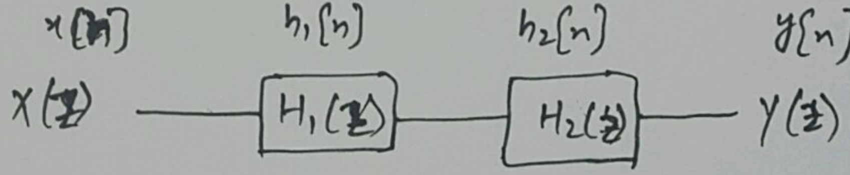
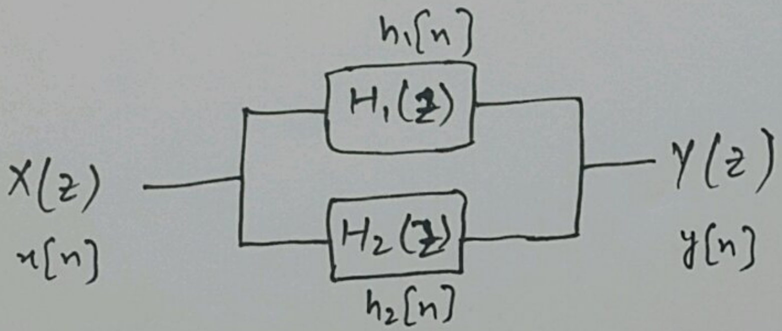
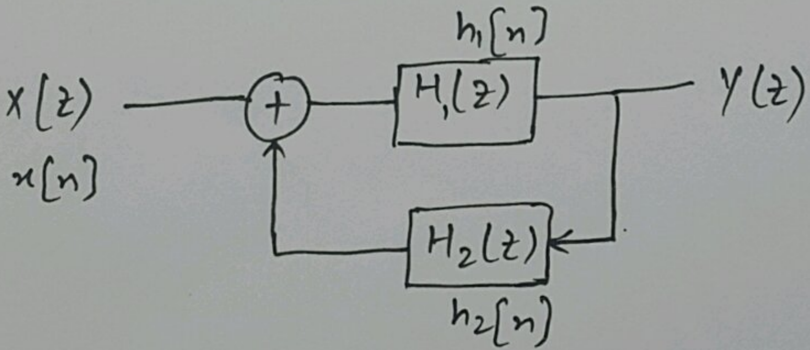
BLOCK Diagram Representation of Causal LTI Systems Described by Difference Equation & Rational System Functions :-



Series / Cascade



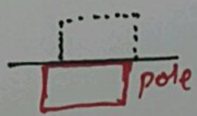
Parallel



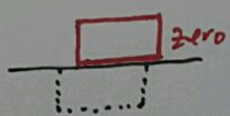
FeedBack

Laplace Transform

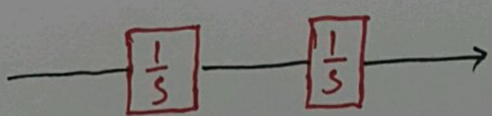
poles → Bottom



zeros → Top

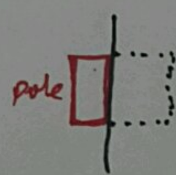


Integrator $\frac{1}{s}$ on Horizontal Axis

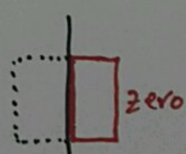


Z-Transform

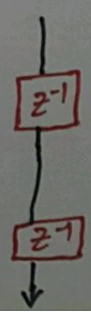
poles → Left Side



zeros → Right Side



Delay z^{-1} on Vertical Axis



$$\Longrightarrow \quad H(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$ (Single Pole)

$$\Rightarrow \quad \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{4}z^{-1}} \Rightarrow Y(z)\left[1-\frac{1}{4}z^{-1}\right] = X(z)$$

