Izpeljava tipov

$M ::= \cdots$

$A ::= \cdots \mid \alpha$

$\mathcal{E} ::= \emptyset \mid A_1 = A_2, \mathcal{E}$

$\mathcal{E}$ bo množica enačb $A_1 = A_1', A_2 = A_2', \dots, A_n = A_n'$

$$\boxed{\Gamma \vdash_F M : A \mid \mathcal{E}}$$

V kontekstu Γ ima izraz M tip A, pri čemer veljajo enačbe $\mathcal{E}$ in smo uporabili parametre iz F.

Neformalen primer

$\lambda f. \lambda x. f(f x)$

$$\dfrac{\dfrac{}{f:\alpha, x:\beta \vdash f : \alpha \mid \emptyset} \qquad \dfrac{\dfrac{}{f:\alpha, x:\beta \vdash f : \alpha \mid \emptyset} \quad \dfrac{}{f:\alpha, x:\beta \vdash x : \beta \mid \emptyset}}{f:\alpha, x:\beta \vdash f x : \gamma \mid \alpha = \beta \to \gamma}}{\dfrac{f:\alpha, x:\beta \vdash f(f x) : \delta \mid \alpha = \beta \to \gamma, \alpha = \gamma \to \delta}{\dfrac{f:\alpha \vdash \lambda x. f(f x) : \beta \to \delta \mid \alpha = \beta \to \gamma, \alpha = \gamma \to \delta}{\emptyset \vdash \lambda f. \lambda x. f(f x) : \alpha \to (\beta \to \delta) \mid \alpha = \beta \to \gamma, \alpha = \gamma \to \delta}}}$$

$$\underline{(\delta \to \delta) \to (\delta \to \delta)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\alpha = \beta \to \gamma \quad \alpha = \gamma \to \delta}{\alpha \mapsto \beta \to \gamma \quad \beta \to \gamma = \gamma \to \delta}}{\alpha \mapsto \beta \to \gamma \quad \beta = \gamma \quad \gamma = \delta}}{\alpha \mapsto \gamma \to \gamma \quad \beta \mapsto \gamma \quad \gamma = \delta}}{\alpha \mapsto \delta \to \delta \quad \beta \mapsto \delta \quad \gamma \mapsto \delta}$$

$$\dfrac{\dfrac{x:\alpha \vdash x : \alpha \qquad x:\alpha \vdash x : \alpha}{x:\alpha \vdash x\,x : \beta}}{\emptyset \vdash \lambda x. x\,x : \alpha \to \beta} \quad \times$$

$$\dfrac{\alpha = \alpha \to \beta}{\times}$$

cikel

sveže $\alpha \notin F$

$$\dfrac{x : A \in \Gamma}{\Gamma \vdash_F x : A \mid \emptyset} \qquad \dfrac{\Gamma, x:\alpha \vdash_{F,\alpha} M : A \mid \mathcal{E}}{\Gamma \vdash_F \lambda x. M : \alpha \to A \mid \mathcal{E}} \qquad \dfrac{\Gamma \vdash M : A_1 \mid \mathcal{E}_1 \quad \Gamma \vdash N : A_2 \mid \mathcal{E}_2}{\Gamma \vdash M N : \alpha \mid \mathcal{E}_1, \mathcal{E}_2, A_1 = A_2 \to \alpha}$$

$$\dfrac{}{\Gamma \vdash \underline{n} : \mathrm{int} \mid \emptyset} \qquad \dfrac{\Gamma \vdash M_1 : A_1 \mid \mathcal{E}_1 \quad \Gamma \vdash M_2 : A_2 \mid \mathcal{E}_2}{\Gamma \vdash M_1 + M_2 : \mathrm{int} \mid \mathcal{E}_1, \mathcal{E}_2, A_1 = \mathrm{int}, A_2 = \mathrm{int}}$$

podobno za $*$ in $<$

$$\dfrac{}{\Gamma \vdash \mathrm{true} : \mathrm{bool} \mid \emptyset} \qquad \dfrac{}{\Gamma \vdash \mathrm{false} : \mathrm{bool} \mid \emptyset}$$

$$\dfrac{\Gamma \vdash M : A \mid \mathcal{E} \quad \Gamma \vdash N_1 : A_1 \mid \mathcal{E}_1 \quad \Gamma \vdash N_2 : A_2 \mid \mathcal{E}_2}{\Gamma \vdash \mathrm{if}\ M\ \mathrm{then}\ N_1\ \mathrm{else}\ N_2 : A_1 \mid \mathcal{E}, \mathcal{E}_1, \mathcal{E}_2, A_1 = A_2, A = \mathrm{bool}}$$

$\sigma ::= \emptyset \mid \sigma, \alpha \mapsto A$ substitucija σ je delna preslikava $\alpha_1 \mapsto A_1, \alpha_2 \mapsto A_2, \ldots, \alpha_n \mapsto A_m$

