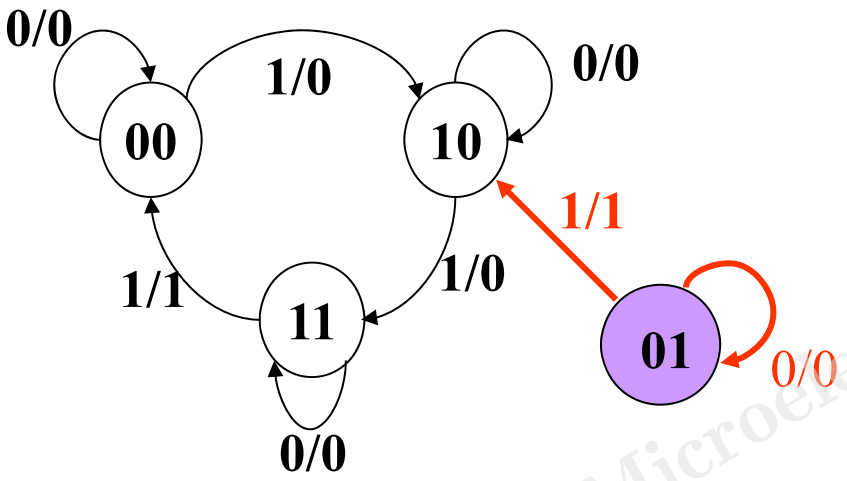
Next state $Q_2^{n+1}Q_1^{n+1} = 01$,

$XQ_2^nQ_1^n = 101$时, (Z=1)

Next state $Q_2^{n+1}Q_1^{n+1} = 10$,

实现自启动

要分析输出的物理意义（即电路功能）是否正确。



此电路为可控模3加法计数器。

X=0，保持原状态；
X=1，作加法计数。

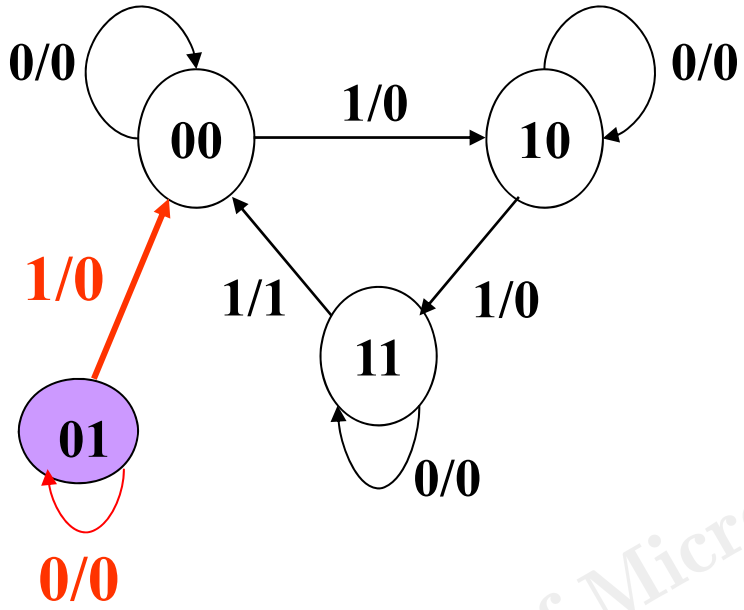
输出Y=1，为进位输出。

显然自启动之后电路功能出现错误。

应该将X=1时01状态的输出设为0。

在设计电路时

在填状态表时不能填 ϕ



X=0，保持原状态；

X=1，作加法计数（从0开始）

状态表

X	Q_2^n	Q_1^n	Q_2^{n+1}	Q_1^{n+1}	Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	1	0	0
1	0	1	0	0	0
1	1	0	1	1	0
1	1	1	0	0	1

才能符合电路要求

例5. 设计一个自动售饮料机的逻辑电路。它的投币口每次只能投入一枚五角或一元的硬币。投入一元五角钱硬币后，机器会自动给出一杯饮料；投入两元（两个一元）硬币后，在给出饮料的同时找回一枚五角硬币。

解:

