Pearson Correlation Coefficient Example

The formula for the Pearson correlation coefficient r is:

$$\rho = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}$$

The covariance formula is:

$$\text{cov}(X, Y) = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

The standard deviation formula is:

$$\sigma_X = \sqrt{\frac{\sum (X_i - \bar{X})^2}{N}}, \quad \sigma_Y = \sqrt{\frac{\sum (Y_i - \bar{Y})^2}{N}}$$

Given the following data points:

X	Y
2	3
4	7
6	9
8	11

Step 1: Calculate the Means

$$\bar{X} = \frac{2 + 4 + 6 + 8}{4} = 5$$
$$\bar{Y} = \frac{3 + 7 + 9 + 11}{4} = 7.5$$

Step 2: Calculate the Deviations from the Mean and the Product of Deviations

$$(X_i - \bar{X}) = (2 - 5), (4 - 5), (6 - 5), (8 - 5) = -3, -1, 1, 3$$
$$(Y_i - \bar{Y}) = (3 - 7.5), (7 - 7.5), (9 - 7.5), (11 - 7.5) = -4.5, -0.5, 1.5, 3.5$$
$$(X_i - \bar{X})(Y_i - \bar{Y}) = (-3)(-4.5), (-1)(-0.5), (1)(1.5), (3)(3.5) = 13.5, 0.5, 1.5, 10.5$$

Step 3: Calculate the Covariance

$$\text{cov}(X, Y) = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{N} = \frac{13.5 + 0.5 + 1.5 + 10.5}{4} = \frac{26}{4} = 6.5$$

Step 4: Calculate the Standard Deviations

$$\sigma_X = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2 + (X_4 - \bar{X})^2}{N}} = \sqrt{\frac{9 + 1 + 1 + 9}{4}} = \sqrt{\frac{20}{4}} = \sqrt{5}$$

$$\sigma_Y = \sqrt{\frac{(Y_1 - \bar{Y})^2 + (Y_2 - \bar{Y})^2 + (Y_3 - \bar{Y})^2 + (Y_4 - \bar{Y})^2}{N}} = \sqrt{\frac{20.25 + 0.25 + 2.25 + 12.25}{4}} = \sqrt{\frac{35}{4}} = \frac{\sqrt{35}}{2}$$

Step 5: Calculate the Pearson Correlation Coefficient

$$r = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{6.5}{\sqrt{5} \times \frac{\sqrt{35}}{2}} = \frac{6.5 \times 2}{\sqrt{5 \times 35}} = \frac{13}{\sqrt{175}} = \frac{13}{5\sqrt{7}}$$

The exact value of r is:

$$\rho = \frac{13}{5 \times \sqrt{7}} \approx 0.968$$

The Pearson correlation coefficient is 0.968. This indicates a strong positive linear relationship between X and Y. As X increases, Y also increases in a linear fashion.

Spearman's Rank Correlation Coefficient Example

The formula for the Spearman's rank correlation coefficient ρ is:

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

Given the following data points:

X	Y
101	210
99	195
103	205
97	195
105	205

Step 1: Rank the Data

Rank the values of X and Y. Assign the average rank in case of ties.

X	Rank(X)	Y	Rank(Y)
101	3	210	5
99	2	195	1.5
103	4	205	3.5
97	1	195	1.5
105	5	205	3.5

Step 2: Calculate the Differences in Ranks and Their Squares

Calculate the difference between the ranks of X and Y for each pair and square the differences:

$$d_i = \text{Rank}(X) - \text{Rank}(Y)$$

$$d_1 = 3 - 5 = -2, \quad d_2 = 2 - 1.5 = 0.5, \quad d_3 = 4 - 3.5 = 0.5, \quad d_4 = 1 - 1.5 = -0.5, \quad d_5 = 5 - 3.5 = 1.5$$

Now calculate the squares of the differences:

$$d_1^2 = 4, \quad d_2^2 = 0.25, \quad d_3^2 = 0.25, \quad d_4^2 = 0.25, \quad d_5^2 = 2.25$$

Step 3: Sum the Squared Differences

$$\sum d_i^2 = 4 + 0.25 + 0.25 + 0.25 + 2.25 = 7$$

Step 4: Calculate Spearman's Rank Correlation Coefficient

Substitute the values into the formula for ρ:

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)} = 1 - \frac{6 \times 7}{5(5^2 - 1)} = 1 - \frac{42}{120} = 1 - 0.35 = 0.65$$

The Spearman rank correlation coefficient ρ is 0.65, indicating a moderate positive monotonic relationship between X and Y. As the values of X increase, the values of Y tend to increase, but not in a perfectly linear fashion.

