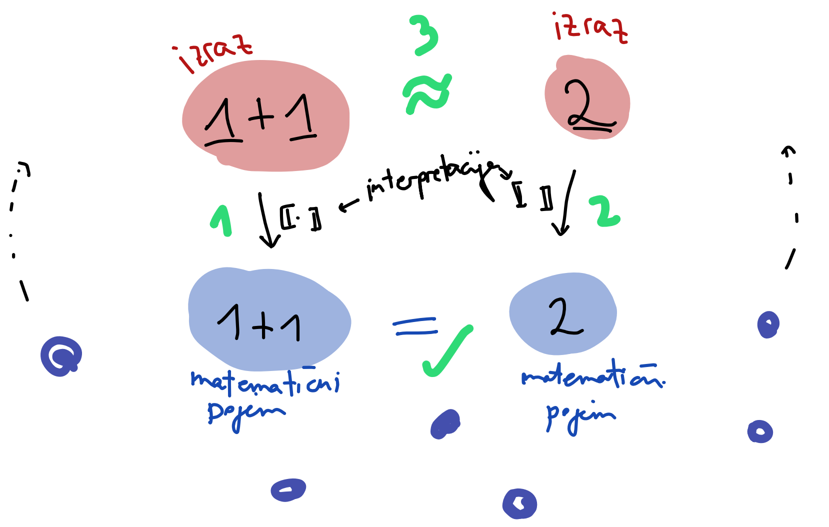

$$M, N ::= x \mid \lambda x.M \mid M N \mid \underline{n} \mid M+N \mid M*N \mid M<N$$
izrazi

$$\mid \text{true} \mid \text{false} \mid \text{if } M \text{ then } N_1 \text{ else } N_2$$

$$\mid \text{rec } f\, x.M$$

$$A, B ::= \text{int} \mid \text{bool} \mid A \to B$$
tipi

$$\boxed{\Gamma \vdash M : A}$$

tipi

kontekst – seznam, ki vsaki izmed spr. $x_1, \dots, x_n$ priredi natanko en tip.
$x_1 : A_1, x_2 : A_2, \dots, x_n : A_n$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x : A} \qquad \frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \to B} \qquad \frac{\Gamma \vdash M : A \to B \quad \Gamma \vdash N : A}{\Gamma \vdash M N : B}$$

$$\Gamma \vdash \underline{n} : \text{int} \qquad \frac{\Gamma \vdash M : \text{int} \quad \Gamma \vdash N : \text{int}}{\Gamma \vdash M + N : \text{int}} \qquad \frac{\Gamma \vdash M : \text{int} \quad \Gamma \vdash N : \text{int}}{\Gamma \vdash M * N : \text{int}} \qquad \frac{\Gamma \vdash M : \text{int} \quad \Gamma \vdash N : \text{int}}{\Gamma \vdash M < N : \text{bool}}$$

$$\frac{}{\Gamma \vdash \text{true} : \text{bool}} \qquad \frac{}{\Gamma \vdash \text{false} : \text{bool}} \qquad \frac{\Gamma \vdash M : \text{bool} \quad \Gamma \vdash N_1 : A \quad \Gamma \vdash N_2 : A}{\Gamma \vdash \text{if } M \text{ then } N_1 \text{ else } N_2 : A}$$

$$\frac{\Gamma, f : A \to B, x : A \vdash M : B}{\Gamma \vdash \text{rec } f\, x.M : A \to B}$$

Za vsak tip A definirajmo množico $[\![A]\!]$

$$[\![\text{int}]\!] = \mathbb{Z} \qquad [\![\text{bool}]\!] = \mathbb{B} = \{ff, tt\} \qquad [\![A \to B]\!] = [\![B]\!]^{[\![A]\!]}$$

Izrazom brez tipov ne bomo pripisovali pomena

Izrazom s tipi priredimo preslikave

$$y : \text{int}, x : \text{int} \vdash x > 2 + y : \text{bool}$$

$$[\![\underbrace{\Gamma \vdash M : A}_{\text{drevo izpeljave}}]\!] : [\![\Gamma]\!] \to [\![A]\!]$$

$$[\![x_1 : A_1, x_2 : A_2, \dots, x_n : A_n]\!] = [\![A_1]\!] \times [\![A_2]\!] \times \dots \times [\![A_n]\!]$$

$$[\![x_1 : A_1, \dots, x_n : A_n \vdash x_i : A_i]\!](a_1, \dots, a_n) = a_i$$

$$[\![\Gamma \vdash \lambda x.M : A \to B]\!](\eta) = a \in [\![A]\!] \mapsto [\![\Gamma, x : A \vdash M : B]\!](\eta, a)$$

$$[\![\Gamma \vdash M N : B]\!](\eta) = \Big(\underset{\in [\![B]\!]^{[\![A]\!]}}{[\![M]\!](\eta)}\Big)\Big(\underset{\in [\![A]\!]}{[\![N]\!](\eta)}\Big)$$

$$[\![\Gamma \vdash \underline{m} : \text{int}]\!](\eta) = m$$

$$[\![\Gamma \vdash M + N : \text{int}]\!](\eta) = [\![\Gamma \vdash M : \text{int}]\!](\eta) + [\![\Gamma \vdash N : \text{int}]\!](\eta)$$

$$[\![\Gamma \vdash M * N : \text{int}]\!](\eta) = [\![M]\!](\eta) \cdot [\![N]\!](\eta)$$

$$\vdots$$

$$[\![\Gamma \vdash \text{if } M \text{ then } N_1 \text{ else } N_2]\!](\eta) = \begin{cases} [\![N_1]\!](\eta) & [\![M]\!](\eta) = tt \\ [\![N_2]\!](\eta) & [\![M]\!](\eta) = ff \end{cases}$$

<u>Trditev</u> Če velja $\emptyset \vdash M : A$ in $M \rightsquigarrow M'$, tedaj velja $[\![M]\!](\cdot) = [\![M']\!](\cdot)$.

Dokaz Doma.

$$(\mathrm{rec}\, f x.\, M)\, V \rightsquigarrow M[V/x, (\mathrm{rec}\, f x.\, M)/f]$$

$$[\![\mathrm{rec}\, f x.\, M]\!]\,(\cancel{[\![V]\!]}) = [\![M]\!]\,(\cancel{[\![V]\!]}, [\![\mathrm{rec}\, f x.\, M]\!])$$