AE4878 - Mission Geometry and Orbit Design
Part 4
Monte Carlo and grid search optimisation on orbital observations

Ali Nawaz
3ME, LR, Delft University of Technology.

September 2, 2018

1 Introduction

The goal of this assignment is to run Monte Carlo and grid search optimisation on a set of orbital observations. The observations are presented with the aid of Figure 1. X axis values are shifted and do not represent true distance from the focal point. This will be corrected for in the estimations.

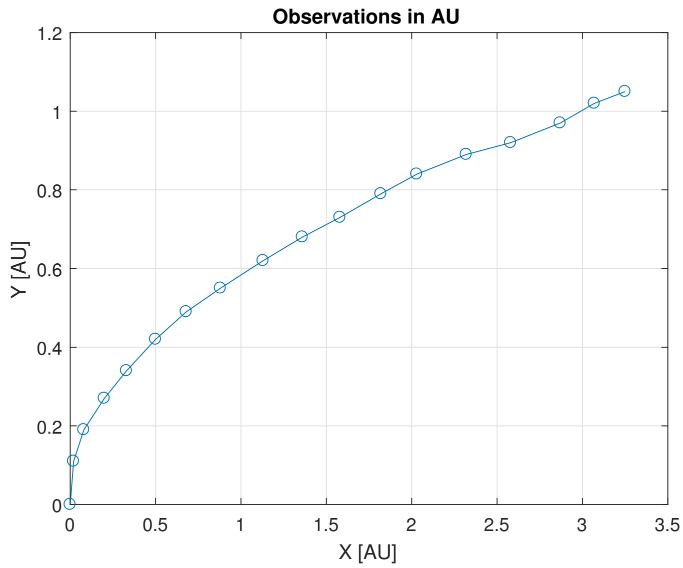


Figure 1: Observation x and y in AU.

Table 1 outlines the discrete X and Y parameters used to plot the observations in Figure 1[1][Lecture- Optimisation Slide- 28].

Table 1: X and Y observations in AU.

X [AU]	0.00	0.02	0.08	0.20	0.33	0.50	0.68	0.88	1.13	1.36	1.58	1.82	2.03	2.32	2.58	2.87	3.07	3.25
Y [AU]	0.00	0.11	0.19	0.27	0.34	0.42	0.49	0.55	0.62	0.68	0.73	0.79	0.84	0.89	0.92	0.97	1.02	1.05

Why Monte Carlo or grid search optimisation? These optimisation techniques rely on sampling or varying parameters to fit a function. The solutions obtained with these approximation are in most cases not optimal but rather starting point localisers. The initial point for optimisation is determined with these search techniques and later more advanced optimisation techniques are used around these sub-optimal solutions to find the optimal solution.

Monte Carlo offers a choice for sampling distribution. For this assignment a uniform sampling distribution is chosen. While grid search as the name suggests places a uniform/non-uniform grid on a topography to estimate parameters. For this assignment uniform spacing is used for the grid search criteria.

First the observation is fit through the assumption that the observations resemble to that of a parabolic orbit. Both Monte Carlo and grid search methods are implemented for parabolic orbit in Section 2. Section 3 focuses on application of Monte Carlo and grid search methods from the perspective of hyperbolic orbits. Section 4 presents the Matlab scripts used to facilitate the estimations.

2 Parabolic orbit

The objective of this Section is to assume that the orbital observations are parabolic. Monte Carlo and grid search sampling based optimization can be applied easily on the observations if an analytical expression of x and y positions of the orbital body wrt the focal point is known. Wertz [2][pg.51, 845] provides an analytic expression relating x and y parameters of a parabola. However, Wertz ignores the need to describe the process of deriving those parameters. As a result Wertz ends up deriving the parabolic expression for a regular parabolic function. However, for the purpose of orbital calculation sideways parabolic function is rather attractive.

Figure 2 orbit outlines a sideways parabolic orbit[1]. Here q is the perifocal distance, which is the distance between the vertex of the parabola and the focal point. q is also the distance between directrix and the vertex point. The observations start from [0,0]. However, Figure 2 shows that from the origin along the directrix line, the vertex has an offset of q.

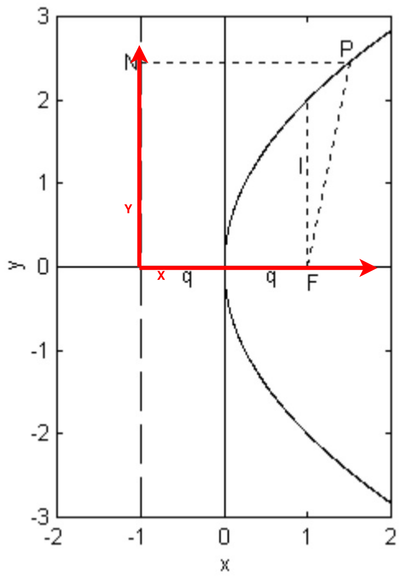


Figure 2: Observation x and y in AU.

Parametric expression for a parabolic orbit can be expressed with the aid of Equation 1 [2].

$$4q(x–h) = (y–k)^2 \tag{1}$$

Where h is the x directional offset of the vertex from the origin.
While k is the y directional offset of the vertex from the origin.
Origin lies on the intersection of directrix and a perpendicular line passing through the focal point.

For the observations shown in Figure1, the parametric equation can be simplified to Equation 2. Where q is the distance of the vertex from the focal point. q is also known as the peri-apsis/peri-focal distance.

[1]Parabolic and Hyperbolic orbit parameters: `http://astrowww.phys.uvic.ca/~tatum/celmechs/celm2.pdf`
[2]Parabolic orbit parameters: `http://www.purplemath.com/modules/parabola.html`

$$x = \frac{y^2}{4q} + q \tag{2}$$

Equation 2 is the final analytical expression where the parameter to be estimated is the perifocal distance q.

2.1 Monte Carlo

For Monte Carlo sampling the objective is to use the Y observations outlined in Table 1. q is generated in the range 0 to 0.5 AU. This is reasonably in line with the verification figure[1] [Lecture - Optimisation, Slide - 28]. q is generated with uniform random distribution. Different samples of 50, 100 and 500 are generated. For each no. of samples, 3 runs are conducted. Based on the values of Y observation and generated q, values of X are obtained. Optimum value of parameter q is chosen such that values of resulting X has the minimum standard deviation from the observations. It is important to note that observations are not perfect either, this is observed by the non-smooth curve in Figure1. Value of q for which minimum standard deviation is observed is chosen as the optimum. Script used for Monte Carlo sampling is presented in Section 4.1.

Figures 3, 4 and 5 represents the X and Y observations for different number of samples.

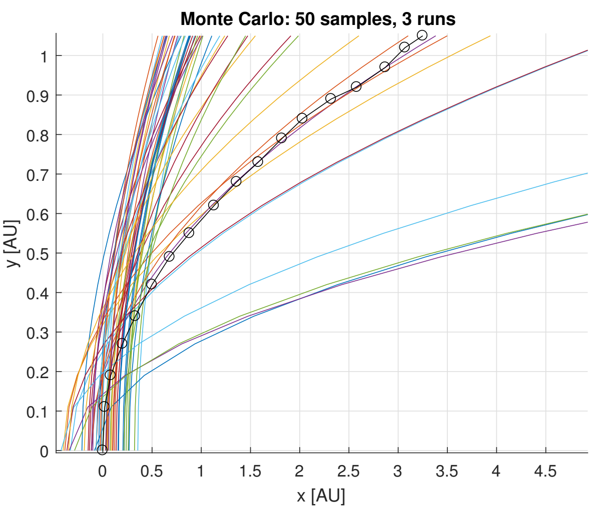


Figure 3: Parabolic parameter estimations with Monte Carlo for 50 samples.

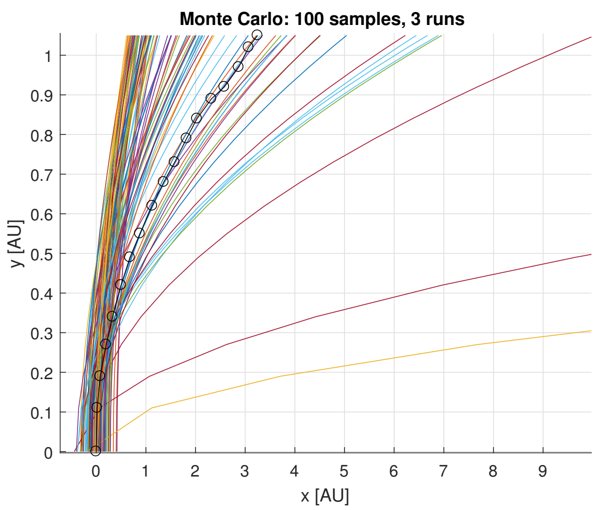


Figure 4: Parabolic parameter estimations with Monte Carlo for 100 samples.

