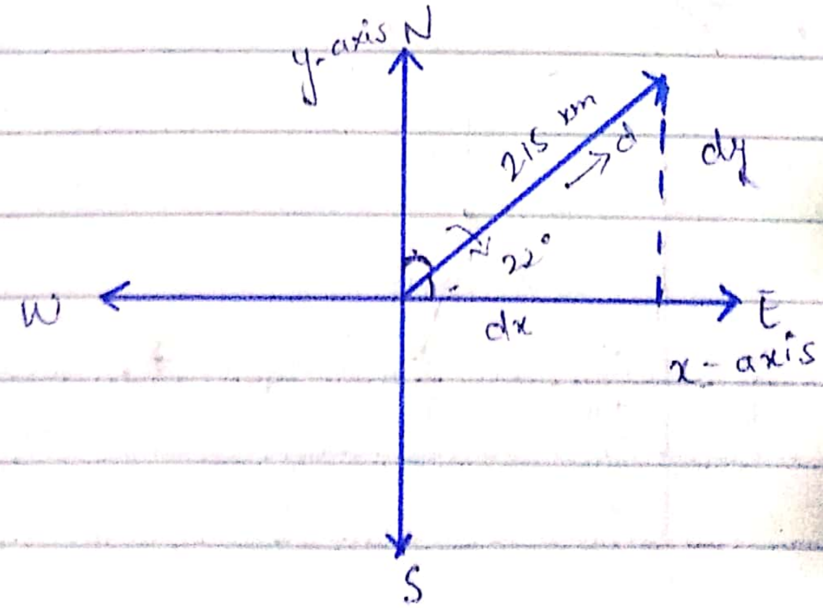
NUMERICAL PROBLEM

Problem no 1:-

A small airplane leaves an airport over cast day and is later sighted 215 km away in a direction making an angle of 22° last of due North. This means that the direction is not due north but it is rotated 22° toward the east ~~from~~ from due North. How far east and north is the airplane from the airport if sighted?



along east

$dx = d\cos\theta = (215)\cos 68^\circ = 80.54$

$dy = d\sin\theta = (215)(\sin 68^\circ) = 200$ km

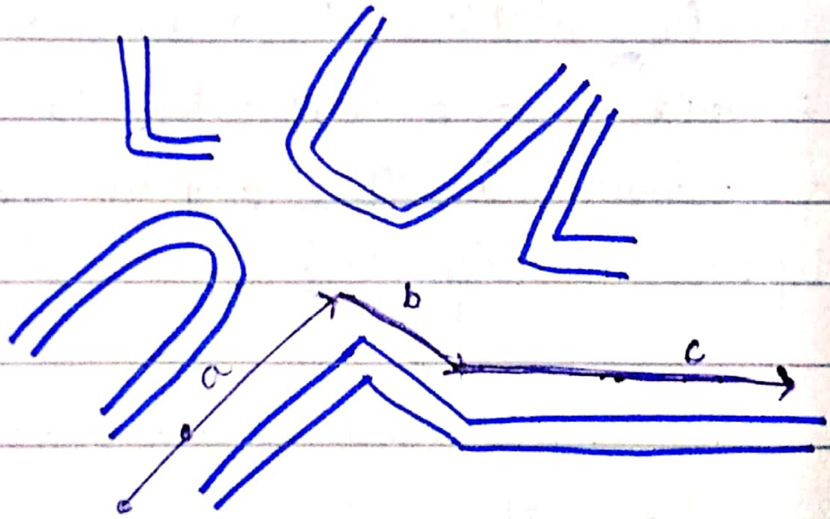
along north

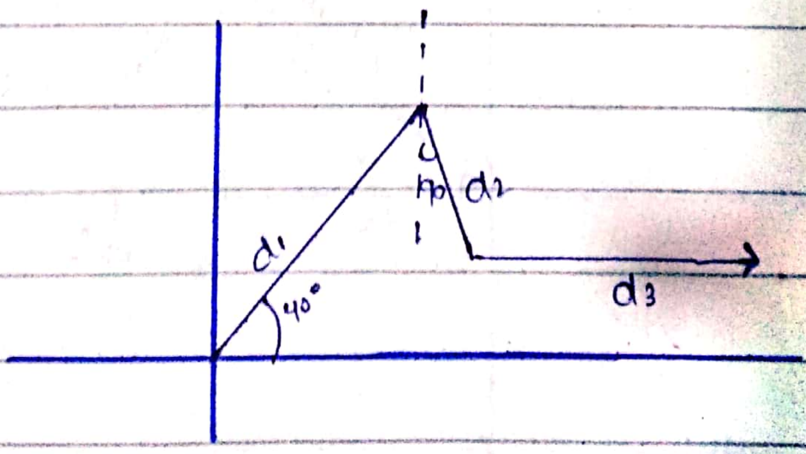
we have to find angle w.r.t x-axis

so ; $\theta = 90^\circ - 22$

$\theta = 68^\circ$

2nd Numerical:-





$d_1 = 6m$, $\theta_1 = 40°$
$d_2 = 8m$, $\theta_2 = 30°$
$d_3 = 5m$, $\theta_3 = 0°$

