LEC-23

LTI System Characterized by Difference Equation

Prob. An LTI system characterized by following DE:

$$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n]$$

Taking z-transform,

$$Y(z).z^{-1} - \frac{5}{2}Y(z) + Y(z).z = X(z)$$

$$Y(z)\left[z^{-1} - \frac{5}{2} + z\right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{z - \frac{5}{2} + z^{-1}} = \frac{1}{z\left(1 - \frac{5}{2}z^{-1} + z^{-2}\right)}$$

$$\boxed{H(z) = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}}$$ Transfer Function

To analyze different System Responses;

$$H(z) = \frac{z^{-1}}{1 - 2z^{-1} - \frac{1}{2}z^{-1} + z^{-2}} = \frac{z^{-1}}{1(1-2z^{-1}) - \frac{1}{2}z^{-1}(1-2z^{-1})}$$

$$H(z) = \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-2z^{-1}}$$

$$z^{-1} = A(1-2z^{-1}) + B\left(1 - \frac{1}{2}z^{-1}\right)$$

Comparing powers;

$$z^{-1} \Rightarrow 1 = -2A - \frac{1}{2}B \Rightarrow 2 = -4A - B \Rightarrow (1)$$

$$z^{0} \Rightarrow 0 = A + B \quad -(2)$$

$$(1) + (2) \Rightarrow \quad \begin{aligned} 2 &= -4A - B \\ 0 &= A + B \\ \hline 2 &= -3A \Rightarrow \boxed{A = -\frac{2}{3}} \end{aligned}$$

put in (2) $\Rightarrow -\frac{2}{3} + B = 0 \Rightarrow \boxed{B = \frac{2}{3}}$

$$H(z) = \frac{-2/3}{1-\frac{1}{2}z^{-1}} + \frac{2/3}{1-2z^{-1}}$$

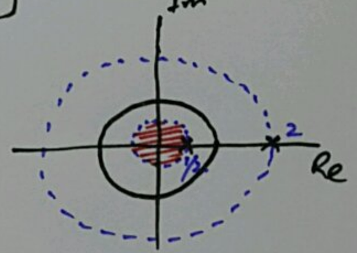
Since no information provided about the system (like ~~causa~~ causality, stability), therefore based on pole locations, we can characterize the system types.

1

$$H(z) = \frac{-2/3}{1-\frac{1}{2}z^{-1}} + \frac{2/3}{1-2z^{-1}}, \quad |z| < 1/2 \text{ (all LSS)}$$

$$h[n] = \frac{2}{3}\left(\frac{1}{2}\right)^n u[-n-1] - \frac{2}{3}(2)^n u[-n-1]$$

Stable = NO
Causal = NO
Anti-Causal = YES

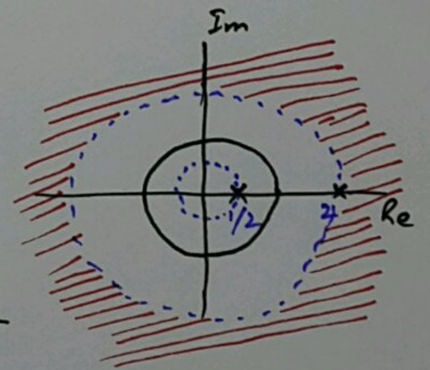


2

$$H(z) = \frac{-2/3}{1-\frac{1}{2}z^{-1}} + \frac{2/3}{1-2z^{-1}}, \quad |z| > 2 \text{ (all RSS)}$$

$$h[n] = -\frac{2}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{3}(2)^n u[n]$$

Stable = NO
Causal = YES
Anti Causal = NO

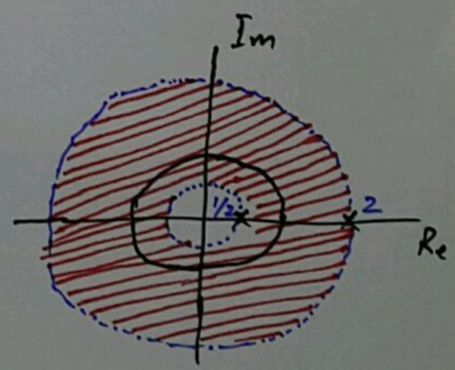


3

$$H(z) = \underbrace{\frac{-2/3}{1-\frac{1}{2}z^{-1}}}_{RSS} + \underbrace{\frac{2/3}{1-2z^{-1}}}_{LSS}, \quad 1/2 < |z| < 2 \text{ (Disc)}$$

$$h[n] = -\frac{2}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{2}{3}(2)^n u[-n-1]$$

Stable = YES
Causal = NO
Anti Causal = NO



If we are asked to seek for different combinations, like;

1 - Stable System = YES (Case $\boxed{3}$)

2- Causal System = YES (Case $\boxed{2}$)

3- Anti-Causal System = YES (Case $\boxed{1}$)

4 - Causal and Stable = NO (Not possible because of pole at 2)

5- Causal and Unstable = YES (Case $\boxed{2}$)

6 - Anticausal and Stable = NO (NOT POSSIBLE) because of pole at 1/2

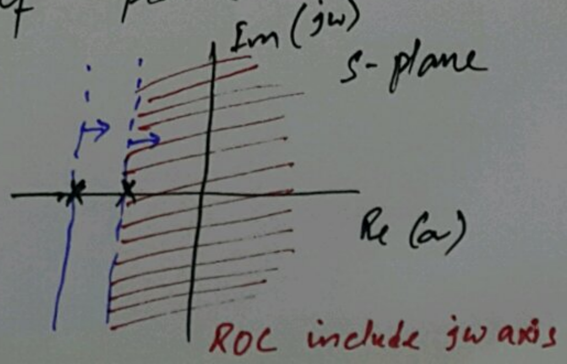
7- Anticausal and Unstable = YES (Case $\boxed{1}$)

General Requirement of LTI System:

"Causal and Stable"

Laplace Transform

poles should be on left half plane.



Z-transform

poles should be inside the unit circle.