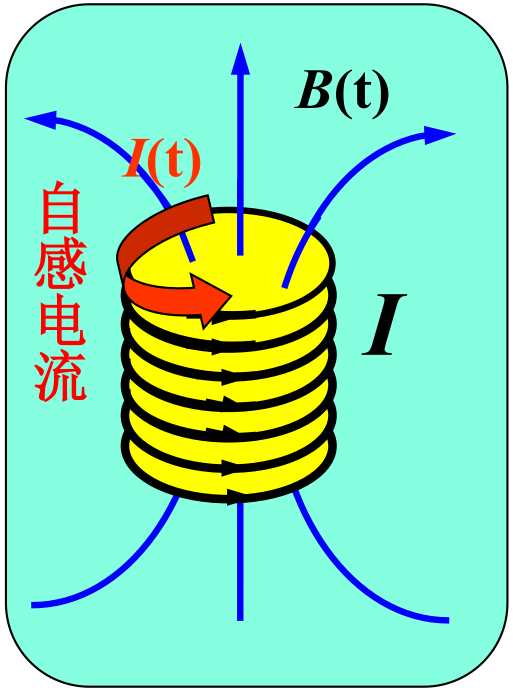
§ 3. 自感与互感

一、自感应



$$\underbrace{\Psi \propto B \propto I}$$

$$I(t) \to B(t) \to \Phi(t) \to \varepsilon_i$$

$$\Psi = LI$$

L —自感系数

*L*与线圈大小、形状、周围介质的磁导率有关；与**线圈是否通电流无关。**

单位 ： H（亨利）

$$\varepsilon_L = -\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}(LI) = -L\frac{\mathrm{d}I}{\mathrm{d}t} - I\frac{\mathrm{d}L}{\mathrm{d}t}$$

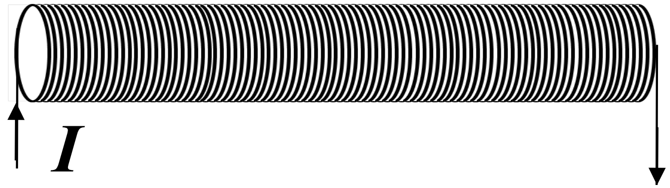
$$\varepsilon_L = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

1. 自感电流反抗线圈中电流变化

2. *L*越大回路中电流越难改变

例 求单层密绕长直螺线管的自感（已知 l、N、S、μ）.

$$I \to B \to \Phi \to L = \Phi / I$$



解：设回路中通有电流 I

$$B = \mu n I = \mu \frac{N}{l} I \quad \to \quad \Phi = BS = \mu \frac{N}{l} IS$$

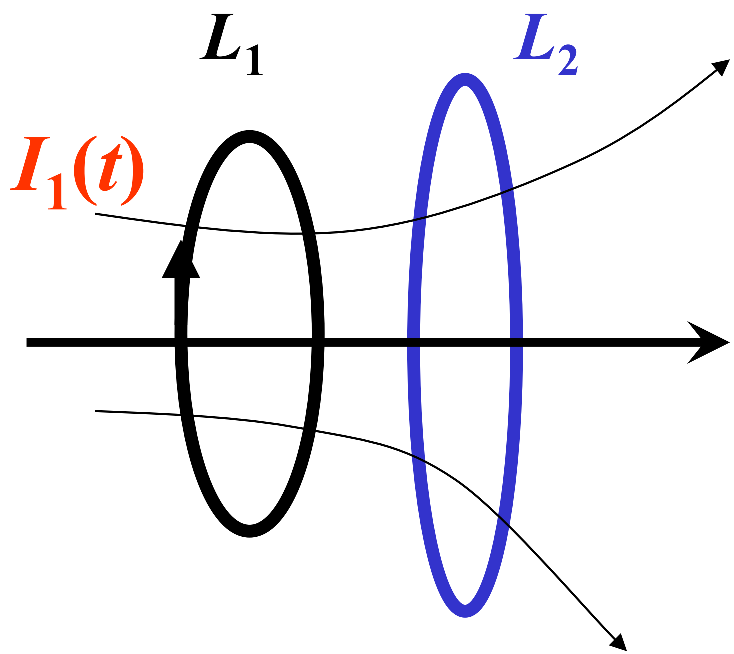
$$\Psi = N\Phi = LI \qquad L = \mu \frac{N^2}{l} S = \mu n^2 l s$$

