# Assignment #1

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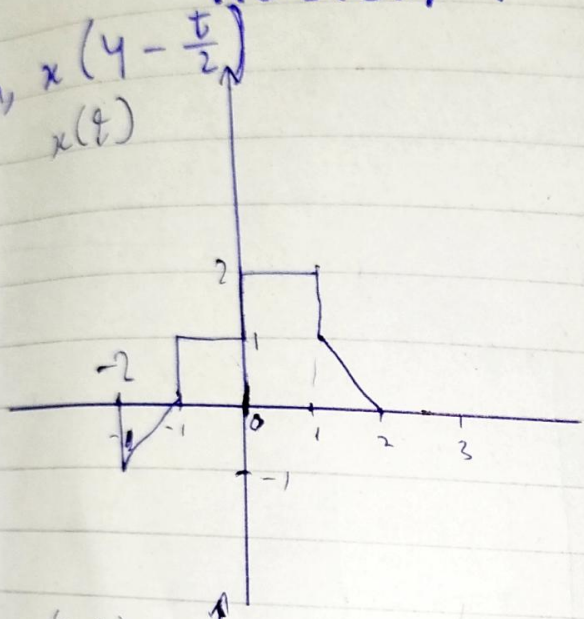
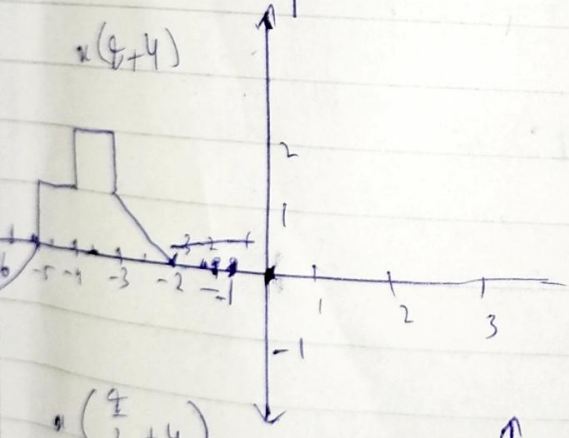
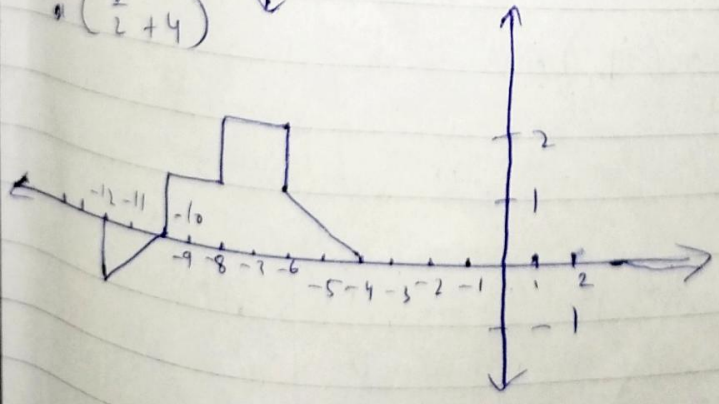
## Problem 1

