Lecture 3

Chapter 2: Basic Laws

Ohm's Law

Kirchhoff's Current Law

Kirchhoff's Voltage Law

Objectives of the Lecture

- Present Kirchhoff’s Current and Voltage Laws.
- Demonstrate how these laws can be used to find currents and voltages in a circuit.
- Explain how these laws can be used in conjunction with Ohm’s Law.

Resistivity, ρ

- Resistivity is a material property
  - Dependent on the number of free or mobile charges (usually electrons) in the material.
    - In a metal, this is the number of electrons from the outer shell that are ionized and become part of the 'sea of electrons'
  - Dependent on the mobility of the charges
    - Mobility is related to the velocity of the charges.
    - It is a function of the material and magnitude of the voltage applied to make the charges move, and temperature.

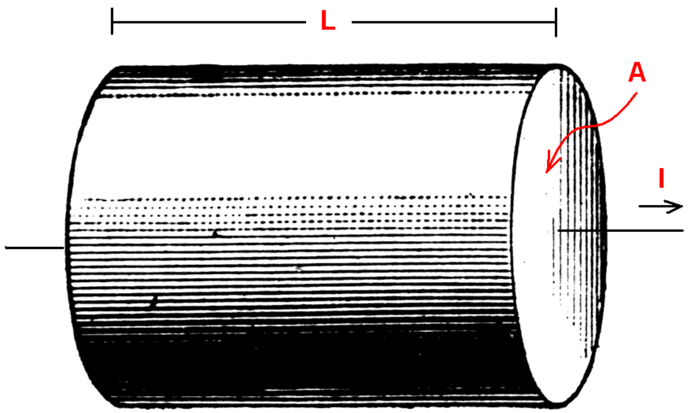
Resistivity of Common Materials at Room Temperature (300K)

Material	Resistivity (Ω-cm)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon (Graphite)	4×10^{-5}	Conductor
Germanium	0.47	Semiconductor
Silicon	640	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Resistance, R

- Resistance takes into account the physical dimensions of the material

$$R = \rho \frac{L}{A}$$



– where:
  - L is the length along which the carriers are moving
  - A is the cross sectional area that the free charges move through.

Ohm's Law

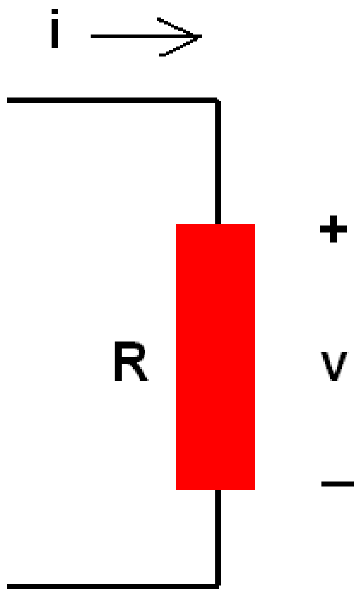
- Voltage drop across a resistor is proportional to the current flowing through the resistor

$$V = iR$$

Units: $V = A\Omega$

where $A = C/s$

Short Circuit

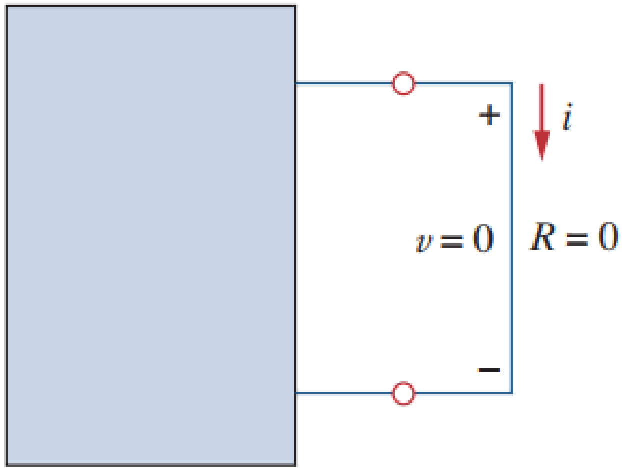


- If the resistor is a perfect conductor (or a short circuit)

  $R = 0\ \Omega,$

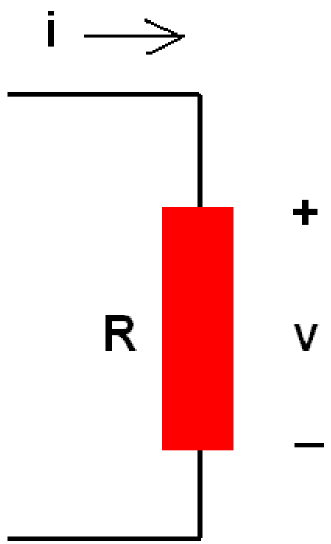
- then

  $v = iR = 0\ V$



  – no matter how much current is flowing through the resistor

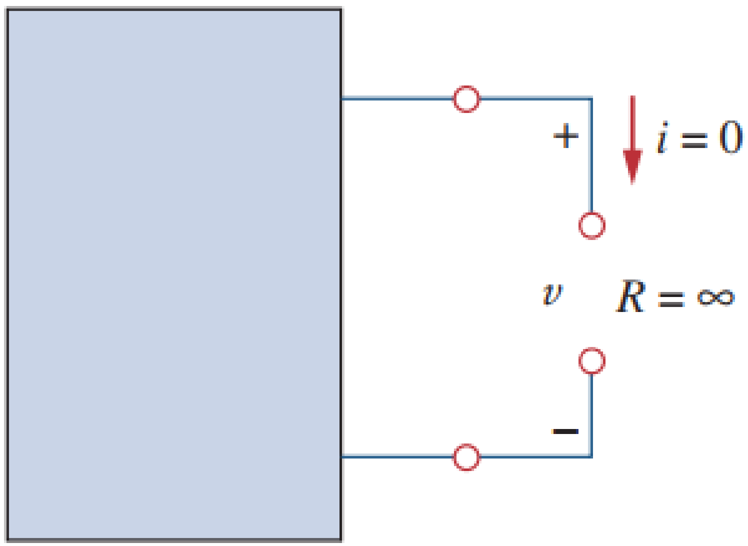
Open Circuit



- If the resistor is a perfect insulator, R = ∞ Ω

- then

$$\mathbf{i} = \lim_{\mathbf{R} \to \infty} \frac{\mathbf{v}}{\mathbf{R}} = 0$$



  – no matter how much voltage is applied to (or dropped across) the resistor.

Conductance, G

- Conductance is the reciprocal of resistance

$G = R^{-1} = i/v$

– Unit for conductance is S (siemens) or (mhos, ℧)

$G = A\sigma/L$

where σ is conductivity,

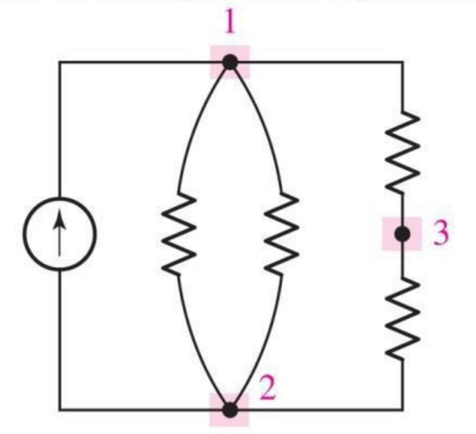
which is the inverse of resistivity, ρ

Power Dissipated by a Resistor

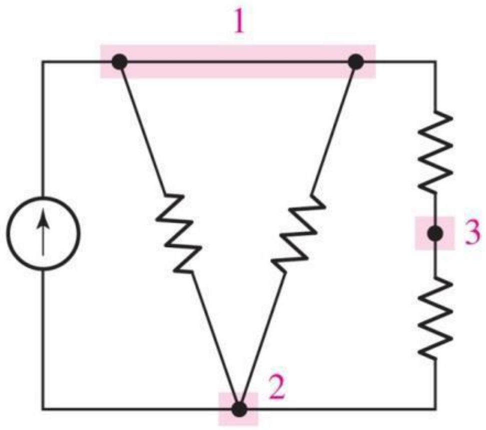
- $p = iv = i(iR) = i^2R$
- $p = iv = (v/R)v = v^2/R$
- $p = iv = i(i/G) = i^2/G$
- $p = iv = (vG)v = v^2G$

Circuit Terminology

- Node
  - point at which 2+ elements have a common connection
    - e.g., node 1, node 2, node 3
- Path
  - a route through a network, through nodes that never repeat
    - e.g., 1→3→2, 1→2→3
- Loop
  - a path that starts & ends on the same node
    - e.g., 3→1→2→3
- Branch
  - a single path in a network; contains one element and the nodes at the 2 ends
    - e.g., 1→2, 1→3, 3→2



