刘宇帛 微2005

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习题7.1.

~~解~~ 证明：$p = -\sum_l a_l \frac{\partial \varepsilon_l}{\partial V} = -\sum_l a_l \frac{\partial}{\partial V}\left[\frac{1}{2m}\left(\frac{2\pi\hbar}{L}\right)^2 (n_x^2+n_y^2+n_z^2)\right]$

$= -\sum_l a_l \frac{\partial}{\partial V}\left[\frac{1}{2m}(2\pi\hbar)^2 \cdot \frac{1}{L^2}(n_x^2+n_y^2+n_z^2)\right]$

令 $U = \frac{1}{V}\sum a_l \varepsilon_l$，有 $p = -\sum_l a_l \frac{\partial}{\partial V}\left[\frac{1}{2m}\frac{(2\pi\hbar)^2}{V^{\frac{2}{3}}}(n_x^2+n_y^2+n_z^2)\right]$

$= -\sum_l a_l \frac{1}{2m}(2\pi\hbar)^2 (n_x^2+n_y^2+n_z^2) V^{-\frac{5}{3}}(-\frac{2}{3})$

$= -\sum_l a_l \frac{(2\pi\hbar)^2}{L^2}(n_x^2+n_y^2+n_z^2) V^{\frac{2}{3}} \cdot V^{-\frac{5}{3}} \cdot (-\frac{2}{3})$

整理得：

$p = \frac{2}{3}\frac{U}{V}$

由于玻耳兹曼分布、玻色分布、费米分布均可应用上述推导，结论对三种分布都成立

7.2. 证明：$\begin{cases} \varepsilon = cp = c\frac{2\pi\hbar}{L}(n_x^2+n_y^2+n_z^2)^{\frac{1}{2}} \\ p = -\sum_l a_l \frac{\partial \varepsilon_l}{\partial V} \end{cases}$

可与上问同理得

$p = -\sum_l a_l \frac{\partial}{\partial V}(2\pi\hbar)(\sum n_i^2)^{\frac{1}{2}} V^{-\frac{1}{3}}$

整理得：

$p = \frac{1}{3}\frac{U}{V}$

对三种分布都成立

7.4. 证明：设 $s_1, s_2, \cdots$ 是状态对应能级 $\varepsilon_{s'}$

$s_{k+1}, s_{k+2}, \cdots s_{\omega}$ 状态对应能级 $\varepsilon_{s''}$

由 $P_s = \dfrac{e^{-\alpha-\beta\varepsilon}}{N}$ 得 $NP_s = e^{-\alpha-\beta\varepsilon_s}$

$\sum e^{-\alpha-\beta\varepsilon_s}$ 表示处在s状态下的粒子数，

据
$$\begin{cases} P = \prod\limits_{s=s'}^{s} P(s) = P_{s'}\cdot\left(\sum\limits_{s=s_1}^{s_k} e^{-\alpha-\beta\varepsilon_{s'}}\right)\cdot P_{s''}\left(\sum\limits_{s=s_1}^{s_k} e^{-\alpha-\beta\varepsilon_{s''}}\right) \\ P(s') = P_{s'}\cdot\left(\sum\limits_{s=s_1}^{s_k} e^{-\alpha-\beta\varepsilon_{s'}}\right) \\ P(s'') = P_{s''}\cdot\left(\sum\limits_{s=s_1}^{s_k} e^{-\alpha-\beta\varepsilon_{s''}}\right) \end{cases}$$

由
$$\begin{cases} \Omega = \dfrac{1}{P} \\ S = k\ln\Omega \end{cases}$$
将上式代入方程

解得：$S = -kN\sum\limits_{s} P_s \ln P_s$

7.16, 解: $\varepsilon = \frac{1}{2m}(p_x^2+p_y^2+p_z^2)+ax^2+bx$

$$= \frac{p^2}{2m} + a\left(x^2+\frac{bx}{a}+\frac{b^2}{4a^2}\right)-\frac{b^2}{4a}$$

$$= \frac{(p_x^2+p_y^2+p_z^2)}{2m} + a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}$$

由能量均分定理

$$\bar{\varepsilon} = 2kT - \frac{b^2}{4a}$$

7.17, 证明: $C_V = \left(\frac{\partial U}{\partial T}\right)_V$

$$Z = \frac{1}{h^3}\int e^{-\frac{\beta}{2m}(p_x^2+p_y^2+p_z^2)-\beta mgz}\,dx\,dy\,dz\,dp_x\,dp_y\,dp_z$$

$$Z = \frac{S}{h^3}\left(\int e^{-\frac{\beta}{2m}p_x^2}dp_x\right)^3\int_0^H e^{-\beta mgz}dz$$

$$Z = \frac{S}{h^3}\left[(2m\pi)^{\frac{3}{2}}\frac{1}{mg}\right]\beta^{-\frac{5}{2}}\left(1-e^{-\beta mgH}\right)$$

两边取 ln 得:

$$\ln Z = \left(\ln\frac{S}{h^3}(2m\pi)^{\frac{3}{2}}\frac{1}{mg}\right)-\frac{5}{2}\ln\beta+\ln\left(1-e^{-\beta mgH}\right)$$

$$N\frac{\ln Z}{\ln\beta} = -NkT + \frac{NmgH}{e^{\frac{mgH}{kT}}-1} = U_0 - U$$

$$U = U_0 + NkT - \frac{NmgH}{e^{\frac{mgH}{kT}}-1}$$

于是有

$$C_V = C_V^0 + Nk - \frac{N(mgH)^2 e^{\frac{mgH}{kT}}}{kT^2\left(e^{\frac{mgH}{kT}}-1\right)^2}$$

7.18. 解: $Z = \dfrac{e^{-\frac{\beta\hbar\omega}{2}}}{1-e^{-\beta\hbar\omega}}$

$$\ln Z = -\frac{\beta\hbar\omega}{2} - \ln(1-e^{-\beta\hbar\omega})$$

$$U-U_0 = N\frac{\partial \ln Z}{\partial\beta} = -N\frac{\hbar\omega}{2} - \frac{N\hbar\omega e^{-\beta\hbar\omega}}{(1-e^{-\beta\hbar\omega})}$$

$S = Nk + Nk\ln\left(\frac{T}{\theta_v}\right)$, 有 $\theta_v = \frac{\hbar\omega}{k}$

7.19. 解: $Z = \dfrac{2I}{\beta\hbar^2}$

$$\ln Z = \ln\left(\frac{2I}{\beta\hbar^2}\right)$$

$$\frac{\partial \ln Z}{\partial\beta} = -\frac{1}{\beta}$$

$S = Nk + Nk\ln\left(\frac{T}{\theta_r}\right)$, 有 $\theta_r = \frac{\hbar^2}{2kI}$