Lecture 11

Linearity

Superposition

Objectives of Lecture

- Introduce the property of linearity
- Introduce the superposition principle
- Provide step-by-step instructions to apply superposition when calculating voltages and currents in a circuit that contains two or more power sources.
- Describe the differences between ideal and real voltage and current sources
  - Demonstrate how a real voltage source and real current source are equivalent so one source can be replaced by the other in a circuit.

Linearity

A Requirement for Superposition

Linear Systems

- The homogeneity property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant.

- If x is multiplied by any constant, a

  $y = f(ax) = af(x)$

- then the system is linear.

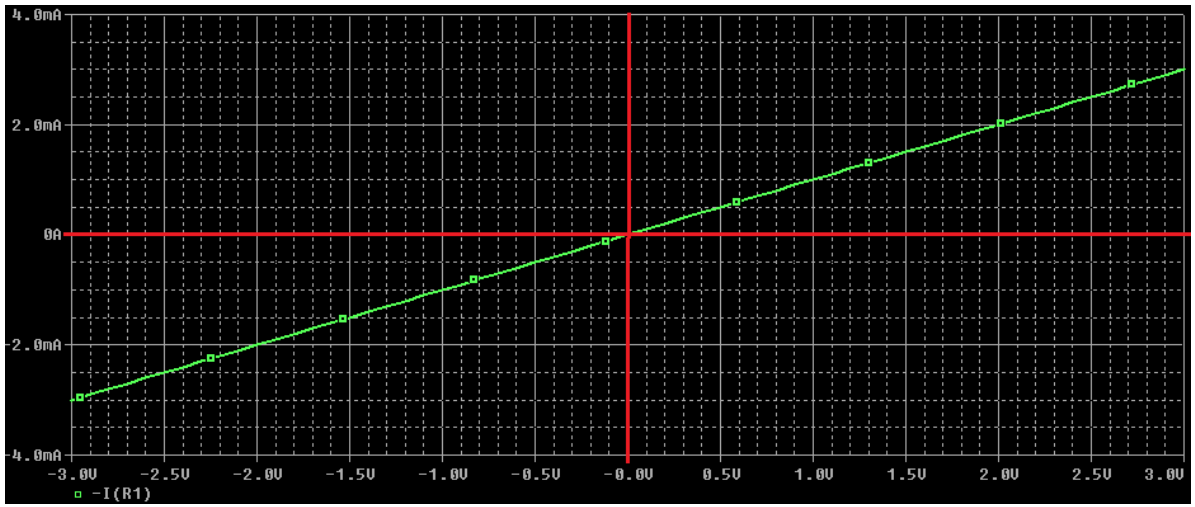
Linearity

- Ohm's Law is a linear function.

  $V = I \times R$

- If the current is increased by a constant k, then the voltage increases correspondingly by k;

  $k \times I \times R = k \times V$



Linearity

- The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.
- If $x = x_1 + x_2$

  $y = f(x) = f(x_1 + x_2) = f(x_1) + f(x_2)$
- then the system is linear.

- Using the voltage-current relationship of a resistor, if

  $V_1 = I_1 \times R$ and $V_2 = I_2 \times R$
- then applying $(I_1 + I_2)$ gives

  $V = (I_1 + I_2) \times R = I_1 \times R + I_2 \times R = V_1 + V_2$

Nonlinear Systems and Parameters

- In a linear resistive circuit power is

  $P = IV$

- Is power linear with respect to current and voltage?

- Power is nonlinear with respect to current and voltage.
  - As either voltage or current increase by a factor of a, P increases by a factor of a^2.

