Applied Physics for Engineers

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Lecture # 16,

Introduction to electromagnetism, The Biot-Savart law, Ampere’s law, solenoid

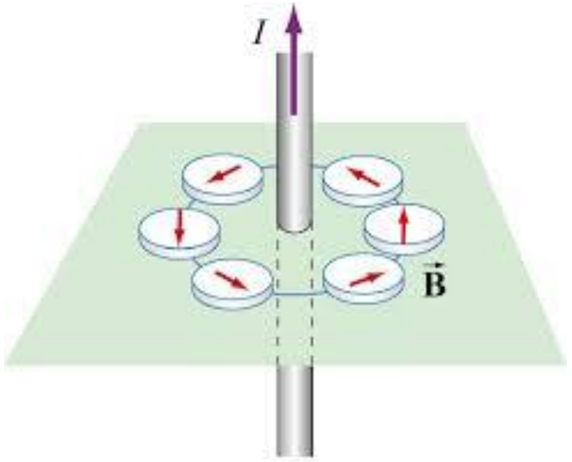
Electromagnetism

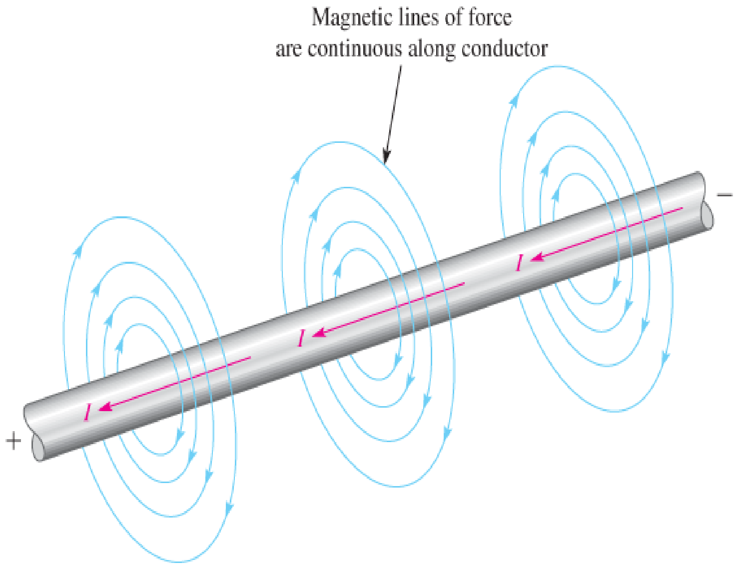
The study of interaction between electric currents or fields and magnetic fields is called electromagnetism.

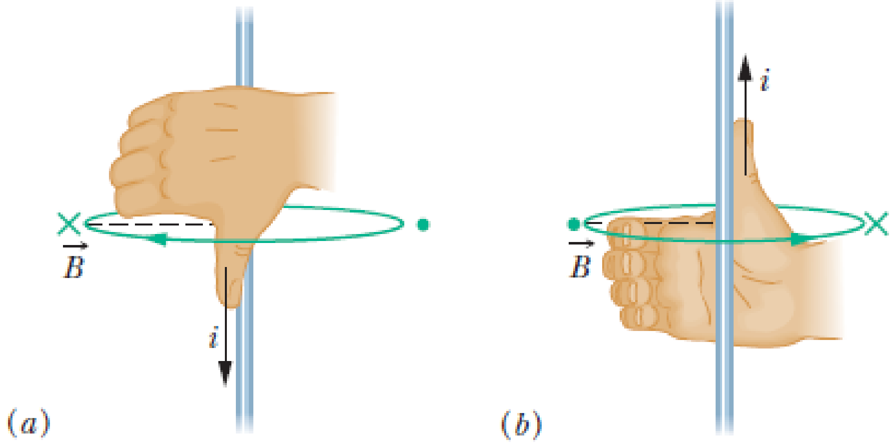
In this study we can convert electric energy into magnetic energy and magnetic energy into electric energy

Magnetism and electricity

- The relationship between magnetism and electricity was discovered in 1819 when Hans Christian Oersted found that an electric current in a wire deflected a nearby compass needle.
- Oersted concluded that current in the wire produces a magnetic field and the needle responds to that magnetic field.