$$L = \mu n^2 V$$ 仅与回路、介质有关

螺线管：长度 $l = 50$ cm, 截面积 $S = 10\ \text{cm}^2$, 总匝数 $N = 3000$

$$L = \mu_0 n^2 V = 23\ \text{mH}$$

二、互感应



$$\Psi_{12} \propto B \propto I_1 \qquad \Psi_{12} = M_{12} I_1$$

$$\varepsilon_{12} = -\frac{\mathrm{d}\Psi_{12}}{\mathrm{d}t} = -M_{12}\frac{\mathrm{d}I_1}{\mathrm{d}t}$$

同理 $\Psi_{21} = M_{21} I_2$

$$\varepsilon_{21} = -\frac{\mathrm{d}\Psi_{21}}{\mathrm{d}t} = -M_{21}\frac{\mathrm{d}I_2}{\mathrm{d}t}$$

可以证明： $M_{12} = M_{21} = M$ $\qquad$ ***M*—互感系数**

与两个回路的大小、形状 、相对位置及周围介质的磁导率有关，与回路中是否通有电流无关。单位：H (亨利)

§ 3. 自感与互感

综合考虑：当两个线圈同时分别通电流 $I_1(t), I_2(t)$

感应电动势: $\varepsilon_1, \varepsilon_2$

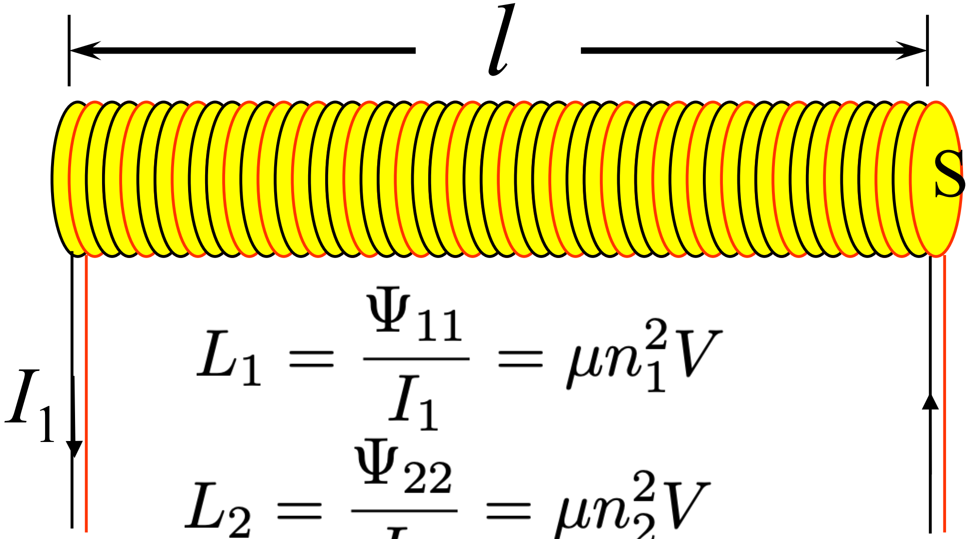
每个线圈中都有自感电动势和互感电动势！

线圈1 $$\varepsilon_1 = \varepsilon_{1L} + \varepsilon_{1M} = -L_1 \frac{\mathrm{d}I_1}{\mathrm{d}t} - M \frac{\mathrm{d}I_2}{\mathrm{d}t}$$

线圈2 $$\varepsilon_2 = \varepsilon_{2L} + \varepsilon_{2M} = -L_2 \frac{\mathrm{d}I_2}{\mathrm{d}t} - M \frac{\mathrm{d}I_1}{\mathrm{d}t}$$

