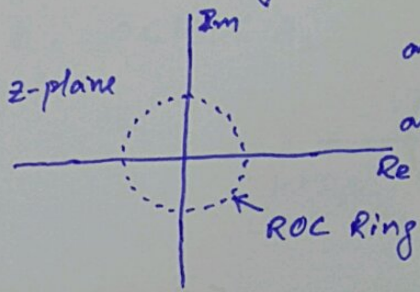
Properties of ROC for Z-transform

① The ROC of $X(z)$ consists of a ring in the z-plane centered about the origin.

As already discussed in previous lecture, ROC depends on r, and not ω. There ROC always forms a circle around the origion



any value of r ($|z|$)
all values of ω

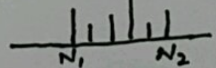
② The ROC does not contain any pole.

e.g ⇒ $X(z) = \dfrac{1}{z-a} \Rightarrow X(z)\Big|_{z=a} = \dfrac{1}{0} = \infty$

$X(z)$ does not converge at pole.

Therefore we use < or > sign ⇒ e.g $|z| > |a|$ ✓

$|z| \geq |a|$ ✗



③ If $x[n]$ is of finite duration, the ROC is entire z-plane except $z = 0$ and/or $z = \pm\infty$

⇒ if $x[n] = \delta[n] \Rightarrow X(z) = \sum_{n=-\infty}^{\infty} \delta[n] z^{-n} = \underset{n<0}{0} + 1 \cdot z^{0} + \underset{n>0}{0} = 1$

So, $\delta[n] \overset{z}{\longleftrightarrow} 1$, ROC is entire z-plane including $z=0$ & $z=\infty$

⇒ if $x[n] = \delta[n-1] \Rightarrow X(z) = 1 \cdot z^{-1} \Rightarrow X(z) = \dfrac{1}{z}$ → pole at zero.

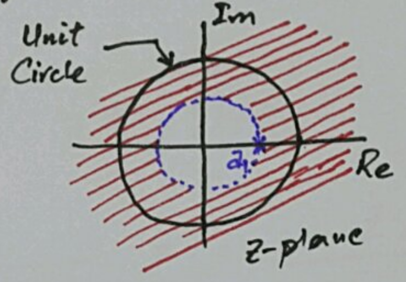
ROC is entire z-plane except $z=0$
↳ because there is a pole. (see property 2)

⇒ if $\delta[n+1] \overset{z}{\longleftrightarrow} z \Rightarrow X(z) = \dfrac{1}{1/z}$ → pole at infinity

↳ ROC = entire z-plane except $z=\infty$

(4) If $x[n]$ is Right Sided Sequence, the ROC will be outside the pole location.

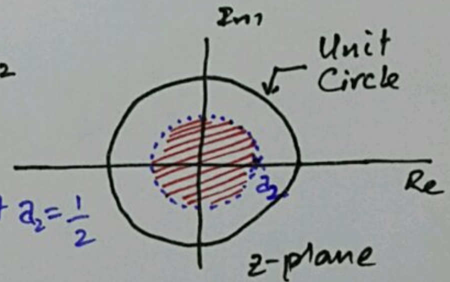
$\Rightarrow$ ROC includes all values of $|z| > a_1$



let $a_1 = \frac{1}{3}$

(5) If $x[n]$ is Left Sided Sequence, the ROC will be inside of the pole location.

$\Rightarrow$ ROC includes all values of z, $0 < |z| < a_2$



let $a_2 = \frac{1}{2}$

(6) If $x[n]$ is two-sided sequence [e.g (4) + (5)], the ROC will be a ring bounded by poles of (4) & (5).

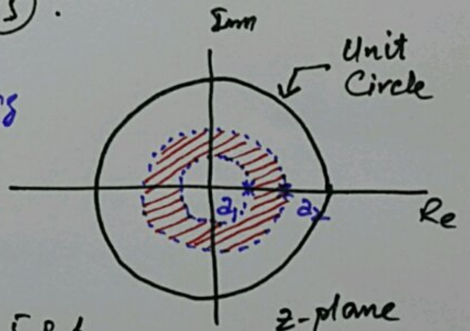
SEE EXAMPLE 10.7

& discuss with your teacher.

Assuming

$a_1 < a_2$

$\frac{1}{3} < \frac{1}{2}$

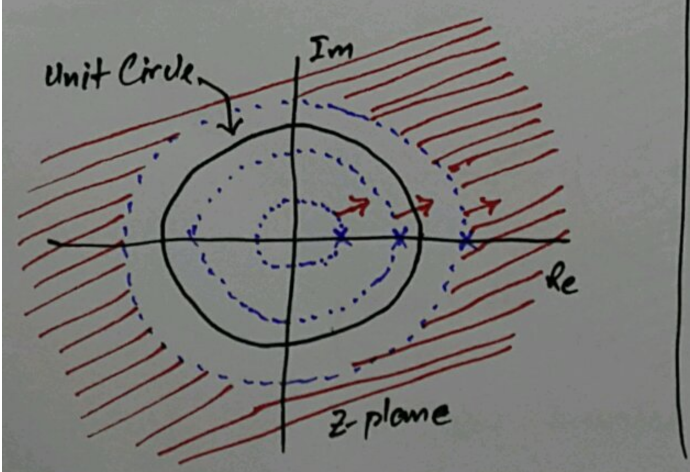


(7) This property is the combination of property 4, 5 & 6. If you understand property 4, 5 & 6, you don't need to worry about property 7.

