# Lecture 9

Mesh Analysis with voltage sources

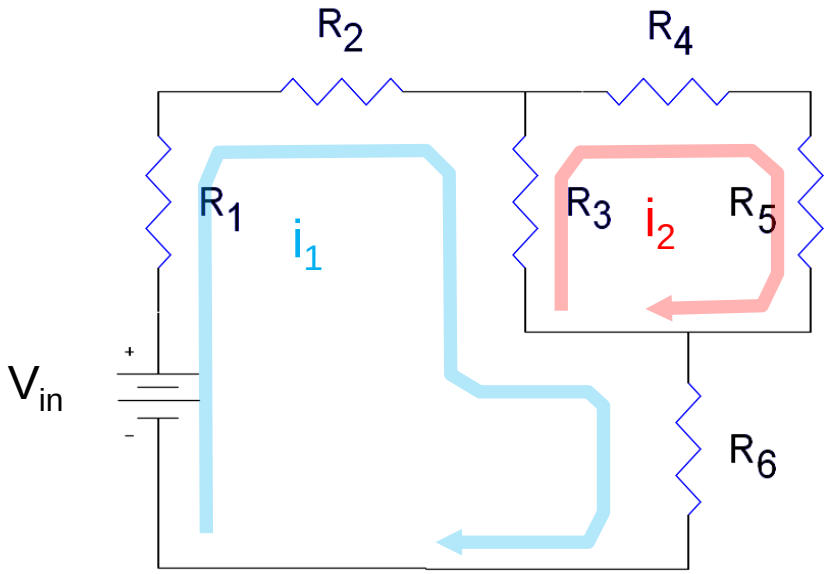
# Objectives of Lecture

- Provide step-by-step instructions for mesh analysis, which is a method to calculate voltage drops and mesh currents that flow around loops in a circuit.

# Mesh Analysis

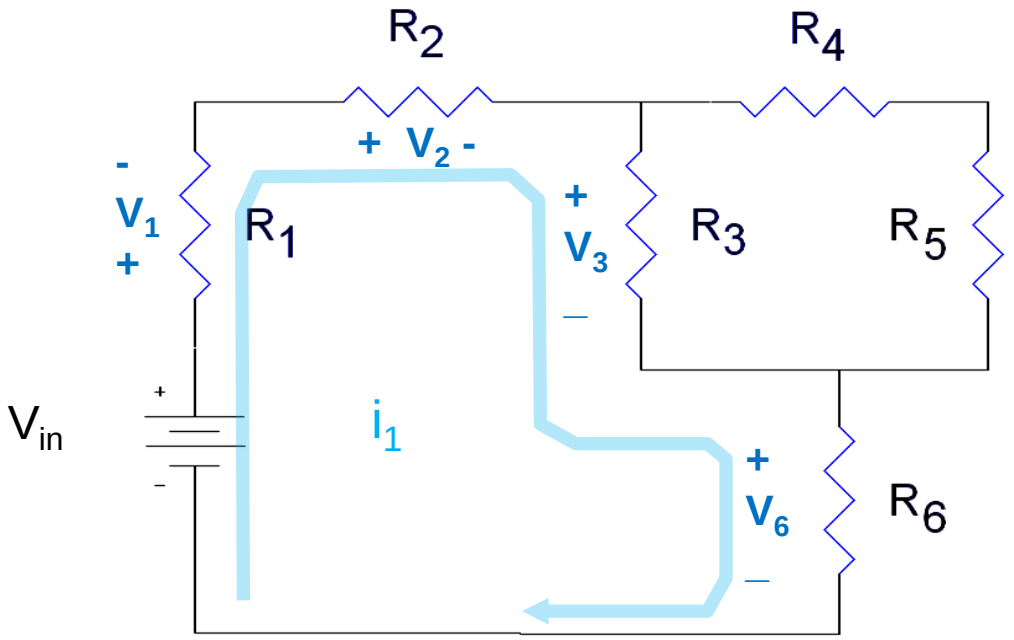
- Technique to find voltage drops within a loop using the currents that flow within the circuit and Ohm's Law
  - First result is the calculation of the current through each component
  - Second result is a calculation of either the voltages across the components or the voltage at the nodes.
- Mesh
  - the smallest loop around a subset of components in a circuit
    - Multiple meshes are defined so that every component in the circuit belongs to one or more meshes