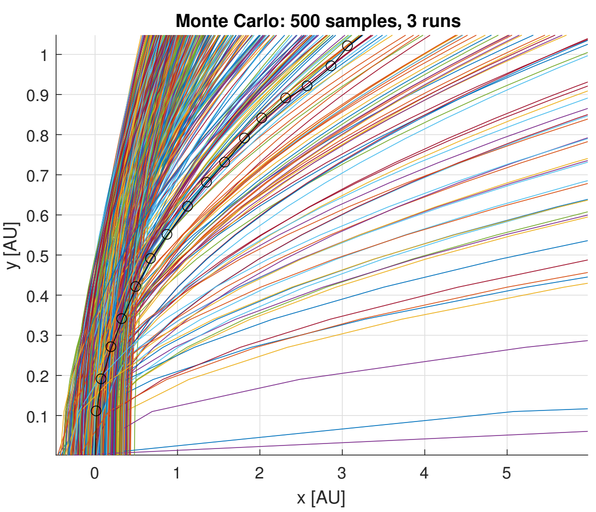


Figure 5: Parabolic parameter estimations with Monte Carlo for 500 samples.

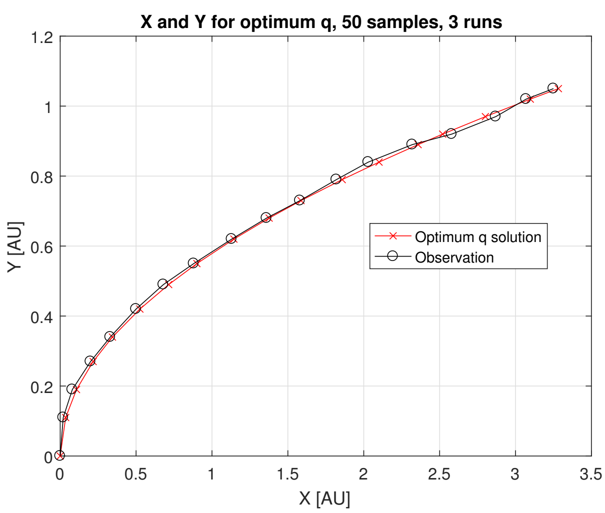


Figure 6: Optimal parabolic parameter estimation with Monte Carlo for 50 samples. q =0.0832 AU and std of 0.0327 AU.

While Figures 6, 7 and 8 represent the observations for optimum values of q.

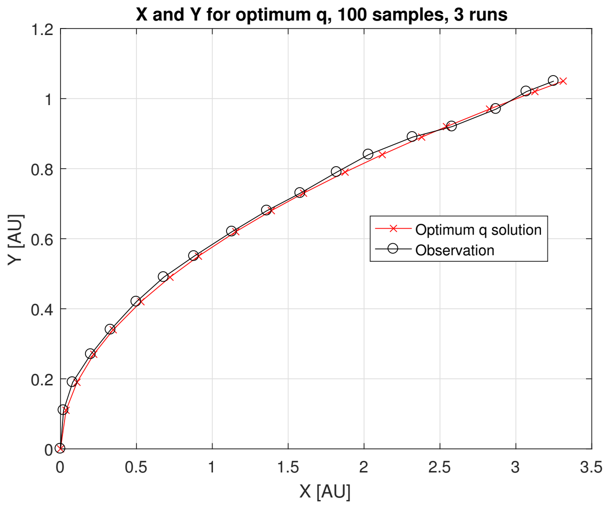


Figure 7: Optimal parabolic parameter estimation with Monte Carlo for 50 samples. q = 0.0828 AU and std of 0.0339 AU.

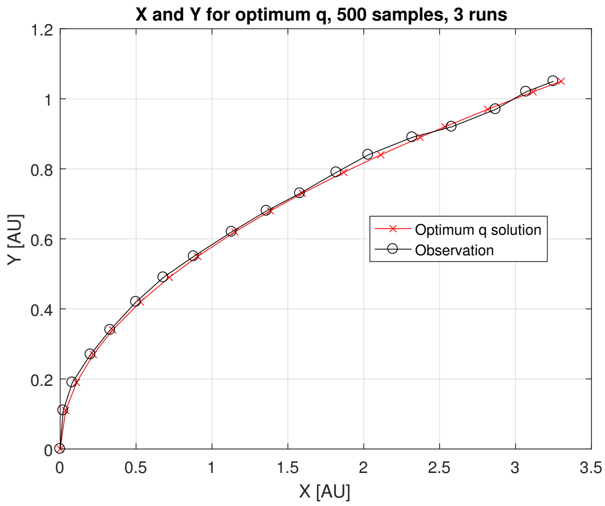


Figure 8: Optimal parabolic parameter estimation with Monte Carlo for 50 samples. q = 0.0836 AU and std of 0.0324 AU.

To conclude this section, table 2 outlines the minimum standard deviation for different number of samples along with the corresponding value of q obtained. From table 2 it can be concluded that, increasing the number of samples increases the accuracy of solution. This can be verified by the decreasing standard deviations for increasing number of samples. However, since the samples are generated in random manner, it could easily result in a scenario that lower no. of samples provide better fit than higher no. of samples. This is seen in this specific scenario when the no. of samples were increased from 50 to 100.

Table 2: Variation of q and standard deviation for Monte Carlo sampling.

No. of Samples	**Perifocal distance q [AU]**	**Standard deviation [AU]**
50	0.0832	0.0327
100	0.0828	0.0339
500	0.0836	0.0324

2.2 Grid Search

The idea behind grid sample is analogous to that of Monte Carlo. Instead of using a random number generator, arithmetic progression of stencils is used. Equation 2 is used as the parametric expression. Similar to Monte Carlo estimations, values of Y are feed into Equation 2, while incrementally varying the grid values for q in the same range [0AU, 0.5AU]. Running once instead of three times is sufficient, since both the function and the grid meshes are constant. Optimum value of parameter q is chosen such that resulting values of X have the minimum standard deviation from the observations. Value of q for which minimum standard deviation is observed is chosen as the optimum. Script used for grid search sampling is outlined in Section 4.1.

Figures 9, 10 and 11 represents the X and Y observations for different number of samples.

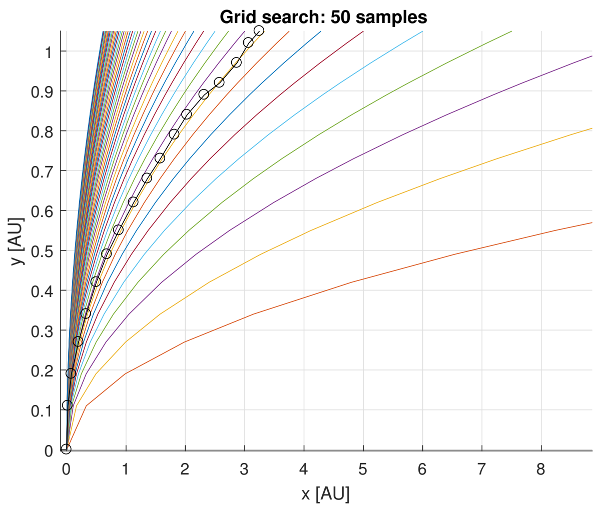


Figure 9: Parabolic parameter estimations with grid search for 50 samples.

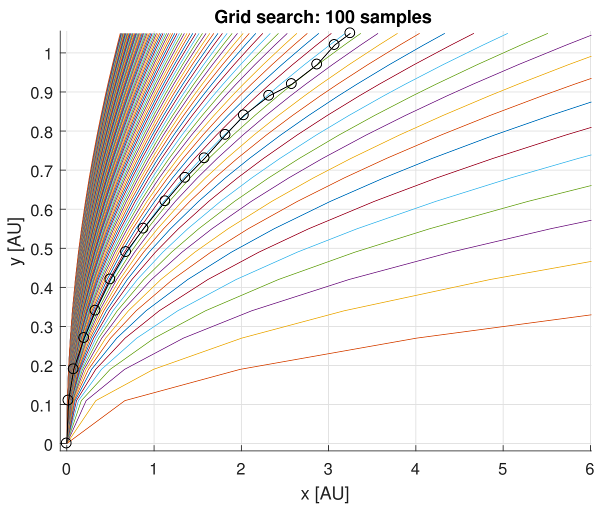


Figure 10: Parabolic parameter estimations with grid search for 100 samples.

