# Applied Physics for Engineers

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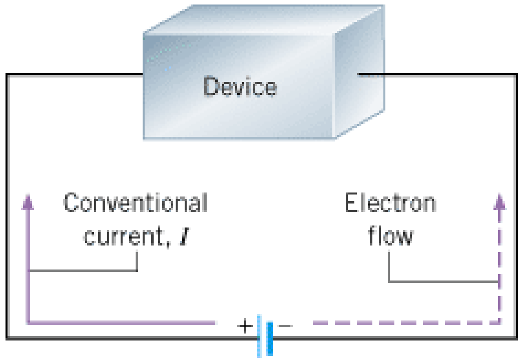
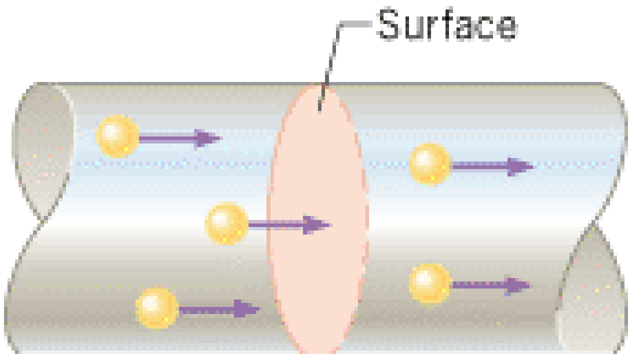
Electric Current, Current density, Ohm’s Law, Resistivity

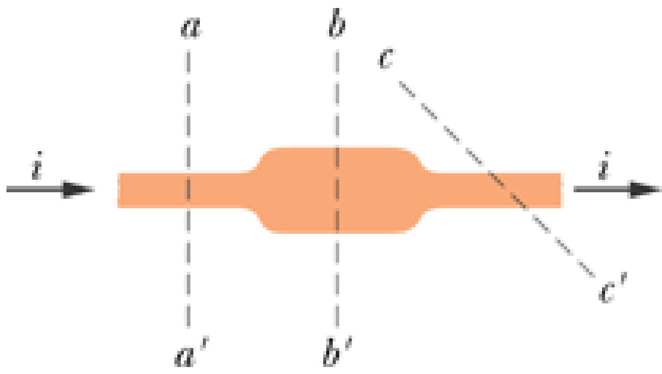
# Electric Current




The electric current is the amount of charge per unit time that passes through a surface area.

