Lecture 6

Voltage Division

Current Division

Series Resistors

Parallel Resistors

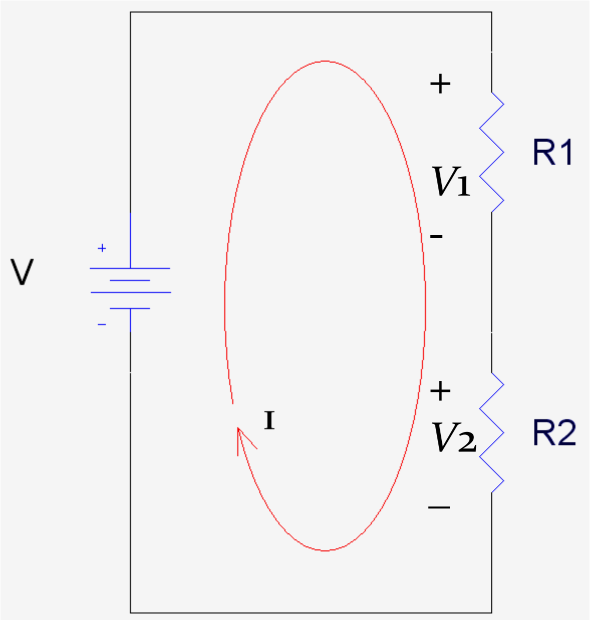
Wye and Delta Networks

Objectives of the Lecture

- Explain mathematically how resistors in series are combined and their equivalent resistance.
- Explain mathematically how resistors in parallel are combined and their equivalent resistance.
- Rewrite the equations for conductances.
- Explain mathematically how a voltage that is applied to resistors in series is distributed among the resistors.
- Explain mathematically how a current that enters the a node shared by resistors in parallel is distributed among the resistors.
- Describe the equations that relate the resistances in a Wye (Y) and Delta (Δ) resistor network.
- Describe a bridge circuit in terms of wye and delta sub-circuits.

Voltage Division

- All resistors in series share the same current



- From KVL and Ohm's Law :

$$0 = -V + V_1 + V_2$$
$$V = I \times R1 + I \times R2$$
$$V = I \times (R1 + R2) = I \times R_{eq}$$
$$R_{eq} = R1 + R2 = V/I \qquad I = V/R_{eq}$$

$$V_1 = I \times R1 = \frac{V}{R_{eq}} \times R1 = \frac{R1}{R1 + R2} \times V$$
$$V_2 = I \times R2 = \frac{V}{R_{eq}} \times R2 = \frac{R2}{R1 + R2} \times V$$

- the source voltage V is divided among the resistors in direct proportion to their resistances;
  - the larger the resistance, the larger the voltage drop.
- This is called the principle of voltage division, and the circuit is called a voltage divider.

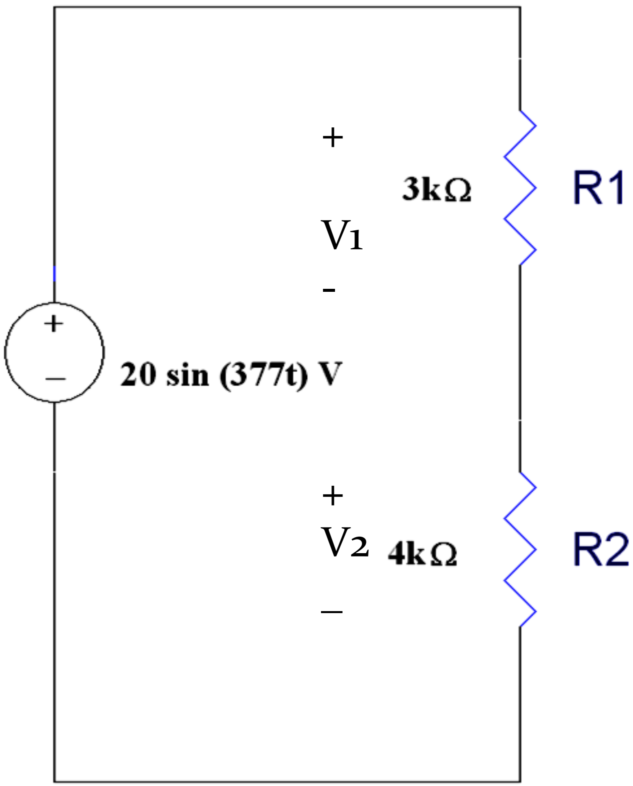
Voltage Division

- In general, if a voltage divider has N resistors (R_1, R_2, . . . , R_N) in series with the source voltage V_{total}, the nth resistor (R_n) will have a voltage drop of

$$V_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} \times V_{total} = \left[\frac{R_n}{R_{eq}}\right] \times V_{total}$$

where V_{total} is the total of the voltages applied across the resistors and R_{eq} is equivalent series resistance.

Example 01



- Find the V_1, the voltage across $R1$, and V_2, the voltage across $R2$

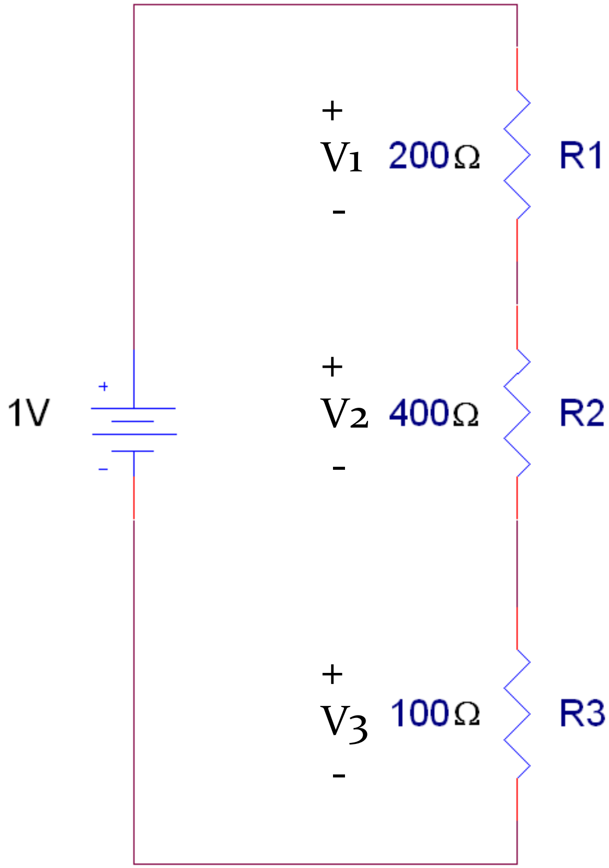
$$V_1 = [R_1/(R_1+R_2)]V_{total}$$
$$V_1 = [3k\Omega/(3k\Omega+4k\Omega)][20V\sin(377t)]$$
$$V_1 = 8.57V\sin(377t)$$

$$V_2 = [R_2/(R_1+R_2)]V_{total}$$
$$V_2 = [4k\Omega/(3k\Omega+4k\Omega)][20V\sin(377t)]$$
$$V_2 = 11.4V\sin(377t)$$

– Check: $V_1 + V_2$ should equal V_{total}
  - 8.57sin(377*t*) + 11.4sin(377*t*) = 20sin(377t) V

Example 02

- Find the voltages listed in the circuit below.



