LEC-22

Analysis & Characterization of LTI Systems using Z-Transform :-

$$Y(z) = H(z) \cdot X(z) \overset{z}{\rightleftharpoons} y[n] = h[n] * x[n]$$

&

$$H(z) = \frac{Y(z)}{X(z)}$$

$Y(z)$ ← z-trans of system output

$X(z)$ ← z-trans of system input

z-transform of Impulse response → $H(z)$

Causality

A discrete-time LTI system is causal if and only if the ROC of its system function is the exterior of the circle, including infinity.

We also know $x[n]$ = causal if $x(n)=0$ for $n<0$

RSS ←

so $H(z)$ is causal if all individual components are RSS.

Two conditions:

① ROC being outside the ~~outside~~ outermost pole.

② Order of numerator can't be greater than order of the denominator when $H(z)$ expressed as ratio of polynomials of z.

→ However, both can have same order.

Example 10.20

$$H(z) = \frac{z^3 - 2z^2 + z}{z^2 + \frac{1}{4}z + \frac{1}{8}}$$

→ Order of Numer = 3

→ Order of Denom. = 2

if $H(z)$ expressed like this & it is required to tell whether the system is causal ?

We will say its not causal became

Example 10.21

Given that $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}$ $|z| > 2$

$$H(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-2} \qquad |z| > 2$$

(Numerator & Denominator have same order = 1)

pole 1 $\Rightarrow$ $z - \frac{1}{2} = 0 \Rightarrow z = \frac{1}{2}$

pole 2 $\Rightarrow$ $z - 2 = 0 \Rightarrow z = 2$

& given that $|z| > 2$

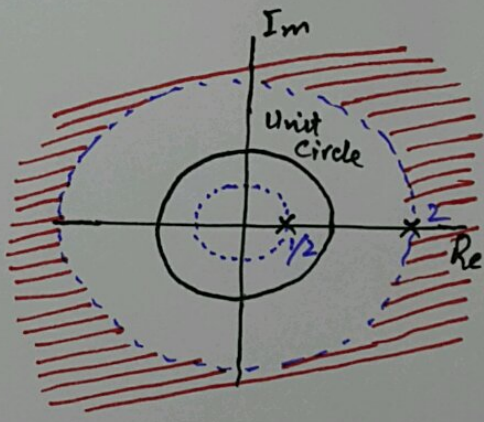
so both are RSS.

and therefore $H(z)$ is causal.

~~h[n]~~

$$\frac{1}{1-\frac{1}{2}z^{-1}} \xrightarrow[\text{RSS}]{z^{-1}} \left(\frac{1}{2}\right)^n u[n]$$

$$\frac{1}{1-2z^{-1}} \xrightarrow[\text{RSS}]{z^{-1}} (2)^n u[n]$$



Hence

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[n]$$

$$\boxed{h[n] = \left[\left(\frac{1}{2}\right)^n + (2)^n\right] u[n]}$$

Stable = NO

Causal = YES (all RSS)

Anti-Causal = NO

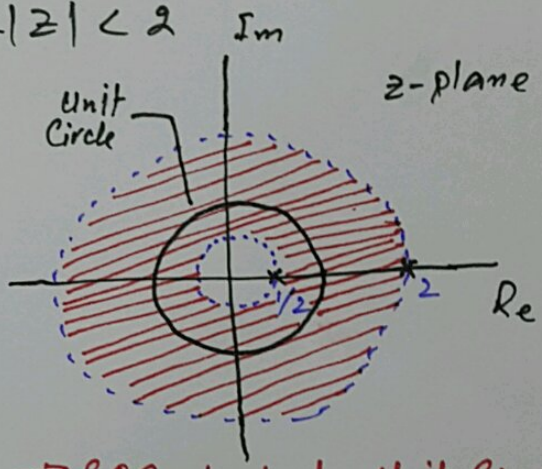
Stability :

An LTI system is stable if and only if ROC of $H(z)$ includes the unit circle.

(1) $$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}, \quad \frac{1}{2} < |z| < 2$$

$\frac{1}{1-\frac{1}{2}z^{-1}}$, $|z| > \frac{1}{2}$ (RSS)

$\frac{1}{1-2z^{-1}}$, $|z| < 2$ (LSS)



Stable = YES
Causal = NO
Anti-causal = NO

→ ROC include Unit Circle System is Stable.
→ Not Causal, since one of fn. is LSS.

(2) $$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}$$

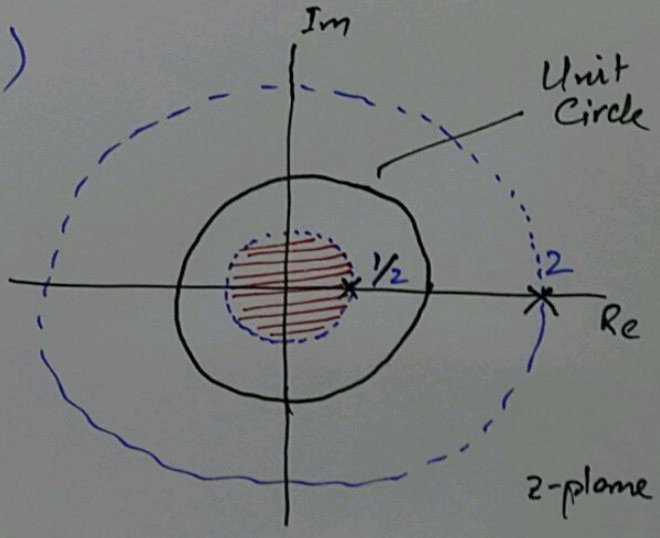
$|z| < \frac{1}{2}$

↓

Both are LSS (inside of innermost pole)

$\frac{1}{1-\frac{1}{2}z^{-1}}$, $|z| < \frac{1}{2}$ (LSS)

$\frac{1}{1-2z^{-1}}$, $|z| < 2$ (LSS)



Stable = NO
Causal = NO
Anti Causal = YES (all LSS)