Applied Physics for Engineers

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The flux of electric field, Gauss' law and its applications

ELECTRIC FLUX

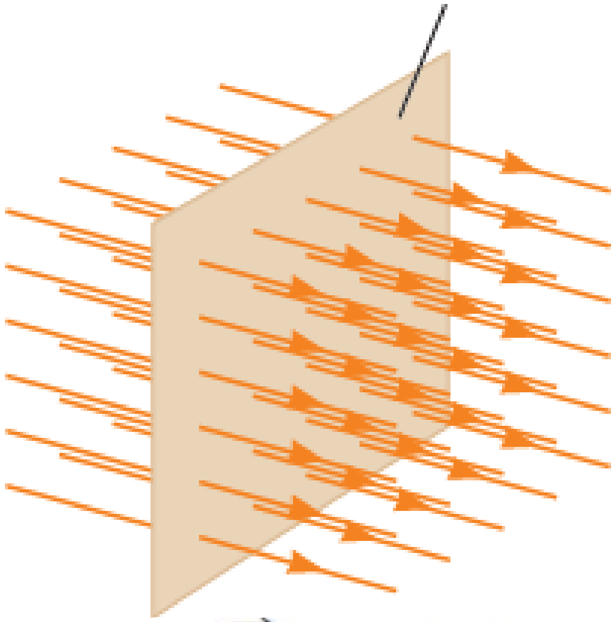
- The electric flux ϕ through a surface is the amount of electric field that penetrates the surface.

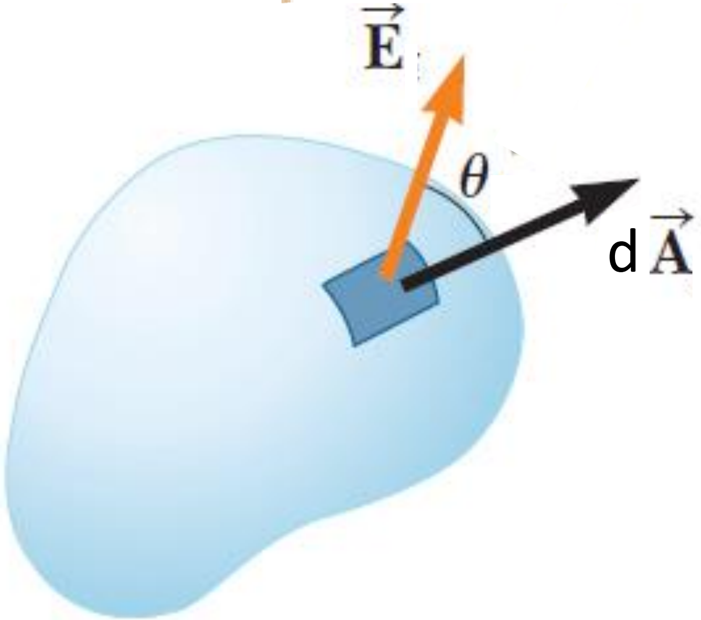
$$d\phi = \mathbf{E} \, . \, d\mathbf{A}$$

