Lambda račun

$x \mapsto M$
fun $x \to M$
$\lambda x. M$
$\backslash x \to M$
$\backslash x. M$

izraz $M, N ::= x \mid \underbrace{\lambda x. M}_{\text{abstrakcija}} \mid \underbrace{M\, N}_{\text{aplikacija}}$

$e ::= \underline{m} \mid e_1 + e_2 \mid \dots$

Primeri

identiteta $\lambda x. x$
fst $\lambda x. \lambda y. x$
dvakratUporabi: $\lambda f. \lambda x. f\,(f\,x)$
kompozitum $\lambda f. \lambda g. \lambda x. f\,(g\,x)$
Ω $(\lambda x. x\,x)\,(\lambda x. x\,x)$

$\boxed{M \rightsquigarrow M'}$

vrednosti $V ::= \lambda x. M$

$$\frac{M \rightsquigarrow M'}{M\,N \rightsquigarrow M'\,N} \qquad \frac{N \rightsquigarrow N'}{(\lambda x. M)\,N \rightsquigarrow (\lambda x. M)\,N'}$$

$$\frac{}{(\lambda x. M)\,V \rightsquigarrow \underbrace{M[V/x]}_{\text{substitucija}}}$$

$x[N/x] := N$
$y[N/x] := y \quad (x \neq y)$
$(\lambda y. M)[N/x] := \lambda y. M[N/x] \quad (x \neq y,\ y \notin fv(N))$
$(M_1\,M_2)[N/x] := (M_1[N/x])\,(M_2[N/x])$

$fv(x) = \{x\}$
$fv(\lambda x. M) = fv(M) \setminus \{x\}$
$fv(M\,N) = fv(M) \cup fv(N)$

$$\frac{M \leadsto M'}{MN \leadsto M'N} \qquad \frac{N \leadsto N'}{VN \leadsto VN'} \qquad \frac{}{(\lambda x.M)V \leadsto M[V/x]}$$

NEUČAKANO IZVAJANJE / EAGER EVALUATION / CALL-BY-VALUE

CbV

$$\frac{M \leadsto M'}{MN \leadsto M'N} \qquad \frac{}{(\lambda x.M)N \leadsto M[N/x]}$$

LENO IZVAJANJE / LAZY EVALUATION / CALL-BY-NAME

CbN

$$(\lambda x. x+x)(1+2) \overset{CBV}{\leadsto} (\lambda x. x+x)\,3 \leadsto 3+3 \leadsto 6$$

$$(\lambda x. x+x)(1+2) \overset{CBN}{\leadsto} (1+2)+(1+2) \leadsto 3+(1+2) \leadsto 3+3 \leadsto 6$$

$$\Omega = (\lambda x. xx)(\lambda x. xx) \leadsto (\lambda x. xx)(\lambda x. xx) \leadsto \dots$$

$$(\lambda x. id)\,\Omega \overset{CBV}{\leadsto} (\lambda x. id)\,\Omega \leadsto \dots$$

$$(\lambda x. id)\,\Omega \overset{CBN}{\leadsto} id$$

Churchevo kodiranje

$\underline{n} := \lambda f . \lambda x . \underbrace{f(f(\cdots(f\,x)\cdots))}_{n}$ $\qquad (\alpha \to \alpha) \to \alpha \to \alpha$

$\underline{0} := \lambda f . \lambda x . x$

$\underline{1} := \lambda f . \lambda x . f\,x$

plus $:= \lambda m . \lambda n . \lambda f . \lambda x . m\,f\,(n\,f\,x)$

krat $:= \lambda m . \lambda n . \lambda f . \lambda x . m\,(n\,f)\,x$

par $:= \lambda x . \lambda y . \lambda p . p\,x\,y$

fst $:= \lambda p . p\,(\lambda x . \lambda y . x)$

snd $:= \lambda p . p\,(\lambda x . \lambda y . y)$

pred

$x \quad f$

$(a, b) \to (b, f\,b)$

(x, x)

$\downarrow$

$(x, f\,x)$

$\downarrow$

$(f\,x, f(f\,x))$

$\downarrow$

$$X = \ldots\ldots X \ldots\ldots$$

$$\Phi(y) = \ldots\ldots y \ldots\ldots$$

$$X = \Phi(X)$$

$$fac = \lambda n.\ \text{if } n=0 \text{ then } 1 \text{ else } n * fac\,(n-1)$$

$$\Phi = \lambda g.\ \lambda n.\ \text{if } n=0 \text{ then } 1 \text{ else } n * g\,(n-1)$$

$$\overbrace{(\lambda x.f(xx))\,(\lambda x.f(xx))}^{M} \rightsquigarrow \underbrace{f\,(\lambda x.f(xx))\,(\lambda x.f(xx))}_{f\,M}$$

$$(\lambda x.\,xx)\,(\lambda x.\,xx) \rightsquigarrow (\lambda x.\,xx)\,(\lambda x.\,xx)$$

$$(\lambda f.\,\Psi(ff))\,(\lambda f.\,\Psi(ff)) \rightsquigarrow \Psi\big((\lambda f.\,\Psi(ff))\,(\lambda f.\,\Psi(ff))\big)$$

$$\big((\lambda f.\,\Psi(ff))\,(\lambda f.\,\Psi(ff))\big)\ 7$$

$$\rightsquigarrow \Psi\big((\lambda f.\,\Psi(ff))\,(\lambda f.\,\Psi(ff))\big)\ 7$$

$$\rightsquigarrow \ldots\ldots$$

$$(\lambda f.\,\lambda x.\,\Psi(ff)\,x)\,(\lambda f.\,\lambda x.\,\Psi(ff)\,x)$$

$$\rightsquigarrow \lambda x.\,\Psi\big((\lambda f.\,\lambda x.\,\Psi(ff)\,x)\,(\lambda f.\,\lambda x.\,\Psi(ff)\,x)\big)\,x$$