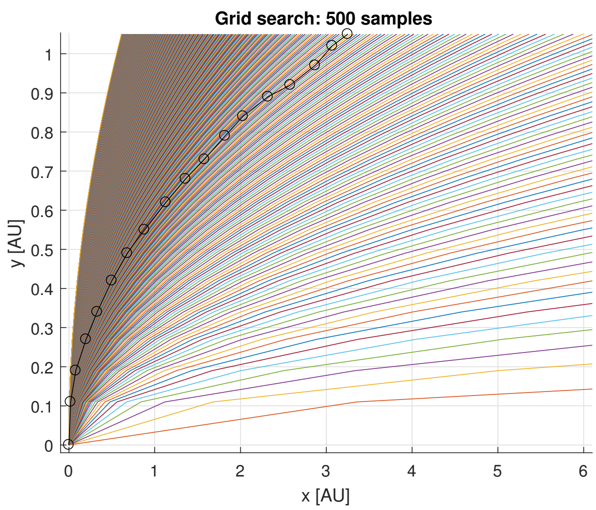


Figure 11: Parabolic parameter estimations with grid search for 500 samples.

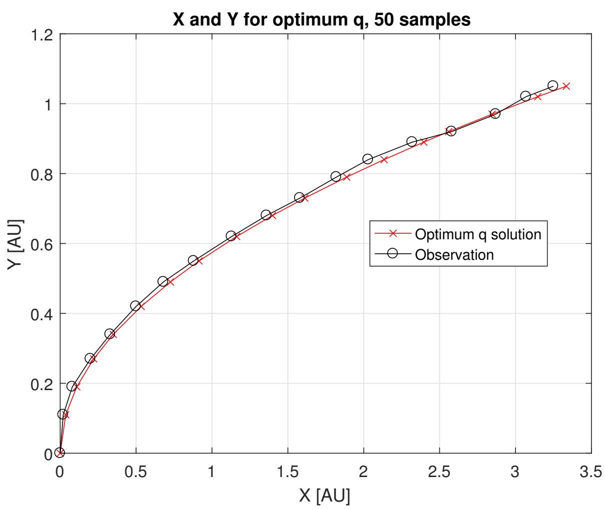
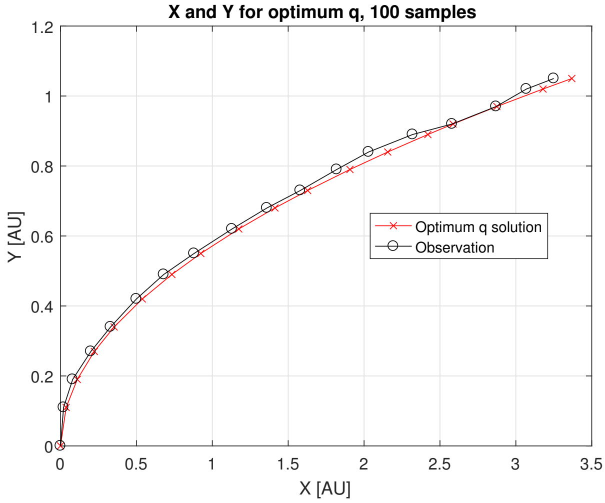


Figure 12: Optimal parabolic parameter estimation with grid search for 50 samples. q = 0.0827 AU and std of 0.0346 AU.

While Figures 12, 13 and 14 represent the observations for optimum values of q. Table 3 outlines the minimum standard deviation for different number of samples along with the corresponding value of q obtained.

Table 3: Values of optimum q and the corresponding standard deviations for different grid samples.

No. of Samples	**Perifocal distance q [AU]**	**Standard deviation [AU]**
50	0.0827	0.0346
100	0.0818	0.0402
500	0.0839	0.0326



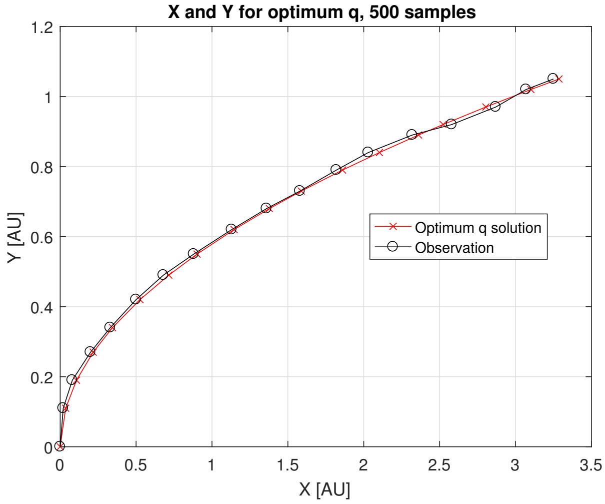


Figure 13: Optimal parabolic parameter estimation with grid search for 100 samples. q = 0.0818 AU and std of 0.0402 AU.

Figure 14: Optimal parabolic parameter estimation with grid search for 500 samples. q =0.0839 and std of 0.0326 AU.

From table 3 it can be concluded that, even if one might expect that increasing the number of samples increases the accuracy of solution this is not always true. This is observed when the no. of samples is increased from 50 to 100. But in general when the no of grid samples cross a certain threshold, the standard deviations either go down or remain constant for a convex function.

2.3 Comparison: Quality vs Time

For Monte Carlo process the computational effort is at least 3 times higher than the computational effort of grid search method. This is justified by the fact that there are three runs. This excludes the fact that generating random number by Monte Carlo adds more computational effort on top of that, compared to a predefined grid. The term computational effort indicate calculation to get estimations for x. Results for one sample is defined as one calculation unit. Based on this a plot can be generated with no. of calculations (function of time) on the x axis and the corresponding standard deviation (function of accuracy) on the y axis. This is shown with the aid of Figure15[Sample points: 50, 100 and 500 order left to right].

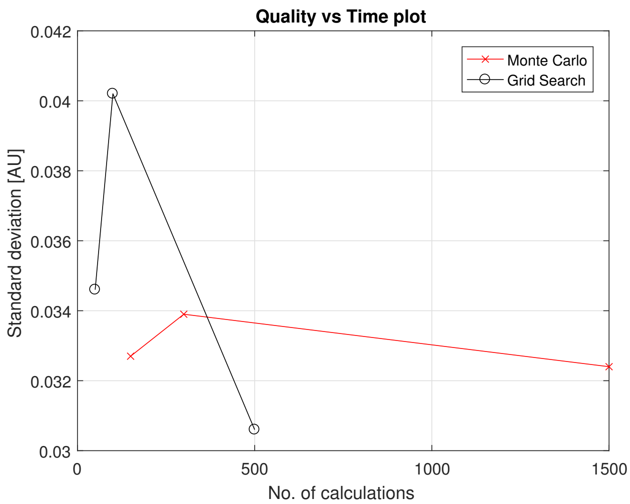


Figure 15: Standard deviations against no. of calculations for Monte Carlo and grid search sampling methods.

In conclusion, for lower number of samples the quality to time ratio for Monte Carlo could be higher than that of grid search. However for increasing samples, grid search demonstrates higher quality to time ratio than Monte Carlo. This is true at least for the problem under consideration.

3 Hyperbolic orbit

The objective of this Section is to assume that the observations are hyperbolic. This is rather unlikely, observing the results for the parabolic functions in the previous section. Since the observations were well submerged within parabolic estimation. But no conclusion can be made yet. The aim of this chapter is to make use of a Hyperbolic parametric equation and use Monte Carlo and grid searching to observe how well the estimations fit the observations. An analytic expression for x and y positions is expressed with the aid of Equation 3 [3]. Figure 16 presents an overview for the co-ordinate system for a hyperbolic orbit [4].

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{3}$$

Where a is the semi-tranverse axis. a<0 for Hyperbolic orbit.

a can be considered as the distance between directrix and the vertex of hyperbola.

c is the distance from the origin to the focal point.

While $b^2 = c^2 - a^2$. Where b = semi-conjugate axis.

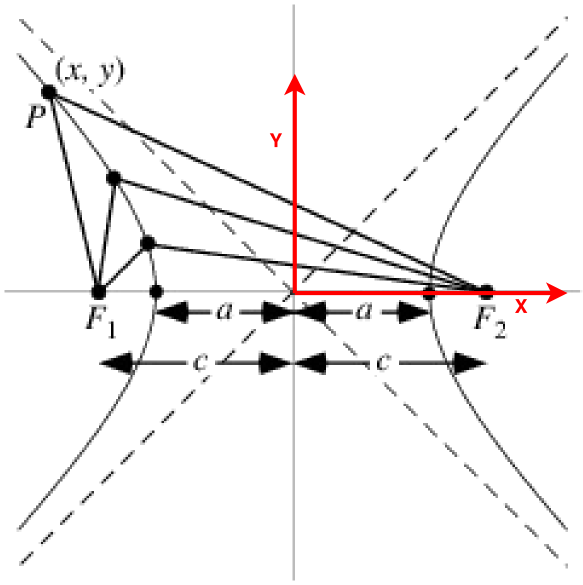


Figure 16: Co-ordinate frame for a hyperbolic orbit.

