Lecture 8

Nodal Analysis with Voltage Sources

Supernodes

Objectives of Lecture

- Provide step-by-step instructions for nodal analysis, which is a method to calculate node voltages and currents that flow through components in a circuit.

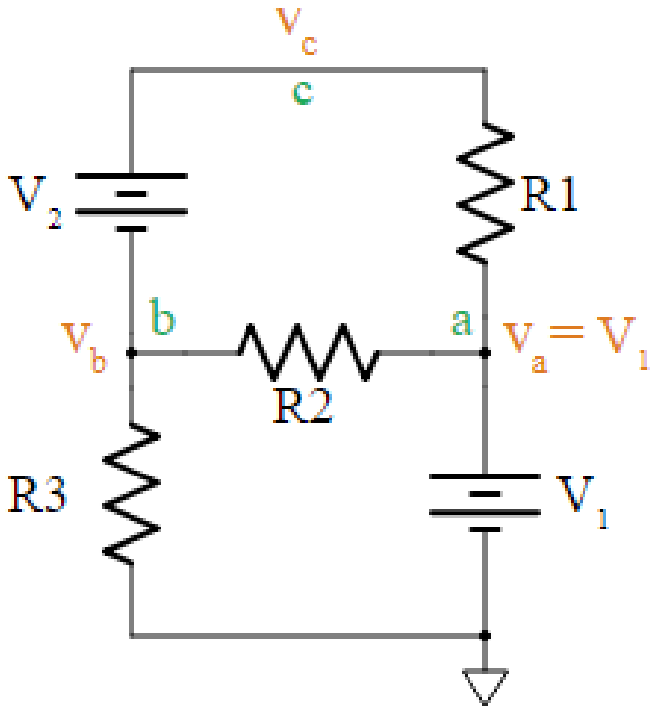
Nodal Analysis with Supernodes

- Floating voltage source

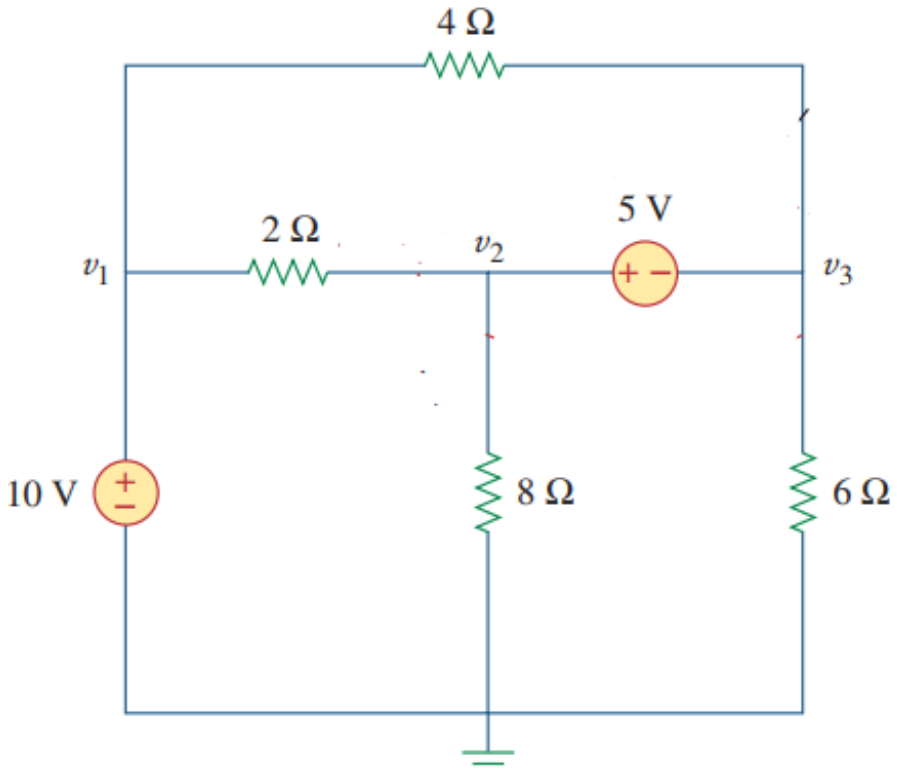
$$v_a = V_1$$

$$i_{R2} + i_{R3} + i_{V_2} = 0$$

$$\frac{v_b - v_a}{R_2} + \frac{v_b - 0}{R_2} + i_{V2} = 0$$



- A voltage source that does not have either of its terminals connected to the ground node.
  - A floating source is a problem for the Nodal Analysis
    - In this circuit, battery V_2 is floating
  - Applying Nodal Analysis