Point Biserial Correlation Coefficient Example

The formula for the point biserial correlation coefficient ρ is:

$$\rho = \frac{M_1 - M_0}{\sigma} \times \sqrt{p_1 \times p_0}$$

Given the following data on gender (binary) and test scores (continuous):

Gender(X)	Test Score(Y)
Male (1)	85
Female (0)	78
Male (1)	90
Female (0)	75
Male (1)	88
Female (0)	80

Step 1: Calculate the Means

First, calculate the mean test scores for males (M_1) and females (M_0):

$$M_1 = \frac{85 + 90 + 88}{3} = 87.67 \quad \text{(Mean for Males)}$$
$$M_0 = \frac{78 + 75 + 80}{3} = 77.67 \quad \text{(Mean for Females)}$$
$$\text{Overall Mean} = \frac{85 + 78 + 90 + 75 + 88 + 80}{6} = 82.67$$

Step 2: Calculate the Overall Standard Deviation

The formula for the standard deviation is:

$$\sigma = \sqrt{\frac{\sum (Y_i - \bar{Y})^2}{n}}$$

Substitute the values:

$$\begin{aligned}
\sigma &= \sqrt{\frac{(85 - 82.67)^2 + (78 - 82.67)^2 + (90 - 82.67)^2 + (75 - 82.67)^2 + (88 - 82.67)^2 + (80 - 82.67)^2}{6}} \\
&= \sqrt{\frac{5.44 + 21.78 + 53.44 + 58.78 + 28.44 + 7.11}{6}} \\
&= \sqrt{\frac{175}{6}} \\
&= \sqrt{29.17} \approx 5.4
\end{aligned}$$

Step 3: Calculate the Proportions

The proportions of males (p_1) and females (p_0) are:

$$p_1 = \frac{3}{6} = 0.5 \quad \text{(Males)}$$
$$p_0 = \frac{3}{6} = 0.5 \quad \text{(Females)}$$

Step 4: Calculate the Point Biserial Correlation Coefficient

Substitute the values into the formula for ρ:

$$\rho = \frac{87.67 - 77.67}{5.4} \times \sqrt{0.5 \times 0.5} = \frac{10}{5.4} \times \sqrt{0.25} = 1.85 \times 0.5 = 0.925$$

The point biserial correlation coefficient $\rho = 0.925$, indicating a strong positive relationship between gender and test scores. Males tend to have higher test scores than females in this dataset.

ANOVA Example

We are given test scores for three teaching methods:

Test Scores	Teaching Method
85	A
90	A
88	A
75	B
78	B
82	C
85	C
87	C
90	C

We split the data into groups based on teaching methods:

Teaching Method	Test Scores
A	$85, 90, 88$
B	$75, 78$
C	$82, 85, 87, 90$

Step 1: Calculate the mean for each group:

$$\text{Mean of A} = \frac{85 + 90 + 88}{3} = 87.67$$

$$\text{Mean of B} = \frac{75 + 78}{2} = 76.5$$

$$\text{Mean of C} = \frac{82 + 85 + 87 + 90}{4} = 86$$

Step 2: Calculate the grand mean:

$$\text{Grand Mean} = \frac{85 + 90 + 88 + 75 + 78 + 82 + 85 + 87 + 90}{9} = 84.44$$

Step 3: Calculate the between-group sum of squares (SSB):

$SSB = n_A(\text{Mean of A} - \text{Grand Mean})^2 + n_B(\text{Mean of B} - \text{Grand Mean})^2 + n_C(\text{Mean of C} - \text{Grand Mean})^2$

Where $n_A = 3$, $n_B = 2$, and $n_C = 4$.

