Lecture 10

Mesh Analysis with current sources

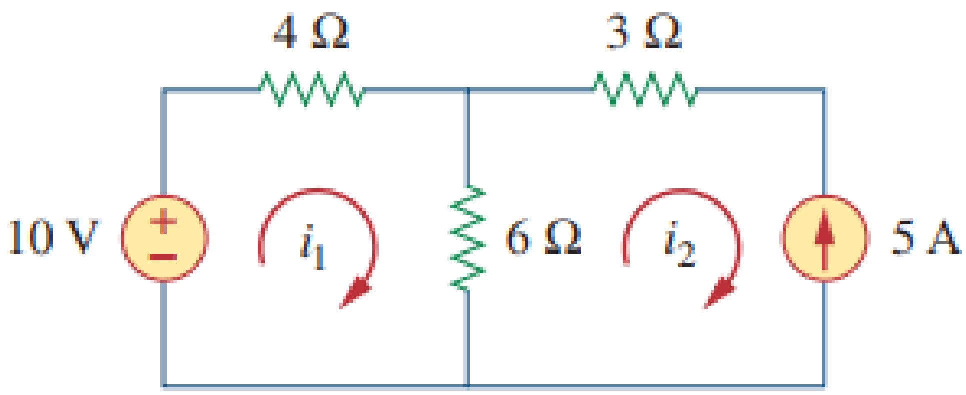
Objectives of Lecture

- Provide step-by-step instructions for mesh analysis, which is a method to calculate voltage drops and mesh currents that flow around loops in a circuit.

Mesh Analysis with Current Sources

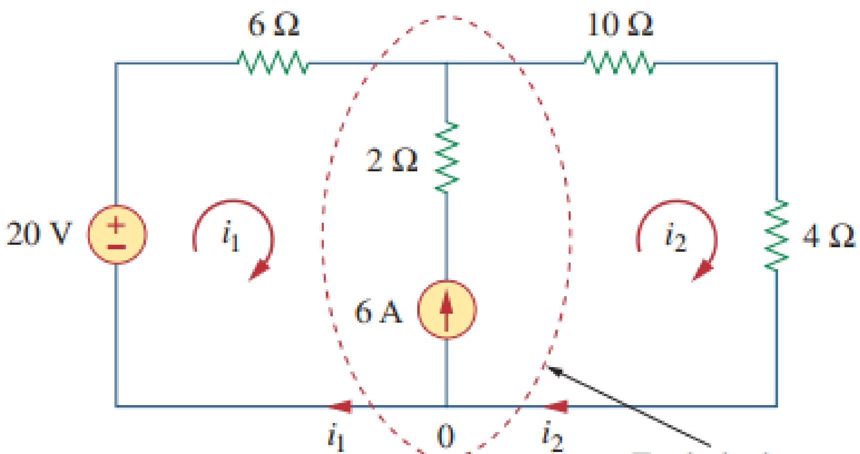
CASE 1 When a current source exists only in one mesh: Consider the circuit in Fig. 3.22, for example. We set $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way; that is,

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \qquad \Rightarrow \qquad i_1 = -2 \text{ A} \qquad \textbf{(3.17)}$$

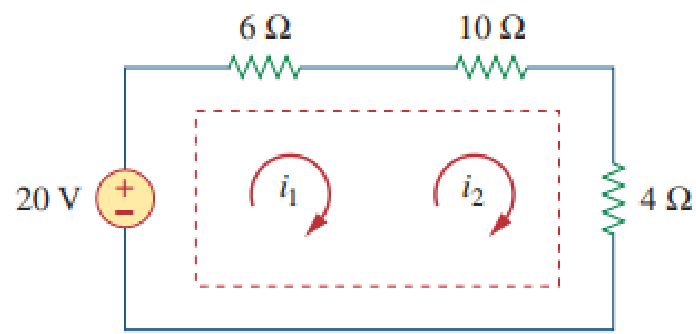


■ **CASE 2** When a current source exists between two meshes: Consider the circuit in Fig. 3.23(a), for example. We create a *supermesh* by excluding the current source and any elements connected in series with it, as shown in Fig. 3.23(b). Thus,

> A supermesh results when two meshes have a (dependent or independent) current source in common.