(a)



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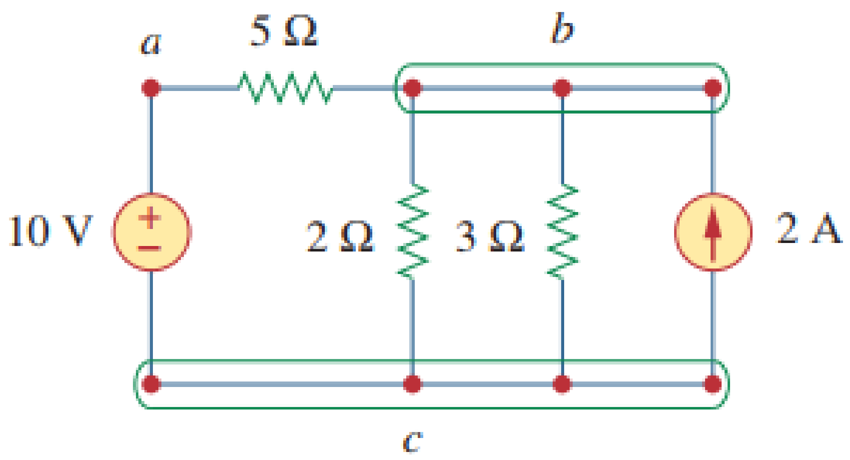
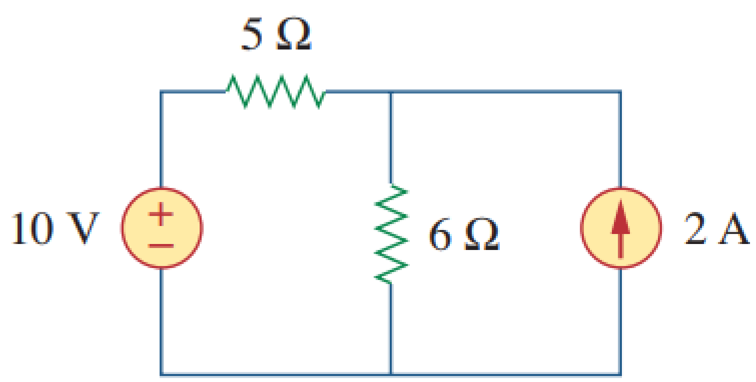
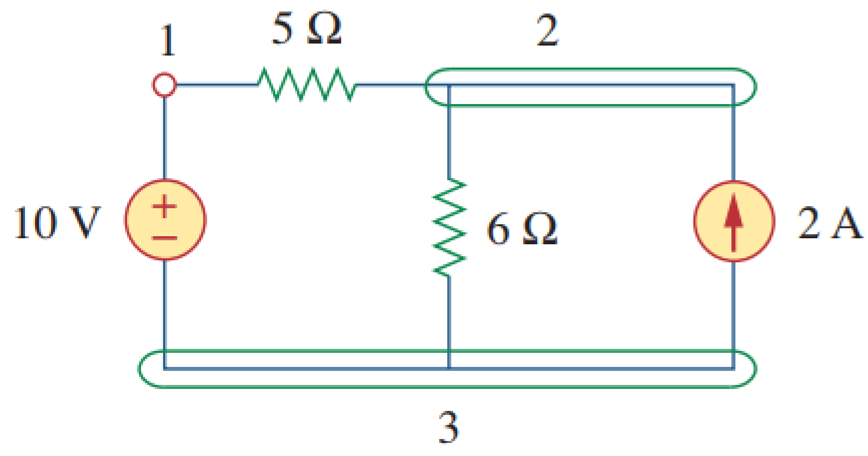


Nodes: 3
Branches: 5

Example 2.4

Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identify which elements are in series and which are in parallel.



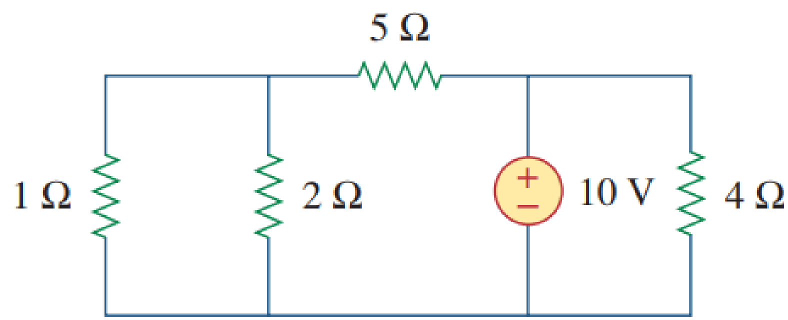


Solution:

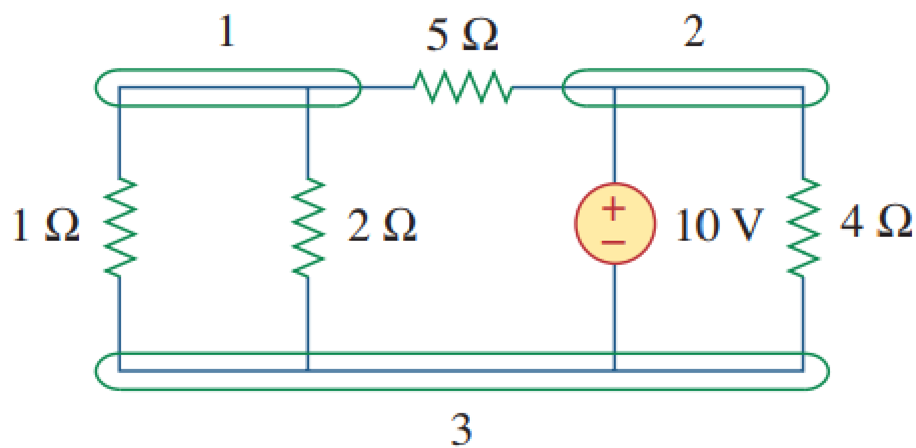
Since there are four elements in the circuit, the circuit has four branches: 10 V, 5 Ω, 6 Ω, and 2 A. The circuit has three nodes as identified in Fig. 2.13. The 5-Ω resistor is in series with the 10-V voltage source because the same current would flow in both. The 6-Ω resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.

Practice Problem 2.4

How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

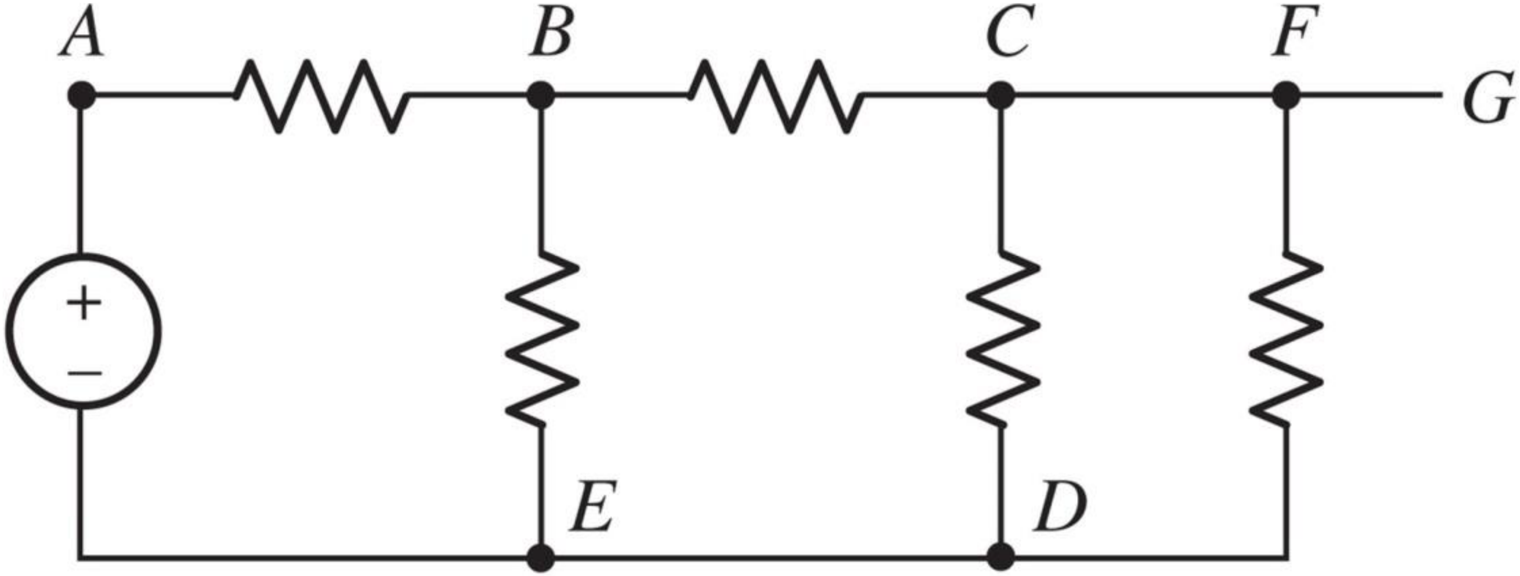


Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.