# Mesh Analysis




- Identify all of the meshes in the circuit
- Label the currents flowing in each mesh
- Label the voltage across each component in the circuit

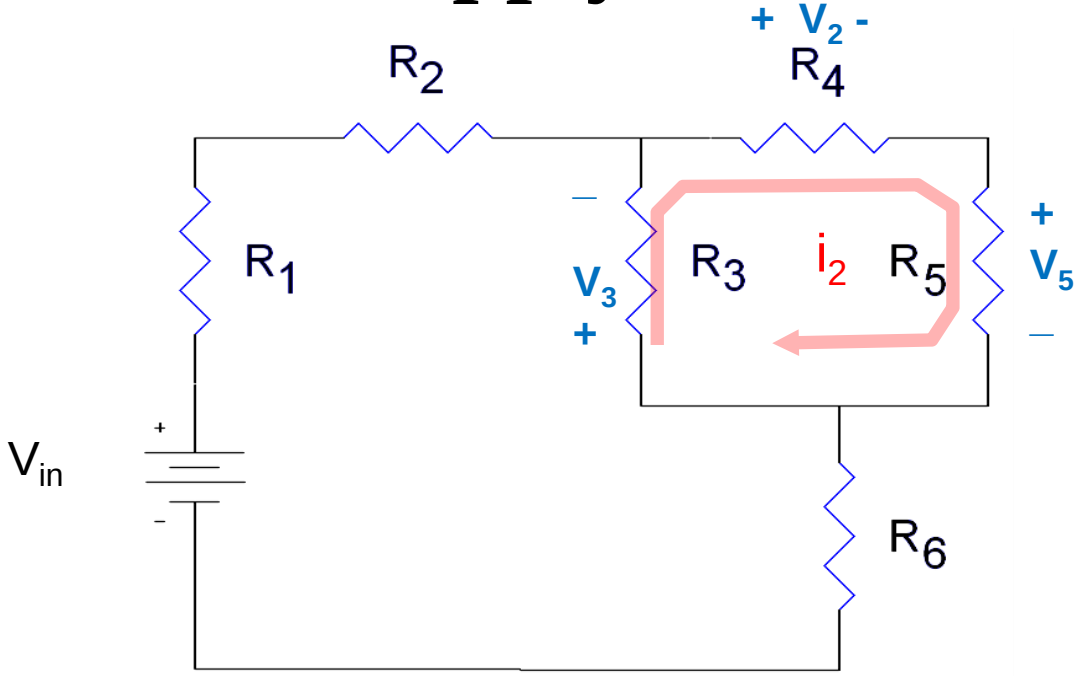
# Apply KVL on Mesh 1




- Use Kirchoff's Voltage Law for Mesh 1

$$-V_{in} + V_1 + V_2 + V_3 + V_6 = 0$$

# Apply KVL on Mesh 2



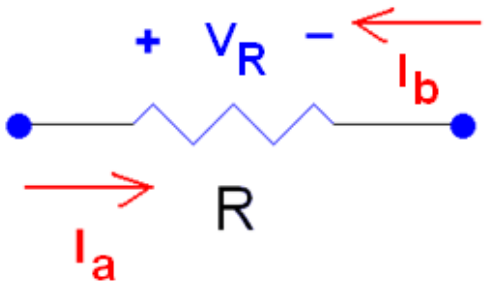

- Use Kirchoff's Voltage Law for Mesh 2

$$+V_3 + V_4 + V_5 = 0$$

# Mesh Analysis

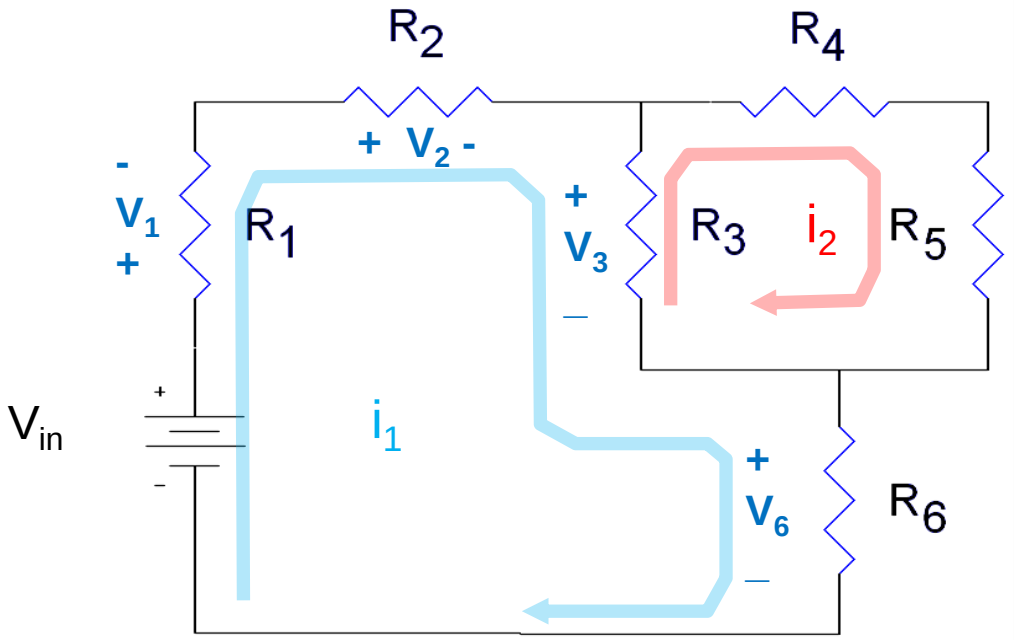
- Use Ohm's Law to relate the voltage drops across each component to the sum of the currents flowing through them.

– Follow the sign convention on the resistor's voltage.




