Applied Physics for Engineers

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Potential due to dipole

- We know that

$$V = k\frac{q}{r}$$

If we want to find the potential at a point due a group of charged particles, then we will just take summation of all the potentials (with the help of super position principle) i.e for n charges the net potential is

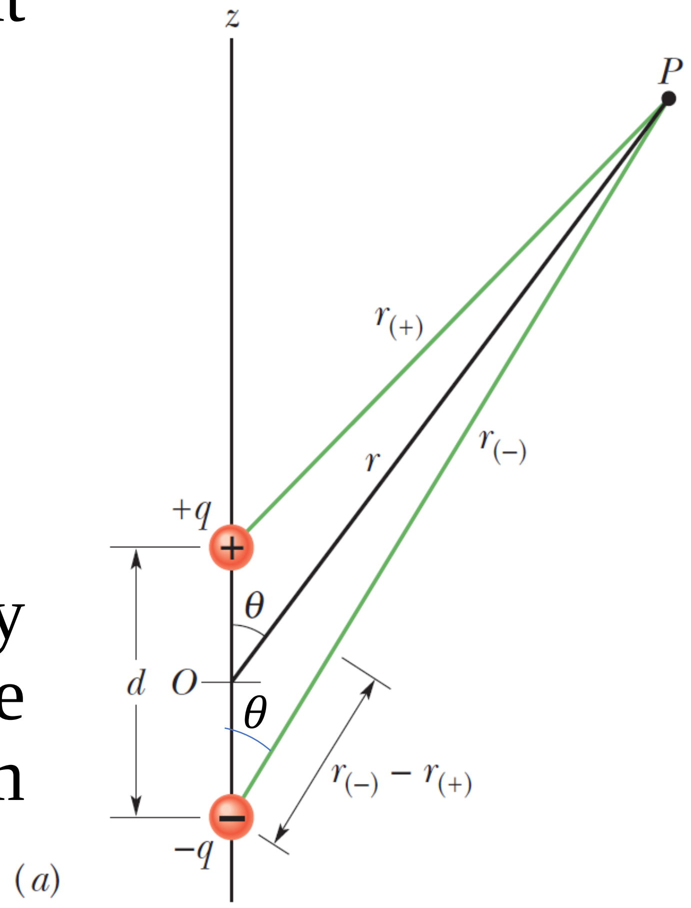
$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

Now let us apply above equation to an electric dipole at an arbitrary point p in figure below

At P, the positively charged particle (at distance $r_{(+)}$) sets up potential $V_{(+)}$ and the negatively charged particle (at distance $r_{(-)}$) sets up potential $V_{(-)}$. Then the net potential at P is

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

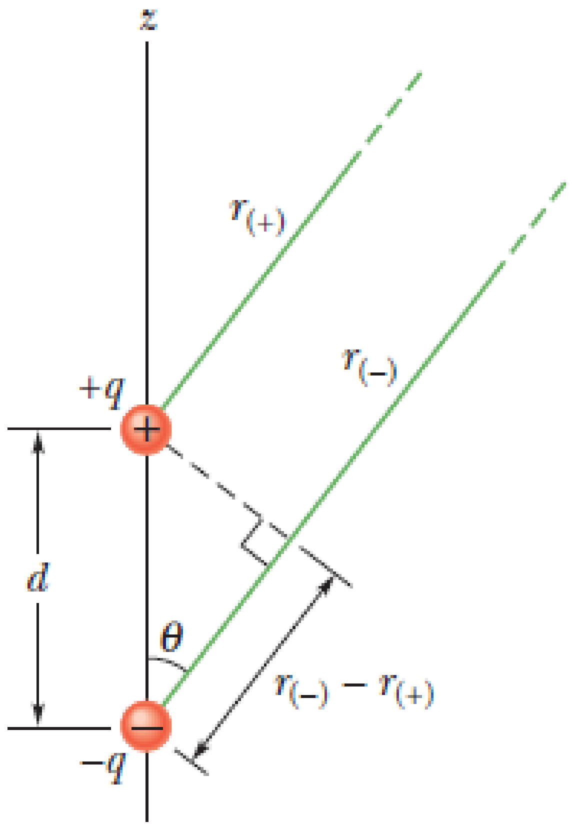
$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}. \qquad \text{(i)}$$



we are usually interested only in points that are relatively far from the dipole, such that r ≫ d, where d is the distance between the charges and r is the distance from the dipole's midpoint to P.