$$R_{eq} = 200\Omega + 400\Omega + 100\Omega$$
$$R_{eq} = 700\Omega$$

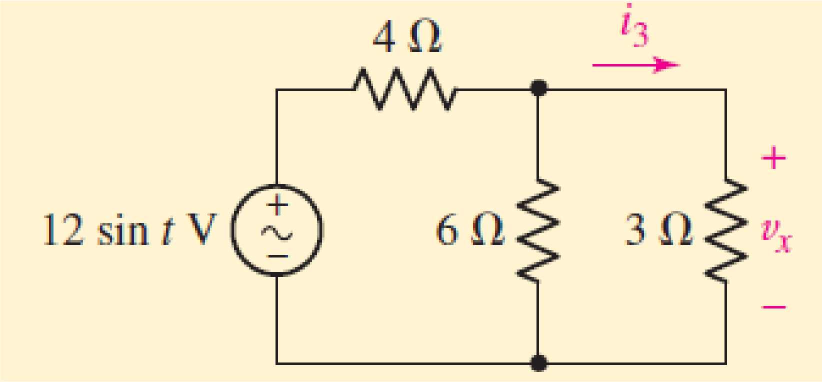
$$V_1 = [200\Omega / 700\Omega](1V)$$
$$V_1 = 0.286V$$

$$V_2 = [400\Omega / 700\Omega](1V)$$
$$V_2 = 0.571V$$

$$V_3 = [100\Omega / 700\Omega](1V)$$
$$V_3 = 0.143V$$

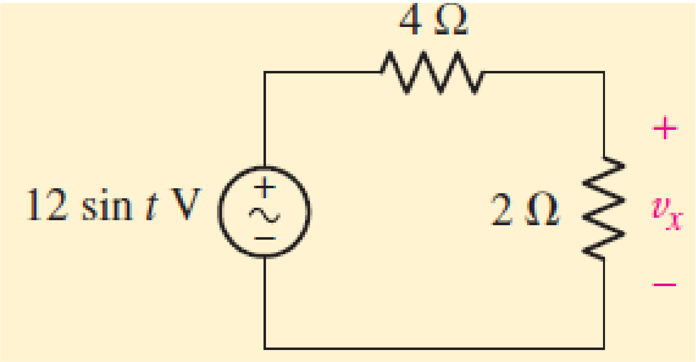
  – Check: $V_1 + V_2 + V_3 = 1\text{V}$

Example 03



- Determine v_x in this circuit:

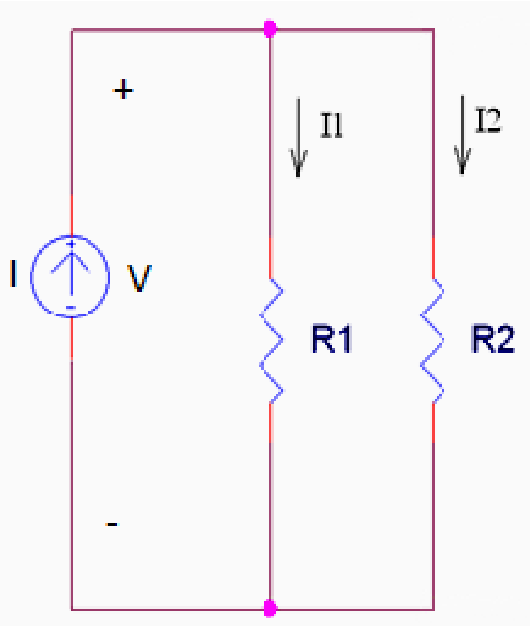
6 Ω || 3 Ω = 2 Ω



$$v_x = (12 \sin t)\frac{2}{4+2} = 4 \sin t$$

Symbol for Parallel Resistors

- To make writing equations simpler, we use a symbol to indicate that a certain set of resistors are in parallel.



  – Here, we would write

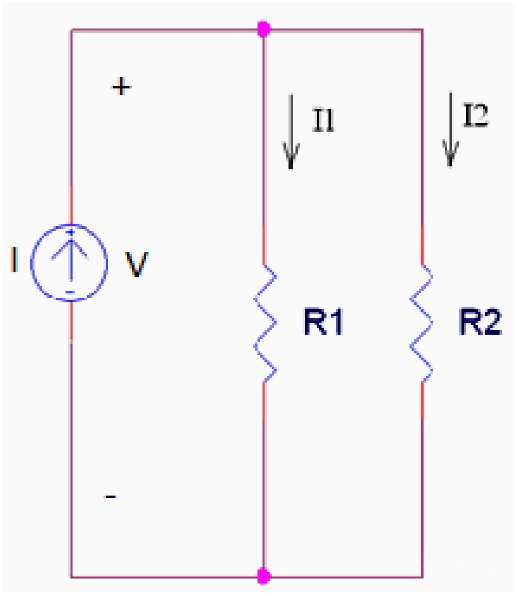
$$R1 \parallel R2$$

  to show that R1 is in parallel with R2.

  – This also means that we should use the equation for equivalent resistance if this symbol is included in a mathematical equation.

Current Division

- All resistors in parallel share the same voltage



  – From KCL and Ohm's Law :

$$0 = -I + I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} = V \times \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$I = \frac{V}{R_{eq}} = \frac{V}{R_1 \| R_2} \qquad V = I \times R_{eq}$$

$$R_{eq} = R_1 \| R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = \frac{I \times R_{eq}}{R_1} = \frac{R_1 \| R_2}{R_1} \times I = \frac{R_2}{R_1 + R_2} \times I$$

$$I_2 = \frac{V}{R_2} = \frac{I \times R_{eq}}{R_2} = \frac{R_1 \| R_2}{R_2} \times I = \frac{R_1}{R_1 + R_2} \times I$$

  – The total current *I* is shared by the resistors in inverse proportion to their resistances
    - the smaller the resistance, the larger the current flow.
  – This is called the principle of current division, and the circuit is called a current divider.

Current Division

- In general, if a current divider has N resistors (R_1, R_2, . . . , R_N) in parallel with the source current I_{total}, the nth resistor (R_n) will have a current flow

$$I_n = \frac{1/R_n}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \times I_{total} = \left[\frac{R_{eq}}{R_n}\right] \times I_{total}$$

where I_{total} is the total of the currents applied to the resistors and R_{eq} is equivalent parallel resistance.

Current Division

- If a current divider circuit with N resistors (having conductances $G_1, G_2, \ldots, G_N$) in parallel with the source current I_{total}, the nth resistor (with conductance G_n) will have a current flow

$$I_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} \times I_{total} = \left[\frac{G_n}{G_{eq}}\right] \times I_{total}$$

where I_{total} is the total of the currents applied to the resistors and G_{eq} is equivalent parallel conductance.

Current Division

The current associated with one resistor R_1 in parallel with one other resistor is:

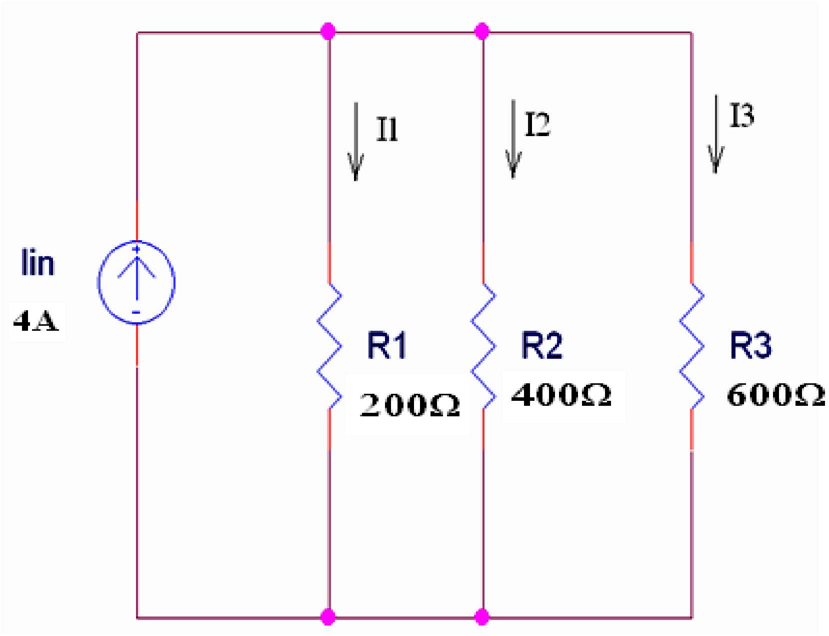
$$I_1 = \left[\frac{R_2}{R_1 + R_2} \right] I_{total}$$

The current associated with one resistor R_m in parallel with two or more resistors is:

$$I_m = \left[\frac{R_{eq}}{R_m} \right] I_{total}$$

where I_{total} is the total of the currents entering the node shared by the resistors in parallel.