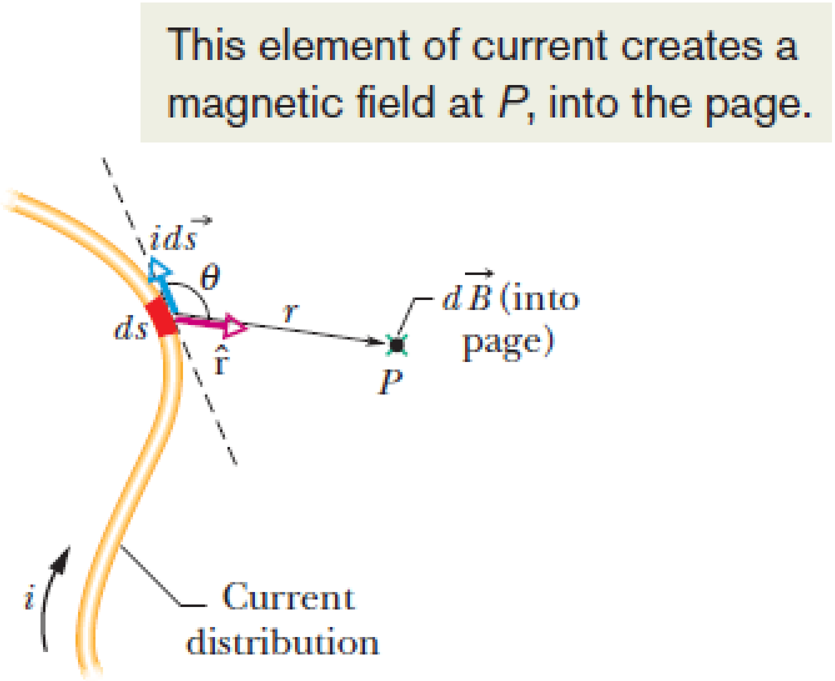
Biot-Savart Law

Calculating the Magnetic Field Due to a Current

- Figure shows a wire of arbitrary shape carrying a current i. We want to find the magnetic field **B** at a nearby point P. We first mentally divide the wire into differential elements ds and then define for each element a length vector d**s** that has length ds and whose direction is the direction of the current in ds.
- We can then define a differential *current-length element* to be *i***d*s*** ; we want to calculate the field d**B** produced at *P* by a typical current-length element.
- ***Magnitude.*** The magnitude of the field d**B** produced at point *P* at distance *r* by a current-length element *i***d*s*** turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{ids\ sin\theta}{r^2}$$

where θ is the angle between the directions of $d\boldsymbol{s}$ and $\hat{\mathrm{r}}$, a unit vector that points from ds toward P.



- Symbol μ_0 is a constant, called the permeability constant, whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T}$$

In vector form,

$$dB = \frac{\mu_0}{4\pi}\frac{id\boldsymbol{s} \times \hat{\mathrm{r}}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{id\boldsymbol{s} \times \boldsymbol{r}}{r^3}$$

$\therefore$ **r** = r$\hat{\mathrm{r}}$

$\hat{\mathrm{r}} = \frac{\boldsymbol{r}}{r}$

- This vector equation and its scalar form are known as the **law of Biot and Savart**
- We can use this law to calculate the net magnetic field produced at a point by various distributions of current.
- If current in a wire is either directly toward or directly away from a point P of measurement, the magnetic field at P from the current is simply zero (the angle θ is either 0° for toward or 180° for away, and both result in sin θ = 0)

Magnetic Field Due to a Current in a Long Straight Wire

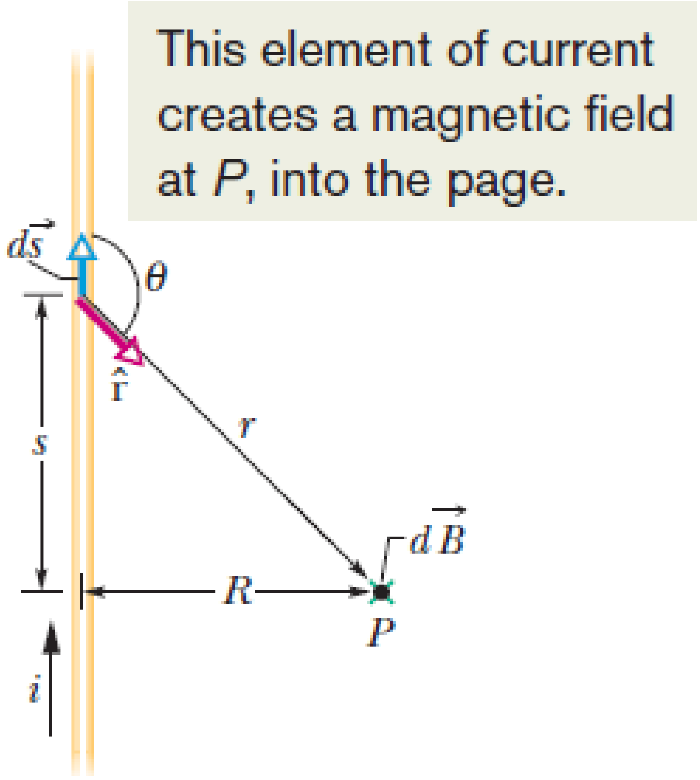
- Figure shows a straight wire of infinite length carrying a current i. We want to find the field **B** at point P, at a perpendicular distance R from the wire. The magnitude of the differential magnetic field produced at P by the current-length element id**s** located a distance r from P is given by Eq:

$$dB = \frac{\mu_0}{4\pi}\frac{ids\ sin\theta}{r^2}$$

The direction of d**B** is directly into the page

- To find total magnetic field we will have to integrate dB over the entire length

$$B = \int dB = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{sin\theta ds}{r^2}$$



- From the figure we can deduce that

$$r^2 = s^2 + R^2$$
$$r = \sqrt{s^2 + R^2}$$

And

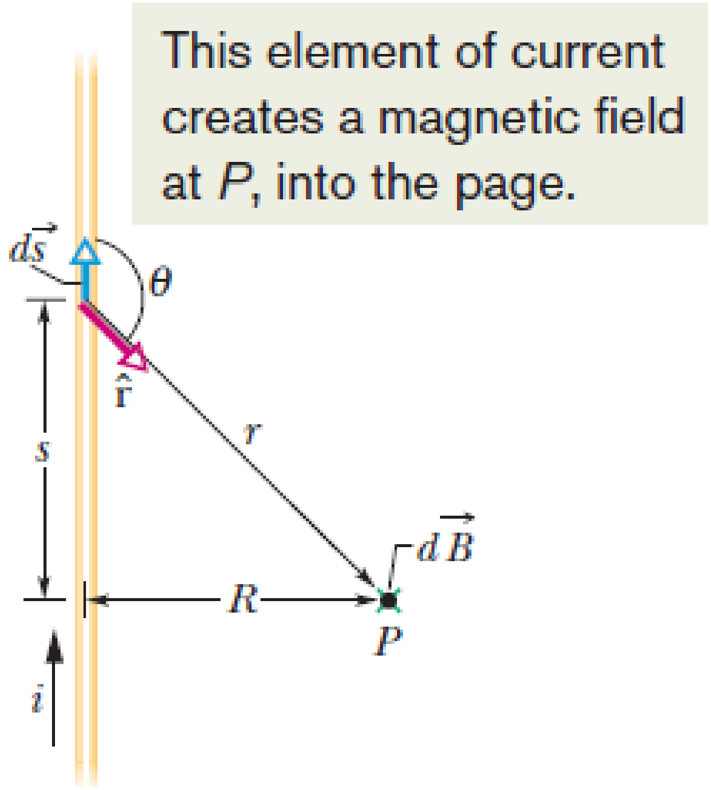
$$\sin\theta = \frac{R}{r} = \frac{R}{\sqrt{s^2 + R^2}}$$

$$B = \int dB = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{+\infty} \frac{\sin\theta ds}{r^2}$$

$$B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{0} \frac{\sin\theta ds}{r^2} + \int_{0}^{+\infty} \frac{\sin\theta ds}{r^2}$$

$$B = \frac{\mu_0 i}{4\pi} 2 \int_{0}^{+\infty} \frac{\sin\theta ds}{r^2}$$

$$B = \frac{\mu_0 i}{4\pi} 2 \int_{0}^{+\infty} \frac{\frac{R}{\sqrt{s^2 + R^2}} ds}{s^2 + R^2}$$



$$B = \frac{\mu_0 i}{4\pi} 2 \int_0^{+\infty} \frac{R ds}{(s^2+R^2)^{\frac{3}{2}}}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

$$B = \frac{\mu_0 i R}{2\pi} \left[\frac{s}{R^2(s^2 + R^2)^{\frac{1}{2}}} \right]_0^{\infty}$$

$$B = \frac{\mu_0 i}{2\pi R} \left[\frac{s}{(s^2+R^2)^{\frac{1}{2}}} \right]_0^{\infty}$$

The limit it applied on the next page

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

- The field magnitude B in above equation depends only on the current and the perpendicular distance R of the point from the wire.