输入: {投入 ¥1.0, A=1；投入 ¥0.5, B=1}

输出: {出饮料, Y=1；找钱, Z=1}

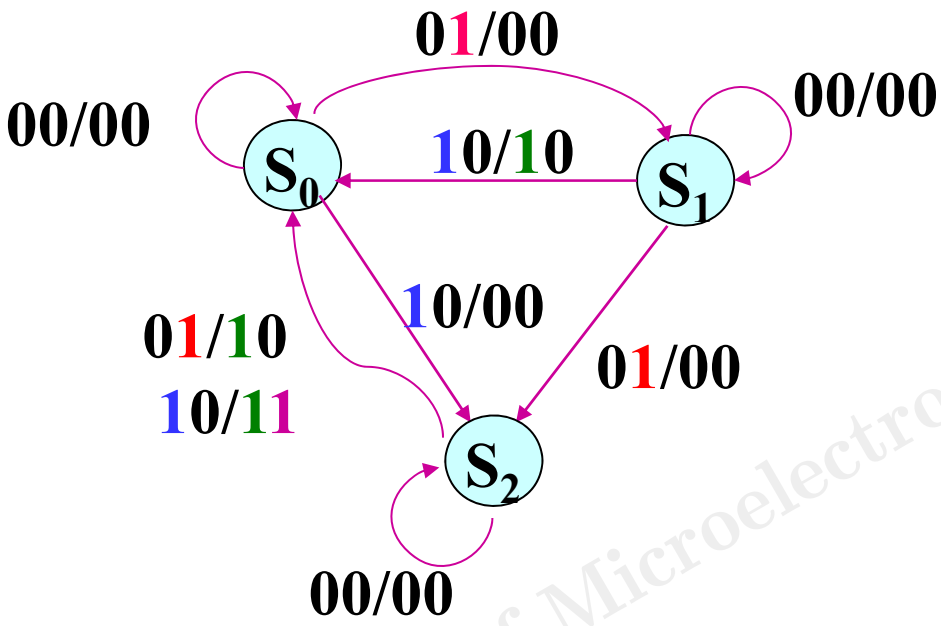
状态:

3个状态

- S_0 : 初始 (未投币)
- S_1 : 投入 ¥0.5,
- S_2 : 投入 ¥1.0 (一个 ¥1.0 或两个 ¥0.5)
  - 再投入 ¥0.5, 返回 S_0, 输出 Y=1, Z=0;
  - 再投入¥1.0, 返回 S_0, 输出 Y=1, Z=1 (找回 ¥0.5).

