Applied Physics for Engineers

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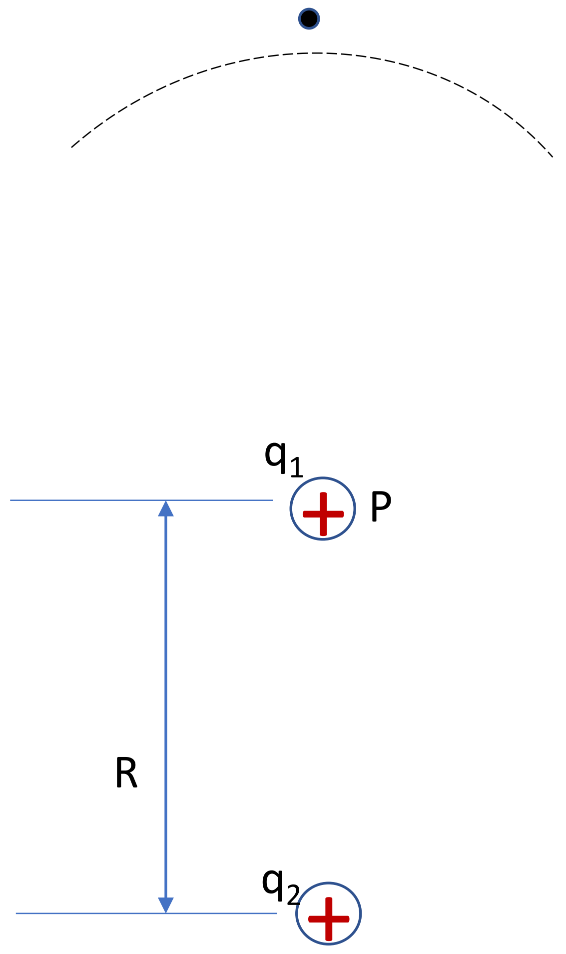
Electric potential, electric potential energy, equipotential surfaces, calculating potential from the field. Potential due to a point charge

Electric potential energy

- A charge q_2 is placed at infinity, outside the field of charge q_1
- To bring q_2 at point P some work needs to be done on the charge q_2
- This work done is stored in the amount of electric potential energy which is given by

$$U = -W_\infty$$

where W_∞ is now the work done by the electric force to bring the charge from infinity



- The work and thus the potential energy can be positive or negative depending on the sign of the rod’s charge.
- Electric potential energy is a scalar quantity having unit of joules
- This energy is conservative

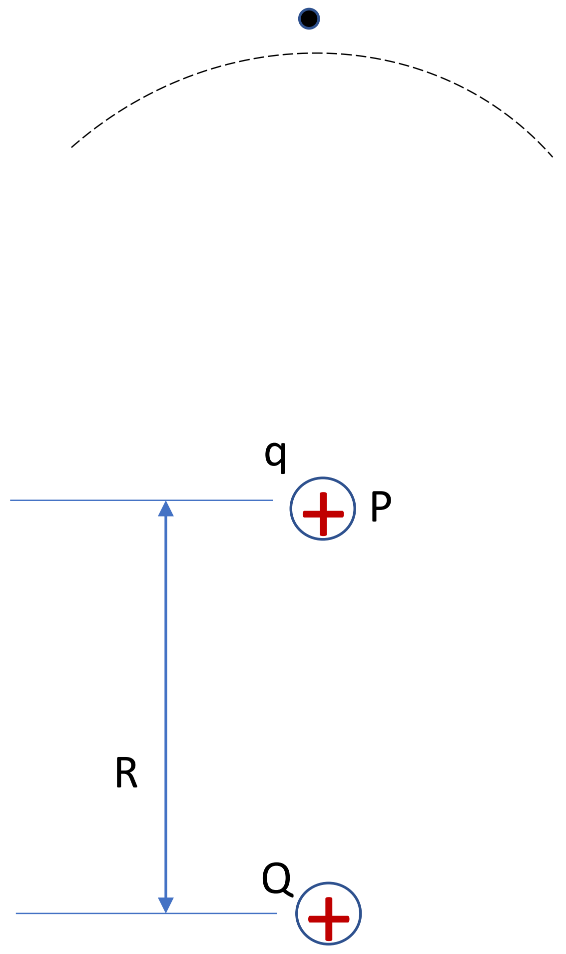
“Electric potential energy is the energy possessed by a charge in an electric field” .