    $P = IV = I^2R = V^2/R$

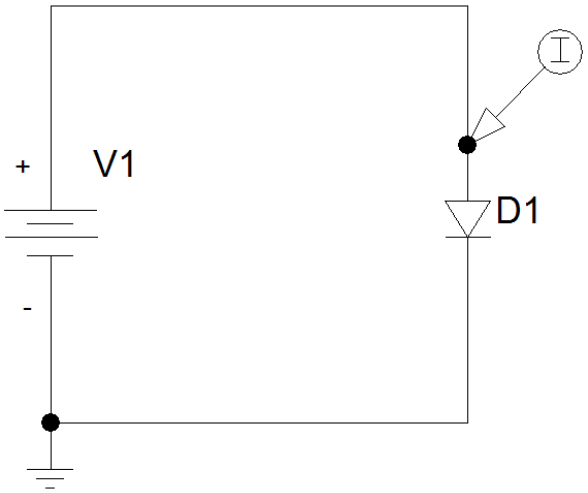
Linear Components

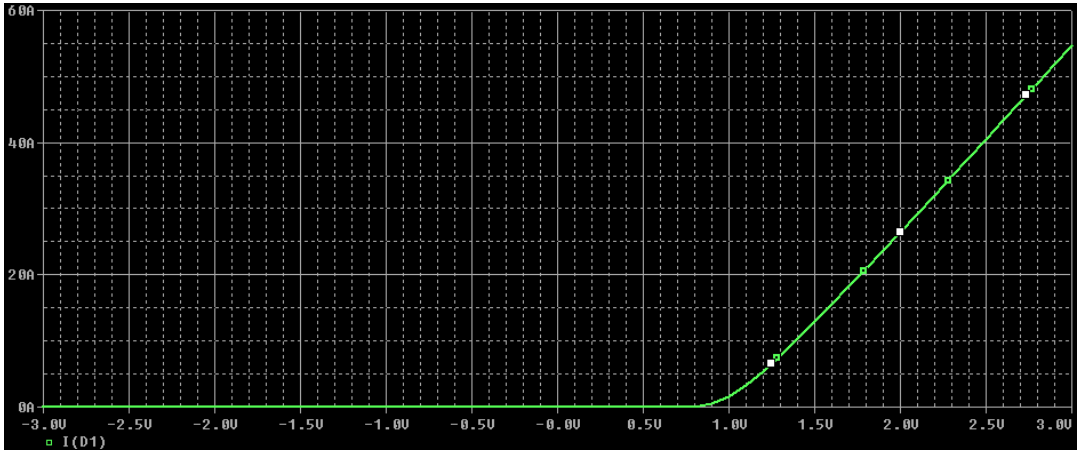
- Resistors
- Inductors
- Capacitors
- Independent voltage and current sources
- Certain dependent voltage and current sources that are linearly controlled

Nonlinear Components

- Diodes including Light Emitting Diodes
- Transistors
- SCRs
- Magnetic switches
- Nonlinearily controlled dependent voltage and current sources

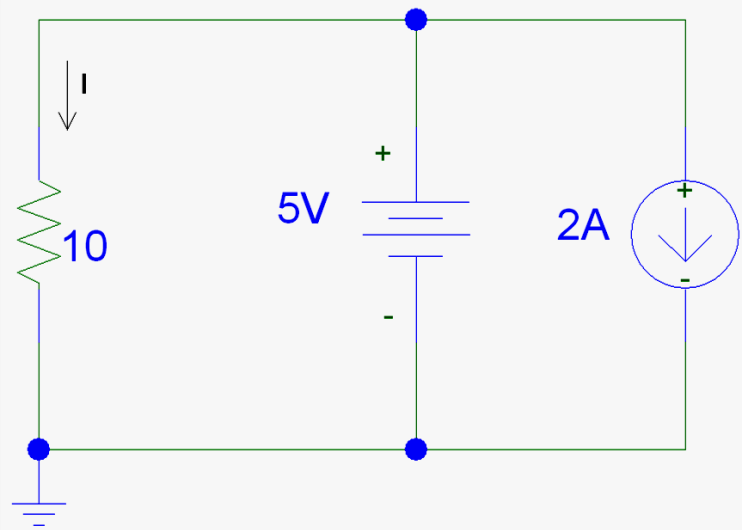
Diode Characteristics





- An equation for a line can not be used to represent the current as a function of voltage.

Example 01…

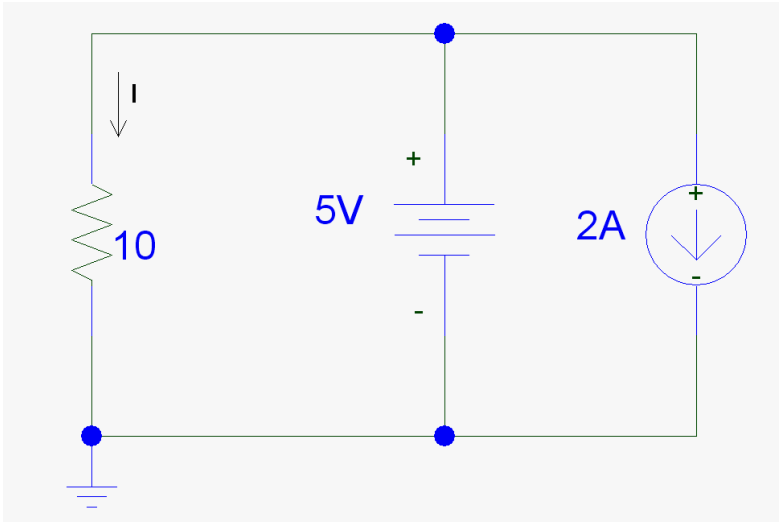


- Find I
  - This circuit can be separated into two different circuits
    - one containing the 5V source
    - the other containing the 2A source.