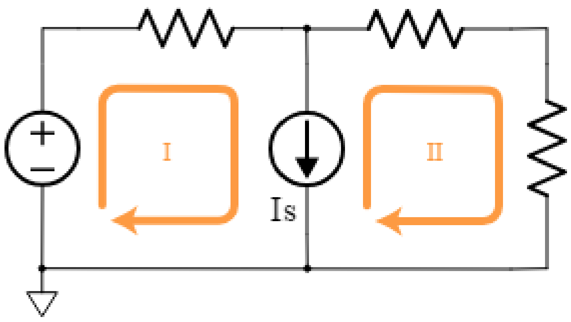
(a)



(b)

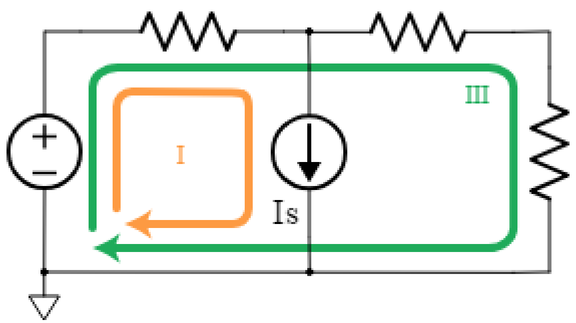
Mesh Analysis with Supermeshes

- Consider the following circuit.



  – Both mesh I and mesh II go through the current source.
    - It is possible to write and solve mesh equations for this configuration.

- Using supermesh



  – You can drop one of the meshes and replace it with the loop that goes around both meshes, as shown here for loop III.
  – You then solve the system of equations exactly the same as the Mesh Analysis

Mesh Analysis with Supermeshes

supermesh = a mesh that contains multiple meshes with a shared current source

For **nodal** analysis, we joined nodes near a **voltage** source. → super<u>node</u>
For **mesh** analysis, we join meshes near a **current** source. → super<u>mesh</u>

→ Reduces the number of simultaneous equations by the number of current sources.

Analysis Steps

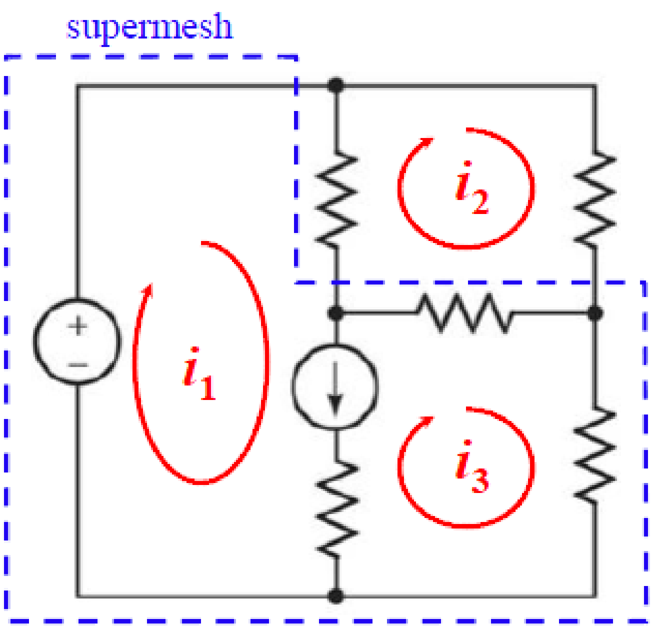
(1) Draw a mesh current for each mesh.

(2) Identify supermeshes.

(3) Write KVL around each supermesh, then KVL for each mesh that is not part of a supermesh.