Def $\sigma(A)$ ... A, v katerem smo vse parametre zamenjali glede na σ

$$\sigma(\mathrm{int}) := \mathrm{int}$$
$$\sigma(\mathrm{bool}) := \mathrm{bool}$$
$$\sigma(A \to B) := \sigma(A) \to \sigma(B)$$
$$\sigma(\alpha) := \begin{cases} A & (\alpha \mapsto A) \in \sigma \\ \alpha & \text{sicer} \end{cases}$$

Def $\boxed{\sigma \models \mathcal{E}}$ ... substitucija σ reši enačbe $\mathcal{E}$

$$\frac{}{\sigma \models \emptyset} \qquad \frac{\sigma \models \mathcal{E} \quad \sigma(A_1) = \sigma(A_2)}{\sigma \models \mathcal{E}, A_1 = A_2}$$

enakost tipov

del sintakse

$\alpha \mapsto \mathrm{int} \models \alpha \to \mathrm{int} = \mathrm{int} \to \alpha$

$\alpha \mapsto \mathrm{int} \models \beta = \beta$

$\alpha \mapsto \mathrm{int} \not\models \beta = \alpha$

Def $\boxed{\mathcal{E} \rightsquigarrow \sigma}$ ... σ je najbolj splošna rešitev $\mathcal{E}$

$FV(\alpha) = \{\alpha\}$
$FV(\mathrm{int}) = \emptyset$
$FV(\mathrm{bool}) = \emptyset$
$FV(A \to B) = FV(A) \cup FV(B)$

prosti parametri tipa A

$$\frac{}{\emptyset \rightsquigarrow \emptyset} \qquad \frac{\mathcal{E} \rightsquigarrow \sigma}{\mathcal{E}, A = A \rightsquigarrow \sigma}$$

$$\frac{\mathcal{E}, A_1 = A_1', A_2 = A_2' \rightsquigarrow \sigma}{\mathcal{E}, A_1 \to A_2 = A_1' \to A_2' \rightsquigarrow \sigma} \qquad \frac{\mathcal{E}[A/\alpha] \rightsquigarrow \sigma \quad ! \alpha \notin FV(A)}{\mathcal{E}, \alpha = A \rightsquigarrow \sigma, \alpha \mapsto \sigma(A)}$$

in podobno za $\mathcal{E}, A = \alpha$

Trditev Če velja $\Gamma \vdash M : A \mid \mathcal{E}$ in $\sigma \models \mathcal{E}$, potem $\sigma(\Gamma) \vdash M : \sigma(A)$.

Trditev Če velja $\mathcal{E} \rightsquigarrow \sigma$, potem $\sigma \models \mathcal{E}$.

Dokaz V obeh primerih rutinska indukcija. D.N.

Trditev Če velja $\Gamma \vdash M : A$, tedaj $\Gamma \vdash M : A' \mid \mathcal{E}$ in obstaja $\sigma \models \mathcal{E}$, da velja $\sigma(A') = A$.

Trditev Če velja $\sigma \models \mathcal{E}$, tedaj $\mathcal{E} \rightsquigarrow \sigma'$ in obstaja σ'', da velja $\sigma = \sigma'' \circ \sigma'$.

Dokaz Bo objavljen v zapiskih.

$A ::= \ldots \mid \mathrm{unit} \mid \mathrm{empty} \mid A + B$

$M ::= \ldots \mid () \mid \mathrm{inl}\, M \mid \mathrm{inr}\, M \mid \mathrm{match}\, M \,\mathrm{with}\, \mathrm{inl}\, x \to N_1 \mid \mathrm{inr}\, x \to N_2 \mid \mathrm{match}\, M \,\mathrm{with}\, /$

$$\frac{}{\Gamma \vdash () : \mathrm{unit}} \qquad \frac{\Gamma \vdash M : A}{\Gamma \vdash \mathrm{inl}\, M : A + B} \qquad \frac{\Gamma \vdash M : A + B \quad \Gamma, x : A \vdash N_1 : C \quad \Gamma, x : B \vdash N_2 : C}{\Gamma \vdash \mathrm{match}\, M \,\mathrm{with}\, \mathrm{inl}\, x \to N_1 \mid \mathrm{inr}\, x \to N_2 : C}$$

$$\frac{\Gamma \vdash M : \mathrm{empty}}{\Gamma \vdash \mathrm{match}\, M \,\mathrm{with}\, / : C}$$

absurd M

$$\begin{array}{lll}
\sigma_1 & \sigma_2 & \sigma_1 \circ \sigma_2 \\
\alpha_3 \mapsto A_3 & \alpha_1 \mapsto A_1 & \alpha_1 \mapsto \sigma_1(A_1) \\
\alpha_4 \mapsto A_4 & \alpha_2 \mapsto A_2 & \alpha_2 \mapsto \sigma_1(A_2) \\
 & & \alpha_3 \mapsto A_3 \\
 & & \alpha_4 \mapsto A_4
\end{array}$$