# LEC 4:
# ELEMENTARY SIGNALS
# +
# DIFFERENTIATION OPERATION

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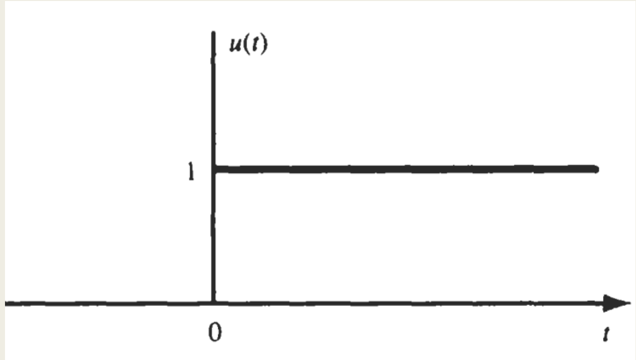
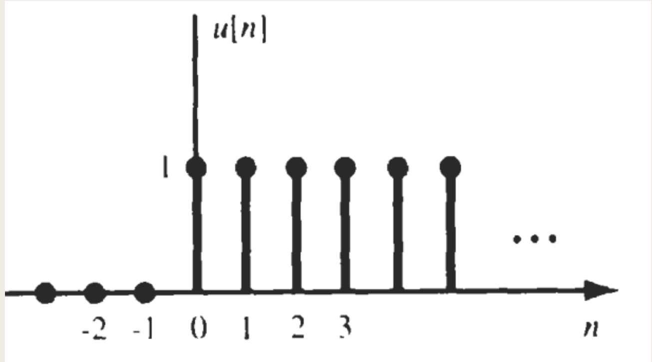
# Basic / Elementary Signals

- Standard signals are used for the analysis of systems. These signals are;
  - *Unit step function*
  - *Unit impulse or Delta function*
  - *Unit ramp function*
  - *Complex exponential function*
  - *Sinusoidal function*

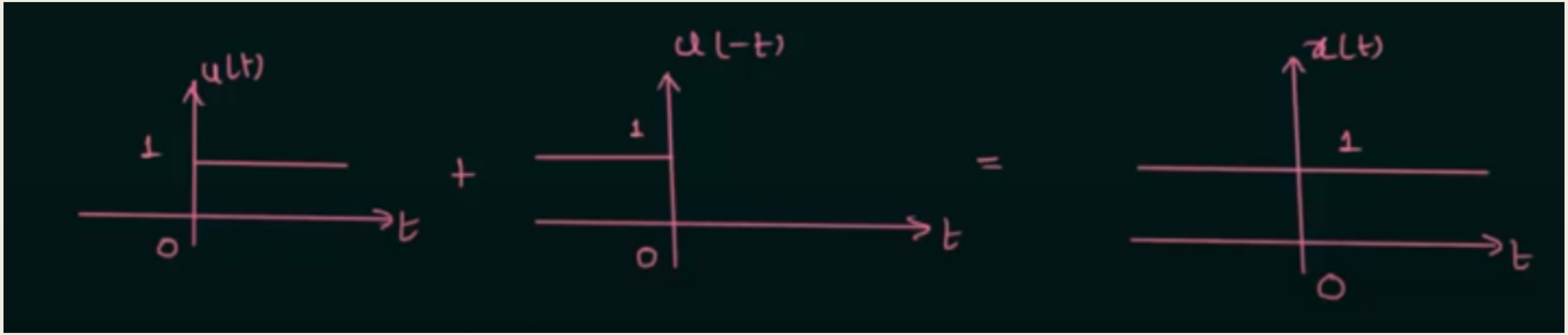
# 1. Unit Step function $(u(t)\ or\ u[n])$



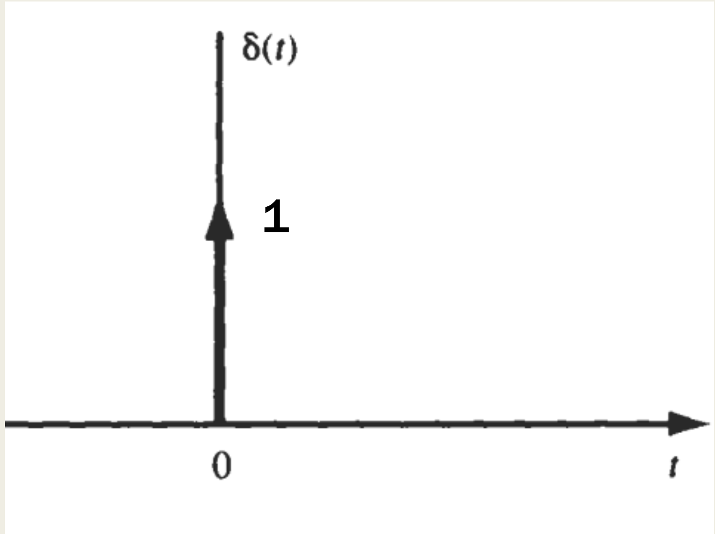



Plot $x(t)=u(t)+u(-t)$ ???