- When you remove a voltage source from the circuit, it should be replaced by a short circuit.

- When you remove a current source from the circuit, it should be replaced by an open circuit.

…Example 01…



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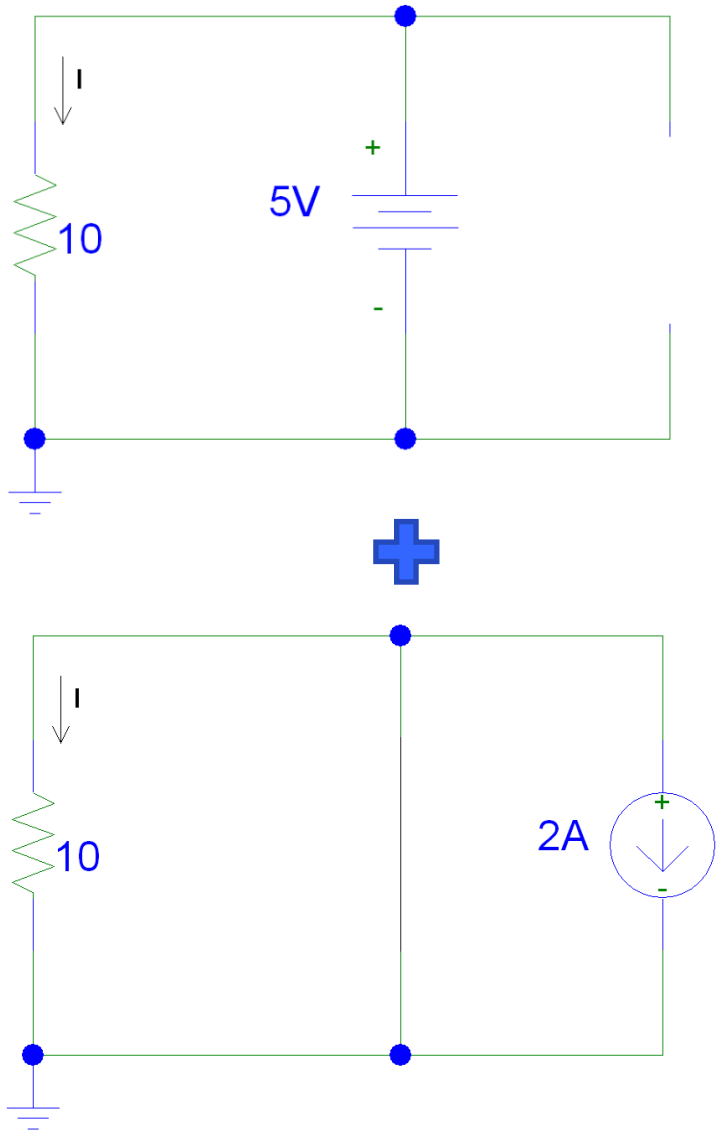
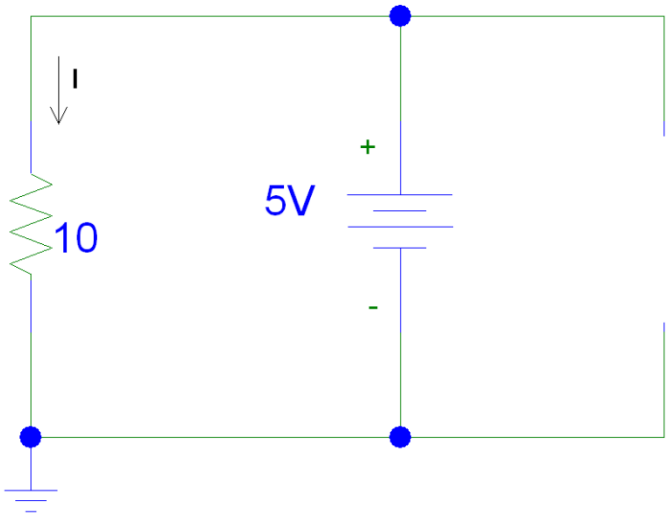
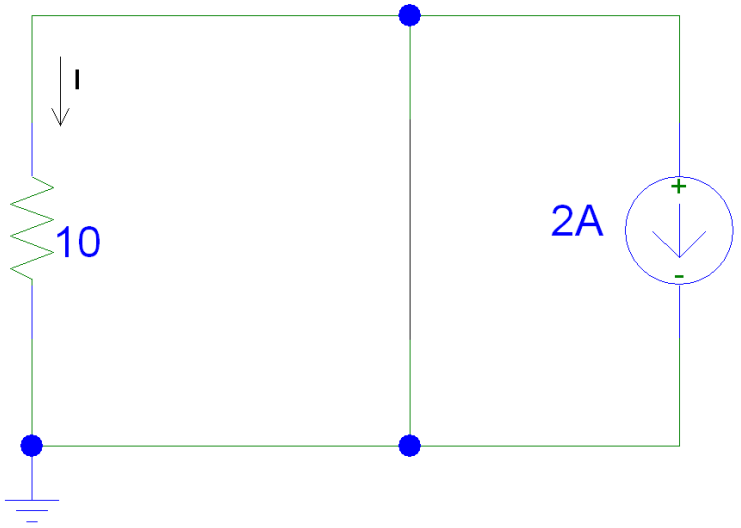


