

Lecture 7: Expected Values of Random Variables

CPE251 Probability Methods in Engineering

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Characteristics of Random Data

Measures of central tendency: Mean, Average Median, Expected Value

Measures of dispersion: Variance, Standard Deviation, Mode

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Expected Value

Let X be a random variable with probability distribution $f(x)$. The mean, or expected value, of X is

$$\mu = E(X) = \sum_x x f(x)$$

if X is discrete, and

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

if X is continuous.

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Example

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

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Example

Let X be the random variable that denotes the life in hours of a certain electronic device. The probability density function is

$$f(x) = \begin{cases} \frac{20,000}{x^3}, & x > 100, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected life of this type of device.

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Variance

Let X be a random variable with probability distribution $f(x)$ and mean μ . The variance of X is

$$\sigma^2 = E[(X - \mu)^2] = \sum_x (x - \mu)^2 f(x), \quad \text{if } X \text{ is discrete, and}$$

$$\sigma^2 = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx, \quad \text{if } X \text{ is continuous.}$$

The positive square root of the variance, σ , is called the **standard deviation** of X .

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Standard Deviation

The square root of variance σ_X^2 of a random variable X is known as standard deviation, denoted by σ .

$$\sqrt{\text{Var}(X)} = \sqrt{\sigma_X^2} = SD(X) = \sigma$$

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Variance: Alternative Expression

The variance of a random variable X is


$$\sigma^2 = E(X^2) - \mu^2.$$

For the discrete case, we can write

$$\begin{aligned}\sigma^2 &= \sum_x (x - \mu)^2 f(x) = \sum_x (x^2 - 2\mu x + \mu^2) f(x) \\ &= \sum_x x^2 f(x) - 2\mu \sum_x x f(x) + \mu^2 \sum_x f(x).\end{aligned}$$

Since $\mu = \sum_x x f(x)$ by definition, and $\sum_x f(x) = 1$ for any discrete probability distribution, it follows that

$$\sigma^2 = \sum_x x^2 f(x) - \mu^2 = E(X^2) - \mu^2.$$

For the continuous case the proof is step by step the same, with summations replaced by integrations. 

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Example (DRV)

Let the random variable X represent the number of defective parts for a machine when 3 parts are sampled from a production line and tested. The following is the probability distribution of X .

| x | 0 | 1 | 2 | 3 |
|--------|------|------|------|------|
| $f(x)$ | 0.51 | 0.38 | 0.10 | 0.01 |

Using Theorem 4.2, calculate σ^2 .

Example (CRV)

The weekly demand for a drinking-water product, in thousands of liters, from a local chain of efficiency stores is a continuous random variable X having the probability density

$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the mean and variance of X .

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References

1. Walpole, R.E., Myers, R.H., Myers, S.L. and Ye, K. (2007) *Probability & Statistics for Engineers & Scientists*. 9th Edition, Pearson Education, Inc.
2. Leon-Garcia, A. (2008). *Probability, Statistics, and Random Processes for Electrical Engineering*. 3rd Edition, Pearson/Prentice Hall.

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