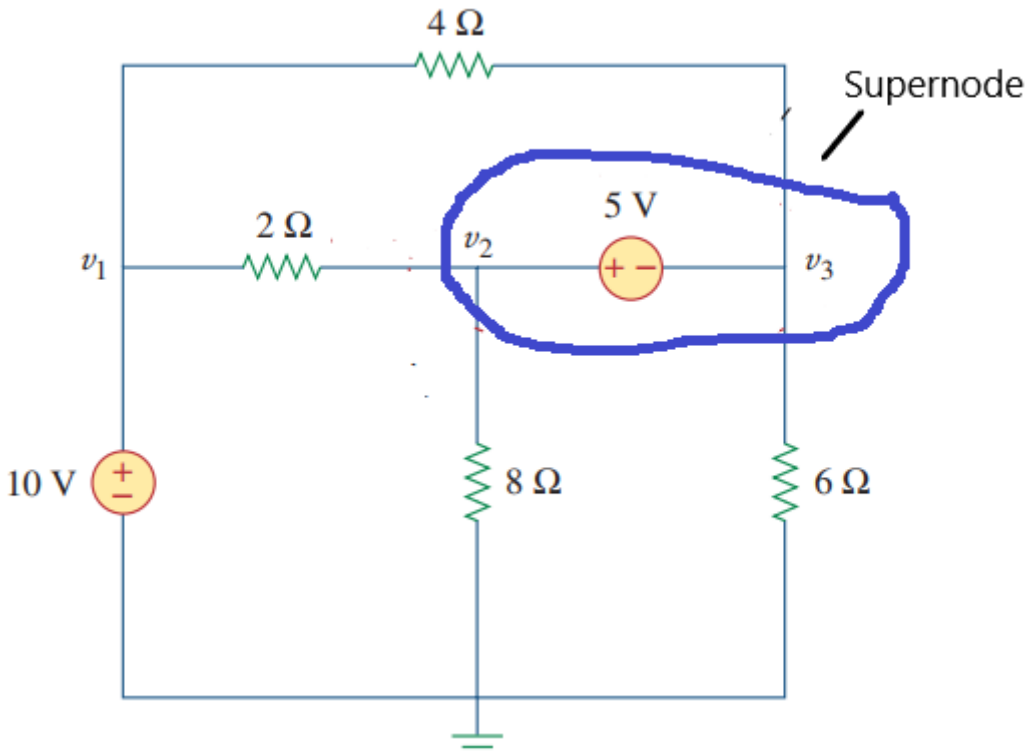
CASE 1 If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. 3.7, for example,

$$v_1 = 10 \text{ V} \tag{3.10}$$

Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.

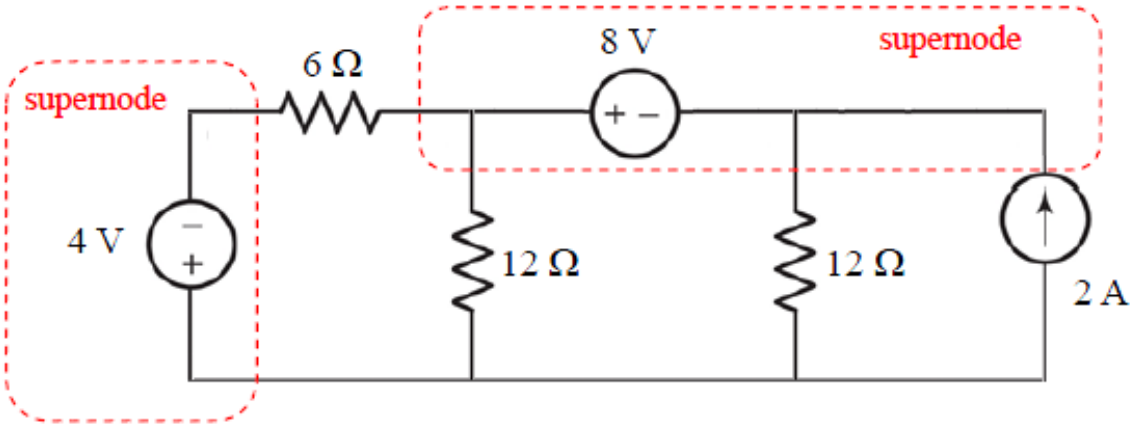
CASE 2 If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a *generalized node* or *supernode*; we apply both KCL and KVL to determine the node voltages.

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.



Steps

supernode:
a collection of multiple nodes separated by voltage sources



Analysis Steps

(1) Choose a reference node (usually ground or the bottom node) to have a voltage of *zero*.

(2) Assign a unique voltage variable to each node that is *not* the reference (v_1, v_2, v_3, … v_{N-1}).