…Example 01…



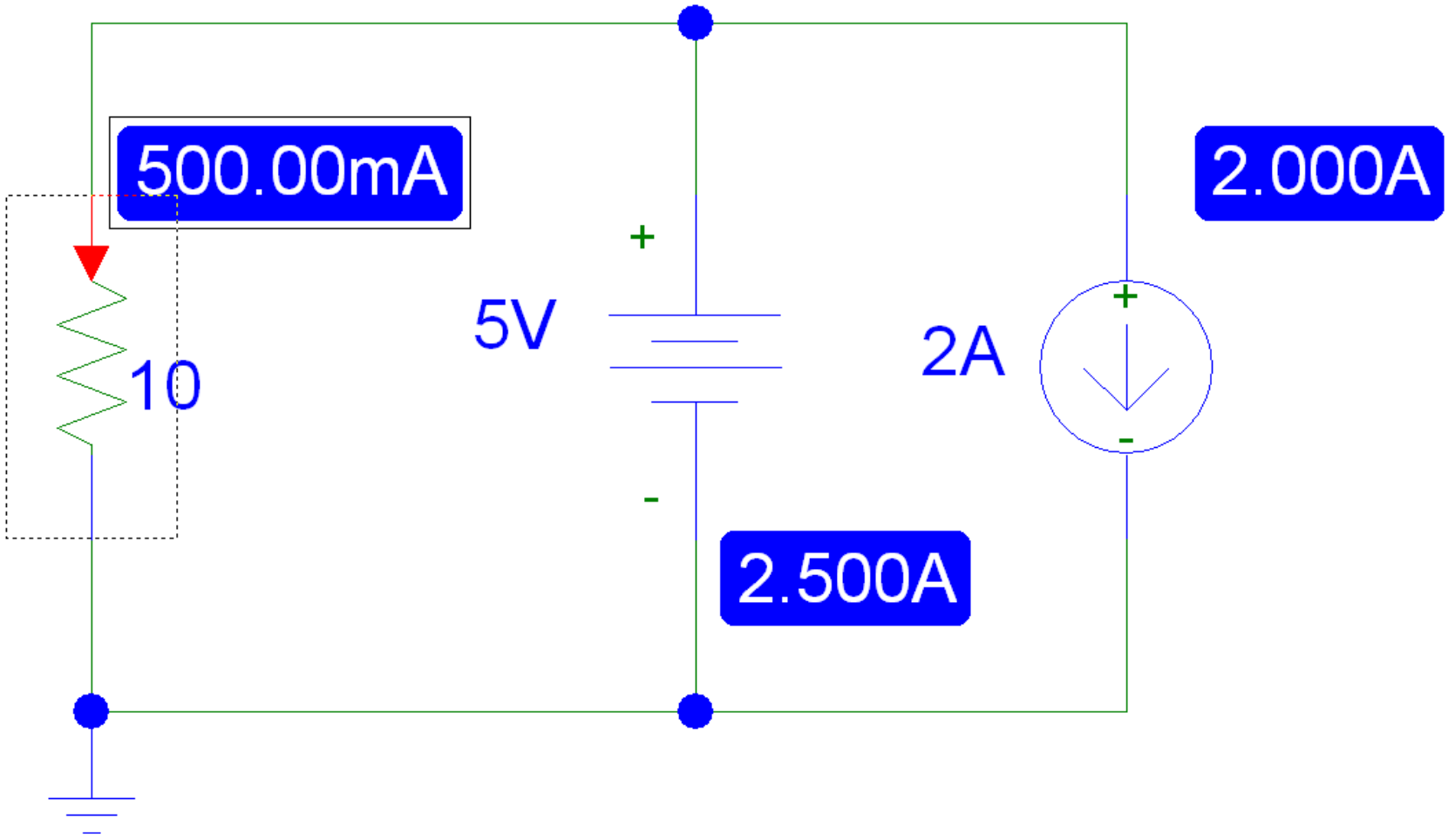


$I_1 = 5V/10\Omega = 0.5A$

$I_2 = 0A$

$I = I_1 + I_2 = 0.5 + 0 = 0.5A$

…Example 01

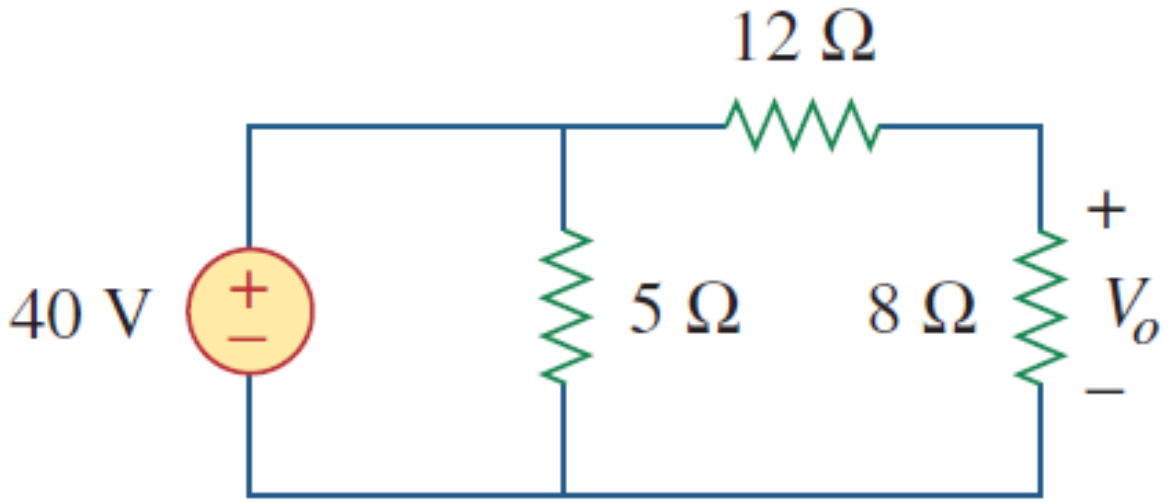


Summary

- The property of linearity can be applied when there are only linear components in the circuit.
  - Resistors, capacitors, inductors
  - Linear voltage and current supplies
- The property is used to separate contributions of several sources in a circuit to the voltages across and the currents through components in the circuit.
  - Superposition

Example 2

- For the circuit, assume that $V_0 = 1V$ and use linearity to calculate the actual value of V_0 .



Answer: 16 V.

Superposition

Superposition

- The voltage across a component is the algebraic sum of the voltages across the component due to each independent source acting upon it.
