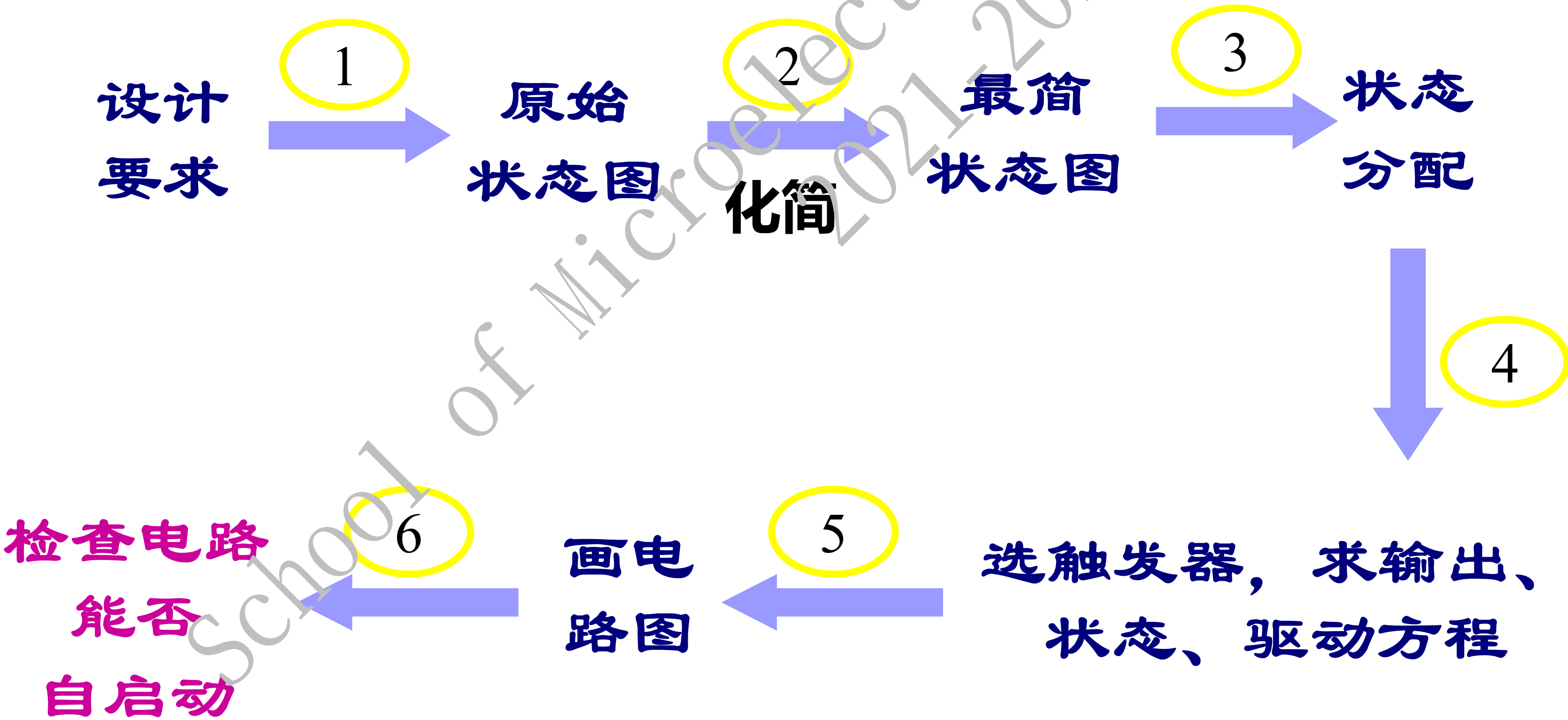
§6.3 同步时序电路设计

Synchronous Sequential Circuit Design

已知 → 功能或状态图

求 → 电路



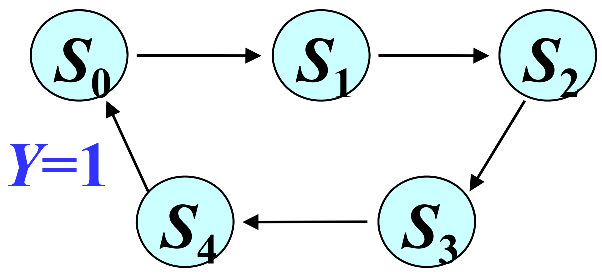
例 1. 设计同步5进制加法计数器

1) 确定状态及状态图

M-5 计数器，5 个状态：S_0, S_1, S_2, S_3, S_4

在计数脉冲*CLK*作用下

- 5 个状态周期性变换
- 在 S_4 状态下进位输出 $Y = 1$



2) 状态化简

M-5, 5 个状态， 不须再化简

3) 状态分配、编码

$2^{n-1} \le$ 状态数 $\le 2^n$　　n：二进制位数　　3位

$S_0 \rightarrow 000$

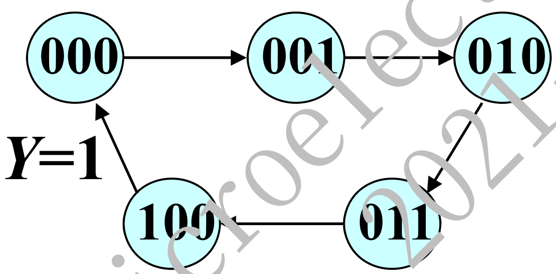
$S_1 \rightarrow 001$

$S_2 \rightarrow 010$

$S_3 \rightarrow 011$

$S_4 \rightarrow 100$

状态图



状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

4) 选择 FF，确定驱动方程、状态方程Q^{n+1}及输出方程

方法 1：先不确定用哪种触发器

由状态表填卡诺图

Q_2^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	0
1	0	1	Φ	Φ

Q_1^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	1	Φ	0
1	1	0	Φ	Φ

Q_0^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	1	1	Φ	0
1	0	0	Φ	Φ

状态表

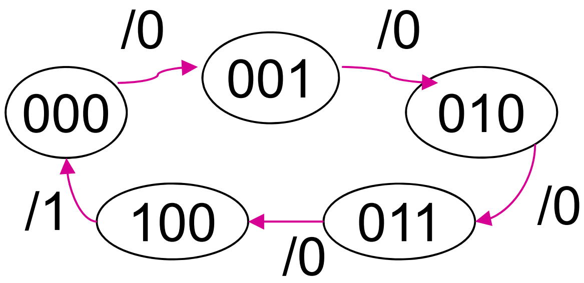
Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