Since x coordinate of observations bear an offset of amount a, this is included into the parametric equation in Equation 3. Equation 4 expresses the analytic parametric equation for hyperbolic orbit. Where the parameters to be estimated are a and b.

$$\frac{(x-a)^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{4}$$

3.1 Monte Carlo

For Monte Carlo sampling the objective is to use the Y observations outlined in Table 1. a is generated in the range -0.5 to 0.5 AU. This is reasonably in line with the verification figure[1] [Lecture - Optimisation, Slide - 28]. a is generated with uniform random distribution. b is related to the eccentricity and a as shown in Equation 5.

[3] Hyperbolic parametric equation: http://www.nabla.hr/PC-ParametricEqu4.htm
[4] Hyperbolic co-ordinate: http://mathworld.wolfram.com/Hyperbola.html

$$
\begin{aligned}
&b^2 = a^2 \cdot (e^2 - 1) \\
&\text{for Hyperbolic orbit e} > 1. \text{ Since } a^2 = (\pm 0.5)^2 \\
&\text{If eccentricity is chosen between 1 and 1.4. The range for b, lies in the range:} \\
&b^2 = [0, 0.24]
\end{aligned} \tag{5}
$$

b is also generated by uniform distribution in the given range. Different samples of 50, 100 and 500 are generated. For each no. of samples, 3 runs are conducted. Based on the values of Y observation and generated a and b, values of X are obtained. Optimum value of parameters (a,b) is chosen such that values of resulting X has the minimum standard deviation from the observations. It is important to note that observations are not perfect either, this is observed by the non-smooth curve in Figure1. Value of (a,b) for which minimum standard deviation is observed is chosen as the optimum combination. Script used for Monte Carlo sampling is presented in Section 4.2.

Figures 17, 18 and 19 represents the X and Y observations for different number of samples. While Figures 20, 21 and 22 represent the observations for optimum values of a and e.

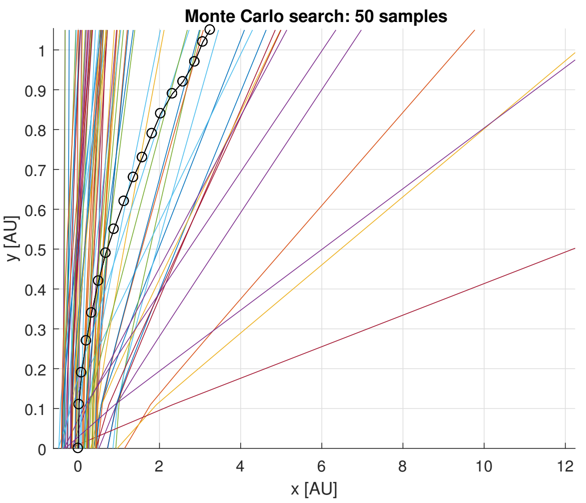


Figure 17: Hyperbolic parameter estimations with Monte Carlo for 50 samples.

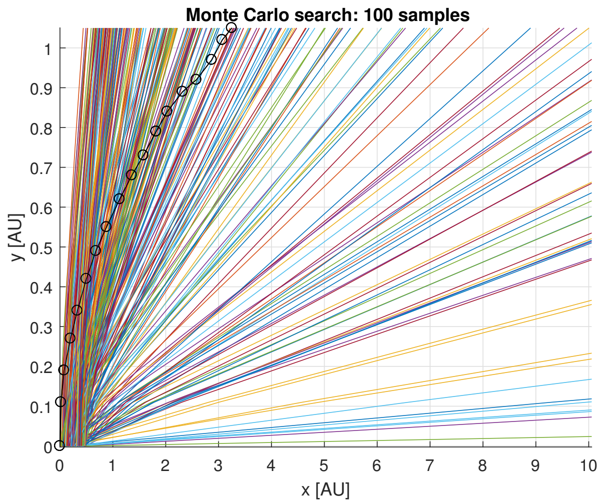


Figure 18: Hyperbolic parameter estimations with Monte Carlo for 100 samples.

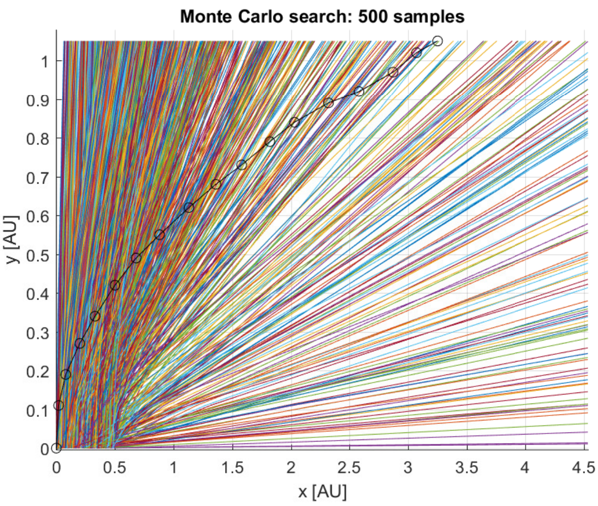


Figure 19: Hyperbolic parameter estimations with Monte Carlo for 500 samples.

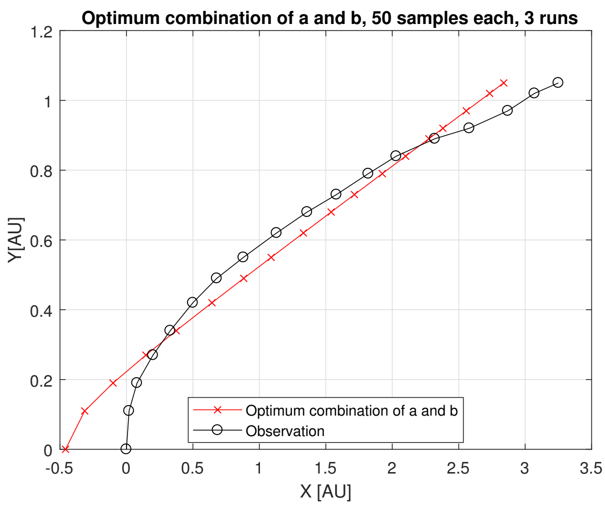
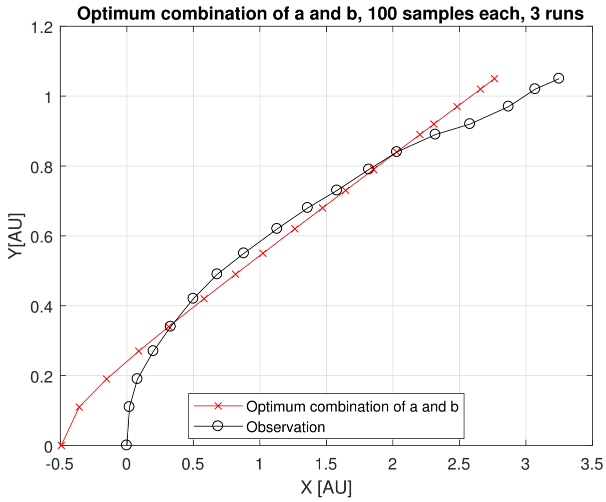


Figure 20: Optimal hyperbolic parameter estimation with Monte Carlo for 50 samples. a = -0.4578 AU, b = 0.1290 AU and std of 0.2344 AU.



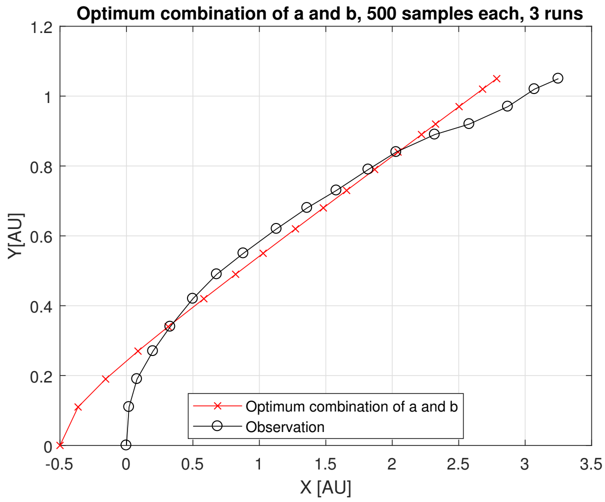


Figure 21: Optimal hyperbolic parameter estimation with Monte Carlo for 50 samples. a = -0.4931 AU, b = 0.1393 AU and std of 0.2311 AU .

Figure 22: Optimal hyperbolic parameter estimation with Monte Carlo for 50 samples. a = -0.4996 AU, b = 0.1398 AU and std of 0.2303 AU .

