

# Hilton-Milnor splitting in homotopy type theory

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## Abstract

We show in HoTT that for pointed types  $X, Y$  we have a pointed equivalence:

$$\Omega X(X \vee Y) \simeq \Omega X \times \Omega Y \times \Omega \Sigma(\Omega X \wedge \Omega Y)$$

known as the Hilton-Milnor splitting.

## 1 Introduction

## 2 Homotopy pullbacks and descent

## 3 The Hilton-Milnor splitting

**Lemma 3.1** (Splitting lemma). Let  $f : X \rightarrow_* Y$  be a pointed map. Let  $g : \Omega Y \rightarrow_* \Omega X$  be a section of  $\Omega f$ . Then the composition of pointed maps

$$\Omega B \times \Omega \mathrm{hfiber}(f) \xrightarrow{g \times \mathrm{pr}_1}_* \Omega A \times \Omega A \xrightarrow{\mathrm{concat}}_* \Omega A$$

is an equivalence of pointed types.

**Theorem 3.1** (Hilton-Milnor Splitting). Let  $X, Y$  be pointed types. Then there is a pointed equivalence:

$$\Omega(X \vee Y) \simeq \Omega X \times \Omega Y \times \Omega \Sigma(\Omega X \wedge \Omega Y)$$