Applied Physics for Engineers

Samra Syed

Lecture # 19

Induced electric field, Gauss's law for magnetic fields, Induced magnetic field,

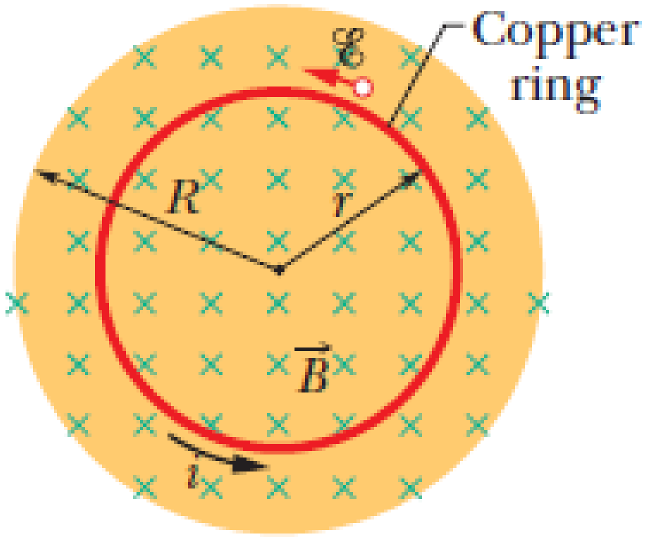
INDUCED ELECTRIC FIELDS

An emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field **E** at every point of such a loop; the induced emf is related to **E** by

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}$$

- Let us place a copper ring of radius r in a uniform external magnetic field, as in Fig. a.
- Suppose that we increase the strength of this field at a steady rate, perhaps by increasing—in an appropriate way—the current in the windings of the electromagnet that produces the field.
- The magnetic flux through the ring will then change at a steady rate and—by Faraday's law—an induced emf and thus an induced current will appear in the ring.
- If there is a current in the copper ring, an electric field must be present along the ring because an electric field is needed to do the work of moving the conduction electrons.
- Moreover, the electric field must have been produced by the changing magnetic flux. This *induced electric field* **E** is just as real as an electric field produced by static charges; either field will exert a force q_0**E** on a particle of charge q_0. So this led to a restatement of Faraday's law

"A changing magnetic field produces an electric field."



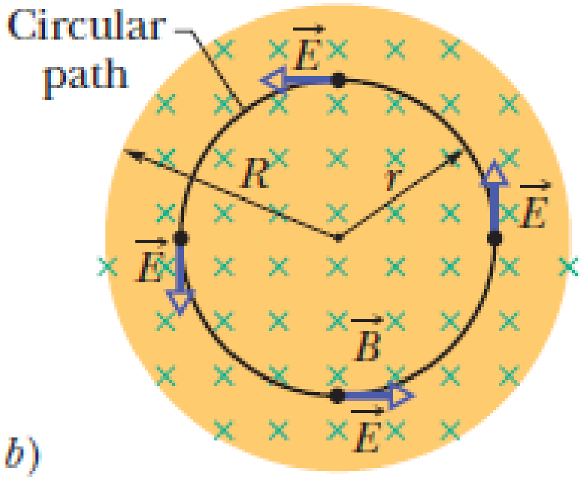
(a)

- Above statement tells us that the electric field is induced even if there is no copper ring. Thus, the electric field would appear even if the changing magnetic field were in a vacuum.

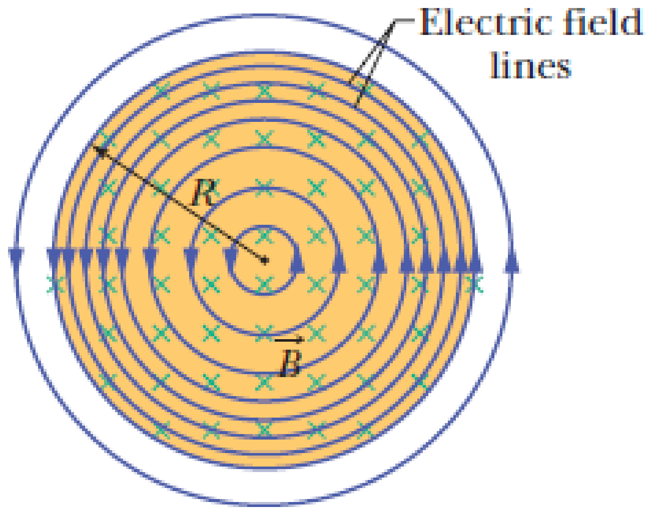
- In figure (b) the copper ring has been replaced by a hypothetical circular path of radius r. We assume, as previously, that the magnetic field **B** is increasing in magnitude at a constant rate dB/dt. The electric field induced at various points around the circular path must be tangent to the circle, as Fig. b shows. Hence, the circular path is an electric field line.