Y

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	1
1	0	0	Φ	Φ

也可直接填卡诺图

直接填卡诺图



5个 有效状态 **3位二进制数 3 个FF**

$$Q_2^nQ_1^nQ_0^n \xrightarrow{/Z} Q_2^{n+1}Q_1^{n+1}Q_0^{n+1}$$

Y

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	1
1	0	0	Φ	Φ

Q_2^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	0	Φ	0
1	0	1	Φ	Φ

Q_1^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	0	1	Φ	0
1	1	0	Φ	Φ

Q_0^{n+1}

Q_0^n \ $Q_2^nQ_1^n$	00	01	11	10
0	1	1	Φ	0
1	0	0	Φ	Φ

Q_2^{n+1} (Q_0^n \ $Q_2^nQ_1^n$)	00	01	11	10
0	0	0	Φ	0
1	0	1	Φ	Φ

$$Q_2^{n+1} = Q_1^nQ_0^n = D_2$$

$$D_2 = Q_1^nQ_0^n$$

Q_1^{n+1} (Q_0^n \ $Q_2^nQ_1^n$)	00	01	11	10
0	0	1	Φ	0
1	1	0	Φ	Φ

$$Q_1^{n+1} = Q_0^n\overline{Q}_1^n + \overline{Q}_0^nQ_1^n = Q_0^n \oplus Q_1^n = T_1 \oplus Q_1^n$$

$$T_1 = Q_0^n$$

Q_0^{n+1} (Q_0^n \ $Q_2^nQ_1^n$)	00	01	11	10
0	1	1	Φ	0
1	0	0	Φ	Φ

$$Q_0^{n+1} = \overline{Q}_2^n\overline{Q}_0^n = D_0$$

或

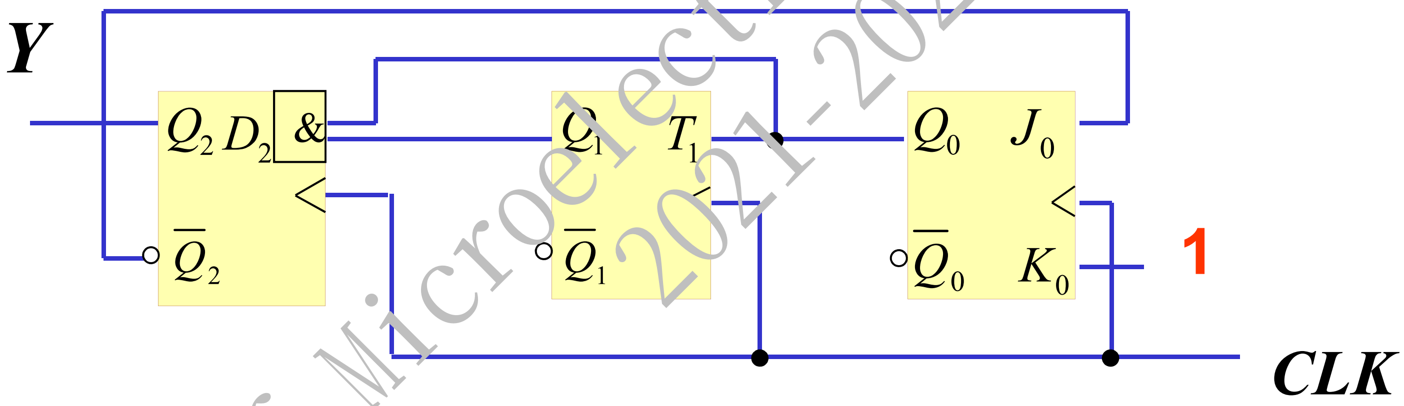
$$\begin{cases} J_0 = \overline{Q_2^n} \\ K_0 = 1 \end{cases}$$

Y (Q_0^n \ $Q_2^nQ_1^n$)	00	01	11	10
0	0	0	Φ	1
1	0	0	Φ	Φ

$$Y = Q_2^n$$

$D_2 = Q_1^n Q_0^n$ $T_1 = Q_0^n$ $\begin{cases} J_0 = \overline{Q}_2^n \\ K_0 = 1 \end{cases}$ $Y = Q_2^n$

5）电路



与门可以省略

6)检查是否可以自启动

$$Q_2^{n+1} = Q_1^n Q_0^n$$

$$Q_1^{n+1} = Q_0^n \overline{Q}_1^n + \overline{Q}_0^n Q_1^n$$

$$= Q_0^n \oplus Q_1^n$$

$$Q_0^{n+1} = \overline{Q}_2^n \overline{Q}_0^n$$

可以自启动

状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1
1	0	1	0	1	0	1
1	1	0	0	1	0	1
1	1	1	1	0	0	1

方法 2：确定用哪种触发器

4) 选择 FF　　　选 JK-FFs

5) 状态方程Q^{n+1}及控制输入– J, K

状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

JK-FF 激励表

$Q^n \rightarrow Q^{n+1}$		J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

Q_2^n	$\Rightarrow$ Q_2^{n+1}	J_2
0	0	0
0	0	0
0	0	0
0	1	1
1	0	X
X	X	X
X	X	X
X	X	X