$$\left[\frac{s}{\sqrt{s^2+R^2}}\right]_0^\infty = \lim_{s\to\infty} \frac{s}{\sqrt{s^2+R^2}} - \cancel{\frac{0}{\sqrt{0^2+R^2}}}^{\,0}$$

$$= \lim_{s\to\infty} \frac{s}{\sqrt{s^2+R^2}}$$

divide the numerator and denominator with $\sqrt{s^2}$

$$= \lim_{s\to\infty} \frac{s/\sqrt{s^2}}{\sqrt{\frac{s^2}{s^2}+\frac{R^2}{s^2}}} = \lim_{s\to\infty} \frac{1}{\sqrt{1+R^2/s^2}}$$

apply limit

$$= \frac{1}{\sqrt{1+R^2/\frac{1}{0}}} = \frac{1}{\sqrt{1+0}} = \underline{\underline{1}}.$$

so

$$\left[\frac{s}{\sqrt{s^2+R^2}}\right]_0^\infty = 1-0 = \underline{1} \,//$$

Ampere's Law

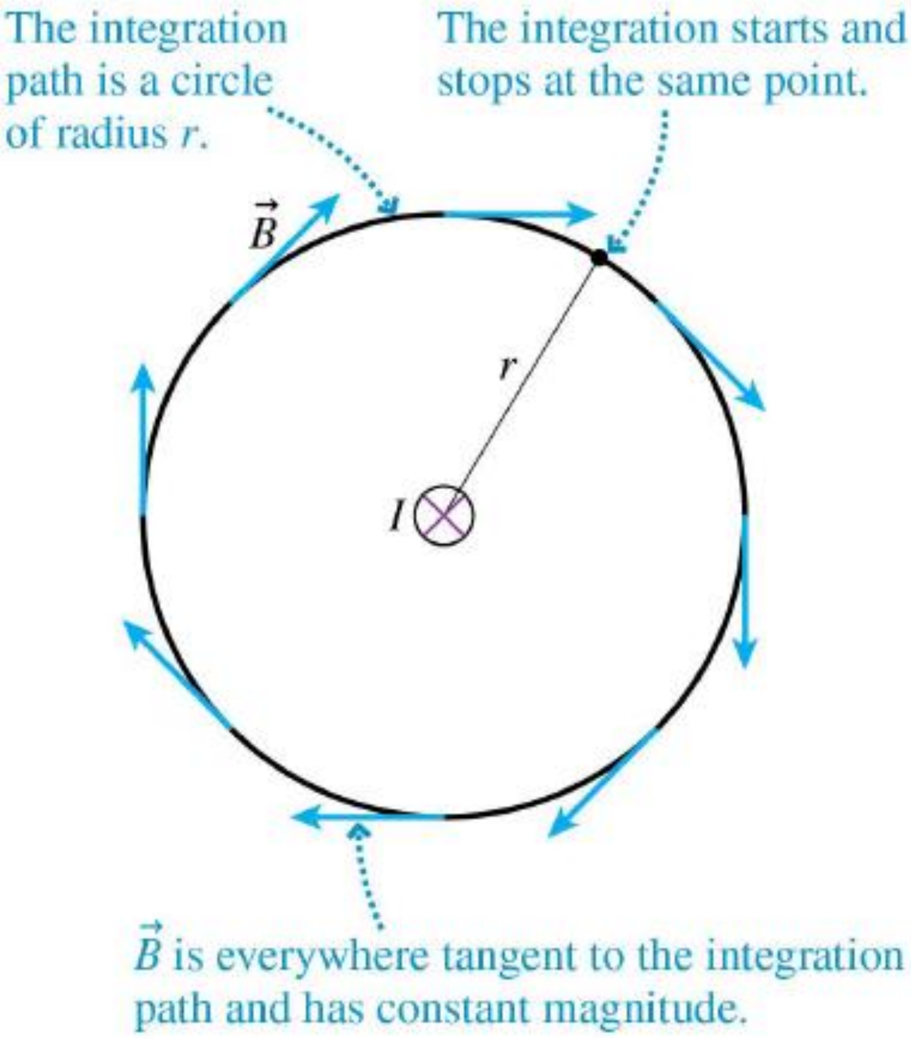
- Consider a line integral of **B** evaluated along a circular path all the way around a wire carrying current *I*.
- This is the line integral around a *closed curve*, which is denoted as

$$\oint \vec{B} \cdot d\vec{s}$$

- Because **B** is tangent to the circle and of constant magnitude at every point on the circle, we can write

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

- the magnitude of **B** Is Constant on this circle and is given by $B = \mu_0 I/2\pi r$ where I is the current *through* this loop, hence