- So the electric field lines produced by the changing magnetic field must be a set of concentric circles, as in Fig. c.

- As long as the magnetic field is increasing with time, the electric field represented by the circular field lines in Fig. c will be present. If the magnetic field remains constant with time, there will be no induced electric field and thus no electric field lines. If the magnetic field is decreasing with time (at a constant rate), the electric field lines will still be concentric circles as in Fig. c, but they will now have the opposite direction.



(b)



(c)

A Reformulation of Faraday's Law

- Consider a particle of charge q_0 moving around the circular path of Fig. b. The work W done on it in one revolution by the induced electric field is $W = \varepsilon q_0$, where ε is the induced emf—that is, the work done per unit charge in moving the test charge around the path.

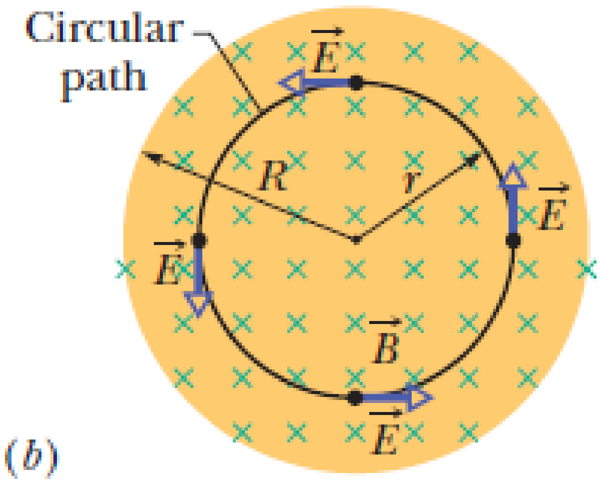
- The work done on a particle of charge q_0 moving along any closed path:

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}.$$

- Substituting εq_0 for W, we find that

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}.$$

- An induced emf is the sum—via integration—of quantities **E**.d**s** around a closed path, where **E** is the electric field induced by a changing magnetic flux and is a differential length vector d**s** along the path.



(b)

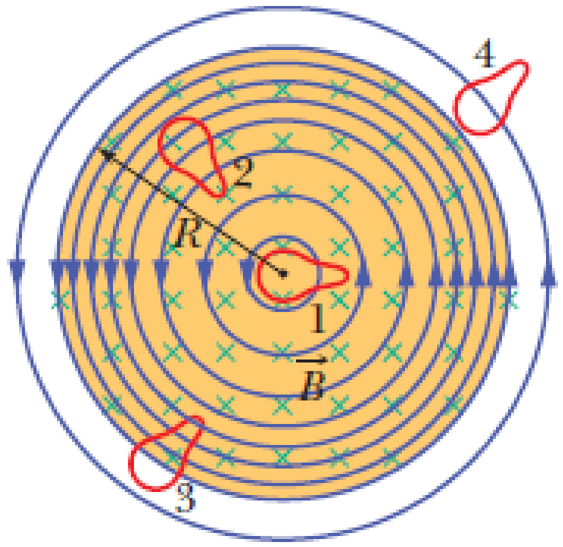
- If we combine above equation with Faraday's law that is,

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)},$$

- can rewrite Faraday's law as

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}.$$

- This equation says simply that a changing magnetic field induces an electric field.



(d)

Gauss' Law for Magnetic Fields

"The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist."