Exercise

- For the circuit below:
  a. Count the number of circuit elements.
  b. If we move from *B* to *C* to *D*, have we formed a path and/or a loop?
  c. If we move from *E* to *D* to *C* to *B* to *E*, have we formed a path and/or a loop?



Kirchhoff's Current Law (KCL)

- Robert Kirchhoff: German university professor, born while Ohm was experimenting
- Based upon conservation of charge
  - the algebraic sum of the currents entering and exiting any node is zero.

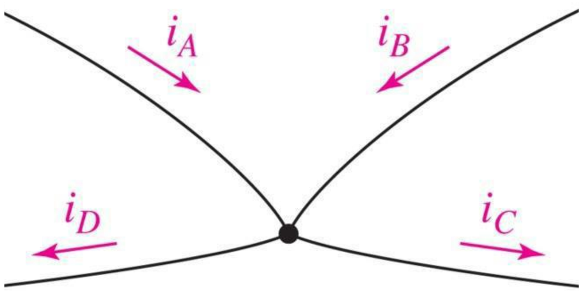
$$\sum_{n=1}^{N} i_n = 0$$

Where N is the total number of branches connected to a node.

$$\sum_{\text{node}} i_{enter} = \sum_{\text{node}} i_{leave}$$

$$i_A + i_B - i_C - i_D = 0$$

$$-i_A - i_B + i_C + i_D = 0$$



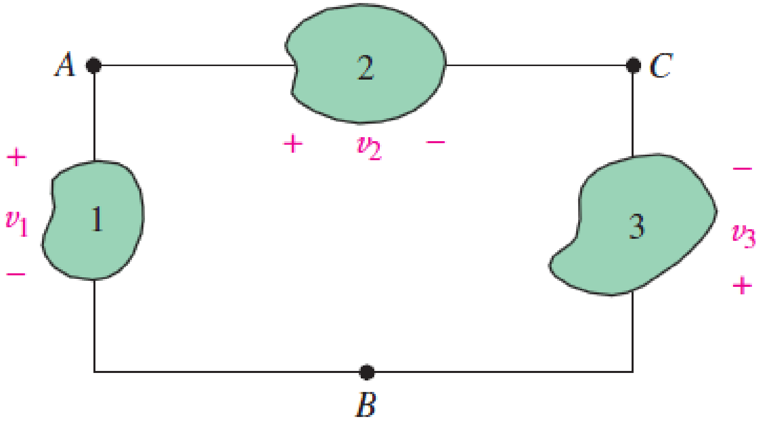
Kirchhoff's Voltage Law (KVL)

- Based upon conservation of energy
  - the algebraic sum of voltages dropped across components around a loop is zero.

$$\sum_{m=1}^{M} v = 0$$

Where M is the total number of branches in the loop.

$$\sum v_{\text{drops}} = \sum v_{\text{rises}}$$



$$-v_1 + v_2 - v_3 = 0$$

$$v_1 - v_2 + v_3 = 0$$

To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.

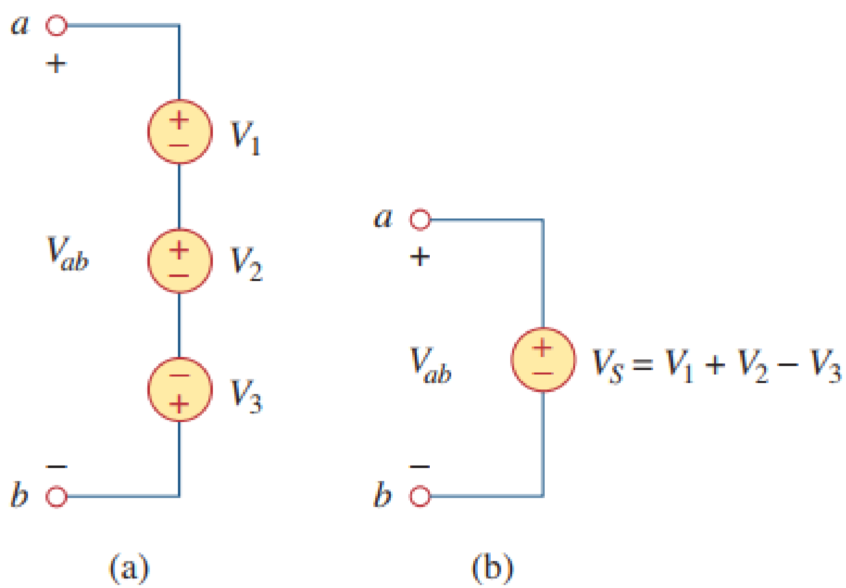
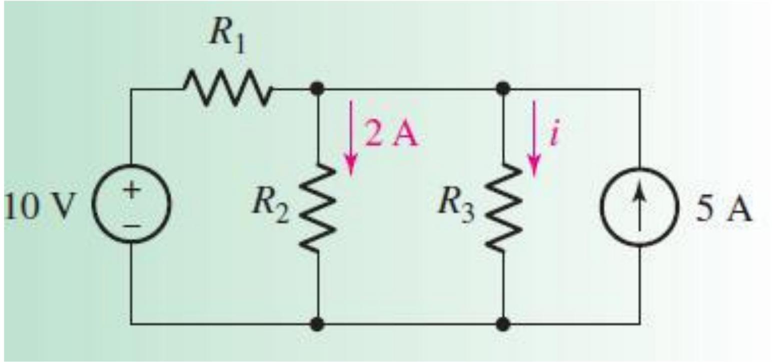
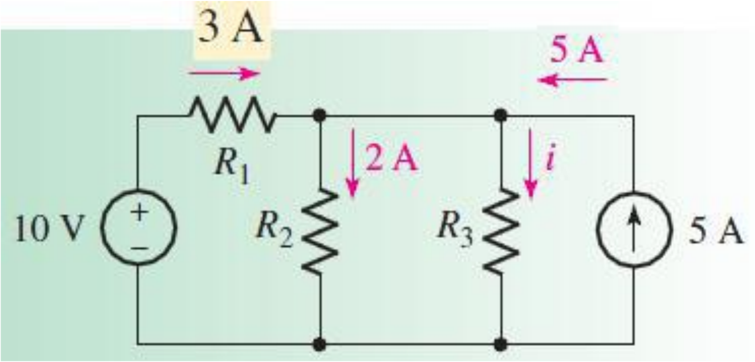


Figure 2.20
Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Example-01

- For the circuit, compute the current through R_3 if it is known that the voltage source supplies a current of 3 A.
- Use KCL





$$3 - 2 - i + 5 = 0$$

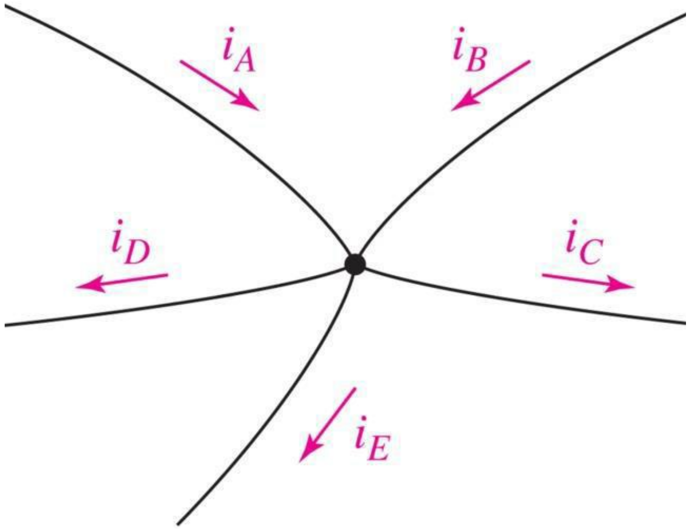
$$i = 3 - 2 + 5 = 6 \text{ A}$$

Example-02

- Referring to the single node below, compute:

a. i_B, given $i_A = 1$ A, $i_D = –2$ A, $i_C = 3$ A, and $i_E = 4$ A

b. i_E, given $i_A = –1$ A, $i_B = –1$ A, $i_C = –1$ A, and $i_D = –1$ A



- Use KCL

$$i_A + i_B - i_C - i_D - i_E = 0$$

a. $i_B = -i_A + i_C + i_D + i_E$

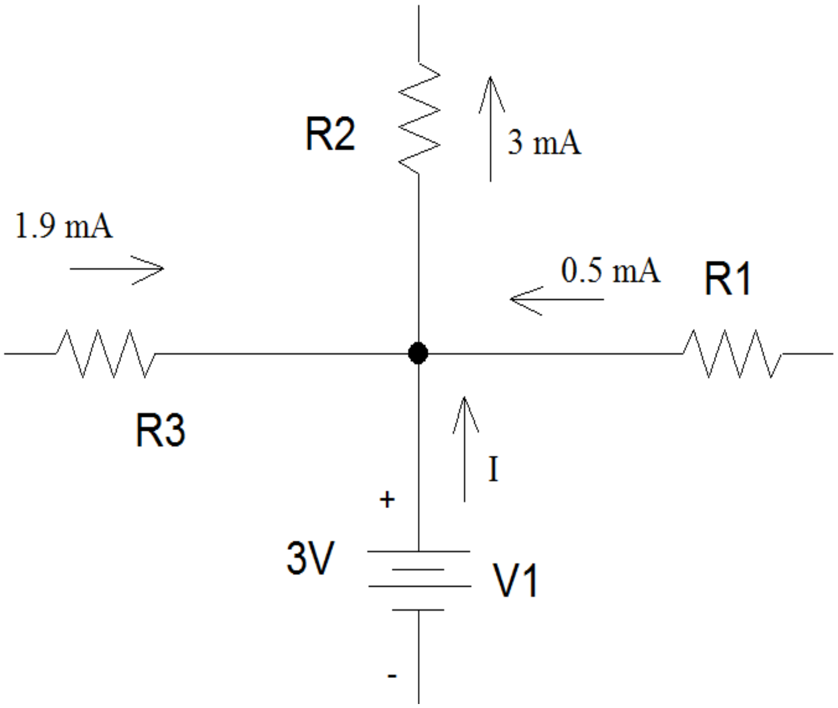
$i_B = -1 + 3 - 2 + 4 = 4$ A

b. $i_E = i_A + i_B - i_C - i_D$

$i_E = -1 - 1 + 1 + 1 = 0$ A

Example-03

- Determine I, the current flowing out of the voltage source.



  - Use KCL
    - 1.9 mA + 0.5 mA + I are entering the node.
    - 3 mA is leaving the node.

$$1.9mA + 0.5mA + I = 3mA$$

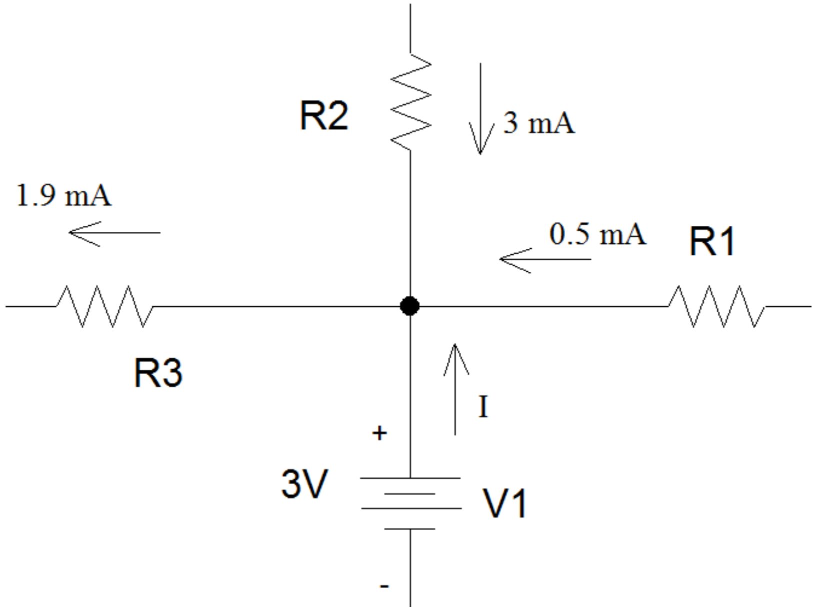
$$I = 3mA - (1.9mA + 0.5mA)$$

$$I = 0.6mA$$

V1 is generating power.

Example-04

- Suppose the current through R2 was entering the node and the current through R3 was leaving the node.



– Use KCL
  - 3 mA + 0.5 mA + I are entering the node.
  - 1.9 mA is leaving the node.

$$3mA + 0.5mA + I = 1.9mA$$

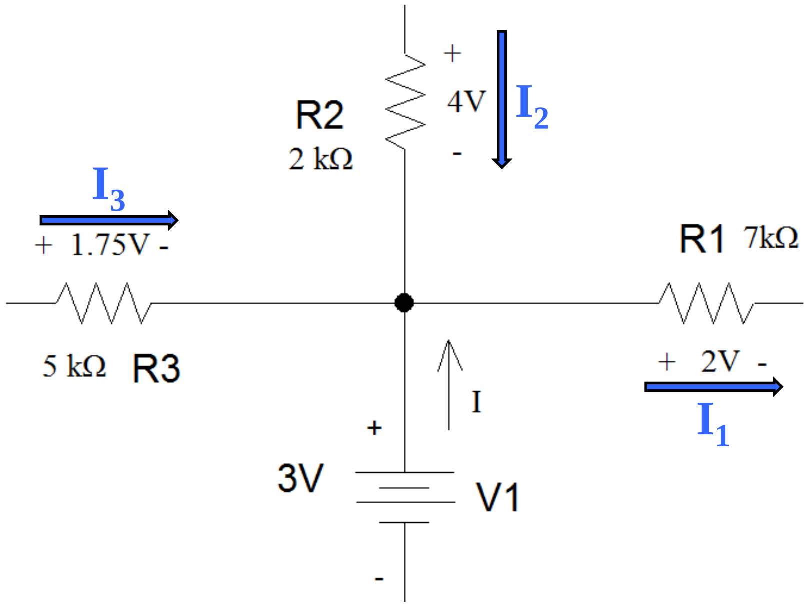
$$I = 1.9mA - (3mA + 0.5mA)$$

$$I = -1.6mA$$

V1 is dissipating power.

Example-05

- If voltage drops are given instead of currents,
  - you need to apply Ohm's Law to determine the current flowing through each of the resistors before you can find the current flowing out of the voltage supply.
    - I_1 is leaving the node.
    - I_2 is entering the node.
    - I_3 is entering the node.
    - I is entering the node.

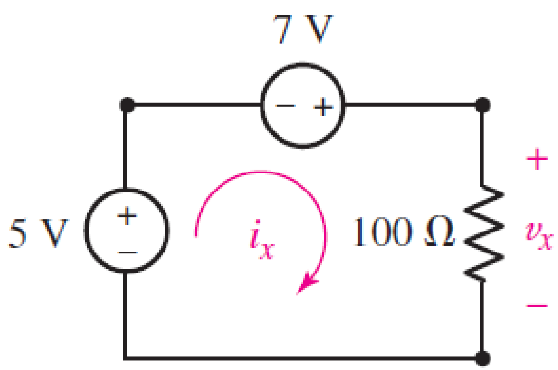


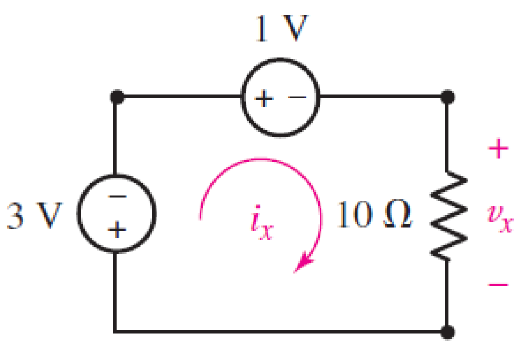
$$I_1 = 2V / 7k\Omega = 0.286mA$$
$$I_2 = 4V / 2k\Omega = 2mA$$
$$I_3 = 1.75V / 5k\Omega = 0.35mA$$

$$I_2 + I_3 + I = I_1$$
$$2mA + 0.35mA + I = 0.286mA$$
$$I = 0.286mA - 2.35mA = -2.06mA$$

Example-06

- For each of the circuits in the figure below, determine the voltage v_x and the current i_x.