$$i = \frac{dq}{dt} \quad \text{(definition of current).}$$

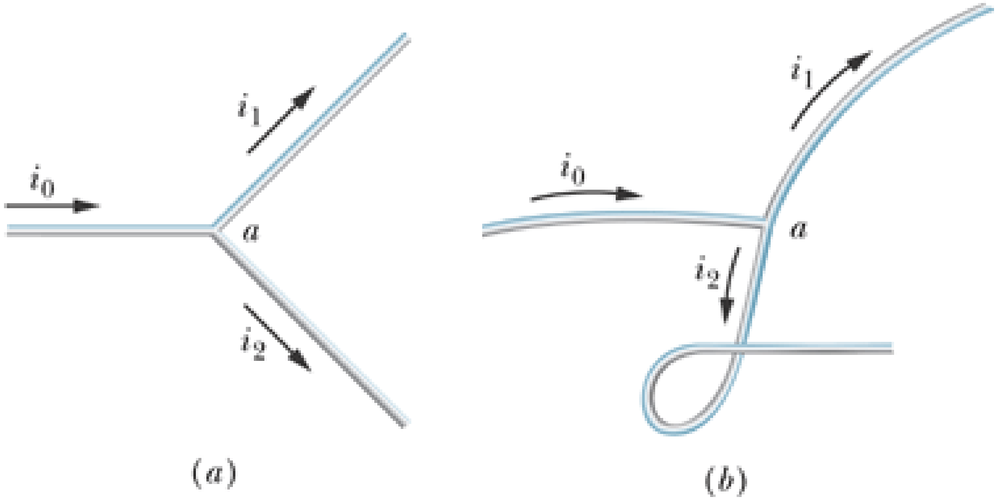


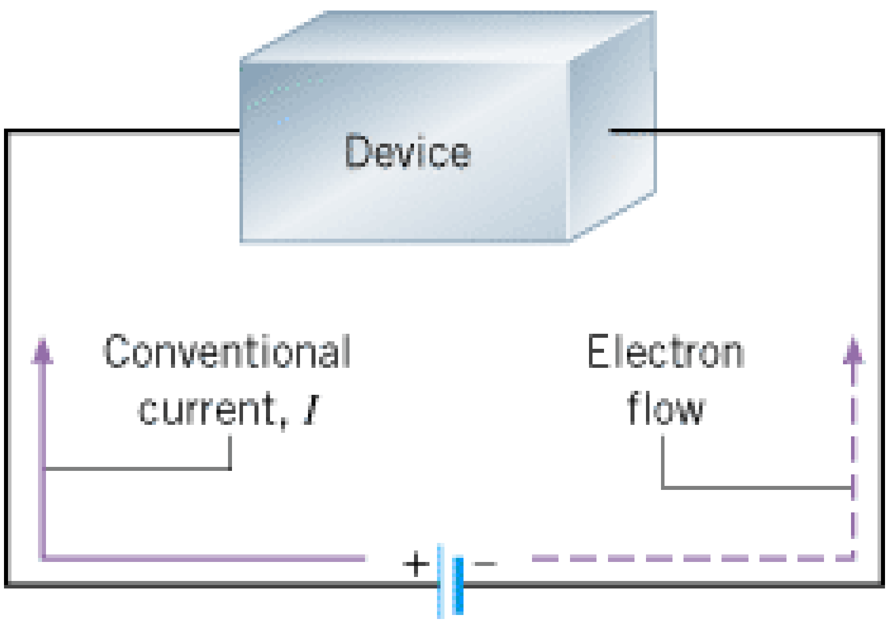

The charge that passes through the plane in a time interval extending from 0 to *t* by integration

$$q = \int dq = \int_0^t i\, dt,$$

The SI unit for current is a coulomb per second (C/s), called as an ***ampere*** (A)

# Direction of current



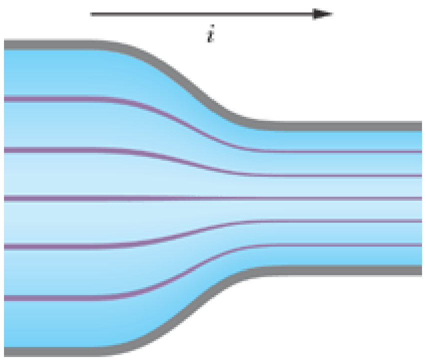

- Current is a scalar quantity.
- As charge is a conserved quantity so current also remains conserved

$$i_0 = i_1 + i_2.$$

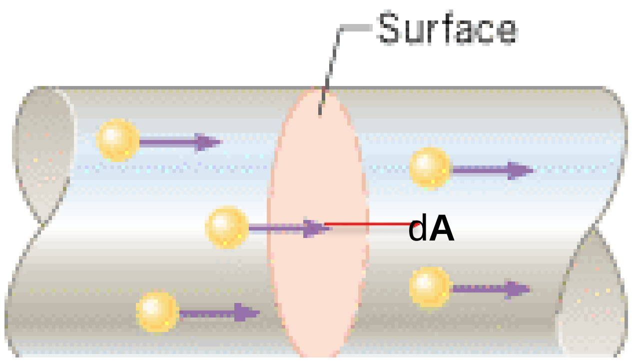
- A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.
- The direction of conventional current is always from a point of higher potential toward a point of lower potential—that is, from the positive toward the negative terminal.

