第三章 逻辑代数 作业答案

3.8 用公式法化简下列逻辑函数。

（1） $A\bar{B}+B\bar{C}+\bar{B}C+\bar{A}B$

解：

方法一：
$$\begin{aligned}F_1 &= A\bar{B}+B\bar{C}+\bar{B}C+\bar{A}B+A\bar{C}\\ &= \bar{B}C+\bar{A}B+A\bar{C}\end{aligned}$$

方法二：
$$\begin{aligned}F_1 &= A\bar{B}+B\bar{C}+\bar{B}C+\bar{A}B+\bar{A}C\\ &= A\bar{B}+B\bar{C}+\bar{A}C\end{aligned}$$

（2） $\overline{\bar{A}\,\bar{B}\,\overline{BC}\;\overline{BCD}\;\overline{\bar{A}\,BCD}}+\bar{A}\,\bar{B}\,\bar{C}D$

解： $F_2 = A+B+B\bar{C}+B\bar{C}D+\bar{A}\bar{B}CD+\bar{A}\bar{B}\bar{C}D = A+B+\bar{A}\bar{B}D = A+B+D$

（3） $(A+B)(B+D)(\bar{C}+\bar{D})(A+C+\bar{D})(\bar{B}+\bar{C}+D)$

解：取对偶式

$$\begin{aligned}F'_3 &= AB+BD+\bar{C}\bar{D}+AC\bar{D}+\bar{B}\bar{C}D\\ &= AB+BD+\bar{C}\bar{D}+A\bar{D}+\bar{B}\bar{C}D\\ &= AB+BD+\bar{C}\bar{D}+A\bar{D}+\bar{B}\bar{C}\\ &= AB+BD+\bar{C}\overline{BD}+A\bar{D}\\ &= AB+BD+\bar{C}+A\bar{D} = BD+\bar{C}+A\bar{D}\end{aligned}$$

再对偶
$$\begin{aligned}F_3 &= (B+D)(A+\bar{D})\bar{C}\\ &= (AD+B\bar{D})\bar{C}\\ &= A\bar{C}D+B\bar{C}\bar{D}\end{aligned}$$

（8） $\overline{AB+A\bar{B}+\bar{A}B}\cdot(\bar{A}\,\bar{B}+CD)$

解： $F_8 = \overline{A+B}\cdot(\bar{A}\,\bar{B}+CD) = \bar{A}\,\bar{B}\cdot(\bar{A}\,\bar{B}+CD) = \bar{A}\,\bar{B}$

（9） $(A+C+D)(A+C+\bar{D})(A+\bar{C}+D)(A+\bar{B})$

解：取对偶式

$F_9' = ACD+AC\bar{D}+A\bar{C}D+A\bar{B} = AC+AD+A\bar{B}$

再对偶

$F_9 = (A+C)(A+D)(A+\bar{B}) = A+\bar{B}CD$

（10） $ABC+\overline{\bar{A}\,\bar{C}(B+\bar{D})\bar{C}D}$

解： $F_{10} = ABC+A+C+\bar{B}D+C+\bar{D} = A+\bar{B}+C+\bar{D}$

（11） $\overline{X+Y}\cdot\overline{\bar{X}+\bar{Y}}$

解： $F_{11} = \bar{X}\bar{Y}\cdot XY = 0$

（18）$\overline{(A+B\bar{C})(\bar{A}+\bar{D}E)}$

解：$F_{18}=\bar{A}\overline{B\bar{C}}+A\overline{\bar{D}E}=\bar{A}\bar{B}+\bar{A}C+AD+A\bar{E}$

（19）$A\bar{B}CD+ABD+A\bar{C}D$

解：$F_{19}=ABD+ACD+A\bar{C}D=AD$

（20）$AC\left(\bar{C}D+\bar{A}B\right)+BC\overline{\left(\bar{B}+\overline{AD}+CE\right)}$

解：$F_{20}=BC\left(\bar{B}+AD\right)\left(\bar{C}+\bar{E}\right)=ABCD\bar{E}$

3.11　化简下列各式为最简或与式。

（1）$X=(\bar{A}+B)(B+\bar{C})(\bar{A}+C)(A+\bar{C})(\bar{B}+C)$

解：

$X'=\bar{A}B+B\bar{C}+\bar{A}C+A\bar{C}+\bar{B}C$

$=\bar{A}B+A\bar{C}+\bar{B}C$

$X=(\bar{A}+B)(A+\bar{C})(\bar{B}+C)$

（3）$X=(B+C+D)(A+\bar{C}+\bar{D})(\bar{A}+\bar{C}+\bar{D})(\bar{A}+\bar{B}+\bar{D})$