$$SSB = 3(87.67 - 84.44)^2 + 2(76.5 - 84.44)^2 + 4(86 - 84.44)^2$$

$$SSB = 3(3.23)^2 + 2(-7.94)^2 + 4(1.56)^2$$

$$SSB = 3(10.43) + 2(63.03) + 4(2.43) = 31.29 + 126.06 + 9.72 = 167.07$$

Step 4: Calculate the within-group sum of squares (SSW):
For each group, calculate the sum of squared deviations from the group mean.
For Method A:

$$(85 - 87.67)^2 + (90 - 87.67)^2 + (88 - 87.67)^2 = 7.13 + 5.43 + 0.11 = 12.67$$

For Method B:

$$(75 - 76.5)^2 + (78 - 76.5)^2 = 2.25 + 2.25 = 4.5$$

For Method C:

$$(82 - 86)^2 + (85 - 86)^2 + (87 - 86)^2 + (90 - 86)^2 = 16 + 1 + 1 + 16 = 34$$

Now, sum the within-group sum of squares:

$$SSW = 12.67 + 4.5 + 34 = 51.17$$

Step 5: Calculate the degrees of freedom:

$$df_B = (\text{number of groups} - 1) = 3 - 1 = 2$$

$$df_W = (\text{total number of observations} - \text{number of groups}) = 9 - 3 = 6$$

Step 6: Calculate the mean squares:

$$MSB = \frac{SSB}{df_B} = \frac{167.07}{2} = 83.54$$

$$MSW = \frac{SSW}{df_W} = \frac{51.17}{6} = 8.53$$

Step 7: Calculate the F-statistic:

$$F = \frac{MSB}{MSW} = \frac{83.54}{8.53} \approx 9.79$$

Step 8: Compare the F-statistic to the critical value:

At $\alpha = 0.05$, the critical value for $df_B = 2$ and $df_W = 6$ is 5.14.

Since $F = 9.79$ is greater than the critical value, this indicates that there is a statistically significant difference in test scores between the teaching methods.

Table of critical values for the F distribution (for use with ANOVA):

How to use this table:

There are two tables here. The first one gives critical values of F at the p = 0.05 level of significance. The second table gives critical values of F at the p = 0.01 level of significance.
1. Obtain your F-ratio. This has (x,y) degrees of freedom associated with it.
2. Go along x columns, and down y rows. The point of intersection is your critical F-ratio.
3. If your obtained value of F is equal to or larger than this critical F-value, then your result is significant at that level of probability.
An example: I obtain an F ratio of 3.96 with (2, 24) degrees of freedom.
I go along 2 columns and down 24 rows. The critical value of F is 3.40. My obtained F-ratio is larger than this, and so I conclude that my obtained F-ratio is likely to occur by chance with a p<.05.

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17
31	4.16	3.31	2.91	2.68	2.52	2.41	2.32	2.26	2.20	2.15
32	4.15	3.30	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14
33	4.14	3.29	2.89	2.66	2.50	2.39	2.30	2.24	2.18	2.13
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11