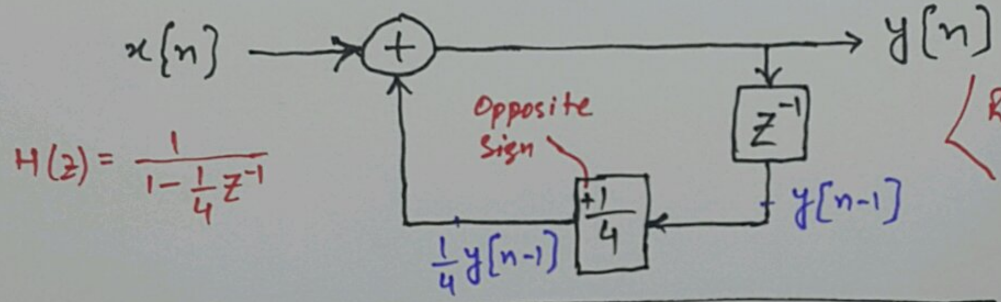
$$\Rightarrow \quad Y(z) - \frac{1}{4}z^{-1}Y(z) = X(z)$$

taking inverse z-transform;

$$y[n] - \frac{1}{4}y[n-1] = x\{n\} \Rightarrow y[n] = x[n] + \frac{1}{4}\underbrace{y[n-1)}_{z^{-1}}$$

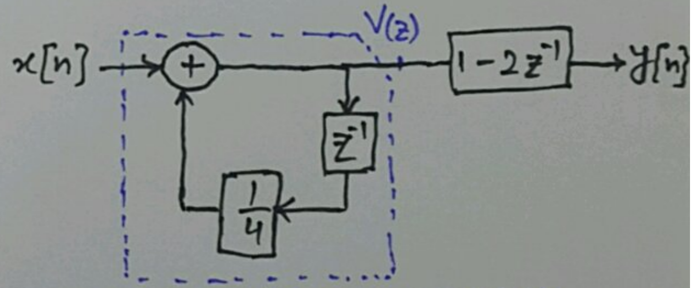
unit delay

$$\boxed{y[n] = x[n] + \frac{1}{4}z^{-1}\cdot y[n]}$$



⟨ Rember this for Single pole ⟩

$$\Longrightarrow \quad H(z) = \left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\left(1-2z^{-1}\right)$$



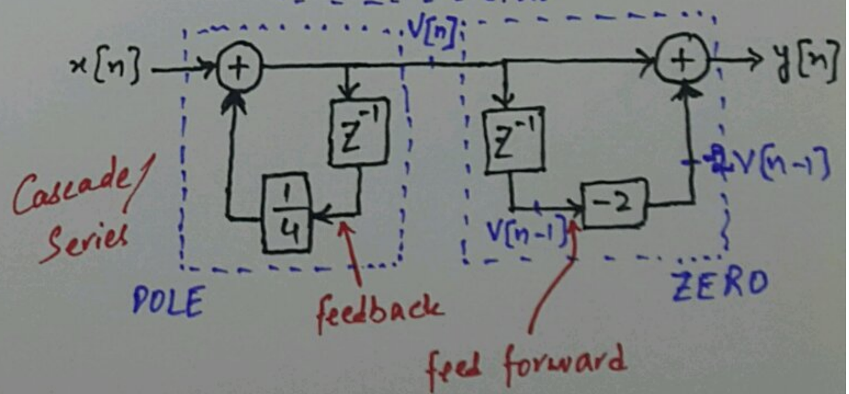
$$Y(z) = V(z)\left(1-2z^{-1}\right)$$

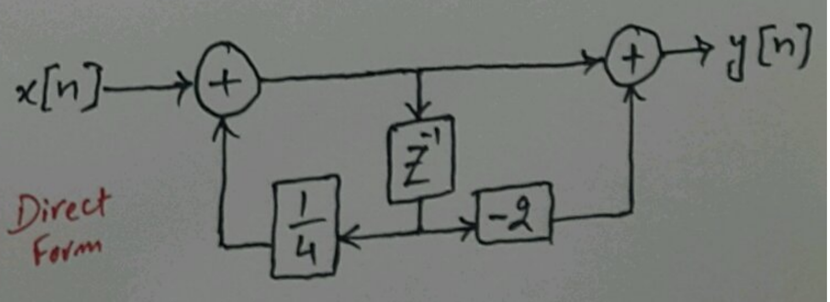
$$Y(z) = V(z) - 2z^{-1}V(z)$$

taking Inverse z-transform;