解：$X'=BCD+A\bar{C}\bar{D}+\bar{A}C\bar{D}+\bar{A}\bar{B}\bar{D}=BCD+\bar{C}\bar{D}+\bar{A}\bar{B}\bar{D}$

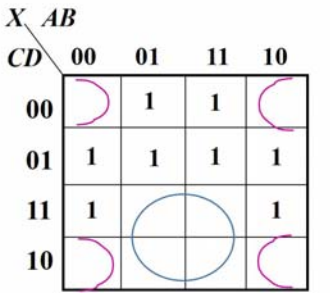
$X=(B+C+D)(\bar{C}+\bar{D})(\bar{A}+\bar{B}+\bar{D})$

（7）$X=A\bar{B}D+\bar{A}\bar{B}\bar{C}D+\bar{B}CD+\left(\overline{A\bar{B}+C}\right)(B+D)$

解：$X=A\bar{B}D+\bar{A}\bar{B}\bar{C}D+\bar{B}CD+\left(\bar{A}+B\right)\bar{C}\left(B+D\right)$

$=A\bar{B}D+\bar{A}\bar{B}\bar{C}D+\bar{B}CD+B\bar{C}+\bar{A}\bar{C}D$

$=\left(B+D\right)\left(\bar{B}+\bar{C}\right)$



3.12　化简下列各式成最简与或式。

（1）$G=\overline{AB+\bar{B}C+AC}$

解：$G=\overline{AB+\bar{B}C}=(\bar{A}+\bar{B})(B+\bar{C})=\bar{A}\bar{B}+\bar{B}\bar{C}$

（3）$G=\overline{(A\oplus B)C+(B\oplus\bar{C})D}$

$$G=\left(A\odot B+\overline{C}\right)\left(B\odot\overline{C}+\overline{D}\right)$$

$$=(AB+\overline{A}\,\overline{B}+\overline{C})(B\overline{C}+\overline{B}C+\overline{D})$$

解：
$$=AB\overline{C}+AB\overline{D}+\overline{A}\,\overline{B}C+\overline{A}\,\overline{B}\,\overline{D}+B\overline{C}+\overline{C}\,\overline{D}$$

$$=\overline{A}\,\overline{B}C+AB\overline{D}+\overline{A}\,\overline{B}\,\overline{D}+B\overline{C}+\overline{C}\,\overline{D}$$

$$=\overline{A}\,\overline{B}C+AB\overline{D}+B\overline{C}+\overline{C}\,\overline{D}$$

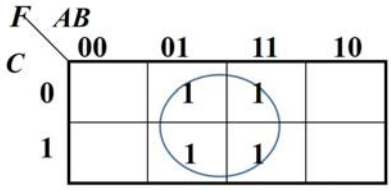
（5）$G=B\overline{C}+ABCE+B\left(\overline{\overline{A}\overline{D}+AD}\right)+B\left(A\overline{D}+\overline{A}D\right)$

