(Lecture #16)

# :- Analysis and Characterization of LTI systems Using Laplace Transform :-

$$y(t) = x(t) * h(t)$$

$$Y(s) = X(s) \cdot H(s)$$

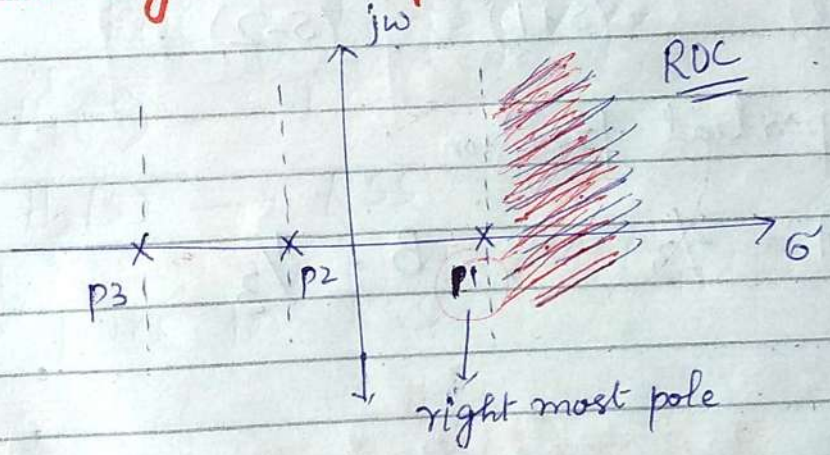
$$H(s) = \frac{Y(s)}{X(s)}$$

$H(s) \Rightarrow$ System transfer function

## ① Causality :-

For an LTI system to be causal :

"For a system with rational system function, causality of the system is equivalent to the ROC being the right half plane to the right of the right most pole."

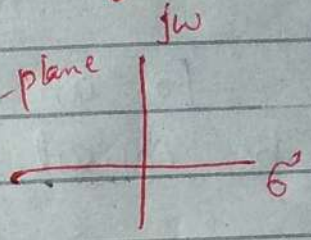



ROC : $Re\{s\} > p_1$

## ② Stability :-

ROC of H(s) can also be related to the stability of a system.

An LTI system is stable if and only if the ROC of its system function H(s) includes the jω-axis.




**Exp 9.20**

$$H(S) = \frac{S-1}{(S+1)(S-2)}$$

$$\frac{S-1}{(S+1)(S-2)} = \frac{A}{(S+1)} + \frac{B}{(S-2)}$$

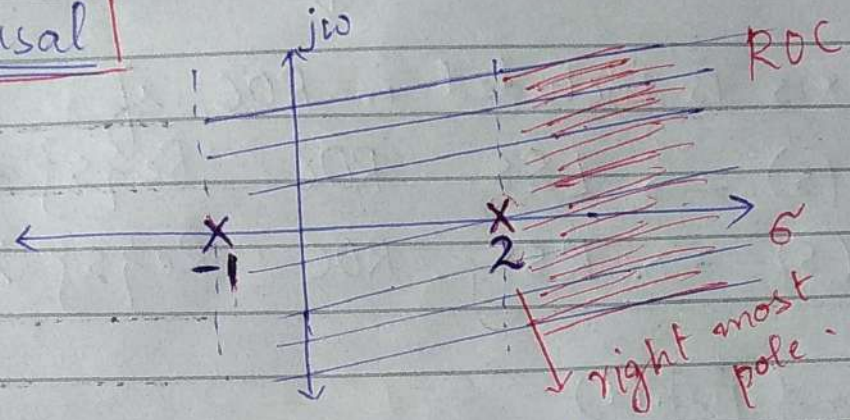
After partial fraction

$$A = 2/3 \quad , \quad B = 1/3$$

$$H(s) = \underbrace{\frac{2/3}{s+1}}_{H_1(s)} + \underbrace{\frac{1/3}{s-2}}_{H_2(s)}$$ (3)

poles $s+1=0 \Rightarrow s=-1$ $\quad$ $s-2=0 \Rightarrow s=2$

(1) If Causal




For this system to be causal both $H_1(s)$ and $H_2(s)$ should have ROC right sided.