驱动方程 $J=F(Q_2^n,Q_1^n,Q_0^n)$

得到 2#-FF 控制输入 J_2 驱动卡诺图

J_2: Q_2^n \ $Q_1^nQ_0^n$	00	01	11	10
0	0	0	1	0
1	×	×	×	×

状态图

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

JK-FF 激励表

$Q^n \rightarrow Q^{n+1}$		J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

Q_1^n	$\Rightarrow$ Q_1^{n+1}	K_1
0	0	X
0	1	X
1	1	0
1	0	1
0	0	X
X	X	X
X	X	X
X	X	X

得到 1#-FF 控制输入 K_1 驱动卡诺图

K_1: Q_2^n \ $Q_1^nQ_0^n$	00	01	11	10
0	×	×	1	0
1	×	×	×	×

得到各个触发器控制输入驱动卡诺图及控制输入

J_2: Q_2^n \ $Q_1^n Q_0^n$	00	01	11	10
0	0	0	1	0
1	×	×	×	×

$$J_2 = Q_1^n Q_0^n$$

J_1: Q_2^n \ $Q_1^n Q_0^n$	00	01	11	10
0	0	1	×	×
1	0	×	×	×

$$J_1 = Q_0^n$$

J_0: Q_2^n \ $Q_1^n Q_0^n$	00	01	11	10
0	1	×	×	1
1	0	×	×	×

$$J_0 = \overline{Q_2^n}$$

K_2: Q_2^n \ $Q_1^n Q_0^n$	00	01	11	10
0	X	X	X	X
1	1	X	X	X

$$K_2 = 1$$

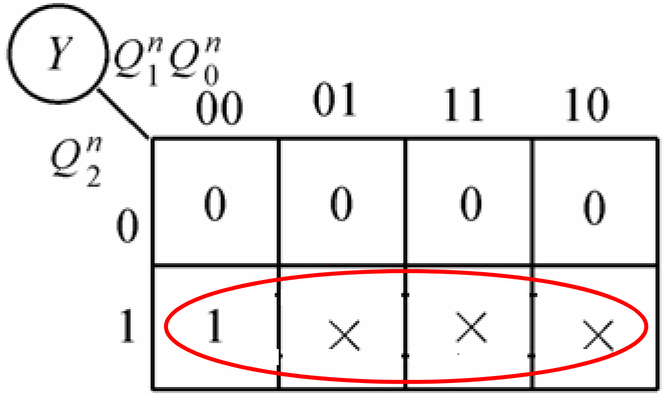
K_1: Q_2^n \ $Q_1^n Q_0^n$	00	01	11	10
0	×	×	1	0
1	×	×	×	×

$$K_1 = Q_0^n$$

K_0: Q_2^n \ $Q_1^n Q_0^n$	00	01	11	10
0	×	1	1	×
1	×	×	×	×

$$K_0 = 1$$

输出卡诺图



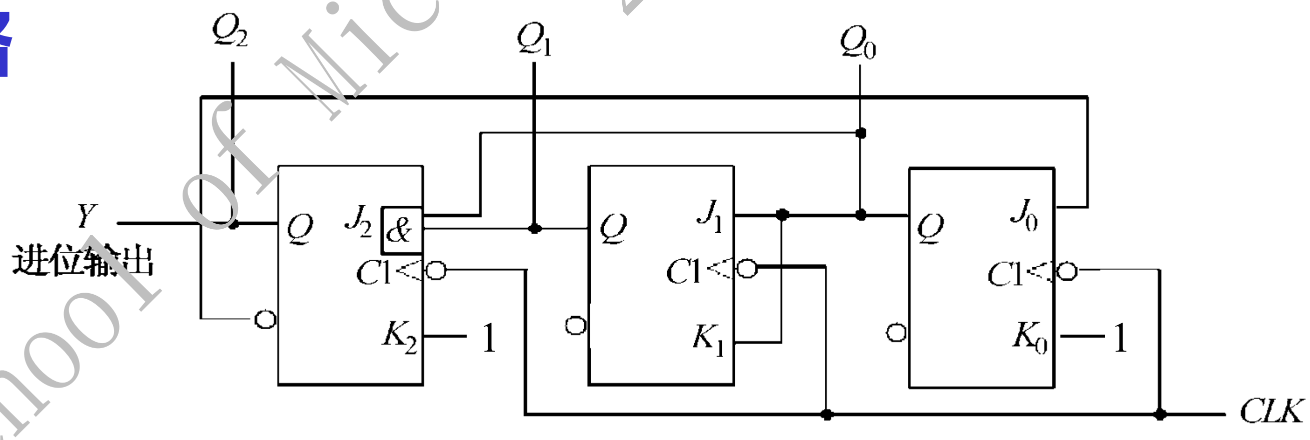
状态表

Q_2^n	Q_1^n	Q_0^n	Q_2^{n+1}	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1

$$Y = Q_2^n$$

$$\begin{cases} J_2 = Q_1^n Q_0^n \\ K_2 = 1 \end{cases} \quad \begin{cases} J_1 = Q_0^n \\ K_1 = Q_0^n \end{cases} \quad \begin{cases} J_0 = \overline{Q_2^n} \\ K_0 = 1 \end{cases}$$