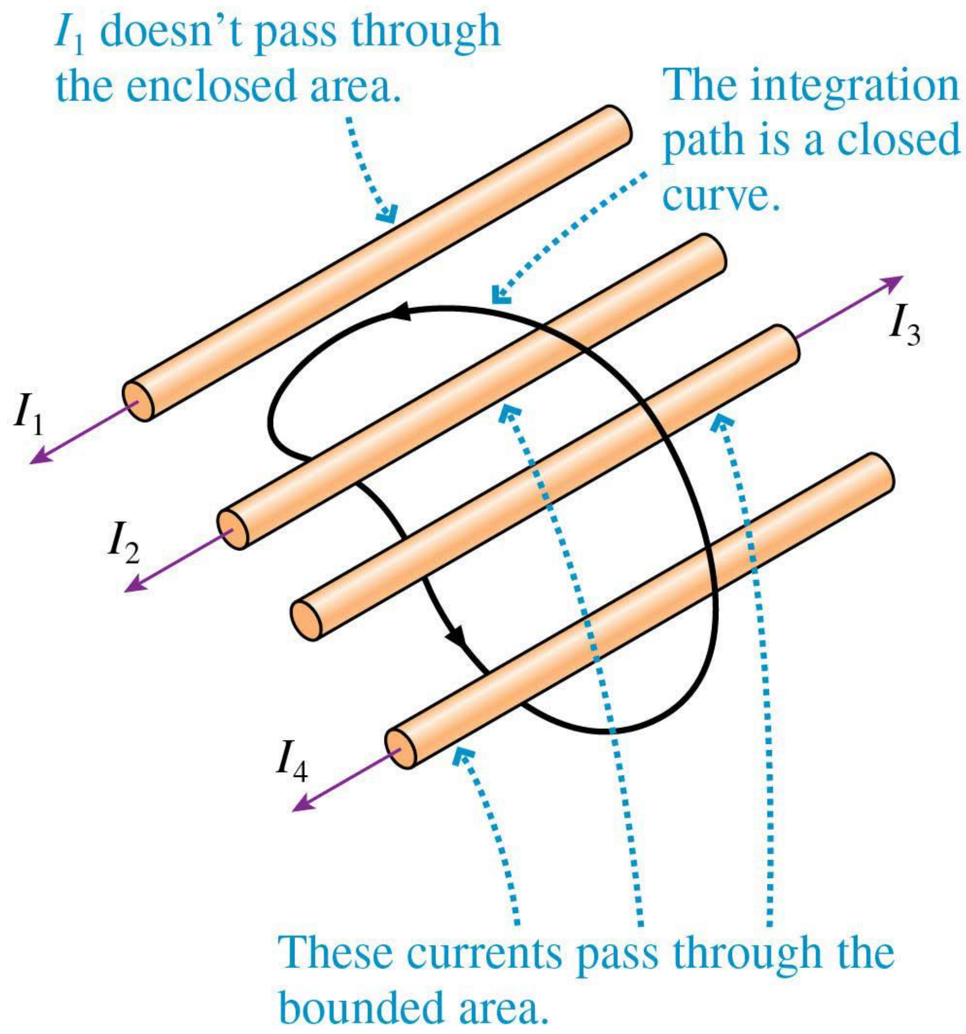


- The line integral of B.d**s** around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \boldsymbol{B} \,.\, d\boldsymbol{s} = \mu_0 I_{enc}$$

This is Ampere's law

- This law, which can be derived from the Biot–Savart law, has traditionally been credited to André-Marie Ampère (1775–1836), for whom the SI unit of current is named. However, the law actually was advanced by English physicist James Clerk Maxwell.



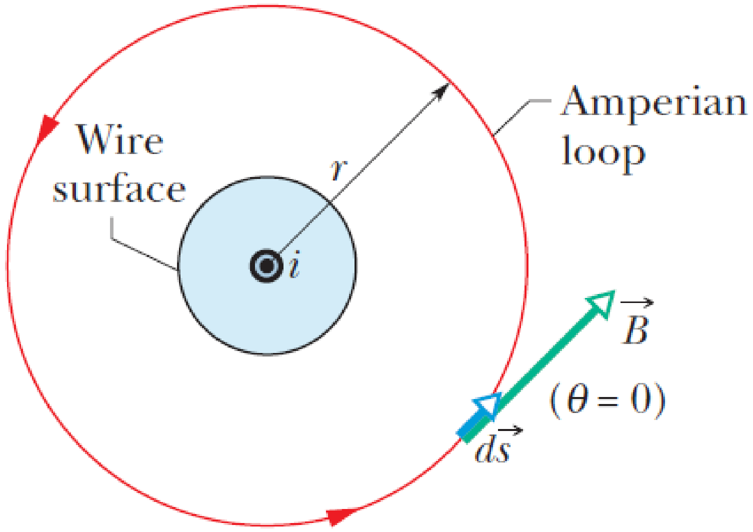
Magnetic Field Outside a Long Straight Wire with Current