多输入，多输出

AB /YZ



S_0: 投入 ¥ 0.5, S_1

S_0:投入¥ 1.0, S_2

S_1:投入¥ 0.5, S_2

S_1:投入¥ 1.0, S_0

饮料

S_2:投入¥ 0.5, S_0

饮料

S_2:投入¥ 1.0, S_0

饮料 和 找钱

A : ¥ 1.0

B : ¥ 0.5

Y : 饮料

Z : 找钱

S_0 : 初始

S_1 : ¥ 0.5

S_2 : ¥ 1.0

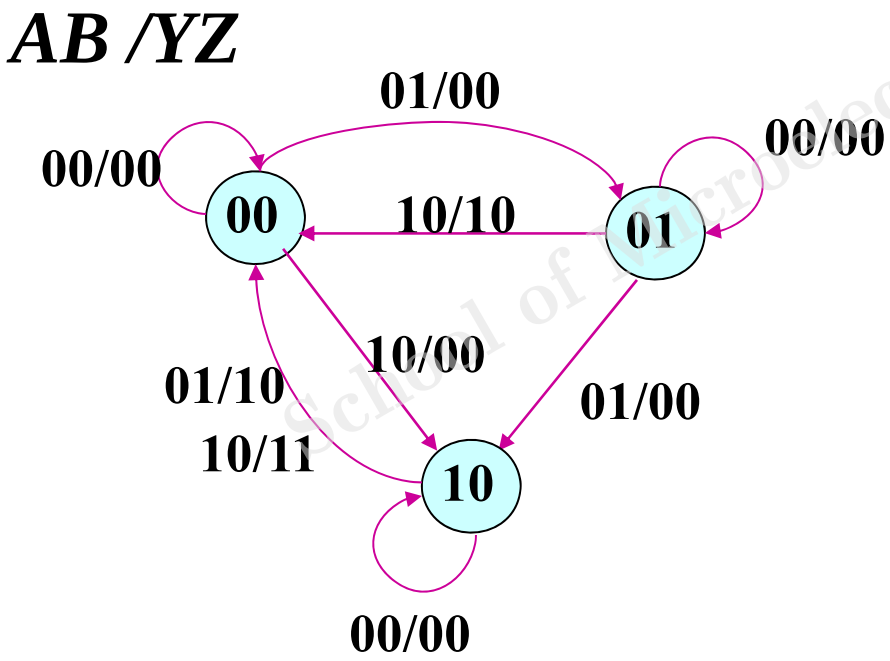
$Q_1^n Q_0^n$

编码

$S_0 \rightarrow 00$

$S_1 \rightarrow 01$

$S_2 \rightarrow 10$



状态表

A B Q_1^n Q_0^n	Q_1^{n+1} Q_0^{n+1} Y Z
0 0 0 0	0 0 0 0
0 0 0 1	0 1 0 0
0 0 1 0	1 0 0 0
0 0 1 1	Φ Φ Φ Φ
0 1 0 0	0 1 0 0
0 1 0 1	1 0 0 0
0 1 1 0	0 0 1 0
0 1 1 1	Φ Φ Φ Φ
1 0 0 0	1 0 0 0
1 0 0 1	0 0 1 0
1 0 1 0	0 0 1 1
1 0 1 1	Φ Φ Φ Φ
1 1 0 0	Φ Φ Φ Φ
1 1 0 1	Φ Φ Φ Φ
1 1 1 0	Φ Φ Φ Φ
1 1 1 1	Φ Φ Φ Φ

Q_1^{n+1} / $Q_1^nQ_0^n$	AB = 00	01	11	10
00	0	0	Φ	1
01	0	1	Φ	0
11	Φ	Φ	Φ	Φ
10	1	0	Φ	0

$$Q_1^{n+1} = BQ_0 + A\overline{Q}_1\overline{Q}_0 + \overline{A}\cdot\overline{B}Q_1$$

Q_0^{n+1} / $Q_1^nQ_0^n$	AB = 00	01	11	10
00	0	1	Φ	0
01	1	0	Φ	0
11	Φ	Φ	Φ	Φ
10	0	0	Φ	0

$$Q_0^{n+1} = B\overline{Q}_1\cdot\overline{Q}_0 + \overline{A}\cdot\overline{B}Q_0$$