例 均匀密绕长直螺线管（无漏磁）已知：n_1 , n_2 , S, l , μ 求：M

解：设螺线管1通稳恒电流I_1



$$L_1 = \frac{\Psi_{11}}{I_1} = \mu n_1^2 V$$

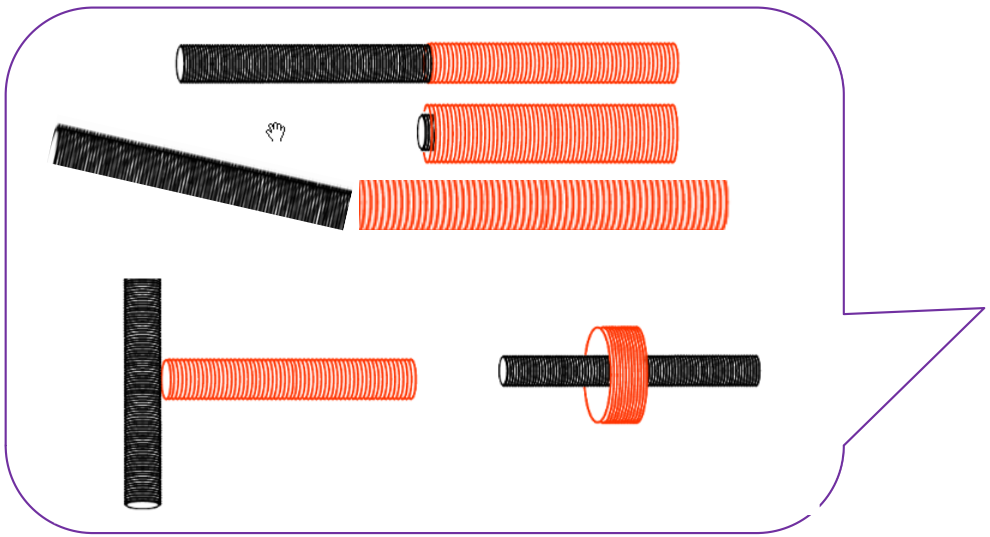
$$L_2 = \frac{\Psi_{22}}{I_2} = \mu n_2^2 V$$