Example 04



- Find currents I_1, I_2, and I_3 in the circuit

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2}}$$

$$= \frac{1}{\frac{1}{200} + \frac{1}{400} + \frac{1}{600}} = 109\ \Omega$$

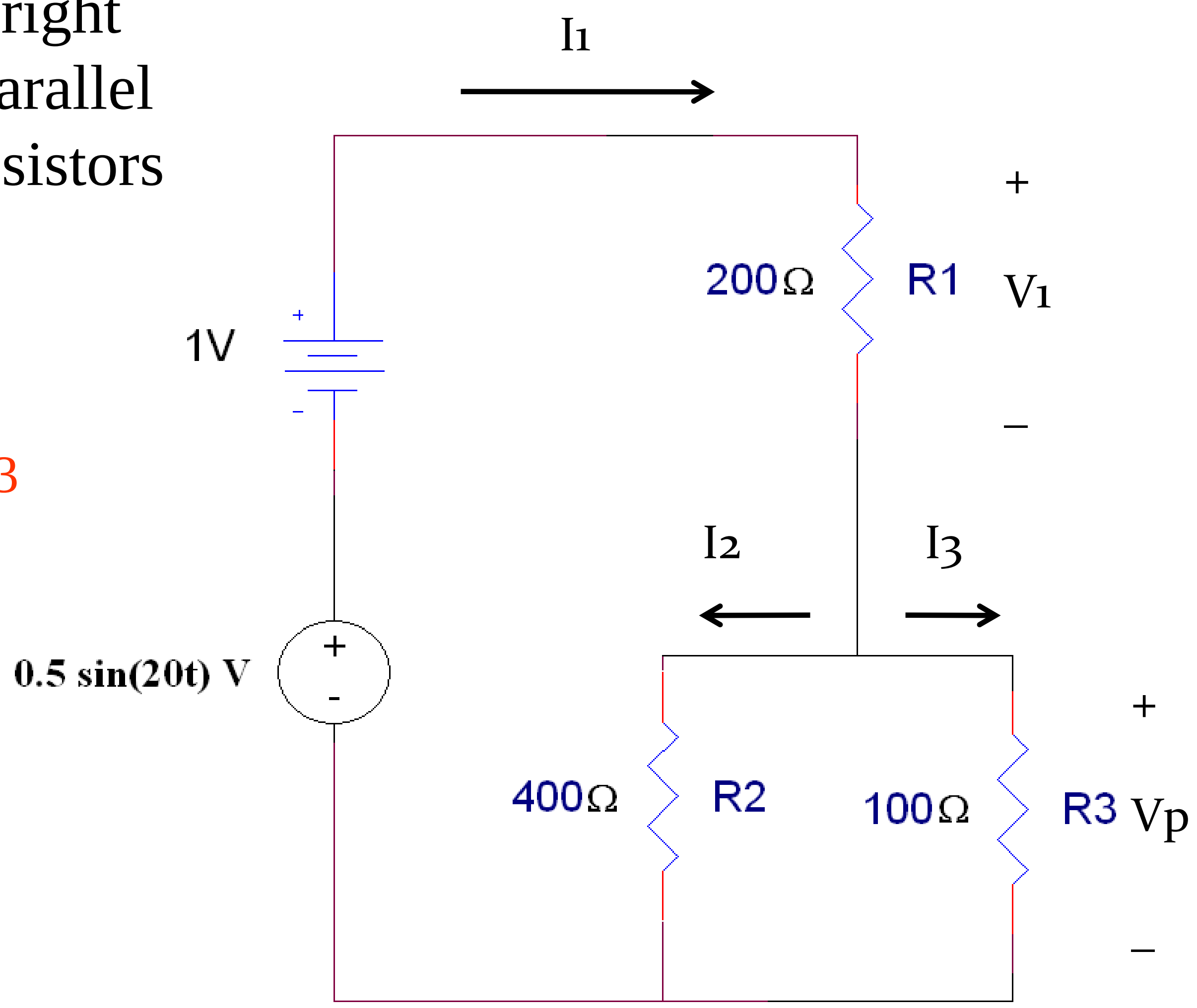
$$I_1 = \frac{R_{eq}}{R_1} \times I_{in} = \frac{109}{200} \times 4 = 2.18\text{ A}$$

$$I_2 = \frac{R_{eq}}{R_2} \times I_{in} = \frac{109}{400} \times 4 = 1.09\text{ A}$$

$$I_3 = \frac{R_{eq}}{R_3} \times I_{in} = \frac{109}{600} \times 4 = 0.727\text{ A}$$

Example 05…

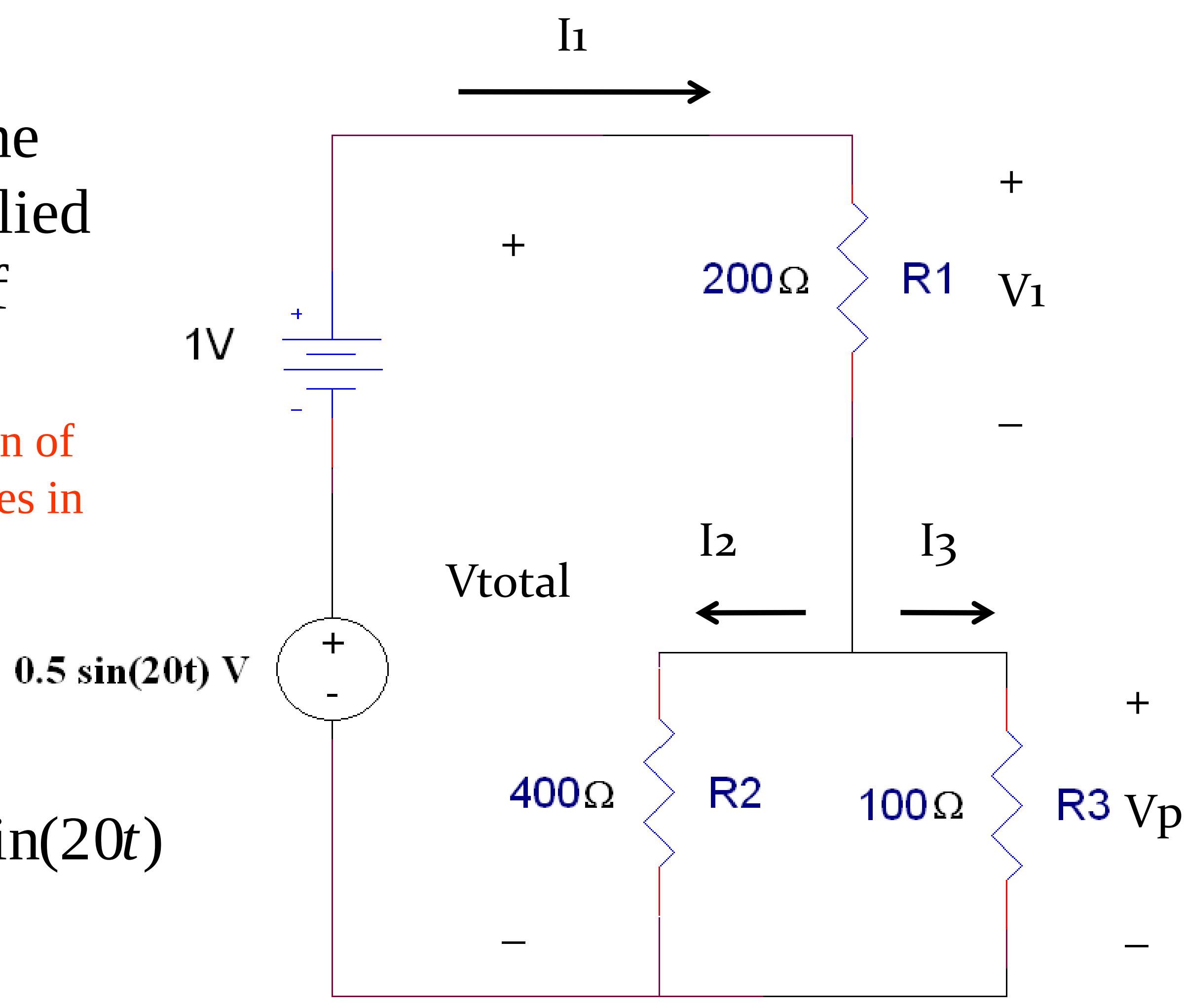
- The circuit to the right has a series and parallel combination of resistors plus two voltage sources.
  - Find V1 and Vp
  - Find I1, I2, and I3