$$y[n] = V[n] - 2V[n-1]$$

$$\boxed{y[n] = V[n] - 2z^{-1}V[n]}$$

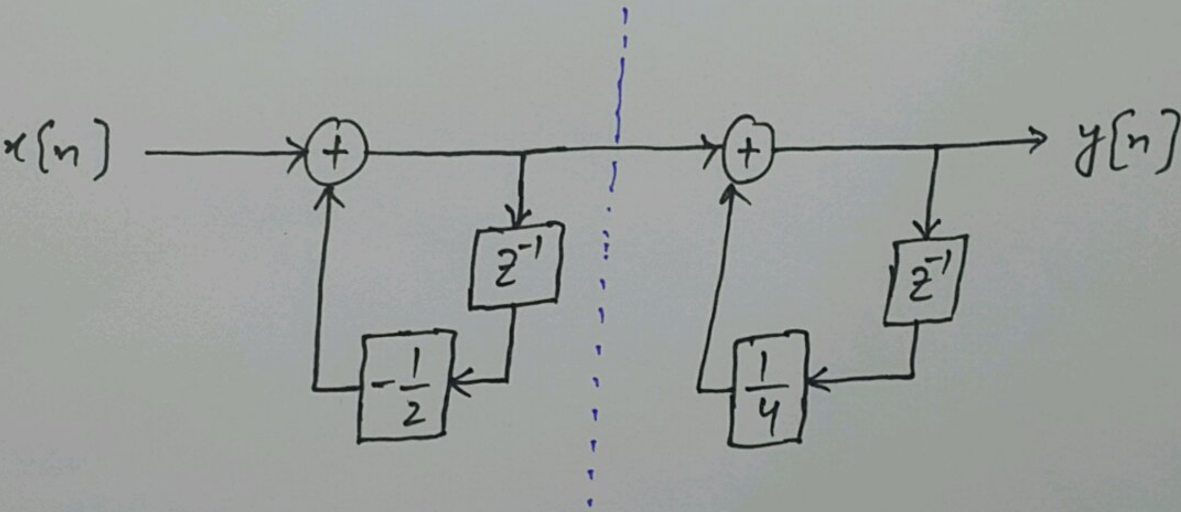




$$\Rightarrow \quad H(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$ (Two Poles)

Cascade / Series Form :-

$$H(z) = \left(\frac{1}{1+\frac{1}{2}z^{-1}}\right)\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)$$ (multiplicative form)



Parallel Form :-

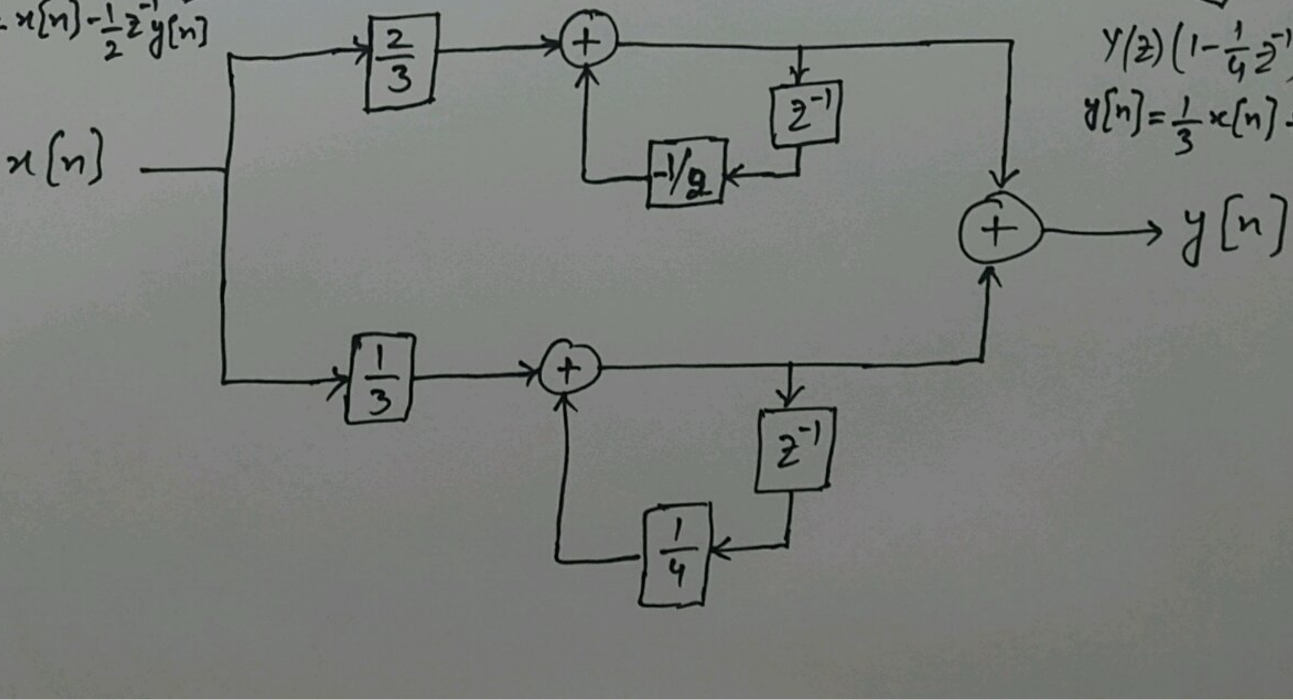
By partial fraction $\Rightarrow$ $H(z) = \dfrac{2/3}{1+\frac{1}{2}z^{-1}} + \dfrac{1/3}{1-\frac{1}{4}z^{-1}}$