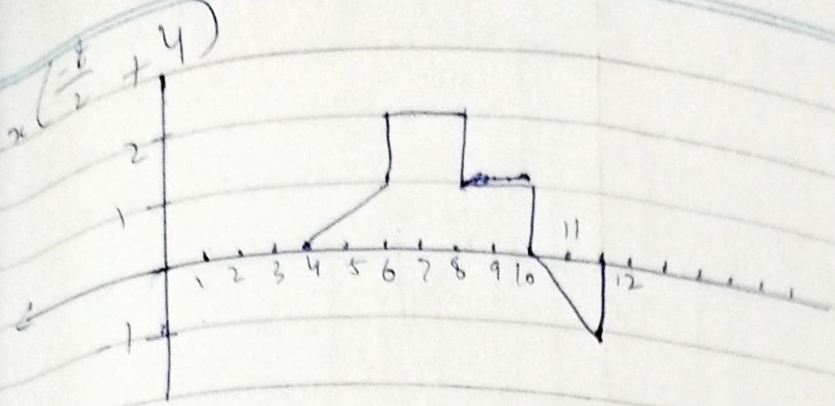
(a) $x\left(4-\frac{t}{2}\right)$




(b) $\left(\frac{t}{2}+4\right)$






$[x(t) + x(-t)]\, u(t)$

$$x(t) = \begin{cases} t+1 & \text{for } -2 \le t < -1 \\ 1 & \text{for } -1 \le t < 0 \\ 2 & \text{for } 0 \le t < 1 \\ -t+2 & \text{for } 1 \le t < 2 \end{cases}$$

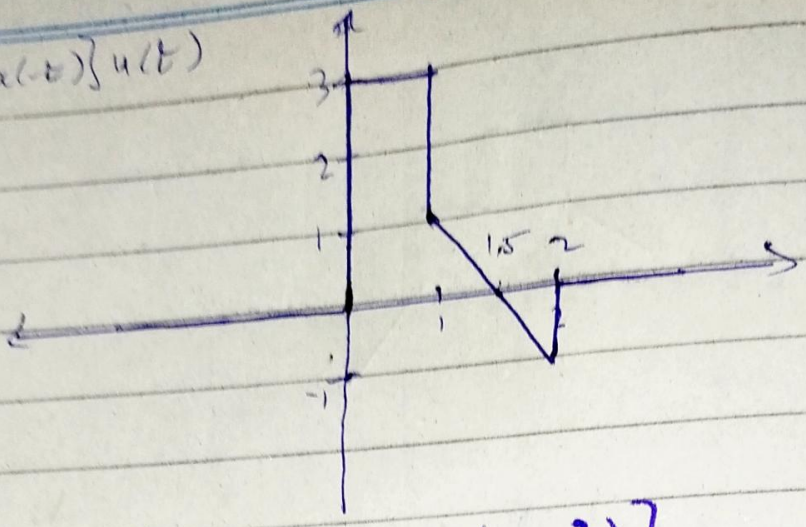
$$x(-t) = \begin{cases} t+2 & \text{for } -2 \le t < -1 \\ 2 & \text{for } -1 \le t < 0 \\ 1 & \text{for } 0 \le t < 1 \\ -t+1 & \text{for } 1 \le t < 2 \end{cases}$$

$$[x(t) + x(-t)] = \begin{cases} 2t+3 & \text{for } -2 \le t < -1 \\ 3 & \text{for } -1 \le t < 1 \\ -2t+3 & \text{for } 1 \le t < 2 \end{cases}$$

$$u(t) = \begin{cases} 1, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

$$[x(t) + x(-t)]\, u(t) = \begin{cases} 0, & t < 0 \\ 3 & 0 \le t < 1 \\ -2t+3, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

$\{x(t)+x(-t)\}u(t)$




(c) $x(t)\left[\delta\left(t+\frac{3}{2}\right)-\delta\left(t-\frac{3}{2}\right)\right]$

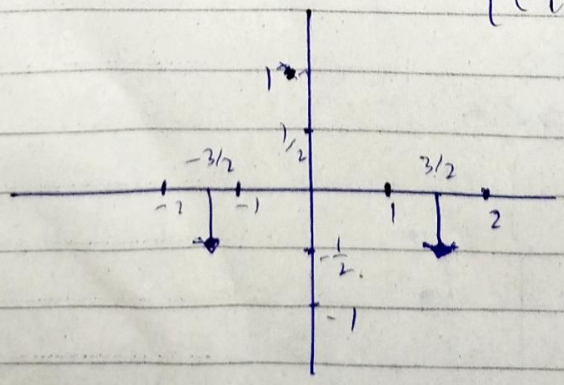
$$\delta\left(t+\tfrac{3}{2}\right)=\begin{cases}\int_{-\infty}^{\infty}\delta\left(t+\frac{3}{2}\right)dt=1, & t=-3/2\\ 0 & \text{elsewhere}\end{cases}$$

$$\delta\left(t-\tfrac{3}{2}\right)=\begin{cases}\int_{-\infty}^{\infty}\delta\left(t-\frac{3}{2}\right)dt=1, & t=3/2\\ 0, & \text{elsewhere}\end{cases}$$

$$\delta\left(t+\tfrac{3}{2}\right)-\delta\left(t-\tfrac{3}{2}\right)=\begin{cases}1, & t=-3/2\\ -1, & t=3/2\\ 0, & \text{elsewhere}\end{cases}$$