$$B_1 = \mu n_1 I_1$$

$$\Psi_{12} = N_2 \Phi_{12} = n_2 l S B_1$$

$$= \mu n_1 n_2 I_1 V$$

$$M = \frac{\Psi_{12}}{I_1} = \mu n_1 n_2 V$$

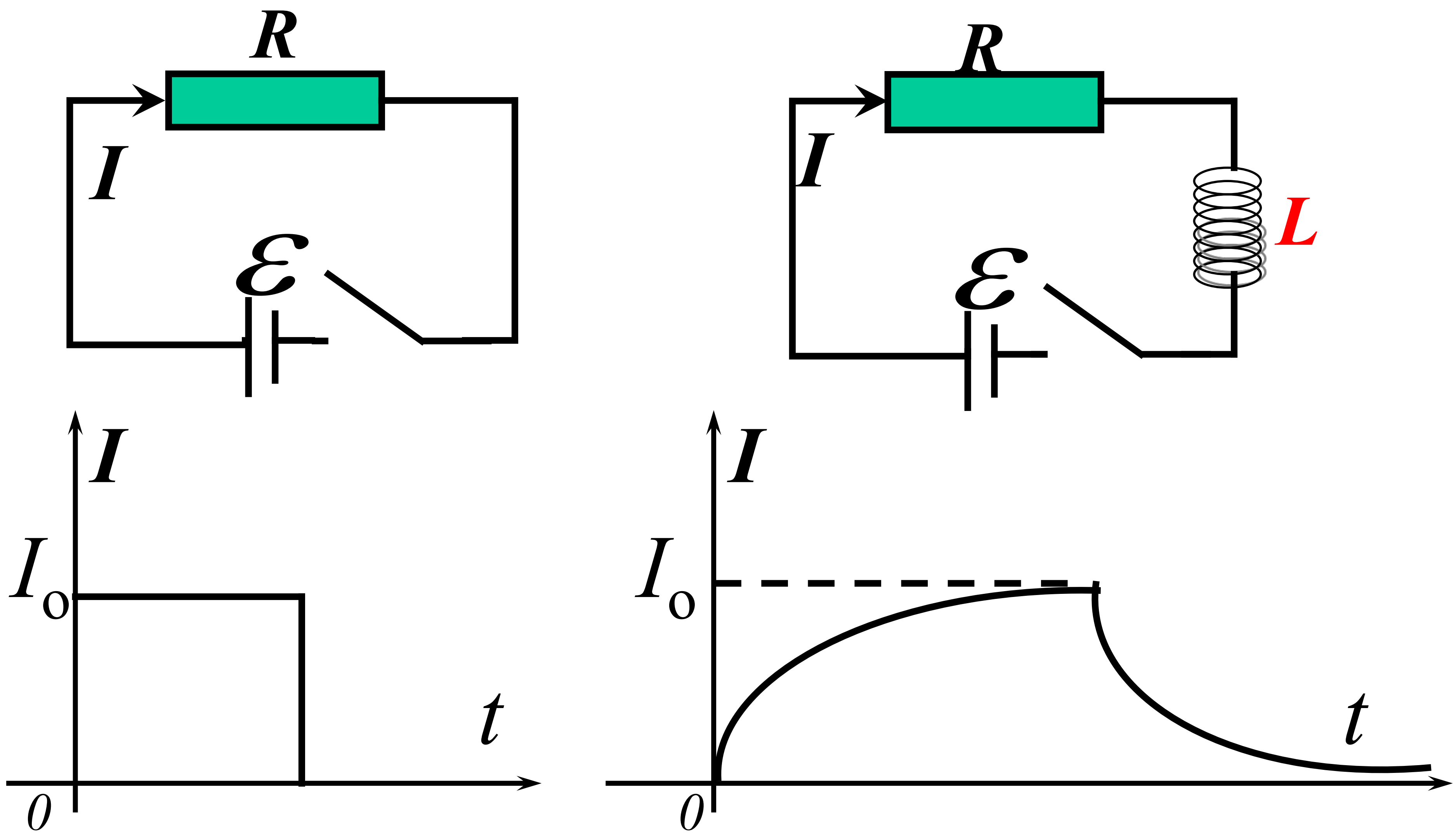


理想耦合 $M = \sqrt{L_1 L_2}$

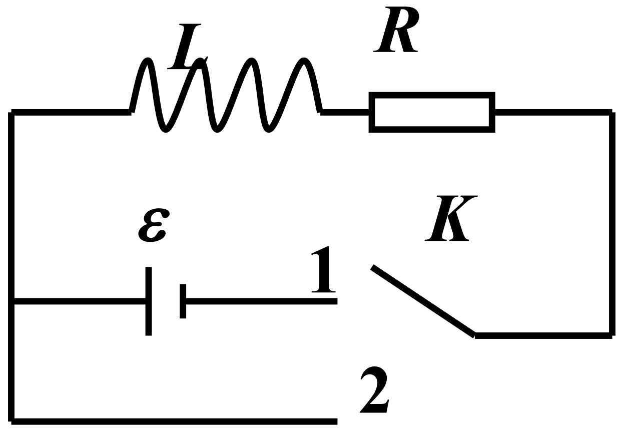
非理想耦合 $M^2 \leq L_1 L_2$

§3. 自感与互感

三、*R-L*电路的暂态过程
（线圈对回路电流的影响）