- The current flowing through across a component is the algebraic sum of the current flowing through component due to each independent source acting upon it.

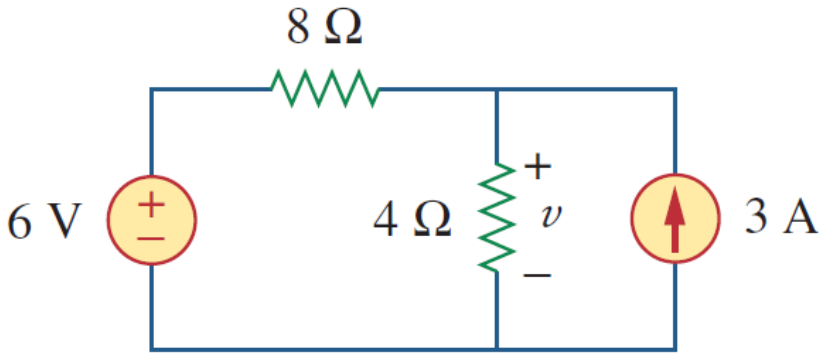
Steps

1. Turn off all independent sources except one.
   - Voltage sources should be replaced with short circuits
   - Current sources should be replaced with open circuits
2. Keep all dependent sources **on**
3. Solve for the voltages and currents in the new circuit.
4. Turn off the active independent source and turn on one of the other independent sources.
5. Repeat Step 3.
6. Continue until you have turned on each of the independent sources in the original circuit.
7. To find the total voltage across each component and the total current flowing, add the contributions from each of the voltages and currents found in Step 3.