$$x(t)$$

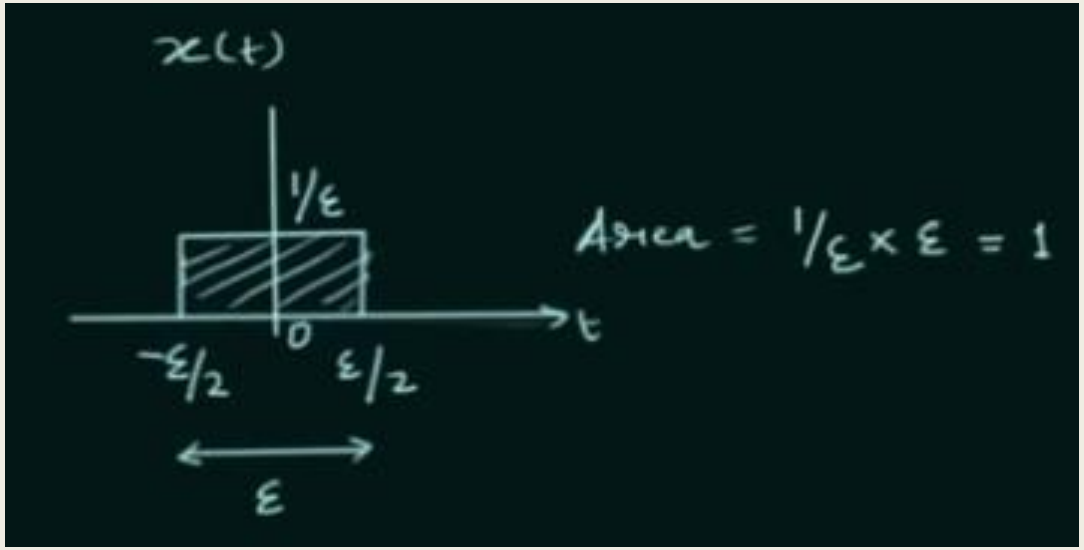


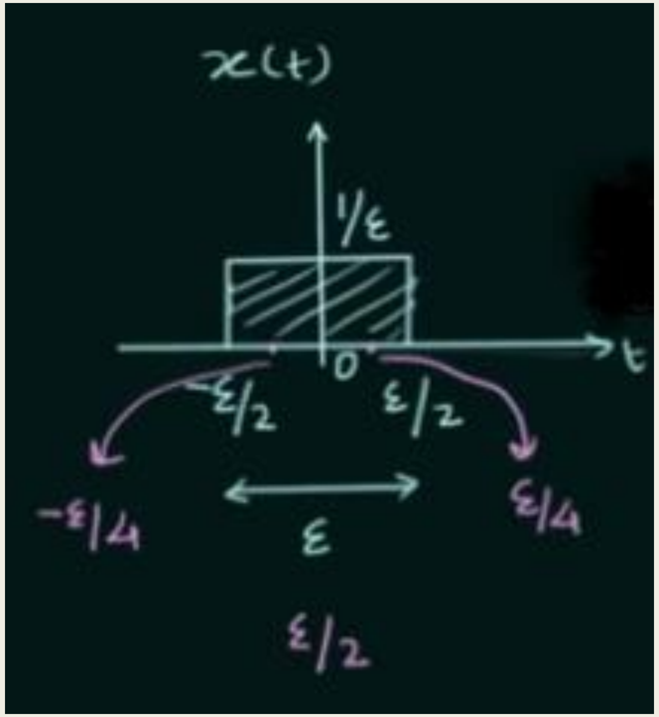
# 2. Unit Impulse Signal

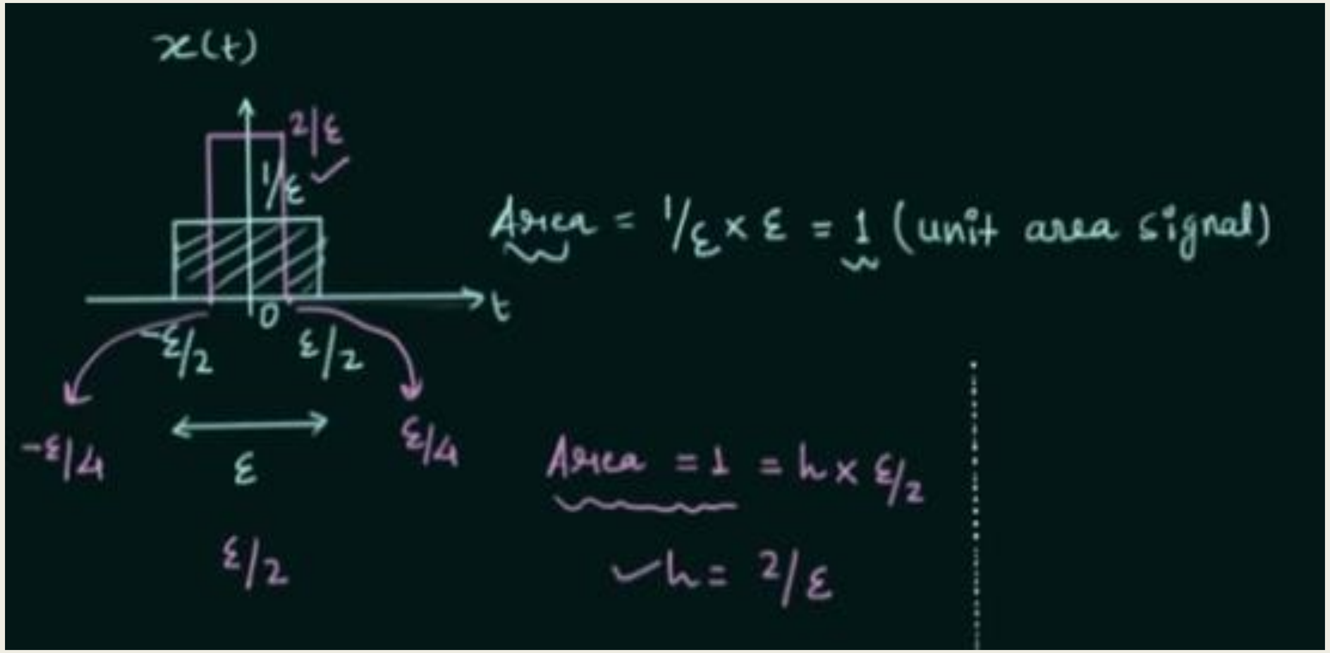



- Continuous Time Unit Impulse Signal is $\delta(t)$
- It is also known as dirac delta
- It is defined as “Area under unit impulse is ‘1’ as its width approaches zero. Thus, it has zero value everywhere except $t = 0$”
- Thus, coefficient with $\delta(t)$ shows its strength or area not amplitude

- $$\delta(t) = \begin{cases} \int_{-\infty}^{\infty} \delta(t)\,dt = 1 & for\ t = 0 \\ 0 & for\ t \neq 0 \end{cases}$$

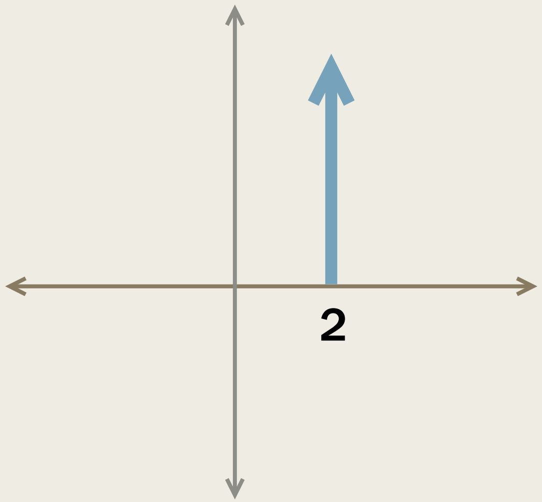

x(t)
1/ε
0
t
-ε/2
ε/2
ε
Area = 1/ε × ε = 1


x(t)
1/ε
0
t
-ε/2
ε/2
-ε/4
ε/4
ε
ε/2

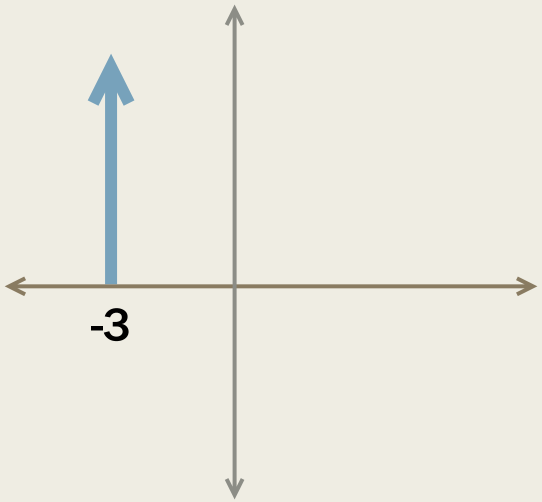

x(t)
2/ε
1/ε
0
t
-ε/2
ε/2
-ε/4
ε/4
ε
ε/2
Area = 1/ε × ε = 1 (unit area signal)
Area = 1 = h × ε/2
h = 2/ε

# A) Time Shifting

i) $\delta(t-2)$

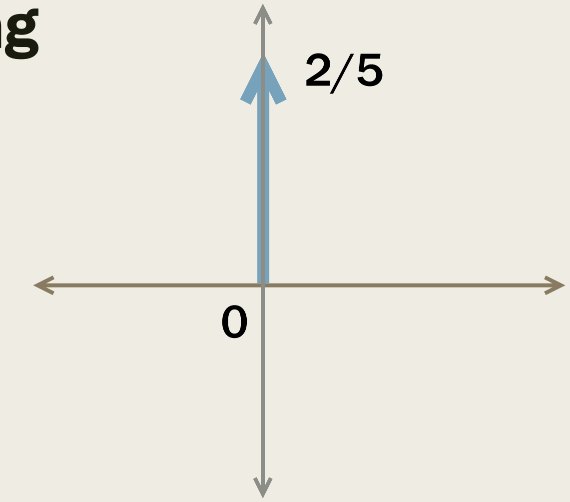



ii) $\delta(t+3)$




# B) Amplitude Scaling

iii) $\frac{2}{5}\delta(t)$

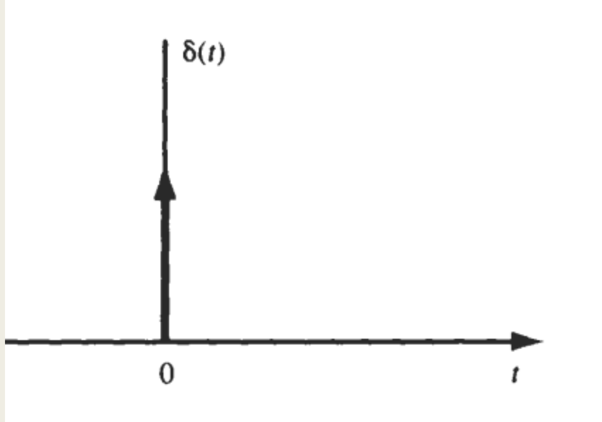


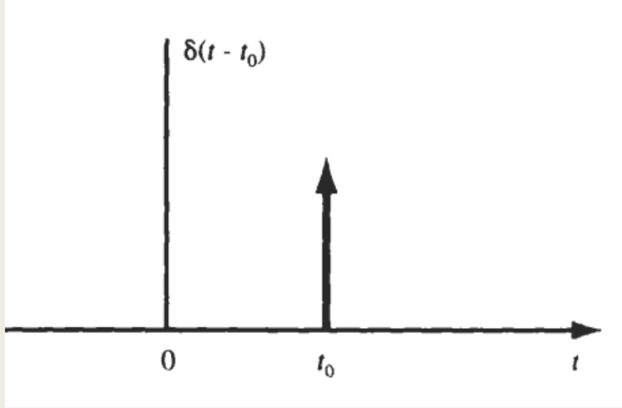

# C) Time Scaling

- $\delta(at) = \frac{1}{|a|}\delta(t)$

# Properties of CT Unit Impulse or Delta function $\delta(t)$

- 1) Integrating a unit impulse function results in '1'
  - $$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$
  - $$\int_{-\infty}^{+\infty} A\delta(t)dt = A$$
- 2) The scaled version of $\delta(at)$ is
  - $$\delta(at) = \frac{1}{|a|}\delta(t)$$
- 3) The flipped version of $\delta(t)$ is
  - $$\delta(-t) = \delta(t)$$
- 4) When an arbitrary function $f(t)$ is multiplied by a shifted impulse function, the product is given by;
  - $$\int^{+\infty} f(t)\delta(t)dt = f(t)|_{t=0}\,\delta(t)$$