Electric potential

"electric potential is the electric potential energy per unit charge"

$$V = \frac{-W_{\infty}}{q} = \frac{U}{q}$$

- We can say that electric potential is the amount of electric potential energy per unit charge when a positive test charge is brought in from infinity
- The charge Q sets up this potential V at P regardless of whether the charge q (or anything else) happens to be there



- Like electric potential energy ‘U’ electric potential is also a scalar quantity and can be positive or negative
- Electric potential is set up at every point in the charge’s electric field. We can say that every charged object sets up electric potential *V* at points throughout its electric field
- If we place a particle with, say, charge *q* at a point where electric potential due to charge Q is known then we can measure the electric potential energy

$$U = qV$$

Unit of electric potential is joules/coulomb or volts

Electric potential difference

- ***Change in Electric Potential.*** If we move from an initial point *i* to a second point *f* in the electric field of a charged object, the electric potential changes by

$$\Delta V = V_f - V_i$$

- If we move a particle with charge q from i to f, then, the potential energy of the system changes by

$$\Delta U = q\,\Delta V = q(V_f - V_i)$$

The change can be positive or negative, depending on the signs of q and ΔV. It can also be zero, if there is no change in potential from i to f (the points have the same value of potential).

Conservation of Energy.

- If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved.
- The conservation of mechanical energy of the particle that moves from point i to point f can be written as