Y / $Q_1^nQ_0^n$	AB = 00	01	11	10
00	0	0	Φ	0
01	0	0	Φ	1
11	Φ	Φ	Φ	Φ
10	0	1	Φ	1

$$Y = AQ_0 + AQ_1 + BQ_1$$

Z / $Q_1^nQ_0^n$	AB = 00	01	11	10
00	0	0	Φ	0
01	0	0	Φ	0
11	Φ	Φ	Φ	Φ
10	0	0	Φ	1

$$Z = AQ_1$$

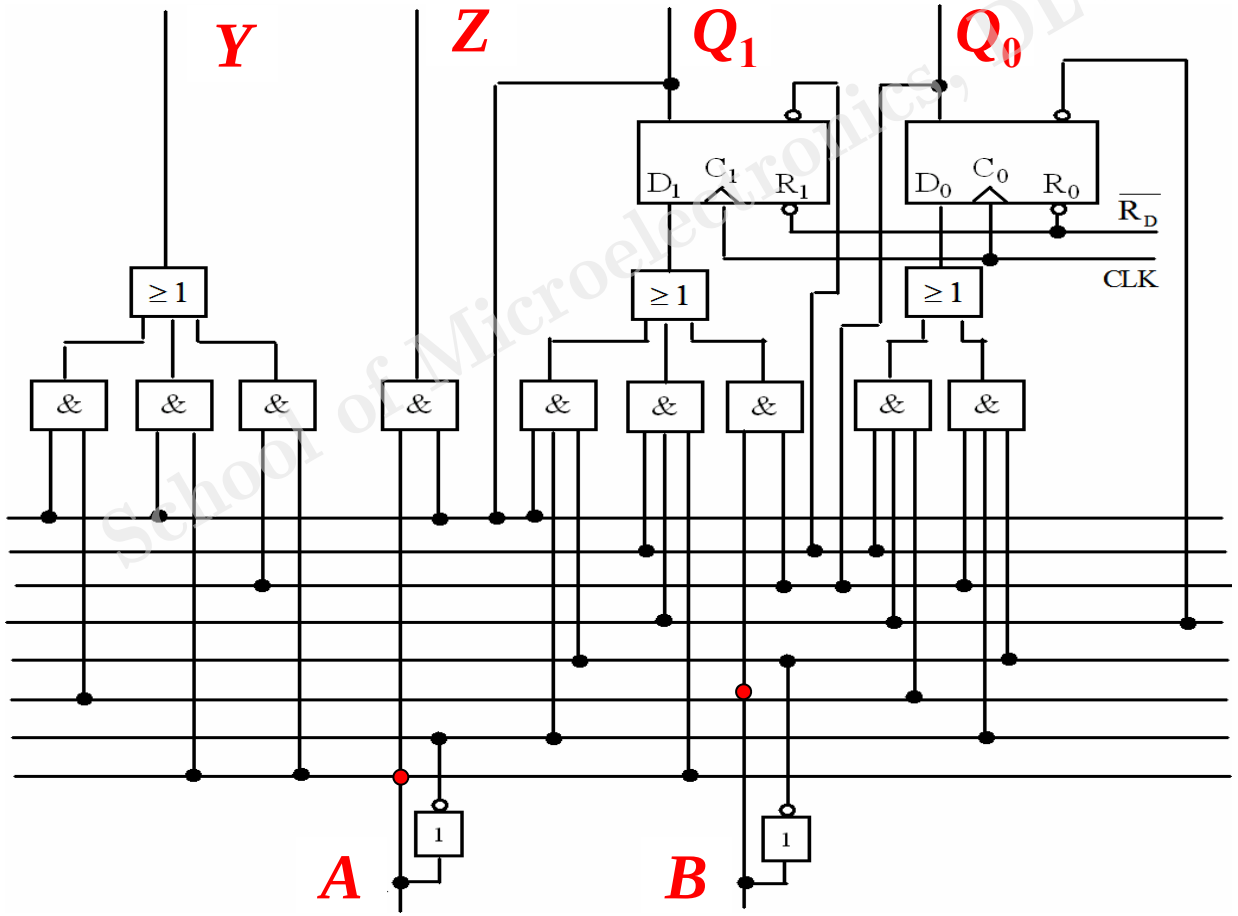
用 D-FF

$$D_1 = BQ_0 + A\overline{Q}_1\overline{Q}_0 + \overline{A}\cdot\overline{B}Q_1$$

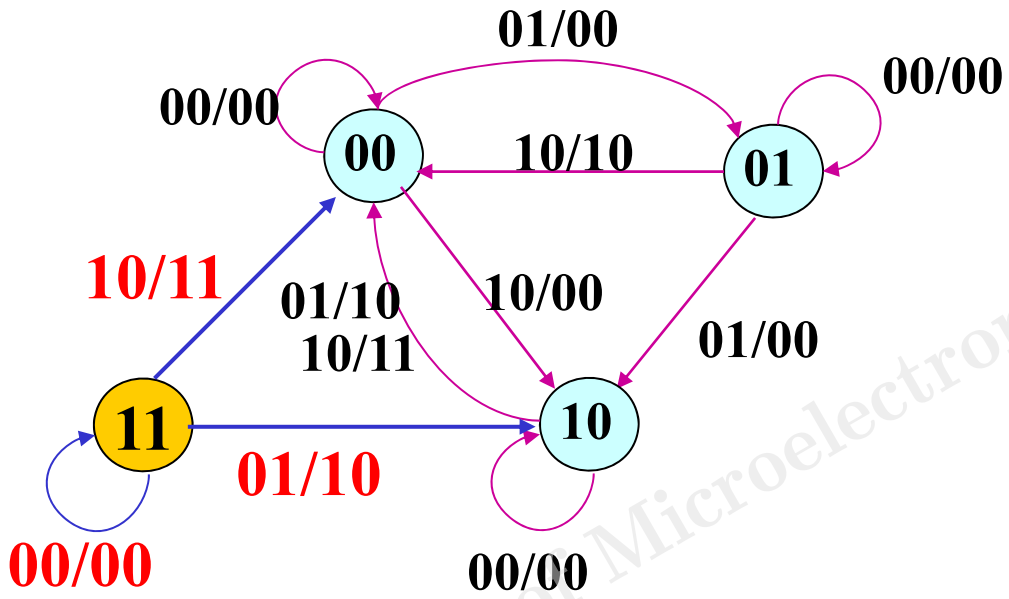
$$D_0 = B\overline{Q}_1\cdot\overline{Q}_0 + \overline{A}\cdot\overline{B}Q_0$$

