Almuthanna Aljajan

Oblig 1

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oppgave 2-12:

$$R_{z,\alpha} \cdot R_{x,\varphi} \cdot R_{z,\theta} \cdot R_{x,\psi}$$

oppgave 2-13:

$$R_{z,\alpha} \cdot R_{z,\theta} \cdot R_{x,\varphi} \cdot R_{x,\psi}$$

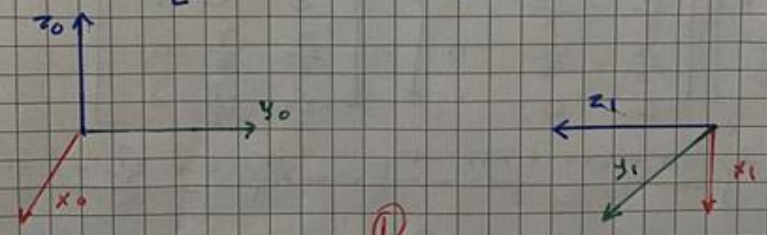
oppgave 2-14:

$$R_{y,\frac{\pi}{2}} \cdot R_{x,\frac{\pi}{2}} = \begin{bmatrix} \cos 90 & 0 & \sin 90 \\ 0 & 1 & 0 \\ -\sin 90 & 0 & \cos 90 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 \\ 0 & \sin 90 & \cos 90 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cdot1 + 0\cdot0 + 1\cdot0 & 0\cdot0 + 0\cdot0 + 1\cdot1 & 0\cdot0 + 0\cdot(-1) + 1\cdot0 \\ 0\cdot1 + 1\cdot0 + 0\cdot0 & 0\cdot0 + 1\cdot0 + 0\cdot1 & 0\cdot0 + 1\cdot(-1) + 0\cdot0 \\ -1\cdot1 + 0\cdot0 + 0\cdot0 & -1\cdot0 + 0\cdot0 + 0\cdot1 & -1\cdot0 + 0\cdot0 + 0\cdot0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$



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oppgave 2-15:

$$R_3^2 = (R_2^1)^T \cdot R_3^1 = R_1^2 \cdot R_3^1$$

$$R_1^2 = (R_2^1)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$R_3^2 = R_1^2 \cdot R_3^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1\cdot 0+0\cdot 0+0\cdot -1 & 1\cdot 0+0\cdot 1+0\cdot 0 & 1\cdot(-1)+0\cdot 0+0\cdot 0 \\ 0\cdot 0+\frac{1}{2}\cdot 0+\frac{\sqrt{3}}{2}\cdot 1 & 0\cdot 1+\frac{1}{2}\cdot 1+\frac{\sqrt{3}}{2}\cdot 0 & 0+(-1)+\frac{1}{2}\cdot 0+\frac{\sqrt{3}}{2}\cdot 0 \\ 0\cdot 0+\frac{\sqrt{3}}{2}\cdot 0+\frac{1}{2}\cdot 1 & 0\cdot 0+(-\frac{\sqrt{3}}{2})\cdot 1+\frac{1}{2}\cdot 0 & 0\cdot(-1)+(-\frac{\sqrt{3}}{2})\cdot 0+\frac{1}{2}\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}$$

oppgave 2-23

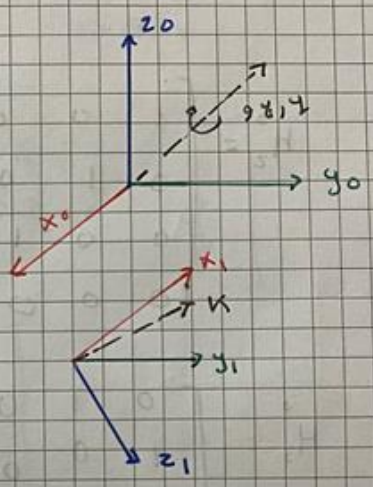
$$R = R_{y,\frac{\pi}{2}} \cdot R_{z,\frac{\pi}{3}} = \begin{bmatrix} \cos\frac{\pi}{2} & 0 & \sin\frac{\pi}{2} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} & 0 \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

$$\theta = \cos^{-1}\left(\frac{1 - R_{11} + R_{22} + R_{33}}{2}\right) = \cos^{-1}\left(\frac{0 + \frac{\sqrt{2}}{2} + 0 - 1}{2}\right) = 98.42°$$

$$K = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} = \frac{1}{2\sin\theta}\begin{bmatrix} \frac{\sqrt{2}}{2} - 0 \\ 1 - (-\frac{\sqrt{2}}{2}) \\ \frac{\sqrt{2}}{2} - 0 \end{bmatrix}$$

$$\begin{bmatrix} 0,3572 \\ 0,8628 \\ 0,3572 \end{bmatrix}$$



oppgave 2-2y

ZYZ

$$R_{ZYZ} = R_{z,\varphi} \cdot R_{y,\theta} \; R_{z,\psi} =$$

$$\begin{bmatrix} \cos\varphi\cos\theta\cos\psi - \sin\varphi\sin\psi & -\cos\varphi\cos\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\sin\theta \\ \sin\varphi\cos\theta\cos\psi + \cos\varphi\sin\psi & -\sin\varphi\cos\theta\sin\psi + \cos\varphi\cos\psi & \sin\varphi\sin\theta \\ -\sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} 0 - \sin\frac{\pi}{2}\sin\frac{\pi}{2} & 0 - \sin\frac{\pi}{2}\cos\frac{\pi}{2} & 0 \\ \sin\frac{\pi}{2}\cos 0\cos\frac{\pi}{4} + 0 & -\sin\frac{\pi}{2}\cos 0\sin\frac{\pi}{4} + 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

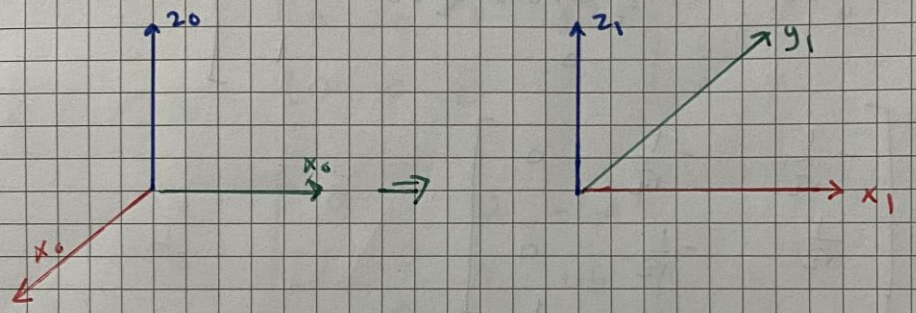
$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Retning til x i forhold til baseplan er $-\frac{\sqrt{2}}{2}$ i retning x_0 og $\frac{\sqrt{2}}{2}$ i retning y_0.

oppgave · 2-36

$$\text{Trans}_{y,1}\ \text{Trans}_{x,3}\ \text{Rot}_{z,\frac{\pi}{2}} = \begin{bmatrix} c_y & -s_y & 0 & 3 \\ s_y & c_y & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Oppgave 2-38

$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} 1 & 0 & 0 & -0{,}5 \\ 0 & 1 & 0 & 1{,}5 \\ 0 & 0 & 1 & 1{,}1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{bmatrix} 0 & 1 & 0 & -0{,}5 \\ 1 & 0 & 0 & 1{,}5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1{,}9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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