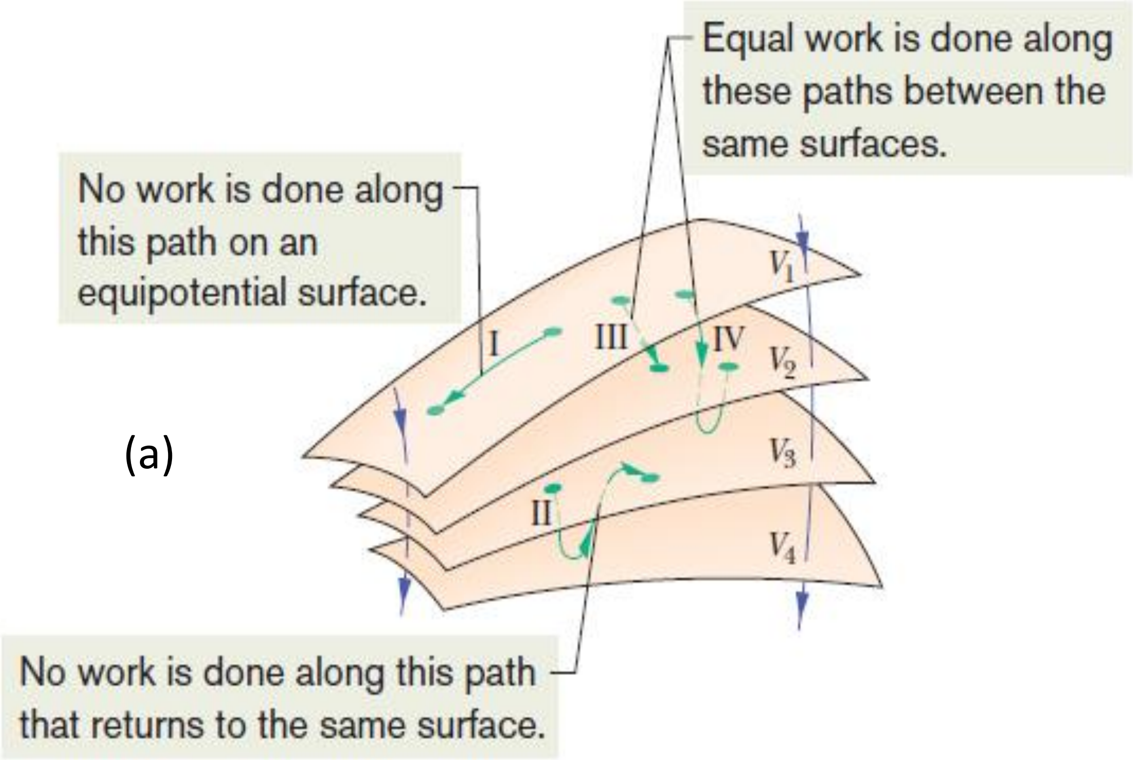
$$U_i + K_i = U_f + K_f,$$

$$\Delta K = -\Delta U.$$

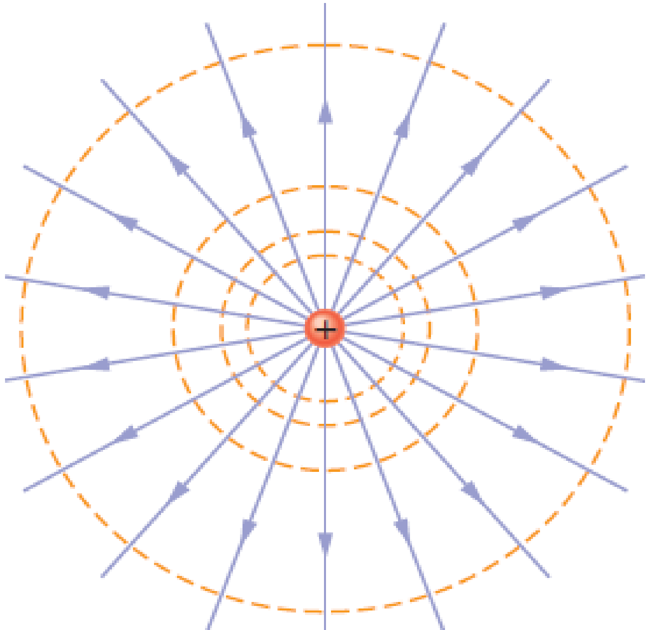
$$\Delta K = -q\,\Delta V = -q(V_f - V_i).$$

Equipotential Surfaces

- Adjacent points that have the same electric potential form an **equipotential surface,** which can be either an imaginary surface or a real, physical surface. No net work W is done on a charged particle by an electric field when the particle moves between two points i and f on the same equipotential surface.
- Because of the path independence of work (and thus of potential energy and potential), W=0 for *any* path connecting points i and f on a given equipotential surface regardless of whether that path lies entirely on that surface.
- For a uniform electric field, the surfaces are a family of planes perpendicular to the field lines. In fact, equipotential surfaces are always perpendicular to electric field lines and thus to **E**, which is always tangent to these lines.



(a)



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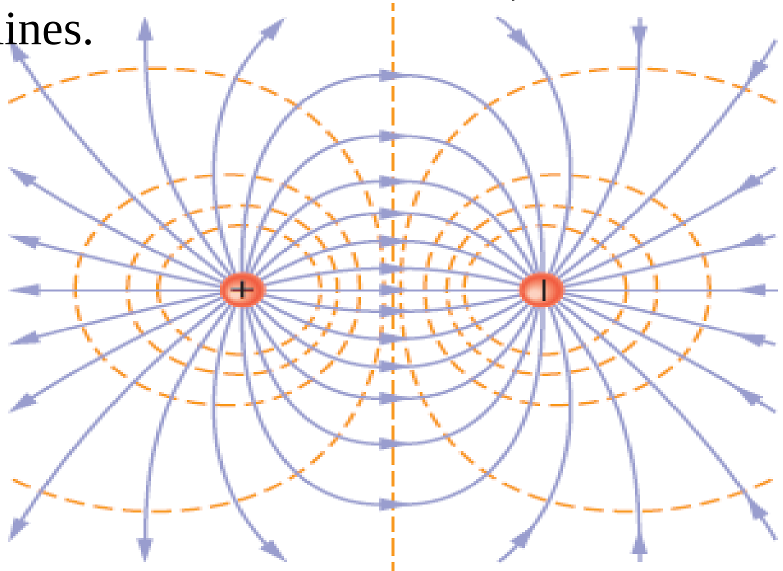


(c)

Calculating the Potential from the Field

We can calculate the potential difference between any two points *i* and *f* in an electric field if we know the electric field **E** vector all along any path connecting those points.

Consider an arbitrary electric field, represented by the field lines in Fig below, and a positive test charge q_0 that moves along the path shown from point *i* to point *f*. At any point on the path, an electric force q_0**E** acts on the charge as it moves through a differential displacement d**s**

The differential work *dW* done on a particle by a force during a displacement d**s** is given by the dot product of the force **F** and the displacement d**s** .