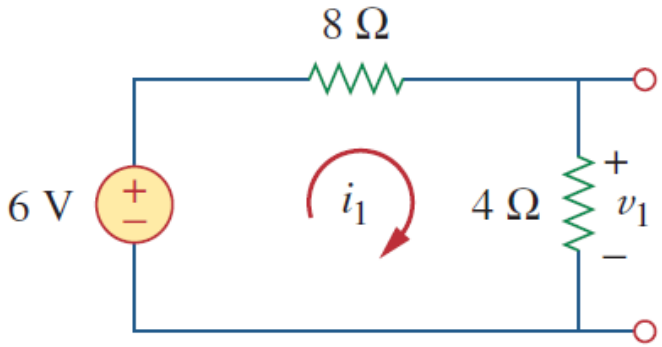
A Requirement for Superposition

- Once you select a direction for current to flow through a component and the direction of the + /– signs for the voltage across a component, you must use the same directions when calculating these values in all of the subsequent circuits.

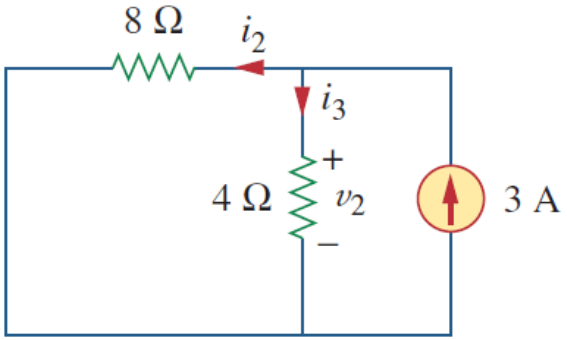
Example 3



- Use the superposition theorem to find v in the circuit.



$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

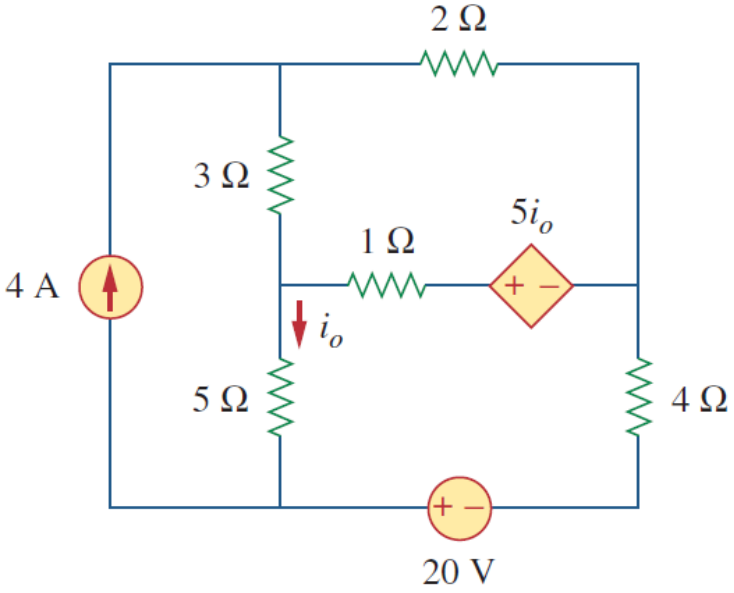


$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

$$v_2 = 4i_3 = 8 \text{ V}$$

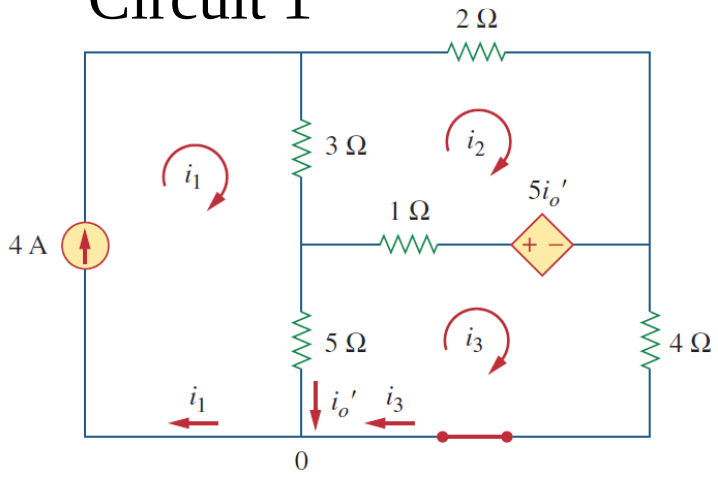
$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Example 4…