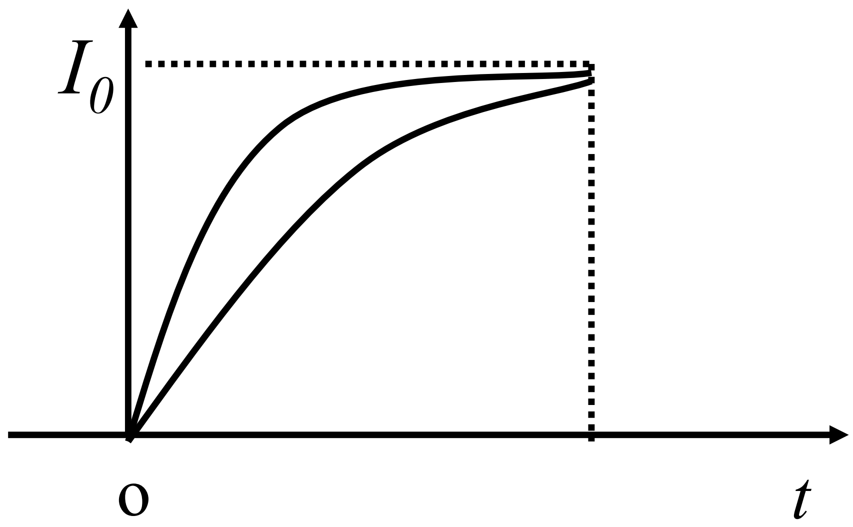
三、*R*-*L*电路的暂态过程



1、由2→1 电路接通

$$\varepsilon_L = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

$$\varepsilon - L\frac{\mathrm{d}I}{\mathrm{d}t} = RI$$



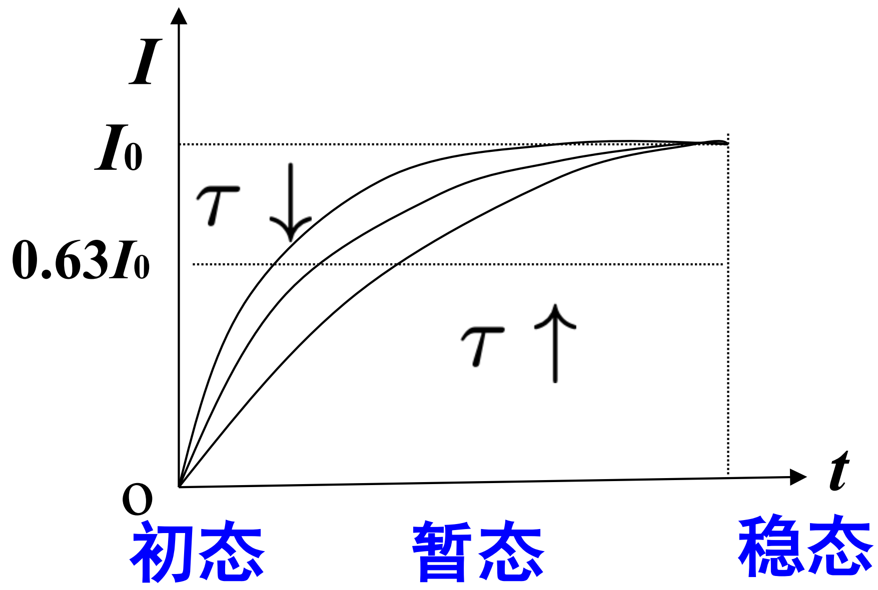
初始条件 $t=0, I=0$ $\quad I = \frac{\varepsilon}{R}(1-e^{-\frac{R}{L}t})$