$|d\,net| = ?$, $\theta = ?$

$\vec{d}\,net = \vec{d_1} + \vec{d_2} + \vec{d_3}$

resolve vectors (d_1) into its components.

$d_1 = dx_1 = d_1 \cos\theta_1 = 6\cos 40° = 4.5$
$dy_1 = d_1 \sin\theta = 6 \sin 40° = 3.85$

$d_2 = dx_2 = d_2 \cos\theta_2 = 8\cos 300° = 4$
$= dy_2 = d_2 \sin\theta_2 = 8 \sin 300° = -6.9$

$d_3 = dx_3 = d_3 \cos\theta_3 = 5\cos 0° = 5$
$= dy_3 = d_3 \sin\theta_3 = 5 \sin 0° = 0$

$d_{net\,x} = 4.5 + 4 + 5 = 13.5$
$d_{net\,y} = 3.85 + (-6.9) + 0 = -3.05$

$$\vec{d_{net}} = 13.5\,m\,\hat{i} - 3.05\,m\,\hat{j}$$

$$|d_{net}| = \sqrt{(d_{net\,x})^2 + (d_{net\,y})^2}$$

$$|d_{net}| = \sqrt{(13.5)^2 + (-3.05)^2}$$

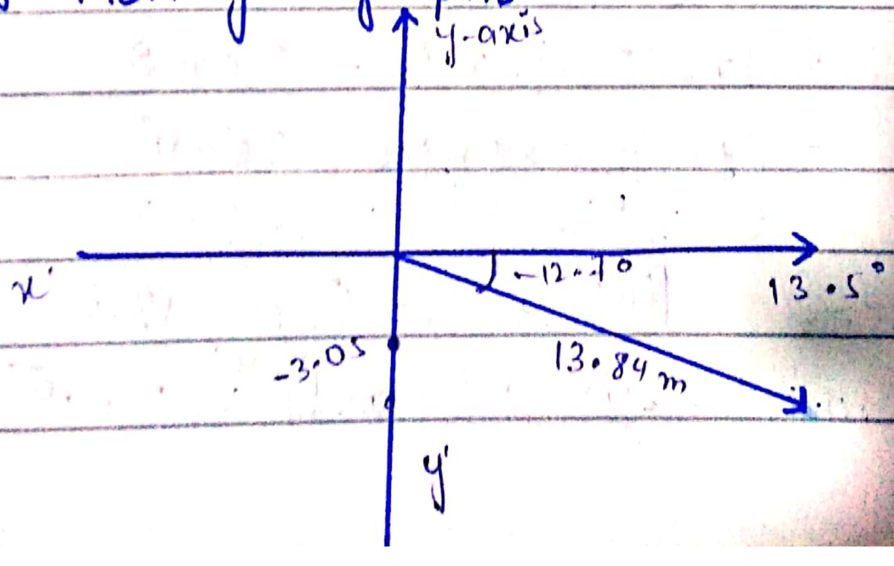
$$|d_{net}| = 13.84\,m$$

$$\theta = \tan^{-1}\left(\frac{d_{net\,y}}{d_{net\,x}}\right)$$

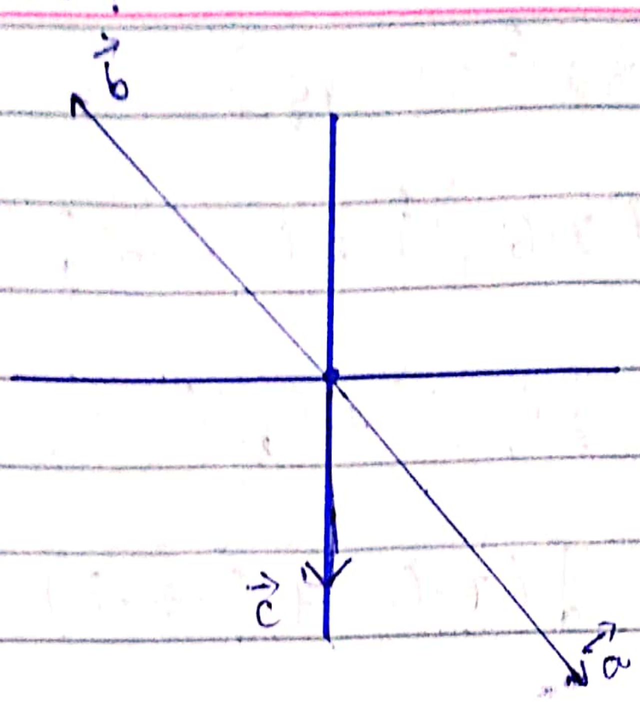
$$\theta = \tan^{-1}\left(\frac{-3.05}{13.5}\right)$$