解：$G=B\overline{C}+A\overline{D}+\overline{A}D$

3.15　用卡诺图化简下列函数，并求出最简与或表达式。

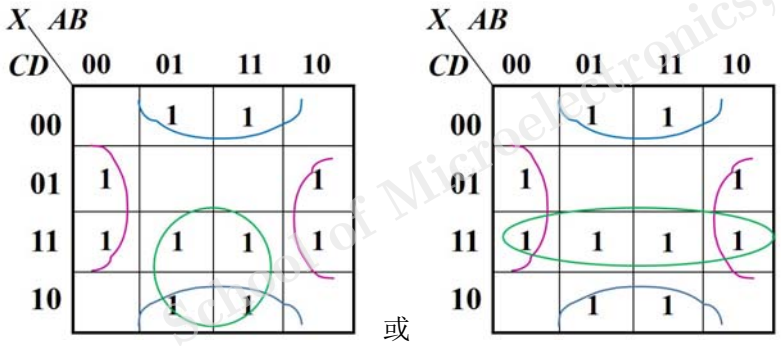
（1）$F_1(A,B,C)=m\sum(2,\ 3,\ 6,\ 7)$

解：$F_1(A,B,C)=B$



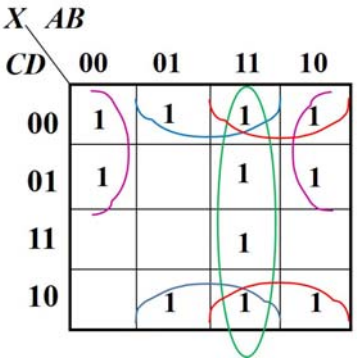
（3）$F_3(A,B,C,D)=m\sum(1,\ 3,\ 4,\ 6,\ 7,\ 9,\ 11,\ 12,\ 14,\ 15)$

解：$F_3(A,B,C,D)=\overline{B}D+B\overline{D}+BC$ 或 $F_3(A,B,C,D)=\overline{B}D+B\overline{D}+CD$



（6）$F_6(A,B,C,D)=m\sum(0,1,4,6,8,9,10,12,13,14,15)$

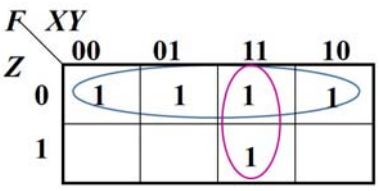
解：$F_6(A,B,C,D)=AB+\overline{B}\overline{C}+B\overline{D}+A\overline{D}$



3.18　用卡诺图化简下列各式，并求出函数的最简与或式、最简或与式。

（1）$F_1=\overline{X}\,\overline{Z}+\overline{Y}\,\overline{Z}+Y\overline{Z}+XYZ$

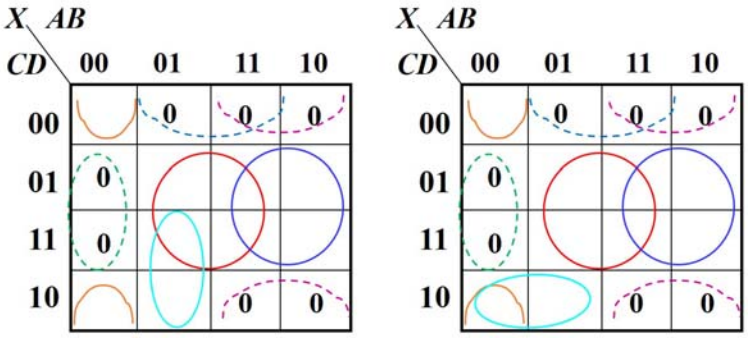
解：$F_1=\overline{Z}+XY=(X+\overline{Z})(Y+\overline{Z})$



（3）$F_3=(\overline{A}+\overline{B}+D)(\overline{A}+D)(A+B+\overline{D})(A+\overline{B}+C+D)$

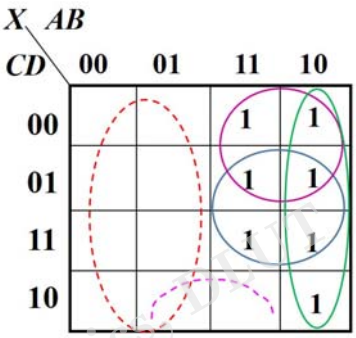
解：$F_3 = BD + AD + \overline{A}\ \overline{B}\ \overline{D} + \overline{A}BC$　或 $= BD + AD + \overline{A}\ \overline{B}\ \overline{D} + \overline{A}C\overline{D}$

$= (A + B + \overline{D})(\overline{B} + C + D)(\overline{A} + D)$



（7）$F_7 = A\overline{BC} + \overline{\overline{A\overline{B}} + \overline{A}\overline{B}} + BC + AD$

解：$F_7 = A\overline{B} + A\overline{C} + AD = A\left(\overline{B} + \overline{C} + D\right)$



3.19　试用最少与非门实现下列逻辑函数。

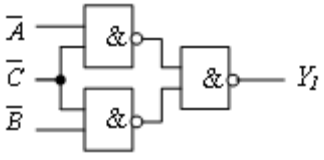
（1）$Y = \overline{A}\,\overline{C} + A\overline{B}\,\overline{C} + \overline{A}B\overline{C}$

