Introduction to dependent type theory

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February 28, 2019

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1 Introduction

1.1 Curry-Howard correspondance

At the beginning of the 20th century, Whitehead and Russell pubished their *Principia Mathematica* [4], demonstrating to mathematicians of the time that formal logic could express much of mathematics. It served to popularise modern mathematical logic leading to many mathematicians taking a more serious look at topic such as the foundations of mathematics.

One of the most influencial mathematicians of the time was David Hilbert. Inspired by Whitehead and Russell's vision, Hilbert and his coleagues at Göttingen became leading researchers in formal logic. Hilbert proposed the *Entscheidungsproblem* (decision problem), that is, to develop an "effectually calculable procedure" to determine the truth or falsehood of any logical statement. At the 1930 Mathematical Congress in Königsberg, Hilbert affirmed his belief in the conjecture, concluding with his famous words "Wir müssen wissen, wir werden wissen" ("We must know, we will know"). At the very same conference, Kurt Gödel announced his proof that arithmetic is incomplete [2], not every statement in arithmetic can be proven.

This however did not deter logicians, who were still interested in understanding why the *Entscheidungsproblem* was undecidable, for this a formal efinition of "effectively calculable" was required. So along came three proposed definitions of what it meant to be "effectively calculable": lambda calculus, pusblished in 1936 by Alonzo Church [1]; *recursive functions*, proposed by Gödel in 1934 later published in 1936 by Stephen Kleene [3]; and finally *Turing machines* in 1937 by Alan Turing [5]

References

[1] Alonzo Church. An unsolvable problem of elementary number theory. *American Journal of Mathematics*, 58(2):345–363, April 1936.

[2] Kurt Gödel. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. *Monatshefte für Mathematik und Physik*, 38(1):173–198, 1931.

[3] S.C. Kleene. General recursive functions of natural numbers. *Mathematische Annalen*, 112:727–742, 1936.

[4] Alfred North Whitehead and Bertrand Russell. *Principia Mathematica*, volume 1. Cambridge University Press, Cambridge, 1910. Nachdruck: GlossarWiki:Whitehead$_R$$ussell$: 2009.

[5] Alan M. Turing. On computable numbers, with an application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society*, 2(42):230–265, 1936.