Posplošimo operacije, da ne bodo n-mestne, ampak da bodo parametrizirane z množico A ter bodo vračale izide iz množice B.

Prej je bilo $A = \{*\}$ in $B = \{1, \dots, n\}$.

Pišemo $op : A \to B$.

Recimo, da imamo družino operacij $\{op_i : A_i \to B_i\}_i$.

(enačbe se da podobno posplošiti)

Model teorije bo tedaj množica M ter družina preslikav

$$op_i^M : A_i \times M^{B_i} \to M$$

(ki zadošča enačbam)

Homomorfizem med modeloma M in N je preslikava $f : M \to N$

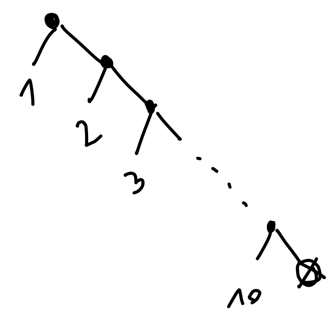
$$f\left(op_i^M(a, k)\right) = op_i^N\left(a, f \circ k\right)$$

Da dobimo homomorfizem iz prostega modela, potrebujemo:

- model na M (preslikavo $op_i^M : A_i \times M^{B_i} \to M$)
- preslikavo $X \to M$

To zapišemo kot

```
handler
| op1 (x, k) -> ...
| ...
| opn (x, k) -> ...
| ret x -> ...
```



$V ::= x \mid \underline{n} \mid \mathsf{true} \mid \mathsf{false} \mid \lambda x.M \mid \mathsf{handler}\ \{ \mathsf{op}_{1\,x}(k) \to M_1, \ldots, \mathsf{op}_{n\,x}(k) \to M_n, \mathsf{return}\ x \to M \}$

$M ::= V_1 + V_2 \mid V_1 * V_2 \mid \ldots$
$\quad \mathsf{if}\ V\ \mathsf{then}\ M_1\ \mathsf{else}\ M_2 \mid V_1\ V_2 \mid$
$\quad \mathsf{return}\ V \mid \mathsf{let}\ x = M\ \mathsf{in}\ N \mid$
$\quad \mathsf{op}_V(x.M) \mid \mathsf{handle}\ M\ \mathsf{with}\ V$

$\mathsf{perform}\ (\mathsf{op}\ V) = \mathsf{op}_V(x.\mathsf{ret}\ x)$

handle M with ...

$A ::= \mathsf{int} \mid \mathsf{bool} \mid A \to B \mid A \Rightarrow B$

$$\frac{\Gamma, x:A \vdash_c M : B}{\Gamma \vdash_v \lambda x.M : A \to B} \qquad \frac{\Gamma \vdash_v V_1 : A \to B \qquad \Gamma \vdash_v V_2 : A}{\Gamma \vdash_c V_1 V_2 : B}$$

$$\frac{\Gamma \vdash_v V : A}{\Gamma \vdash_c \mathsf{return}\ V : A} \qquad \frac{\Gamma \vdash_c M : A \qquad \Gamma, x:A \vdash_c N : B}{\Gamma \vdash_c \mathsf{let}\ x = M\ \mathsf{in}\ N : B}$$

v signaturi

$$\frac{\mathsf{op} : A \to B \qquad \Gamma \vdash_v V : A \qquad \Gamma, x:B \vdash M : C}{\Gamma \vdash \mathsf{op}_V(x.M) : C} \qquad \frac{\Gamma \vdash_c M : A \qquad \Gamma \vdash_v V : A \Rightarrow B}{\Gamma \vdash_c \mathsf{handle}\ M\ \mathsf{with}\ V : B}$$

$$\frac{\Gamma, x:A \vdash M : B \qquad \Gamma, x:A_i, k:B_i \to B \vdash M_i : B \qquad \mathsf{op}_i : A_i \to B_i}{\Gamma \vdash \{ (\mathsf{op}^i_x(k) \to M_i)_i, \mathsf{return}\ x \to M \} : A \Rightarrow B}$$

$\boxed{M \rightsquigarrow M'}$

$$\frac{M \rightsquigarrow M'}{\mathsf{let}\ x = M\ \mathsf{in}\ N \rightsquigarrow \mathsf{let}\ x = M'\ \mathsf{in}\ N}$$

$$\frac{}{\mathsf{let}\ x = \mathsf{return}\ V\ \mathsf{in}\ N \rightsquigarrow N[V/x]}$$

$$\frac{}{\mathsf{let}\ x = \mathsf{op}_V(y.M)\ \mathsf{in}\ N \rightsquigarrow \mathsf{op}_V(y.\ \mathsf{let}\ x = M\ \mathsf{in}\ N)}$$

$$\frac{M \rightsquigarrow M'}{\text{handle } M \text{ with } V \rightsquigarrow \text{handle } M' \text{ with } V}$$

$$\frac{}{\text{handle ret } V \text{ with } \{\text{ret } x \to M, \dots\} \rightsquigarrow M[V/x]}$$

$$\frac{}{\begin{array}{l}\text{handle } op^{i}_{V}(y.M) \text{ with } \{\cdots, op^{i}_{x}(k) \to M_i\} \\ \quad \rightsquigarrow M_i[V/x, (\text{fun } y \to \text{handle } M \text{ with } \{\cdots\})/k]\end{array}}$$