In that case, we can approximate the two lines to *P* as being parallel and their length difference as being the leg of a right triangle with hypotenuse *d* (Fig.*b*). Also, that difference is so small that the product of the lengths is approximately r^2.Thus,

$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$



(*b*)

If we substitute these quantities into Eq. (i), we can approximate V to be

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2},$$

where θ is measured from the dipole axis as shown in Fig. (a). We can now write V as

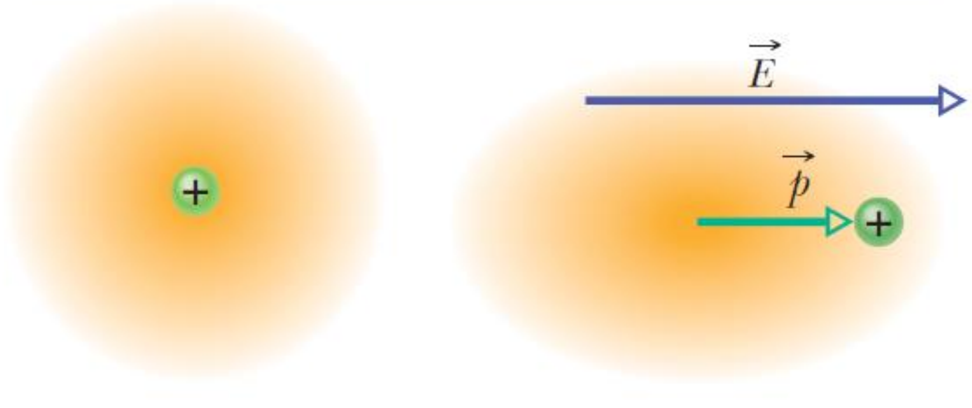
$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(electric dipole)}$$

In which p (= qd) is the magnitude of the electric dipole moment. The vector **p** is directed along the dipole axis, from the negative to the positive charge.

Induced Dipole Moment

- Many molecules, such as water, have permanent electric dipole moments. In other molecules (called nonpolar molecules) and in every isolated atom, the centers of the positive and negative charges coincide (a) and thus no dipole moment is set up.
- However, if we place an atom or a nonpolar molecule in an external electric field, the field distorts the electron orbits and separates the centers of positive and negative charge (b).

The electric field shifts the positive and negative charges, creating a dipole.



(a) (b)

- Because the electrons are negatively charged, they tend to be shifted in a direction opposite the field. This shift sets up a dipole moment that points in the direction of the field.
- This dipole moment is said to be induced by the field, and the atom or molecule is then said to be polarized by the field (that is, it has a positive side and a negative side).
- When the field is removed, the induced dipole moment and the polarization disappear.

Potential Due To A Continuous Charge Distribution

- When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we cannot use the summation to find the potential V at a point P. Instead, we must choose a differential element of charge dq, determine the potential dV at P due to dq, and then integrate over the entire charge distribution.
- Let us again take the zero of potential to be at infinity. If we treat the element of charge dq as a particle, then the potential dV at point P due to dq:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \text{(positive or negative } dq\text{).}$$

- Here r is the distance between P and dq. To find the total potential V at P, we integrate to sum the potentials due to all the charge elements:

$$V = \int dV = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$

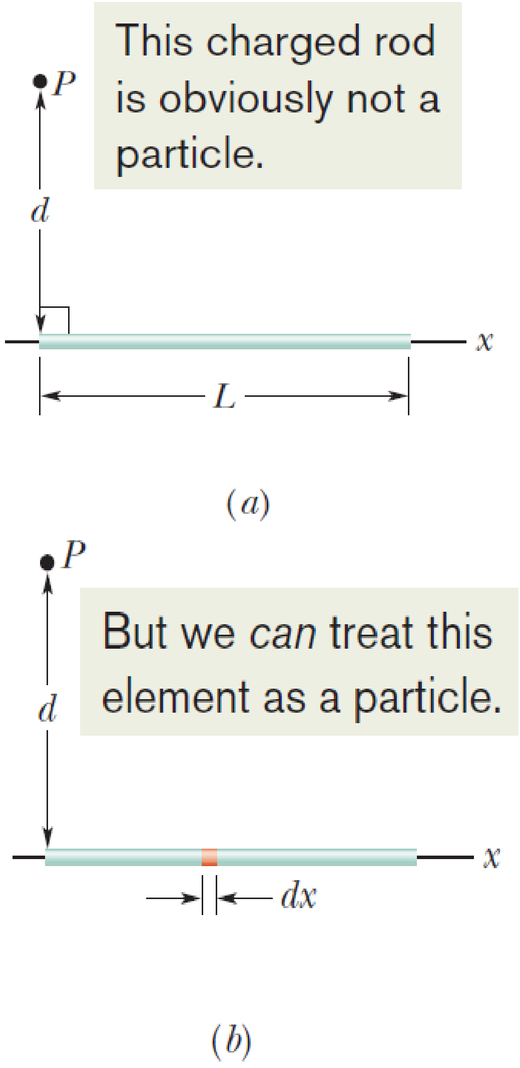
- The integral must be taken over the entire charge distribution. Note that because the electric potential is a scalar, there are no vector components to consider

Electric Potential due to a line of charge

- In Fig. *a*, a thin nonconducting rod of length L has a positive charge of uniform linear density λ. Let us determine the electric potential V due to the rod at point P, a perpendicular distance d from the left end of the rod.
- We consider a differential element dx of the rod as shown in Fig. *b*. This (or any other) element of the rod has a differential charge of

$$dq = \lambda dx$$

This element produces an electric potential dV at point P

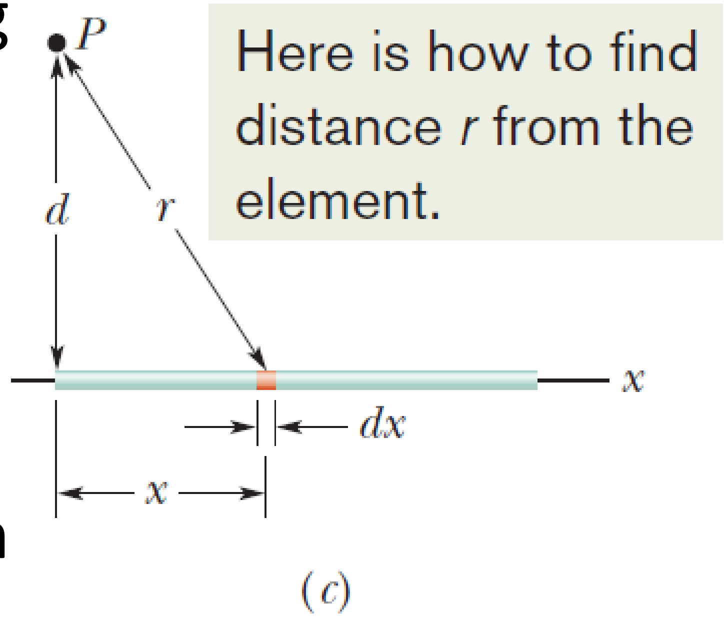


- To find the potential dV we will first find the distance of point P from charge element dq by using Pythagoras theorem.

$$(\text{hypt})^2 = (\text{Base})^2 + (\text{Perp})^2$$

$$r^2 = x^2 + d^2$$

$$r = \sqrt{x^2 + d^2}$$



- If we treat the charge element dq as a particle, then we can write the potential dV at point P as

