properties of Laplace Transform ①

① Linearity :-

$$x_1(t) \longrightarrow X_1(s) \qquad \text{ROC}: R_1$$

$$x_2(t) \longrightarrow X_2(s) \qquad \text{ROC}: R_2$$

$$ax_1(t) + bx_2(t) \longrightarrow a X_1(s) + b X_2(s)$$

$$\text{ROC}: R_1 \cap R_2$$

$$X(s) = X_1(s) + X_2(s)$$

$$R_1 \qquad R_2$$

② Time Shifting :-

$$x(t) \xrightarrow{LT} X(s) \qquad \text{ROC}: R_1$$

$$x(t-t_0) \longrightarrow e^{-st_0} X(s) \qquad \text{ROC}: R_1$$

③ Shifting in S-domain :-

$$x(t) \xrightarrow{LT} X(s) \qquad \text{ROC}: R$$

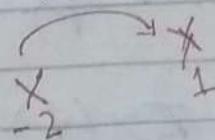$$e^{s_0 t} x(t) \longrightarrow X(s-s_0) \qquad \text{ROC}: R + Re\{s_0$$

If $X(s)$ has poles or zero at $s = a$

then $X(s-s_0)$ has a pole or zero at

$$s - s_0 = a$$

$$s = a + s_0 \qquad -2+3$$

④ Time Scaling :-

$$x(t) \xrightarrow{\;\;d\;\;} X(s)$$

$$x(\alpha t) \longrightarrow \frac{1}{|\alpha|} X\left(\frac{s}{\alpha}\right) \qquad ROC: \frac{R}{\alpha}$$

If $\alpha > 1$ Compression in ROC

$0 < \alpha < 1$ expansion in ROC.

⑤ Convolution :-

$$x_1(t) \longrightarrow X_1(s) \qquad ROC = R_1$$

$$x_2(t) \longrightarrow X_2(s) \qquad ROC = R_2$$

$$x_1(t) * x_2(t) \longrightarrow X_1(s)\, X_2(s) \qquad ROC: R_1 \cap R_2$$

⑥ Differentiation in time domain :-

$$x(t) \longrightarrow X(s) \qquad ROC: R$$

$$\frac{dx(t)}{dt} \longrightarrow s\, X(s) \qquad ROC: R$$

⑦ Differentiation in s-domain :-

$$x(t) \longrightarrow X(s) \qquad ROC = R$$

$$-t\, x(t) \longrightarrow \frac{dX(s)}{ds} \qquad ROC = R$$

⑧ Integration in time Domain :-

$$x(t) \xrightarrow{LT} X(s) \qquad ROC : R$$

$$\int_{-\infty}^{t} x(\tau) d\tau \longrightarrow \boxed{\frac{1}{s}} X(s)$$

$LT\{u(t)\}$

ROC : $R \cap (ROC \text{ of } u(t))$

$R \cap (Re\{s\} > 0)$.