Računski učinki

Drobnozrnati neučakani λ-račun (fine-grained call-by-value)

vrednosti $V ::= x \mid \mathsf{true} \mid \mathsf{false} \mid \underline{n} \mid \lambda x.M$

izračuni $M ::= \mathsf{if}\ V\ \mathsf{then}\ M_1\ \mathsf{else}\ M_2 \mid V_1 + V_2 \mid V_1 * V_2 \mid V_1 < V_2 \mid \ldots$
$\quad V_1\ V_2 \mid \mathsf{let}\ x = M_1\ \mathsf{in}\ M_2 \mid \mathsf{return}\ V$

CBV: $(\underline{2}+\underline{4}) * (\underline{1}+\underline{6}) \rightsquigarrow \underline{6} * (\underline{1}+\underline{6})$
$\rightsquigarrow (\underline{2}+\underline{4}) * \underline{7}$

FGCBV:
$\mathsf{let}\ x = \underline{2}+\underline{4}\ \mathsf{in}$
$\mathsf{let}\ y = \underline{1}+\underline{6}\ \mathsf{in}$
$x + y$

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$\boxed{s, M \rightsquigarrow s', M'}$

$$\frac{}{s, \mathsf{if}\ \mathsf{true}\ \mathsf{then}\ M_1\ \mathsf{else}\ M_2 \rightsquigarrow s, M_1} \qquad \frac{}{\mathsf{if}\ \mathsf{false}\ \mathsf{then}\ M_1\ \mathsf{else}\ M_2 \rightsquigarrow M_2}$$

$$\frac{}{s, \underline{m_1} + \underline{m_2} \rightsquigarrow s, \mathsf{return}\ \underline{m_1 + m_2}}$$

podobno za ostale operacije

$$\frac{}{(\lambda x.M)\ V \rightsquigarrow M[V/x]} \qquad \frac{s, M_1 \rightsquigarrow s', M_1'}{s, \mathsf{let}\ x = M_1\ \mathsf{in}\ M_2 \rightsquigarrow s', \mathsf{let}\ x = M_1'\ \mathsf{in}\ M_2}$$

$$\frac{}{s, \mathsf{let}\ x = \mathsf{return}\ V\ \mathsf{in}\ M_2 \rightsquigarrow s, M_2[V/x]} \qquad \frac{}{s, \mathsf{lookup}\ \ell \rightsquigarrow s, \mathsf{return}\ s[\ell]}$$

$\boxed{\Gamma \vdash_v V : A}$ $\boxed{\Gamma \vdash_c M : A}$ $\Gamma = x_1 : A_1, \ldots, x_n : A_n$

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash_v x : A} \qquad \frac{}{\Gamma \vdash_v \mathsf{true} : \mathsf{bool}} \qquad \frac{}{\Gamma \vdash_v \mathsf{false} : \mathsf{bool}}$$

$$\frac{}{\Gamma \vdash_v \underline{n} : \mathsf{int}} \qquad \frac{\Gamma, x : A \vdash_c M : B}{\Gamma \vdash_v \lambda x. M : A \to B}$$

$$\frac{\Gamma \vdash_v V : \mathsf{bool} \quad \Gamma \vdash_c M_1 : A \quad \Gamma \vdash_c M_2 : A}{\Gamma \vdash_c \mathsf{if}\ V\ \mathsf{then}\ M_1\ \mathsf{else}\ M_2 : A} \qquad \frac{\Gamma \vdash_v V_1 : A \to B \quad \Gamma \vdash_v V_2 : A}{\Gamma \vdash_c V_1\, V_2 : B}$$

$$\frac{\Gamma \vdash_c M_1 : A \quad \Gamma, x : A \vdash_c M_2 : B}{\Gamma \vdash_c \mathsf{let}\ x = M_1\ \mathsf{in}\ M_2 : B} \qquad \frac{\Gamma \vdash_v V : A}{\Gamma \vdash_c \mathsf{return}\ V : A}$$

<u>Trditev</u> (Ohranitev):