## Exp 4.1: Evaluate i) $\int_{-\infty}^{+\infty} e^{-t}\delta(2t-2)\,dt$ ii) $\int_{-5}^{-2} e^{-t}\delta(2t-2)\,dt$

(i) $\int_{-\infty}^{+\infty} e^{-t}\delta(2t-2)\,dt$

ii) $\int_{-5}^{-2} e^{-t}\delta(2t-2)\,dt$

- $\delta(2t-2) = \delta[2(t-1)] = \frac{1}{2}\delta(t-1)$

- $= \int_{-\infty}^{+\infty} e^{-t}\frac{1}{2}\delta(t-1)\,dt$

- $= \frac{1}{2}\int_{-\infty}^{+\infty} e^{-t}\delta(t-1)\,dt$

- $= \frac{1}{2}e^{-t}\Big|_{t=1}$

= 0

*Unit impulse should be present between the limits of integration

# Exp 4.2: Evaluate the following integrals

(a) $\displaystyle\int_{-1}^{1} (3t^2 + 1)\delta(t)\, dt$

(b) $\displaystyle\int_{1}^{2} (3t^2 + 1)\delta(t)\, dt$

(c) $\displaystyle\int_{-\infty}^{\infty} (t^2 + \cos \pi t)\, \delta(t - 1)\, dt$

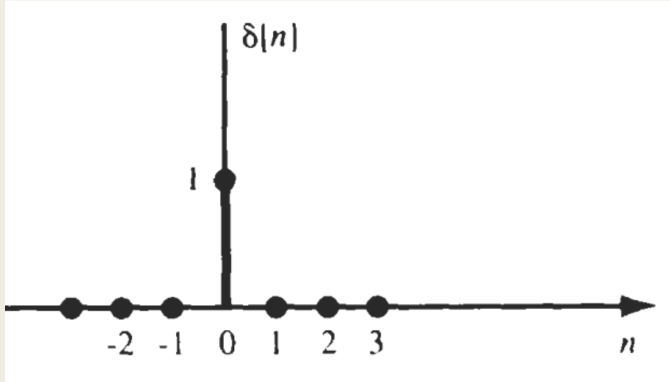
(d) $\displaystyle\int_{-\infty}^{\infty} e^{-t}\delta(2t - 2)\, dt$

# Solution:

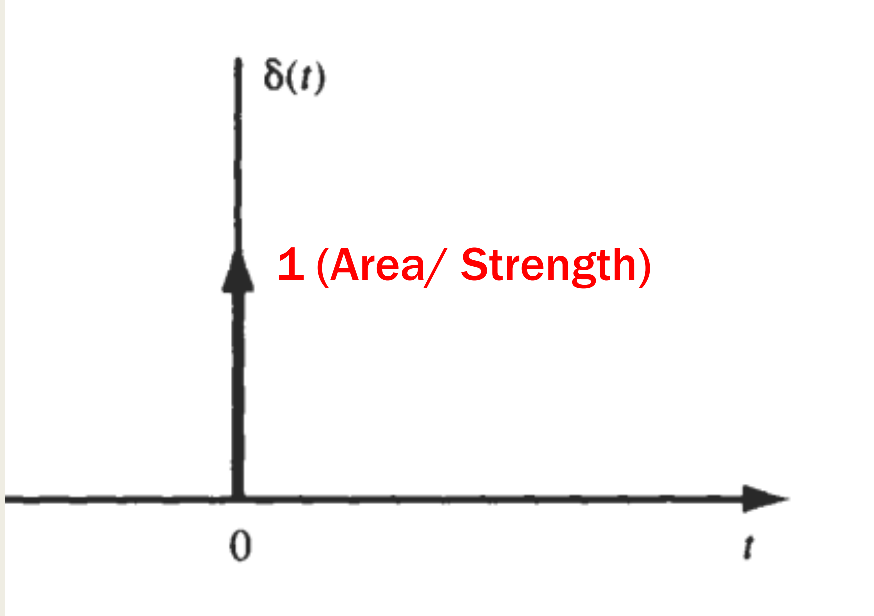
- (a) $$\int_{-1}^{1}(3t^2+1)\delta(t)\,dt=(3t^2+1)\big|_{t=0}=1$$
- (b) $$\int_{1}^{2}(3t^2+1)\delta(t)\,dt=0$$
- (c) $$\int_{-\infty}^{\infty}(t^2+\cos\pi t)\delta(t-1)\,dt=(t^2+\cos\pi t)\big|_{t=1}$$ $$=1+\cos\pi=1-1=0$$
- (d) $$\int_{-\infty}^{\infty}e^{-t}\delta(2t-2)\,dt=\int_{-\infty}^{\infty}e^{-t}\delta[2(t-1)]\,dt$$ $$=\int_{-\infty}^{\infty}e^{-t}\frac{1}{|2|}\delta(t-1)\,dt=\frac{1}{2}e^{-t}\Big|_{t=1}=\frac{1}{2e}$$

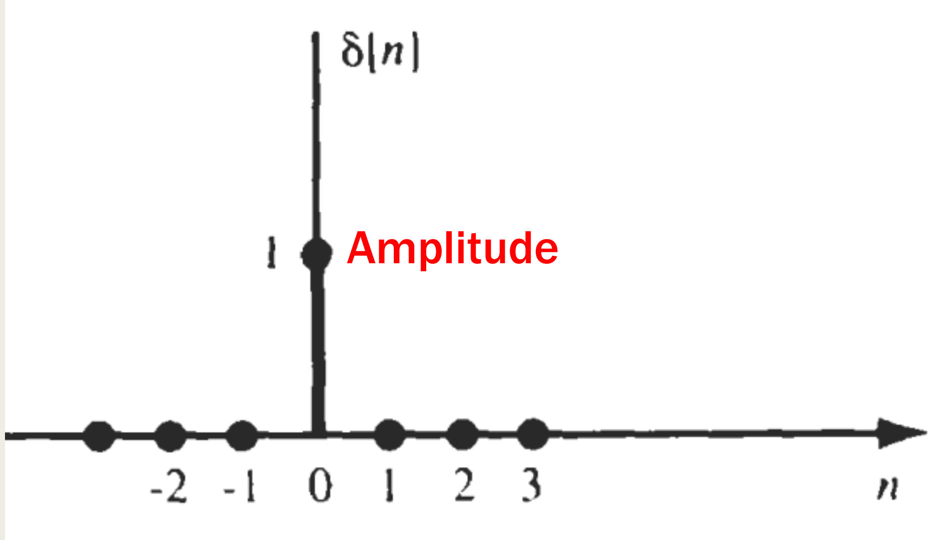
# DT Unit Sample Signal/ Unit Impulse Sequence $\delta[n]$

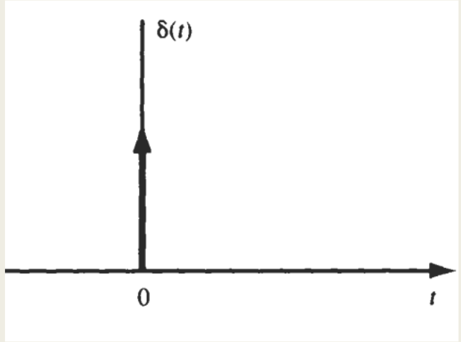
- Amplitude of unit sample is '1' at $n = 0$ and it has zero value at all other values of $n$
- $\delta[n] = \begin{cases} 1 & for\ n = 0 \\ 0 & for\ n \neq 0 \end{cases}$ or $\delta[n] = \{\dots, 0, 0, 0, \underset{\uparrow}{1}, 0, 0, 0, \dots\}$