$$V_R = (I_a - I_b)R$$

# Voltage drops on resistors in Mesh 1

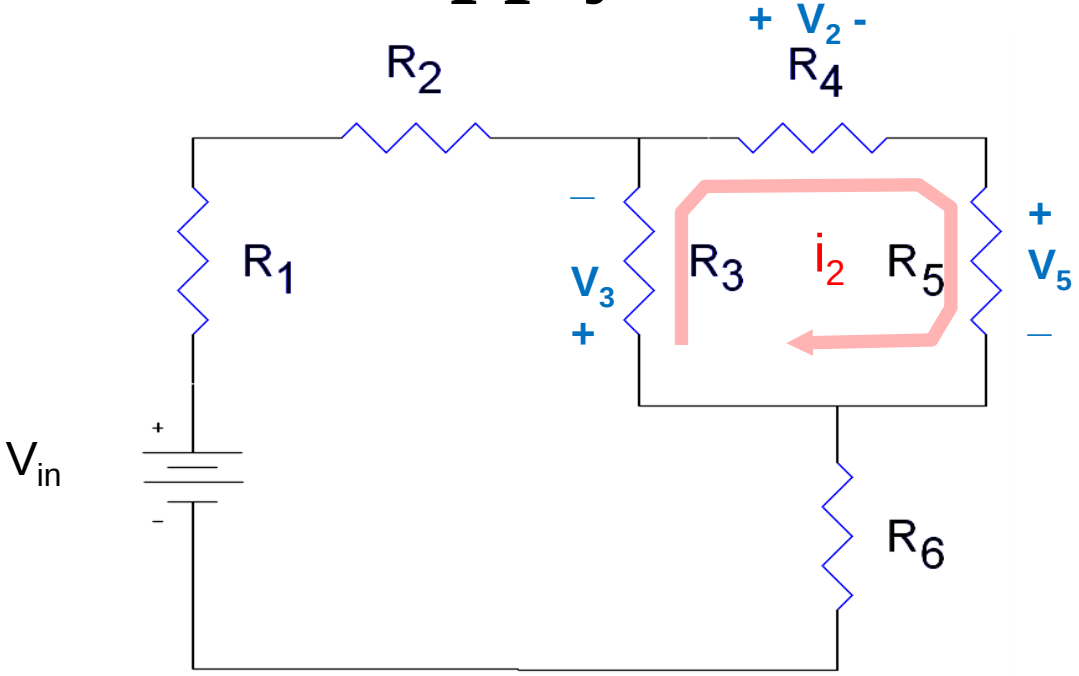



$$V_1 = i_1 R_1$$
$$V_2 = i_1 R_2$$
$$V_3 = (i_1 - i_2) R_3$$
$$V_6 = i_1 R_6$$

- Use Kirchoff's Voltage Law for Mesh 1

$$-V_{in} + V_1 + V_2 + V_3 + V_6 = 0$$

# Apply KVL on Mesh 2




$$V_4 = i_2 R_4$$
$$V_5 = i_2 R_5$$
$$V_3 = (i_2 - i_1) R_3$$

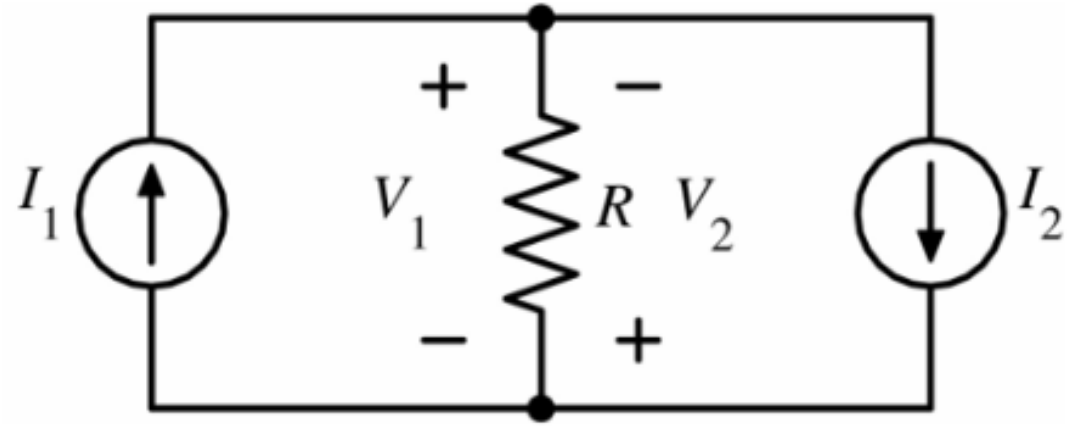
- Use Kirchoff's Voltage Law for Mesh 2

$$+V_3 + V_4 + V_5 = 0$$

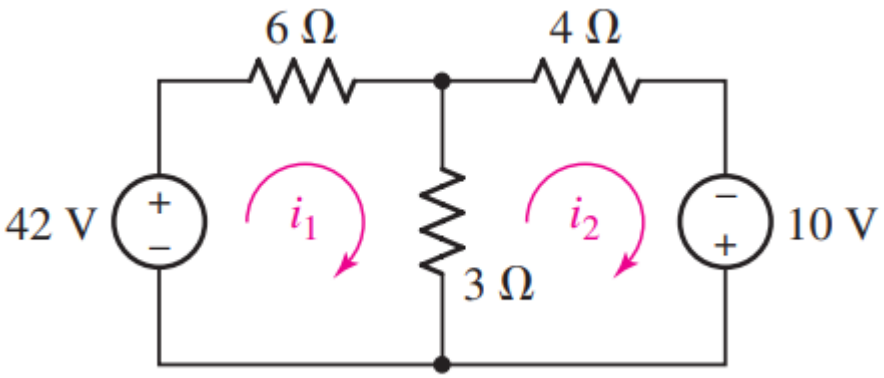
# Summary

- Steps in Mesh Analysis
  1. Identify all of the meshes in the circuit
  2. Label the currents flowing in each mesh
  3. Label the voltage across each component in the circuit