Če velja $M \leadsto M'$ in $\Gamma \vdash_c M : A$, tedaj $\Gamma \vdash_c M' : A$

<u>Trditev</u> (napredek):

Če velja $\emptyset \vdash_c M : A$, tedaj:

- obstaja M', da velja $M \leadsto M'$, ali
- $M = \mathsf{return}\ V$ (za nek $\vdash_v V : A$)

Izjeme

Imejmo množico izjem $\mathbb{E} = \{E_1, \dots, E_n\}$

$M ::= \dots \mid \text{raise } E \mid \text{try } M \text{ with } E_1 \to M_1 \mid \dots \mid E_n \to M_n$

$$\frac{}{\Gamma \vdash_c \text{raise } E : A \,!\, \{E\}} \qquad \frac{\Gamma \vdash_c M : A \,!\, \mathcal{E} \qquad \left(\Gamma \vdash M_i : A \,!\, \mathcal{E}_i\right)_{i=1}^{n}}{\Gamma \vdash_c \text{try } M \text{ with } \{E_i \to M_i\}_i : A \,!\, \bigcup_i \mathcal{E}_i}$$

op. sem.

raise E obravnavamo kot dodatno končno obliko izračunov

$$\frac{}{\text{let } x = \text{raise } E \text{ in } M_2 \rightsquigarrow \text{raise } E}$$

V CBV bi morali dodati še

$\text{raise } E + M_2 \rightsquigarrow \text{raise } E$

$M_1 + \text{raise } E \rightsquigarrow \text{raise } E$

$\vdots$

$$\frac{}{\text{try } (\text{raise } E_j) \text{ with } \{E_i \to M_i\}_i \rightsquigarrow M_j}$$

$$\frac{}{\text{try } (\text{return } V) \text{ with } \{E_i \to M_i\}_i \rightsquigarrow \text{return } V}$$

$$\frac{M \rightsquigarrow M'}{\text{try } M \text{ with } \{E_i \to M_i\}_i \rightsquigarrow \text{try } M' \text{ with } \{E_i \to M_i\}_i}$$

Napredek:

Če velja $\vdash M : A \,!\, \mathcal{E}$, tedaj:

- $M \rightsquigarrow M'$
- $M = \text{return } V$
- $M = \text{raise } E$ za $E \in \mathcal{E}$

Nedeterminizem

$$M ::= \ldots \mid M_1 \oplus M_2 \mid \mathrm{amb}$$

$\mathrm{amb} := \mathrm{return\ true} \oplus \mathrm{return\ false}$

$M_1 \oplus M_2 := \mathrm{let}\ b = \mathrm{amb}\ \mathrm{in}\ \mathrm{if}\ b\ \mathrm{then}\ M_1\ \mathrm{else}\ M_2$

$$\frac{\Gamma \vdash_c M_1 : A \qquad \Gamma \vdash_c M_2 : A}{\Gamma \vdash_c M_1 \oplus M_2 : A} \qquad \frac{}{\Gamma \vdash_c \mathrm{amb} : \mathrm{bool}}$$

op. sem.

1. možnost

$$\frac{}{M_1 \oplus M_2 \rightsquigarrow M_1} \qquad \frac{}{M_1 \oplus M_2 \rightsquigarrow M_2}$$

$$\frac{}{\mathrm{amb} \rightsquigarrow \mathrm{return\ true}} \qquad \frac{}{\mathrm{amb} \rightsquigarrow \mathrm{return\ false}}$$

2. možnost

$\boxed{M \Downarrow \mathcal{V}}$ $\qquad \mathcal{V} = \{V_1, \ldots, V_n\}$

$$\frac{M_1 \Downarrow \mathcal{V}}{\mathrm{if\ true\ then}\ M_1\ \mathrm{else}\ M_2 \Downarrow \mathcal{V}}$$

$$\frac{M_2 \Downarrow \mathcal{V}}{\mathrm{if\ false\ then}\ M_1\ \mathrm{else}\ M_2 \Downarrow \mathcal{V}}$$

$$\frac{}{\underline{m_1} + \underline{m_2} \Downarrow \{\underline{m_1 + m_2}\}}$$