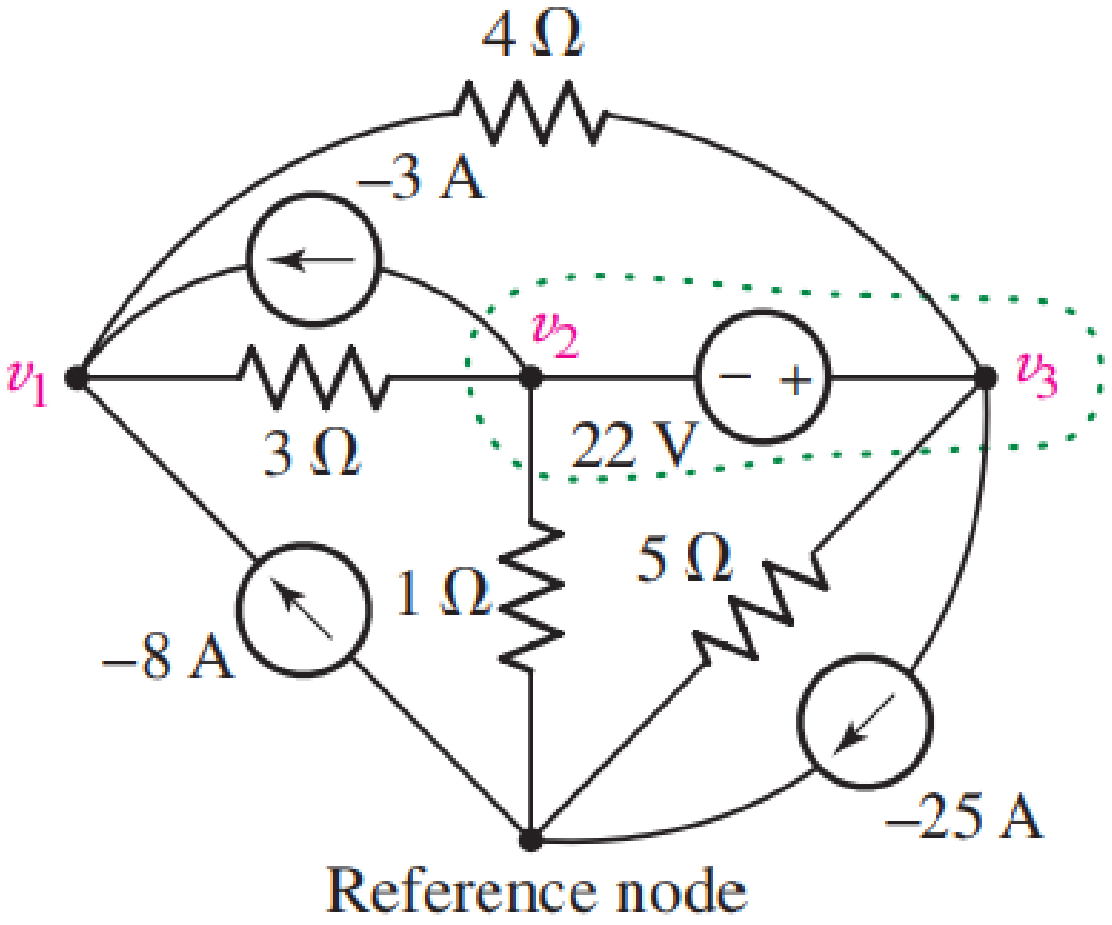
(3) For independent & dependent voltage sources, identify a *supernode* and write the voltage across the supernode in terms of node voltages.

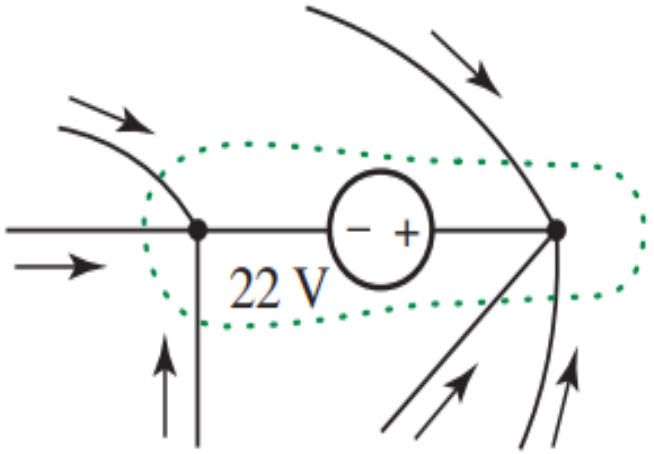
Write a KCL equation *at all N – 1 nodes* ***including the supernode*** (and not the reference, or a supernode which includes the reference).

(4) Solve the $N-1$ node equations + source equations simultaneously.