三、*R-L*电路的暂态过程

$$I = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$$

$$= I_0(1 - e^{-\frac{R}{L}t})$$

$\tau = L/R$ 时间常数



$$t = \tau \rightarrow I = I_0(1 - e^{-1})$$

$$= 0.63 I_0$$

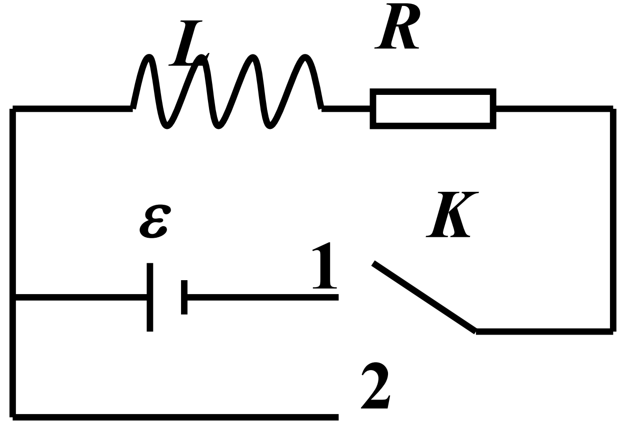
初态由初始条件决定

稳态由电路的物理条件决定

暂态按指数变化，快慢由τ 决定

§ 3. 自感与互感

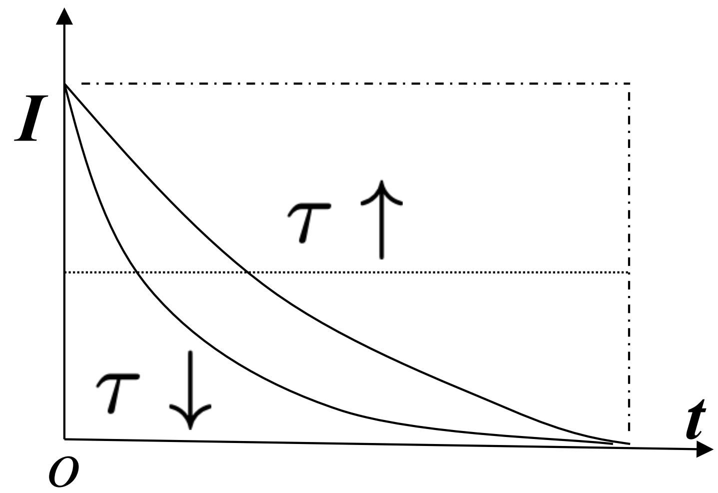
三、*R-L*电路的暂态过程



2、由1→2电路断开

$$\varepsilon_L = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

$$-L\frac{\mathrm{d}I}{\mathrm{d}t} = RI$$

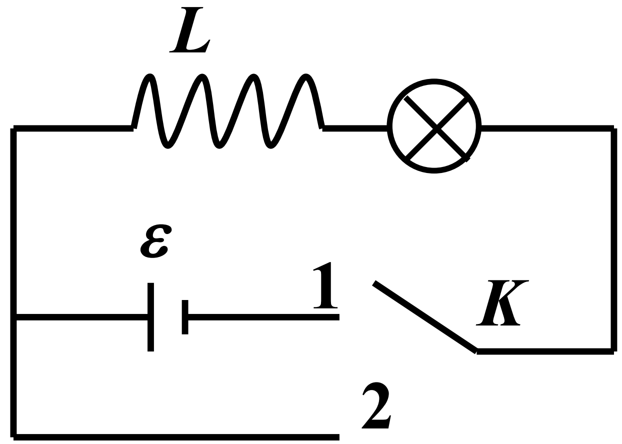


初始条件 $t = 0, I = \varepsilon / R$

$$I = \frac{\varepsilon}{R}e^{-\frac{R}{L}t} \qquad t = \tau \rightarrow I = 0.37I_0$$

§ 4. 磁场的能量

由1→2 电路断开



灯泡强烈的闪亮一下，所消耗的能量从哪里来的？

该能量存储在线圈内的磁场中—磁能

d*t* 时间内通过灯泡的电量：$i\mathrm{d}t$

自感电动势做功：$\mathrm{d}A=\varepsilon_L i\mathrm{d}t=-L\dfrac{\mathrm{d}i}{\mathrm{d}t}i\mathrm{d}t=-Li\mathrm{d}i$

电流由起始值*I*减小到0，总功：$A=\int\mathrm{d}A=\int_I^0 -Li\mathrm{d}i=\dfrac{1}{2}LI^2$

自感为*L*的线圈通有电流*I*时所具有的磁能：$W_m=\dfrac{1}{2}LI^2$

W_m, B, H 之间的关系?