- Figure shows a long straight wire that carries current *i* directly out of the page. The magnetic field produced by the current has the same magnitude at all points that are the same distance *r* from the wire.
- If we encircle the wire with a concentric circular Amperian loop of radius *r*, as in Figure, we can use ampere's law to find the magnetic field at a point at some distance r.
- The magnetic field then has the same magnitude *B* at every point on the loop. By ampere's law

$$\oint \boldsymbol{B} \,.\, d\boldsymbol{s} = \mu_0 I_{enc} \qquad \text{(i)}$$

$$\oint \boldsymbol{B} \,.\, d\boldsymbol{s} = \oint B\cos\theta \; ds = B \oint ds = B(2\pi r) \qquad \therefore\ \theta = 0^{o}$$

All of the current is encircled and thus all is used in Ampere's law.



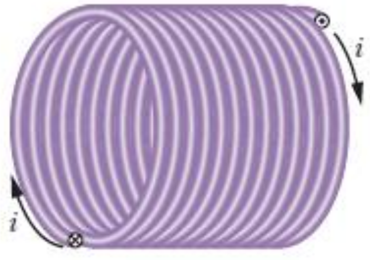
Note that - ds is the summation of all the line segment lengths ds around the circular loop; that is, it simply gives the circumference 2πr of the loop.

- By putting the value in equation (i)