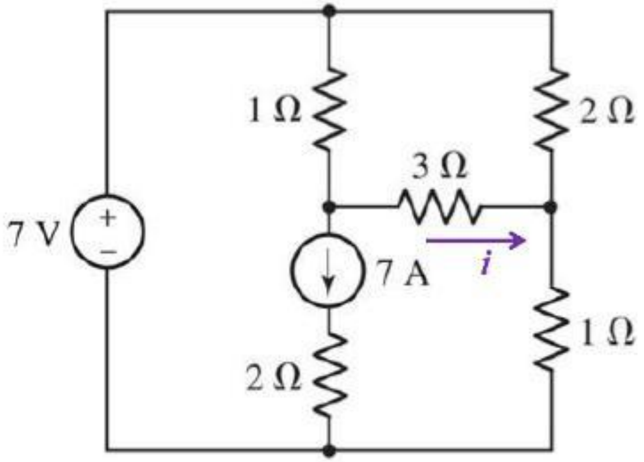
(4) Express additional unknowns (dependent V/I) in terms of mesh currents.

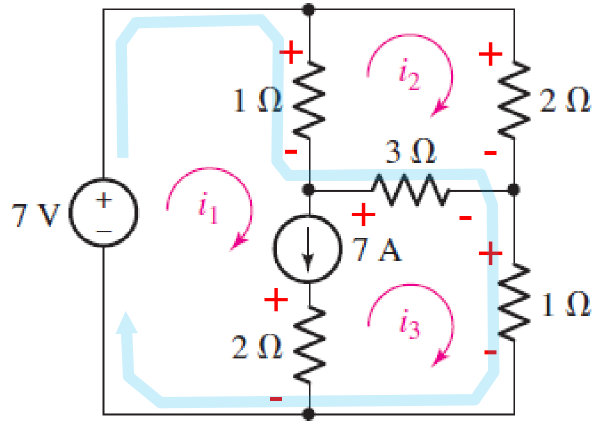
(5) Solve the simultaneous equations.



Example 01

- Determine the current *i* as labeled in the circuit.





- Supermesh $-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$
- Mesh 2 $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$

$$-i_1 + 6i_2 - 3i_3 = 0$$

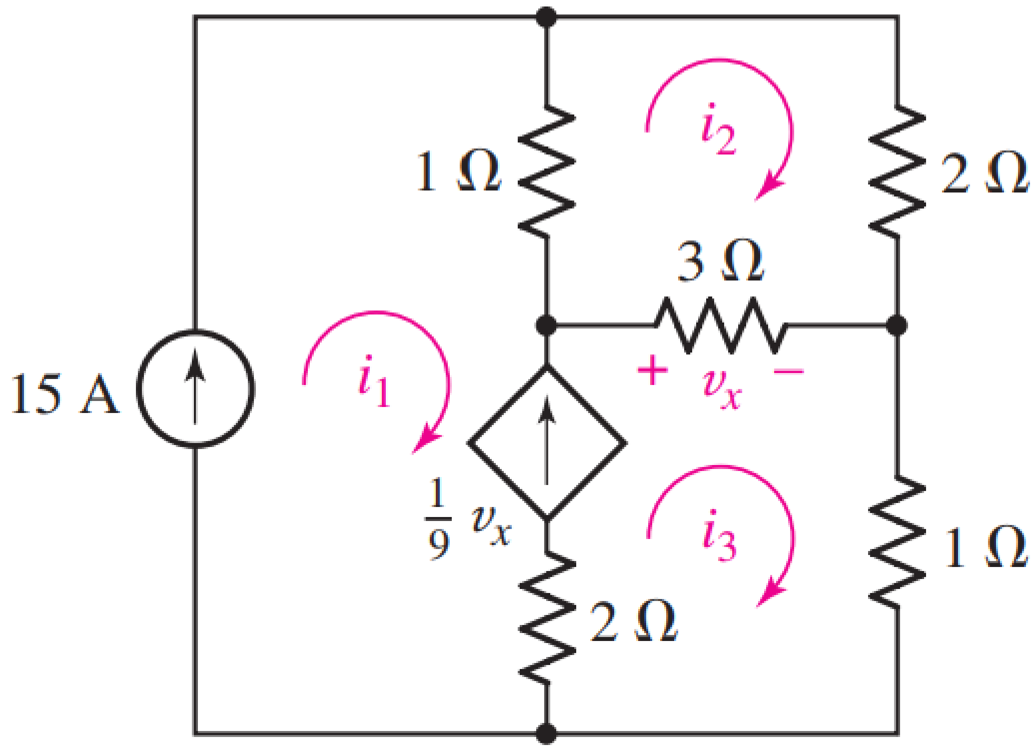
- Independent source current is related to the mesh currents