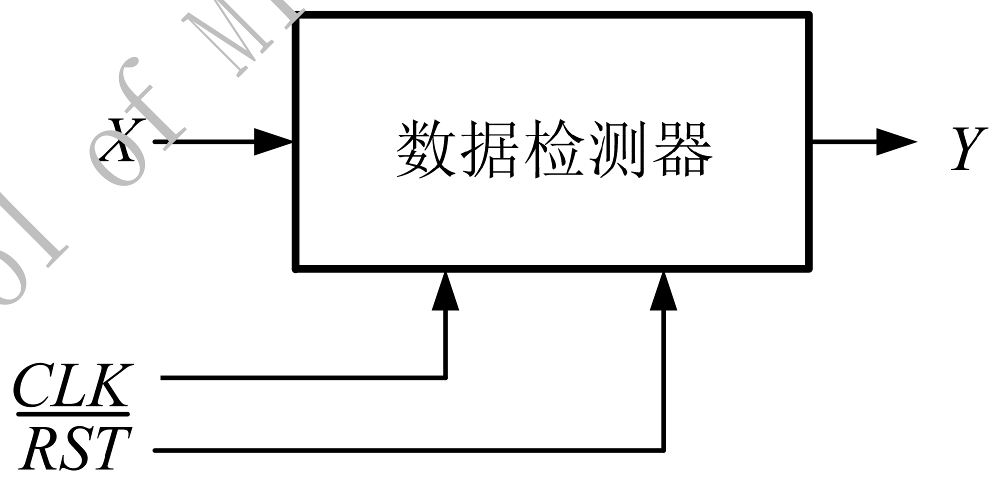
6) 电路



7)检查是否可以自启动

例 2. 设计一个串行数据检测器。该检测器有一个输入端X。电路的功能是对输入信号进行检测。当连续输入三个1（以及三个以上1）时，该电路输出$Y = 1$，否则输出$Y = 0$。

时钟周期	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}
X	0	1	0	1	1	1	0	0	1	1	1	1	0	1	0
Y	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0



1）根据设计要求，设定状态

S_0—初始状态或没有收到1时的状态

S_1—收到一个1后的状态

S_2—连续收到两个1后的状态

S_3—连续收到三个1（以及三个以上1）后的状态

X=1, 收到一个 "1"

输入三个1（以及三个以上1），输出$Y=1$

2）画出状态转换图

S_0—初始状态或没有收到1时的状态；

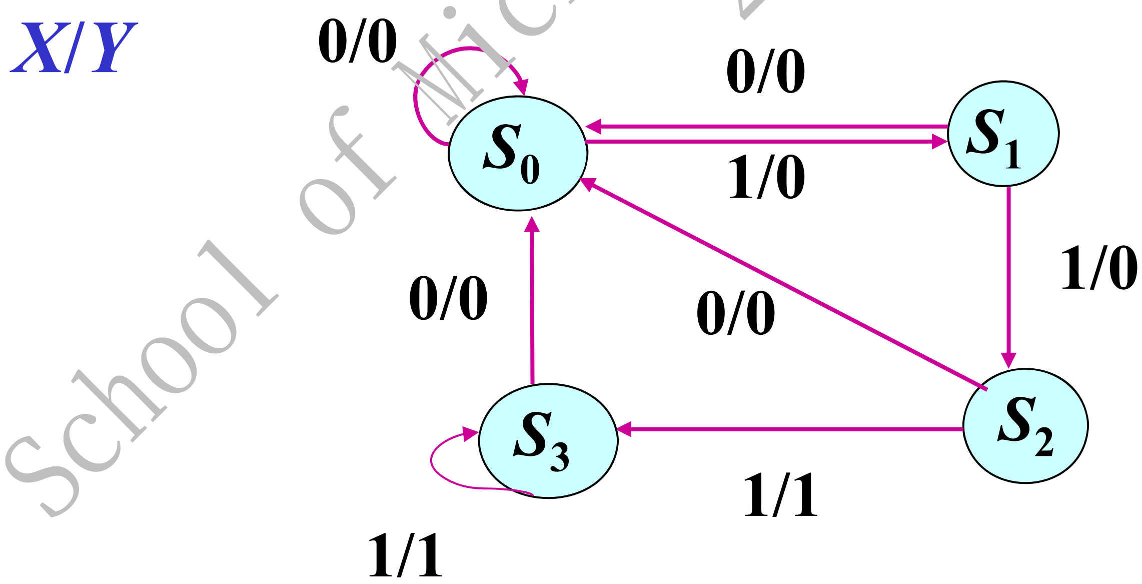
S_1—收到一个1后的状态；

S_2—连续收到两个1后的状态；

S_3—连续收到三个1（以及三个以上1）后的状态。

X=1, 收到一个“1”

输入三个1（以及三个以上1），输出$Y = 1$



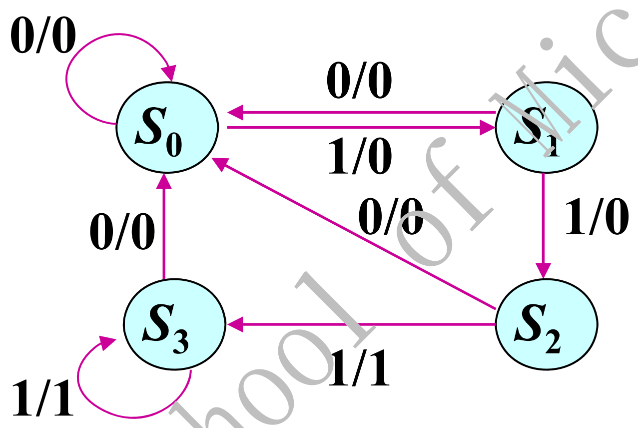
3）状态化简

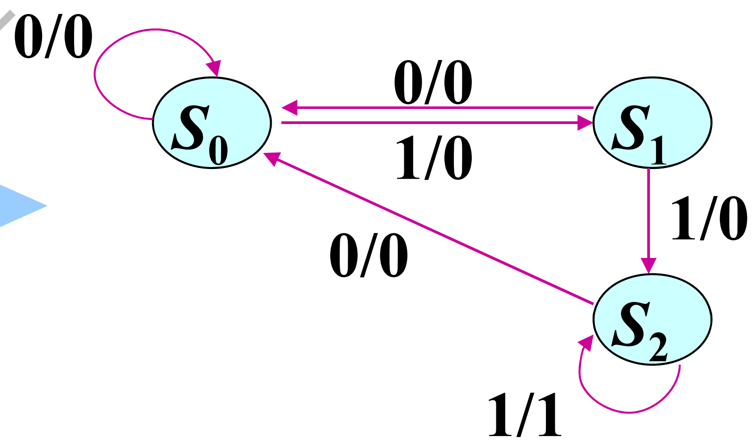
状态化简：合并等效状态

等效状态:

在相同的输入条件下，输出相同、次态也相同的状态

S_2 和 S_3 是等效状态，将 S_2 和 S_3 合并为 S_2





3）状态分配、编码

Set $S_0 = 00$

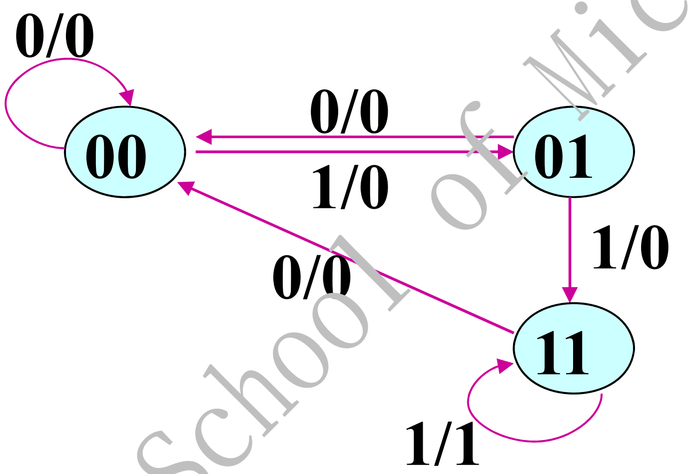
$S_1 = 01$ 编码可以不连续

$S_2 = 11$

状态表

X	Q_1^n	Q_0^n	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	Φ	Φ	Φ
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	Φ	Φ	Φ
1	1	1	1	1	1

编码后的状态图



4）选触发器及控制输入

X	Q_1^n	Q_0^n	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	Φ	Φ	Φ
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	Φ	Φ	Φ
1	1	1	1	1	1

Q_1^{n+1}

Q_0^n \ XQ_1^n	00	01	11	10
0	0	Φ	Φ	0
1	0	0	1	1

$$Q_1^{n+1} = XQ_0^n = D_1 \qquad D_1 = XQ_0^n$$

Q_0^{n+1}

Q_0^n \ XQ_1^n	00	01	11	10
0	0	Φ	Φ	1
1	0	0	1	1

$$Q_0^{n+1} = X = D_0 \qquad D_0 = X$$

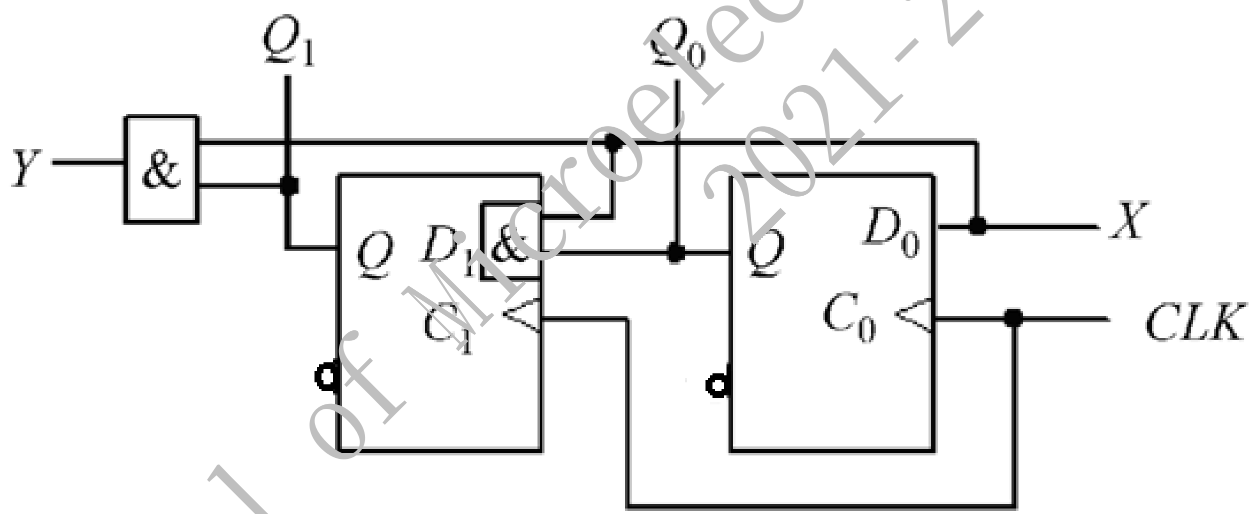
Y

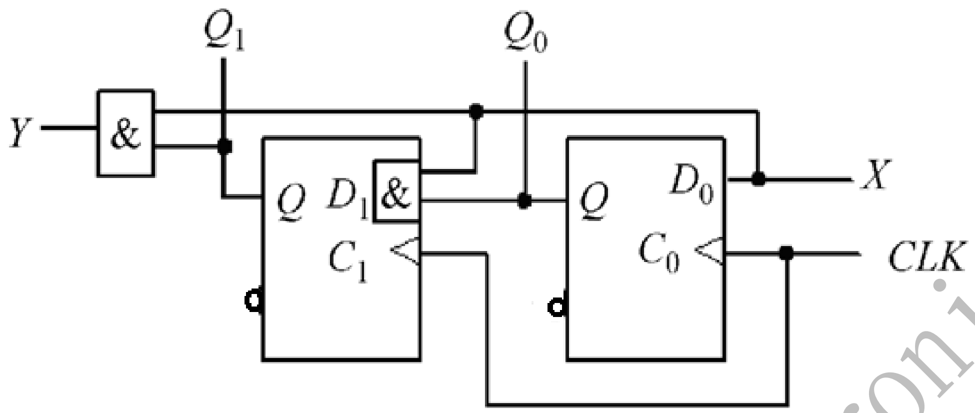
Q_0^n \ XQ_1^n	00	01	11	10
0	0	Φ	Φ	0
1	0	0	1	0