$Y(z)\left[1+\frac{1}{2}z^{-1}\right] = \left(\frac{2}{3}\right)X(z)$

$y[n] = \frac{2}{3}x[n] - \frac{1}{2}z^{-1}y[n]$

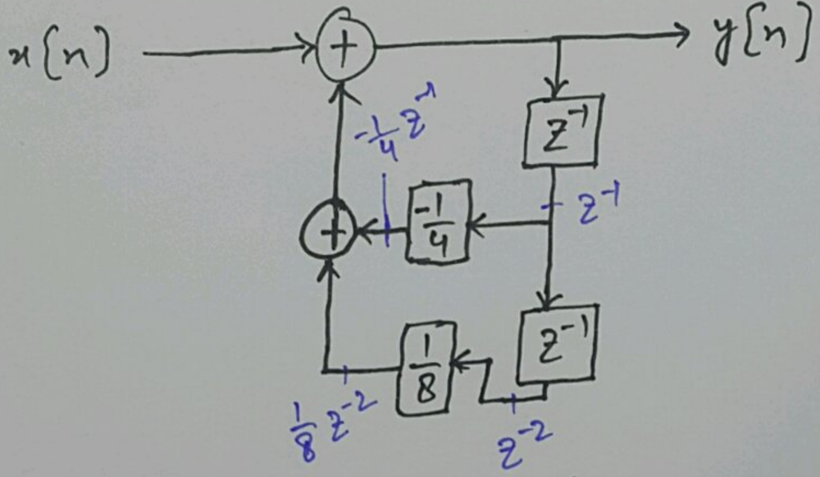
$Y(z)\left(1-\frac{1}{4}z^{-1}\right) = \frac{1}{3}X(z)$

$y[n] = \frac{1}{3}x[n] + \frac{1}{4}z^{-1}y[n]$



Direct Form

$$H(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)} = \frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