$$i_1 - i_3 = 7$$

$i_1 = 9\text{ A}, i_2 = 2.5\text{ A},$ $i_3 = 2\text{ A}$

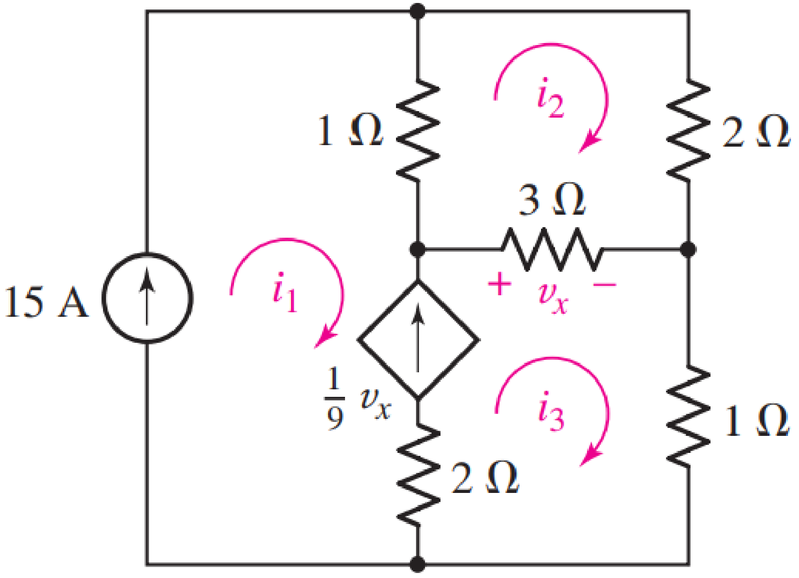
Example 02

- Evaluate three unknown currents in the circuit.



- Mesh 1:

$$i_1 = 15 \text{ A}$$



- Supermesh:

$$\frac{v_x}{9} = i_3 - i_1 = \frac{3(i_3 - i_2)}{9}$$

- Mesh 2

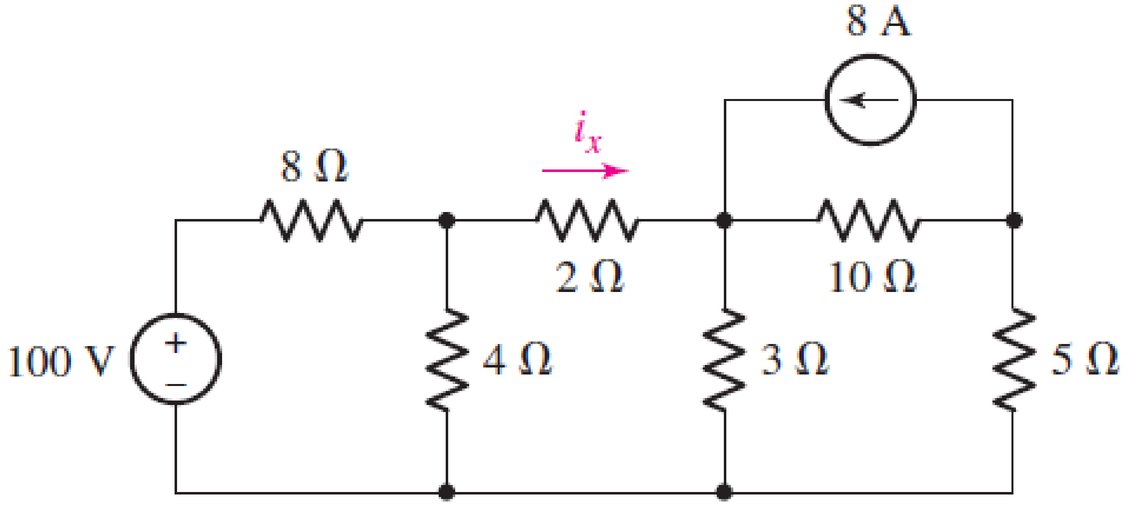
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

