APPLIED PHYSICS FOR ENGINEERS

Lecture # 3

Electric field due to a line of charge,
Electric field due to a disk of charge

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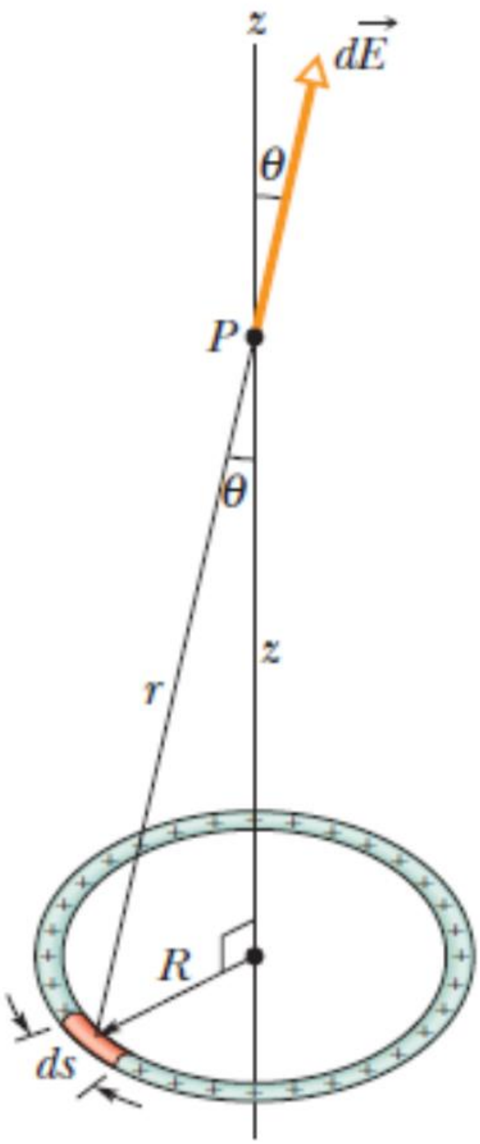
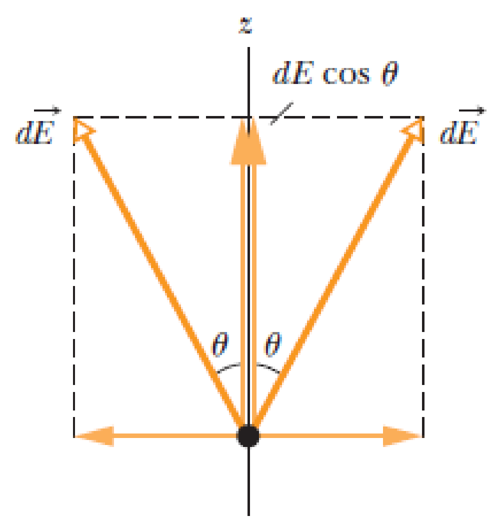
The Electric Field Due to a Line of Charge





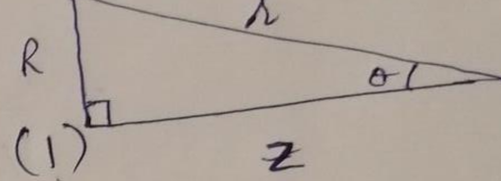
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$\because \lambda = \frac{dq}{ds}$