$$Y = XQ_1^n$$

5) 电路

2 D-FFs $D_1 = XQ_0^n$ $D_0 = X$ $Y = XQ_1^n$





$$Q_1^{n+1} = XQ_0^n$$

$$Q_0^{n+1} = X$$

$$Y = XQ_1^n$$

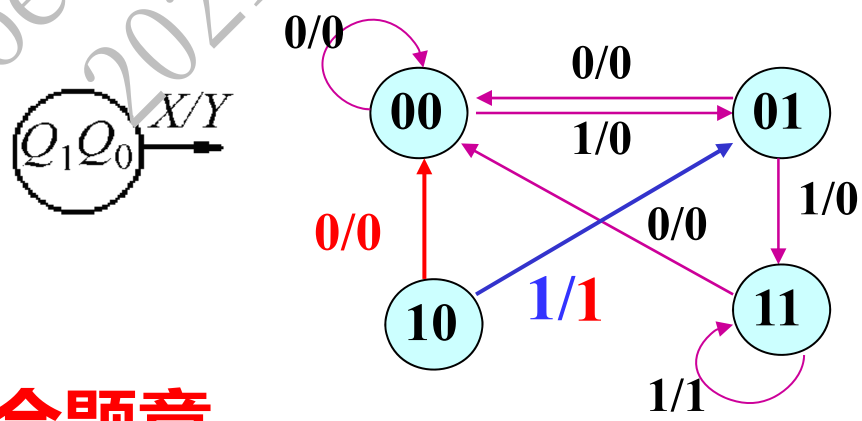
6）自启动

从电路的状态图分析

可以自启动

但其功能错误，

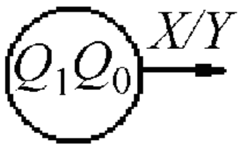
输出应设置为0，才符合题意

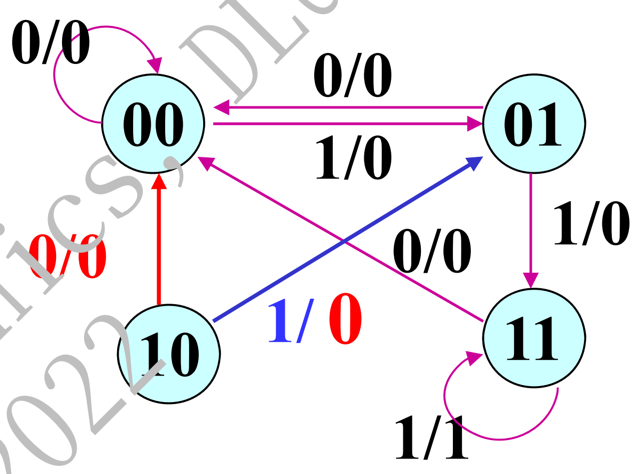


检测连续输入三个及以上个1时，电路输出$Y = 1$。

自启动

让X=1，10对应的输出为0





状态表

X	Q_1^n	Q_0^n	Q_1^{n+1}	Q_0^{n+1}	Y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	1	1	1	1