  4. Use Kirchoff's Voltage Law
  5. Use Ohm's Law to relate the voltage drops across each component to the sum of the currents flowing through them.
  6. Solve for the mesh currents
  7. Once the voltage across all of the components are known, calculate the mesh currents.

- Keep this in your mind



# Example 01




- Determine the loop currents $i_1$ and $i_2$

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$

$$9i_1 - 3i_2 = 42$$

$$-3i_1 + 7i_2 = 10$$

$$\begin{bmatrix} 9 & -3 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix}$$

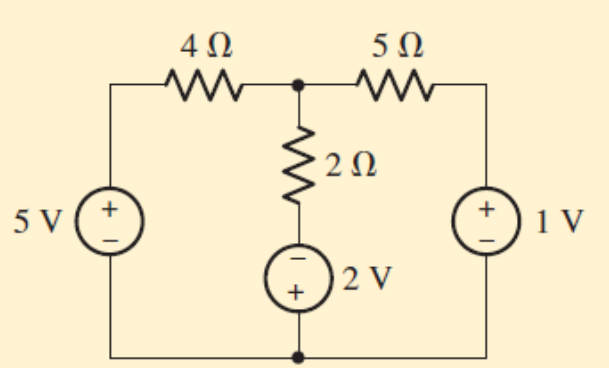
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

