Inverse Laplace Transform :-

$F(s) \xrightarrow{ILT} f(t)$

Method I :-

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{st}\, ds$$

Method II :- Use partial fraction.

$$F(s) = \text{—} \quad \text{—} \quad \text{—} \quad \text{—}$$

then use standard LT from table.

Exp :-

$$X(s) = \frac{1}{(s+1)(s+2)} \qquad Re\{s\} > -1$$

Use partial fractions.

$$X(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

After performing partial fraction

$$A = 1$$
$$B = -1$$

$$X(s) = \frac{1}{s+1} + \frac{(-1)}{s+2}$$

given

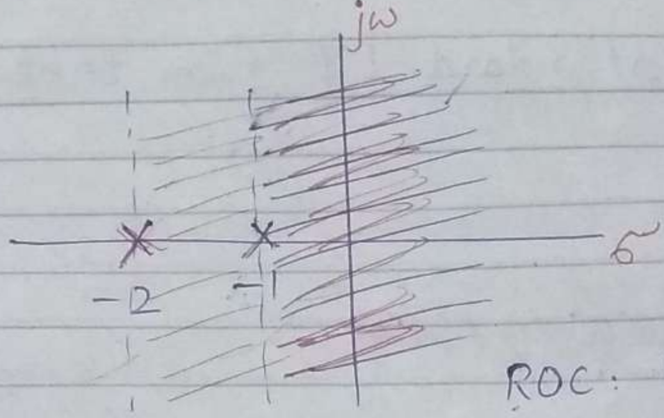
$$X(s) = \frac{1}{(s+1)(s+2)} \qquad Re\{s\} > -1$$

$\downarrow$

$$X(s) = \frac{1}{s+1} + \frac{(-1)}{s+2}$$

$s+1=0 \qquad s+2=0$

poles $\quad s=-1 \qquad s=-2$



ROC: $Re\{s\} > -1$ (common area)

(RSS)

$\therefore$ $X(s)$ is RSS

$$\frac{1}{s+1} \longrightarrow e^{-t}u(t) \qquad Re\{s\} > -1$$

$$\frac{1}{s+2} \longrightarrow e^{-2t}u(t) \qquad Re\{s\} > -2$$

$$\therefore \quad x(t) = \left[e^{-t}u(t)\right] + \left[(-1)e^{-2t}u(t)\right]$$

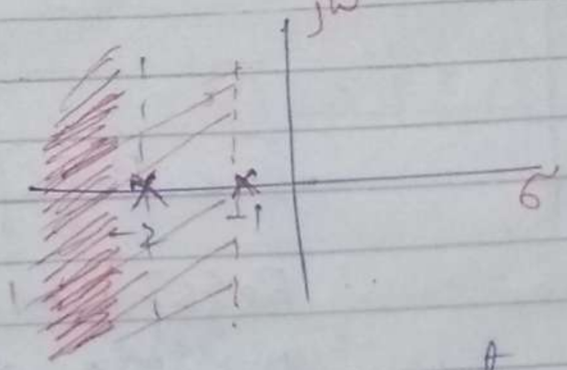
$$\boxed{x(t) = e^{-t}u(t) - e^{-2t}u(t)}$$

Exp 9.10 $\quad X(s) = \dfrac{1}{(s+1)(s+2)} \quad Re\{s\} < -2$ ③

After partial fraction

$$X(s) = \frac{1}{(s+1)} + \frac{(-1)}{s+2}$$

poles $\quad -1 \quad -2$



ROC: $Re\{s\} < -2$

LSS → $X(s)$

$$\frac{1}{s+1} \longrightarrow -e^{-t}u(-t) \quad Re\{s\} < -1$$

$$\frac{1}{s+2} \longrightarrow -e^{-2t}u(-t) \quad Re\{s\} < -2$$

$$\boxed{x(t) = -e^{-t}u(-t) + e^{-2t}u(-t)}$$

Exp 9.11 :-

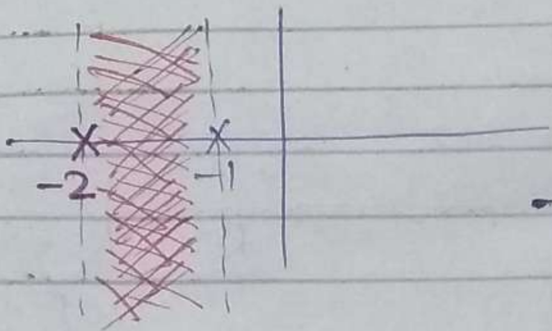
$$X(s) = \frac{1}{(s+1)(s+2)} \qquad -2 < Re\{s\} < -1$$

RSS → LSS

RSS+LSS

After partial fraction

$$X(s) = \frac{1}{s+1} + \frac{(-1)}{s+2}$$



$-2 < Re\{s\} < -1$

$Re\{s\} < -1$ LSS

$Re\{s\} > -2$ RSS

$$\therefore \frac{1}{s+1} \xrightarrow{LSS} -e^{-t}u(-t)$$

$$\frac{1}{s+2} \xrightarrow{RSS} +e^{-2t}u(+t)$$

$$\therefore x(t) = -e^{-t}u(-t) - e^{-2t}u(t)$$