$dq = \lambda ds$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}$$

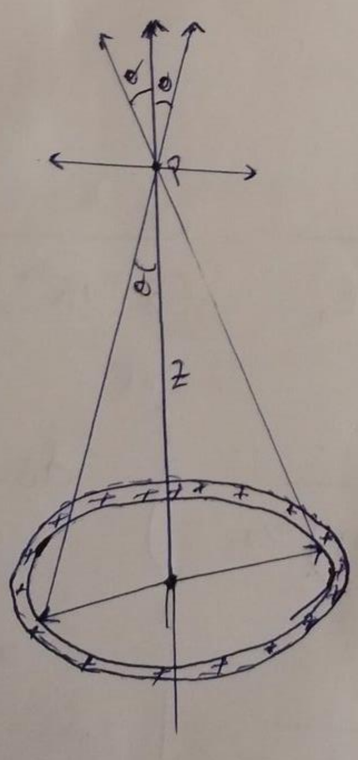
$r^2 = z^2 + R^2$

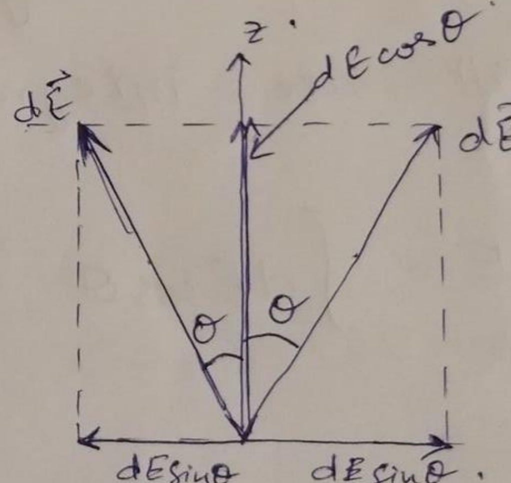


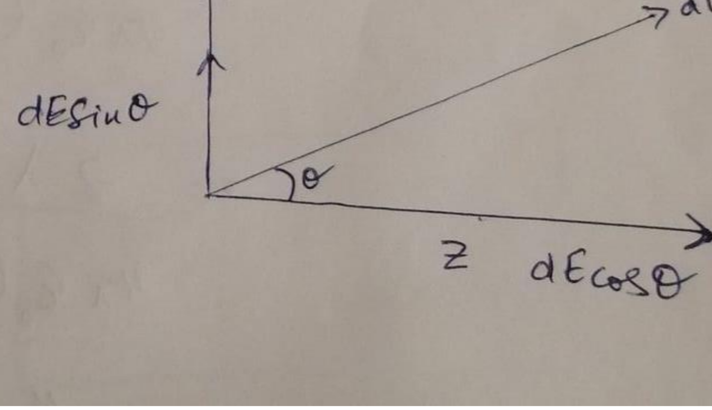
so

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{(z^2+R^2)} \quad (1)$$

This is the field magnitude contributed by each charge element.







$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}} \quad \text{(ii)} \because \quad r^2 = z^2+R^2$$
$$r = \sqrt{z^2+R^2}$$

Multiply (i) and (ii)

$$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2+R^2)} \times \frac{z}{(z^2+R^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2+R^2)^{1+\frac{1}{2}}} ds.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2+R^2)^{3/2}} ds.$$

To sum a huge number of components

we will use integration

$$E = \int dE\cos\theta = \int_0^{2\pi R} \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2+R^2)^{3/2}} ds$$

$$= \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds.$$

$$= \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\Big|\, s \,\Big|_0^{2\pi R}$$

$$= \quad " \quad " \quad (2\pi R - 0)$$

$$=$$

$$E = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$$

as $\lambda = \frac{dq}{ds}$ for infinitesimal element charge (q)