(8) if $x[n]$ = RSS (with 2 or more poles)

ROC is outside the outermost pole.

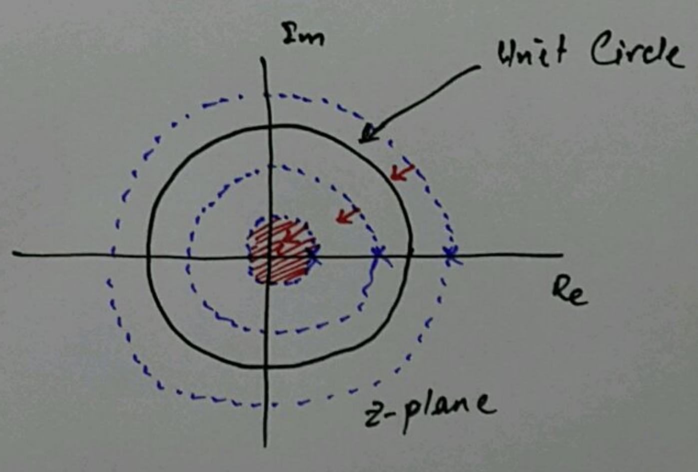
[Intersection of all individual ROCs]



(9) If $x[n]$ = LSS (with 2 or more poles)

ROC is inside of innermost pole

[Intersection of all individual ROCs]



Example 10.8

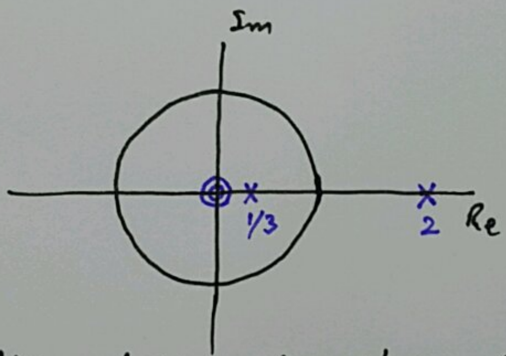
Given $$X(z) = \frac{1}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)}$$

$$X(z) = \frac{z \cdot z}{z\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)z} = \frac{z \cdot z}{\left(z-\frac{1}{3}\right)\left(z-2\right)}$$

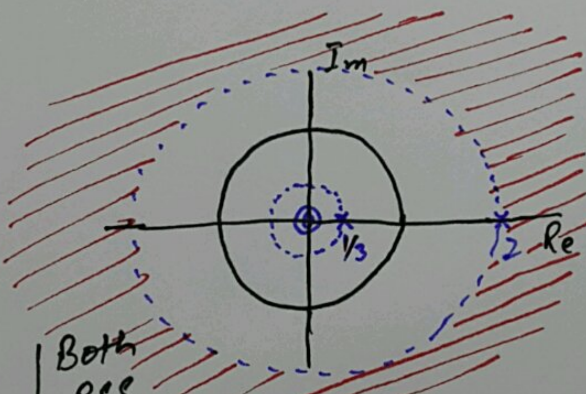
zeros $\Rightarrow$ $z \cdot z \Rightarrow$ 2 zeroes at $z=0$

poles $\Rightarrow$ $z-\frac{1}{3} \Rightarrow z=\frac{1}{3}$ & $z-2=0 \Rightarrow z=2$

pole-zero plot

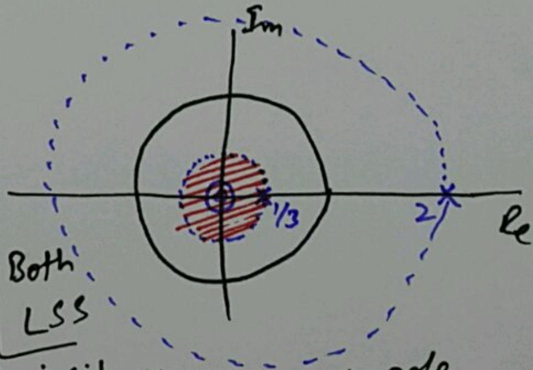


There are 3 possibilities where z-transform converges.



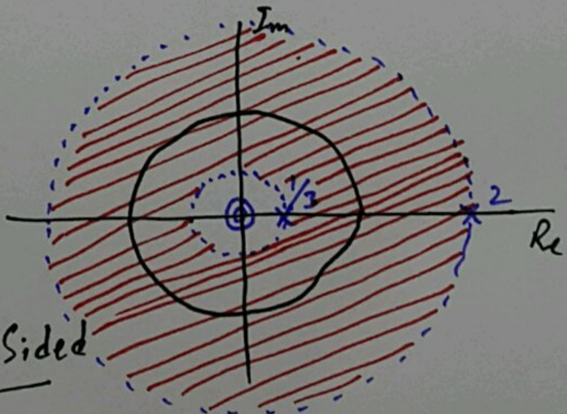
Both RSS

ROC outside the outermost pole



Both LSS

ROC inside the innermost pole



Two-Sided

ROC bounded by poles