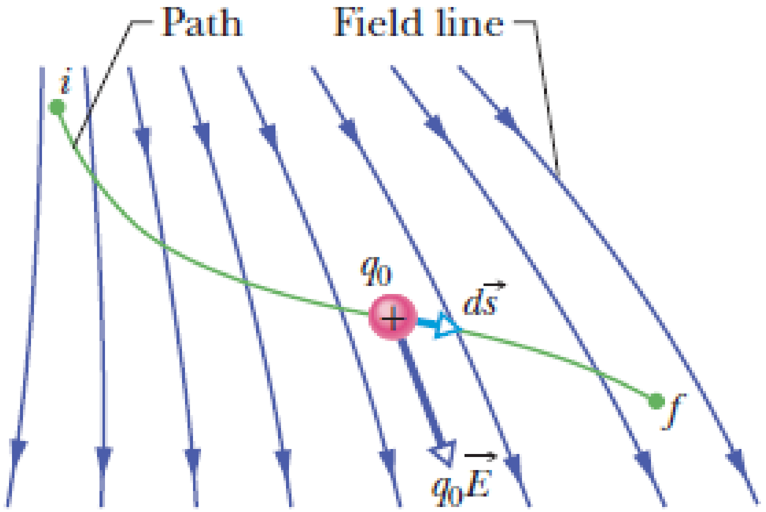
$$dW = \vec{F} \cdot d\vec{s}.$$

$$\vec{F} = q_0\vec{E}$$

$$dW = q_0\vec{E} \cdot d\vec{s}.$$

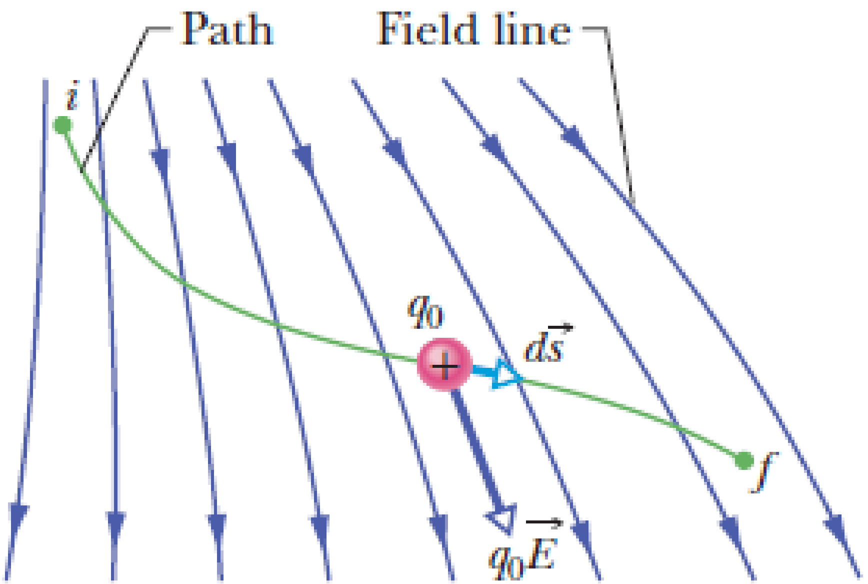
$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}.$$

$$\therefore \text{W}=-\Delta U = -q_0\Delta V = -q_0(V_f - V_i) = q_0(V_i - V_f)$$



$$V_i - V_f = \int_i^f \boldsymbol{E}.d\boldsymbol{s} \qquad \text{(i)}$$

$$V_f - V_i = -\int_i^f \boldsymbol{E}.d\boldsymbol{s} \qquad \text{(ii)}$$



If the line integral $\int_i^f \boldsymbol{E}.d\boldsymbol{s}$ is positive, the electric field does positive work on a positive test charge as it moves from i (the point of higher potential) to f (the point of lower potential). Or, for the term $V_i - V_f$ we can also say that the potential of i with respect to f is high.

If we set potential V_i= 0, then (ii) can be written as

$$V = -\int_i^f \boldsymbol{E}.d\boldsymbol{s}$$

24.6 Potential Due to a Point Charge:

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Consider a point P at distance R from a fixed particle of positive charge q. Imagine that we move a positive test charge q_0 from point P to infinity. The path chosen can be the simplest one— a line that extends radially from the fixed particle through P to infinity.

$$V_f - V_i = -\int_R^\infty E\,dr.$$

If $V_f=0$ (at ∞) and $V_i=V$ (at R). Then, for the magnitude of the electric field at the site of the test charge,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

That gives:

$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}.$$

Switching R to r,

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

To find the potential of the charged particle, we move this test charge out to infinity.

