$\mathbb{1} = \{*\}$

$\{\emptyset\}$

$0 \quad 1 \quad 2 \quad 3$

$0 \quad 0^{+} \quad 0^{++} \quad 0^{+++}$

$$X \xrightarrow{\iota_1} X + Y \xleftarrow{\iota_2} Y$$

$\iota_1(*)$

$\iota_2(\iota_1(*))$

$\iota_2(\iota_2(\iota_1(*)))$

$\ddots$

$\mathbb{N}_{MP}$

$* \quad *^{+}$

$0 = \{\}$

$1 = \{0\}$?

$2 = \{$

$= \mathbb{N}_{vN} \cong 1 + \mathbb{N}_{vN}$

$$\mathbb{N}_{MP} = \mathbb{1} + \mathbb{N}_{MP}$$

$$1 + \mathbb{N}_{MP} = \{\iota_1(*)\} \cup \{\iota_2(x) \mid x \in \mathbb{N}_{MP}\}$$

$$= \mathbb{N}_{MP}$$

$$\underset{\iota_1}{A} + \underset{\iota_2(b)}{B} + \underset{\iota_3(c)}{C}$$

$$\mathbb{N} = 1 + \mathbb{N}$$

$$\mathbb{E} = \mathbb{N} + \mathbb{E} \times \mathbb{E} + \mathbb{E} \times \mathbb{E} + \mathbb{E}$$

$$F : X \longmapsto \mathbb{N} + X \times X + X \times X + X$$

$$F = K_{\mathbb{N}} + id \times id + id \times id + id$$

$$\mathbb{P} : X \longmapsto \mathbb{P}X$$

$$f : X \to Y$$

$$\mathbb{P}f : \mathbb{P}X \to \mathbb{P}Y$$

$$\mathbb{P}f(A) = f_*A = f[A] = \{f(x) \mid x \in A\}$$

$$F, G$$

$$(F \times G)(A) = FA \times GA$$

$$f : A \to B$$

$$(F \times G)(f) : FA \times GA \to FB \times GB$$

$$(x_1, x_2) \longmapsto ((Ff)(x_1), (Gf)(x_2))$$

$$(F + G)(A) = FA + GA$$

$$(F + G)(f) : FA + GA \to FB + GB$$

$$\iota_1(x_1) \longmapsto \iota_1((Ff)x_1)$$

$$\iota_2(x_2) \longmapsto \iota_2((Gf)x_2)$$

$$RX = C^* \longrightarrow (X \times C^* + 1)$$

$$Rf$$

F monoton, Scottovo zvezen

$I_0 = \emptyset \qquad I_{n+1} = F I_n \qquad I = \bigcup_{n=0}^{\infty} I_n$

- $I_n \subseteq I_{n+1}$

  z indukcijo

  - $n=0$: $I_0 = \emptyset \subseteq I_1$ ✓
  - $n \mapsto n+1$: I.P. $I_n \subseteq I_{n+1}$

    $I_{n+1} = F I_n \underset{F \text{ monoton}}{\subseteq} F I_{n+1} = I_{n+2}$ ✓

- $F I \subseteq I$

  $F I = \bigcup_{A \subseteq^k I} F A$.

  vsaka končna $A \subseteq^k I = \bigcup_{n=0}^{\infty} I_n$ leži že v nekem vmesnem členu I_m.

  Po monotonosti je $FA \subseteq F I_m = I_{m+1} \subseteq I$

  Zato tudi $\bigcup_{A \subseteq^k I} FA \subseteq I$. ✓

- $F I = I$ **DN**

- I je najmanjša množica, zaprta za F

  $F X \subseteq X \Rightarrow I \subseteq X$

  z ind.