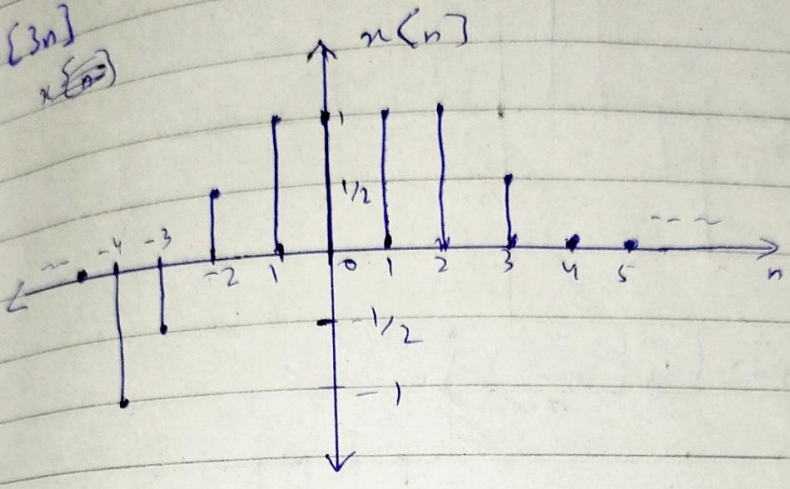
$$x(t)=\begin{cases}t+1, & -2\le t\le -1\\ 1, & -1\le t\le 0\\ 2, & 0\le t\le 1\\ -t+2, & 1\le t\le 2\end{cases}$$

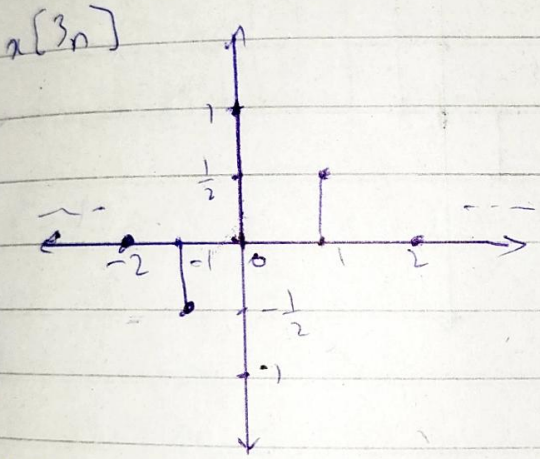
$$x(t)\left[\delta\left(t+\tfrac{3}{2}\right)-\delta\left(t-\tfrac{3}{2}\right)\right]=\begin{cases}(t+1)(1)=\frac{-3}{2}+1=\frac{-1}{2}, & t=\frac{-3}{2}\\ (-t+2)(-1)=(t-2), & t=\frac{3}{2}\end{cases}$$