$$\theta = -12.7^\circ$$

Now Plotting graphs :-



Q# Find Resultant of following vectors?



$\vec{a} = (4.2m)\hat{i} - (1.5m)\hat{j}$

$\vec{b} = (-1.6m)\hat{i} + (2.9m)\hat{j}$

$\vec{c} = (-3.7m)\hat{j}$

$\vec{r} = r_x\hat{i} + r_y\hat{j}$.

$r_x = a_x + b_x + c_x$

$= (4.2) + (-1.6) + 0$

$r_x = (2.6\,m)\hat{i}$

$r_y = a_y + b_y + c_y$

$= (-1.5) + (2.9) + (-3.7)$

$r_y = (-2.3\,m)\hat{j}$

$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$

$$\vec{r} = (2.6m)\hat{i} - (2.3m)\hat{j}$$

$$|r| = \sqrt{(r_x)^2 + (r_y)^2}$$

$$= \sqrt{(2.6)^2 + (-2.3)^2}$$

$$|r| = 3.47m$$

$$\theta = ?$$

$$\vec{a}.\vec{b} = ab\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{ab}\right)$$

$$\theta = \cos^{-1}\frac{\vec{a}.\vec{b}}{|a||b|}$$

$$= \cos^{-1}\frac{(a_x b_x + a_y b_y)}{\sqrt{(a_x^2) + (a_y)^2}\sqrt{(b_x)^2 + (b_y)^2}}$$

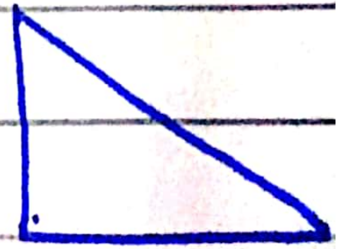
$$= \frac{\cos^{-1}(a_x b_x + a_y b_y)}{\left(\sqrt{(a_x)^2 + (a_y)^2}\right)\left(\sqrt{(b_x)^2 + (b_y)^2}\right)}$$

$$\theta = \frac{\cos^{-1}\left[(4.2)(-1.6) + (-1.5)(2.9)\right]}{\sqrt{(4.2)^2 + (-1.5)^2} \cdot \sqrt{(-1.6)^2 + (2.9)^2}}$$