  – Applying KVL clockwise around the loop and Ohm's law

$$-5-7+v_x=0$$

$$v_x = 12\text{ V}$$

$$i_x = \frac{v_x}{100} = \frac{12}{100}\text{ A} = 120\text{ mA}$$

$$+3+1+v_x=0$$

$$v_x = \underline{-4\text{ V}}$$

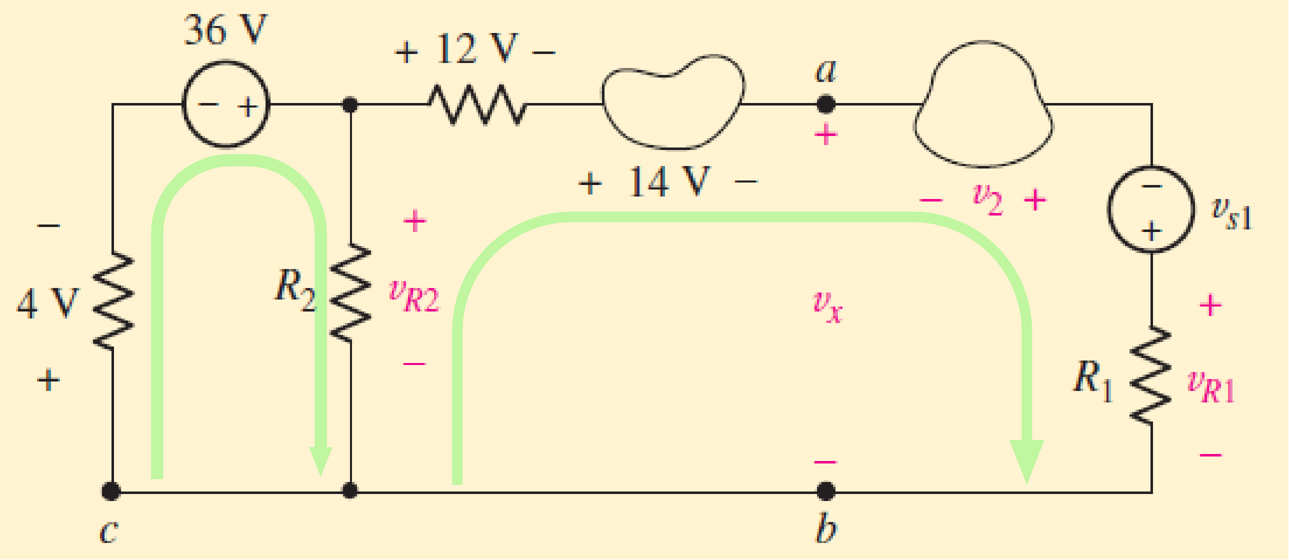
$$i_x = \frac{v_x}{10} = \underline{-400\text{ mA}}$$

Example-07

- For the circuit below, determine

  a. v_{R2}

  b. v_x



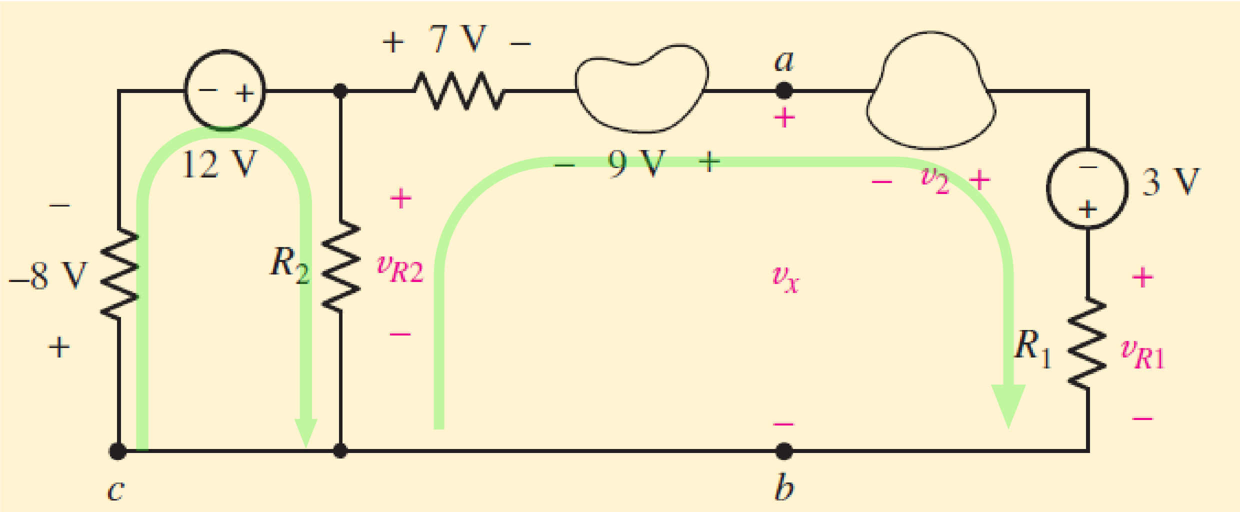
a. $4 - 36 + v_{R2} = 0$ $v_{R2} = 32$ V

b. $-32 + 12 + 14 + v_x = 0$ $v_x = 6$ V

Example-08

- For the circuit below, determine

a. v_{R2} b. v_x if $v_{R1} = 1$ V.



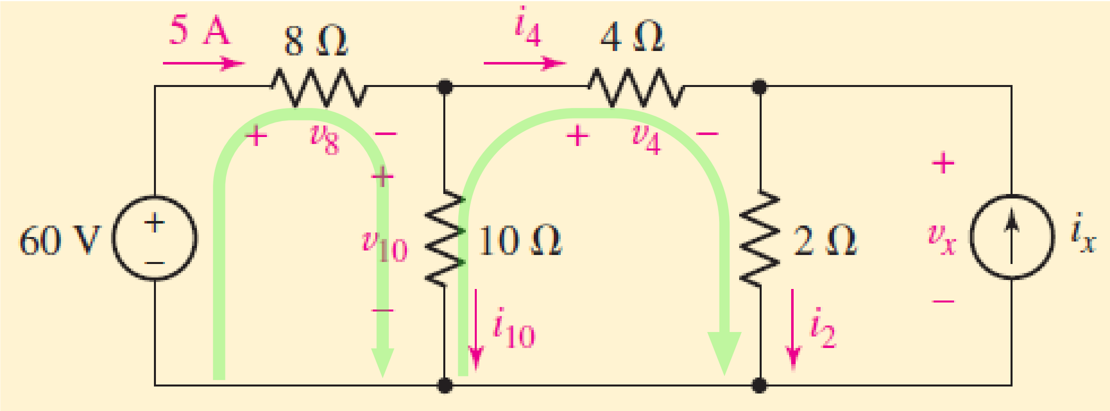
a. KVL yields $-8 - 12 + v_{R2} = 0$ $\quad$ $v_{R2} = 20$ V

b. KVL yields $-20 + 7 - 9 - v_2 - 3 + v_{R1}$

where $v_{R1} = 1$ V. Thus, $v_2 = -24$ V

Example-09

- For the circuit below, determine v_x



$$-60 + v_8 + v_{10} = 0$$

$$v_{10} = 0 + 60 - 40 = 20\ \text{V}$$

$$-v_{10} + v_4 + v_x = 0$$

$$v_x = 20 - v_4$$

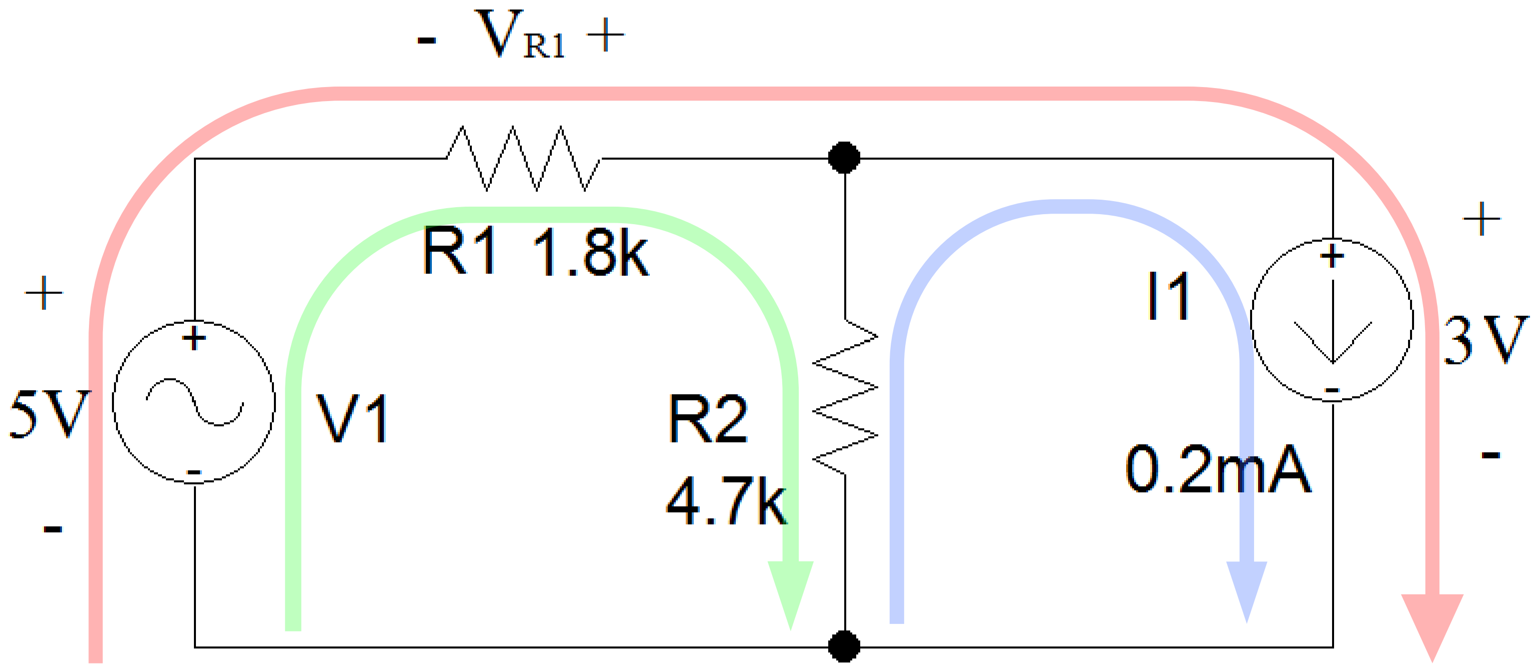
$$i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 5 - \frac{20}{10} = 3$$