...Example 05…

- First, calculate the total voltage applied to the network of resistors.
  - This is the addition of two voltage sources in series.

$$V_{total} = 1V + 0.5V\sin(20t)$$



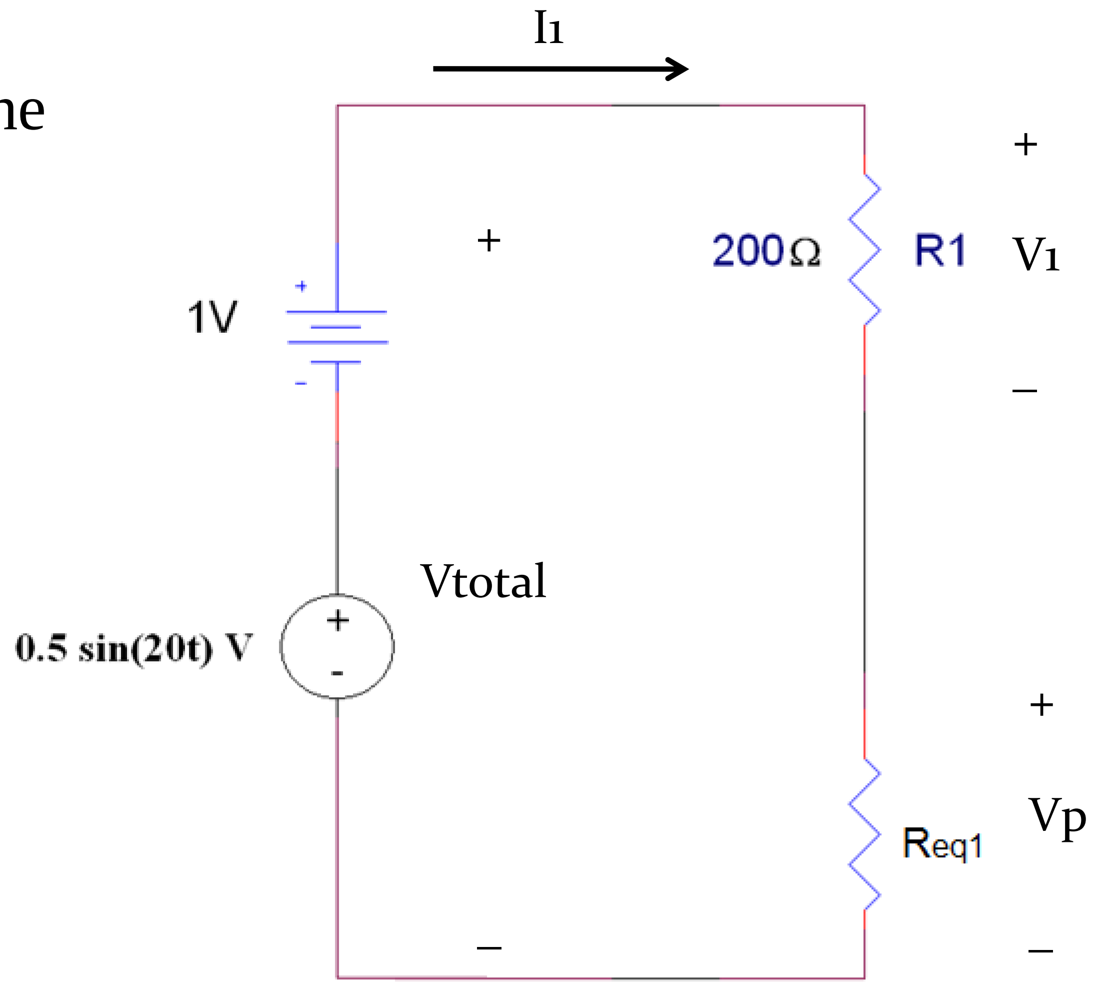
…Example 05…

- Second, calculate the equivalent resistor that can be used to replace the parallel combination of R2 and R3.

$$R_{eq1} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{eq1} = \frac{400\Omega(100\Omega)}{400\Omega + 100\Omega}$$

$$R_{eq1} = 80\Omega$$



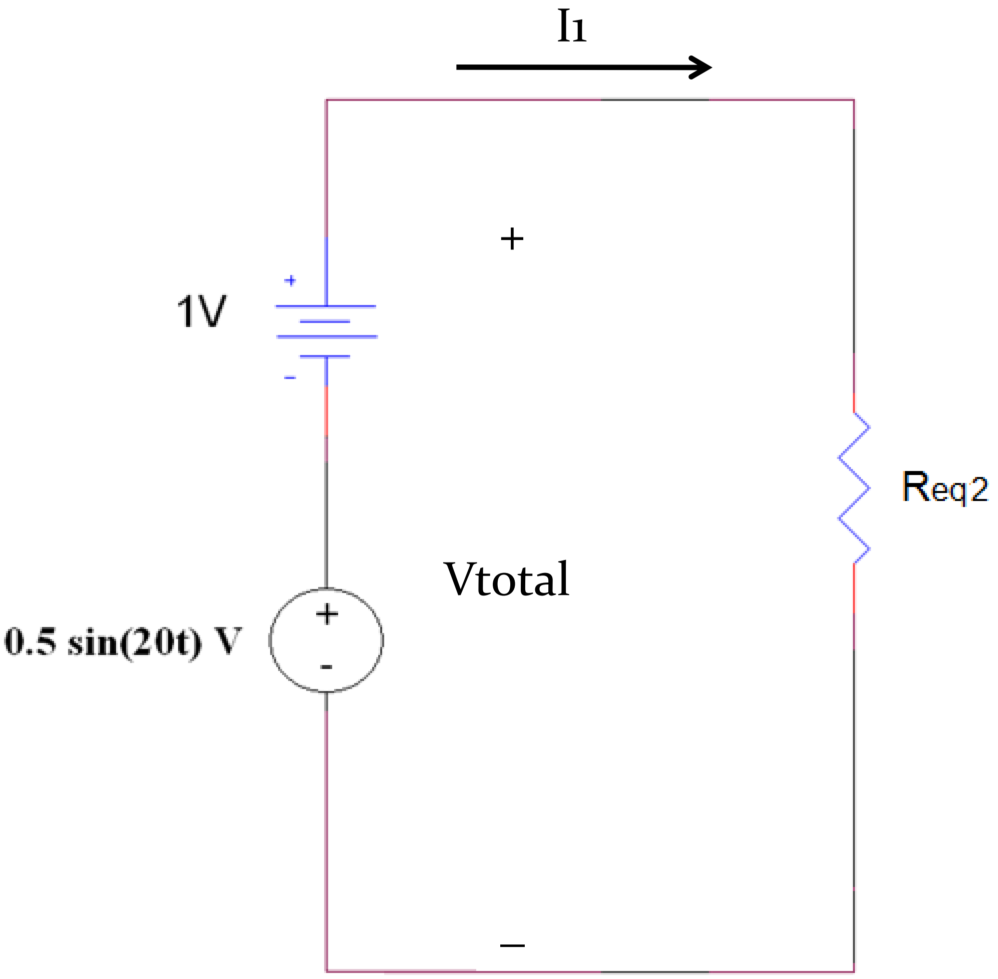
…Example 05…

- To calculate the value for I1, replace the series combination of R1 and Req1 with another equivalent resistor.