To conclude this section, table 4 outlines the minimum standard deviation for different number of samples along with the corresponding value of optimum a and b obtained. From table 2 it can be concluded that, increasing the number of samples increases the accuracy of solution. This can be verified by the decreasing standard deviations for increasing number of samples. However, the standard deviations are quite off compared to the values obtained for a parabolic orbit scenario. The reason is obvious, an orbit cannot be parabolic and hyperbolic at the same time. Based on the observations the orbit is rather parabolic.

Table 4: Values of optimum a and b parameters and the corresponding standard deviations for Monte Carlo simulations.

No. of Samples	Semi major axis, a [AU]	Semi conjugate axis, b [AU]	Standard deviation [AU]
50	-0.4578	0.1290	0.2344
100	-0.4931	0.1393	0.2311
500	-0.4996	0.1398	0.2303

3.2 Grid Search

The idea behind grid sample is analogous to that of Monte Carlo. Instead of using a random number generator, arithmetic progression of stencils is used. Equation 4 is used as the parametric expression. Similar to Monte Carlo estimations, values of Y are feed into Equation 4, while incrementally varying the grid values for (a,b) in the same range as presented for the Monte Carlo case. i.e. a = [-0.5 AU, 0.5 AU] and b^2 = [0 AU, 0.24 AU]. Running once instead of three times is sufficient, since both the function and the grid meshes are constant. Optimum value of the combinations of parameters (a,b) is chosen such that resulting values of X have the minimum standard deviation from the observations. Combinations of (a,b) for which minimum standard deviation is observed is chosen as the optimum. Script used for grid search sampling is outlined in Section 4.2.

Figures 23, 24 and 25 represents the X and Y observations for different number of samples.

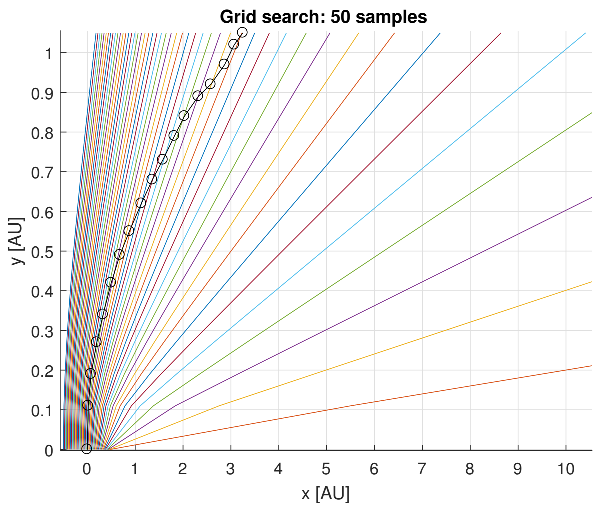


Figure 23: Hyperbolic parameter estimations with grid search for 50 samples.

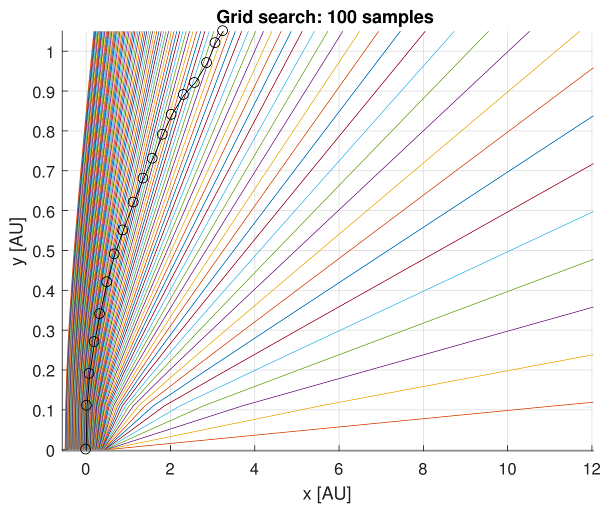


Figure 24: Hyperbolic parameter estimations with grid search for 100 samples.

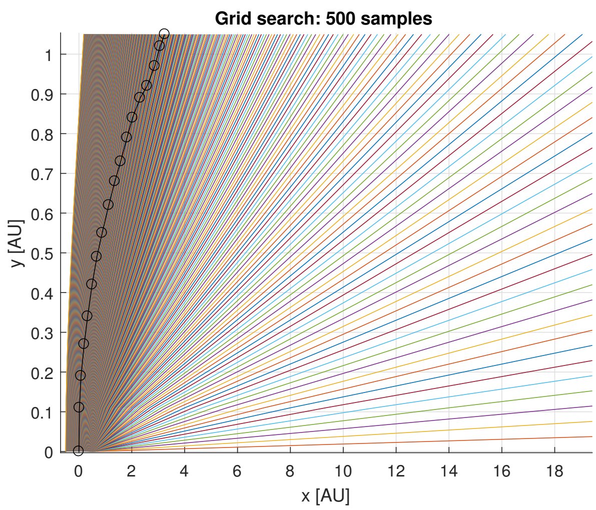


Figure 25: Hyperbolic parameter estimations with grid search for 500 samples.

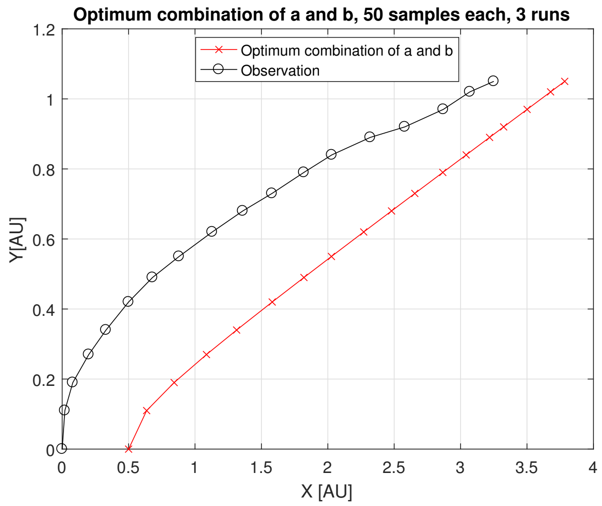
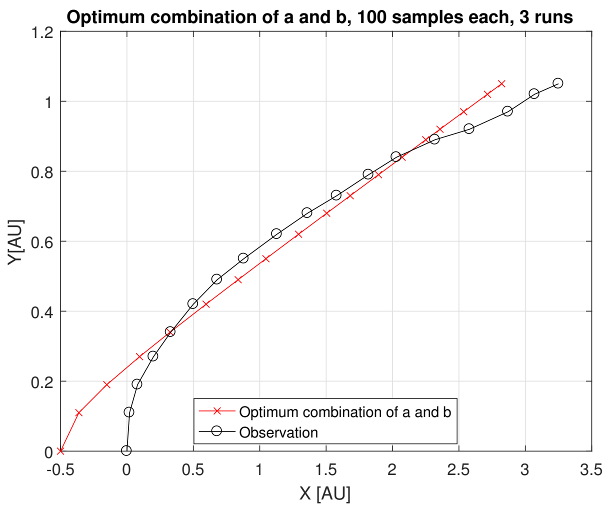


Figure 26: Optimal hyperbolic parameter estimation with grid search for 50 samples. a = 0.500 AU, b =0.140 AU and std of 0.2302 AU.

While Figures 26, 27 and 28 represent the observations for optimum values of q. Table 5 outlines the minimum standard deviation for different number of samples along with the corresponding value of (a,b) obtained.

Table 5: Values of optimum a and b parameters and the corresponding standard deviations for grid search.

No. of Samples	Semi major axis, a [AU]	Semi conjugate axis, b [AU]	Standard deviation [AU]
50	0.5000	0.1400	0.2302
100	-0.5000	0.1386	0.2306
500	-0.5000	0.1404	0.2302



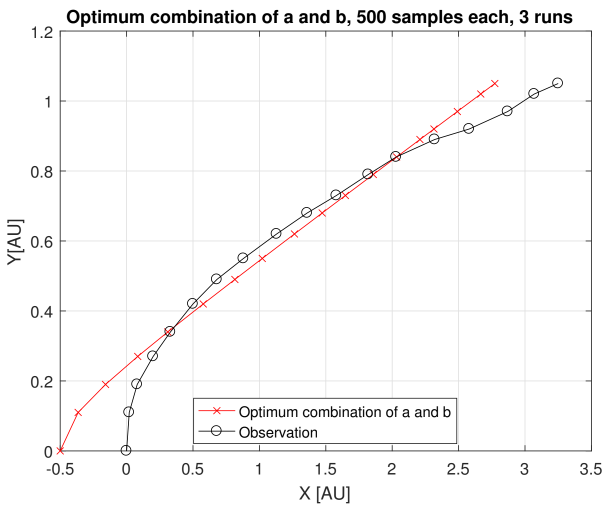


Figure 27: Optimal hyperbolic parameter estimation with grid search for 100 samples. a= -0.5 AU, b = 0.1386 and std of 0.2306 AU.