Example 1

- Determine the value of unknown node voltage v_1





The KCL equation at node 1 is

$$-8 - 3 = \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4}$$

or

$$0.5833v_1 - 0.3333v_2 - 0.2500v_3 = -11$$

Next we consider the 2-3 supernode. Two current sources are connected, and four resistors. Thus,

$$3 + 25 = \frac{v_2 - v_1}{3} + \frac{v_3 - v_1}{4} + \frac{v_3}{5} + \frac{v_2}{1}$$

or

$$-0.5833v_1 + 1.3333v_2 + 0.45v_3 = 28$$

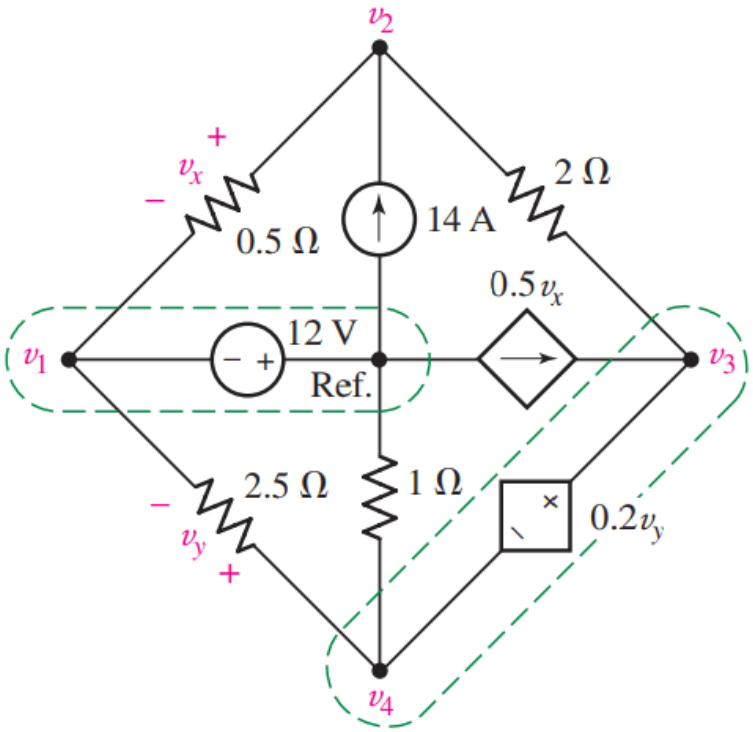
Since we have three unknowns, we need one additional equation, and it must utilize the fact that there is a 22 V voltage source between nodes 2 and 3:

$$v_2 - v_3 = -22$$

Solving Eqs. the solution for v_1 is 1.071 V.

Example 02

- Determine the node-to-reference voltages in the circuit provided.
  - identify the nodes & supernodes
  - write KCL at each node (except the reference)

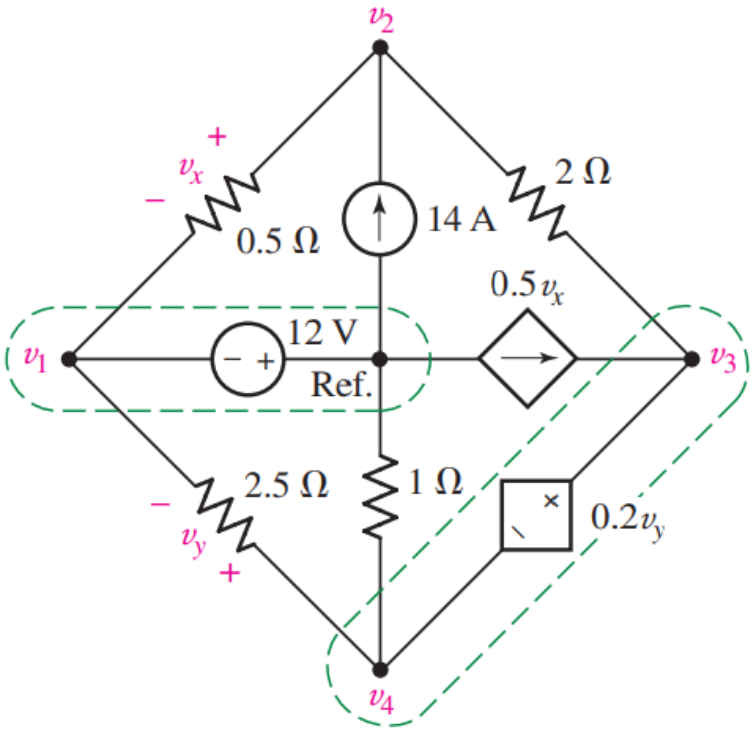


$$v_1 = -12 \text{ V}$$

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} = 14$$

$$0.5v_x = \frac{v_3 - v_2}{2} + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5}$$

…Example 02



- When we relate the source voltages to the node voltages

$$v_3 - v_4 = 0.2v_y$$

$$0.2v_y = 0.2(v_4 - v_1)$$

- When we express the dependent current source in terms of the assigned variables