$$\theta = \cos^{-1}(-0.75)$$

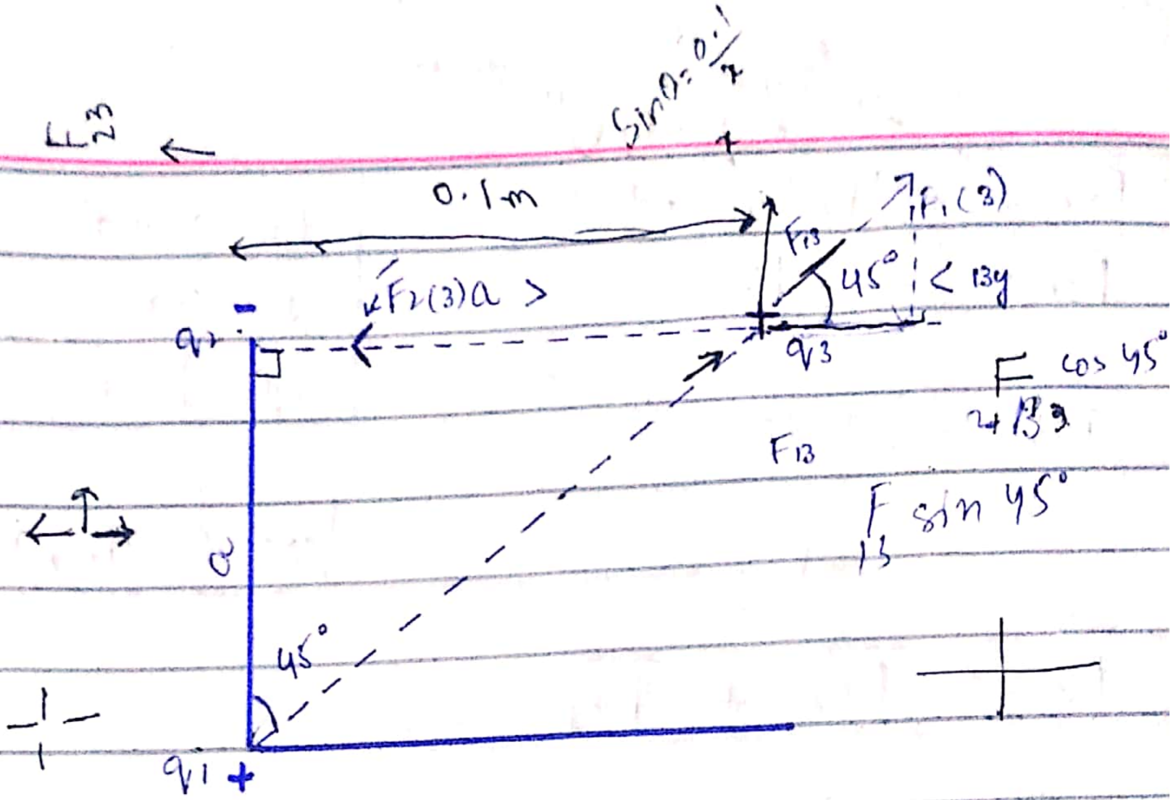
$$\theta = 138.5^\circ$$

NUMERICAL PROBLEM



Question:-

Consider three points charges located at the corner of right angle triangle shown in figure. where $q_1 = q_3 = 5\,\mu C$, $q_2 = -2\,\mu C$ and $a = 0.1\,m$. Final resultant force exerted on q_3.



$$|F_{23}| = \frac{kq_2q_3}{a^2}$$

$$= \frac{9 \times 10^9 \times 5\mu C \times -2\mu C}{(0.1)^2}$$

(we ignore '–' sign)

$$|F_{23}| = 9N$$

$$F_{13x} = |F_{13}|\cos\theta = 11.2\cos 45 = 7.9N$$

$$F_{13y} = |F_{13}|\sin\theta = 11.2\sin 45 = 7.9N$$

$$|F_{13}| = \frac{kq_1q_3}{(\sqrt{2}\,a)^2} = 0.0$$

$$= 11.2N$$

(Forces are opposite to each other)

$F_{3x} = (-9N) + (7.9)$

$F_{3x} = -1.1$

$d = a^2 + a^2$

$d = 2a^2$

$d = \sqrt{2}a$

$F_{3y} = 7.9$

$\because \cos\theta = F_{13x}$

$\vec{F_3} = \vec{F_{3x}} + \vec{F_{3y}}$

$\vec{F_3} = (-1.1N)\hat{i} + (7.9)\hat{j}$

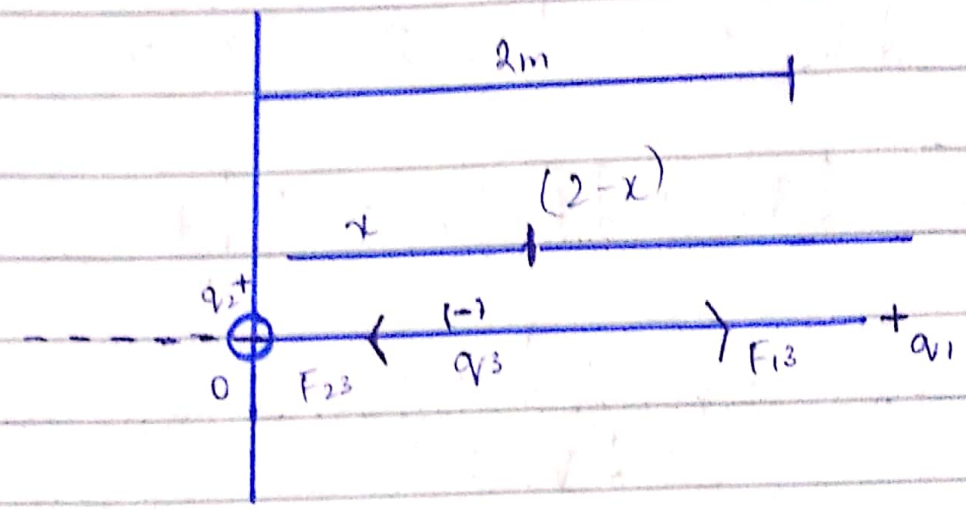
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{7.9}{-1.1}\right)$

