Lecture 3: Finding Probabilities Using Counting Methods

CPE251 Probability Methods in Engineering

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Fundamental Principle of Counting

A pair of experiments A and B, with number of outcomes n_A and n_B respectively, performed together leads to $n_A \times n_B$ number of outcomes.

The number of distinct ordered pairs (x_1, x_2), from sets with n_1 and n_2 numbers of distinct elements respectively, is $n_1 \times n_2$.

The number of distinct ordered k-tuples with components x_i represented by $(x_1, x_2, x_3, \dots, x_k)$ obtained from sets with n_i numbers of distinct elements is $\Pi_{i=1}^{k} n_i$

With Replacement or Without Replacement: using an object again and again?

Permutations or Combinations: order matters or not?

2

Permutations – Ordered Arrangement without Replacement

An arrangement of all or a part of a set of objects; Order does matter

Permutations of all objects each of a different kind:

Permutations of *n* objects taken *n* at a time:

$$(n)_n = n \times (n-1) \times (n-2) \times \cdots \times 1 = n!$$

Permutations of n objects taken $k \leq n$ at a time:

$$P_k^n = (n)_k = \frac{n!}{(n-k)!}$$

Permutations of all objects each belonging to a group of a different kind (partition):
The number of distinct permutations of *n* things of which n_i objects are of kind i is

$$(n)_{n_k} = \frac{n!}{n_1!\, n_2 \ldots n_k!}$$

This is known as multinomial.

Combinations – Un-ordered Arrangement without Replacement

An arrangement of objects without regard to order

The number of combinations of *n* distinct objects taken *r* at a time is:

$$C_r^n = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Examples

Example 2.18: In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, how many possible selections are there?

***Solution*:** Since the awards are distinguishable, it is a permutation problem. The total number of sample points is

$$_{25}P_3 = \frac{25!}{(25-3)!} = \frac{25!}{22!} = (25)(24)(23) = 13,800.$$

Example 2.19: A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if

(a) there are no restrictions;

(b) A will serve only if he is president;

(c) B and C will serve together or not at all;

(d) D and E will not serve together?

***Solution*:** (a) The total number of choices of officers, without any restrictions, is

$$_{50}P_2 = \frac{50!}{48!} = (50)(49) = 2450.$$

(b) Since A will serve only if he is president, we have two situations here: (i) A is selected as the president, which yields 49 possible outcomes for the treasurer's position, or (ii) officers are selected from the remaining 49 people without A, which has the number of choices $_{49}P_2 = (49)(48) = 2352$. Therefore, the total number of choices is $49 + 2352 = 2401$.

(c) The number of selections when B and C serve together is 2. The number of selections when both B and C are not chosen is ${}_{48}P_2 = 2256$. Therefore, the total number of choices in this situation is $2 + 2256 = 2258$.

(d) The number of selections when D serves as an officer but not E is $(2)(48) = 96$, where 2 is the number of positions D can take and 48 is the number of selections of the other officer from the remaining people in the club except E. The number of selections when E serves as an officer but not D is also $(2)(48) = 96$. The number of selections when both D and E are not chosen is ${}_{48}P_2 = 2256$. Therefore, the total number of choices is $(2)(96) + 2256 = 2448$. This problem also has another short solution: Since D and E can only serve together in 2 ways, the answer is $2450 - 2 = 2448$.

Allow Replacement

Permutations: n distinct objects to fill k places with replacement allowed: n^k

Combinations: $\binom{n-1+k}{k} = \binom{n-1+k}{n-1}$

References

1. Walpole, R.E., Myers, R.H., Myers, S.L. and Ye, K. (2007) *Probability & Statistics for Engineers & Scientists.* 9th Edition, Pearson Education, Inc.
2. Leon-Garcia, A. (2008). *Probability, Statistics, and Random Processes for Electrical Engineering*. 3rd Edition, Pearson/Prentice Hall.