$$R_{eq2} = R_1 + R_{eq1}$$

$$R_{eq2} = 200\Omega + 80\Omega$$

$$R_{eq2} = 280\Omega$$



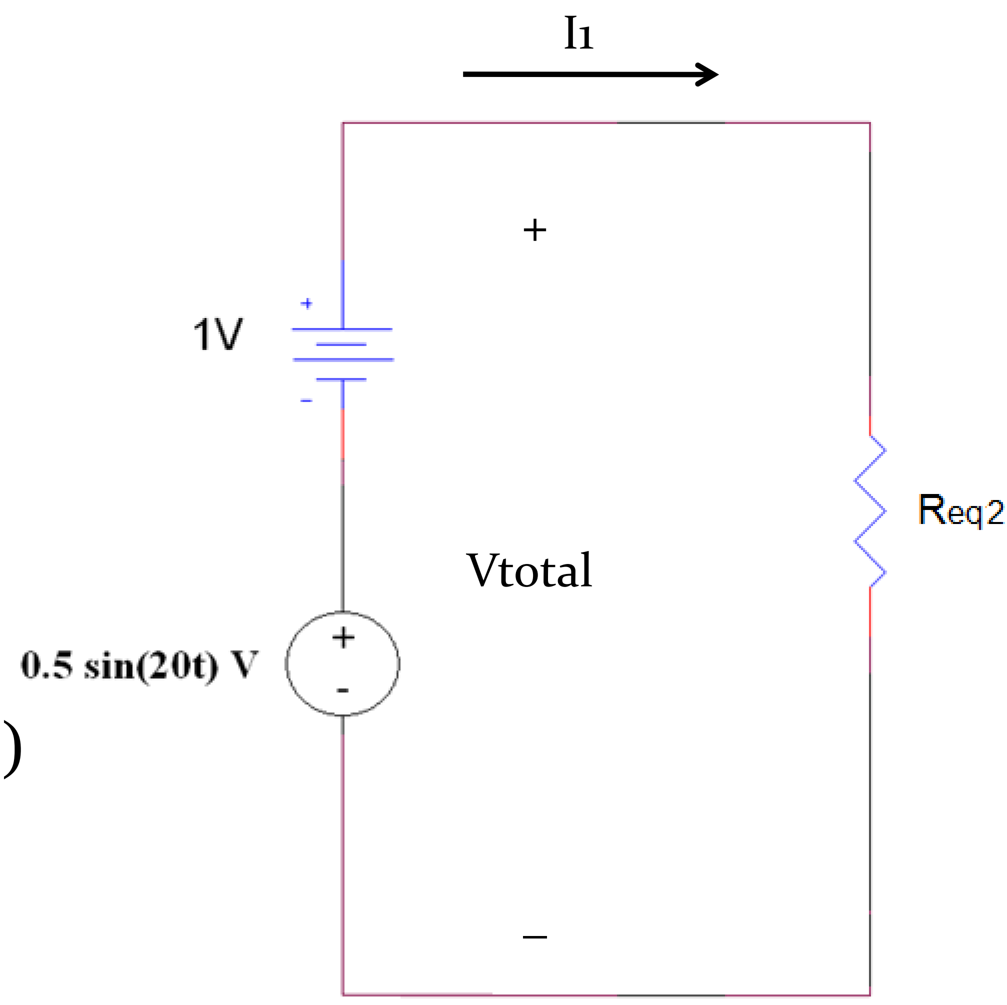
…Example 05…

$$I_1 = \frac{V_{total}}{R_{eq2}}$$

$$I_1 = \frac{1V + 0.5V\sin(20t)}{280\Omega}$$

$$I_1 = \frac{1V}{280\Omega} + \frac{0.5V\sin(20t)}{280\Omega}$$

$$I_1 = 3.57mA + 1.79mA\sin(20t)$$



…Example 05…

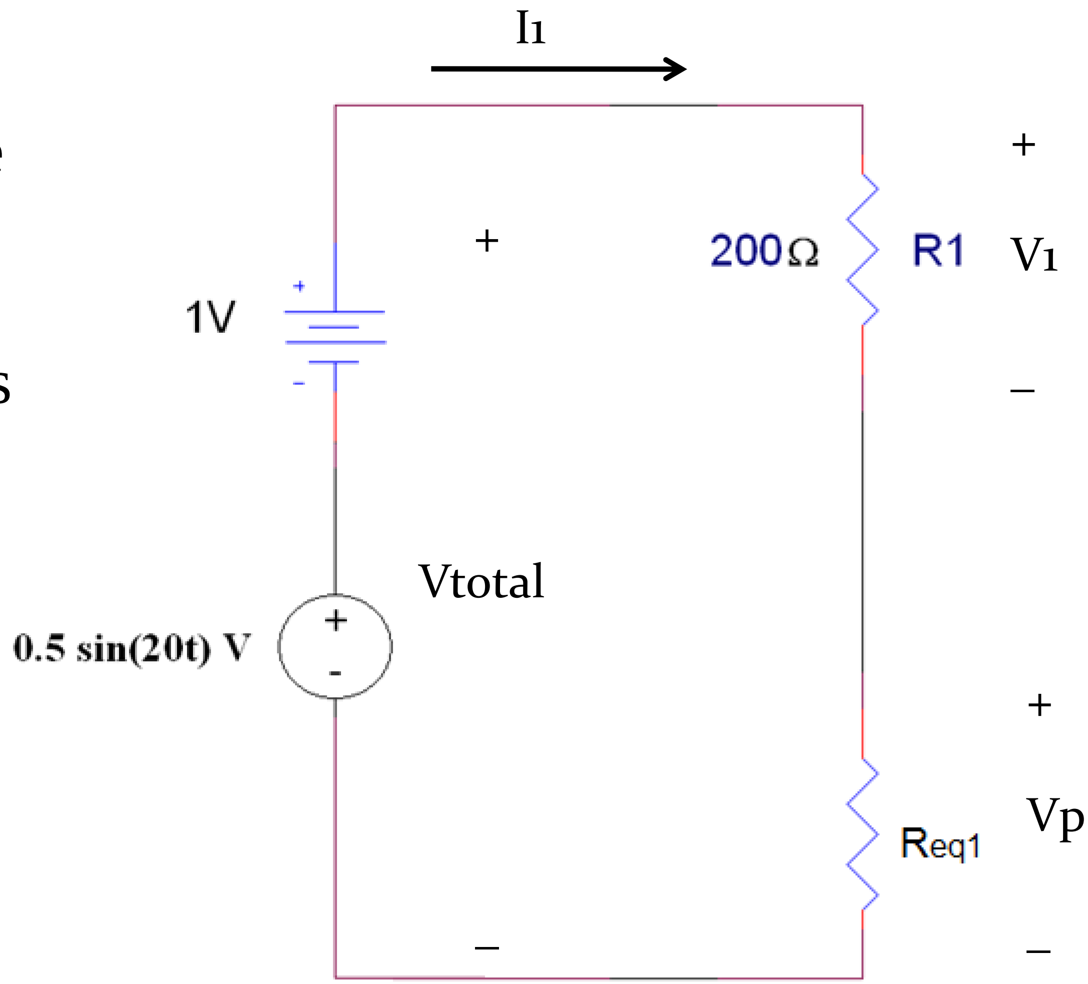
- To calculate V1, use one of the previous simplified circuits where R1 is in series with Req1.