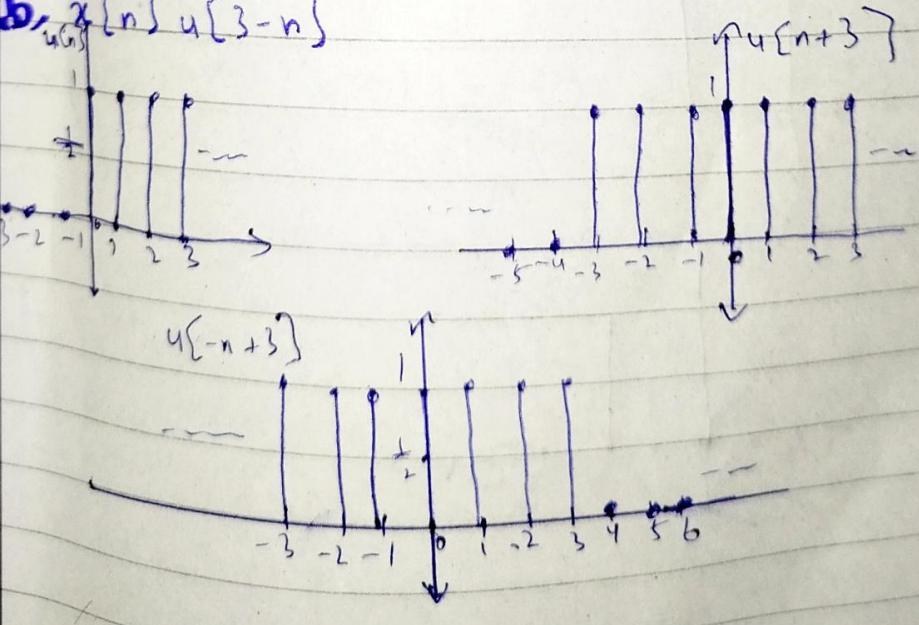
# Problem 2

a) $x[3n]$

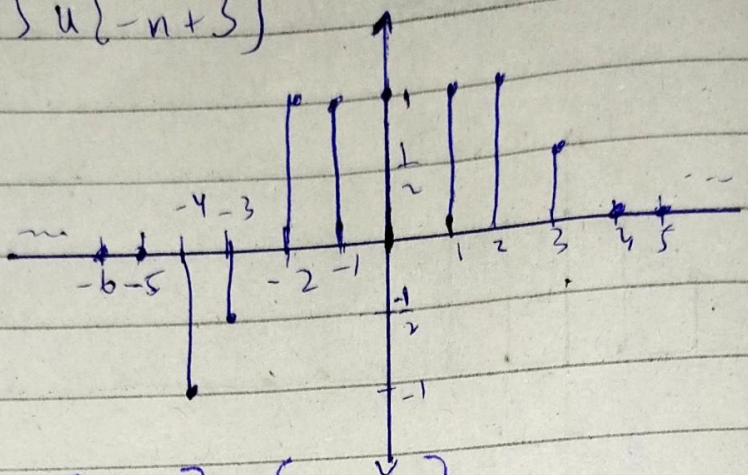



b) $x[n]\,u[3-n]$

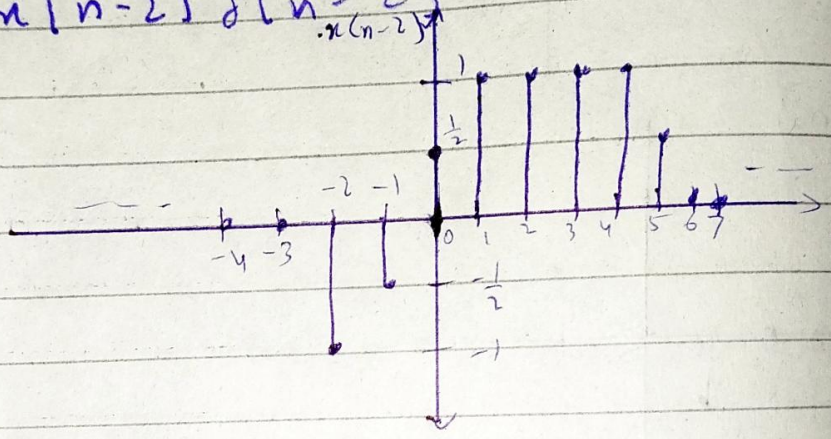


$x[n]\,u[-n+3]$

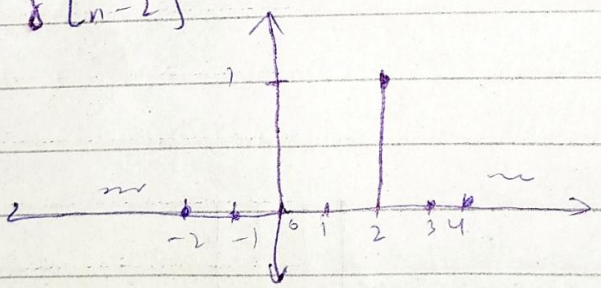



(c) $x[n-2]\,\delta[n-2]$

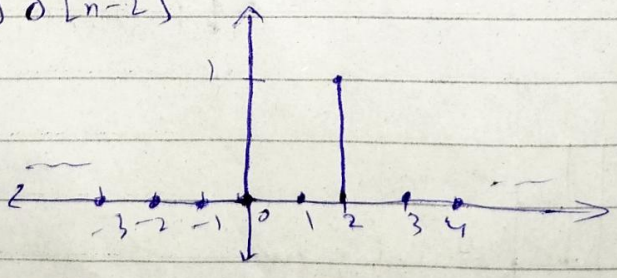
$x(n-2)$




$\delta[n-2]$

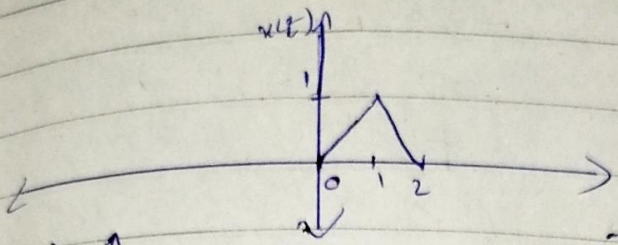


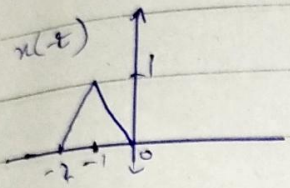

$x[n-2]\,\delta[n-2]$