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

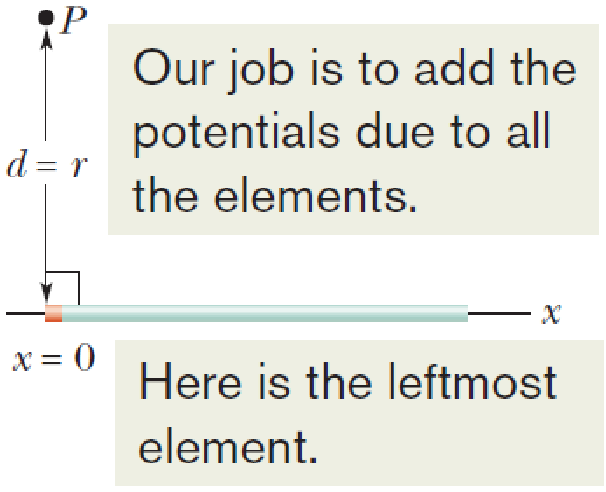
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}} \qquad \text{(i)}$$

- Since charge on the rod is positive and V=0 is at infinity, so dV is positive.
- We now find the total potential *V* produced by the rod at point *P* by integrating Eq. (i) along the length of the rod, from $x = 0$ to $x = L$

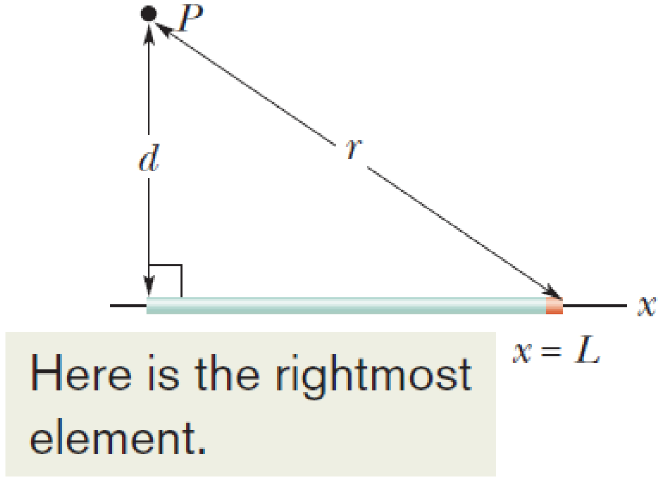
$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln\left(x + (x^2 + d^2)^{1/2} \right) \right]_0^L$$



(d)



(e)

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L + (L^2 + d^2)^{1/2}\right) - \ln d\right]$$

- We can simplify this result by using the general relation

$$\ln A - \ln B = \ln(A/B).$$

We then find

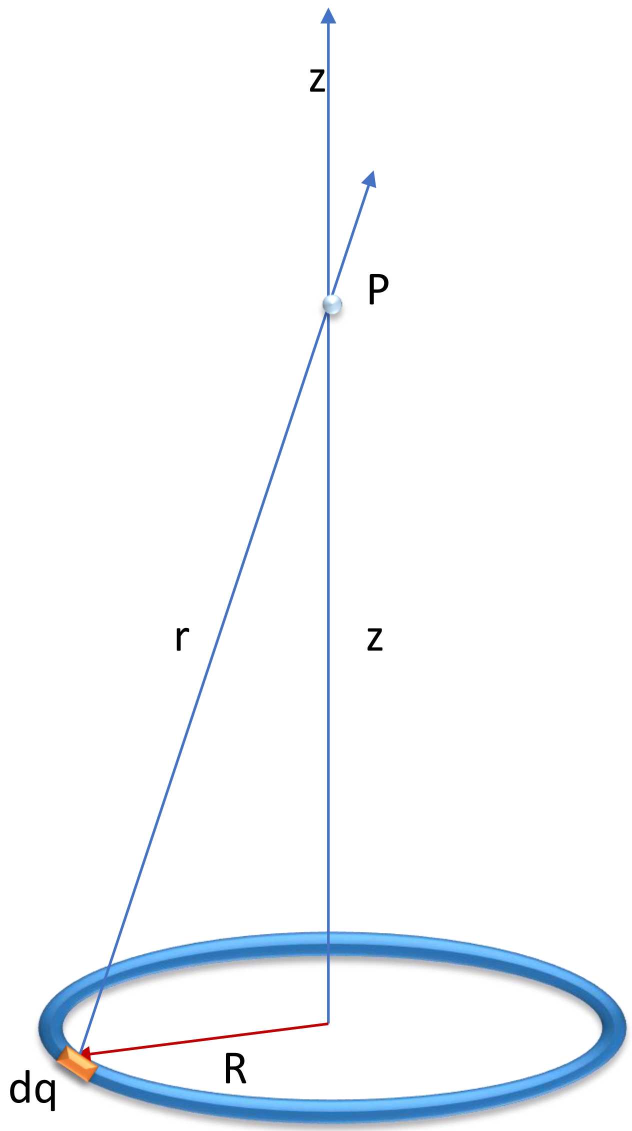
$$\boxed{V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right]}$$