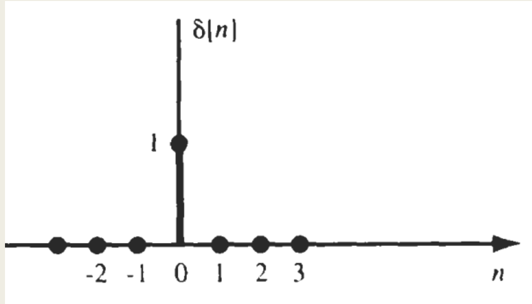



- $\delta[n]$ is not the sampled version of $\delta(t)$. *The main difference is Area under $\delta(t) = 1$ while Amplitude of $\delta[n] = 1$*

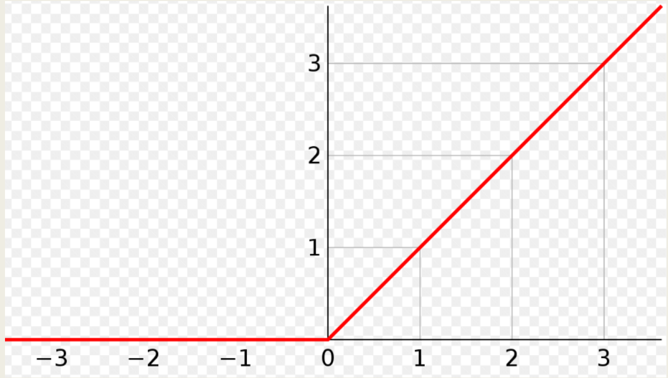

δ(t)
1 (Area/ Strength)
0
t

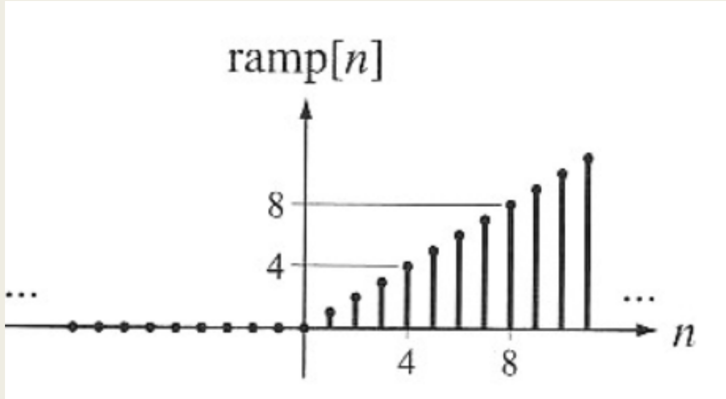

δ[n]
1
Amplitude
-2
-1
0
1
2
3
n


δ(t)
0
t


δ[n]
1
-2 -1 0 1 2 3
n

# 3. Unit Ramp function



# Relationship between the Signals

## 1. Relationship between Unit step and Unit ramp signal

- The unit ramp function is defined as,
  - $r(t) = \begin{cases} t & for\ t \geq 0 \\ 0 & for\ t < 0 \end{cases}$
- Differentiating w.r.t '$t$' gives
- $\frac{d}{dt} r(t) = \begin{cases} \frac{d}{dt}(t) & for\ t \geq 0 \\ 0 & for\ t < 0 \end{cases} \quad = \begin{cases} 1 & for\ t \geq 0 \\ 0 & for\ t < 0 \end{cases} \quad = u(t)$
- $\boxed{\therefore\ \frac{d}{dt} r(t) = u(t) \quad \text{or} \quad r(t) = \int u(t)dt}$