$\boxed{\theta = -82.1^\circ}$

Numerical Problem # 02

Three points charges lie along x-axis as show in figure. The positive charge $q_1 = 15\mu C$ is at $x = 2m$, the positive charge $q_2 = 6\mu C$ is at the origin and the net force acting on q_3 is zero. What is the x-coordinate of q_3?



$$\vec{F_{13}} + (-\vec{F_{23}}) = 0$$

$$|F_{13}| = |F_{23}|$$

$$\frac{\cancel{k} q_1 \cancel{q_3}}{(2-x)^2} = \frac{\cancel{k} q_2 \cancel{q_3}}{(x)^2}$$

$$\frac{q_1}{(2-x)^2} = \frac{q_2}{(x)^2}$$

$$q_1 x^2 = q_2 (2-x)^2$$

$$\pm \sqrt{q_1 x^2} = \sqrt{q_2 (2-x)^2}$$

$$\pm \sqrt{q_1}\, x = \sqrt{q_2}\, 2 - \sqrt{q_2}\, x$$

$$\left(\pm \sqrt{q_1} + \sqrt{q_2}\right) x = 2\sqrt{q_2}$$

$$x = \frac{2\sqrt{q_2}}{\sqrt{q_2} \pm \sqrt{q_1}}$$

$$x = \frac{2\sqrt{q_2}}{\sqrt{q_2} + \sqrt{q_1}} \qquad x = \frac{2\sqrt{q_2}}{\sqrt{q_2} - \sqrt{q_1}}$$

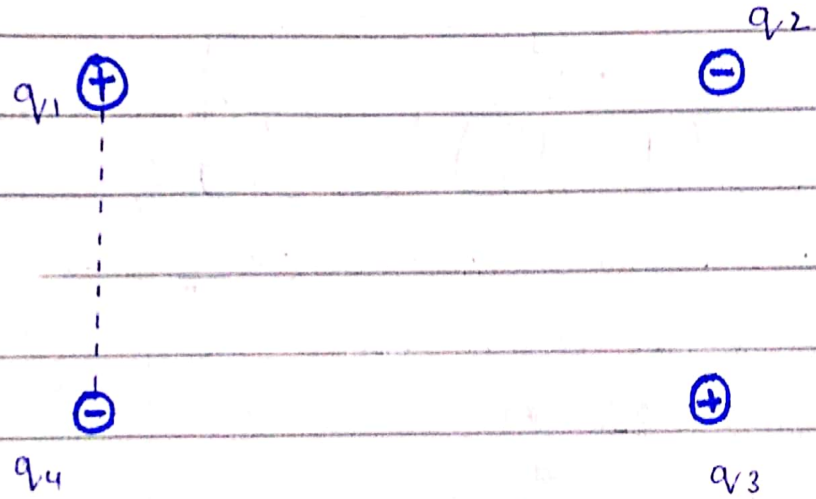
$$x = \frac{2\sqrt{(6\mu C)}}{\sqrt{6\mu} + \sqrt{15}\,\mu} \qquad x = \frac{2\sqrt{6\mu C}}{\sqrt{6\mu} - \sqrt{15\mu}}$$

$$\boxed{x = 0.775\,m} \qquad \boxed{x = -3.442\,m}$$

$$E = \frac{1}{\cancel{4}_{2}\pi\epsilon_0} \cdot \frac{q}{z^2} \cdot \frac{\cancel{2}d}{z} \qquad (\because qd = p)$$

$$\boxed{E = \frac{p}{2\pi\epsilon_0 z^3} \quad \text{Ans.}}$$

Example:



$q_1 = +10\,nC$

$q_2 = -20\,nC$

$q_3 = +20\,nC$

$q_4 = -10\,nC$

$a = 5\,cm = 5 \times 10^{-3}\,m$

$$\vec{E} = E_x\hat{i} + E_y\hat{j}$$

$$\vec{E_x} = E_{x_1} + E_{x_2} + E_{x_3} + E_{x_4}$$

$$\vec{E_y} = E_{y_1} + E_{y_2} + E_{y_3} + E_{y_4}$$

$$E_{x_1} = + \frac{kq_1}{r^2} =$$

$$E_{x_2} = + \frac{kq_2}{r^2} =$$

$$E_{x_3} = - \frac{kq_3}{r^2} =$$

$$E_{x_4} = - \frac{kq_4}{r^2} =$$

$$E_{y_1} = - \frac{kq_1}{r^2} =$$

$$E_{y_2} = + \frac{kq_2}{r^2} =$$

$$E_{y_3} = + \frac{kq_3}{r^2} =$$

$$E_{y_4} = - \frac{kq_4}{r^2} =$$