$$v_4 = (4)(3) = 12\ \text{V}$$

$$v_x = 20 - 12 = 8\ \text{V}$$

Example-10…

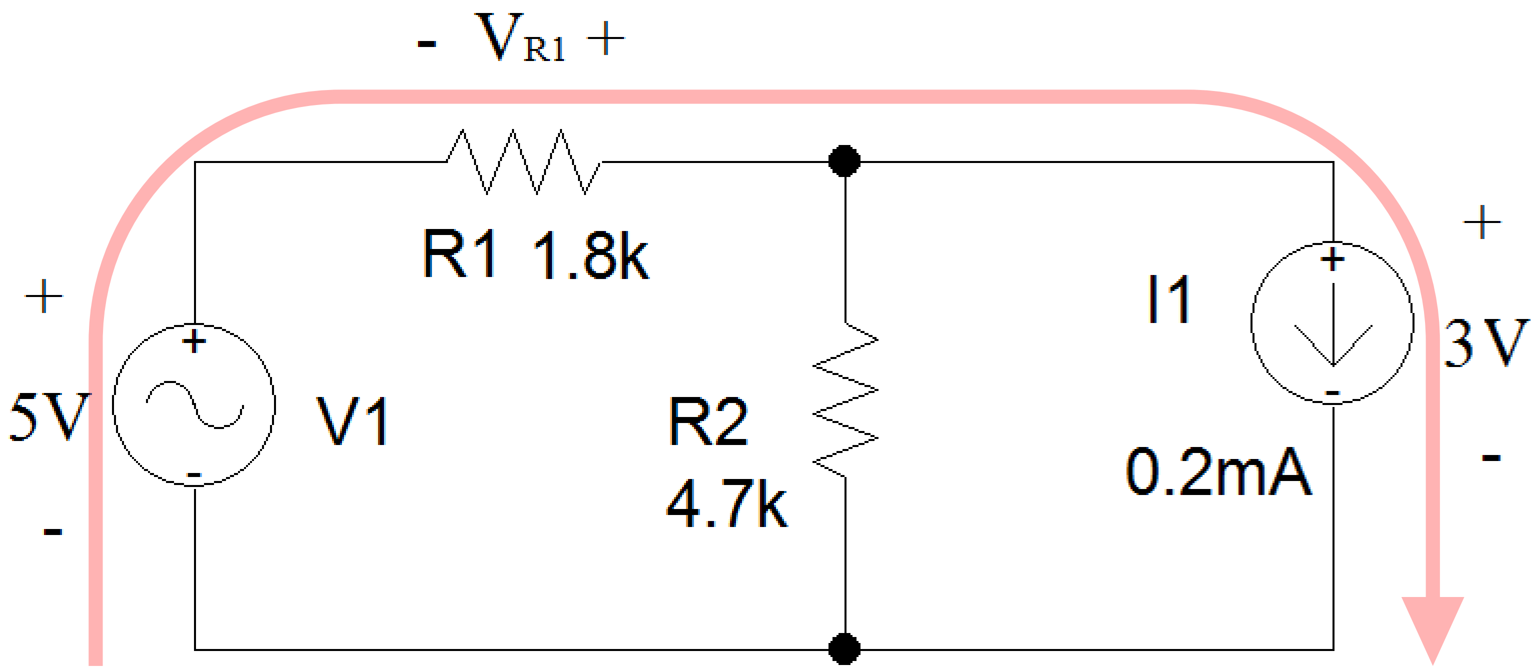
- Find the voltage across R1.
  - Note that the polarity of the voltage has been assigned in the circuit schematic.



– First, define a loop that include R1.

…Example-10…

- If the red loop is considered

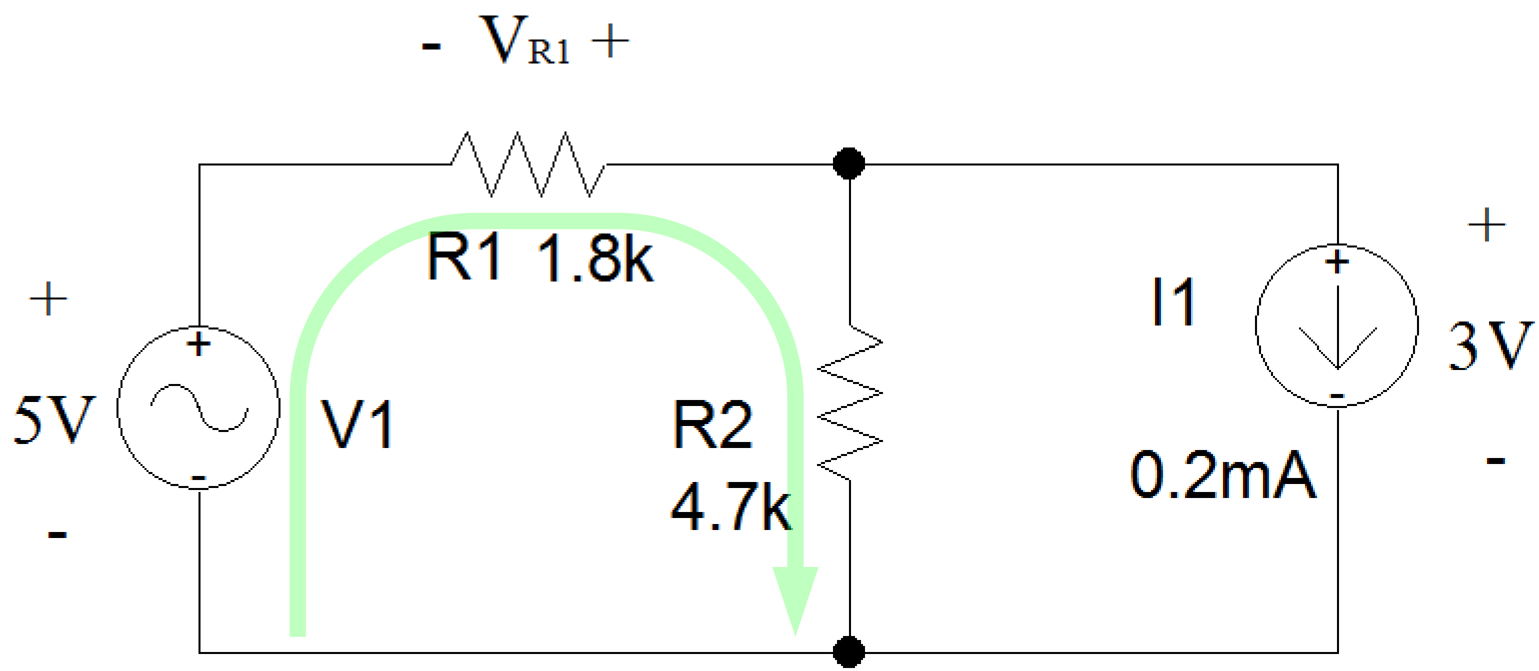


  - By convention, voltage drops are added and voltage rises are subtracted in KVL.