## 2. Relationship between Unit step and Unit Impulse signal

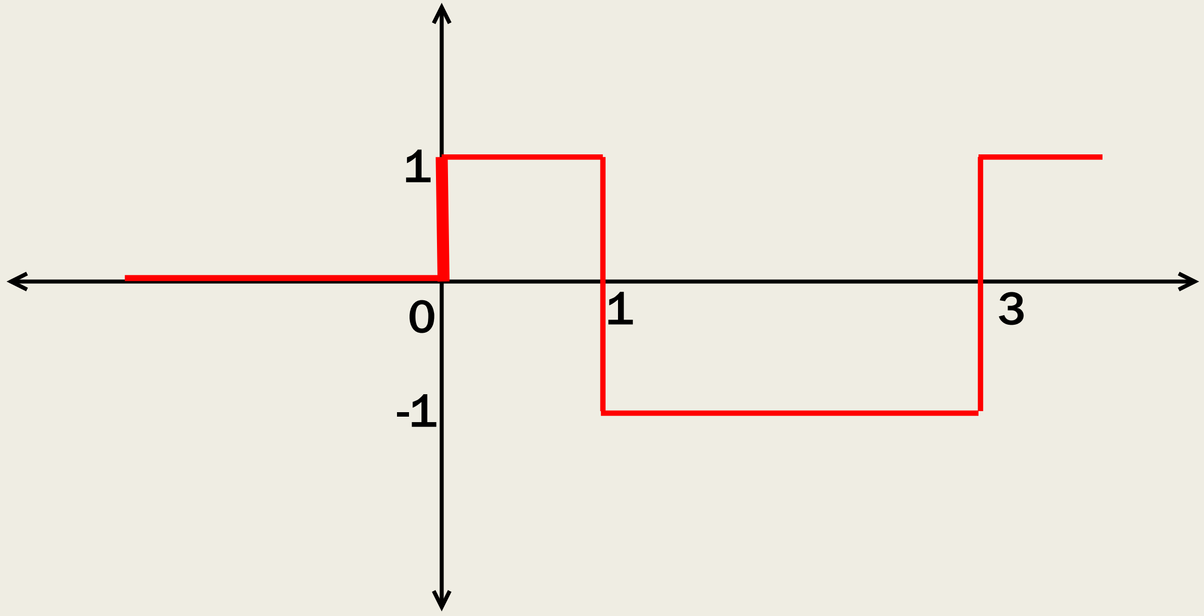
- $\frac{d}{dt}u(t) = \delta(t)$

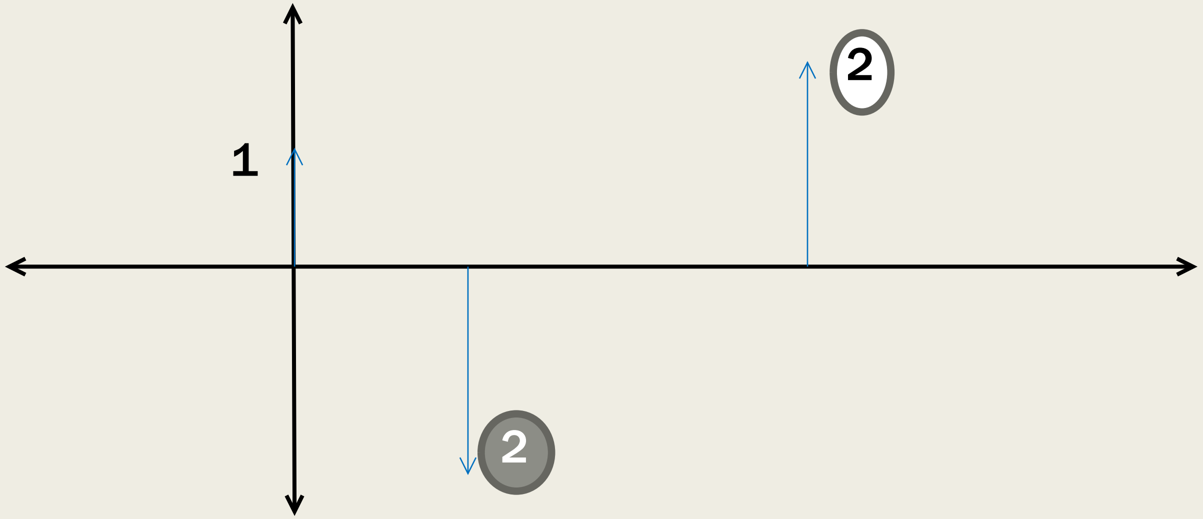
- or $u(t) = \int \delta(t)dt$

## P.P 4.1:

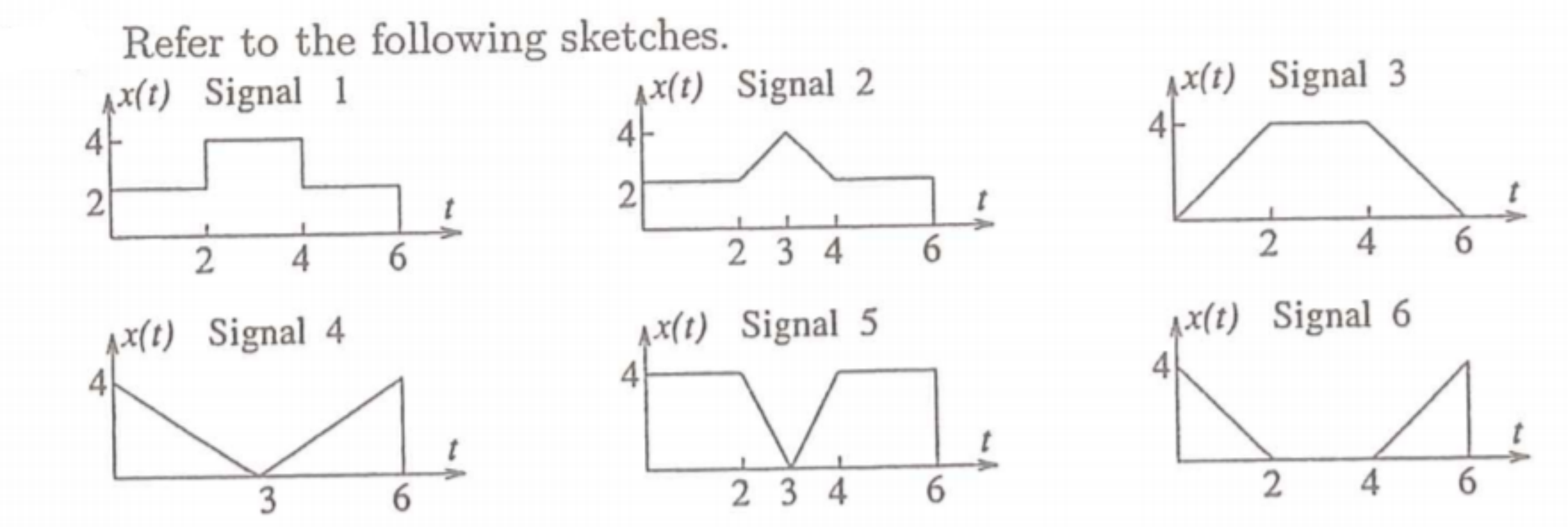
How can we write $\delta[n]$ in terms of $u[n]$.
Also write $u[n]$ in terms of $\delta[n]$

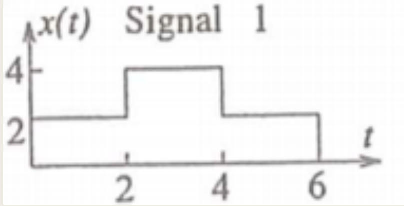
# Exp 4.3: Draw waveform for the differentiated signal (****)



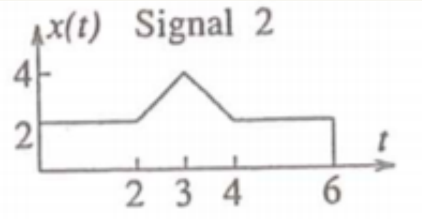



# **Exp 4.4:** Draw waveform for the differentiated version of signals from 1 to 6

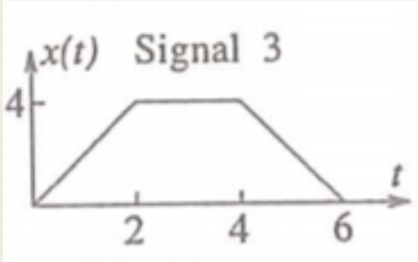




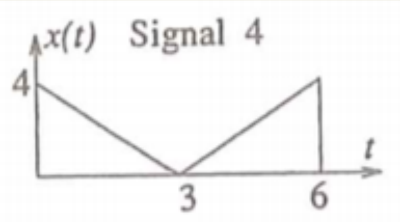

$$\text{(Signal 1:)} \quad x(t) = \begin{cases} 2 & 0 < t < 2 \\ 4 & 2 < t < 4 \\ 2 & 4 < t < 6 \\ 0 & \text{elsewhere} \end{cases}$$



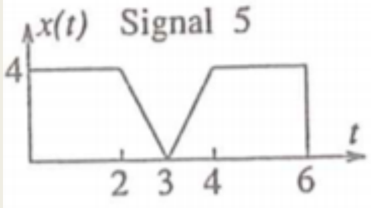

$$\text{(Signal 2:)} \quad x(t) = \begin{cases} 2 & 0 < t \leq 2 \\ 2t - 2 & 2 \leq t \leq 3 \\ -2 + 10 & 3 \leq t \leq 4 \\ 2 & 4 \leq t < 6 \\ 0 & \text{elsewhere} \end{cases}$$



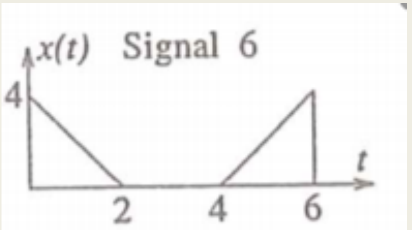

$$\text{(Signal 3:)} \quad x(t) = \begin{cases} 2t & 0 \leq t \leq 2 \\ 4 & 2 \leq t \leq 4 \\ -2t + 12 & 4 \leq t \leq 6 \\ 0 & \text{elsewhere} \end{cases}$$




$$\text{(Signal 4:)} \quad x(t) = \begin{cases} -2t + 4 & 0 < t \leq 2 \\ 2t - 4 & 2 \leq t < 4 \\ 0 & \text{elsewhere} \end{cases}$$

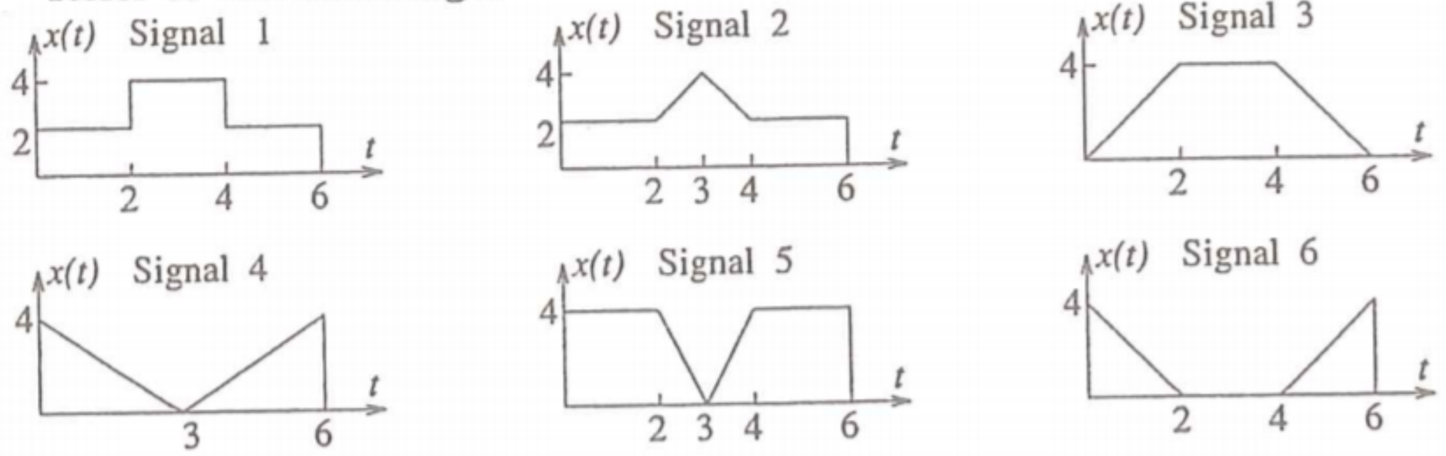



$$\text{(Signal 5:)} \quad x(t) = \begin{cases} 4 & 0 < t \leq 2 \\ -4t + 12 & 2 \leq t \leq 3 \\ 4t - 12 & 3 \leq t \leq 4 \\ 4 & 4 \leq t < 6 \\ 0 & \text{elsewhere} \end{cases}$$