$$V_1 = \frac{R_1}{R_1 + R_{eq}} V_{total}$$

or

$$V_1 = R_1 I_1$$

$$V_1 = 0.714V + 0.357V \sin(20t)$$



…Example 05…

To calculate *Vp*:

$$V_p = \frac{R_{eq1}}{R_1 + R_{eq1}} V_{total}$$

or

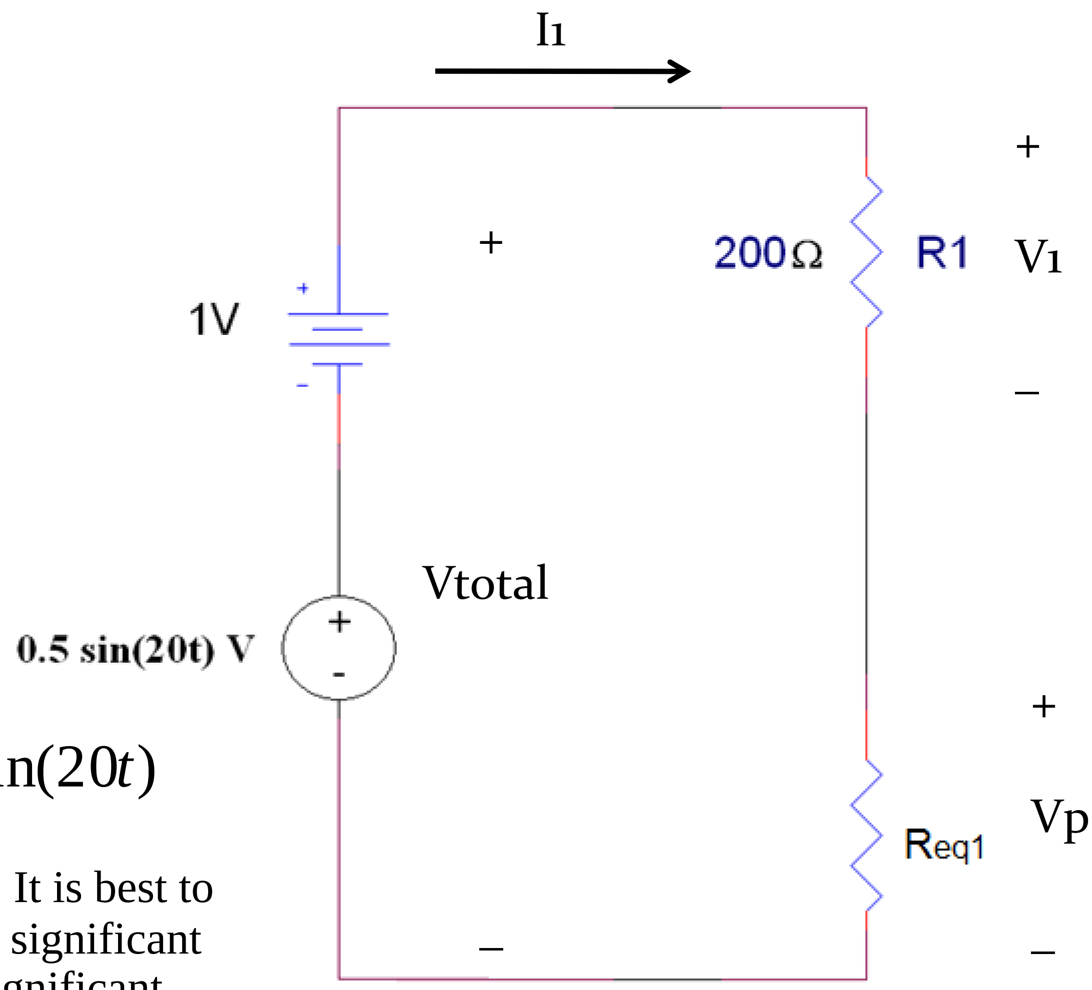
$$V_p = R_{eq1} I_1$$

or

$$V_p = V_{total} - V_1$$

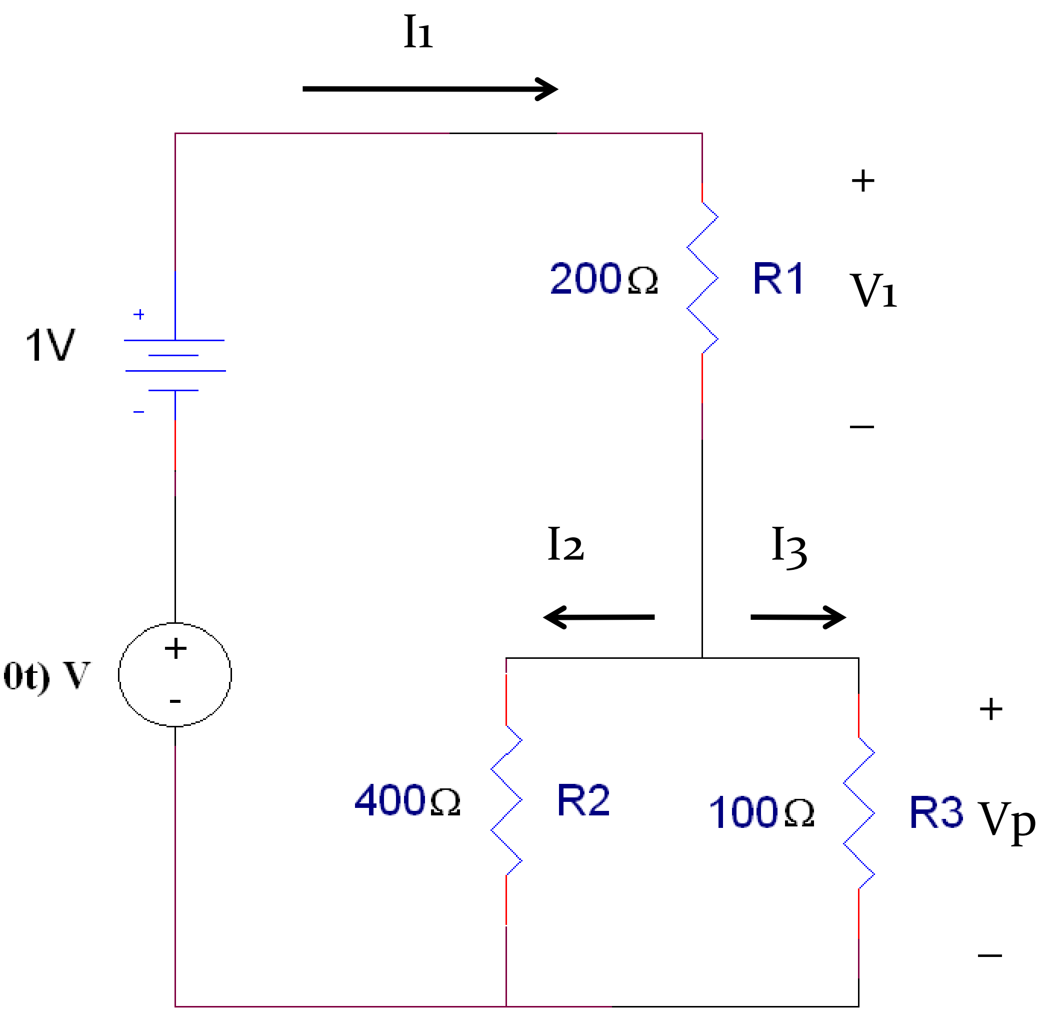
$$V_p = 0.287V + 0.143V \sin(20t)$$

Note: rounding errors can occur. It is best to carry the calculations out to 5 or 6 significant figures and then reduce this to 3 significant figures when writing the final answer.



