Oblig 3

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« oblig 3 »

oppgave 4.10

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \Rightarrow$$

$$v_0 = R_1^0 \cdot v_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

oppgave 4-11

Antar avstand fra O_1 til $O_c = a_c$

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & C_1 a_1 \\ S_1 & C_1 & 0 & S_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2 a_c \\ S_2 & C_2 & 0 & S_2 a_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} C_1C_2 - S_1S_2 & -C_1S_2 - S_1C_2 & 0 & C_1 a_1 + C_1C_2 a_c - S_1S_2 a_c \\ C_1S_2 + S_1C_2 & C_1C_2 - S_1S_2 & 0 & S_1 a_1 + S_1C_2 a_c + C_1S_2 a_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A_1 A_2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & a_1\cos\theta_1 + a_c\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & a_1\sin\theta_1 + a_c\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A_1 A_2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1 C_1 + a_c C_{12} \\ S_{12} & C_{12} & 0 & a_1 S_1 + a_c S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Komponentene for Jakobian matrise

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} C_1 a_1 \\ S_1 a_1 \\ 0 \end{bmatrix}$$

$$O_c = \begin{bmatrix} a_1 C_1 + a_c C_{12} \\ a_1 S_1 + a_c S_{12} \\ 0 \end{bmatrix}, \quad O_c - O_0 = \begin{bmatrix} a_1 C_1 + a_c C_{12} \\ a_1 S_1 + a_c S_{12} \\ 0 \end{bmatrix}$$

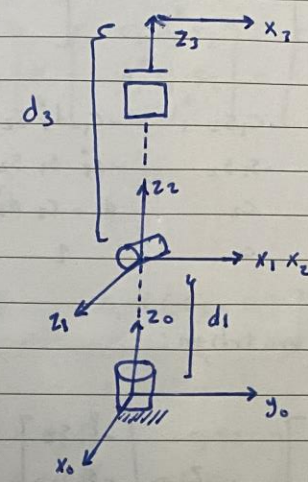
$$z_0 \times (o_c - o_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 c_1 + a_c c_{12} \\ a_1 s_1 + a_c s_{12} \\ 0 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_c s_{12} \\ a_1 c_1 + a_c c_{12} \\ 0 \end{bmatrix}$$

$$o_c - o_1 = \begin{bmatrix} a_c c_{12} \\ a_c s_{12} \\ 0 \end{bmatrix}, \quad z_1 \times (o_c - o_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_c c_{12} \\ a_c s_{12} \\ 0 \end{bmatrix} = \begin{bmatrix} -a_c s_{12} \\ a_c c_{12} \\ 0 \end{bmatrix}$$

Jakobian matrise blir:

$$J = \begin{bmatrix} -a_1 s_1 - a_c s_{12} & -a_c s_{12} & 0 \\ a_1 c_1 + a_c c_{12} & a_c c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

oppgave 4.13



Jakobian matrise med 3ledd fra eksempel 4.10 blir den øverste delen

DH: tabell:

ledd	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1^*
2	0	-90	0	θ_2^*
3	0	0	d_3^*	0

Vi multipleserer først A matrise

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_1 \cdot A_2 = \begin{bmatrix} c_1 c_2 & -s_1 & -c_1 s_2 & 0 \\ c_2 s_1 & c_1 & -s_1 s_2 & 0 \\ s_2 & 0 & c_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 \cdot A_2 \cdot A_3 = \begin{bmatrix} c_1 c_2 & -s_1 & -c_1 s_2 & -c_1 d_3^* s_2 \\ c_2 s_1 & c_1 & -s_1 s_2 & -d_3^* s_1 s_2 \\ s_2 & 0 & c_2 & d_1 + c_2 d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Komponentene for Jacobian matrisen:

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}, \quad z_2 = \begin{bmatrix} -c_1 s_2 \\ -s_2 s_1 \\ c_2 \end{bmatrix}, \quad o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$o_1 = o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad o_3 = \begin{bmatrix} -c_1 d_3^* s_2 \\ -d_3^* s_1 s_2 \\ d_1 + c_2 d_3^* \end{bmatrix}$$

$$z_0 \times (o_3 - o_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -c_1 d_3^* s_2 \\ -d_3^* s_1 s_2 \\ c_2 d_3^* \end{bmatrix} = \begin{bmatrix} d_3^* s_1 s_2 \\ -c_1 d_3^* s_2 \\ 0 \end{bmatrix}$$

$$z_1 \times (o_3 - o_1) = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} -c_1 d_3^* s_2 \\ -d_3^* s_1 s_2 \\ c_2 d_3^* \end{bmatrix} = \begin{bmatrix} -c_1 c_2 d_3^* \\ -s_1 c_2 d_3^* \\ -s_2 d_3^* (s_1^2 + c_1^2) \end{bmatrix} = \begin{bmatrix} -c_1 c_2 d_3^* \\ -s_1 c_2 d_3^* \\ -s_2 d_3^* \end{bmatrix}$$

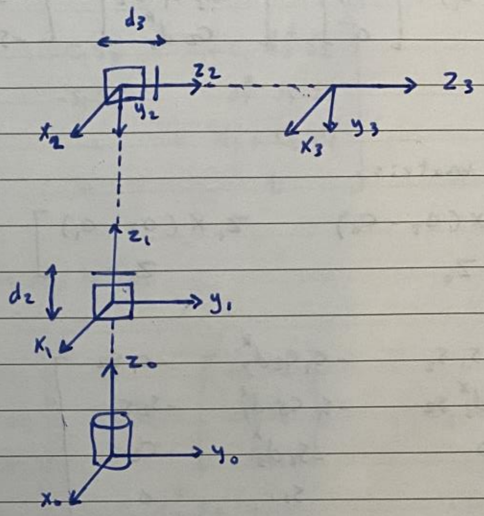
Jakobian matrisi

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) \\ z_0 & z_1 \end{bmatrix}$$

$$= \begin{bmatrix} d_3^* s_1 s_2 & -c_1 c_2 d_3^* & -c_1 s_2 \\ -c_1 d_3^* s_2 & -s_1 c_2 d_3^* & -s_2 s_2 \\ 0 & -s_2 d_3^* & c_2 \\ 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow J_{11} = \begin{bmatrix} d_3^* s_1 s_2 & -c_1 c_2 d_3^* & -c_1 s_2 \\ -c_1 d_3^* s_2 & -s_1 c_2 d_3^* & -s_2 s_2 \\ 0 & -s_2 d_3^* & c_2 \end{bmatrix}$$

oppgave 4-15:



Fra boka får vi A matriser:

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_1 A_2 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Komponentene for Jakobian matrise:

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_2 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, \quad o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad o_3 = \begin{bmatrix} -s_1 d_3^* \\ c_1 d_3^* \\ d_1 + d_2^* \end{bmatrix}$$

$$z_0 \times (o_3 - o_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -s_1 d_3^* \\ c_1 d_3^* \\ d_1 + d_2^* \end{bmatrix} = \begin{bmatrix} -c_1 d_3^* \\ -s_1 d_3^* \\ 0 \end{bmatrix}$$

Jakobian matrise:

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 & z_2 \\ z_0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -c_1 d_3^* & 0 & -s_1 \\ -s_1 d_3^* & 0 & c_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Finner determinanten til matrisen og sjekker når den er 0 for å kunne finne singulariteter.

$$J_{11} = \begin{bmatrix} -c_1 d_3^* & 0 & -s_1 \\ -s_1 d_3^* & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \det J_{11} = d_3^* s_1^2 + c_1^2 d_3^*$$

$$= d_3^* (s_1^2 + c_1^2)$$

$$= d_3^* \Rightarrow d_3^* = 0$$

roboten har bare den singulariteten