$$-5\text{ V} - V_{R1} + 3\text{ V} = 0 \qquad\qquad V_{R1} = -2\text{ V}$$

…Example-10

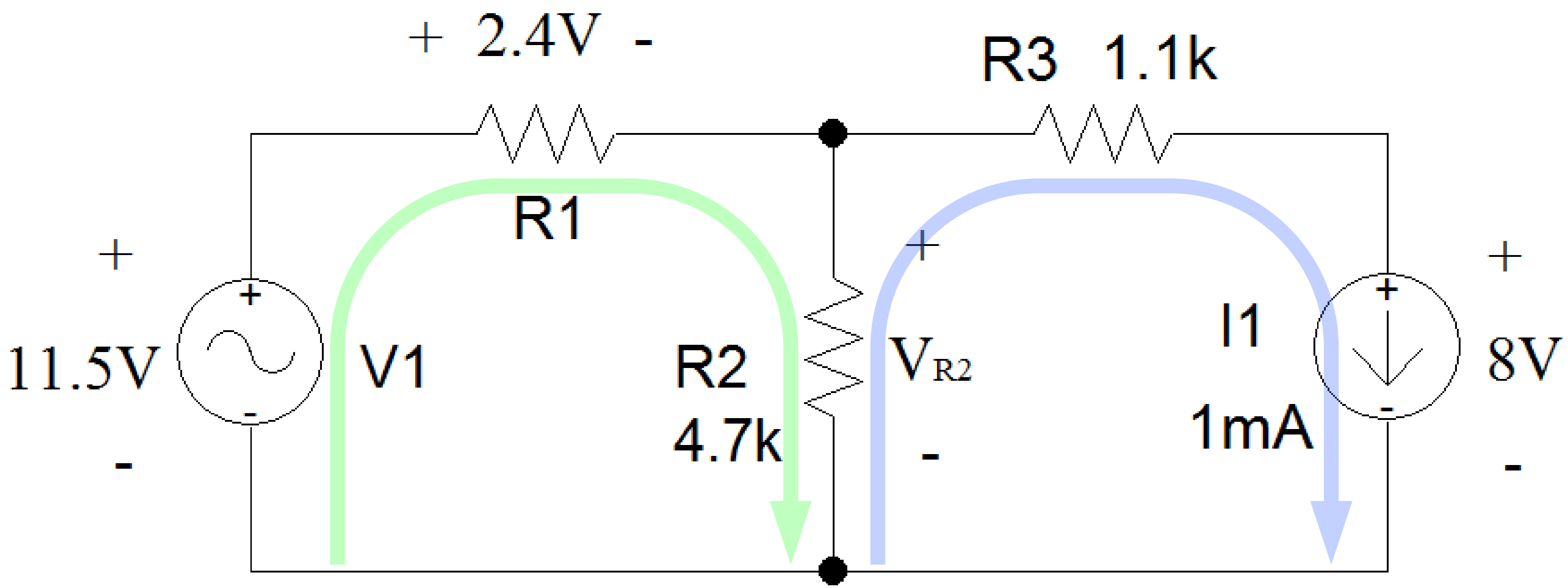
- Suppose you chose the green loop instead.
    - Since R2 is in parallel with I1, the voltage drop across R2 is also 3V.



$$-5\text{ V} - V_{R1} + 3\text{ V} = 0 \qquad V_{R1} = -2\text{ V}$$

Example-11…

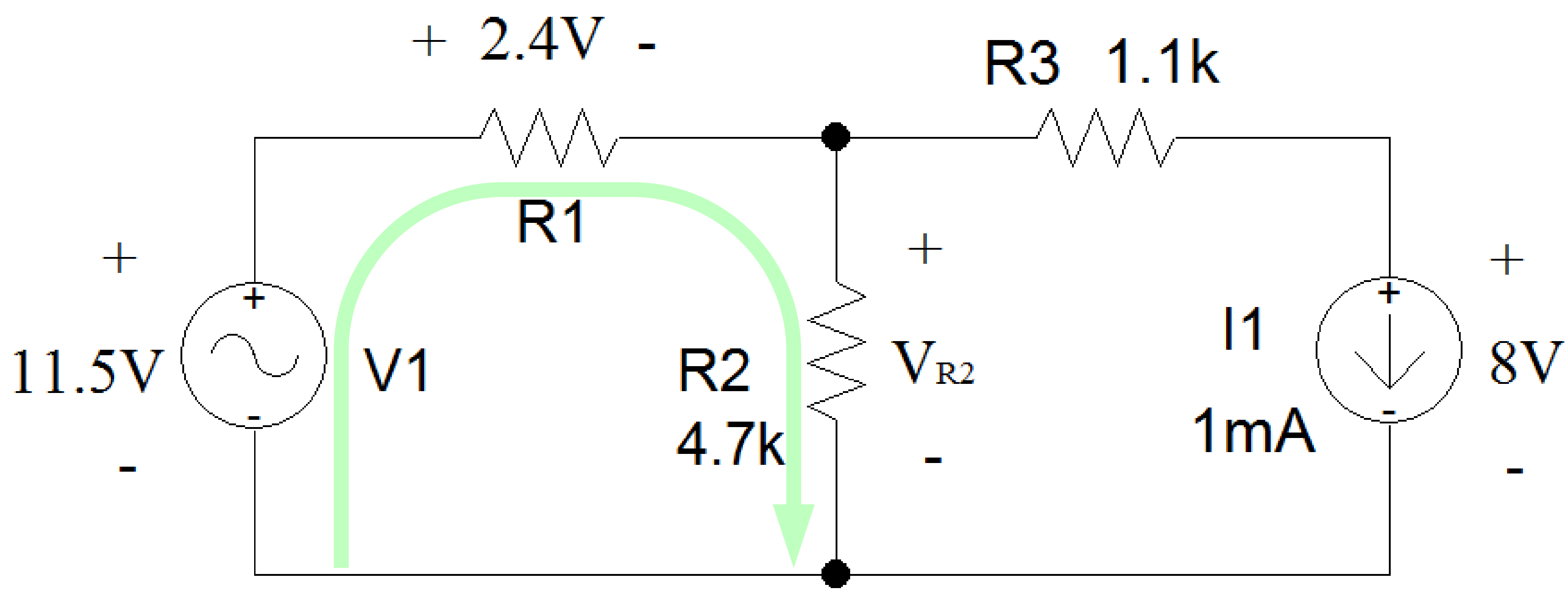
- Find the voltage across R2 and the current flowing through it.



  – First, draw a loop that includes R2.

…Example-11…

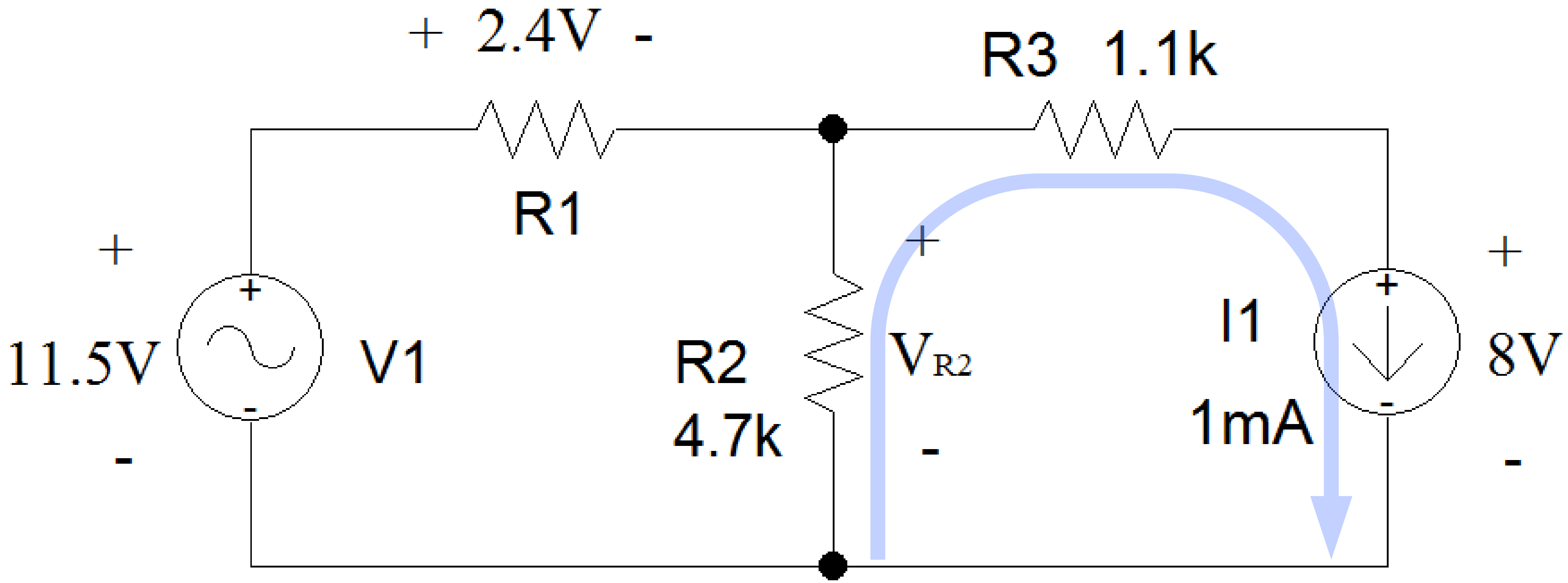
- If the green loop is used:



$-11.5\text{ V} + 2.4\text{ V} + V_{R2} = 0$ $\qquad$ $V_{R2} = 9.1\text{ V}$

…Example-11…

- If the blue loop is used:



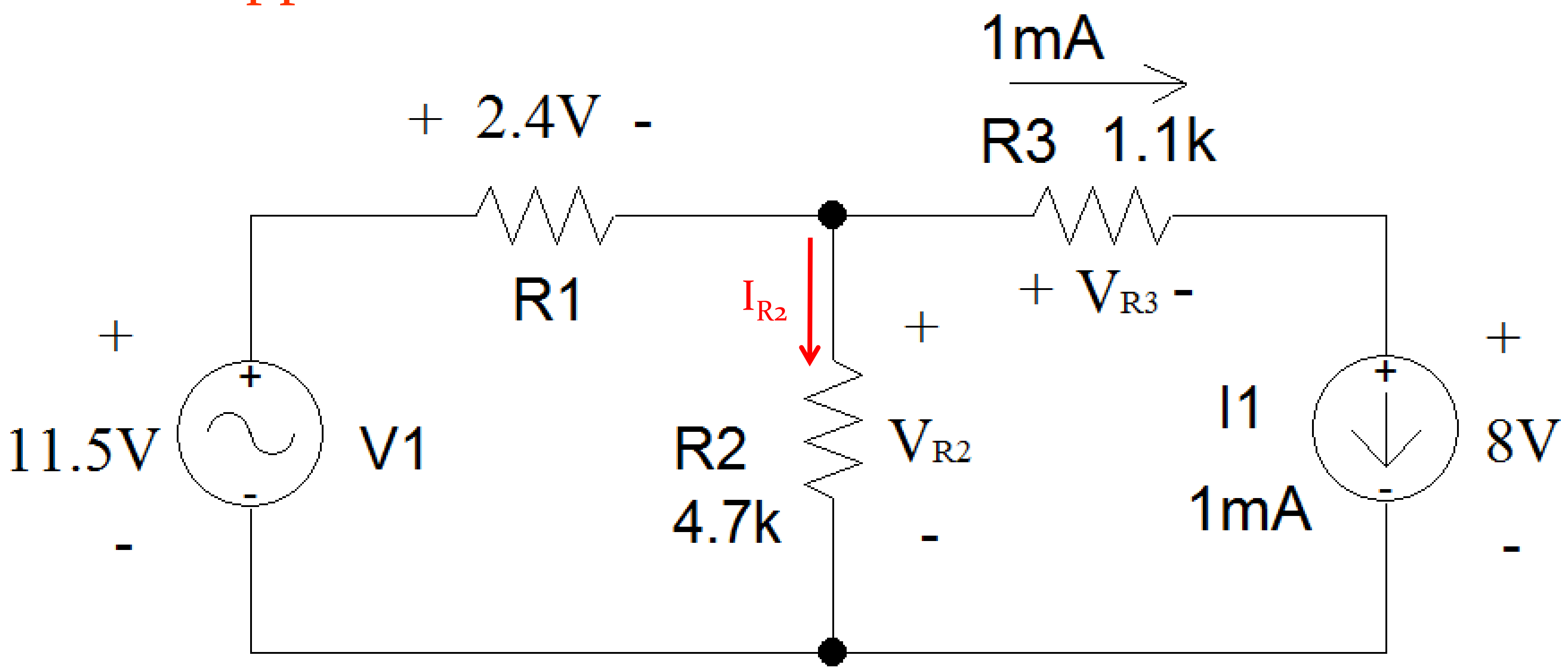
- First, find the voltage drop across R3

$1\ \text{mA} \times 1.1\ \text{k}\Omega = 1\times10^{-3}\ \text{A} \times 1.1\times10^{3}\ \Omega = 1.1\ \text{V}$

$1.1\ \text{V} + 8\ \text{V} - V_{R2} = 0 \qquad V_{R2} = 9.1\ \text{V}$

…Example-11

– Once the voltage across R2 is known, Ohm's Law is applied to determine the current.



I_{R2} = 9.1 V / 4.7 kΩ = 9.1 V / (4.7×10^3 Ω)

I_{R2} = 1.94×10^{-3} A = 1.94 mA

Example 2.5

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2.

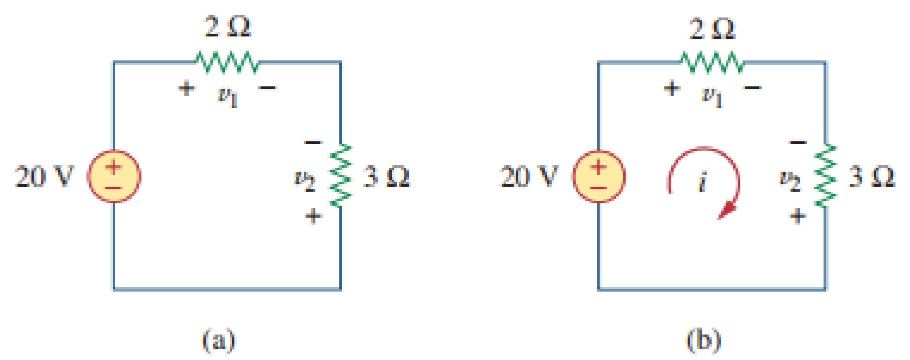


Figure 2.21
For Example 2.5.

Solution:

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \qquad \text{or} \qquad 5i = 20 \qquad \Rightarrow \qquad i = 4 \text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

Determine v_o and i in the circuit shown in Fig. 2.23(a).

Example 2.6

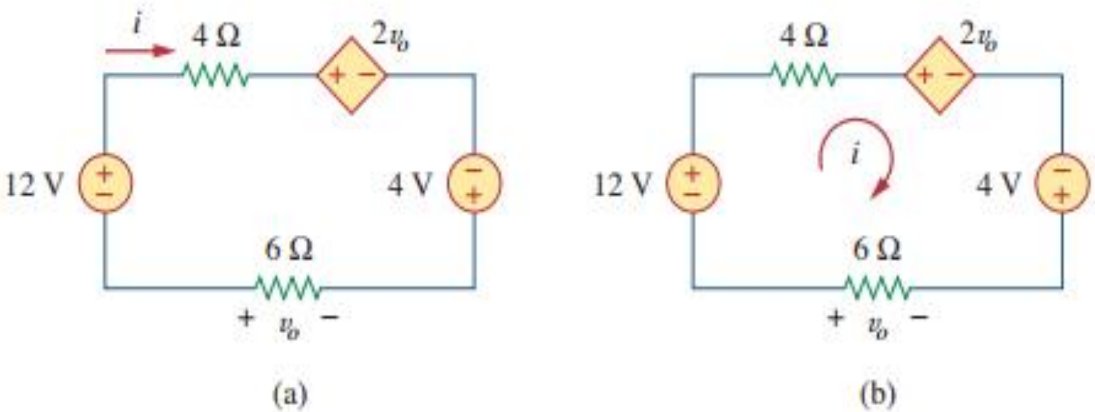


Figure 2.23
For Example 2.6.

Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \textbf{(2.6.1)}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad \textbf{(2.6.2)}$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \qquad \Rightarrow \qquad i = -8 \text{ A}$$

and $v_o = 48$ V.

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

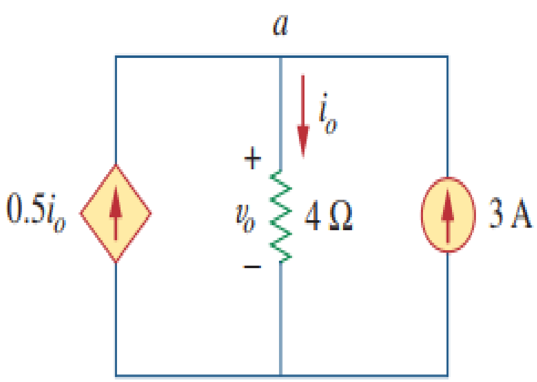


Figure 2.25
For Example 2.7.

Solution:

Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

Thank You