Example A modem transmits 100,000 bits. Each bit 0 or 1. with probability 0.5, with binomial distribution.

(a) Estimate the probability of at least 50,300 ones.

(b) What is the probability that the no. of ones is between 49,700 and 50,300.

Solution

(a) This is a problem of CDF, which can be reduced to Gaussian CDF because the no. of values is large. Howeve this requires the expected value and variance of the required, Gaussian distribution.

$$W_n = X_1 + X_2 + \cdots$$

$$E[W_n] = n\, E[X_i] = n \mu_X \quad (1)$$

$$Var[W_n] = n\, Var[X_i] = n\, \sigma_X^2 \quad (2)$$

$$\mu_X = E[X_i] = n_1 p = 1 \times 0.5 = 0.5$$

$\because$ because each value X_i is taken only once

$$\sigma_X^2 = Var[X_i] = n_1 p(1-p) = 1 \times 0.5(1-0.5)$$

$$= 0.25$$

$\because$ because there is no repetition in the each binomial RV X_i

Note that n and n_1 are different

Now Eq (1) and (2) become

$$E[W_n] = n \times \mu_X$$

$$= 100{,}000 \times 0.5$$

$$= 50{,}000$$

$$Var[W_n] = 100{,}000 \times 0.25$$

$$= 25{,}000$$

We need to find $P[W_n \geq 50,300]$ which can be calculated using complimentary Gaussian CDF as below:

$$P[W_n \geq 50,300] = 1 - P[W_n \leq 50,300]$$

$$\approx 1 - \phi\left[\frac{w_n - E[W_n]}{\sqrt{Var[W_n]}}\right]$$

$$= 1 - \phi\left[\frac{50,300 - 50,000}{\sqrt{25,000}}\right]$$

$$= 1 - \phi(1.9)$$

$$= 1 - 0.9713$$

$$= 0.0287 = 2.87 \times 10^{-2}$$

(b) This part can be solved by taking the difference of two Gaussian CDFs as below.

$$P[49,700 \leq W_n \leq 50,300]$$

$$= P[W_n \leq 50,300] - P[W_n \leq 49,700]$$

$$= \phi\left(\frac{50,300 - E[W_n]}{Var[W_n]}\right) - \phi\left(\frac{49,700 - E[W_n]}{Var[W_n]}\right)$$

$$= \phi\left(\frac{50,300 - 50,000}{\sqrt{25,000}}\right) - \phi\left(\frac{49,700 -}{\sqrt{25,000}}\right)$$

$$= 0.9713 - \phi(-1.9)$$

$$= 0.9713 - [1 - \phi(1.9)]$$

$$= 0.9713 - 0.0287 = 0.9426$$