# Current Density $\vec{J}$

- Current density is to study the flow of charge through a cross section of the conductor at a particular point
- It is a vector which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.
- The magnitude of *J* is equal to the current per unit area through that area element (d**A** is the area vector)




$$\mathbf{J} = \frac{di}{dA}$$

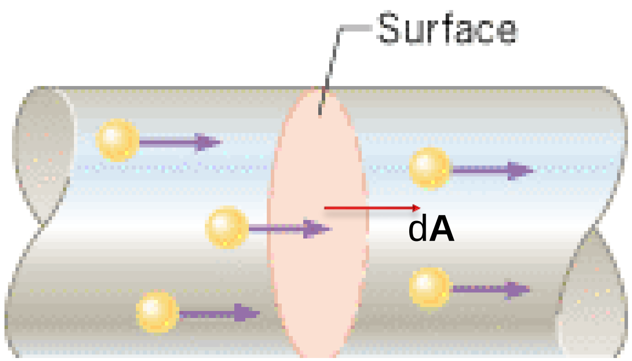


- If the current is uniform across the surface and parallel to d**A** then **J** is also uniform and parallel to d**A**

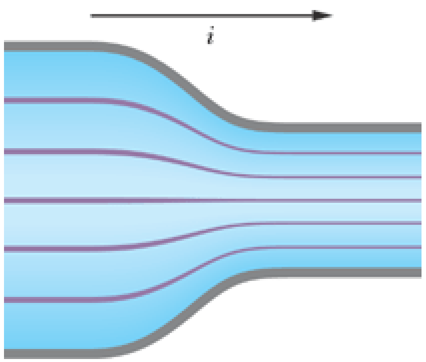
$$i = \int di = \int \boldsymbol{J} . d\boldsymbol{A}$$

$$= J \int dA = JA$$

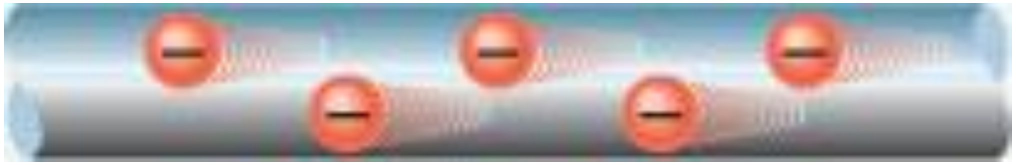
$$J = \frac{i}{A}$$




- where $A$ is the total area of the surface
- the SI unit for current density is the ampere per square meter (A/m$^2$).
- Because charge is conserved during the transition, the amount of charge and thus the amount of current cannot change. However, the current density does change—it is greater in the narrower conductor. The spacing of the streamlines suggests this increase in current density; streamlines that are closer together imply greater current density.



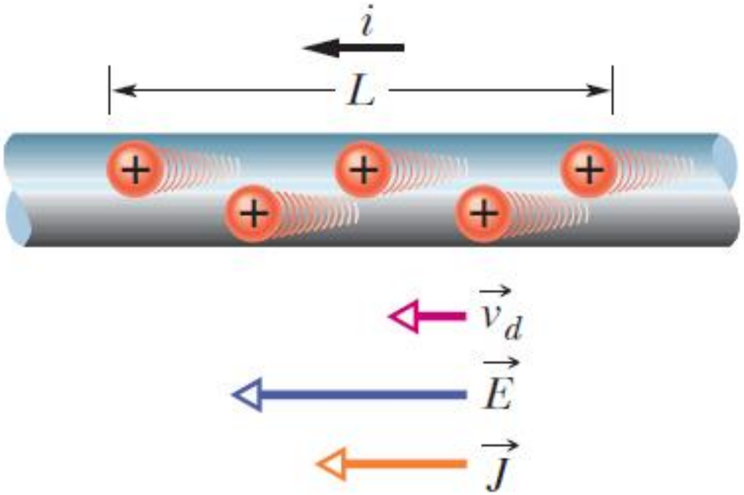
# Drift Speed



When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to *drift* with a **drift speed** $v_d$ in the direction opposite that of the applied electric field that causes the current

*"Drift velocity is the velocity attained by the charged particles, such as electrons, in a material due to an electric field"*

The drift speed is tiny compared with the speeds in the random motion. For example, in the copper conductors of household wiring, electron drift speeds are perhaps $10^{-5}$ or $10^{-4}$ m/s, whereas the random-motion speeds are around $10^6$ m/s.