Because V is the sum of positive values of dV, it too is positive.

Potential Due to Ring of Charge

- In the fig. A thin nonconducting ring of radius R has uniformly distributed positive charge q. Let us determine electric potential V due to the ring at a point P, which is placed at central axis z of the ring.
- We consider an infinitesimally small element dq of charge q on the ring and find the potential dV due to this charge element
- Point P is at a distance r from dq where,

$$r = \sqrt{R^2 + z^2}$$



- Now we can write the potential dV at point P due to dq as,

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$V = \int dV$$

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

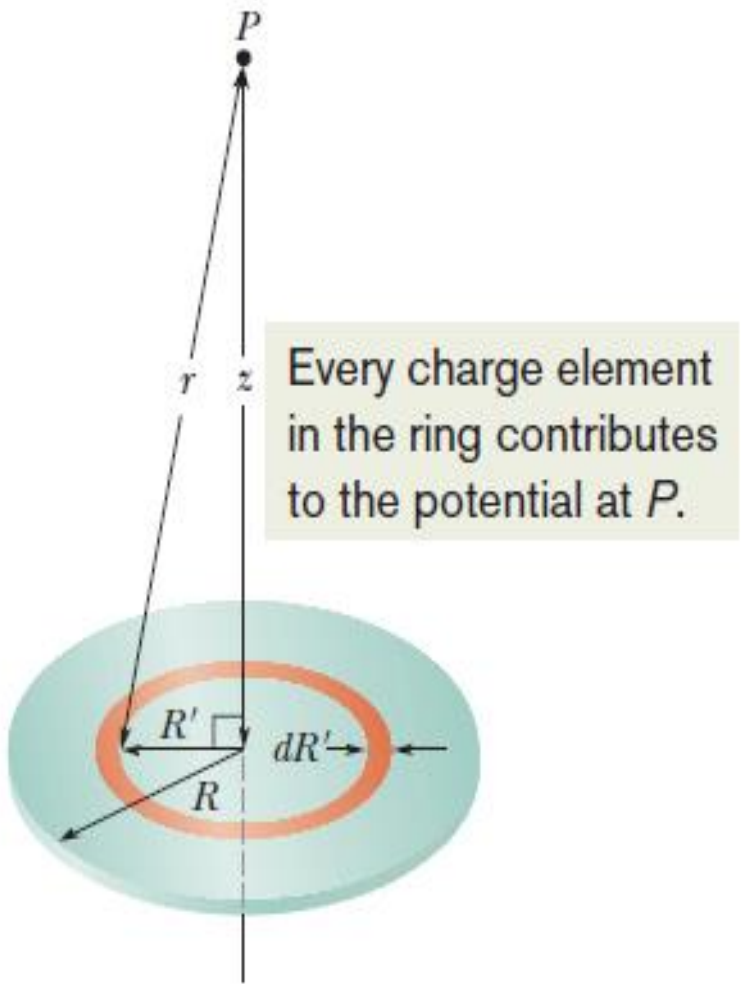
$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\sqrt{R^2+z^2}}$$ r is constant for all the charge elements dq

$$V = \frac{1}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\int dq$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2+z^2}}}$$ *V is potential due to a ring of charge at point P*

Electric Potential Due to Disk of charge

- We have to derive an expression for $V(z)$, the electric potential at any point on the central axis due to a uniformly charged non-conducting disk.
- Because we have a circular distribution of charge on the disk, we could start with a differential element that occupies angle $d\theta$ and radial distance dr. We would then need to set up a two-dimensional integration. However, let's do something easier.