…Example 05…

- Finally, use the original circuit to find I2 and I3.



$$I_2 = \frac{R_3}{R_2 + R_3} I_1$$

or

$$I_2 = \frac{R_{eq1}}{R_2} I_1$$

$$I_2 = 0.714mA + 0.357mA\sin(20t)$$

...Example 05

- Lastly, the calculation for I3.

$$I_3 = \frac{R_2}{R_2 + R_3} I_1$$

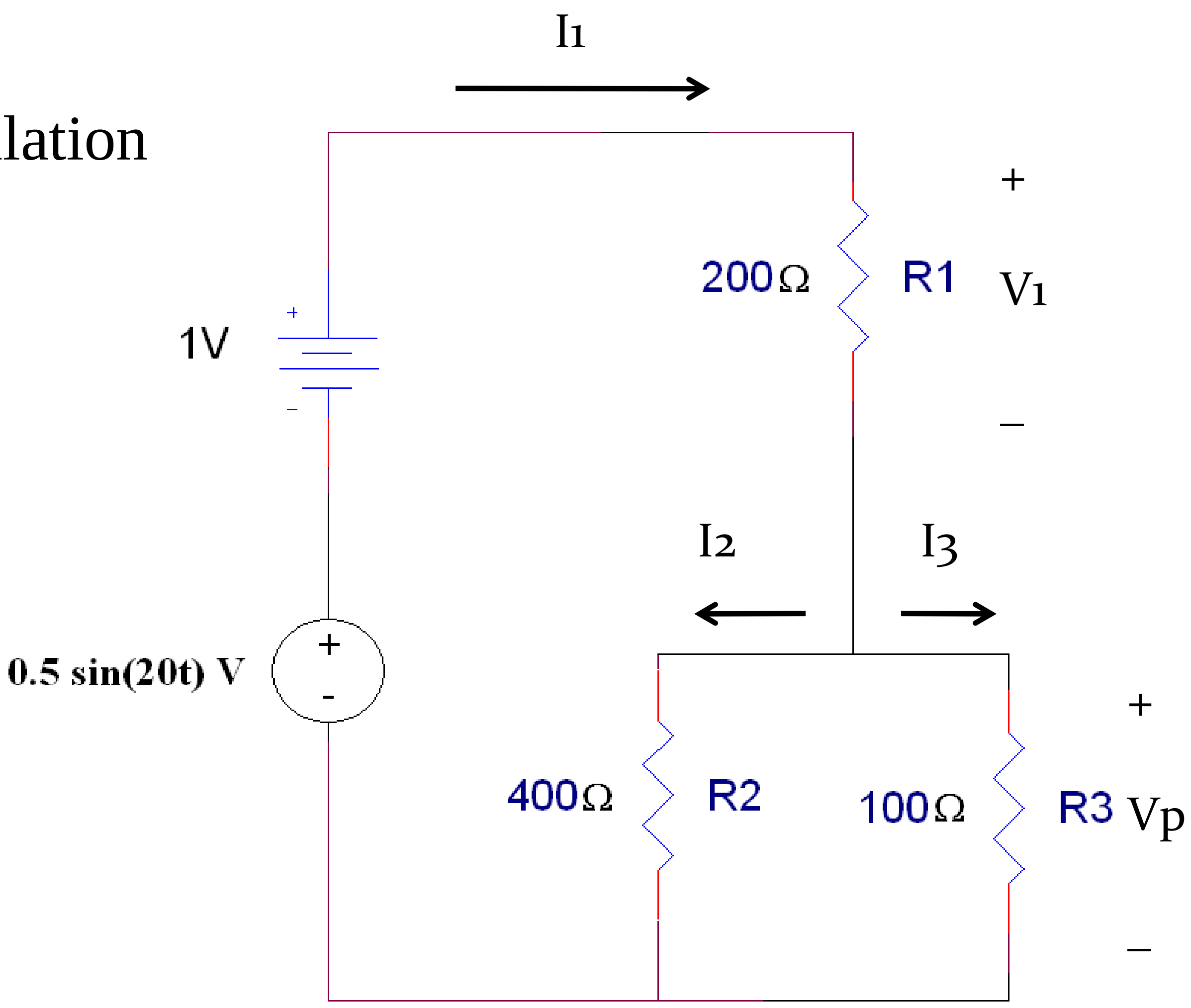
or

$$I_3 = \frac{R_{eq1}}{R_3} I_1$$

or

$$I_3 = I_1 - I_2$$

$$I_3 = 2.86mA + 1.43mA\sin(20t)$$



Summary

- The equations used to calculate the voltage across a specific resistor R_n in a set of resistors in series are:

$$V_n = \left[\frac{R_n}{R_{eq}}\right] V_{total}$$

$$V_n = \left[\frac{G_{eq}}{G_n}\right] V_{total}$$

- The equations used to calculate the current flowing through a specific resistor R_m in a set of resistors in parallel are:

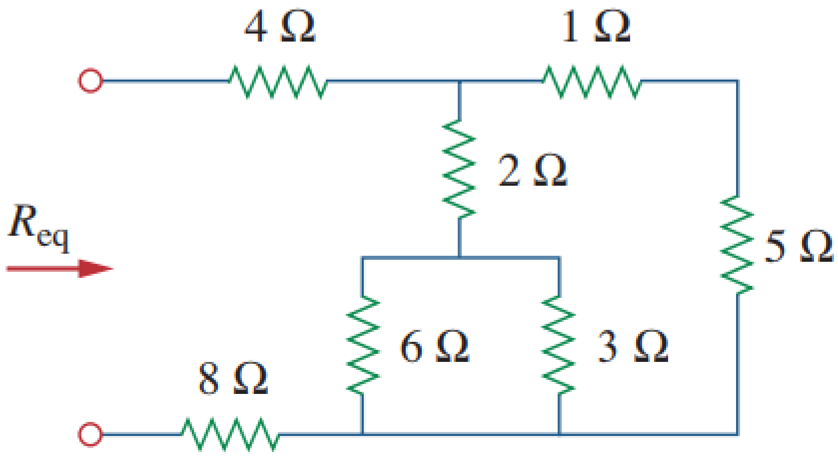
$$I_m = \frac{R_{eq}}{R_m} \mathrm{I}_{\mathrm{total}}$$

$$I_m = \frac{G_m}{G_{eq}} \mathrm{I}_{\mathrm{total}}$$

Summary Table

Series and Parallel Circuits		
Series	**Duality**	**Parallel**
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \rightleftarrows G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \rightleftarrows G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements	$R \rightleftarrows G$	$G_T = G_1 + G_2$
$R_T = R_1 + R_2$		and $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{R_1 R_2}{R_1 + R_2}$
I the same through series elements	$I \rightleftarrows V$	V the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \rightleftarrows I$	$I_T = I_1 + I_2 + I_3$
Largest V across largest R	$V \rightleftarrows I$ and $R \rightleftarrows G$	Greatest I through largest G (smallest R)
$V_x = \dfrac{R_x E}{R_T}$	$E, V \rightleftarrows I$ and $R \rightleftarrows G$	$I_x = \dfrac{G_x I_T}{G_T} = \dfrac{R_T I_T}{R_x}$
		with $I_1 = \dfrac{R_2 I_T}{R_1 + R_2}$ and $I_2 = \dfrac{R_1 I_T}{R_1 + R_2}$
$P = EI_T$	$E \rightleftarrows I$ and $I \rightleftarrows E$	$P = I_T E$
$P = I^2 R$	$I \rightleftarrows V$ and $R \rightleftarrows G$	$P = V^2 G = V^2/R$
$P = V^2/R$	$V \rightleftarrows I$ and $R \rightleftarrows G$	$P = I^2/G = I^2 R$

Example 2.9



Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

(The symbol ∥ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

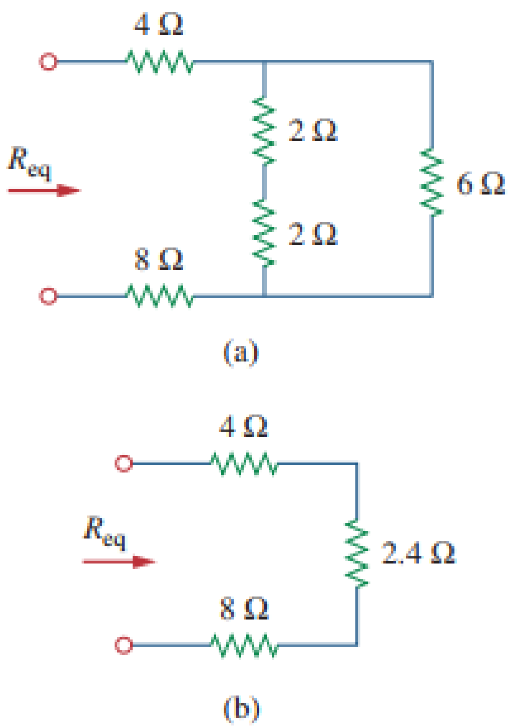


Figure 2.35
Equivalent circuits for Example 2.9.