- We will relate the drift speed $v_d$ of the conduction electrons in a current through a wire to the magnitude $J$ of the current density in the wire.

- Let us assume that the charge carriers in the wire all move with the same drift speed $v_d$ and that the current density $J$ is uniform across the wire's cross-sectional area $A$. The number of charge carriers in a length $L$ of the wire is $nAL$, where $n$ is the number of carriers per unit volume. The total charge of the carriers in the length $L$, each with charge $e$, is then

$$q = (nAL)e.$$

- Because the carriers all move along the wire with speed $v_d$, this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

- the current $i$ is the time rate of transfer of charge across a cross section, so here we have

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d.$$

- Solving for $v_d$ and taking ($J = i/A$)

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

- In the vector form

$$\vec{J} = (ne)\vec{v}_d.$$

- Here the product *ne*, whose SI unit is the coulomb per cubic meter ($C/m^3$), is the *carrier charge density*

# Resistance and Resistivity

- We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$R = \frac{V}{i} \quad \text{(definition of } R\text{).}$$

- The SI unit for resistance is volt per ampere. This combination occurs so often that we give it a special name, the **ohm** that is,

$$\begin{aligned} 1 \text{ ohm} &= 1\ \Omega = 1 \text{ volt per ampere} \\ &= 1 \text{ V/A.} \end{aligned}$$

- As we have done several times in other connections, we often wish to take a general view and deal not with particular objects but with materials. Here we do so by focusing not on the potential difference V across a particular resistor but on the electric field **E** at a point in a resistive material. Instead of dealing with the current i through the resistor, we deal with the current density **J** at the point in question. Instead of the resistance R of an object, we deal with the resistivity ρ of the material:

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{).} \longrightarrow \vec{E} = \rho \vec{J}.$$

# Resistance and Resistivity

If we combine the SI units of E and J, we get, for the unit of ρ, the ohm-meter:

$$\frac{\text{unit }(E)}{\text{unit }(J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\,\text{m} = \Omega\cdot\text{m}.$$

And the conductivity of the material is simply the reciprocal of resistivity:

$$\sigma = \frac{1}{\rho} \quad \text{(definition of } \sigma\text{)}. \qquad \text{And} \qquad \vec{J} = \sigma\vec{E}.$$

## Calculating Resistance from Resistivity

Resistance is a property of an object. Resistivity is a property of a material.

If we know the resistivity of a substance such as copper, we can calculate the resistance of a length of wire made of that substance. Let *A* be the cross-sectional area of the wire, let *L* be its length, and let a potential difference *V* exist between its ends

$$E = V/L \quad \text{and} \quad J = i/A.$$

We can write

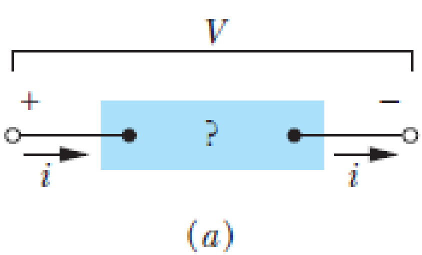
$$\rho = \frac{E}{J} = \frac{V/L}{i/A}. \longrightarrow R = \rho\frac{L}{A}.$$

# Ohm's Law

- ***Ohm's law*** *is an assertion that the current through a device is always directly proportional to the potential difference applied to the device.*
- *A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.*

We can generalize this for conducting materials by using Eq. 26-11 (**E** = ρ**J**):