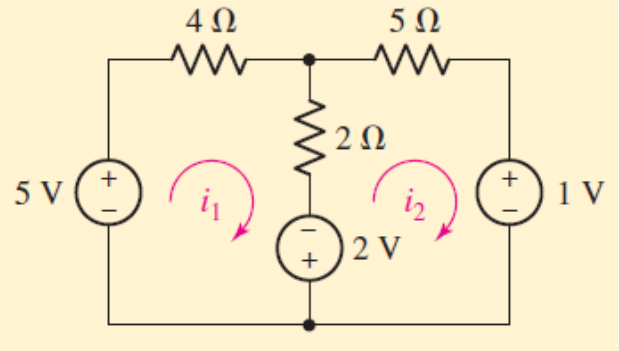
The current through the 6-Ω resistor is 6 A.

The current through the 3-Ω resistor is $(i_1 - i_2) = 2$ A

# Example 02

- Determine the power supplied by the 2 V source




- Mesh 1 $-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$
- Mesh 2 $+2 + 2(i_2 - i_1) + 5i_2 + 1 = 0$

$$6i_1 - 2i_2 = 7$$

$$-2i_1 + 7i_2 = -3$$

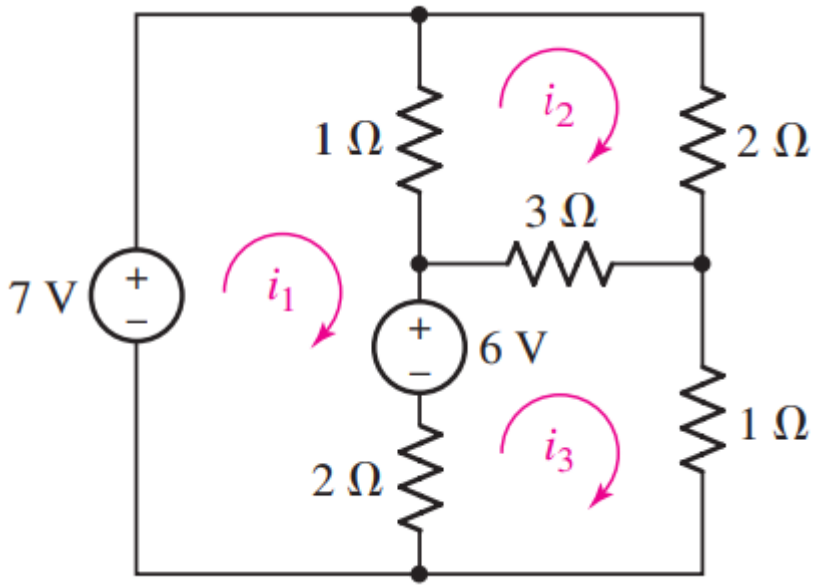
$$i_1 = \frac{43}{38} = 1.132 \text{ A}$$

$$i_2 = -\frac{2}{19} = -0.1053 \text{ A}$$

- Power absorbed by the 2 V source $-(2)(1.237) = -2.474$ W.
  - Actually 2.474 W is supplied

# Example 03

- Use mesh analysis to determine the mesh currents in the circuit.




$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$
$$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