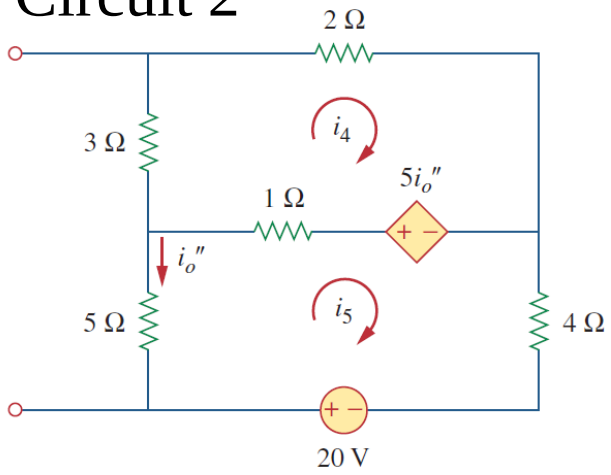


- Using the superposition theorem, find i_0 in the circuit.
  - The circuit involves a dependent source, which must be left intact. We let $i_o = i'_o + i''_o$
  - According to superposition theorem:

Circuit 1

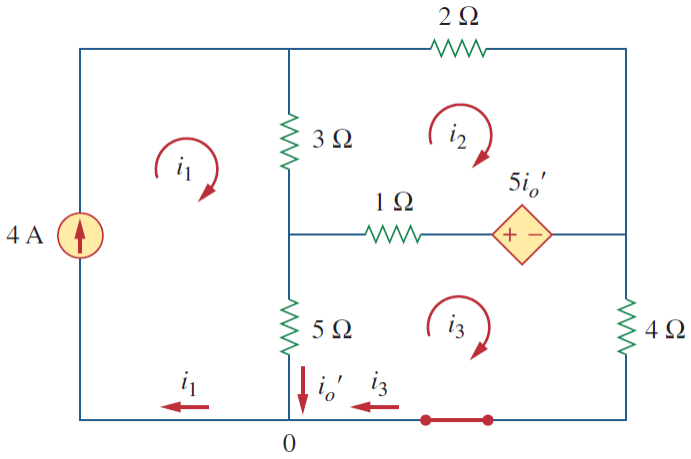


Circuit 2



…Example 4

- Circuit 1



$$i_1 = 4 \text{ A}$$

$$-3i_1 + 6i_2 - 1i_3 - 5i_o' = 0$$

$$-5i_1 - 1i_2 + 10i_3 + 5i_o' = 0$$

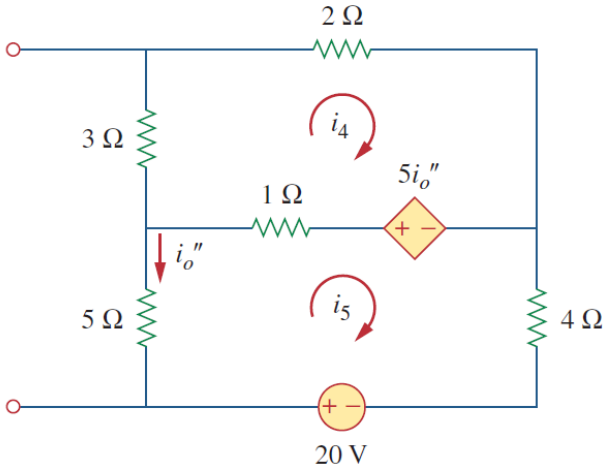
$$i_3 = i_1 - i_o' = 4 - i_o'$$

$$3i_2 - 2i_o' = 8$$