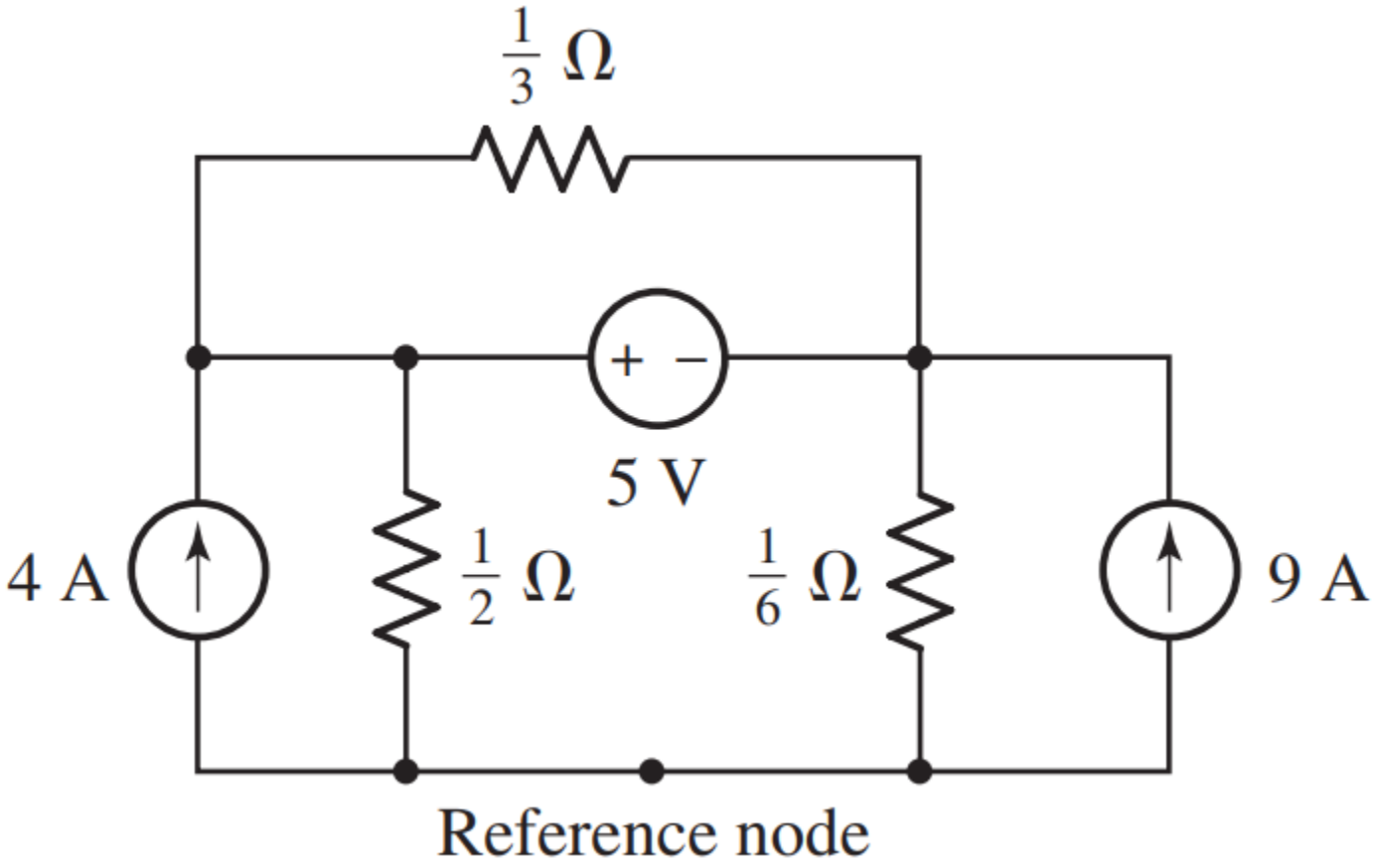
$$0.5v_x = 0.5(v_2 - v_1)$$

$$\begin{aligned} -2v_1 + 2.5v_2 - 0.5v_3 &= 14 \\ 0.1v_1 - v_2 + 0.5v_3 + 1.4v_4 &= 0 \\ v_1 &= -12 \\ 0.2v_1 + v_3 - 1.2v_4 &= 0 \end{aligned}$$

$v_1 = -12$ V, $v_2 = -4$ V, $v_3 = 0$ V, and $v_4 = -2$ V.