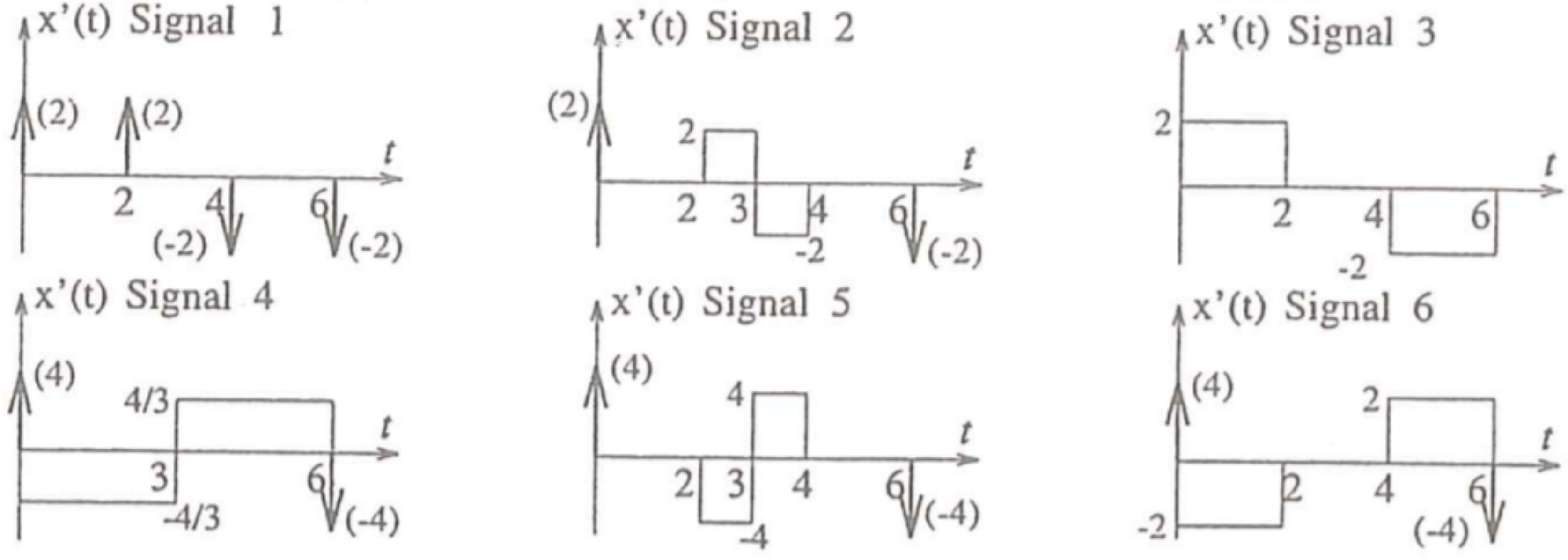



$$\text{(Signal 6:)} \quad x(t) = \begin{cases} -2t + 4 & 0 < t \leq 2 \\ 2t - 8 & 4 \leq t < 6 \\ 0 & \text{elsewhere} \end{cases}$$

Refer to the following sketches.

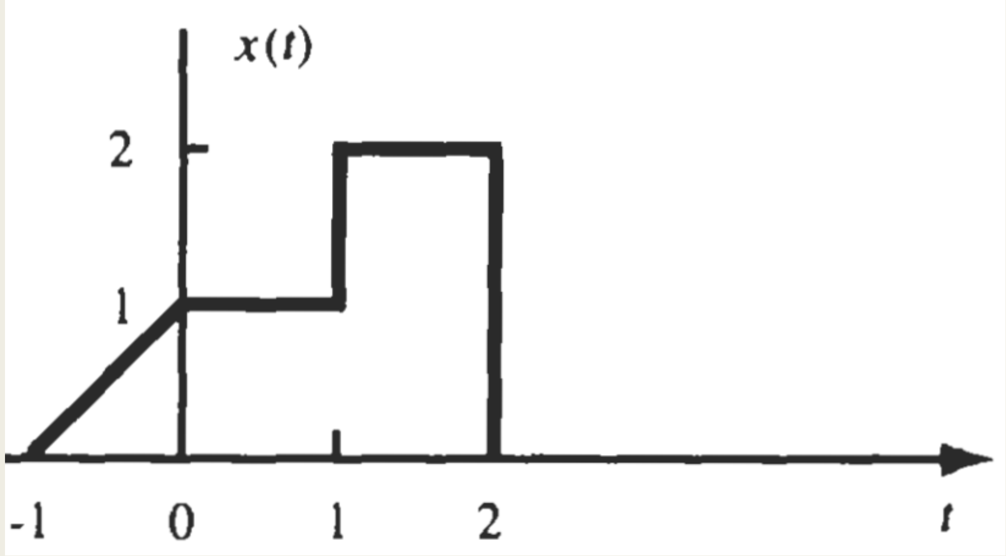


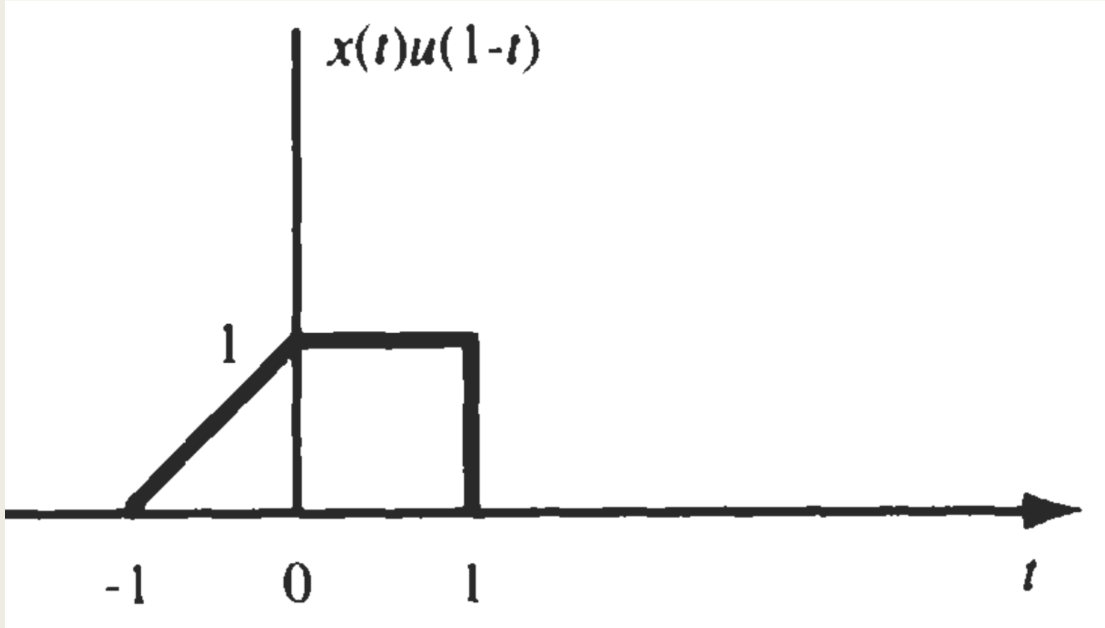

The first derivative $x'(t)$ will contain impulses at the discontinuities:



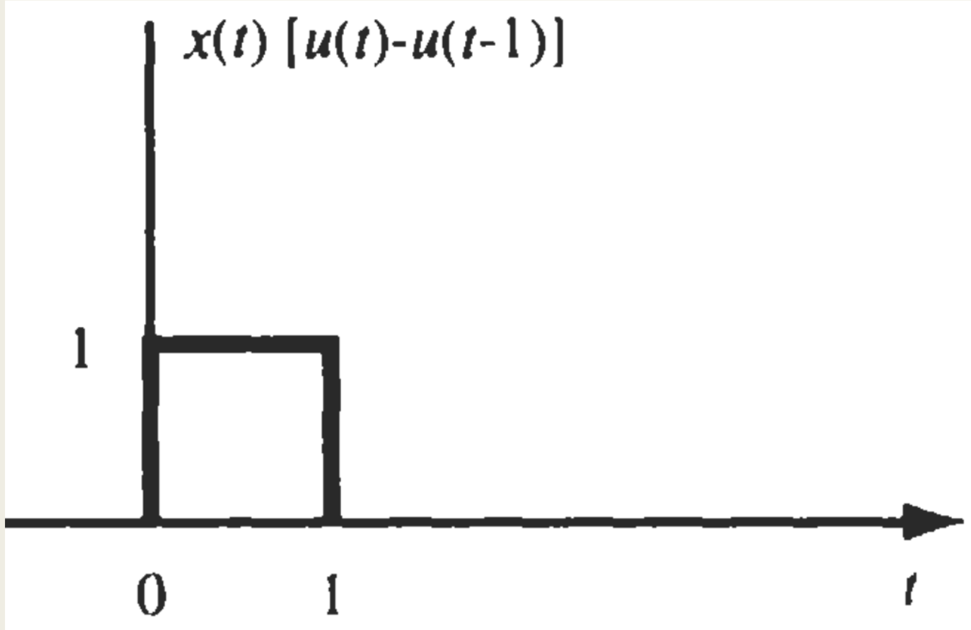
## Exp 4.5: A CTS $x(t)$ is shown in Fig. Sketch and label each of the following signals