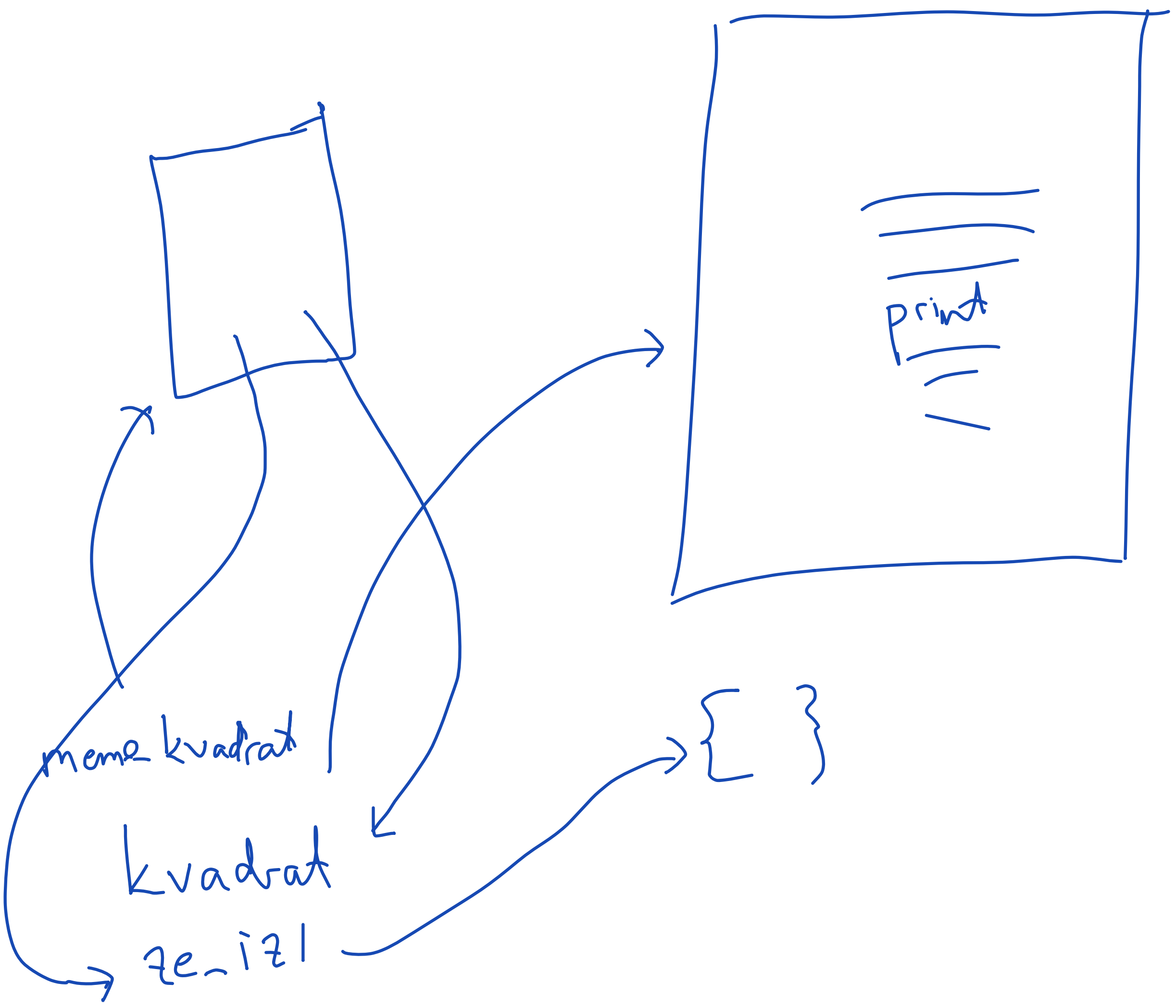
ostali podobno

$$\frac{M[V/x] \Downarrow \mathcal{V}}{(\lambda x. M)\, V \Downarrow \mathcal{V}}$$