or

$$6i_2 - 3i_3 = 15$$

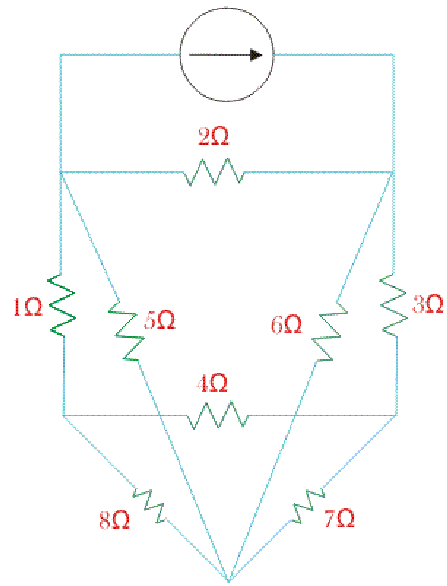
Nodal vs. Mesh Analysis: A Comparison

- The following is a planar circuit with 5 nodes and 4 meshes.
  - Planar circuits are circuits that can be drawn on a plane surface with no wires crossing each other.
- Determine the current i_x

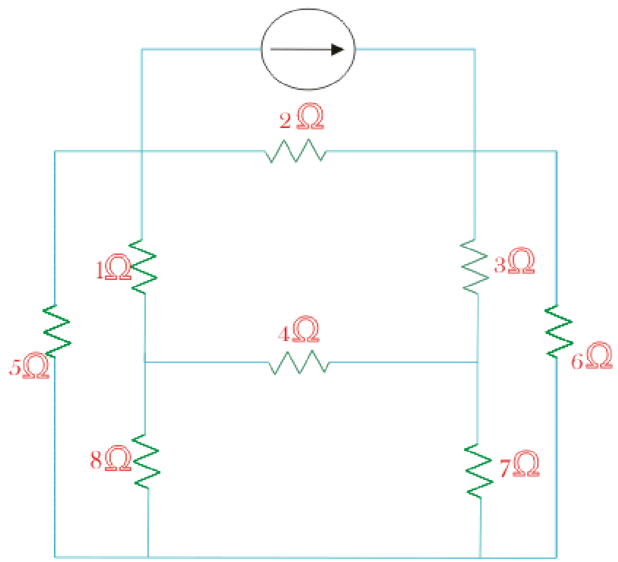


Planar vs Non-planar circuits

- Planar

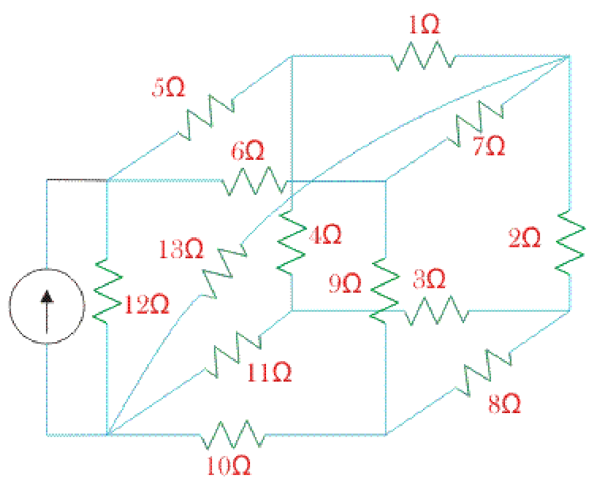


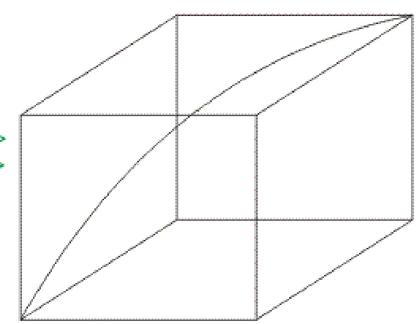
Original Circuit looks non-planar



Modified Circuit appear as planar

- Non-planar

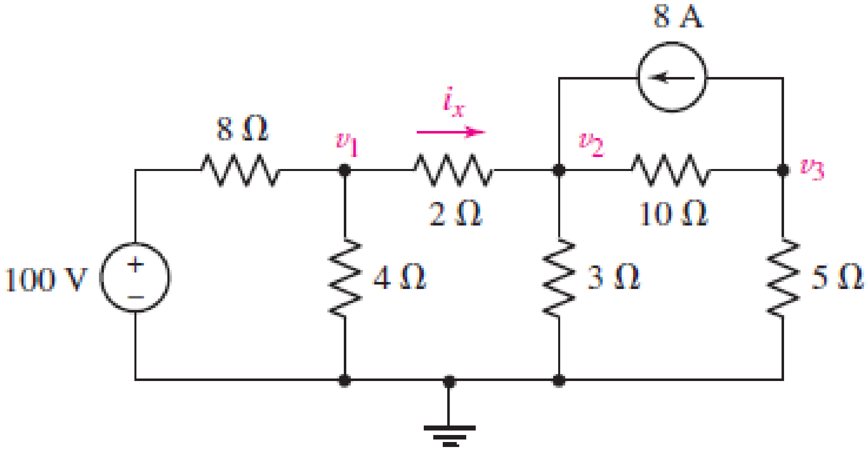




Non-planar Graph

Nodal vs. Mesh Analysis: A Comparison

- Using Nodal Analysis



  - Although we can write four distinct equations, there is no need to label the node between the 100 V source and the 8 ohm resistor, since that node voltage is clearly 100 V.