(a) $x(t)u(1-t)$; (b) $x(t)[u(t)-u(t-1)]$; (c) $x(t)\delta(t-\frac{3}{2})$

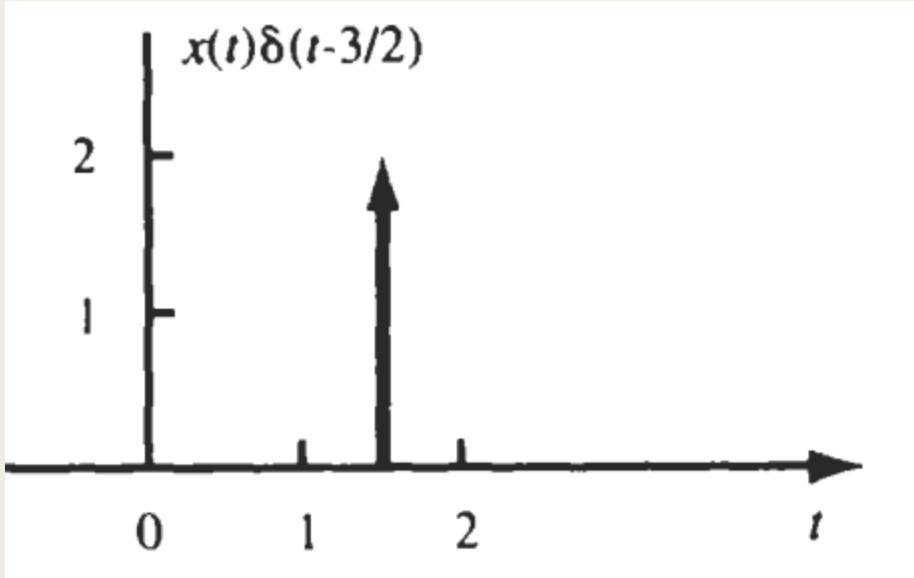




x(t)u(1-t)
1
-1
0
1
t

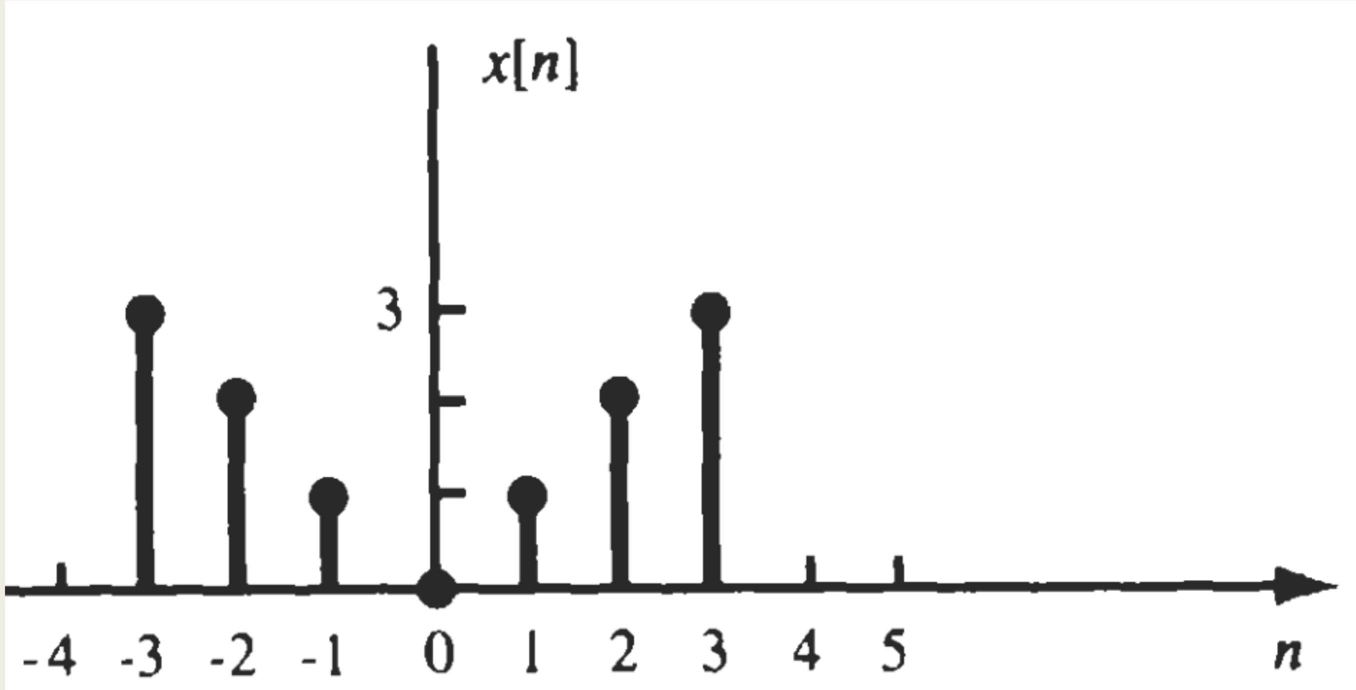
x(t) [u(t)-u(t-1)]
1
0
1
t

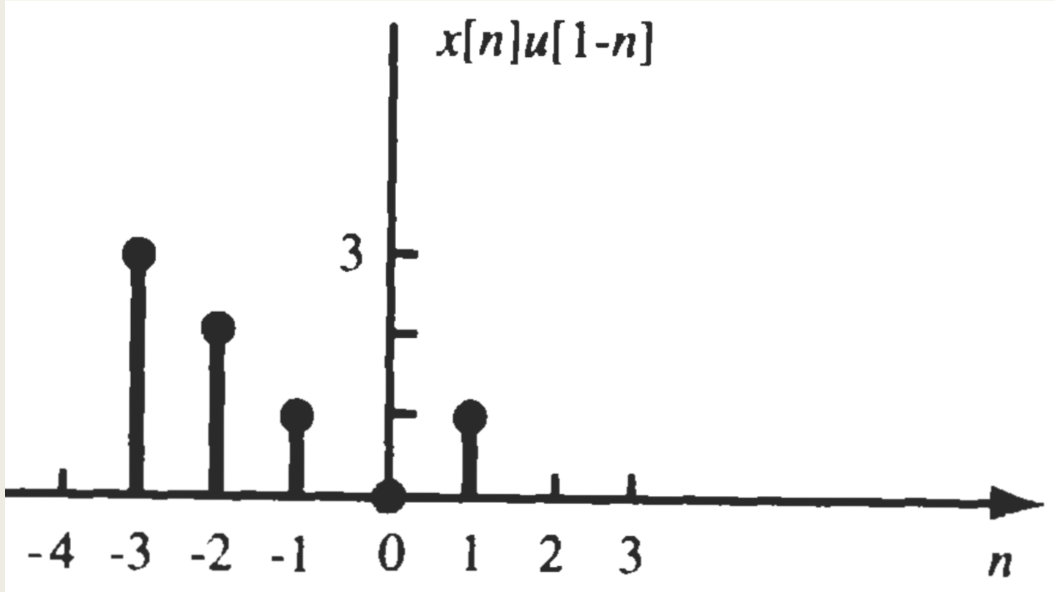



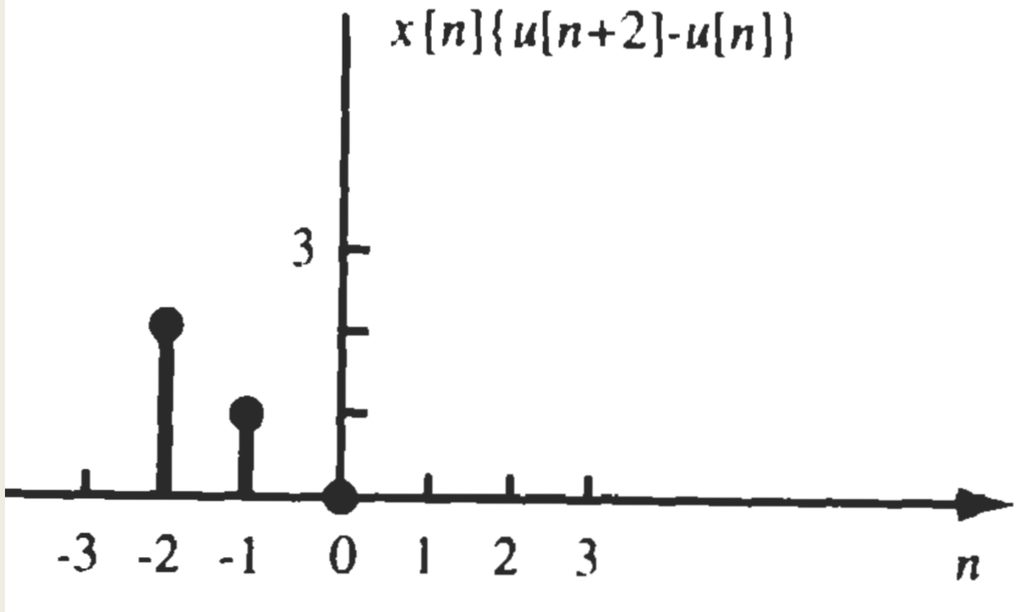
x(t)δ(t-3/2)
2
1
0
1
2
t

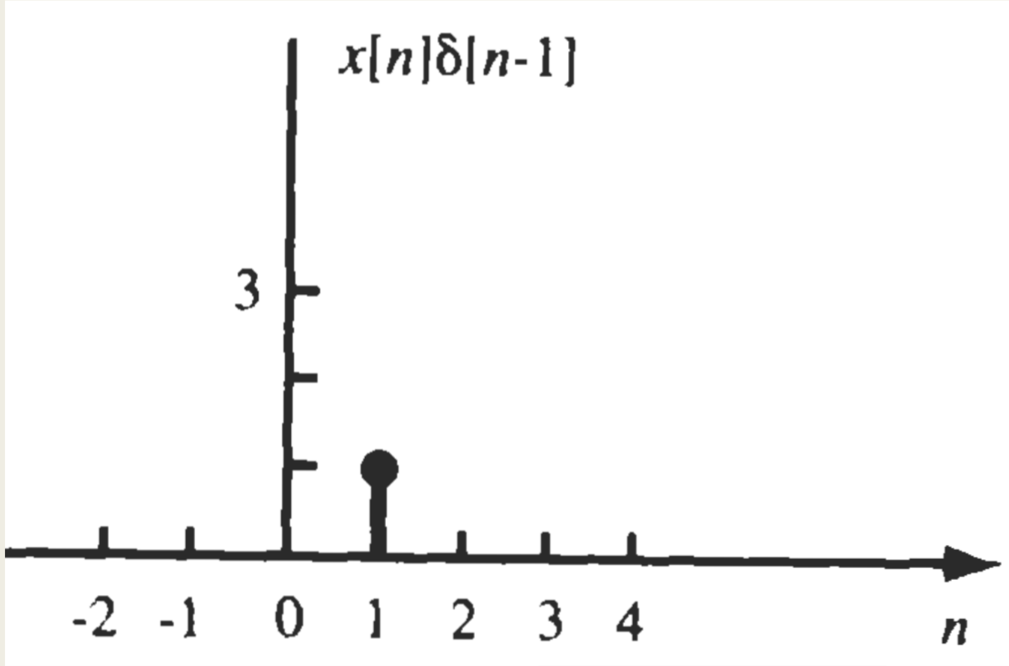
## Exp 4.6: A DTS $x$[n] is shown in Fig. Sketch and label each of the following signals

$$(a)\ x[n]u[1-n];\ (b)\ x[n]\{u[n+2]-u[n]\};\ (c)\ x[n]\delta[n-1]$$

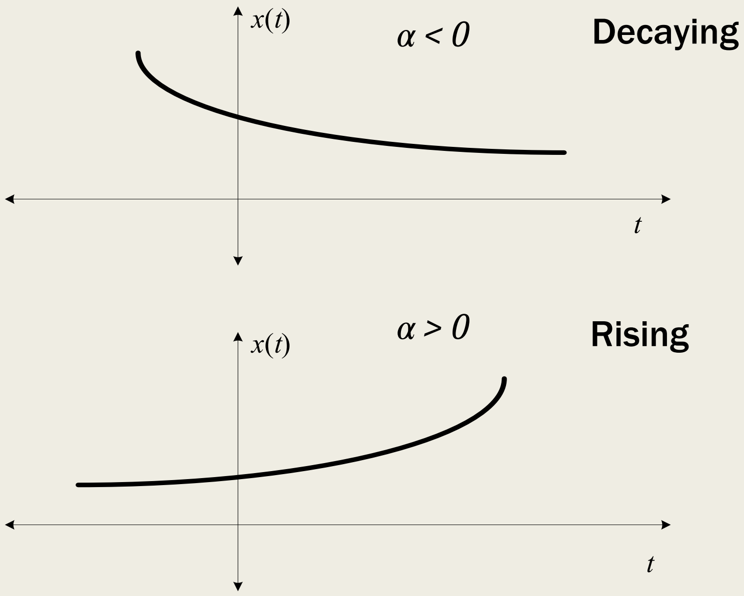


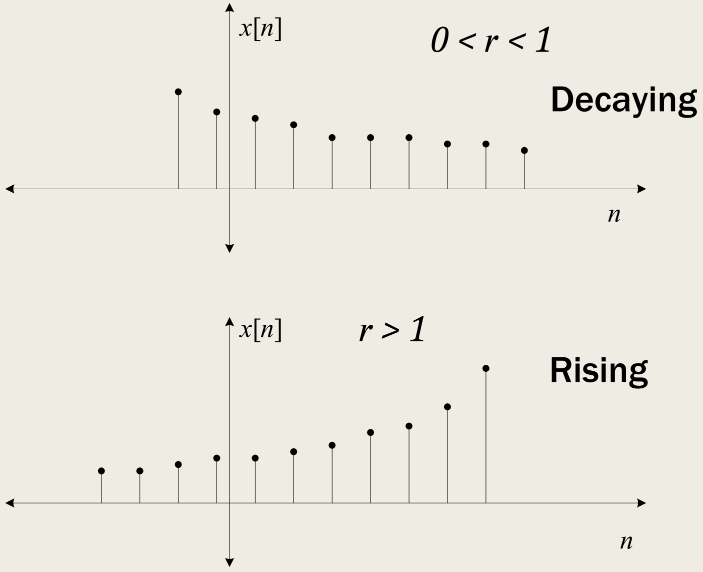

x[n]u[1-n]
3
-4 -3 -2 -1 0 1 2 3
n


x[n]{u[n+2]-u[n]}
3
-3 -2 -1 0 1 2 3
n


x[n]δ[n-1]
3
