# Tipi

## λ-račun s preprostimi tipi (simply-typed λ-calculus / STLC)

$M, N ::= x \mid \lambda x.M \mid M N \mid \underline{n} \mid M + N \mid M * N \mid M < N$

izrazi

$\mid \text{true} \mid \text{false} \mid \text{if } M \text{ then } N_1 \text{ else } N_2$

$\mid \text{rec } f\, x.M$

$$\dfrac{\dfrac{\dfrac{x{:}\text{int} \vdash x{:}\text{int} \qquad x{:}\text{int} \vdash \underline{2}{:}\text{int}}{x{:}\text{int} \vdash x < \underline{2} : \text{bool}}}{\emptyset \vdash (\lambda x. x < \underline{2}) : \text{int} \to \text{bool}} \qquad \dfrac{}{\emptyset \vdash \underline{3} : \text{int}}}{\emptyset \vdash (\lambda x. x < \underline{2})\, \underline{3} : \text{bool}}$$

$A, B ::= \text{int} \mid \text{bool} \mid A \to B$

tipi

$\boxed{\Gamma \vdash M : A}$

kontekst – seznam, ki vsakemu izmed spr. $x_1, \dots, x_n$ priredi natanko en tip.

$x_1{:}A_1, x_2{:}A_2, \dots, x_n{:}A_n$

$$\frac{x{:}A \in \Gamma}{\Gamma \vdash x : A} \qquad \frac{\Gamma, x{:}A \vdash M : B}{\Gamma \vdash \lambda x.M : A \to B} \qquad \frac{\Gamma \vdash M : A \to B \qquad \Gamma \vdash N : A}{\Gamma \vdash M N : B}$$

$$\frac{}{\Gamma \vdash \underline{n} : \text{int}} \qquad \frac{\Gamma \vdash M{:}\text{int} \qquad \Gamma \vdash N{:}\text{int}}{\Gamma \vdash M + N : \text{int}} \qquad \frac{\Gamma \vdash M{:}\text{int} \qquad \Gamma \vdash N{:}\text{int}}{\Gamma \vdash M * N : \text{int}} \qquad \frac{\Gamma \vdash M{:}\text{int} \qquad \Gamma \vdash N{:}\text{int}}{\Gamma \vdash M < N : \text{bool}}$$

$$\frac{}{\Gamma \vdash \text{true} : \text{bool}} \qquad \frac{}{\Gamma \vdash \text{false} : \text{bool}} \qquad \frac{\Gamma \vdash M{:}\text{bool} \qquad \Gamma \vdash N_1{:}A \qquad \Gamma \vdash N_2{:}A}{\Gamma \vdash \text{if } M \text{ then } N_1 \text{ else } N_2 : A}$$

$$\frac{\Gamma, f{:}A \to B, x{:}A \vdash M : B}{\Gamma \vdash \text{rec } f\, x.M : A \to B}$$

$V ::= \lambda x.M \mid \underline{n} \mid \text{true} \mid \text{false} \mid \text{rec } f\, x.M$

$\boxed{M \leadsto M'}$

$$\frac{M \leadsto M'}{M N \leadsto M' N} \qquad \frac{N \leadsto N'}{(\lambda x.M) N \leadsto (\lambda x.M) N'} \qquad \frac{}{(\lambda x.M)\, V \leadsto M[V/x]}$$

$$\frac{M \leadsto M'}{M + N \leadsto M' + N} \qquad \frac{N \leadsto N'}{\underline{m} + N \leadsto \underline{m} + N'} \qquad \frac{}{\underline{m} + \underline{n} \leadsto \underline{m + n}}$$

(podobno za $*$ in $<$)

$$\frac{M \leadsto M'}{\text{if } M \text{ then } N_1 \text{ else } N_2 \leadsto \text{if } M' \text{ then } N_1 \text{ else } N_2}$$

$$\frac{}{\text{if true then } N_1 \text{ else } N_2 \leadsto N_1} \qquad \frac{}{\text{if false then } N_1 \text{ else } N_2 \leadsto N_2}$$

$$\frac{N \leadsto N'}{(\text{rec } f\, x.M)\, N \leadsto (\text{rec } f\, x.M)\, N'} \qquad \frac{}{(\text{rec } f\, x.M)\, V \leadsto M[V/x, (\text{rec } f\, x.M)/f]}$$

ali

$$\frac{N \leadsto N'}{V N \leadsto V N'}$$

## Izrek o varnosti:

<u>Trditev</u> (napredek): Če velja $\emptyset \vdash M : A$, tedaj:
- obstaja $M'$, da velja $M \leadsto M'$
- je $M$ vrednost.

<u>Trditev</u> (ohranitev): Če velja $\emptyset \vdash M : A$ in $M \leadsto M'$, tedaj velja $\emptyset \vdash M' : A$.

<u>Dokaz (napredek):</u>
Z indukcijo na $\emptyset \vdash M : A$.