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1.1 解：$\begin{cases}\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \\ pV = nRT\end{cases}$ $\begin{cases}\beta = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V \\ pV = nRT\end{cases}$ $\begin{cases}K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \\ pV = nRT\end{cases}$

$\alpha = \frac{nR}{pV}$ $\beta = \frac{V}{nR}$ $K_T = -\frac{nRT\ln p}{V}$

1.2 解：由已知，得物态方程 $V=V(T,p)$

$dV = \left(\frac{dV}{dT}\right)_p dT + \left(\frac{dV}{dp}\right)_T dp$

$\frac{dV}{V} = \left(\frac{\partial V}{\partial T}\right)_p \frac{dT}{V} + \left(\frac{dV}{dp}\right)_T \frac{dp}{V}$

由 $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$，$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$，

$\frac{dV}{V} = \alpha dT - K_T dp$

$\ln V = \int \alpha dT - K_T dp$

证毕.

当 $\alpha = \frac{1}{T}$，$K_T = \frac{1}{p}$ 时，代入上式，

得 $V = \frac{T}{p}$

1.8. 证明：$C_n = \frac{dQ}{dT}$

$\begin{cases} dQ = dU + pdV \\ pV = nRT,\ n=1 \text{时有 } pV = RT \end{cases}$

解得：$\begin{cases} C_V + p\frac{dV}{dT} = C_n \\ pdV + Vdp = RdT \end{cases}$

将 $pV^n = Const$ 代入解得：$npV^{n-1}dV + V^n dp = 0$

综上，有 $pdV - npdV = RdT \Rightarrow \frac{dV}{dT} = \frac{R}{(1-n)p}$

代入原式得：

$C_n = \frac{R}{1-n} + C_V = \frac{C_p - C_V}{1-n} + C_V$

由 $\gamma = \frac{C_p}{C_V}$ 得：

$C_n = \frac{n-\gamma}{n-1} C_V$

1.9. 证明：

$$\begin{cases} dU = đQ + đW = C_V dT \\ đW = -P dV \\ đQ = C_n dT \end{cases}$$

解得：$P dV = (C_n - C_V) dT$

$PV = nRT$，取 $n=1$ 有 $PV = RT$

联立得 $(C_n - C_V)\frac{dT}{T} = R\frac{dV}{V}$

由 $\frac{dT}{T} = \frac{dP}{P} + \frac{dV}{V}$，

得 $(C_n - C_V)\frac{dP}{P} + (C_n - C_V)\frac{dV}{V} = 0$

当 $n = \frac{C_n - C_P}{C_n - C_V}$ 时，有 $\frac{dP}{P} + n\frac{dV}{V} = 0$

两边积分得：

$PV^n = Const$ 说明该过程一定是多方过程

1.12. 解：$\gamma = \frac{C_P}{C_V} = \gamma(T)$

$dU = đQ + đW$ 由已知，$đQ = 0$ 即 $dU = đW$

$C_V dT = đW = -P dV$

解得：$C_V = \frac{1}{\gamma - 1}R$

$$\begin{cases} PV = nRT,\ n=1 \text{时 } PV = RT \\ C_V = \frac{1}{\gamma-1}R \end{cases}$$

解得：$\frac{R\,dT}{\gamma - 1} = -\frac{RT\,dV}{V} \Leftrightarrow \frac{dV}{V} + \frac{1}{\gamma-1}\frac{dT}{T} = 0$

由 $\ln F(T) = \int \frac{dT}{(\gamma-1)T} = \int \frac{1}{\gamma-1}\frac{dT}{T}$ 得：$\frac{dF}{F} = \frac{1}{\gamma-1}\frac{dT}{T}$

则有 $d[\ln(V) + \ln F(T)] = 0 \Rightarrow V \cdot F(T) = Const$

1.13. 解：已知 $\gamma=\gamma(T)$ ~~理想气体卡诺循环满足以下条件~~

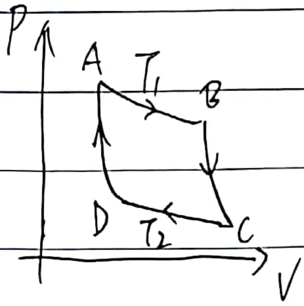
有 $Q_1 = nRT_{AB}\ln\frac{V_B}{V_A}$ （AB段）

$Q_2 = nRT_2\ln\frac{V_C}{V_D}$ （CD段）

卡诺循环 BC、DA 段为绝热过程，有 $dQ=0$

可知 $V\cdot F(T)=$ const

代入数据，




$$\begin{cases} V_B F(T_1) = V_C F(T_2) \\ V_A F(T_1) = V_D F(T_2) \end{cases}$$

有 $\frac{V_B}{V_A}=\frac{V_C}{V_D}$

可知 $W=Q_1-Q_2+\Delta Q$

$\Delta Q=0$

即 $W=Q_1-Q_2$

$\eta=\frac{W}{Q_1}=1-\frac{Q_2}{Q_1}=1-\frac{T_2}{T_1}$

理想气体卡诺循环效率仍为 $1-\frac{T_2}{T_1}$