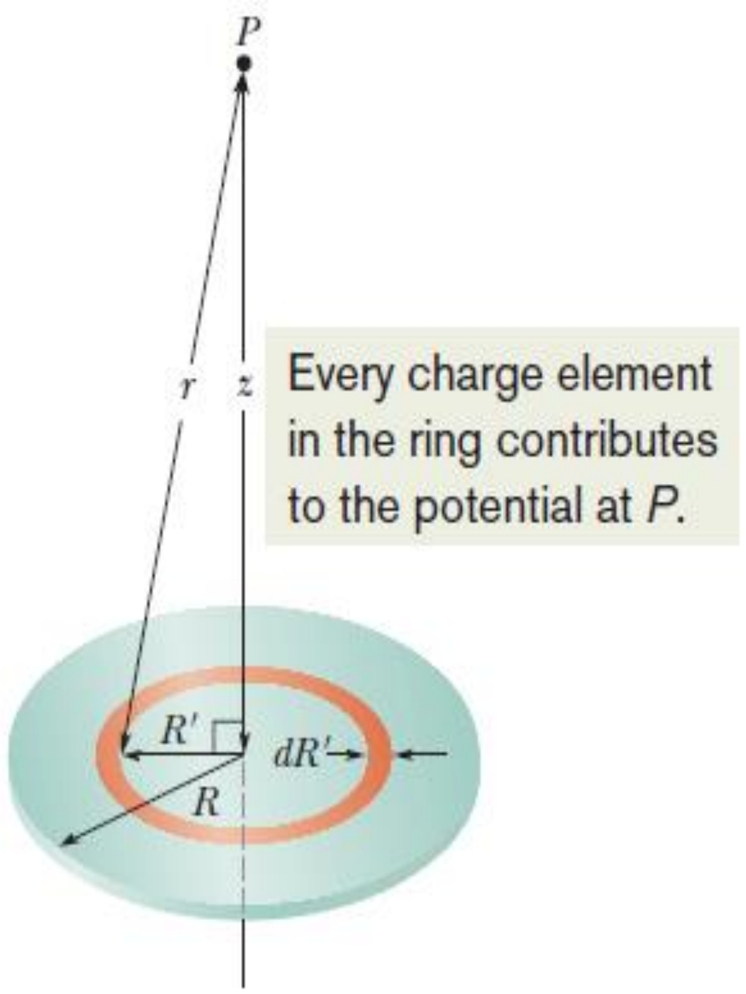


- In the figure, consider a differential element consisting of a flat ring of radius *R*’ and radial width *dR*’.
- We have already derived the expression for potential due to a ring at P, which is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2+z^2}}$$

- Here since we will take ring as differential element so above equation will become