-2 -1 0 1 2 3 4
n

# 4. Real Exponential Signal



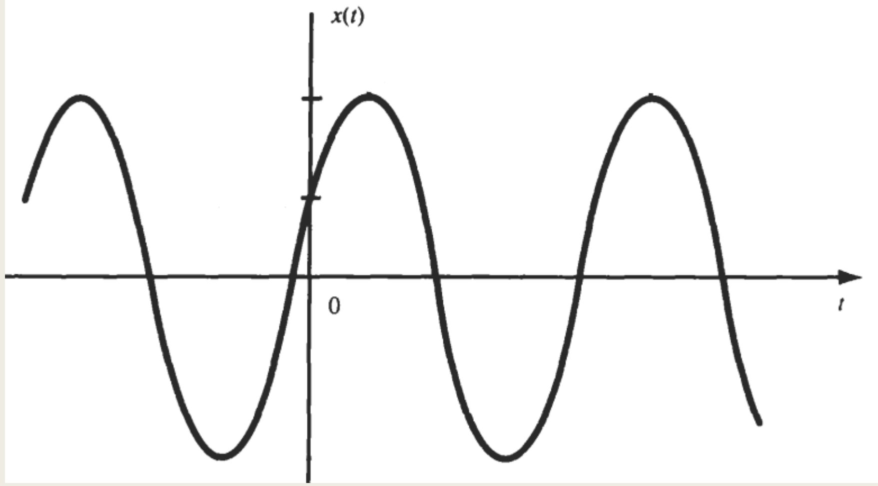



# 5. Complex Exponential Signal

- When exponent is purely imaginary, then signal is said to be complex exponential
- It is given as
  - CT: $x(t) = e^{j\omega t}$
  - DT: $x[n] = e^{j\omega n}$

# 6. Sinusoidal Signal

- It is given as
  - CT: $x(t) = \cos(\omega t + \phi)$
  - DT: $x[n] = \cos(\omega n + \phi)$



## P.P 4.2: Evaluate the following integrals

(a) $\int_{-1}^{8} [u(t+3) - 2\delta(t)u(t)]\,dt$

(b) $\int_{1/2}^{5/2} \delta(3t)\,dt$

- **Solution:**
- (a) Ans: 7

# P.P 4.3: Draw waveforms of the following

(a) $f_1(t) = 3u(t-1)$

(b) $f_2(t) = u(2-t)$

(c) $f(t) = f_1(t)f_2(t)$

# Thank You !!!