以长直螺线管为例

$$\left.\begin{aligned} W_m &= \frac{1}{2}LI^2 \\ L &= \mu n^2 V \end{aligned}\right\} W_m = \frac{1}{2}\mu n^2 I^2 V$$

$$\left.\begin{aligned} B &= \mu n I \\ H &= nI \end{aligned}\right\} BH = \mu n^2 I^2$$

$$\Rightarrow W_m = \frac{1}{2}BHV$$

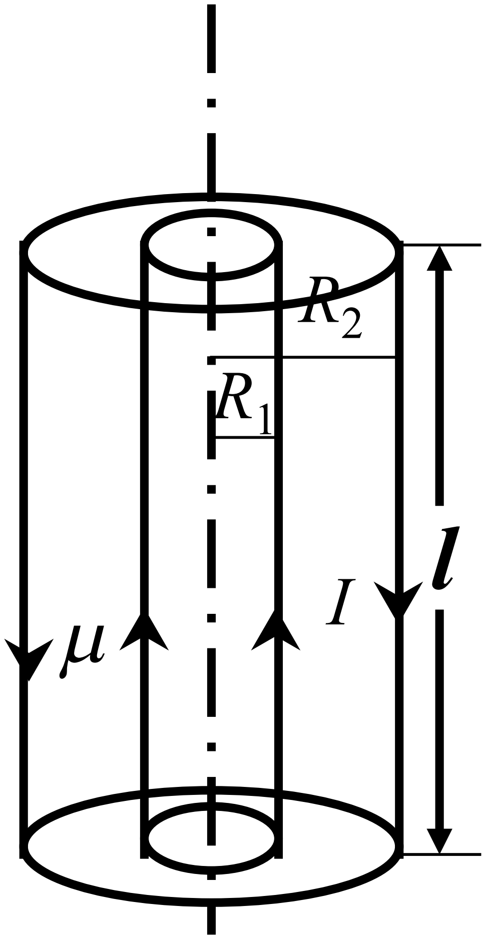
均匀磁场

磁场的能量密度: $w_m = \frac{1}{2}BH = \frac{1}{2\mu}B^2 = \frac{1}{2}\mu H^2$

非均匀磁场:选体积元 $\mathrm{d}V \to \mathrm{d}W_m = w_m \mathrm{d}V = \frac{1}{2}BH\mathrm{d}V$

$$W_m = \int \mathrm{d}W_m = \frac{1}{2}\int_V BH\mathrm{d}V = \frac{1}{2}\int_V \frac{B^2}{\mu}\mathrm{d}V$$

例: 长直同轴电缆. 已知 R_1、R_2， 填充介质均匀各向同性， 电流I在两柱面上均匀分布。求：1. l 长段电缆W_m；2.电缆的自感系数



解： $I \rightarrow H \rightarrow w_m \rightarrow W_m \rightarrow L$

$$H = \begin{cases} \dfrac{I}{2\pi r} \ (R_1 \le r \le R_2) \\ 0 \ (r < R_1, r > R_2) \end{cases}$$

$$w_m = \frac{1}{2}\mu H^2 = \frac{\mu I^2}{8\pi^2 r^2} \qquad \mathrm{d}V = l2\pi r \mathrm{d}r$$

$$W_m = \int w_m \mathrm{d}V = \frac{\mu I^2 l}{4\pi}\int_{R_1}^{R_2}\frac{\mathrm{d}r}{r}$$