$$\oint \boldsymbol{B} \,.\, d\boldsymbol{s} = \mu_0 I_{enc}$$

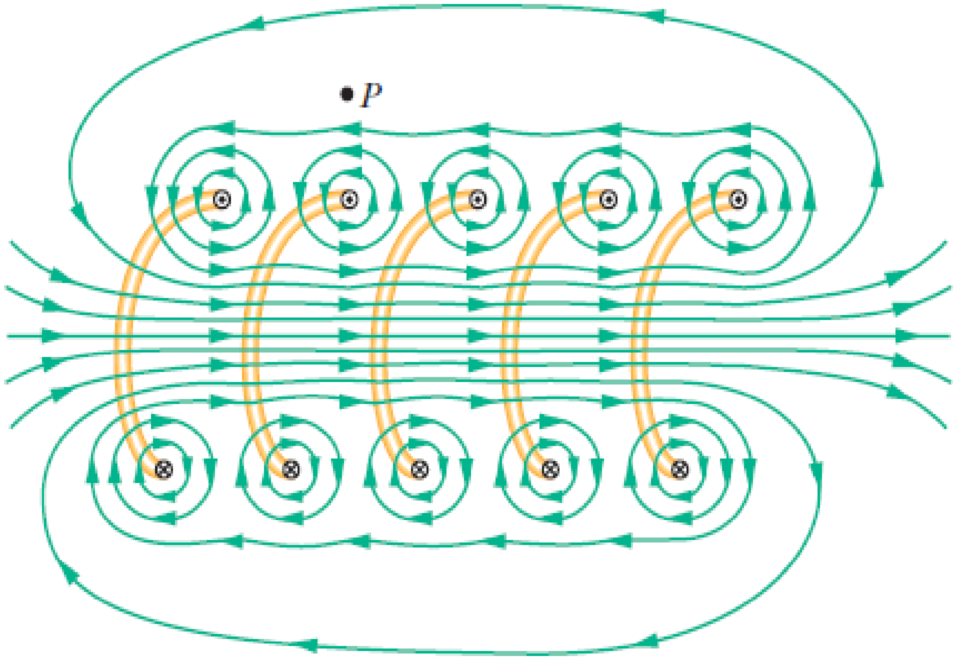
$$B(2\pi r) = \mu_0 I$$

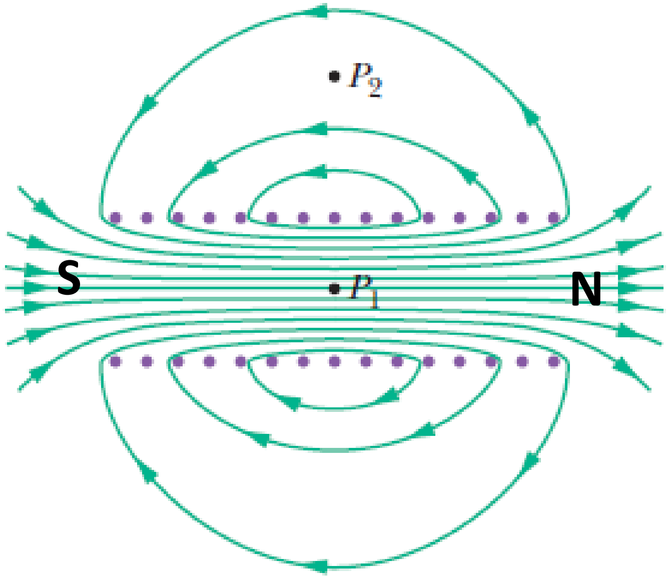
$$B = \frac{\mu_0 I}{2\pi r}$$



Magnetic Field of a Solenoid

- A **uniform magnetic field** can be generated with a **solenoid**. A solenoid is a helical coil of wire with the same current *I* passing through each loop in the coil. They may have hundreds or thousands of coils, often called *turns*.
- Figure shows a section through a portion of a “stretched-out” solenoid. The solenoid’s magnetic field is the vector sum of the fields produced by the individual turns (windings) that make up the solenoid.
- The magnetic field is strongest and most uniform *inside* the solenoid
- With many current loops along the same axis, the field in the center is strong and roughly parallel to the axis, whereas the field outside the loops is very close to zero.
- No real solenoid is ideal, but a very uniform magnetic field can be produced near the center of a tightly wound solenoid whose length is much larger than its diameter



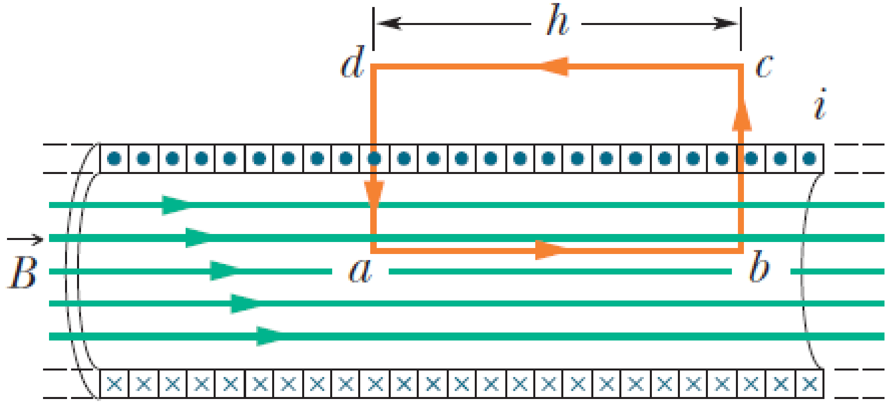


- ***Ampere's Law.*** Let us now apply Ampere's law

$$\oint \boldsymbol{B} \cdot d\boldsymbol{s} = \mu_0 I_{enc}$$

to the ideal solenoid of Figure, where **B** is uniform within the solenoid and zero outside it, using the rectangular Amperian loop abcda. We write $\oint \boldsymbol{B} \cdot d\boldsymbol{s}$ as the sum of four integrals, one for each loop segment:

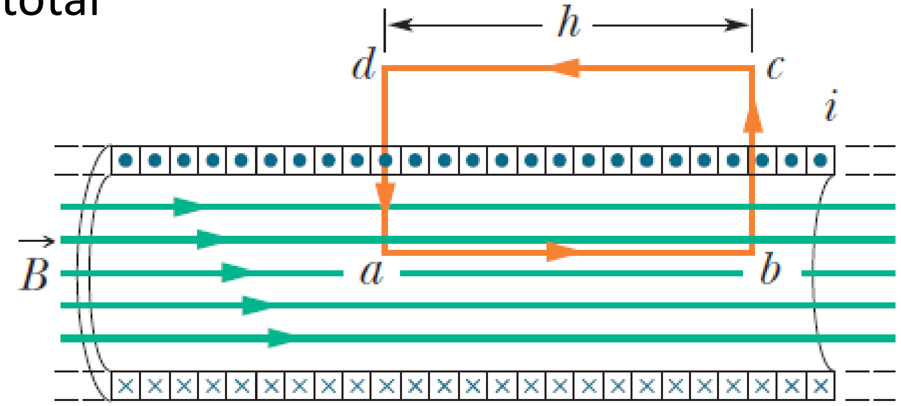
$$\oint \boldsymbol{B} \cdot d\boldsymbol{s} = \underbrace{\int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{B} \cdot d\boldsymbol{s}}_{\boxed{\mathrm{Bh}}} + \underbrace{\int_{\boldsymbol{b}}^{\boldsymbol{c}} \boldsymbol{B} \cdot d\boldsymbol{s}}_{\boxed{0}} + \underbrace{\int_{\boldsymbol{c}}^{\boldsymbol{d}} \boldsymbol{B} \cdot d\boldsymbol{s}}_{\boxed{0}} + \underbrace{\int_{\boldsymbol{d}}^{\boldsymbol{a}} \boldsymbol{B} \cdot d\boldsymbol{s}}_{\boxed{0}}$$