36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15	2.11
37	4.11	3.25	2.86	2.63	2.47	2.36	2.27	2.20	2.15	2.10
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09
39	4.09	3.24	2.85	2.61	2.46	2.34	2.26	2.19	2.13	2.08
40	4.09	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
41	4.08	3.23	2.83	2.60	2.44	2.33	2.24	2.17	2.12	2.07
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.07
43	4.07	3.21	2.82	2.59	2.43	2.32	2.23	2.16	2.11	2.06
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.15	2.09	2.04
47	4.05	3.20	2.80	2.57	2.41	2.30	2.21	2.14	2.09	2.04
48	4.04	3.19	2.80	2.57	2.41	2.30	2.21	2.14	2.08	2.04
49	4.04	3.19	2.79	2.56	2.40	2.29	2.20	2.13	2.08	2.03
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
51	4.03	3.18	2.79	2.55	2.40	2.28	2.20	2.13	2.07	2.02
52	4.03	3.18	2.78	2.55	2.39	2.28	2.19	2.12	2.07	2.02
53	4.02	3.17	2.78	2.55	2.39	2.28	2.19	2.12	2.06	2.02
54	4.02	3.17	2.78	2.54	2.39	2.27	2.19	2.12	2.06	2.01
55	4.02	3.17	2.77	2.54	2.38	2.27	2.18	2.11	2.06	2.01
56	4.01	3.16	2.77	2.54	2.38	2.27	2.18	2.11	2.05	2.01
57	4.01	3.16	2.77	2.53	2.38	2.26	2.18	2.11	2.05	2.00
58	4.01	3.16	2.76	2.53	2.37	2.26	2.17	2.10	2.05	2.00
59	4.00	3.15	2.76	2.53	2.37	2.26	2.17	2.10	2.04	2.00
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
61	4.00	3.15	2.76	2.52	2.37	2.25	2.16	2.09	2.04	1.99
62	4.00	3.15	2.75	2.52	2.36	2.25	2.16	2.09	2.04	1.99
63	3.99	3.14	2.75	2.52	2.36	2.25	2.16	2.09	2.03	1.99
64	3.99	3.14	2.75	2.52	2.36	2.24	2.16	2.09	2.03	1.98
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.03	1.98
66	3.99	3.14	2.74	2.51	2.35	2.24	2.15	2.08	2.03	1.98
67	3.98	3.13	2.74	2.51	2.35	2.24	2.15	2.08	2.02	1.98
68	3.98	3.13	2.74	2.51	2.35	2.24	2.15	2.08	2.02	1.97
69	3.98	3.13	2.74	2.51	2.35	2.23	2.15	2.08	2.02	1.97
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97
71	3.98	3.13	2.73	2.50	2.34	2.23	2.14	2.07	2.02	1.97
72	3.97	3.12	2.73	2.50	2.34	2.23	2.14	2.07	2.01	1.97
73	3.97	3.12	2.73	2.50	2.34	2.23	2.14	2.07	2.01	1.96
74	3.97	3.12	2.73	2.50	2.34	2.22	2.14	2.07	2.01	1.96
75	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96
76	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96
77	3.97	3.12	2.72	2.49	2.33	2.22	2.13	2.06	2.00	1.96
78	3.96	3.11	2.72	2.49	2.33	2.22	2.13	2.06	2.00	1.95
79	3.96	3.11	2.72	2.49	2.33	2.22	2.13	2.06	2.00	1.95
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95
81	3.96	3.11	2.72	2.48	2.33	2.21	2.13	2.06	2.00	1.95
82	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05	2.00	1.95
83	3.96	3.11	2.72	2.48	2.32	2.21	2.12	2.05	2.00	1.95
84	3.96	3.11	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.95
85	3.95	3.10	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.94

86	3.95	3.10	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.94
87	3.95	3.10	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.94
88	3.95	3.10	2.71	2.48	2.32	2.20	2.12	2.05	1.99	1.94
89	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
91	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.98	1.94
92	3.95	3.10	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.94
93	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
94	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
95	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
96	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
97	3.94	3.09	2.70	2.47	2.31	2.19	2.11	2.04	1.98	1.93
98	3.94	3.09	2.70	2.47	2.31	2.19	2.10	2.03	1.98	1.93
99	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.98	1.93
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.98	1.93

Critical values of F for the 0.01 significance level:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	4052.19	4999.52	5403.34	5624.62	5763.65	5858.97	5928.33	5981.10	6022.50	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.93	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.52	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.90	3.81
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.02	3.84	3.71	3.60	3.51
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.77	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.05	3.73	3.50	3.33	3.20	3.09	3.01
30	7.56	5.39	4.51	4.02	3.70	3.47	3.31	3.17	3.07	2.98
31	7.53	5.36	4.48	3.99	3.68	3.45	3.28	3.15	3.04	2.96
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02	2.93