$$d\phi = E \, dA \cos\theta$$

We will integrate to find the total flux

$$\Phi_E \equiv \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



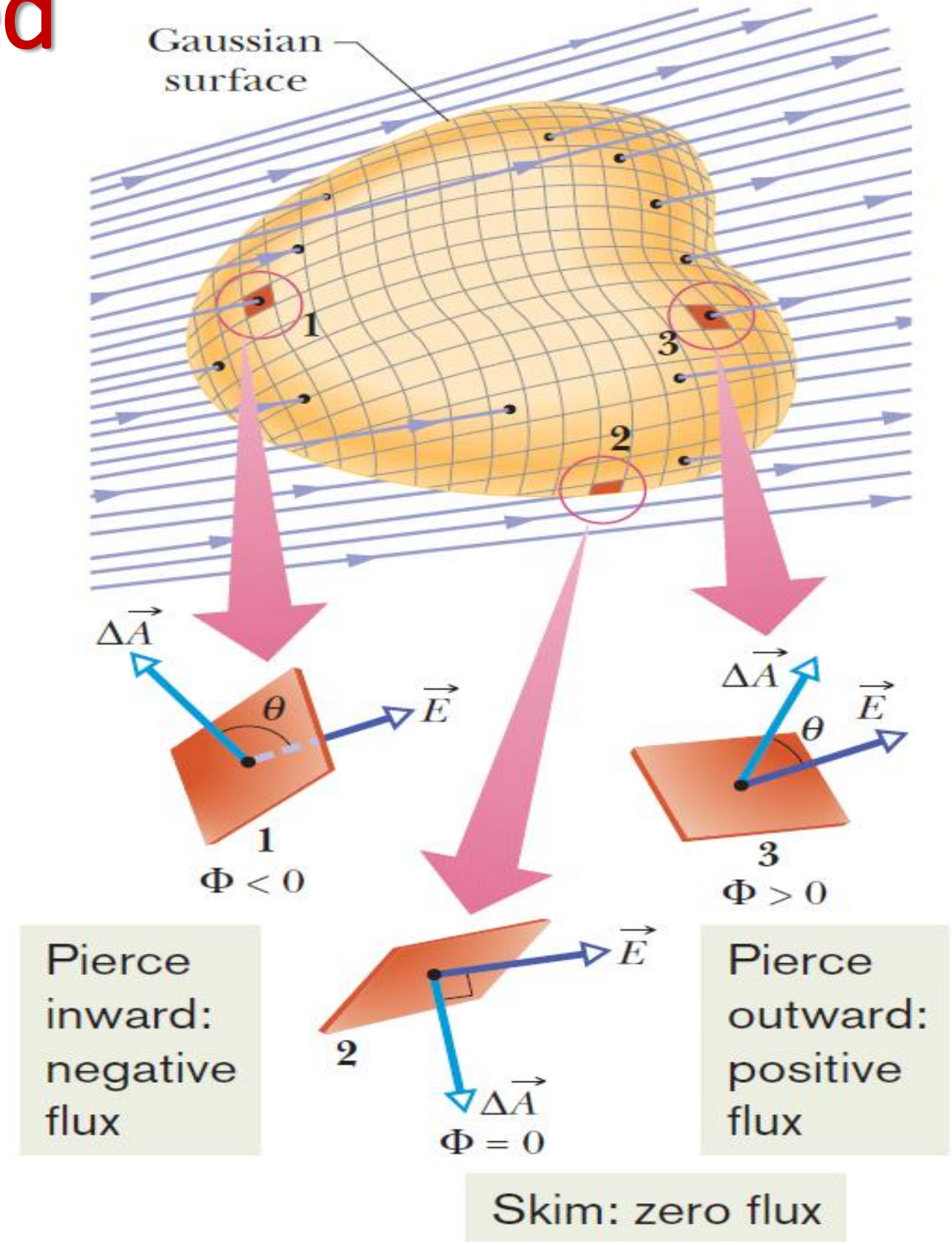


Electric Flux through a closed surface

1. $180° > \theta > 90°$, the flux is negative because cos θ is negative

2. $\theta = 90°$ so, Flux is zero

3. $\theta < 90°$ then, $\Phi = \vec{\mathbf{E}} \cdot \Delta \vec{\mathbf{A}}_1$ is positive



- The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number of lines leaving the surface minus the number of lines entering the surface.
- If more lines are leaving than entering, the net flux is positive.
- If more lines are entering than leaving, the net flux is negative.
- If more lines are entering than leaving, the net flux is negative.

The net flux through a closed surface will be,

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

GAUSS' LAW

- Gauss's law gives the relationship between the net electric flux through a closed surface and the charge enclosed by the surface