$$Y = AQ_0 + AQ_1 + BQ_1$$

$$Z = AQ_1$$



由电路得到状态图



A：¥ 1.0

B：¥ 0.5

Y：饮料

Z：找钱

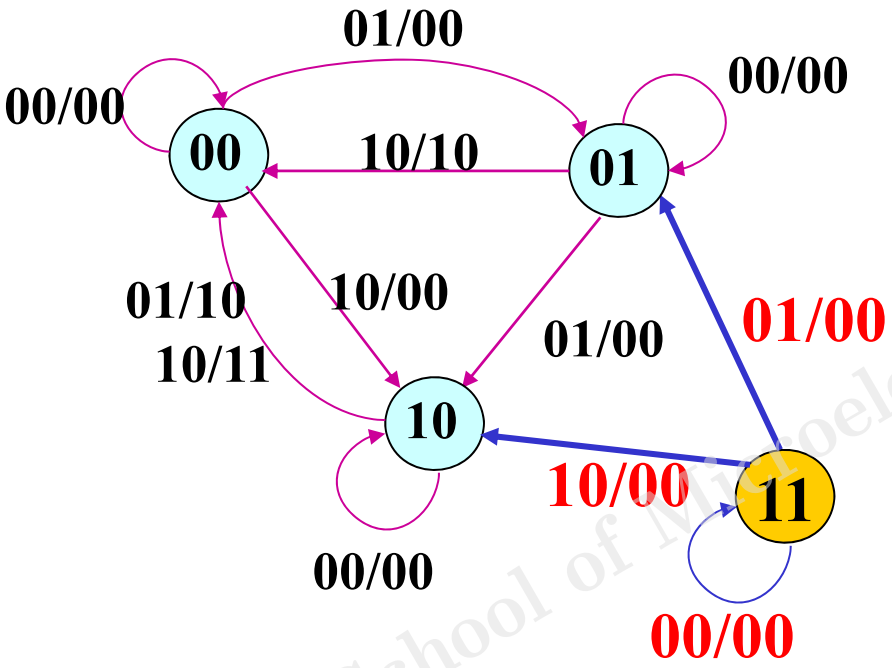
在电路处于“11”状态时，

若 $AB = 00$ (无输入)，$Q_1Q_0 = 11$，电路不能自启动；

若$AB = 01$ 或 10，电路可以自启动，但是找钱系统出错；

所以，电路初始工作时，应首先将 R_D 设置为低电平，电路状态从 “00”开始。

电路状态图应设计成



状态表

A	B	Q_1^n	Q_0^n	Q_1^{n+1}	Q_0^{n+1}	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0
0	0	1	0	1	0	0	0
0	0	1	1	1	1	0	0
0	1	0	0	0	1	0	0
0	1	0	1	1	0	0	0
0	1	1	0	0	0	1	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	0	1	1
1	0	1	1	1	0	0	0
1	1	0	0	Φ	Φ	Φ	Φ
1	1	0	1	Φ	Φ	Φ	Φ
1	1	1	0	Φ	Φ	Φ	Φ
1	1	1	1	Φ	Φ	Φ	Φ