Figure 28: Optimal hyperbolic parameter estimation with grid search for 500 samples. a= -0.500 AU, b = 0.1404 and std of 0.2302 AU.

From table 5 it can be concluded that, even if one might expect that increasing the number of samples increases the accuracy of solution this is not always true. This is observed when the no. of samples is increased from 50 to 100. But in general when the no of grid samples cross a certain threshold, the standard deviations either go down or remain constant for a convex function. However, a very important observation is that for no. of samples = 50, the best fit solution for a is positive. However, for hyperbolic orbit it is known that ”a” should be negative.

3.3 Comparison: Quality vs Time

For Monte Carlo process the computational effort is at least 3 times higher than the computational effort of grid search method. This is justified by the fact that there are three runs. This excludes the fact that generating random number by Monte Carlo adds more computational effort on top of that, compared to a predefined grid. The term computational effort indicate calculation to get estimations for x. Results for one sample is defined as one calculation unit. Based on this a plot can be generated with no. of calculations (function of time) on the x axis and the corresponding standard deviation (function of accuracy) on the y axis. This is shown with the aid of Figure29[Sample points: 50, 100 and 500 order left to right].

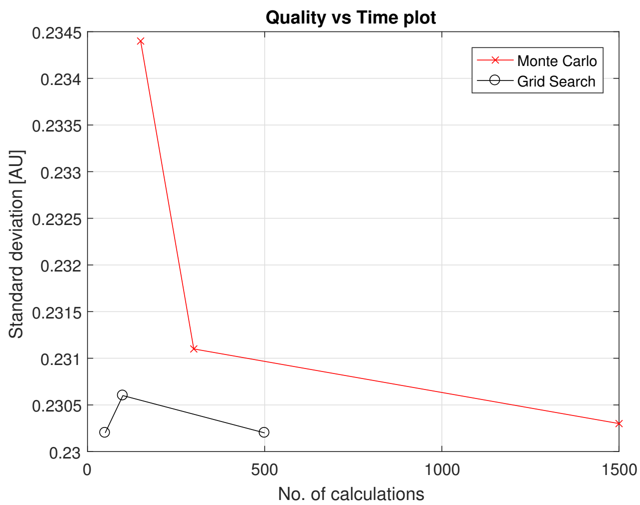


Figure 29: Standard deviations against no. of calculations for Monte Carlo and grid search sampling methods.

In conclusion, grid search demonstrates higher quality to time ratio than Monte Carlo. However, the standard deviations in general are significantly higher than that observed in Figure 15. There are two reason for it;

1.The orbit is parabolic, expecting a parabolic orbit to fit through a hyperbolic model will surely result in high standard deviations. 2.Grid based or Monte Carlo based sampling techniques lose resolution when the dimension of parameter is increased but the no. of samples are kept the same.

It can be concluded that observed X and Y values form a parabolic trajectory.

4 Matlab Scipt

4.1 Parabolic orbit

Following script is used for Monte Carlo sampling of parabolic orbit.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% AE4878 Mission Geometry and Orbit Design %%%%%%%%%%%
%%%%% Assignment 4 - Week 5 - OPTIMIZATION - Version 1 %%%
%%%%% Author Info: Ali Nawaz; Student ID - 4276477 %%%%%%%
%%%%% Assigned tasks : OPTIM_5a, QUESTION 2 %%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% MONTE CARLO FOR PARABOLIC PARAMETER ESTIMATION
close all; clear all;
x = [ 0.00 0.02 0.08 0.20 0.33 0.50 0.68 0.88 1.13 1.36 1.58 1.82 2.03 2.32 2.58 2.87 3.07 ...
    3.25]; % Distance in AU from slide 28
y = [ 0.00 0.11 0.19 0.27 0.34 0.42 0.49 0.55 0.62 0.68 0.73 0.79 0.84 0.89 0.92 0.97 1.02 ...
    1.05]; % Distances in AU from slide 28

ub = 0.50; % Upper bound on perifocal distance q [AU]
lb = 0; % Lower bound on perifocal distance q [AU]

% Store the estimations of x and corresponding standard deviations

% For 50 samples
x_est1 = [];
std_est1 = [];

% For 100 samples
x_est2 = [];
std_est2 = [];

% For 500 samples
x_est3 = [];
std_est3 = [];

q1_lst = [];
q2_lst = [];
q3_lst = [];

for run = 1:3 % Repeating each run three times

    % Random perifocal distance generation
    q1 = lb + (ub).*rand(1,50); % 50 samples
    q2 = lb + (ub).*rand(1,100); % 100 samples
    q3 = lb + (ub).*rand(1,500); % 500 samples

    for t = 1:length(q1)
        x_buf1 = q1(t) + (y.^2)./(4*q1(t)); % store the value of x for q
        std_est1 = [std_est1, std(x - (x_buf1- q1(t)))]; % estimate standard deviation
        x_est1 = [x_est1,x_buf1']; % store the estimated x
    end

    for t = 1:length(q2)
        x_buf2 = q2(t) + (y.^2)./(4*q2(t));% store the value of x for q
        std_est2 = [std_est2, std(x - (x_buf2- q2(t)))];% estimate standard deviation
        x_est2 = [x_est2,x_buf2'];% store the estimated x
    end

    for t = 1:length(q3)
        x_buf3 = q3(t) + (y.^2)./(4*q3(t));% store the value of x for q
        std_est3 = [std_est3, std(x - (x_buf3- q3(t)))];% estimate standard deviation
        x_est3 = [x_est3,x_buf3'];% store the estimated x
    end
```

```
    q1_lst = [q1_lst,q1];
    q2_lst = [q2_lst,q2];
    q3_lst = [q3_lst,q3];
end

% Plot figure for 50 samples, 3 runs case
figure(1)
hold on
for k = 1:length(q1)
        plot(x_est1(:,k)-q1(k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo: 50 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

% Plot figure for 100 samples, 3 runs case
figure(2)
hold on
for k = 1:length(q2)
        plot(x_est2(:,k)-q2(k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo: 100 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

% Plot figure for 500 samples, 3 runs case
figure(3)
hold on
for k = 1:length(q3)
        plot(x_est3(:,k)-q3(k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo: 500 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

% Find the index for minimum standard deviation
index1 = find(std_est1 == min(std_est1));
index2 = find(std_est2 == min(std_est2));
index3 = find(std_est3 == min(std_est3));

% Find the optimum value of q for which min. std happens.
opti_q1 = q1_lst(index1);
opti_q2 = q2_lst(index2);
opti_q3 = q3_lst(index3);

%% Plot X and Y for optimum values of q
figure(11)
plot(x_est1(:,index1) - q1_lst(index1),y','r-x',x,y,'k-o');
grid on
legend('Optimum q solution','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y [AU]');
title('X and Y for optimum q, 50 samples, 3 runs');

figure(22)
plot(x_est2(:,index2)- q2_lst(index2),y','r-x',x,y,'k-o');
grid on
legend('Optimum q solution','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y [AU]');
title('X and Y for optimum q, 100 samples, 3 runs');

figure(33)
plot(x_est3(:,index3)- q3_lst(index3),y','r-x',x,y,'k-o');
grid on
legend('Optimum q solution','Observation','Location','best');
```