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

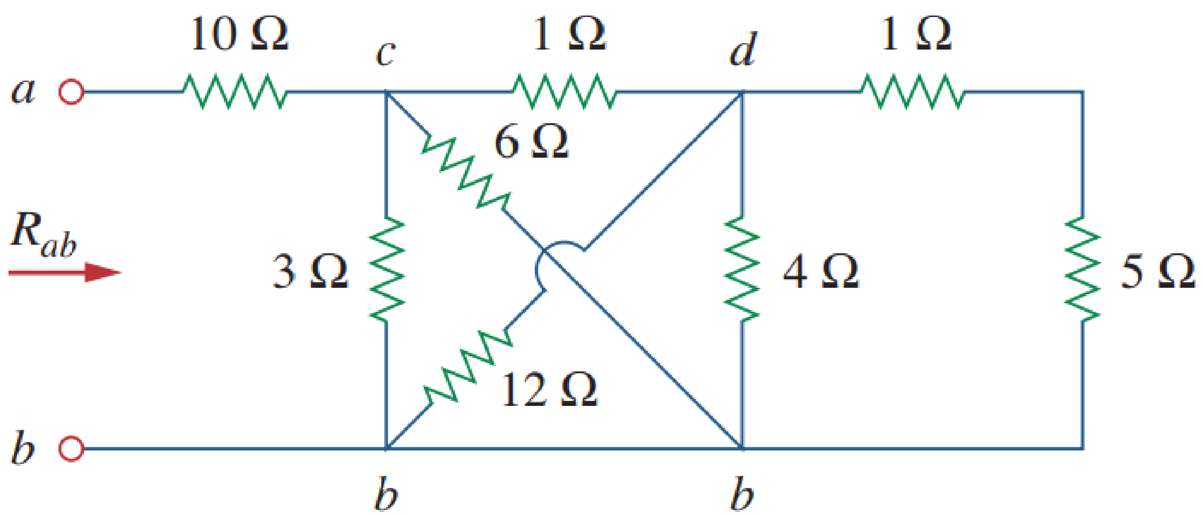


Figure 2.37

Solution:

The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes *c* and *b*. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega \qquad \textbf{(2.10.1)}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega \tag{2.10.2}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \tag{2.10.3}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of 1 Ω + 2 Ω = 3 Ω. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

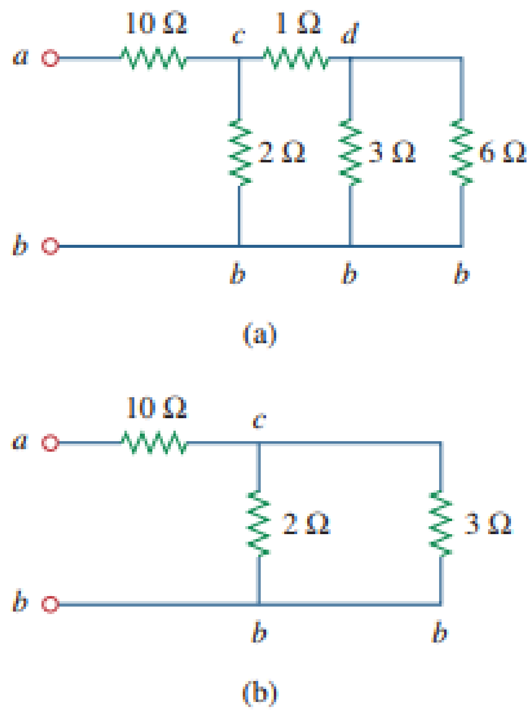


Figure 2.38
Equivalent circuits for Example 2.10.

Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.46. How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.46 can be simplified by using three-terminal equivalent networks. These are

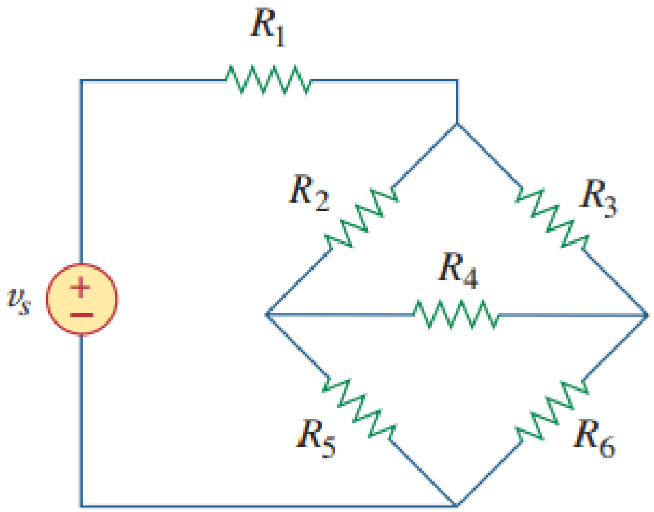


Figure 2.46
The bridge network.

the wye (Y) or tee (T) network shown in Fig. 2.47 and the delta (Δ) or pi (Π) network shown in Fig. 2.48. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

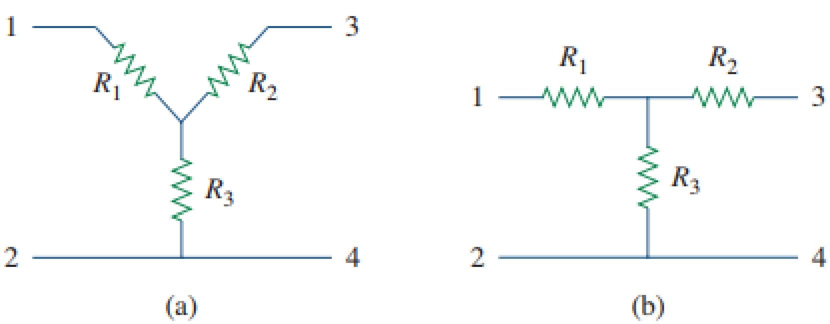


Figure 2.47
Two forms of the same network: (a) Y, (b) T.

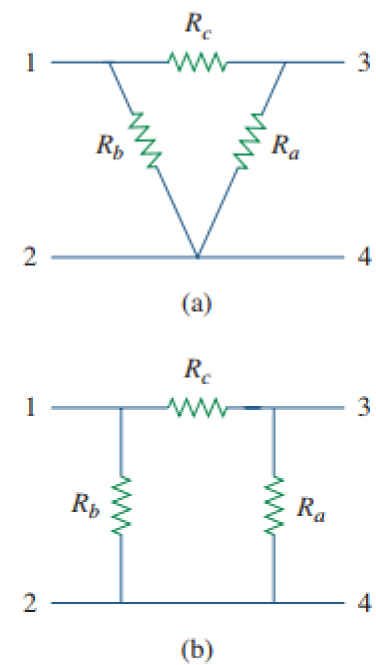


Figure 2.48
Two forms of the same network: (a) Δ, (b) Π.

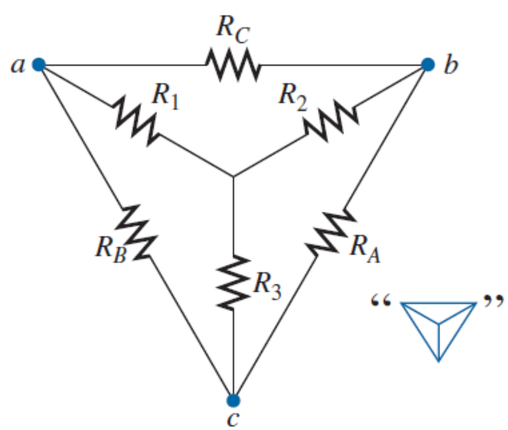
Wye and Delta Networks

To transform a Delta into a Wye

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



To transform a Wye into a Delta

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

If $R_1 = R_2 = R_3 = R$, then Ra = Rb =Rc = 3R

If $R_a = R_b = R_c = R'$, then $R_1 = R_2 = R_3 = R'/3$

Uses

- Distribution of 3 phase power
- Distribution of power in stators and windings in motors/generators.
  - Wye windings provide better torque at low rpm and delta windings generates better torque at high rpm.

Summary

- There is a conversion between the resistances used in wye and delta resistor networks.