Lecture 8: Families of Discrete Random Variables

CPE251 Probability Methods in Engineering

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Some Families of DRV

Family	Definition of X	pmf of X	$E(X)$	$Var(X)$
Bernoulli	Probability of a single success (let $x = 1$)	$p_k = p_X(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & \text{elsewhere} \end{cases}$	p	$p(1-p)$
Binomial	Probability of k successes	$p_k = p_X(x) = \binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$
Geometric	No. of failures *before* or *including* first success	$p_k = p(1-p)^k$ or $p_k = p(1-p)^{k-1}$	$(1-p)/p$ or $1/p$	$(1-p)/p^2$
Discrete Uniform	Equiprobable values of X	$p_X(x) = \begin{cases} 1/(L-K+1) & X \in [K, L], K < L \\ 0 & \text{elsewhere} \end{cases}$	$\frac{K+L}{2}$	$(L-K)(L-K+2)/12$
Poisson	Random number of outcomes/arrivals in time/space	$p_X(x) = \frac{\alpha^k}{k!} e^{-\alpha} \quad X \geq 0$ $\alpha = \lambda t$ $\alpha =$ average number of arrivals $\lambda =$rate of arrival $t =$ interval of time or space	α	α

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Examples

1. The probability that a certain kind of component will survive a shock test is 3/4. Find the probability that exactly 2 of the next 4 components tested survive.
2. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?
3. For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?
4. During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?
5. Probability of a chess card picked up at random

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Examples

1. The probability that a certain kind of component will survive a shock test is 3/4. Find the probability that exactly 2 of the next 4 components tested survive.

$$b\left(2; 4, \frac{3}{4}\right) = \binom{4}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 = \left(\frac{4!}{2!\,2!}\right)\left(\frac{3^2}{4^4}\right) = \frac{27}{128}$$

2. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?

(a) $P(X \geq 10) = 1 - P(X < 10) = 1 - \sum_{x=0}^{9} b(x; 15, 0.4)$

(b) $P(3 \leq X \leq 8) = \sum_{x=3}^{8} b(x; 15, 0.4) = \sum_{x=0}^{8} b(x; 15, 0.4) - \sum_{x=0}^{2} b(x; 15, 0.4)$

(c) $P(X = 5) = b(5; 15, 0.4) = \sum_{x=0}^{5} b(x; 15, 0.4) - \sum_{x=0}^{4} b(x; 15, 0.4)$

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3. A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. (a) The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20? (b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?
   a) $P(X \geq 1) = 1 - P(X = 0) = 1 - b(0; 20, 0.03)$
   b) Consider a) a s a Bernoulli trial with $p = P[X \geq 1]$

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4. For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?
5. During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

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References

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