Y Q_0^n \ XQ_1^n	00	01	11	10
0	0	0	0	0
1	0	0	1	0

$$Y = XQ_1^nQ_0^n$$

既实现自启动，也符合题意

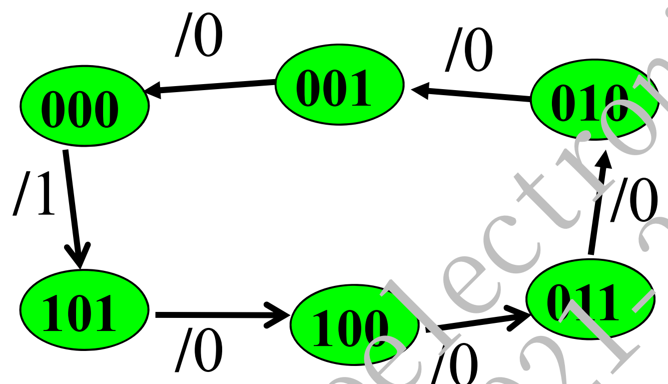
可以在最初设计时考虑自启动（K-map随意项的填写）

例 3. 设计 M-6 减法计数器

6 个状态　　　**直接用3位数编码**　$Q_3Q_2Q_1$

借位输出 Z

$/Z$



Z

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	0

Q_3^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	1

Q_2^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	0	0	Φ	1
1	0	1	Φ	0

Q_1^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	1	Φ	1
1	0	0	Φ	0

Q_3^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	1

$$Q_3^{n+1} = \overline{Q_3}\,\overline{Q_2}\,\overline{Q_1} + Q_3Q_1$$

$$D_3 = \overline{Q_3}\,\overline{Q_2}\,\overline{Q_1} + Q_3Q_1$$

Q_2^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	0	0	Φ	1
1	0	1	Φ	0

$$Q_2^{n+1} = Q_2Q_1 + Q_3\overline{Q_1}$$

$$D_2 = Q_2Q_1 + Q_3\overline{Q_1}$$

Q_1^{n+1}

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	1	Φ	1
1	0	0	Φ	0

$$Q_1^{n+1} = \overline{Q_1}$$

$$D_1 = \overline{Q_1}$$

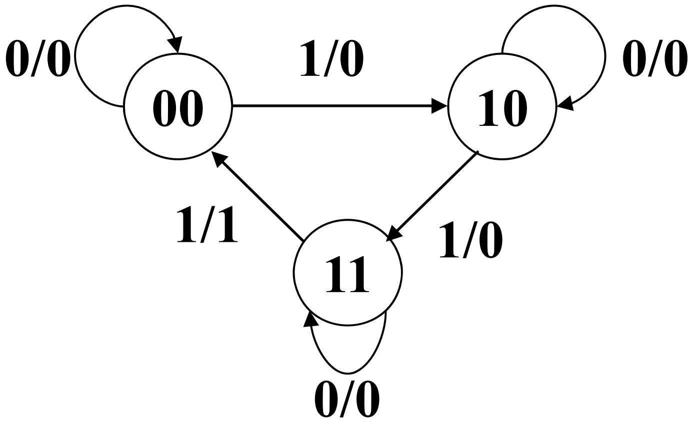
Z

Q_1^n \ $Q_3^nQ_2^n$	00	01	11	10
0	1	0	Φ	0
1	0	0	Φ	0

$$Z = \overline{Q_3}\,\overline{Q_2}\,\overline{Q_1}$$

自启动及电路图略

例 4. 按照下面状态图设计电路



状态表 (根据状态图)

X/Z

X	Q_2^n	Q_1^n	Q_2^{n+1}	Q_1^{n+1}	Z
0	0	0	0	0	0
0	0	1	ϕ	ϕ	ϕ
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	1	0	0
1	0	1	ϕ	ϕ	ϕ
1	1	0	1	1	0
1	1	1	0	0	1

1) 确定状态及状态表

状态数 $\xrightarrow{\text{确定}}$ FF 个数

n FFs $\rightarrow 2^n$ 状态

$2^{n-1} \leq$ 状态数 $\leq 2^n \rightarrow$ n FFs

$3 < 2^2$ 需要 2 个 FF

2)选择 FF (K-map, 圈 1)

Q_2^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10
0	0	1	1	1
1	ϕ	1	0	ϕ

2# FF　　**选择 JK-FF**

$$Q_2^{n+1} = \overline{X}Q_2^n + X\overline{Q}_1^n$$

$$Q_2^{n+1} = J_2\overline{Q}_2^n + \overline{K}_2 Q_2^n$$

找到 $J_2 = ?$　　$K_2 = ?$

不能按上面方法圈，必须圈成 $Q_2^{n+1} = \_\_\_\ \overline{Q}_2^n + \_\_\_\ Q_2^n$

Q_2^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10
0	0	1	1	1
1	ϕ	1	0	ϕ
	$\overline{Q}_2^n$	Q_2^n	Q_2^n	$\overline{Q}_2^n$

$$Q_2^{n+1} = X\overline{Q}_2^n + (\overline{X} + \overline{Q}_1^n)Q_2^n$$

$$= X\overline{Q}_2^n + \overline{XQ_1^n}\,Q_2^n$$

$$\therefore \begin{cases} J_2 = X \\ K_2 = XQ_1^n \end{cases}$$

- **能找到系数（控制变量）时尽量化简**
- **找不到系数时，牺牲化简也要找到系数**

X	Q_2^n	Q_1^n	Q_2^{n+1}	Q_1^{n+1}	Z
0	0	0	0	0	0
0	0	1	φ	φ	φ
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	1	0	0
1	0	1	φ	φ	φ
1	1	0	1	1	0
1	1	1	0	0	1

1# FF

Q_1^{n+1}

Q_1^n \ XQ_2^n	00	01	11	10	
0	0	0	1	0	→ $\overline{Q}_1^n$
1	φ	1	0	φ	→ Q_1^n

JK-FF

$$Q_1^{n+1} = J_1\overline{Q}_1^n + \overline{K}_1 Q_1^n$$
$$= XQ_2^n\overline{Q}_1^n + \overline{X}Q_1^n$$

$$\therefore \begin{cases} J_1 = XQ_2^n \\ K_1 = X \end{cases}$$

输出 Z

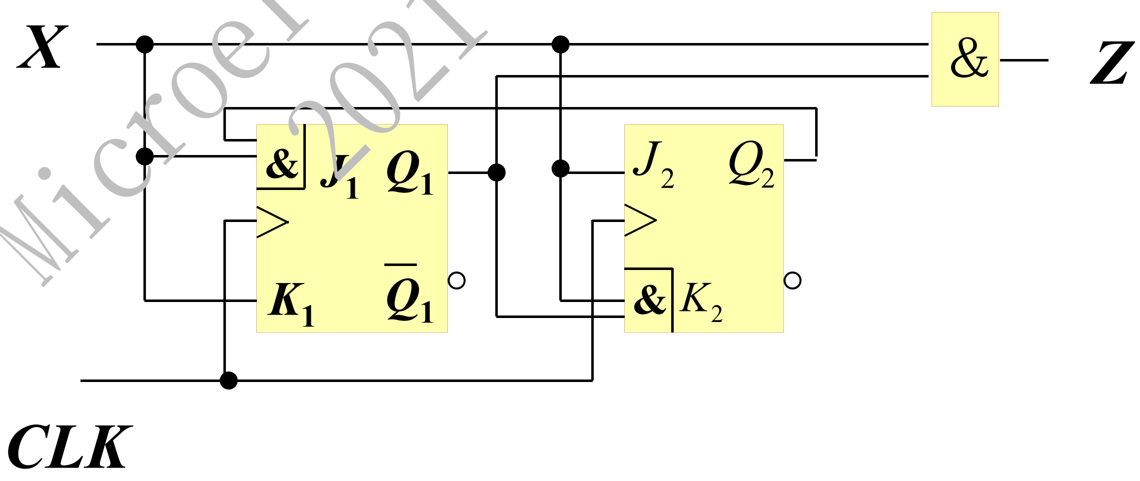
Q_1^n \ XQ_2^n	00	01	11	10
0	0	0	0	0
1	Φ	0	1	Φ