$$= \frac{\mu I^2 l}{4\pi}\ln\frac{R_2}{R_1} \qquad\qquad L = \frac{2W_m}{I^2}$$

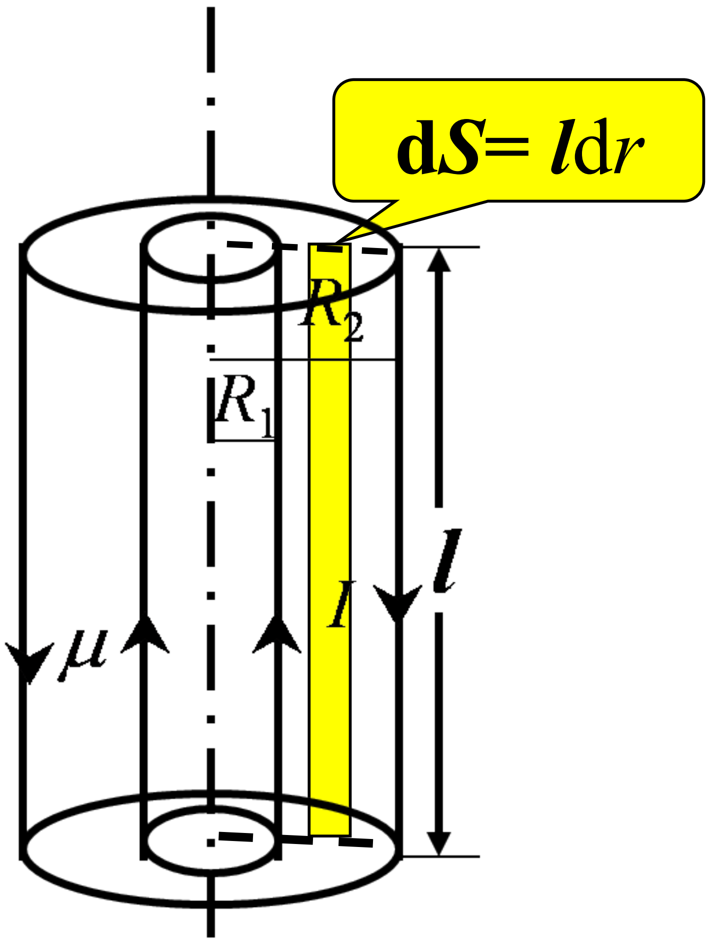
解：方法2 $I \to H \to B \to \Phi \to L \to W_m$

$$B = \begin{cases} \dfrac{\mu I}{2\pi r} \ (R_1 \le r \le R_2) \\ 0 \ (r < R_1, r > R_2) \end{cases}$$

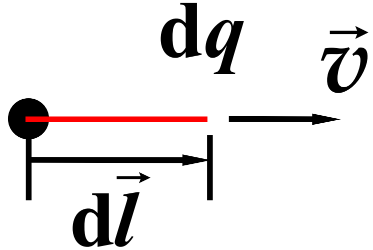
$$\mathrm{d}\Phi = \vec{B} \cdot \mathrm{d}\vec{S} = \frac{\mu I l}{2\pi r}\mathrm{d}r$$

$$\Phi = \int_{R_1}^{R_2} \frac{\mu I l}{2\pi r}\mathrm{d}r = \frac{\mu I l}{2\pi}\ln\frac{R_2}{R_1}$$

$$L = \frac{\Phi}{I} \qquad W_m = \frac{1}{2}LI^2$$



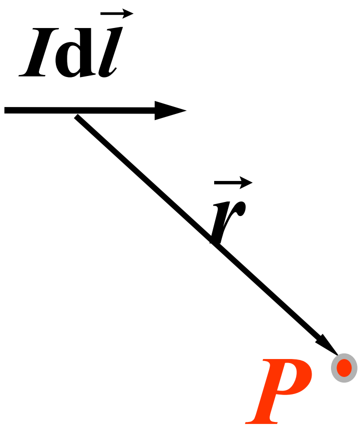
§ 5. 匀速运动点电荷的磁场与电场



dt 时间, 点电荷移动

$$\left.\begin{array}{l} \mathrm{d}l = v\mathrm{d}t \\ I = \dfrac{\mathrm{d}q}{\mathrm{d}t} \end{array}\right\} I\mathrm{d}\vec{l} = \vec{v}\mathrm{d}q$$

移动的点电荷相当于电流元



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{\mathrm{d}q\vec{v}\times\vec{r}}{r^3}$$

比较该点电荷在P点产生的电场

$$\vec{E} = \frac{\mathrm{d}q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

$$\vec{B} = \mu_0\varepsilon_0(\vec{v}\times\vec{E}) = \frac{1}{c^2}(\vec{v}\times\vec{E})$$