Gauss's law asserts that the net magnetic flux Φ_B through any closed Gaussian surface is zero:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

If we compare this law with Gauss's law for electric fields

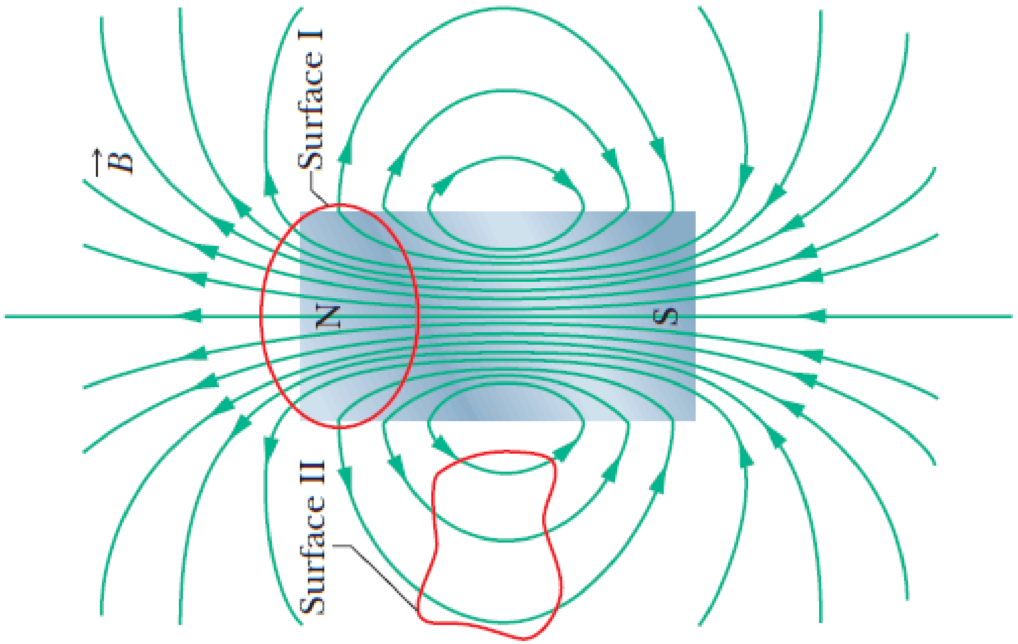
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

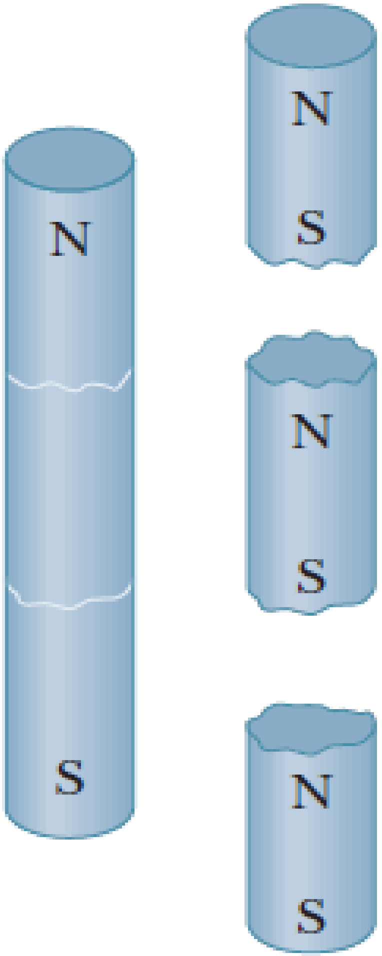
In both equations, the integral is taken over a closed Gaussian surface.

Gauss' law for electric fields says that this integral (the net electric flux through the surface) is proportional to the net electric charge q_{enc} enclosed by the surface.

Gauss' law for magnetic fields says that there can be no net magnetic flux through the surface because there can be no net "magnetic charge" (individual magnetic poles) enclosed by the surface.

- The simplest magnetic structure that can exist and thus be enclosed by a Gaussian surface is a dipole, which consists of both a source and a sink for the field lines.
- Thus, there must always be as much magnetic flux into the surface as out of it, and the net magnetic flux must always be zero.