- We write the following three equations:

$$\frac{v_1 - 100}{8} + \frac{v_1}{4} + \frac{v_1 - v_2}{2} = 0 \quad \text{or} \quad 0.875v_1 - 0.5v_2 = 12.5$$

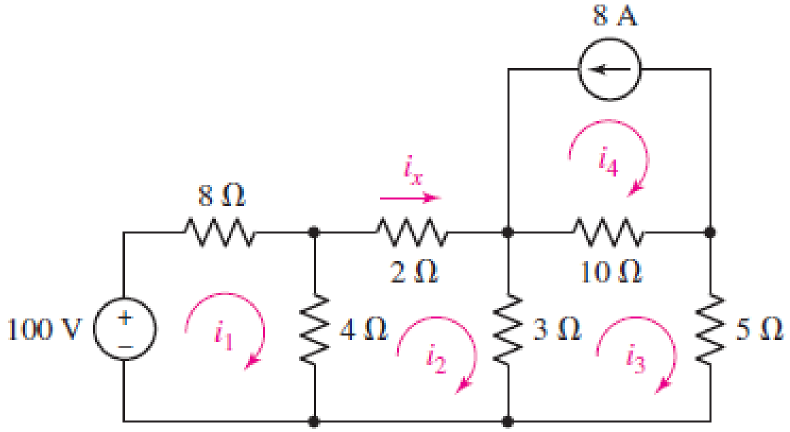
$$\frac{v_2 - v_1}{2} + \frac{v_2}{3} + \frac{v_2 - v_3}{10} - 8 = 0 \quad \text{or} \quad -0.5v_1 - 0.9333v_2 - 0.1v_3 = 8$$

$$\frac{v_3 - v_2}{10} + \frac{v_3}{5} + 8 = 0 \quad \text{or} \quad -0.1v_2 + 0.3v_3 = -8$$

- Solving, we find that $v_1 = 25.89\text{ V}$ $v_2 = 20.31\text{ V}$ $i_x = \frac{v_1 - v_2}{2} = 2.79\text{ A}$

Nodal vs. Mesh Analysis: A Comparison

- Using Mesh Analysis



  - We see that we have four distinct meshes
  - However it is obvious that $i_4 = -8$ A
  - We therefore need to write three distinct equations.