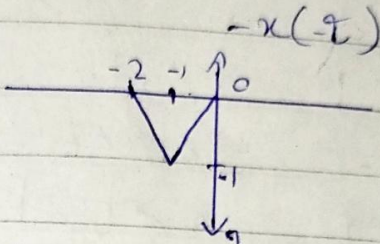


# Problem 3

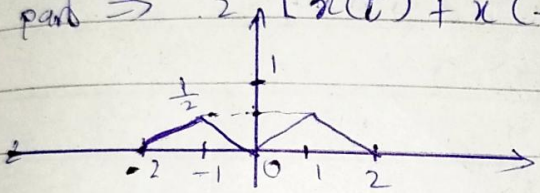
a)



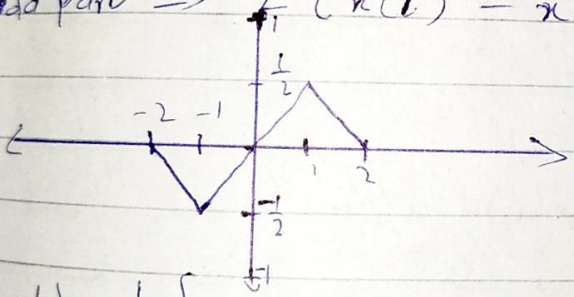




Even part $\Rightarrow \frac{1}{2}\left[x(t) + x(-t)\right]$

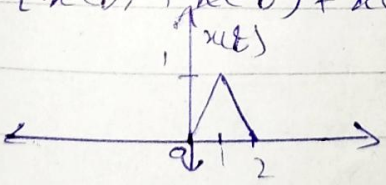



Odd part $\Rightarrow \frac{1}{2}\left\{x(t) - x(-t)\right\}$

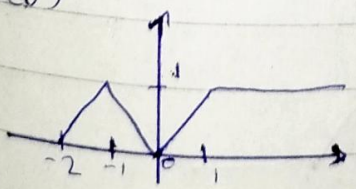



Even + Odd $= \frac{1}{2}\left[x(t) + x(-t) + x(t) - x(-t)\right] = \frac{1}{2}\left\{2x(t)\right.$