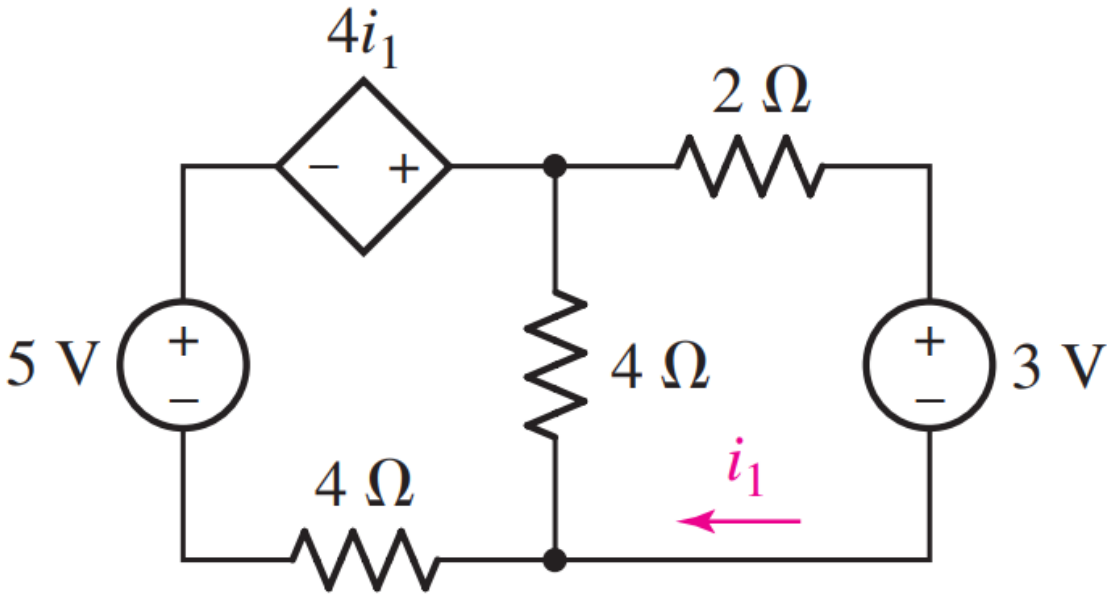
Simplifying,

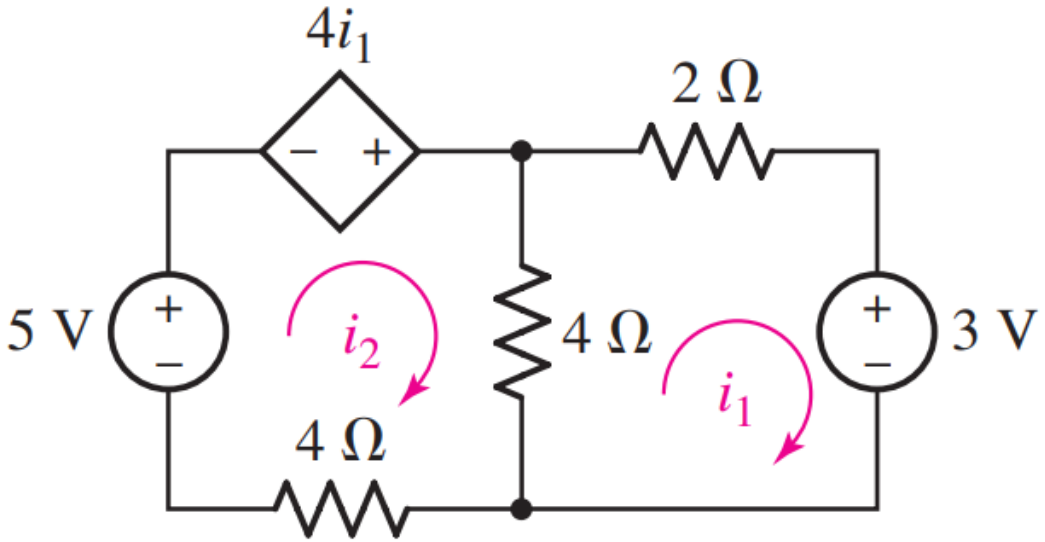
$$3i_1 - i_2 - 2i_3 = 1$$
$$-i_1 + 6i_2 - 3i_3 = 0$$
$$-2i_1 - 3i_2 + 6i_3 = 6$$

and solving, we obtain $i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 3$ A.