$$\frac{M_1 \Downarrow \mathcal{V}_1 \qquad M_2 \Downarrow \mathcal{V}_2}{M_1 \oplus M_2 \Downarrow \mathcal{V}_1 \cup \mathcal{V}_2} \qquad \frac{}{\mathrm{amb} \Downarrow \{\mathrm{true}, \mathrm{false}\}}$$

$$\frac{}{\mathrm{return}\ V \Downarrow \{V\}}$$

$$\frac{M_1 \Downarrow \mathcal{V}_1 \qquad (M_2[W/x] \Downarrow \mathcal{V}_W)_{W \in \mathcal{V}_1}}{\mathrm{let}\ x = M_1\ \mathrm{in}\ M_2 \Downarrow \bigcup_{W \in \mathcal{V}_1} \mathcal{V}_W}$$



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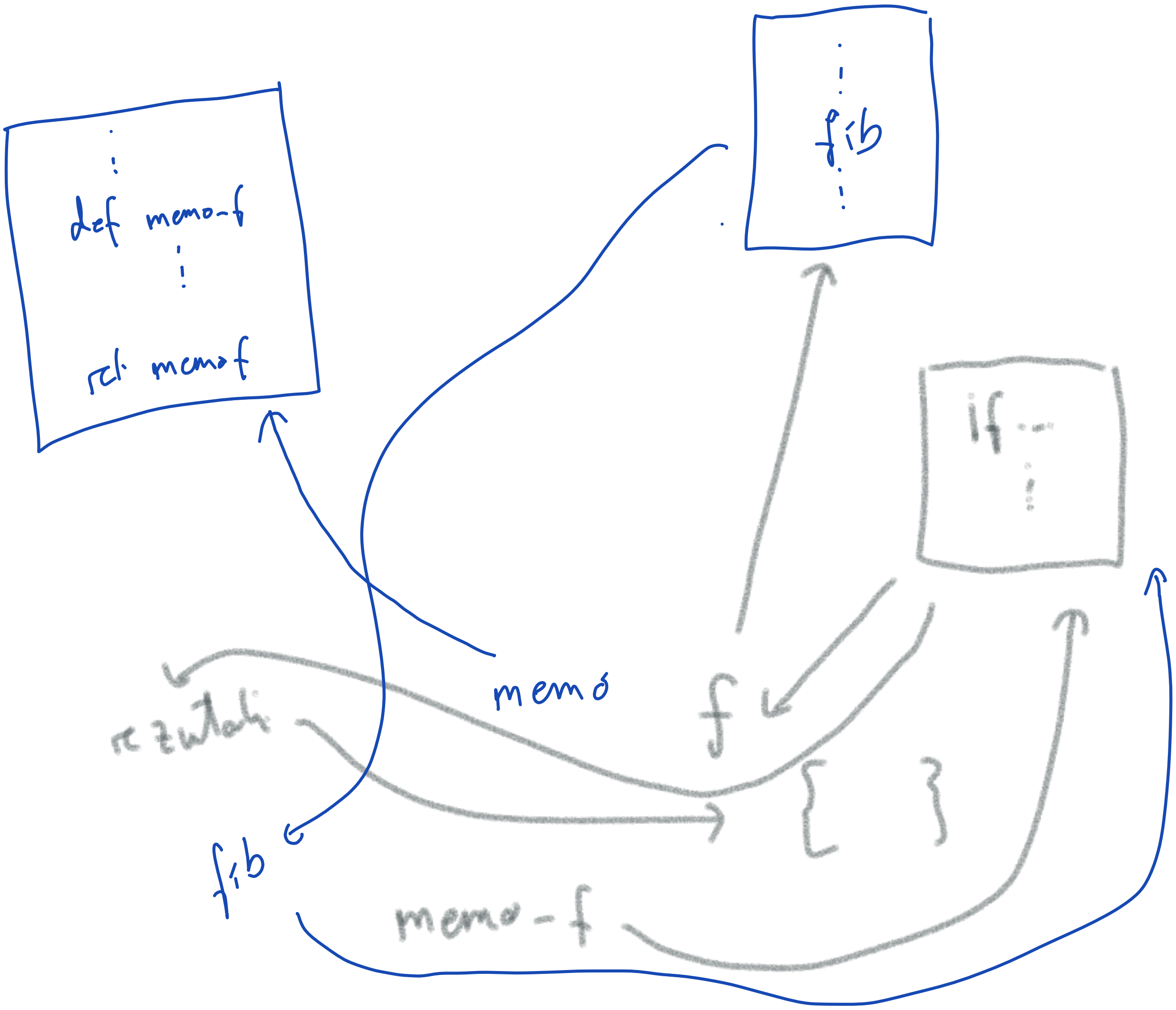
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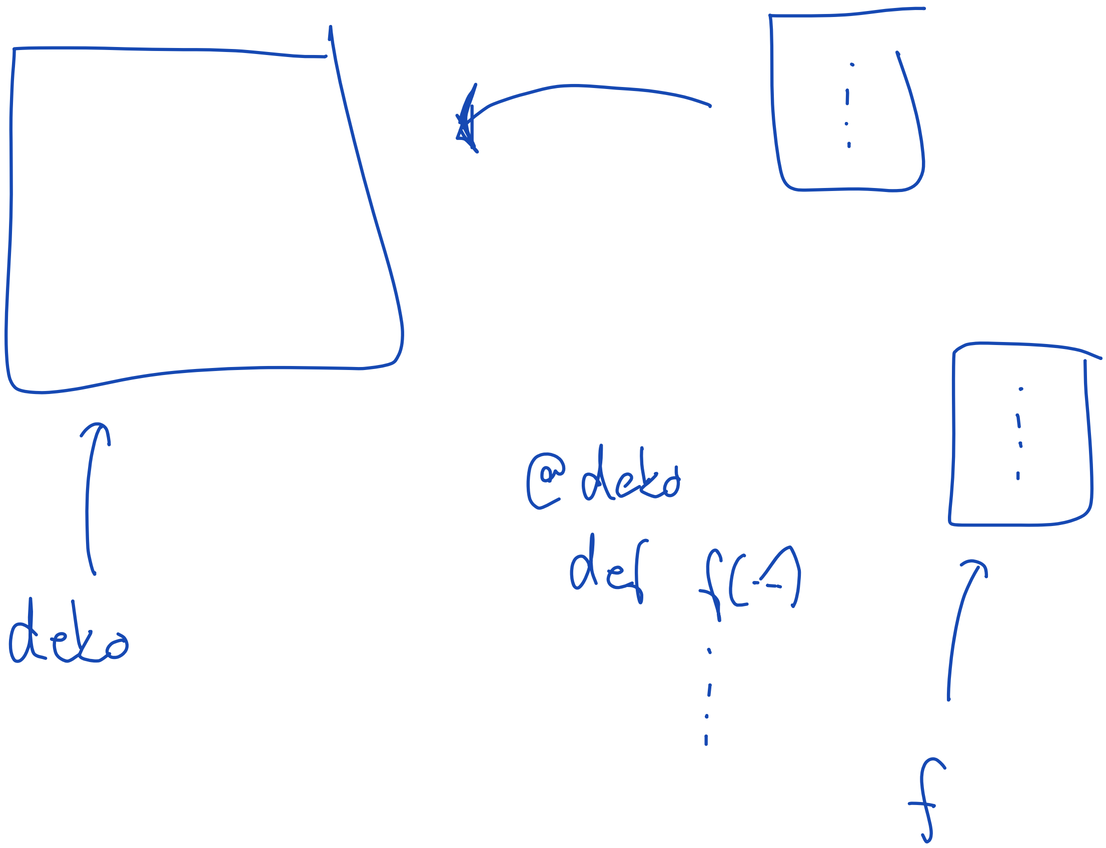
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