Example 03

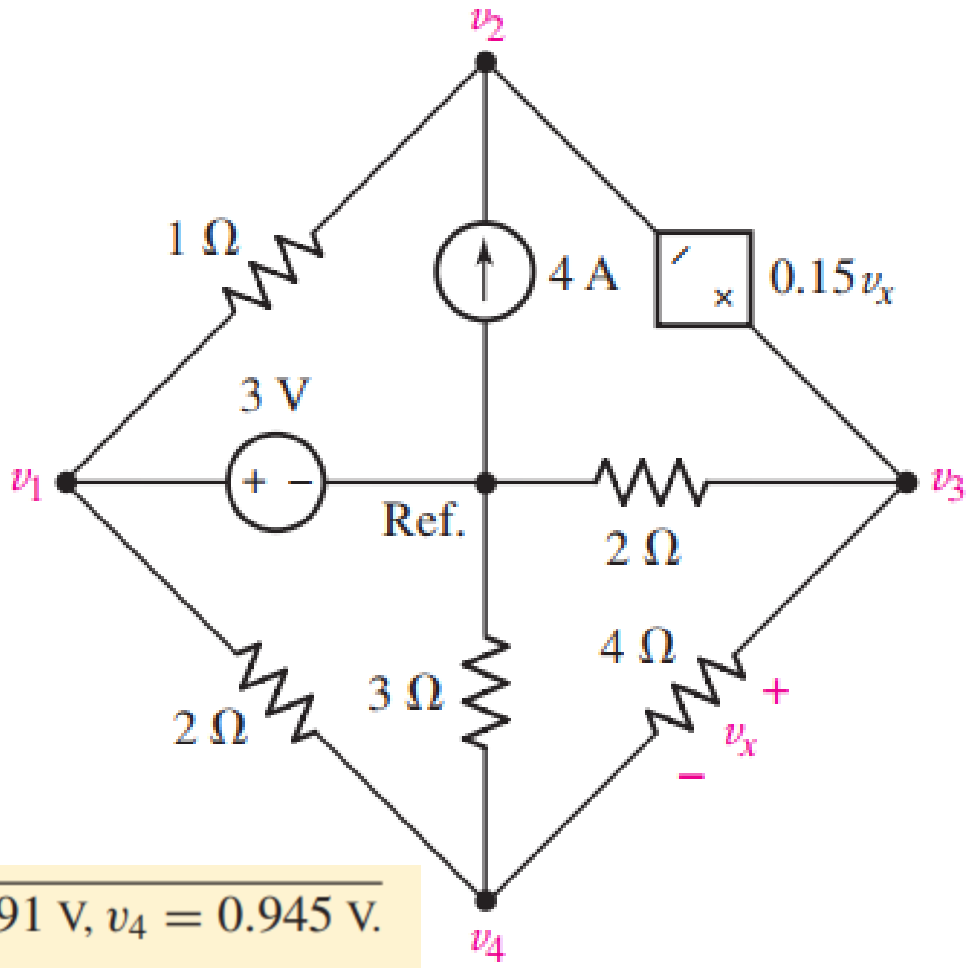
- Compute voltage across each current source



Ans: 5.375 V, 375 mV.