# Example 04

- Determine the current $i_1$






For the left mesh, KVL yields

$$-5 - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0 \qquad [27]$$

and for the right mesh we find

$$4(i_1 - i_2) + 2i_1 + 3 = 0 \qquad [28]$$

Grouping terms, these equations may be written more compactly as

$$-8i_1 + 8i_2 = 5$$

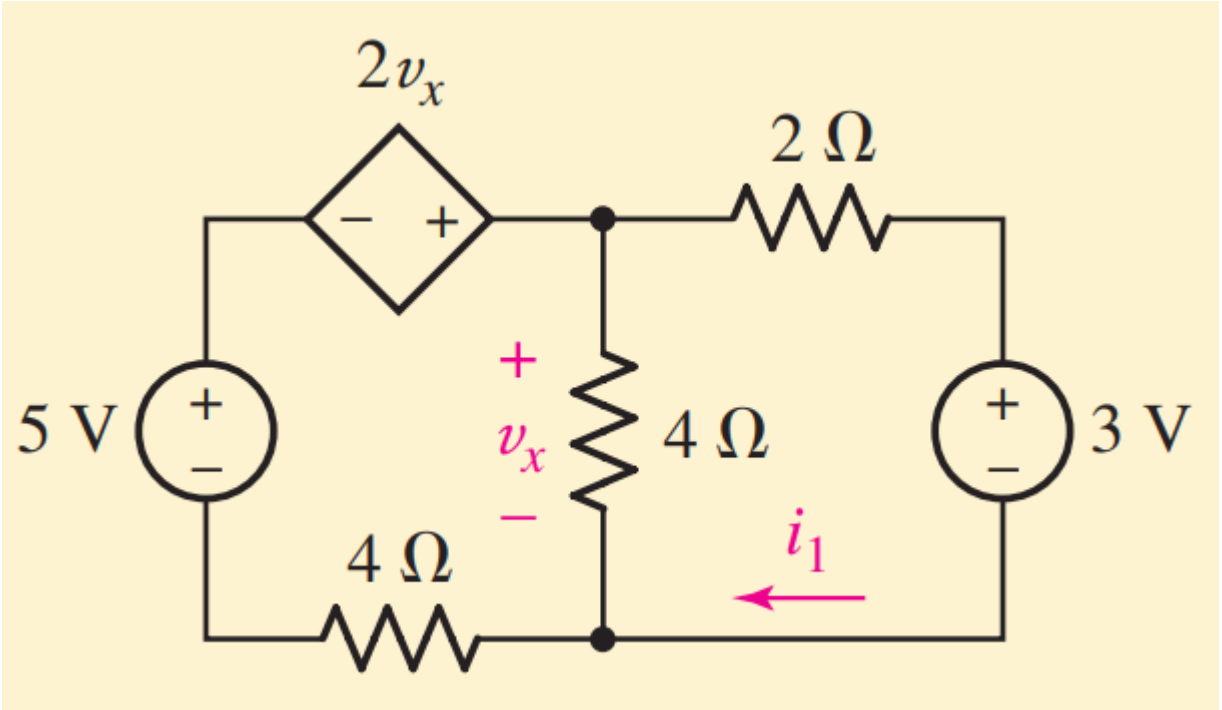
and

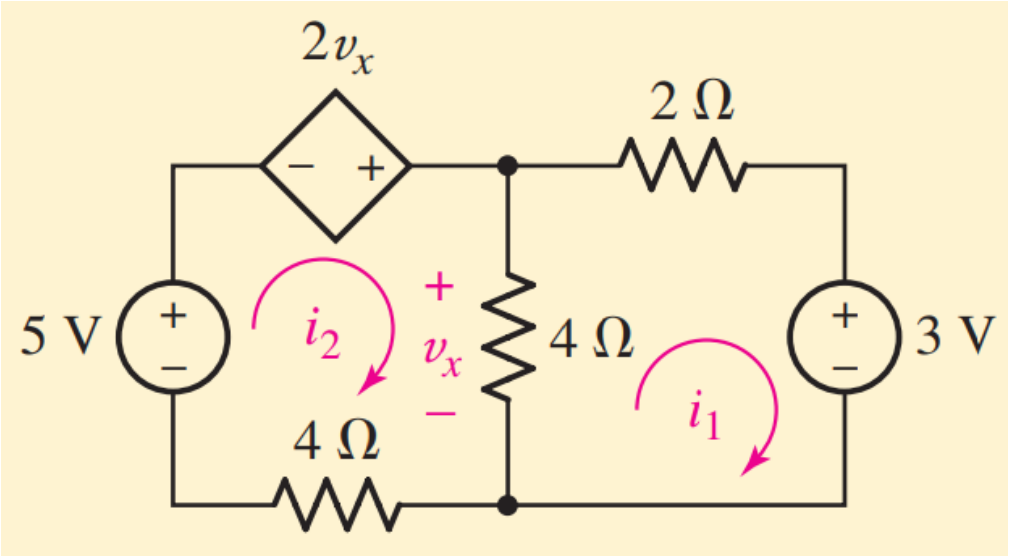
$$6i_1 - 4i_2 = -3$$

Solving, $i_2 = 375$ mA, so $i_1 = -250$ mA.