Induced Magnetic Fields (Maxwell's Law of Induction)

- Faraday's Law of induction

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

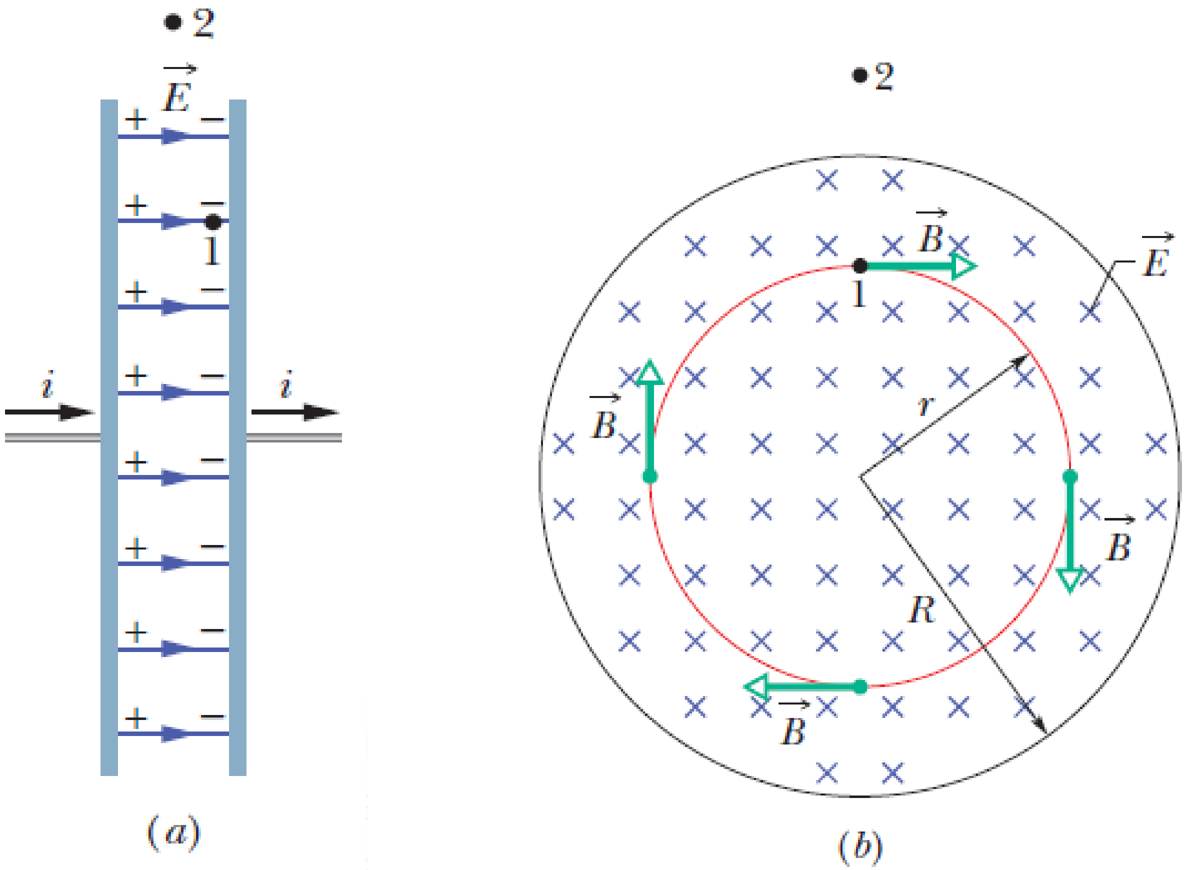
- Maxwell's law of induction

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{(i)}$$

Here B is the magnetic field induced along a closed loop by the changing electric flux Φ_E in the region encircled by that loop.

Charging a Capacitor:

- Figure shows a charging of a parallel-plate capacitor with circular plates.
- We assume that the charge on our capacitor (a) is being increased at a steady rate by a constant current i in the connecting wires. Then the electric field magnitude between the plates must also be increasing at a steady rate.
- Figure (b) is a view of the right-hand plate of (a) from between the plates. The electric field is directed into the page.



Ampere–Maxwell Law

This expression was called displacement current i_d

- Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(ii)}$$

where i_{enc} is the current encircled by the closed loop.

Thus, our two equations that specify the magnetic field produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. We can combine the two equations into the single equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc} \qquad \text{(iii)}$$

- When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of Eq. (iii) is zero, and so Eq. (iii) reduces to Eq. (ii), Ampere's law.
- When there is a change in electric flux but no current (such as inside or outside the gap of a charging capacitor), the second term on the right side of Eq. (iii) is zero, and so Eq. (iii) reduces to Eq. (i), Maxwell's law of induction.