Example 04

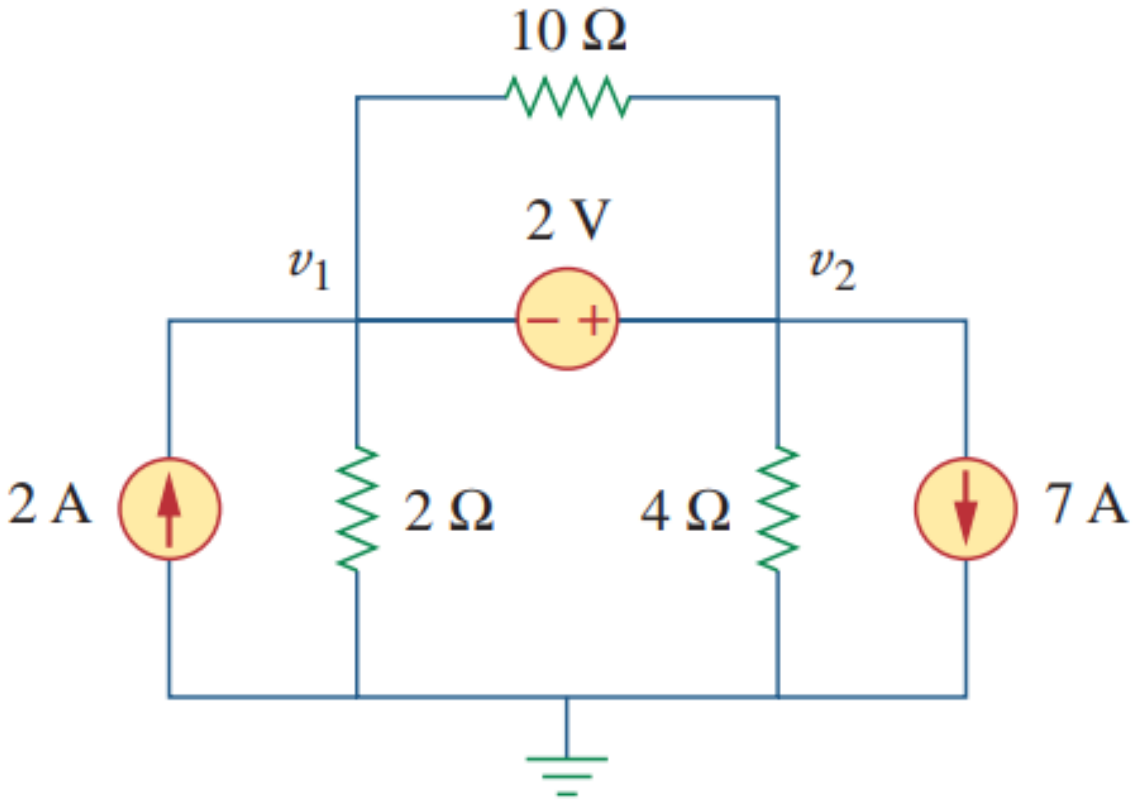
- Determine nodal voltages



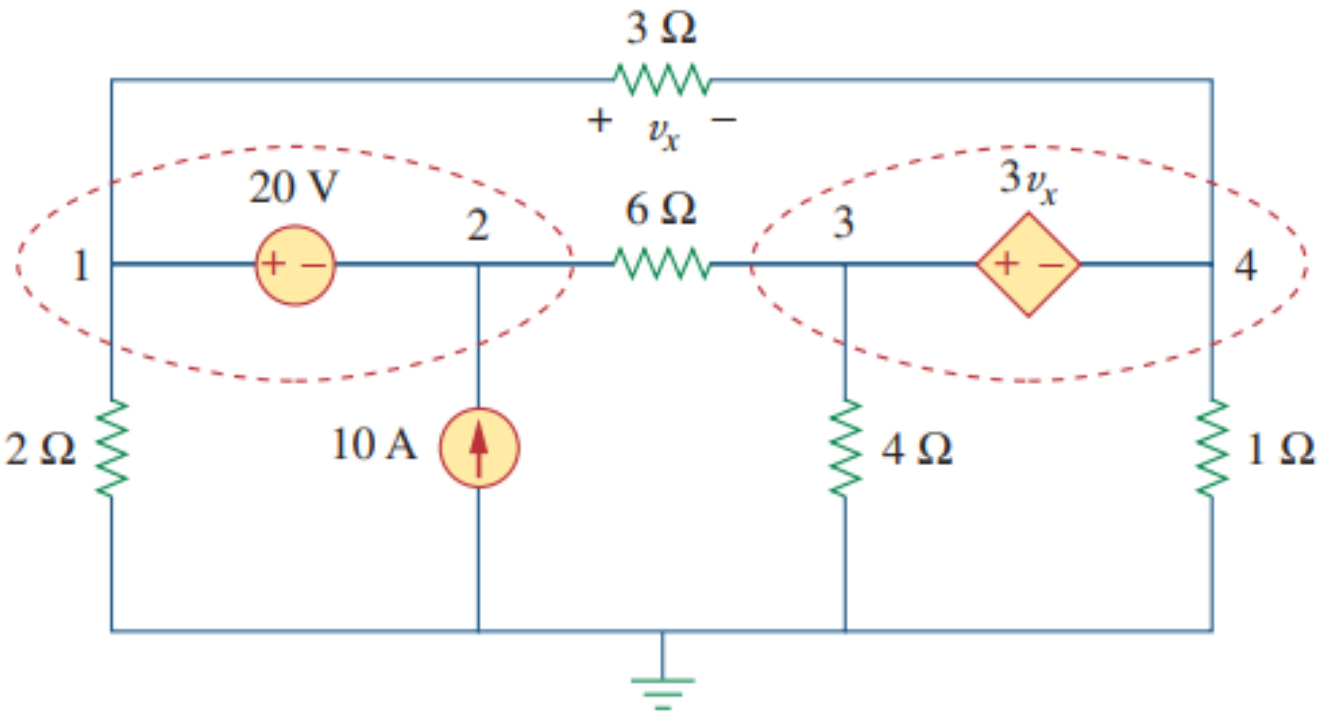
Ans: $v_1 = 3$ V, $v_2 = -2.33$ V, $v_3 = -1.91$ V, $v_4 = 0.945$ V.