$$i_2 + 5i_o' = 20$$

$$i_o' = \frac{52}{17} \text{ A}$$

- Circuit 2



$$6i_4 - i_5 - 5i_o'' = 0 \qquad i_5 = -i_o''$$

$$-i_4 + 10i_5 - 20 + 5i_o'' = 0$$

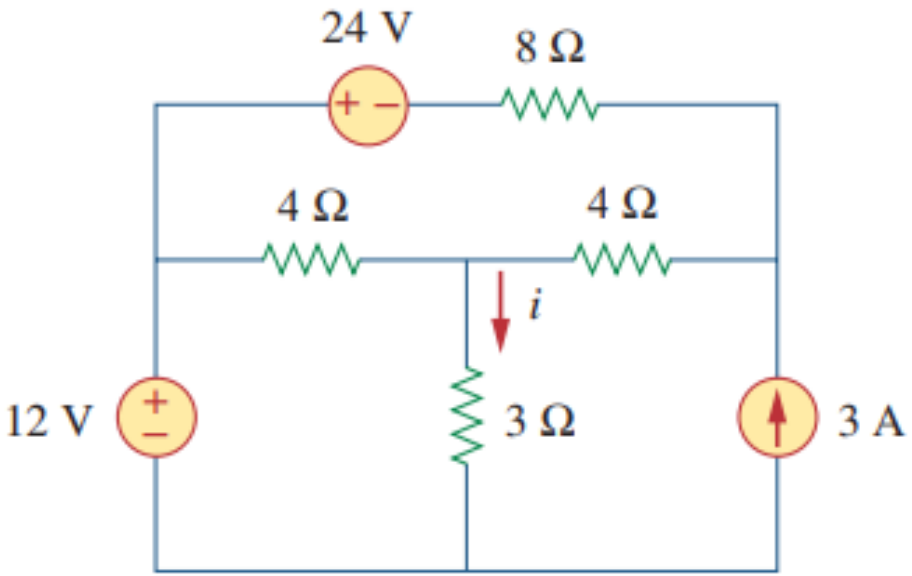
$$6i_4 - 4i_o'' = 0$$

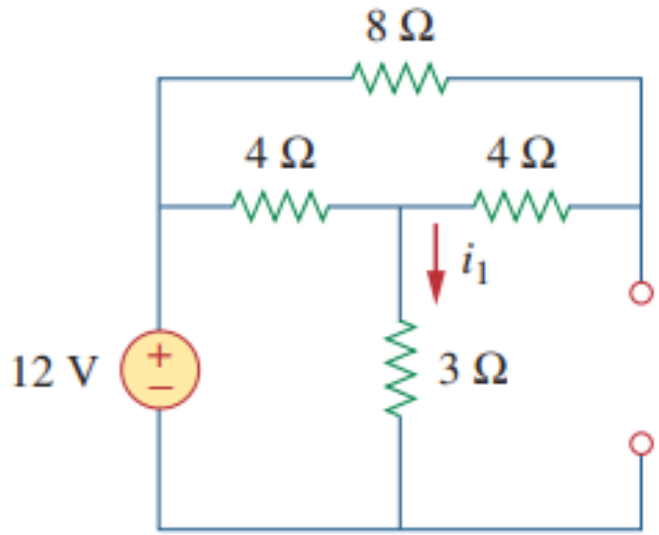
$$i_4 + 5i_o'' = -20$$

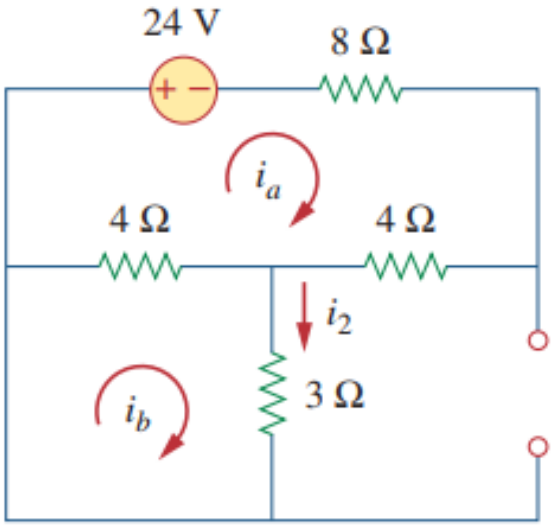
$$i_o'' = -\frac{60}{17} \text{ A}$$

$$i_o = i_o' + i_o'' = -\frac{8}{17} = -0.4706 \text{ A}$$

Example 05







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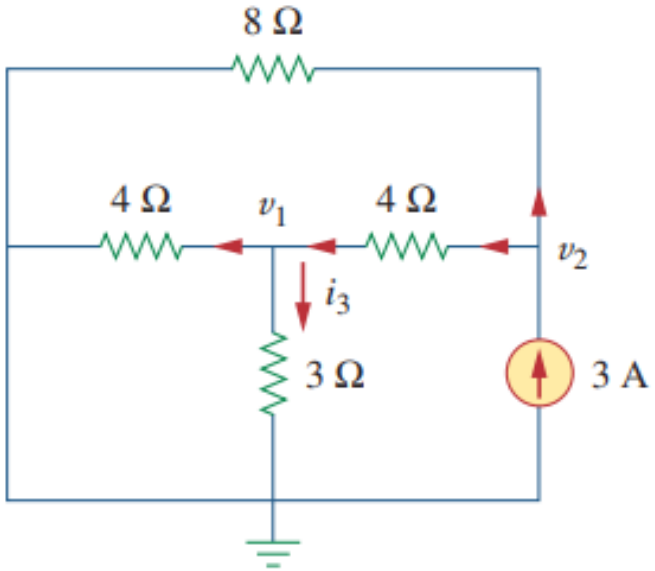

(c)

Thank You