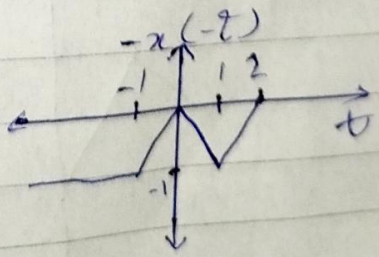
$= x(t)$

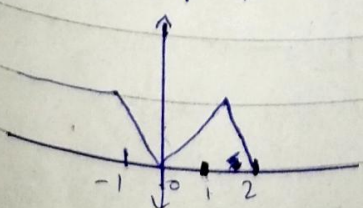



b) $x(t)$




$x(-t)$



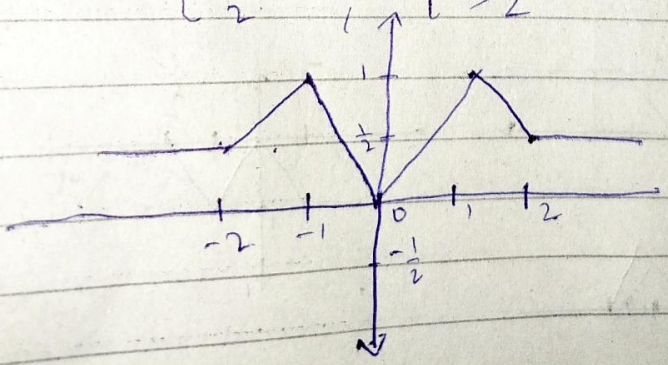
$$x(t) = \begin{cases} t+2, & -2 < t < -1 \\ -t, & -1 < t < 0 \\ t, & 0 < t < 1 \\ 1, & t > 2 \\ 0, & t < -2 \end{cases}$$

$$x(-t) = \begin{cases} 1, & t < -1 \\ -t, & -1 < t < 0 \\ t, & 0 < t < 1 \\ -t+2, & 1 < t < 2 \\ 0, & t > 2 \end{cases}$$

$$-x(-t) \begin{cases} -1, & t < -1 \\ t, & -1 < t < 0 \\ -t, & 0 < t < 1 \\ t-2, & 1 < t < 2 \\ 0, & t > 2 \end{cases}$$

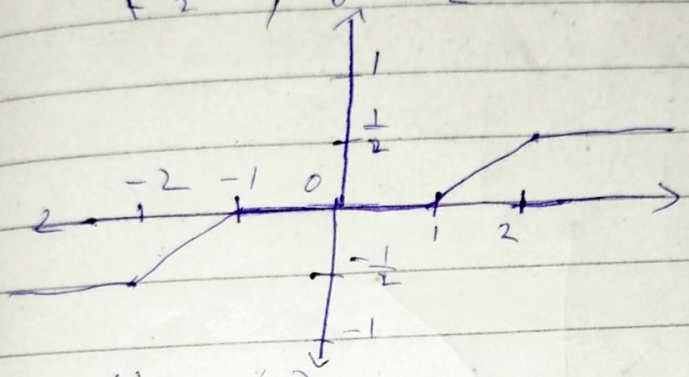
Even Part $\Rightarrow \frac{1}{2}[x(t) + x(-t)]$

$$\Rightarrow \begin{cases} \frac{1}{2}, & t < -2 \\ \frac{(t+3)}{2}, & -2 < t < -1 \\ \frac{-2t}{2} = -t, & -1 < t < 0 \\ \frac{2t}{2} = t, & 0 < t < 1 \\ \frac{(-t+3)}{2}, & 1 < t < 2 \\ \frac{1}{2}, & t > 2 \end{cases}$$



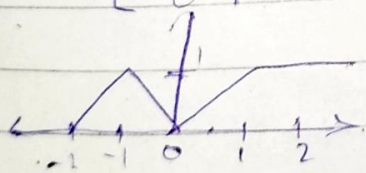
Odd Part $\Rightarrow \frac{1}{2}[x(t) - x(-t)]$

$$\rightarrow \begin{cases} -\frac{1}{2}, & t < -2 \\ \frac{(t+1)}{2}, & -2 < t < -1 \\ 0, & -1 < t < 1 \\ \frac{(t-1)}{2}, & 1 < t < 2 \\ \frac{1}{2}, & t > 2 \end{cases}$$