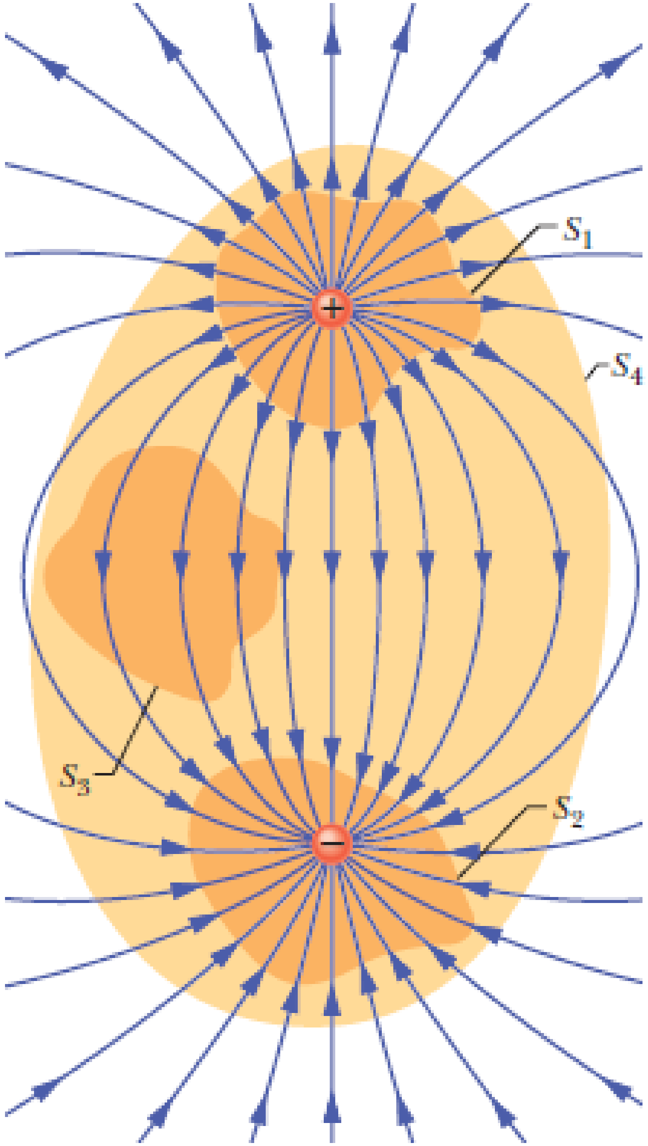
$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0}$$

- By substituting the value of flux in above equation, we get

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}$$

- Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc} in Gauss' law.

- **Surface S_1.** The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive,
- **Surface S_2.** The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge
- **Surface S_3.** This surface encloses no charge, and thus q_{enc} = 0.
- **Surface S_4.** This surface encloses no *net* charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero



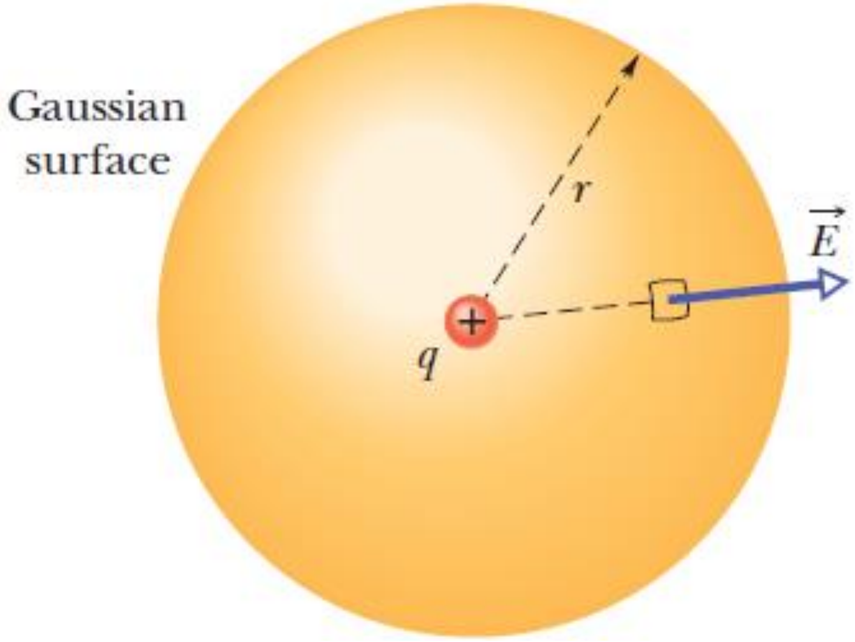
Applications of Gauss's Law

- In the figure we have enclosed a charged particle in a spherically symmetric gaussian surface, that is centered on the particle.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}$$

- Electric field vector **E** and area vector d**A** parallel to each other so θ=0 and,

$$\varepsilon_0 \oint E\, dA = q_{\text{enc}}$$



$$\varepsilon_0 E \oint dA = q$$

$q_{enc} = q$

Electric field is same along the whole area so it will be taken out of the integral

$$\varepsilon_0 E \oint dA = q$$

We know the total area of the sphere to be as $4\pi r^2$, so the above equation can be written as

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}.$$

This is the same equation that we found using coulomb's law