- Writing a KVL equation for meshes 1, 2, and 3:

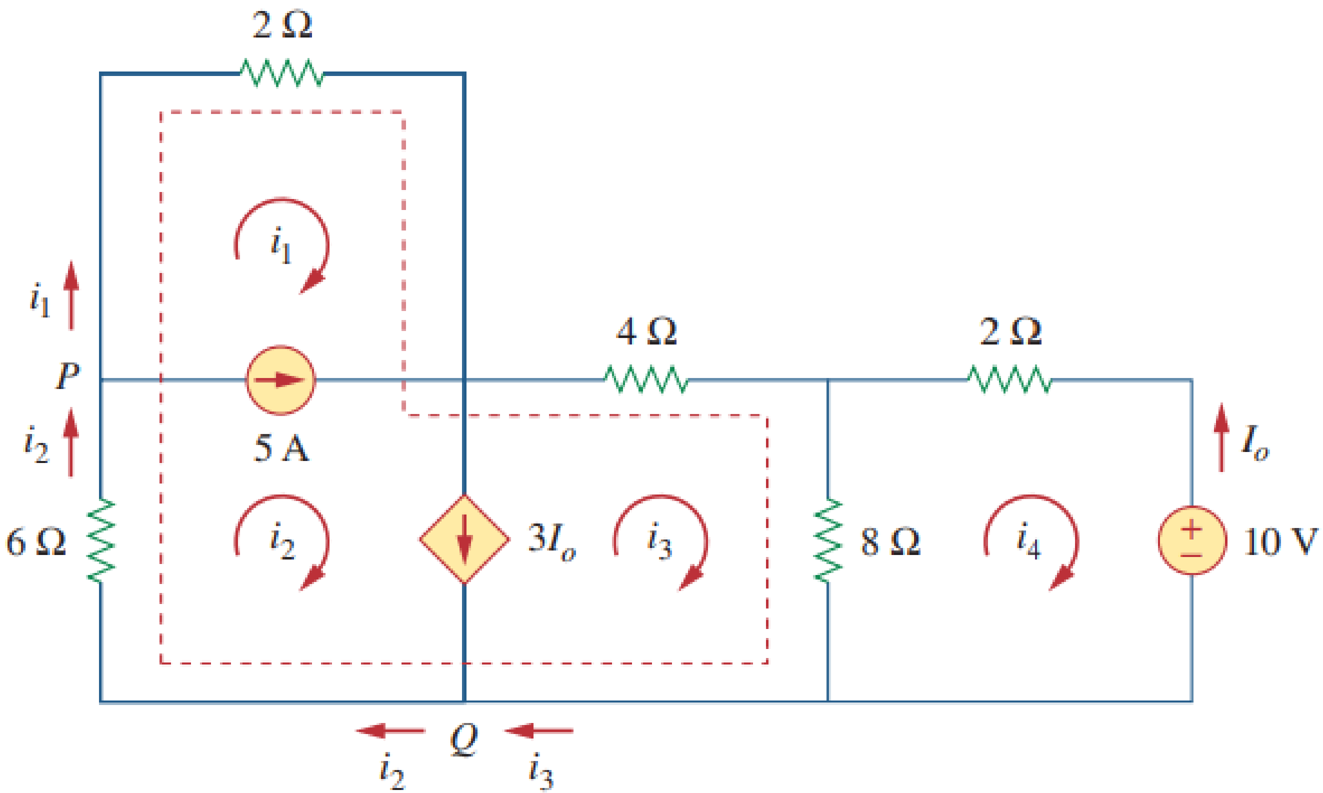
$$-100 + 8i_1 + 4(i_1 - i_2) = 0 \quad \text{or} \quad 12i_1 - 4i_2 = 100$$

$$4(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \quad \text{or} \quad -4i_1 + 9i_2 - 3i_3 = 0$$

$$3(i_3 - i_2) + 10(i_3 + 8) + 5i_3 = 0 \quad \text{or} \quad -3i_2 + 18i_3 = -80$$

- Solving, we find that $i_2\ (= i_x) = 2.79$ A

Example 03



Thank You