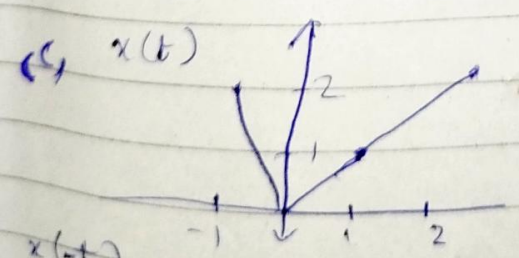



Even + Odd of $x(t)$

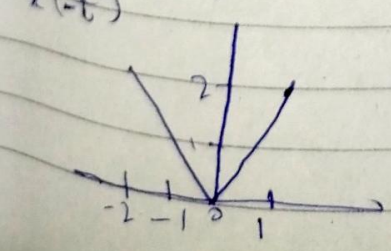
$$x(t) = \begin{cases} t+2, & -2 < t < -1 \\ -t, & -1 < t < 0 \\ t, & 0 < t < 1 \\ 1, & t > 2 \\ 0, & t < -2 \end{cases}$$



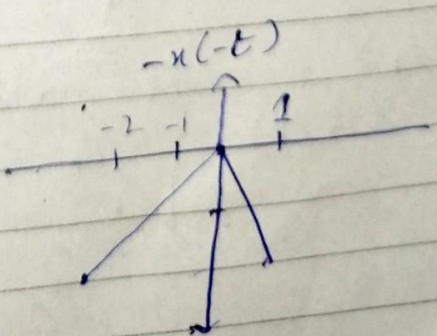

(c) $x(t)$




$x(-t)$




$-x(-t)$



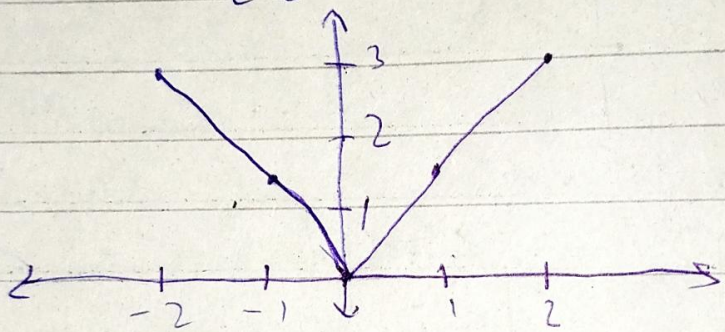
$$x(t) = \begin{cases} -2t, & t<0 \\ t, & t>0 \end{cases}$$

$$x(-t) = \begin{cases} -t, & t<0 \\ 2t, & t>0 \end{cases}$$

$$-x(-t) = \begin{cases} t, & t<0 \\ -2t, & t>0 \end{cases}$$

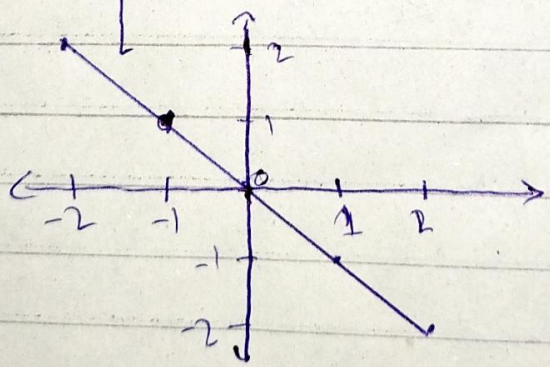
Even part $\Rightarrow \frac{1}{2}[x(t) + x(-t)]$

$$\Rightarrow \begin{cases} \frac{-3t}{2}, & t<0 \\ \frac{3t}{2}, & t>0 \end{cases}$$