So for total charge

$\lambda = \frac{q}{s}$ $\because S = 2\pi R$

$q = \lambda(2\pi R)$

so

$$E = \frac{zq}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$$

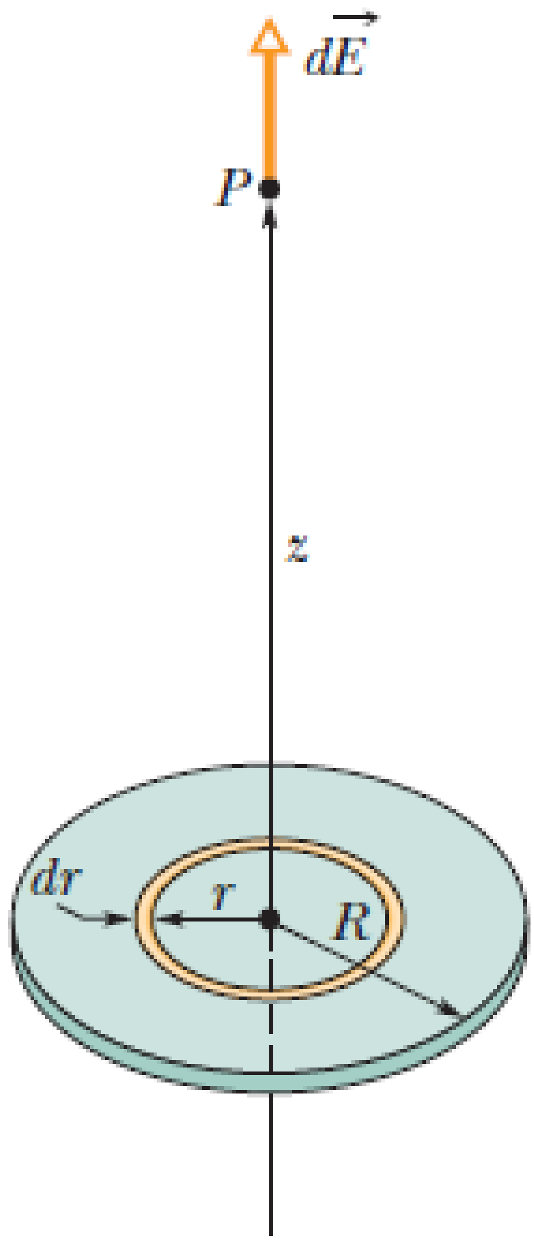
This is the electric field due to a ~~string~~ charged ring.

if $z \gg R$ ~~than~~ Then $R \to 0$ as compared to z so

$$E = \frac{zq}{4\pi\varepsilon_0 (z^2+0)^{3/2}} = \frac{\cancel{z}q}{4\pi\varepsilon_0 z^{\cancel{3}}}$$

This is electric field due to a charged ring at large distance. $\leftarrow$ $\boxed{E = \frac{q}{4\pi\varepsilon_0 z^2}}$

The Electric Field Due to a Charged Disk



The disk has radius R and a uniform surface charge density σ (charge per unit area) on its top surface.

Electric field due to a ring of charge

$$E = \frac{qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \quad (1)$$

Electric field due to a ring on the disk will be dE

$$dE = \frac{dq\, z}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} \quad (2)$$

To find total field at P, we will integrate equation (2)

$\because$ thickness of ring is dr

surface charge density

$$\sigma = \frac{dq}{dA} \qquad \because A = \pi r^2$$
$$dA = 2\pi r\, dr.$$

$$\because dq = \sigma(2\pi r\, dr)$$

$$dE = \frac{z\sigma(2\pi r\, dr)}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R \frac{2r\,dr}{(z^2+r^2)^{3/2}}$$

$$E = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2+r^2)^{-3/2}\, 2r\, dr. \qquad (3)$$

Suppose

$$(z^2+r^2) = X$$

$$m = -3/2$$

$$dX = 2r\,dr$$

$$\cancel{E} = \int X^m\, dX = \frac{X^{m+1}}{m+1}$$

Eq (3) becomes.

$$E = \frac{\sigma z}{4\varepsilon_0} \left| \frac{(z^2+r^2)^{-3/2+1}}{-\frac{3}{2}+1} \right|_0^R$$

$$= \frac{\sigma z}{4\varepsilon_0} \left| \frac{(z^2+r^2)^{-1/2}}{-\frac{1}{2}} \right|_0^R$$

$$E = \frac{\sigma z}{2\cancel{4}\varepsilon_0} \left(-\cancel{2}\left[(z^2+R^2)^{-\frac{1}{2}} - (z^2+0)^{-\frac{1}{2}} \right] \right)$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[-(z^2+R^2)^{-\frac{1}{2}} + (z^2)^{-\frac{1}{2}}\right]$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\frac{\cancel{z}}{\cancel{z}} - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]}$$

if $R \to \infty$, z is finite than above equation will become

$$E = \frac{\sigma}{2\varepsilon_0}\left[1\right]$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

$R \to \infty$ means we are dealing with an infinite sheet of charge.