$$B \int_{a}^{b} ds = Bh$$

- **Net Current:** In the length h there are nh turns, each of which passes once through abcd carrying current *i*. Hence the total current enclosed by the rectangle is

$$I_{enc} = i(nh)$$



Ampere's law then gives us

$$Bh = \mu_0 i(nh)$$

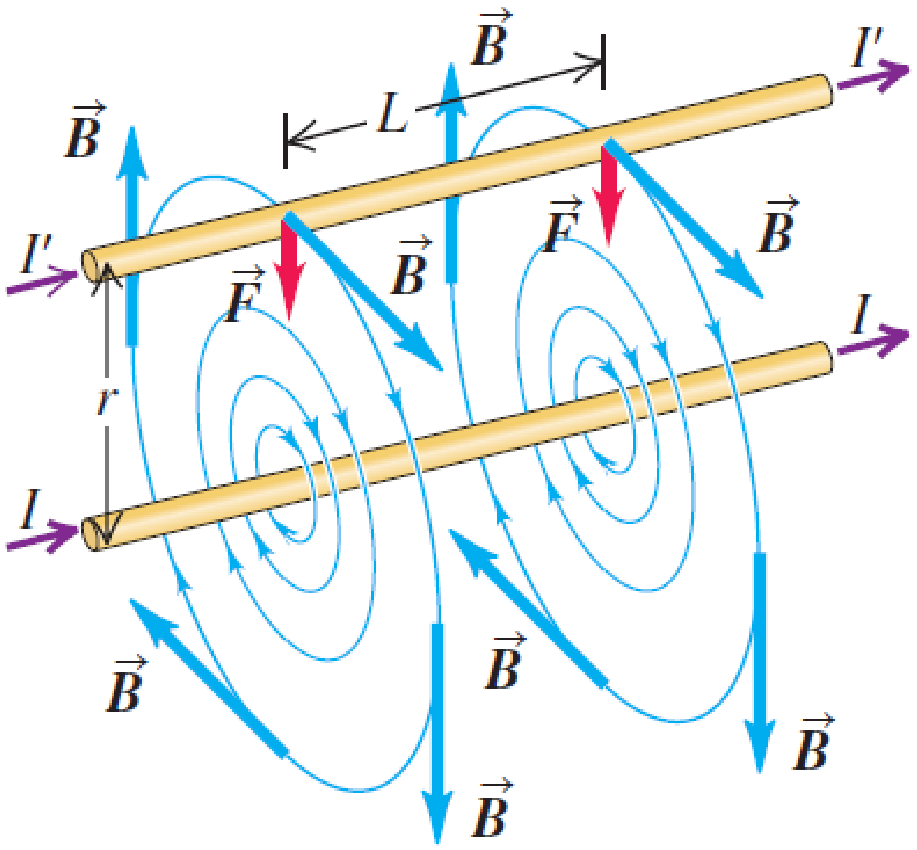
$$\boxed{B = \mu_0 in}$$

Above Equation is consistent with the experimental fact that the magnetic field magnitude B within a solenoid does not depend on the diameter or the length of the solenoid and that B is uniform over the solenoidal cross section.

Magnetic forces between two parallel currents

- Figure shows segments of two long, straight, parallel conductors separated by a distance r and carrying currents I and in the same direction.
- Each conductor lies in the magnetic field **B** set up by the other, so each experiences a force.
- The figure shows some of the field lines set up by the current I'.
- The lower conductor produces a field **B** that, at the position of the upper conductor, has magnitude

$$B = \frac{\mu_0 I}{2\pi r}$$



The force that this field **B** exerts on a length L of the upper conductor

is

$$\vec{F} = I'\vec{L} \times \vec{B}$$

So the magnitude of the magnetic force of upper conductor on lower conductor is

$$F = I'LB$$

$$F = \frac{\mu_0 II'L}{2\pi r}$$

- The force on the lower conductor is upward
- Two parallel conductors carrying current in the same direction attract each other. If the direction of either current is reversed, the forces also reverse. Parallel conductors carrying currents in opposite directions repel each other