Example 05

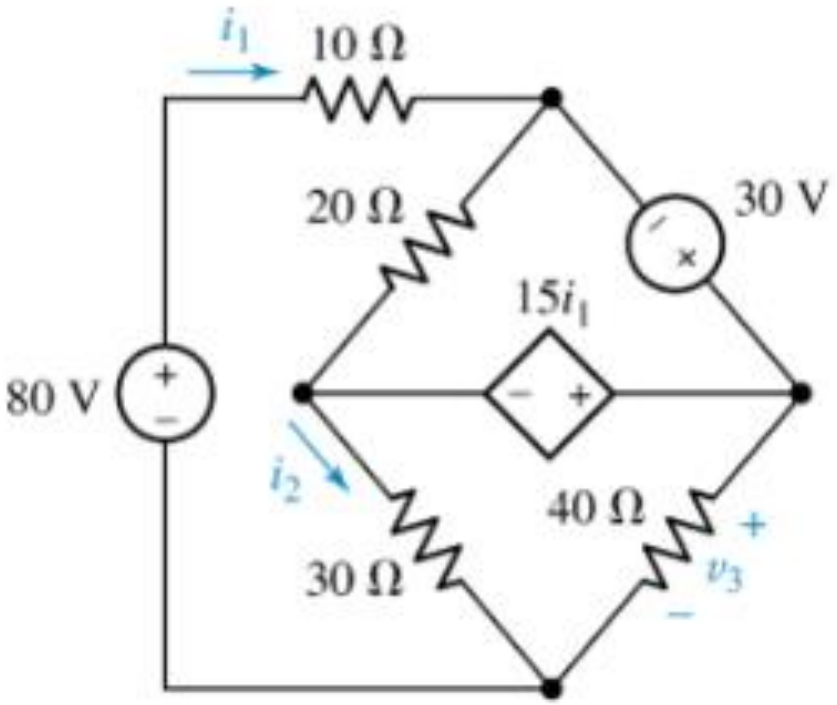
- Determine nodal voltages



Example 06

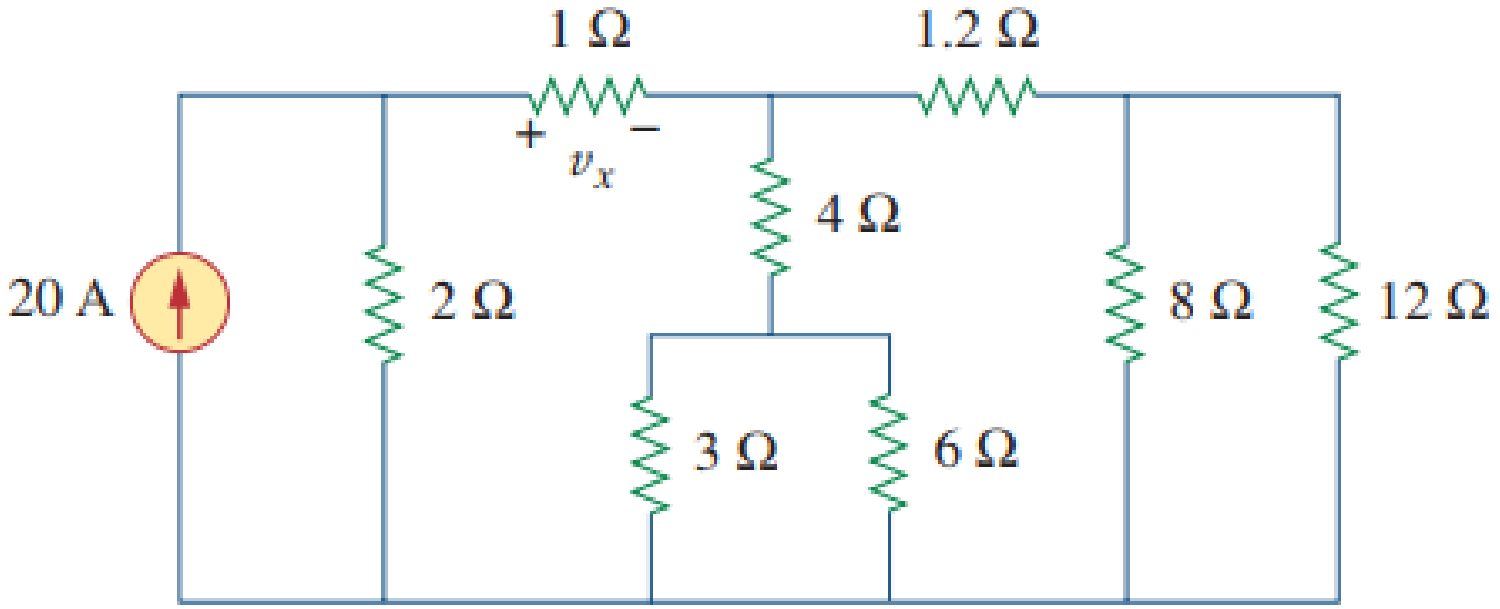


Find the voltage v_3 in the circuit below.



Quiz 1 (FA22-BCE-B)

- In the circuit shown below, determine v_x and the power absorbed by 12Ω resistor



Thank You