# Example 05

- Determine the current $i_1$






For the left mesh, KVL now yields

$$-5 - 2v_x + 4(i_2 - i_1) + 4i_2 = 0 \qquad [29]$$

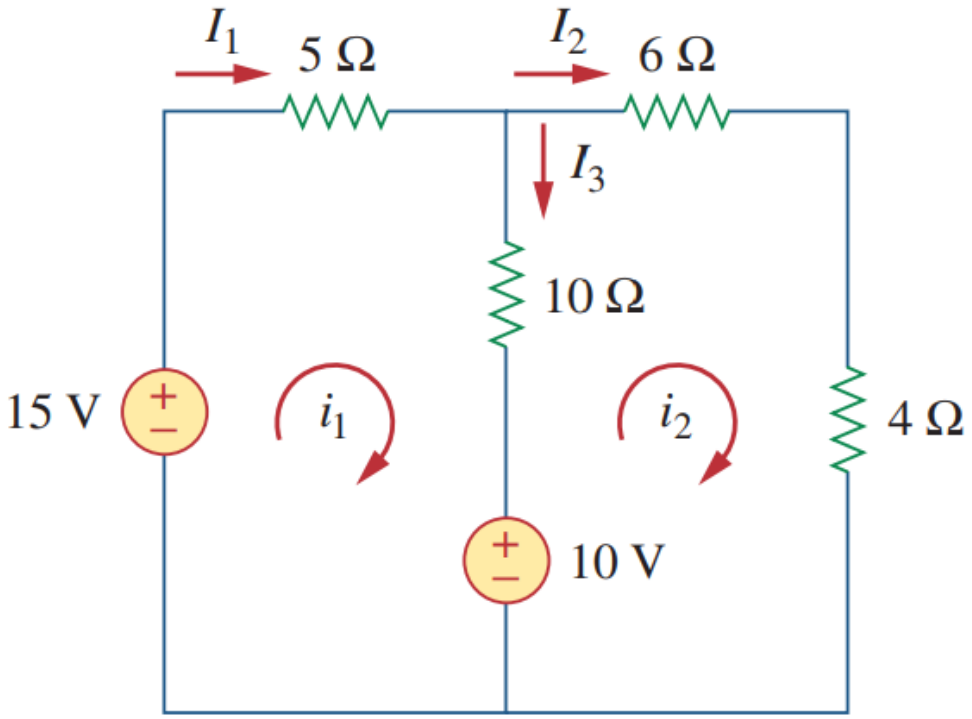
and for the right mesh we find the same as before, namely,

$$4(i_1 - i_2) + 2i_1 + 3 = 0 \qquad [30]$$

our dilemma is to construct an equation for $v_x$ in terms of mesh currents, such as

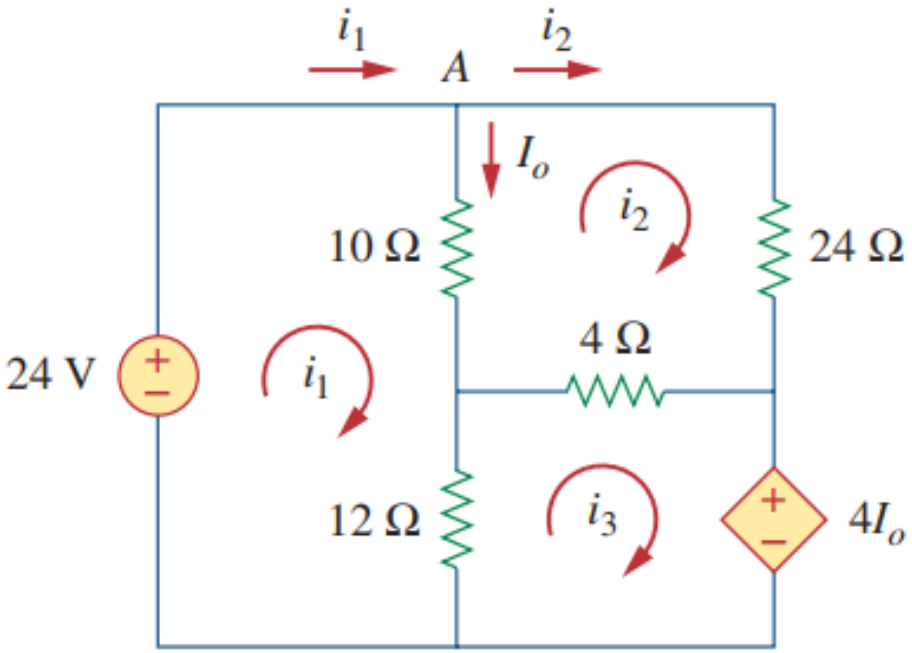
$$v_x = 4(i_2 - i_1) \qquad [31]$$

# Example 3.5



# Example 3.6

- Determine $I_o$



# Thank You