$$Z = XQ_1^n$$

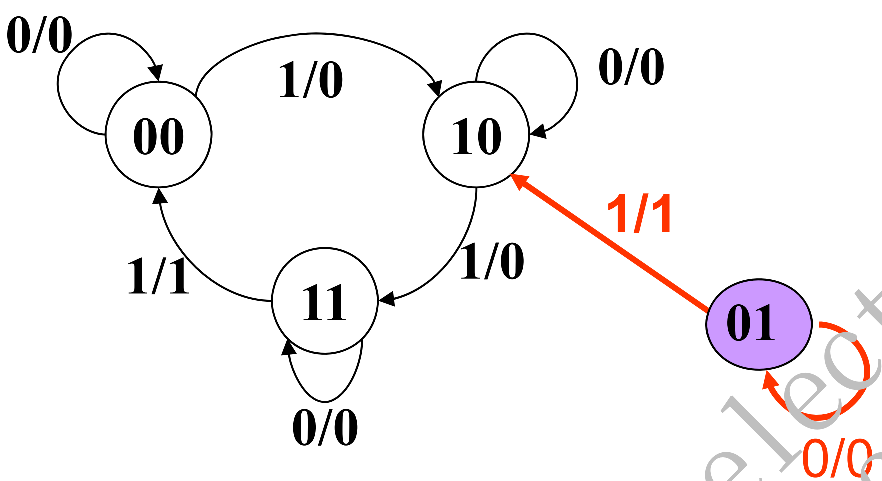
电路

$$\begin{cases} J_2 = X \\ K_2 = XQ_1^n \end{cases}$$

$$\begin{cases} J_1 = XQ_2^n \\ K_1 = X \end{cases}$$



3）讨论：01 状态



Q_2^{n+1} (Q_1^n \ XQ_2^n)	00	01	11	10
0	0	1	1	1
1	0	1	0	1

Q_1^{n+1} (Q_1^n \ XQ_2^n)	00	01	11	10
0	0	0	1	0
1	1	1	0	0

分析卡诺图 K-map

$XQ_2^nQ_1^n = 001$, $(Z=0)$

Next state $Q_2^{n+1}Q_1^{n+1} = 01$,

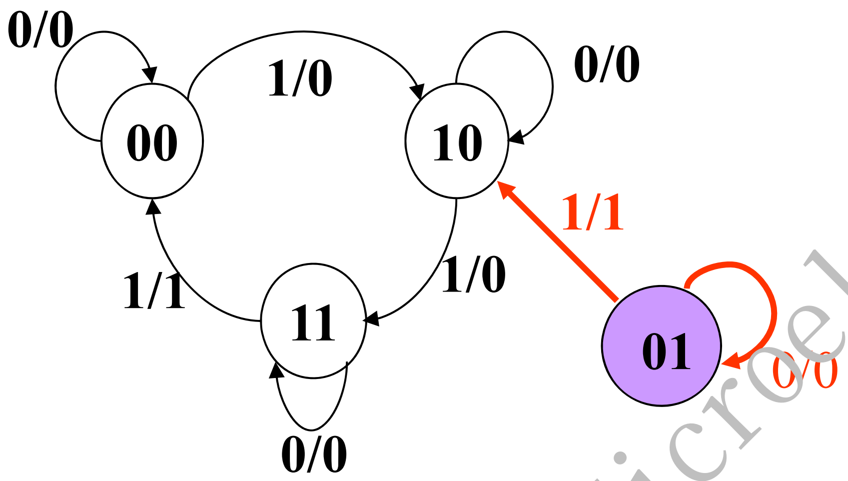
$XQ_2^nQ_1^n = 101$时, $(Z=1)$

Next state $Q_2^{n+1}Q_1^{n+1} = 10$,

Z (Q_1^n \ XQ_2^n)	00	01	11	10
0	0	0	0	0
1	0	0	1	1

实现自启动

要分析输出的物理意义（即电路功能）是否正确



此电路为可控模3加法计数器

X=0，保持原状态

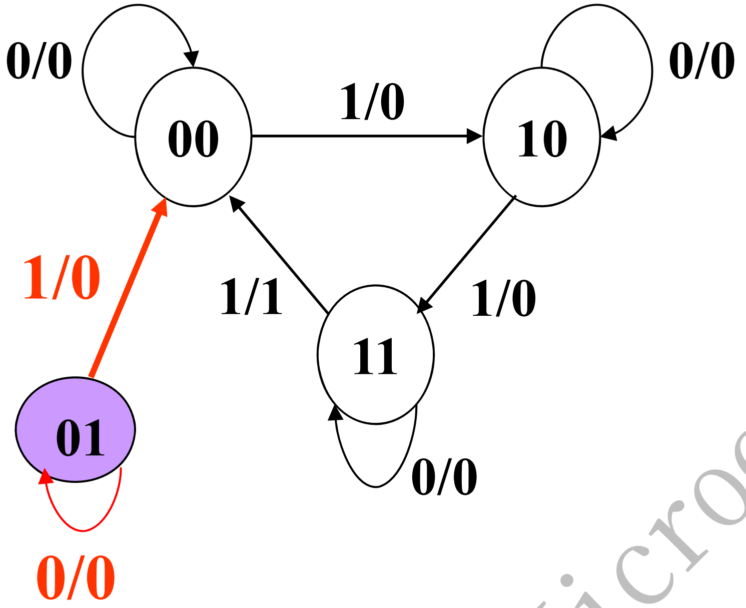
X=1，作加法计数

输出Y=1，为进位输出

显然自启动之后电路功能出现错误

应该将X=1时01状态的输出设为0，次态为00

在设计电路时



X=0，保持原状态

X=1，作加法计数（从0开始）

在填状态表时不能填 ϕ

状态表

X	Q_2^n	Q_1^n	Q_2^{n+1}	Q_1^{n+1}	Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	1	0	0
1	0	1	0	0	0
1	1	0	1	1	0
1	1	1	0	0	1