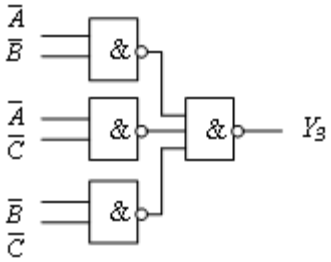
解：$Y_1 = \overline{A\overline{C}} + A\overline{B}\overline{C} + \overline{A}B\overline{C} = \overline{A\overline{C}} + \overline{B\overline{C}} = \overline{\overline{A\overline{C}} \cdot \overline{B\overline{C}}}$

或 $Y_1 = (\overline{A} + A\overline{B} + \overline{A}B)\overline{C} = (\overline{A} + \overline{B})\overline{C} = \overline{\overline{A}\,\overline{B} \cdot \overline{C}}$

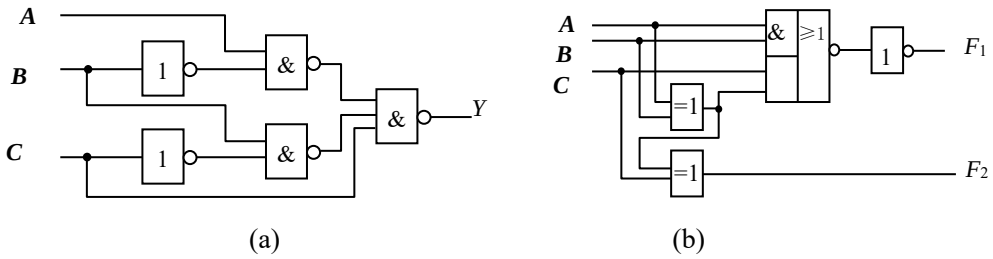
（3）$Y = \overline{A\overline{B} + AC + \overline{A}BC}$

解：$Y_3 = \overline{A\overline{B} + (A + B)C} = (\overline{A} + \overline{B})(\overline{A} \cdot \overline{B} + \overline{C}) = \overline{A}\ \overline{B} + \overline{A}\ \overline{C} + \overline{B}\ \overline{C} = \overline{\overline{A}\ \overline{B} \cdot \overline{A}\ \overline{C} \cdot \overline{B}\ \overline{C}}$





3.20　写出题图 3.20 中各逻辑图的逻辑函数式，并化简为最简与或式。



(a) (b)

题图 3.20

解：(a) $Y=\overline{\overline{A\overline{B}}\cdot\overline{B\overline{C}}\cdot C}=A\overline{B}+B\overline{C}+\overline{C}=A\overline{B}+\overline{C}$

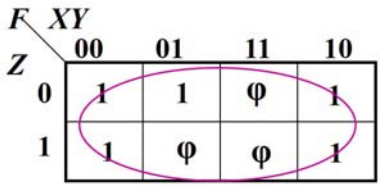
(b) $F_1=\overline{\overline{AB}+\overline{A\oplus B}\cdot C}=AB+(A\overline{B}+\overline{A}B)C=AB+A\overline{B}C+\overline{A}BC=AB+AC+BC$