  $I_0 \subseteq X$

  $I_{n+1} = F I_n \subseteq F X \subseteq X$

  Zato je tudi: $I = \bigcup_{n=0}^{\infty} I_n \subseteq X$ ✓

$$P(0) \wedge \left(\forall n.\, P(n) \Rightarrow P(n^+)\right) \Rightarrow \forall n.\, P(n)$$

$$\forall n \in \mathbb{N}.\, P(n) \wedge$$
$$\forall e_1, e_2 \in \mathbb{E}.\, P(e_1) \wedge P(e_2) \Rightarrow P(e_1 + e_2) \wedge$$
$$\forall e_1, e_2 \in \mathbb{E}.\, P(e_1) \wedge P(e_2) \Rightarrow P(e_1 * e_2) \wedge$$
$$\forall e \in \mathbb{E}.\, P(e) \Rightarrow P(-e)$$
$$\Longrightarrow$$
$$\forall e \in \mathbb{E}.\, P(e)$$

$$A \Rightarrow C \wedge B \Rightarrow C \iff (A \vee B \Rightarrow C)$$

$$C^A \times C^B \cong C^{A+B}$$

$$FX = \mathbb{N} + X \times X + X \times X + X$$

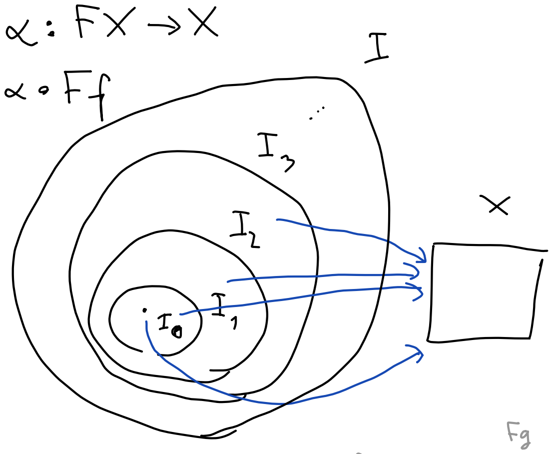
$f: X_1 \to Y_1$
$g: X_2 \to Y_2$
$f \times g: X_1 \times X_2 \to Y_1 \times Y_2$
$f + g: X_1 + X_2 \to Y_1 + Y_2$

$$\begin{tikzcd}
\mathbb{E} \arrow[d, "ev"'] & = & \mathbb{N} + \mathbb{E} \times \mathbb{E} + \mathbb{E} \times \mathbb{E} + \mathbb{E} = F\mathbb{E} \arrow[d, "id_{\mathbb{N}} + ev \times ev + ev \times ev + ev = Fev"] \\
\mathbb{Z} & & \mathbb{N} + \mathbb{Z} \times \mathbb{Z} + \mathbb{Z} \times \mathbb{Z} + \mathbb{Z} = F\mathbb{Z} \arrow[ll, "\alpha"]
\end{tikzcd}$$

$$ev = \alpha \circ Fev$$

Trditev

(monoton, zvezen) funktor F, I najm. množ. zaprta za F, $\alpha: FX \to X$
Potem obstaja enoličen $f: I \to X$, da velja $f = \alpha \circ Ff$



Dokaz

$f_0 : I_0 \to X = !_X$ ali $\emptyset_X : \emptyset \to X$
$f_{n+1} : I_{n+1} = FI_n \to X$
$f_{n+1} = \alpha \circ Ff_n$
$f_{n+1}|_{I_n} = f_n$

Dokažemo z indukcijo
- $n = 0$ ✓
- $n \rightsquigarrow n+1$

$$f_{n+2}|_{I_{n+1}} \overset{def}{=} (\alpha \circ Ff_{n+1})|_{FI_n} \overset{\text{očitno}}{=} \alpha \circ (Ff_{n+1}|_{FI_n}) = \alpha \circ F(f_{n+1}|_{I_n}) \overset{IP}{=} \alpha \circ Ff_n \overset{def}{=} f_{n+1}$$

$g: X \to Y$ $\quad A \subseteq X$
$Fg|_{FA} = F(g|_A)$

$$\begin{tikzcd}
X \arrow[r, "g"] & Y \\
A \arrow[u, phantom, "\subseteq" rotate=90] \arrow[ur, "g|_A"'] &
\end{tikzcd}
\qquad
\begin{tikzcd}
FX \arrow[r, "Fg"] & FY \\
FA \arrow[u, phantom, "\subseteq" rotate=90] \arrow[ur, "F(g|_A)"'] &
\end{tikzcd}$$

Ker se f_i ujemajo, lahko definiramo $f = \bigcup_n f_n$.