33	7.47	5.31	4.44	3.95	3.63	3.41	3.24	3.11	3.00	2.91
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98	2.89
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88
36	7.40	5.25	4.38	3.89	3.57	3.35	3.18	3.05	2.95	2.86
37	7.37	5.23	4.36	3.87	3.56	3.33	3.17	3.04	2.93	2.84
38	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.92	2.83
39	7.33	5.19	4.33	3.84	3.53	3.31	3.14	3.01	2.90	2.81
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
41	7.30	5.16	4.30	3.82	3.50	3.28	3.11	2.98	2.88	2.79
42	7.28	5.15	4.29	3.80	3.49	3.27	3.10	2.97	2.86	2.78
43	7.26	5.14	4.27	3.79	3.48	3.25	3.09	2.96	2.85	2.76
44	7.25	5.12	4.26	3.78	3.47	3.24	3.08	2.95	2.84	2.75
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74
46	7.22	5.10	4.24	3.76	3.44	3.22	3.06	2.93	2.82	2.73
47	7.21	5.09	4.23	3.75	3.43	3.21	3.05	2.92	2.81	2.72
48	7.19	5.08	4.22	3.74	3.43	3.20	3.04	2.91	2.80	2.72
49	7.18	5.07	4.21	3.73	3.42	3.20	3.03	2.90	2.79	2.71
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.79	2.70
51	7.16	5.05	4.19	3.71	3.40	3.18	3.01	2.88	2.78	2.69
52	7.15	5.04	4.18	3.70	3.39	3.17	3.01	2.87	2.77	2.68
53	7.14	5.03	4.17	3.70	3.38	3.16	3.00	2.87	2.76	2.68
54	7.13	5.02	4.17	3.69	3.38	3.16	2.99	2.86	2.76	2.67
55	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66
56	7.11	5.01	4.15	3.67	3.36	3.14	2.98	2.85	2.74	2.66
57	7.10	5.00	4.15	3.67	3.36	3.14	2.97	2.84	2.74	2.65
58	7.09	4.99	4.14	3.66	3.35	3.13	2.97	2.84	2.73	2.64
59	7.09	4.98	4.13	3.66	3.35	3.12	2.96	2.83	2.72	2.64
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
61	7.07	4.97	4.12	3.64	3.33	3.11	2.95	2.82	2.71	2.63
62	7.06	4.97	4.11	3.64	3.33	3.11	2.94	2.81	2.71	2.62
63	7.06	4.96	4.11	3.63	3.32	3.10	2.94	2.81	2.70	2.62
64	7.05	4.95	4.10	3.63	3.32	3.10	2.93	2.80	2.70	2.61
65	7.04	4.95	4.10	3.62	3.31	3.09	2.93	2.80	2.69	2.61
66	7.04	4.94	4.09	3.62	3.31	3.09	2.92	2.79	2.69	2.60
67	7.03	4.94	4.09	3.61	3.30	3.08	2.92	2.79	2.68	2.60
68	7.02	4.93	4.08	3.61	3.30	3.08	2.91	2.79	2.68	2.59
69	7.02	4.93	4.08	3.60	3.30	3.08	2.91	2.78	2.68	2.59
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59
71	7.01	4.92	4.07	3.60	3.29	3.07	2.90	2.77	2.67	2.58
72	7.00	4.91	4.07	3.59	3.28	3.06	2.90	2.77	2.66	2.58
73	7.00	4.91	4.06	3.59	3.28	3.06	2.90	2.77	2.66	2.57
74	6.99	4.90	4.06	3.58	3.28	3.06	2.89	2.76	2.66	2.57
75	6.99	4.90	4.05	3.58	3.27	3.05	2.89	2.76	2.65	2.57
76	6.98	4.90	4.05	3.58	3.27	3.05	2.88	2.76	2.65	2.56
77	6.98	4.89	4.05	3.57	3.27	3.05	2.88	2.75	2.65	2.56
78	6.97	4.89	4.04	3.57	3.26	3.04	2.88	2.75	2.64	2.56
79	6.97	4.88	4.04	3.57	3.26	3.04	2.87	2.75	2.64	2.55
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55
81	6.96	4.88	4.03	3.56	3.25	3.03	2.87	2.74	2.63	2.55
82	6.95	4.87	4.03	3.56	3.25	3.03	2.87	2.74	2.63	2.55