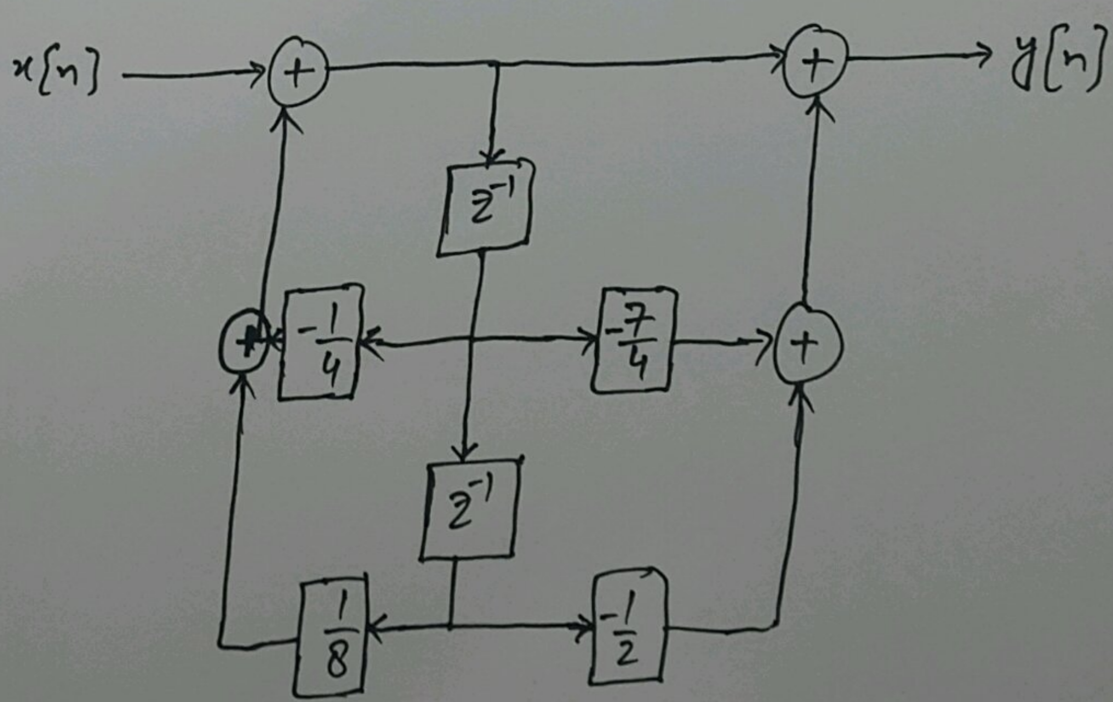


$$y[n] = x[n] - \frac{1}{4}z^{-1}y[n] + \frac{1}{8}z^{-2}y[n]$$

Ex # 10.31

$$H(z) = \frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

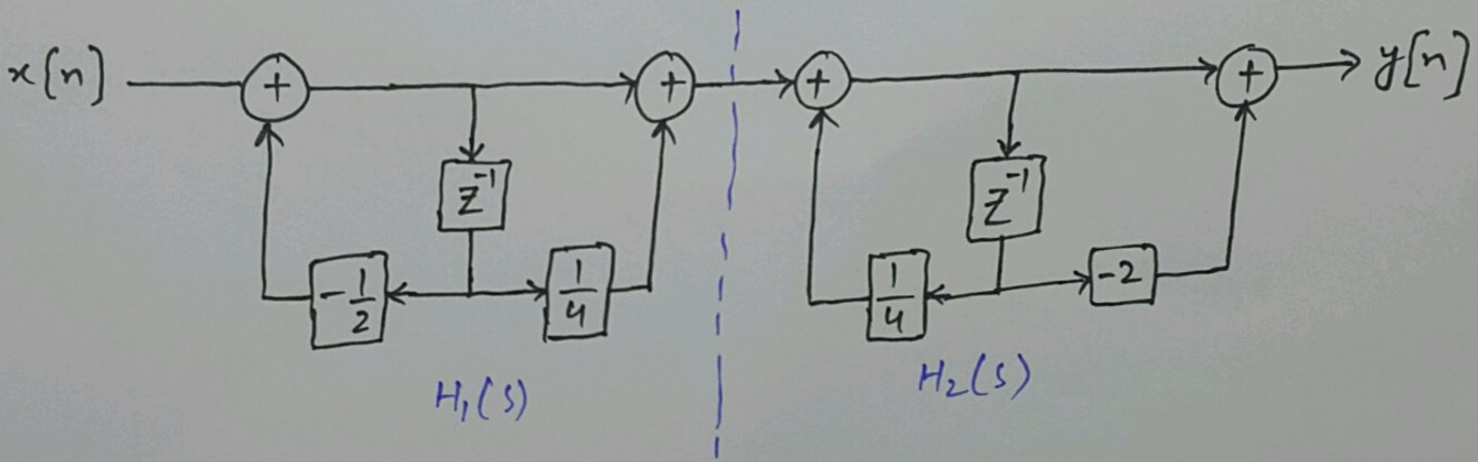
Direct Form



Series Form

$$H(z) = \frac{1}{\left(1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}\right)}\left(1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}\right)$$

$$H(z) = \underbrace{\left(\frac{1+\frac{1}{4}z^{-1}}{1+\frac{1}{2}z^{-1}}\right)}_{H_1(s)}\underbrace{\left(\frac{1-2z^{-1}}{1-\frac{1}{4}z^{-1}}\right)}_{H_2(s)}$$



Parallel Form (Need to do partial Fraction) to get additive form

~~H(z)=~~

$$H(z) = \frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

C
M | A
R

After long division ;

$$H(z) = 4+\frac{A}{1+\frac{1}{2}z^{-1}}+\frac{B}{1-\frac{1}{4}z^{-1}} \quad \Big| \; C+\frac{R}{M}$$

After Partial Frac. ;

$$H(z) = 4+\frac{5/3}{1+\frac{1}{2}z^{-1}}+\frac{14/3}{1-\frac{1}{4}z^{-1}}$$