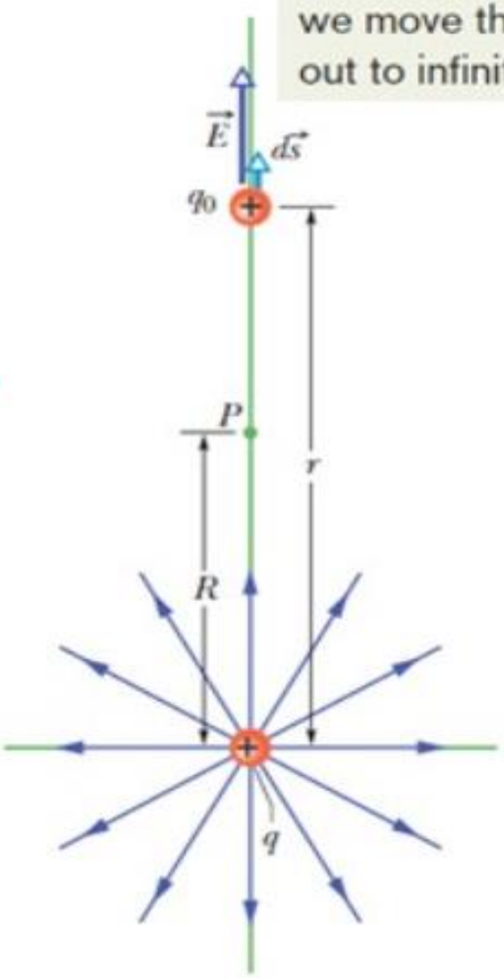


Fig. 24-6 The positive point charge q produces an electric field $\vec{E}$ and an electric potential V at point P. We find the potential by moving a test charge q_0 from P to infinity. The test charge is shown at distance r from the point charge, during differential displacement $d\vec{s}$.

24.7 Potential Due to a Group of Point Charges:

The net potential at a point due to a group of point charges can be found with the help of the superposition principle. First the individual potential resulting from each charge is considered at the given point. Then we sum the potentials.

For n charges, the net potential is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \qquad (n \text{ point charges}).$$

Electric potential difference

- ***Change in Electric Potential.*** If we move from an initial point *i* to a second point *f* in the electric field of a charged object, the electric potential changes by

$$\Delta V = V_f - V_i$$

$$\Delta V = k\frac{Q}{r_f} - k\frac{Q}{r_i}$$

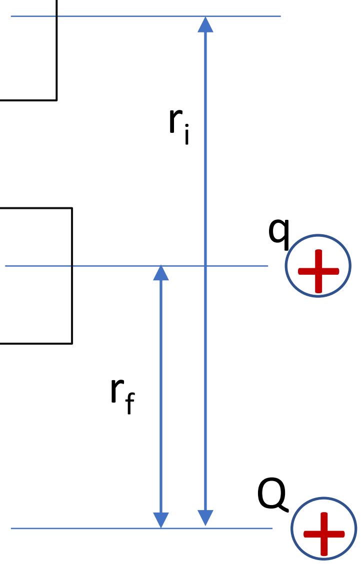
$$\Delta V = kQ\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

- If we move a charge q form point i to f, the potential energy of the system will be,

$$\Delta U = q\,\Delta V = q(V_f - V_i)$$

Lower electric potential energy and Lower electric potential

Higher electric potential energy and higher electric potential



$U_i = k\frac{Qq}{r_i}$

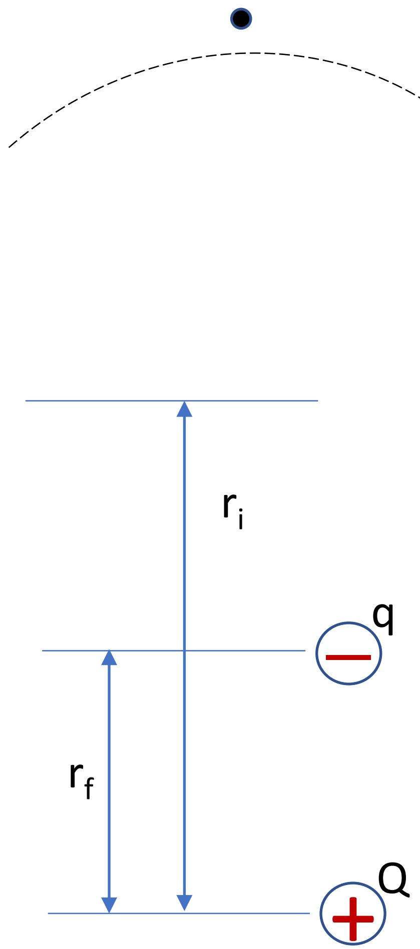
$U_f = k\frac{Qq}{r_f}$

Electric potential difference

Higher electric potential energy and lower electric potential

$$U_i = -k\frac{Qq}{r_i}$$

$$V_i = k\frac{Q}{r_i}$$



Lower electric potential energy and higher electric potential

$$U_f = -k\frac{Qq}{r_f}$$

$$V_f = k\frac{Q}{r_f}$$