$F_2=A\oplus B\oplus C$

3.21 利用函数的随意状态化简函数，并求出最简与或式。

（1）$G=\overline{Y}+\overline{X}\,\overline{Z},\quad d=YZ+XY$

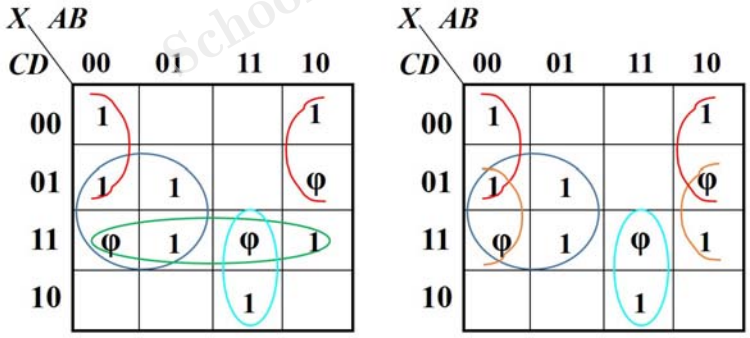
解：$G_1=1$



（3）$G(A,B,C,D)=\sum m(0,\ 1,\ 5,\ 7,\ 8,\ 11,\ 14)+\sum d(3,\ 9,\ 15)$

解：$G_3=\overline{B}\ \overline{C}+\overline{A}D+ABC+CD$

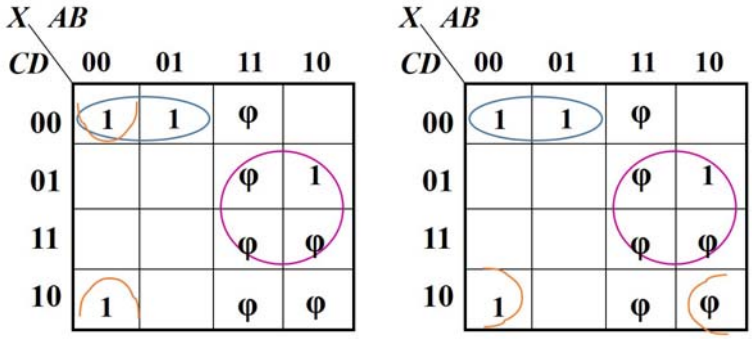
或 $G_3=\ \overline{B}\ \overline{C}+\overline{A}D+ABC+\overline{B}D$



（5）$G(A,B,C,D)=\overline{A+C+D}+\overline{A}\overline{B}C\overline{D}+A\overline{B}\overline{C}D\ ,d=AB+AC$

解：$G_5=AD+\overline{A}C\overline{D}+\overline{A}\overline{B}\overline{D}$

或 $G_5=AD+\overline{A}C\overline{D}+\overline{B}C\overline{D}$



3.22　化简下列具有约束条件为 $d=AB+AC$ 的逻辑函数。

（1）$Z_1=\overline{A}\,\overline{C}+\overline{A}B$，$d=AB+AC$

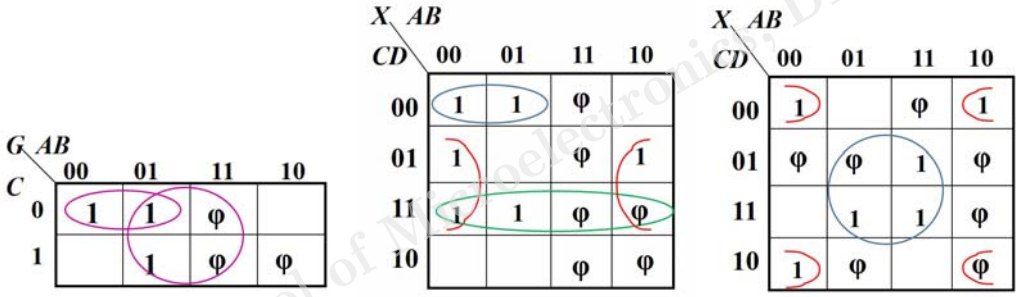
解：$Z_1=\overline{A}\ \overline{C}+B$

（3）$Z_3=\overline{A}\,\overline{C}\,\overline{D}+\overline{A}BCD+\overline{A}\,\overline{B}D+A\overline{B}\,\overline{C}D$，$d=AB+AC$

解：$Z_3=\overline{A}\ \overline{C}\ \overline{D}+\overline{B}D+CD$

（5）$Z_5=\Sigma m(0,2,7,8,13,15)+\Sigma d(1,5,6,9,10,11,12)$

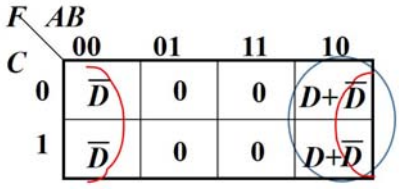
解：$Z_5=BD+\overline{B}\overline{D}=B\odot D$



3.23　用 VEM 化简逻辑函数。

（2）$X=\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}+\overline{A}\,\overline{B}C\overline{D}+A\overline{B}\,\overline{C}D+A\overline{B}\,\overline{C}\,\overline{D}+A\overline{B}C\overline{D}+A\overline{B}CD$，将变量 D 作为引入卡诺图的变量。

解：$X_2=\overline{B}\ \overline{D}+A\overline{B}$



3.24　用 VEM 化简下列逻辑函数，将变量 *C*、*D* 作为引入卡诺图的变量。

（2）$Y=A\overline{B}CD+AB\overline{C}D+ABC\overline{D}+AB\overline{C}\ \overline{D}+\overline{A}BCD+A\overline{B}C\overline{D}+\overline{A}\ \overline{B}\ \overline{C}D+ABCD$

解：$Y_2=\overline{A}\ \overline{B}\ \overline{C}D+AC+BCD+AB$