```
xlabel('X [AU]');
ylabel('Y [AU]');
title('X and Y for optimum q, 500 samples, 3 runs');
```

Following is the script used for grid search sampling of parabolic orbit.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% AE4878 Mission Geometry and Orbit Design %%%%%%%%%%%
%%%%% Assignment 4 - Week 5 - OPTIMIZATION - Version 1 %%%
%%%%% Author Info: Ali Nawaz; Student ID - 4276477 %%%%%%%
%%%%% Assigned tasks : OPTIM_5a, Question 3 %%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% GRID SEARCH FOR PARABOLIC PARAMETER ESTIMATION
close all; clear all;
x = [ 0.00 0.02 0.08 0.20 0.33 0.50 0.68 0.88 1.13 1.36 1.58 1.82 2.03 2.32 2.58 2.87 3.07 ...
    3.25]; % Distance in AU from slide 28
y = [ 0.00 0.11 0.19 0.27 0.34 0.42 0.49 0.55 0.62 0.68 0.73 0.79 0.84 0.89 0.92 0.97 1.02 ...
    1.05]; % Distances in AU from slide 28

ub = 0.45; % Upper bound on perifocal distance q [AU]
lb = 0; % Lower bound on perifocal distance q [AU]

% Store the estimations of x and corresponding standard deviations

% For 50 samples
x_est1 = [];
std_est1 = [];

% For 100 samples
x_est2 = [];
std_est2 = [];

% For 500 samples
x_est3 = [];
std_est3 = [];

q1_lst = [];
q2_lst = [];
q3_lst = [];

% Grided perifocal distance generation
q1 = 0:(ub/(50-1)):ub; % 50 samples
q2 = 0:(ub/(100-1)):ub; % 100 samples
q3 = 0:(ub/(500-1)):ub; % 500 samples

for t = 1:length(q1)
    x_buf1 = q1(t) + (y.^2)./(4*q1(t)); % store the value of x for q
    std_est1 = [std_est1, std(x - (x_buf1- q1(t)))]; % estimate standard deviation
    x_est1 = [x_est1,x_buf1']; % store the estimated x
end

for t = 1:length(q2)
    x_buf2 = q2(t) + (y.^2)./(4*q2(t));% store the value of x for q
    std_est2 = [std_est2, std(x - (x_buf2- q2(t)))];% estimate standard deviation
    x_est2 = [x_est2,x_buf2'];% store the estimated x
end

for t = 1:length(q3)
    x_buf3 = q3(t) + (y.^2)./(4*q3(t));% store the value of x for q
    std_est3 = [std_est3, std(x - (x_buf3- q3(t)))];% estimate standard deviation
    x_est3 = [x_est3,x_buf3'];% store the estimated x
end

q1_lst = [q1_lst,q1];
q2_lst = [q2_lst,q2];
q3_lst = [q3_lst,q3];


% Plot figure for 50 samples, 3 runs case
figure(1)
hold on
for k = 1:length(q1)
        plot(x_est1(:,k)-q1(k),y');
end
```

```
plot(x,y,'k-o');
grid on
title('Grid search: 50 samples');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

% Plot figure for 100 samples, 3 runs case
figure(2)
hold on
for k = 1:length(q2)
        plot(x_est2(:,k)-q2(k),y');
end
plot(x,y,'k-o');
plot(x,y,'k-o');
grid on
title('Grid search: 100 samples');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

% Plot figure for 500 samples, 3 runs case
figure(3)
hold on
for k = 1:length(q3)
        plot(x_est3(:,k)-q3(k),y');
end
plot(x,y,'k-o');
grid on
title('Grid search: 500 samples');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

% Find the index for minimum standard deviation
index1 = find(std_est1 == min(std_est1));
index2 = find(std_est2 == min(std_est2));
index3 = find(std_est3 == min(std_est3));

% Find the opitmum value of q for which min. std happens.
opti_q1 = q1_lst(index1);
opti_q2 = q2_lst(index2);
opti_q3 = q3_lst(index3);

%% Plot X and Y for optimum values of q
figure(11)
plot(x_est1(:,index1) - q1_lst(index1),y','r-x',x,y,'k-o');
grid on
legend('Optimum q solution','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y [AU]');
title('X and Y for optimum q, 50 samples');

figure(22)
plot(x_est2(:,index2)- q2_lst(index2),y','r-x',x,y,'k-o');
grid on
legend('Optimum q solution','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y [AU]');
title('X and Y for optimum q, 100 samples');

figure(33)
plot(x_est3(:,index3)- q3_lst(index3),y','r-x',x,y,'k-o');
grid on
legend('Optimum q solution','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y [AU]');
title('X and Y for optimum q, 500 samples');
```

Following is the script for quality vs time comparison of parabolic orbit:

```
%% COMPARING MONTE CARLO AND GRID SEARCH COMPUTATIONAL EFFORT VS QUALITY
%no. of samples
samples = [ 50 100 500 ];

```

```
%no. of calculations, m = Monte Carlo, g = grid search
calc_m = 3.*samples;
calc_g = samples;

%corresponding standard deviations
std_m = [0.0327 0.0339 0.0324];
std_g = [ 0.0346 0.0402 0.0306];

% Plot the results
figure(41)
plot(calc_m,std_m,'r-x',calc_g,std_g,'k-o');
grid on
legend('Monte Carlo','Grid Search');
ylabel('Standard deviation [AU]');
xlabel('No. of calculations');
title('Quality vs Time plot');
```

4.2 Hyperbolic orbit

Following is the script used for Monte Carlo sampling of Hyperbolic orbit.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% AE4878 Mission Geometry and Orbit Design %%%%%%%%%%%
%%%%% Assignment 4 - Week 5 - OPTIMIZATION - Version 1 %%%
%%%%% Author Info: Ali Nawaz; Student ID - 4276477 %%%%%%%
%%%%% Assigned tasks : OPTIM_5a, QUESTION 5 %%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% MONTE CARLO FOR HYPERBOLIC PARAMETER ESTIMATION, PARAMETERS : semi transverse axis a and ...
    semi-conjugate axis b
% close all; clear all;
x = [ 0.00 0.02 0.08 0.20 0.33 0.50 0.68 0.88 1.13 1.36 1.58 1.82 2.03 2.32 2.58 2.87 3.07 ...
    3.25]; % Distance in AU from slide 28
y = [ 0.00 0.11 0.19 0.27 0.34 0.42 0.49 0.55 0.62 0.68 0.73 0.79 0.84 0.89 0.92 0.97 1.02 ...
    1.05]; % Distances in AU from slide 28

uba = 0.5; % Upper bound on a, semi transverse axis
lba = -0.5; % Lower bound on a, semi conjugate a

ubb = sqrt(0.240); % Upper bound on b
lbb = 0; % Lower bound on b
% Store the estimations of x and corresponding standard deviations

% For 50 samples
x_est1 = [];
std_est1 = [];

% For 100 samples
x_est2 = [];
std_est2 = [];

% For 500 samples
x_est3 = [];
std_est3 = [];

% List of semi major axis a for sample size 50, 100 and 500
a1_lst = [];
a2_lst = [];
a3_lst = [];

% List of semi conjugagte axis b for sample size 50, 100 and 500
b1_lst = [];
b2_lst = [];
b3_lst = [];

% Storing a and b parameters for each calculation
para_lst1 =[]; % 50 samples
para_lst2 = [];% 100 samples
para_lst3 = [];% 500 samples

for run = 1:3 % Repeating each run three times

```

```
    % Random perifocal distance generation
    a1 = lba + (uba + uba).*rand(1,50); % 50 samples
    a2 = lba + (uba).*rand(1,100); % 100 samples
    a3 = lba + (uba).*rand(1,500); % 500 samples

    % Random eccentricity generation
    b1 = lbb + (ubb).*rand(1,50); % 50 samples
    b2 = lbb + (ubb).*rand(1,100); % 100 samples
    b3 = lbb + (ubb).*rand(1,500); % 500 samples


    for t = 1:length(a1)
        for j = 1:length(b1)
            x_buf1 = sqrt( (1 + (y.^2)./(b1(j)^2) ).*a1(t)^2) + a1(t); % Estimate the value ...
                of x for given q,e and y
            std_est1 = [std_est1, std(x - (x_buf1- a1(t)))]; % estimate standard deviation
            x_est1 = [x_est1,x_buf1']; % store the estimated x
            para_lst1 = [ para_lst1, [a1(t);b1(j)] ]; % Store parameters a and b in a list
        end
    end

    for t = 1:length(a2)
        for j = 1:length(b2)
            x_buf2 = sqrt( (1 + (y.^2)./(b2(j)^2) ).*a2(t)^2) + a2(t); % Estimate the value ...
                of x for given q,e and y
            std_est2 = [std_est2, std(x - (x_buf2- a2(t)))]; % estimate standard deviation
            x_est2 = [x_est2,x_buf2']; % store the estimated x
            para_lst2 = [ para_lst2, [a2(t);b2(j)] ]; % Store parameters a and b in a list
        end
    end

    for t = 1:length(a3)
        for j = 1:length(b3)
            x_buf3 = sqrt( (1 + (y.^2)./(b3(j)^2) ).*a3(t)^2) + a3(t); % Estimate the value ...
                of x for given q,e and y
            std_est3 = [std_est3, std(x - (x_buf3- a3(t)))]; % estimate standard deviation
            x_est3 = [x_est3,x_buf3']; % store the estimated x
            para_lst3 = [ para_lst3, [a3(t);b3(j)] ]; % store parameters a and b in a list
        end
    end
end

%% Plot figure for 50 samples, 3 runs case
figure(1)
hold on
for k = 1:length(a1)
        plot(x_est1(:,k) - a1(:,k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo search: 50 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

figure(2)
hold on
for k = 1:length(a2)
        plot(x_est2(:,k) - a2(:,k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo search: 100 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

figure(3)
hold on
for k = 1:length(a3)
        plot(x_est3(:,k) - a3(:,k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo search: 500 samples, 3 runs');
xlabel('x [AU]');
```