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R'^2+z^2}}$$



As q=σA

q=$\sigma\pi R'^2$

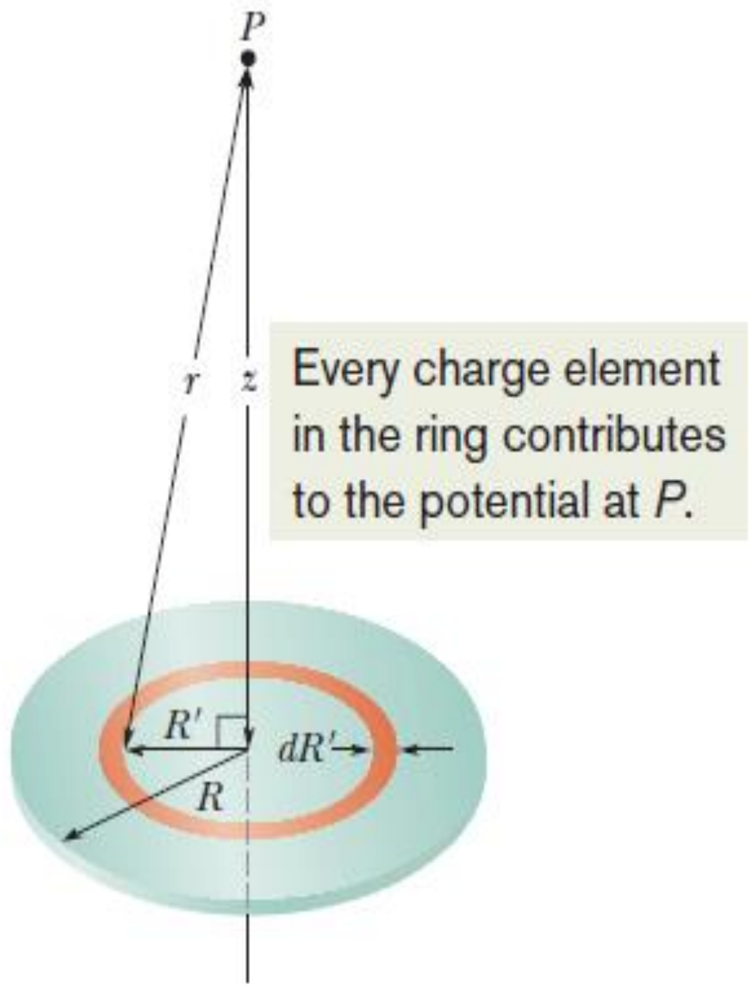
So the charge dq has magnitude

dq = $\sigma\ 2\pi R' dR'$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R'^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\ 2\pi R' dR'}{\sqrt{R'^2+z^2}}$$

- We find the net potential at *P* by adding (via integration) the contributions of all the rings from *R*′ = 0 to *R*′ = *R*:

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2+R'^2}}$$



$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2 + R'^2}\right]_0^R$$

$$\boxed{V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)}$$

V is the electric potential due to disk of charge

24.10 Calculating the Field from the Potential:

Suppose that a positive test charge q_0 moves through a displacement from one equipotential surface to the adjacent surface. The work the electric field does on the test charge during the move is $-q_0\, dV$.

The work done by the electric field may also be written as the scalar product or $(q_0\vec{E})\cdot d\vec{s}$ $= q_0 E(\cos\theta)\, ds$.

Therefore, $$-q_0\, dV = q_0 E(\cos\theta)\, ds,$$

That is, $$E\cos\theta = -\frac{dV}{ds}$$

Since $E\cos\theta$ is the component of E in the direction of **ds**,

$$E_s = -\frac{\partial V}{\partial s}$$

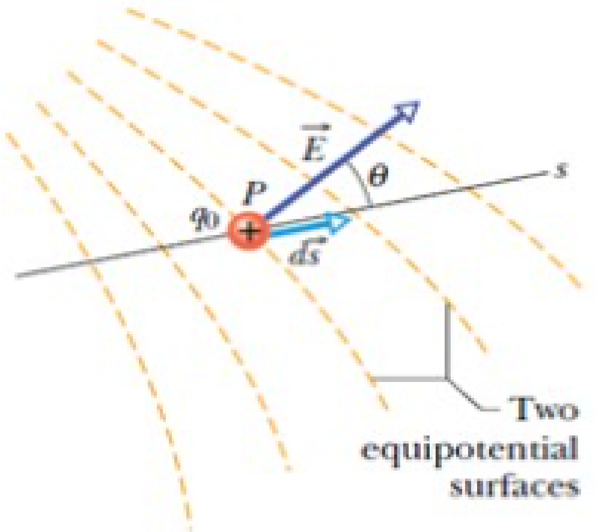


Fig. 24-14 A test charge q_0 moves a distance $d\vec{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\vec{s}$ makes an angle θ with the direction of the electric field $\vec{E}$.

If we take the s axis to be, in turn, the x, y, and z axes, the x, y, and z components of E at any point are

$$E_x = -\frac{\partial V}{\partial x};\qquad E_y = -\frac{\partial V}{\partial y};\qquad E_z = -\frac{\partial V}{\partial z}.$$

Therefore, the component of E in any direction is the negative of the rate at which the electric potential changes with distance in that direction.