- *A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.*



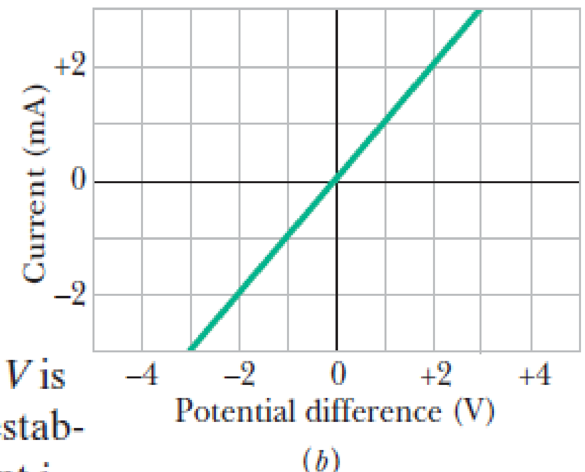
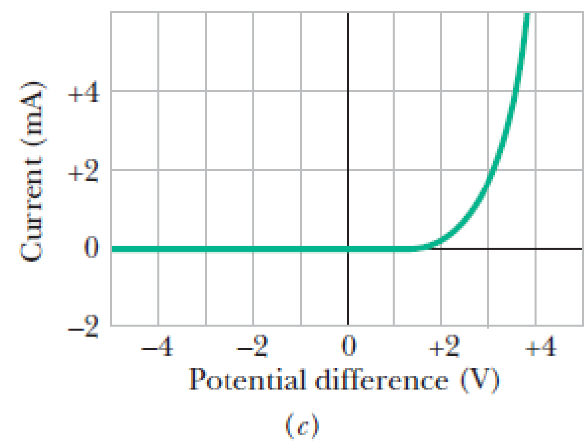




**Figure 26-11** (*a*) A potential difference $V$ is applied to the terminals of a device, establishing a current $i$. (*b*) A plot of current $i$ versus applied potential difference $V$ when the device is a 1000 Ω resistor. (*c*) A plot when the device is a semiconducting *pn* junction diode.



# Microscopic form of Ohm's Law

* It is often assumed that the conduction electrons in a metal move with a single effective speed $v_{eff}$, and this speed is essentially independent of the temperature. For copper, $v_{eff}=1.6 \times 10^6 m/s$.

* When we apply an electric field to a metal sample, the electrons modify their random motions slightly and drift very slowly—in a direction opposite that of the field—with an average drift speed $v_d$. The drift speed in a typical metallic conductor is about 5 x10$^{-7}$ m/s, less than the effective speed (1.6 x10$^6$ m/s) by many orders of magnitude.

* The motion of conduction electrons in an electric field is a combination of the motion due to random collisions and that due to ***E***.

* If an electron of mass *m* is placed in an electric field of magnitude *E*, the electron will experience an acceleration: $$a = \frac{F}{m} = \frac{eE}{m}.$$

* In the average time $\tau$ between collisions, the average electron will acquire a drift speed of $$v_d = a\tau = \frac{eE\tau}{m}.$$

* $\vec{J} = ne\vec{v}_d$ → $v_d = \dfrac{J}{ne} = \dfrac{eE\tau}{m}$ → $E = \left(\dfrac{m}{e^2 n\tau}\right) J.$ → $\boxed{\rho = \dfrac{m}{e^2 n\tau}.}$

(1)

Above equation may be taken as a statement that metals obey Ohm's law if we can show that, for metals, their resistivity is a constant, independent of the strength of the applied electric field. Let's consider the quantities in Eq. (1). We can reasonably assume that $n$, the number of conduction electrons per volume, is independent of the field, and $m$ and $e$ are constants. Thus, we only need to convince ourselves that t, the average time (or *mean free time*) between collisions, is a constant, independent of the strength of the applied electric field. Indeed, t can be considered to be a constant because the drift speed $v_d$ caused by the field is so much smaller than the effective speed *$v_{\mathrm{eff}}$ that the electron speed— and thus t—is hardly affected by the field. Thus, because the right side of Eq. (1) is independent of the field magnitude, metals obey Ohm's law.

* v effective is the speed of electrons without electric field