```
ylabel('y [AU]');
hold off

%% FINDING OPTIMUM PARAMETERS
% Finding the index for minimum standard deviation
index1 = find(std_est1 == min(std_est1));
index2 = find(std_est2 == min(std_est2));
index3 = find(std_est3 == min(std_est3));


% Finding the optimum parameter combination for minimum standard deviation
opti_a1b1 =  para_lst1(:,index1);
opti_a1b2 =  para_lst2(:,index2);
opti_a1b3 =  para_lst3(:,index3);


%% PLOT X AND Y FOR OPTIMUM VALUES OF A AND B
figure(11)
plot(x_est1(:,index1) + para_lst1(1,index1),y','r-x',x,y,'k-o');
grid on
legend('Optimum combination of a and b','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y[AU]');
title('Optimum combination of a and b, 50 samples each, 3 runs');

figure(22)
plot(x_est2(:,index2) + para_lst2(1,index2),y','r-x',x,y,'k-o');
grid on
legend('Optimum combination of a and b','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y[AU]');
title('Optimum combination of a and b, 100 samples each, 3 runs');

figure(33)
plot(x_est3(:,index3) + para_lst3(1,index3),y','r-x',x,y,'k-o');
grid on
legend('Optimum combination of a and b','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y[AU]');
title('Optimum combination of a and b, 500 samples each, 3 runs');
```

Following is the script used for grid search of Hyperbolic orbit:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% AE4878 Mission Geometry and Orbit Design %%%%%%%%%%%
%%%%% Assignment 4 - Week 5 - OPTIMIZATION - Version 1 %%%
%%%%% Author Info: Ali Nawaz; Student ID - 4276477 %%%%%%%
%%%%% Assigned tasks : OPTIM_5a, QUESTION 6 %%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% GRID SEARCH FOR HYPERBOLIC PARAMETER ESTIMATION, PARAMETERS : semi transverse axis a and ...
    semi-conjugate axis b
% close all; clear all;
x = [ 0.00 0.02 0.08 0.20 0.33 0.50 0.68 0.88 1.13 1.36 1.58 1.82 2.03 2.32 2.58 2.87 3.07 ...
    3.25]; % Distance in AU from slide 28
y = [ 0.00 0.11 0.19 0.27 0.34 0.42 0.49 0.55 0.62 0.68 0.73 0.79 0.84 0.89 0.92 0.97 1.02 ...
    1.05]; % Distances in AU from slide 28

uba = 0.5; % Upper bound on a, semi transverse axis
lba = -0.5; % Lower bound on a, semi conjugate a

ubb = sqrt(0.240); % Upper bound on b
lbb = 0; % Lower bound on b
% Store the estimations of x and corresponding standard deviations

% For 50 samples
x_est1 = [];
std_est1 = [];

% For 100 samples
x_est2 = [];
std_est2 = [];

% For 500 samples
x_est3 = [];
```

```
std_est3 = [];

% List of semi major axis a for sample size 50, 100 and 500
a1_lst = [];
a2_lst = [];
a3_lst = [];

% List of semi conjugagte axis b for sample size 50, 100 and 500
b1_lst = [];
b2_lst = [];
b3_lst = [];

% Storing a and b parameters for each calculation
para_lst1 =[]; % 50 samples
para_lst2 = [];% 100 samples
para_lst3 = [];% 500 samples

% GRID SEARCH NO MULTIPLE RUNS REQUIRED

% Grid semi major axis distance generation
a1 = lba:((uba-lba)/(50-1)):uba; % 50 samples
a2 = lba:((uba-lba)/(100-1)):uba; % 100 samples
a3 = lba:((uba-lba)/(500-1)):uba; % 500 samples

% Grid semi conjugate axis generation
b1 = lbb:((ubb-lbb)/(50-1)):ubb; % 50 samples
b2 = lbb:((ubb-lbb)/(100-1)):ubb; % 100 samples
b3 = lbb:( (ubb - lbb)/(500-1)):ubb; % 500 samples

disp('Running the set of  50 samples...');

for t = 1:length(a1)
    for j = 1:length(b1)
        x_buf1 = sqrt( (1 + (y.^2)./(b1(j)^2) ).*a1(t)^2) + a1(t); % Estimate the value of x ...
            for given q,e and y
        std_est1 = [std_est1, std(x - (x_buf1- a1(t)))]; % estimate standard deviation
        x_est1 = [x_est1,x_buf1']; % store the estimated x
        para_lst1 = [ para_lst1, [a1(t);b1(j)] ]; % Store parameters a and b in a list
    end
end
disp('Running the set of  100 samples...');
for t = 1:length(a2)
    for j = 1:length(b2)
        x_buf2 = sqrt( (1 + (y.^2)./(b2(j)^2) ).*a2(t)^2) + a2(t); % Estimate the value of x ...
            for given q,e and y
        std_est2 = [std_est2, std(x - (x_buf2- a2(t)))]; % estimate standard deviation
        x_est2 = [x_est2,x_buf2']; % store the estimated x
        para_lst2 = [ para_lst2, [a2(t);b2(j)] ]; % Store parameters a and b in a list
    end
end
disp('Running the set of 500 samples');
for t = 1:length(a3)
    for j = 1:length(b3)
        x_buf3 = sqrt( (1 + (y.^2)./(b3(j)^2) ).*a3(t)^2) + a3(t); % Estimate the value of x ...
            for given q,e and y
        std_est3 = [std_est3, std(x - (x_buf3- a3(t)))]; % estimate standard deviation
        x_est3 = [x_est3,x_buf3']; % store the estimated x
        para_lst3 = [ para_lst3, [a3(t);b3(j)] ]; % store parameters a and b in a list
    end
end


%% Plot figure for 50 samples, 3 runs case
figure(1)
hold on
for k = 1:length(a1)
        plot(x_est1(:,k) - a1(:,k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo search: 50 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

figure(2)
```

```
hold on
for k = 1:length(a2)
        plot(x_est2(:,k) - a2(:,k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo search: 100 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

figure(3)
hold on
for k = 1:length(a3)
        plot(x_est3(:,k) - a3(:,k),y');
end
plot(x,y,'k-o');
grid on
title('Monte Carlo search: 500 samples, 3 runs');
xlabel('x [AU]');
ylabel('y [AU]');
hold off

%% FINDING OPTIMUM PARAMETERS
% Finding the index for minimum standard deviation
index1 = find(std_est1 == min(std_est1));
index2 = find(std_est2 == min(std_est2));
index3 = find(std_est3 == min(std_est3));


% Finding the optimum parameter combination for minimum standard deviation
opti_a1b1 =  para_lst1(:,index1);
opti_a1b2 =  para_lst2(:,index2);
opti_a1b3 =  para_lst3(:,index3);



%% PLOT X ADN Y FOR OPTIMUM VAUES OF A AND B
figure(11)
plot(x_est1(:,index1(1,1)) - para_lst1(1,index1(1,1)),y','r-x',x,y,'k-o');
grid on
legend('Optimum combination of a and b','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y[AU]');
title('Optimum combination of a and b, 50 samples each, 3 runs');

figure(22)
plot(x_est2(:,index2(1,1)) - para_lst2(1,index2(1,1)),y','r-x',x,y,'k-o');
grid on
legend('Optimum combination of a and b','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y[AU]');
title('Optimum combination of a and b, 100 samples each, 3 runs');

figure(33)
plot(x_est3(:,index3(1,1)) - para_lst3(1,index3(1,1)),y','r-x',x,y,'k-o');
grid on
legend('Optimum combination of a and b','Observation','Location','best');
xlabel('X [AU]');
ylabel('Y[AU]');
title('Optimum combination of a and b, 500 samples each, 3 runs');
```

Following is the script for quality vs time comparison of hyperbolic orbit:

```
%% COMPARING MONTE CARLO AND GRID SEARCH COMPUTATIONAL EFFORT VS QUALITY
%no. of samples
samples = [ 50 100 500 ];

%no. of calculations, m = Monte Carlo, g = grid search
calc_m = 3.*samples;
calc_g = samples;

%corresponding standard deviations
std_m = [0.2344 0.2311 0.2303];
std_g = [ 0.2302 0.2306 0.2302];
```

```

% Plot the results
figure(41)
plot(calc_m,std_m,'r-x',calc_g,std_g,'k-o');
grid on
legend('Monte Carlo','Grid Search');
ylabel('Standard deviation [AU]');
xlabel('No. of calculations');
title('Quality vs Time plot');
```

References

[1] R. Noomen. *AE4878 Mission Geometry and Orbit Design*. TU Delft, 2017.

[2] James R. Wertz. *Orbit Constellation Design Management*. Springer, 2009.