Odd part $\Rightarrow \frac{1}{2}[x(t) - x(-t)]$

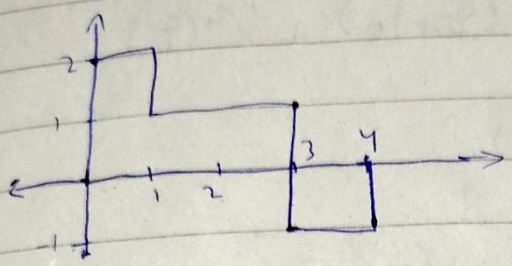
$$\Rightarrow \begin{cases} -t, & -2 < t < 2 \end{cases}$$




$x(t)$ : even + odd

$$x(t) = \begin{cases} -2t, & t<0 \\ t, & t>0 \end{cases}$$

# Problem 4




$$x(t) = \begin{cases} 2, & 0<t<1 \\ 1, & 1<t<3 \\ -1, & 3<t<4 \\ 0, & \text{elsewhere} \end{cases}$$

Now, $x(t)$ in term of unit signal

$$x(t) = \begin{cases} 2u(t) - 2u(t-1), & 0<t<1 \\ u(t-1) - u(t-3), & 1<t<3 \\ -u(t-3) + u(t-4), & 3<t<4 \\ 0, & \text{elsewhere otherwise} \end{cases}$$

Taking derivative,

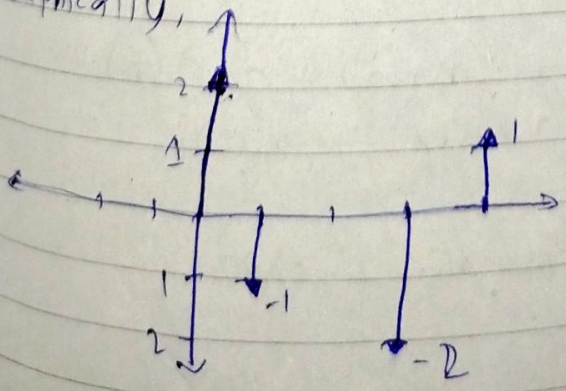
$$\frac{d}{dt}x(t) = \begin{cases} 2\delta(t) - 2\delta(t-1), & 0<t<1 \\ \delta(t-1) - \delta(t-3), & 1<t<3 \\ -\delta(t-1) + \delta(t-4), & 3<t<4 \\ 0, & \text{otherwise} \end{cases}$$

So,

$$\frac{d}{dt}x(t) = 2\delta(t) - 2\delta(t-1) + \delta(t-1) - \delta(t-3) - \delta(t-3) + \delta(t-4)$$

$$= 2\delta(t) - \delta(t-1) - 2\delta(t-3) + \delta(t-4)$$

Graphically,



# Problem 5

$$y(t) = x(t+10) + x^2(t)$$

R.H.S

$\Rightarrow x(t+10) + x^2(t)$

we put delay in it

$= x(t-t_1+10) + x^2(t-t_1)$

L.H.S

$\Rightarrow x(t) \xrightarrow{\text{delay}} x(t-t_1)$

$= x(t-t_1) \xrightarrow{\text{systems}} x(t-t_1+10) + x^2(t-t_1)$

L.H.S = R.H.S, system is time invariant

For Linearity,

R.H.S

$y_1(t) = x_1(t+10) + x_1^2(t)$

$y_2(t) = x_2(t+10) + x_2^2(t)$

$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$

$y_3(t) = a_1 x_1(t+10) + a_1 x_1^2(t) + a_2 x_2(t+10) + a_2 x_2^2(t)$

L.H.S

$\Rightarrow f\{a_1 x_1(t) + a_2 x_2(t)\}$

$= [a_1 x_1(t+10) + a_2 x_2(t+10)] + [a_1 x_1(t) + a_2 x_2(t)]^2$

$= a_1 x_1(t+10) + a_2 x_2(t+10) + a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2(a_1 x_1(t))\{a_2 x_2(t)\}$

$\neq$ R.H.S

So, the system is non-linear