83	6.95	4.87	4.03	3.55	3.25	3.03	2.86	2.73	2.63	2.54
84	6.95	4.87	4.02	3.55	3.24	3.03	2.86	2.73	2.63	2.54
85	6.94	4.86	4.02	3.55	3.24	3.02	2.86	2.73	2.62	2.54
86	6.94	4.86	4.02	3.55	3.24	3.02	2.85	2.73	2.62	2.53
87	6.94	4.86	4.02	3.54	3.24	3.02	2.85	2.72	2.62	2.53
88	6.93	4.86	4.01	3.54	3.23	3.01	2.85	2.72	2.62	2.53
89	6.93	4.85	4.01	3.54	3.23	3.01	2.85	2.72	2.61	2.53
90	6.93	4.85	4.01	3.54	3.23	3.01	2.85	2.72	2.61	2.52
91	6.92	4.85	4.00	3.53	3.23	3.01	2.84	2.71	2.61	2.52
92	6.92	4.84	4.00	3.53	3.22	3.00	2.84	2.71	2.61	2.52
93	6.92	4.84	4.00	3.53	3.22	3.00	2.84	2.71	2.60	2.52
94	6.91	4.84	4.00	3.53	3.22	3.00	2.84	2.71	2.60	2.52
95	6.91	4.84	4.00	3.52	3.22	3.00	2.83	2.70	2.60	2.51
96	6.91	4.83	3.99	3.52	3.21	3.00	2.83	2.70	2.60	2.51
97	6.90	4.83	3.99	3.52	3.21	2.99	2.83	2.70	2.60	2.51
98	6.90	4.83	3.99	3.52	3.21	2.99	2.83	2.70	2.59	2.51
99	6.90	4.83	3.99	3.52	3.21	2.99	2.83	2.70	2.59	2.51
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50

Chi-Square Test of Independence Example

We are given the following data on gender and teaching method preference:

Gender	Teaching Method Preference
Male	A
Male	A
Male	A
Male	B
Male	C
Male	C
Male	C
Female	B
Female	B
Female	B
Female	A
Female	C
$\vdots$	$\vdots$

Now, we split the data into groups for a contingency table:

	Method A	Method B	Method C	Total
Male	30	10	20	60
Female	10	30	50	90
Total	40	40	70	150

Step 1: Calculate the expected frequencies for each cell. The expected frequency formula is:

$$\text{Expected Frequency} = \frac{(\text{Row Total} \times \text{Column Total})}{\text{Grand Total}}$$

For Males preferring Method A:

$$\text{Expected Frequency} = \frac{(60 \times 40)}{150} = 16$$

Now calculate the expected frequencies for all cells:

	Method A	Method B	Method C	Total
Male	16	16	28	60
Female	24	24	42	90

Step 2: Use the chi-square formula:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Where O is the observed frequency, and E is the expected frequency. We now calculate the chi-square statistic for each cell:

For Males preferring Method A:

$$\frac{(30 - 16)^2}{16} = \frac{14^2}{16} = 12.25$$

For Males preferring Method B:

$$\frac{(10 - 16)^2}{16} = \frac{(-6)^2}{16} = 2.25$$

For Males preferring Method C:

$$\frac{(20 - 28)^2}{28} = \frac{(-8)^2}{28} = 2.29$$

For Females preferring Method A:

$$\frac{(10-24)^2}{24} = \frac{(-14)^2}{24} = 8.17$$

For Females preferring Method B:

$$\frac{(30-24)^2}{24} = \frac{6^2}{24} = 1.50$$

For Females preferring Method C:

$$\frac{(50-42)^2}{42} = \frac{8^2}{42} = 1.52$$

Step 3: Sum the chi-square values:

$$\chi^2 = 12.25 + 2.25 + 2.29 + 8.17 + 1.50 + 1.52 = 27.98$$

Step 4: Determine the degrees of freedom:

$$df = (r-1) \times (c-1) = (2-1) \times (3-1) = 2$$

Step 5: Compare the chi-square statistic to the critical value from the chi-square table at $\alpha = 0.05$:

The critical value for $df = 2$ and $\alpha = 0.05$ is 5.99. Since $\chi^2 = 27.98$, this indicates there is statistically significant correlation between gender and teaching methods.

Critical values of chi-square (right tail)

Degrees of freedom (*df*)	Significance level (α)							
	.99	.975	.95	.9	.1	.05	.025	.01
1	-------	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892
40	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691
50	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154
60	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379
70	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425
80	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329
100	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116
1000	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807