$\therefore$ ROC : $Re\{s\} > 2$

$H_1(s) \longrightarrow$ RSS

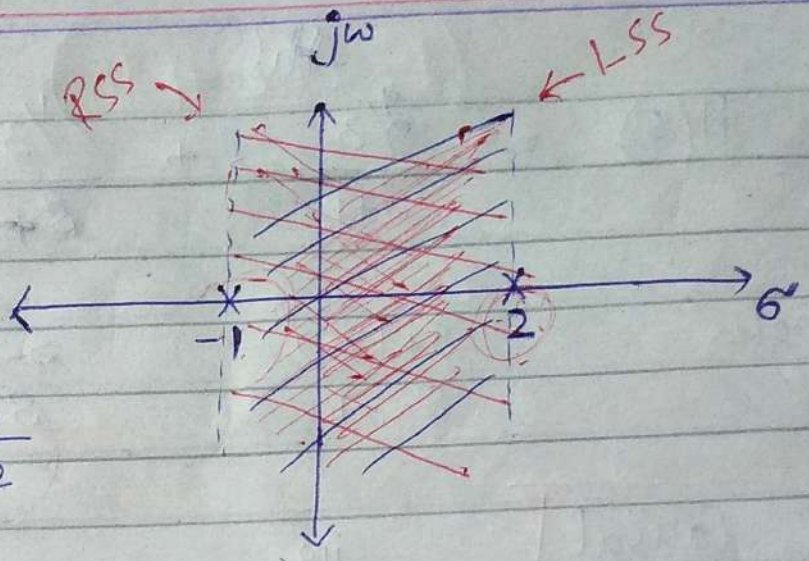
$H_2(s) \longrightarrow$ RSS

Use pairs for RSS $\quad \frac{2}{3}\cdot\frac{1}{s+1} \quad \frac{1}{3}\cdot\frac{1}{s-2}$

$$h(t) = \underbrace{\frac{2}{3}e^{-t}u(t)}_{RSS} + \underbrace{\frac{1}{3}e^{+2t}u(t)}_{RSS}$$

system is causal and unstable.

(2) **Stable :-**




$H_1(s)$ $H_2(s)$

$$H(s) = \frac{2/3}{s+1} + \frac{1/3}{s-2}$$

jω-axis is included in ROC if

$H_2(s) \longrightarrow$ LSS ROC : $Re\{s\} < 2$

$H_1(s) \longrightarrow$ RSS ROC : $Re\{s\} > -1$

To find $h(t)$

$$\frac{2}{3}\frac{1}{s+1} \xrightarrow{RSS} \frac{2}{3} e^{-t} u(t)$$

($u(t)$ : RSS)

$$\frac{1}{3}\frac{1}{s-2} \xrightarrow{LSS} \frac{1}{3}\left[-e^{2t} u(-t)\right]$$

$$\therefore h(t) = \frac{2}{3} e^{-t} u(t) - \frac{1}{3} e^{2t} u(-t)$$

System is stable and non-causal.

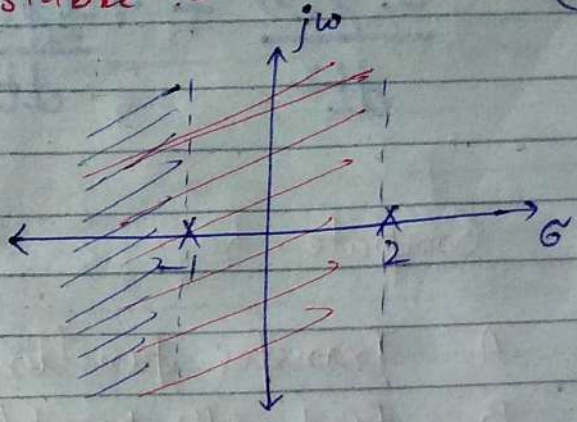
LSS+RSS

→ LSS

(3) Anti-causal and Unstable :- (Non-Causal)

(5)

$$H(s) = \frac{2/3}{s+1} + \frac{1/3}{s-2}$$




ROC : $Re\{s\} < -1$

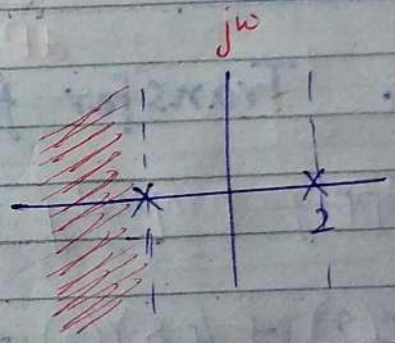
$$2/3 \quad \frac{1}{s+1} \xrightarrow{LSS} \frac{2}{3}\left[-e^{-t}u(-t)\right]$$

$$1/3 \quad \frac{1}{s-2} \xrightarrow{LSS} 1/3\left[-e^{2t}u(-t)\right]$$

$$h(t) = -\frac{2}{3}e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$$

(4) Anti Causal and Stable (Non-Causal)

Not possible




(5) Causal and Stable

Not possible