Practical - 1

Aim : To analyse the data and draw conclusions of Completely randomized data (CRD)

Problem : The following table shows the lives (in hours) of 4 batches of electric lamps.

Batches	Life of Bulb (in hrs)						
1	1600	1610	1650	1680	1700	1720	1800
2	1580	1640	1700	1750	1640		
3	1480	1550	1620	1640	1660	1740	1820 1600
4	1510	1520	1570	1600	1680	1530	

Perform analysis and draw appropriate conclusion about the the equality of treatment means.

Theory :-

Model for C.R.D :

$$Y_{ij} = \mu + \alpha_i + e_{ij} \quad , \quad i = 1, 2, 3, \ldots, v \quad j = 1, 2, 3, \ldots, r_i$$

where

v - No. of treatments

r_i - No. of times i^{th} treatment is replicated

y_{ij} = j^{th} observation corresponding to i^{th} treatment

μ - General mean

α_i - Fixed effect due to i^{th} treatment

e_{ij} - Random error corresponding to j^{th} replication of i^{th} treatment

and $e_{ij} \sim N(0, \sigma_e^2)$

Also $\sum_i \sum_j \alpha_i = 0$

Sum of squares due to treatment, $SST = \sum_{i}^{v} \frac{T_{i.}^2}{r_i} - \frac{T_{..}^2}{n}$

Sum of squares due to error, $SSE = \sum_{j} \sum_{i} y_{ij}^2 - \sum_{i} \frac{T_{i.}^2}{r_i}$

Total sum of squares, $TSS = \sum_{j} \sum_{i} y_{ij}^2 - \frac{T_{..}^2}{n}$

or $TSS = SST + SSE$

where $T_{i.} = \sum_{j=1}^{r_i} y_{ij}$ and $T_{..} = \sum_{i} \sum_{j} y_{ij}$

Hypothesis :

Null hypothesis (H_0) : $\alpha_1 = \alpha_2 = \ldots = \alpha_v = 0$

Alternate hypothesis (H_1) : Atleast one of the α_i's not 0.

ANOVA Table

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares	variance Ratio
Treatment	$v-1$	SST	$MST = \frac{SST}{v-1}$	$\frac{MST}{MSE} \sim F_{v-1, n-v}$
Error	$n-v$	SSE	$MSE = \frac{SSE}{n-v}$	
Total	$n-1$	TSS		

Decision rule :

Reject H_0 at α% l.o.s if $F_{cal} > F_{tab}$ at $(v-1, n-v)$ d.f otherwise accept H_0.

If null hypothesis is rejected we further compare the treatments pairwise using t-test.

Calculations :

$n = 26 \quad v = 4$

Remaining calculations in excel table we get,

$F_{cal} = 2.1756$

$F_{tab} = 2.9752$

Result :

- Since calculated value of $F_{(3,22)}$ is less than tabulated value, we accept H_0, and conclude that the treatments do not differ significantly.

PRACTICAL-2

Aim: To analyse the data and draw conclusions of completely randomised data (C.R.D).

Problem: For testing the variety effect in a completely randomised experiment, the data (yield of barley in grams) are as shown in following table:

320	340	398	360	350	372	455	417	420	358
v_1	v_4	v_5	v_4	v_3	v_2	v_2	v_3	v_1	v_5
400	353	334	331	359	370	340	375	320	325
v_3	v_1	v_3	v_1	v_4	v_4	v_5	v_2	v_5	v_3
430	358	378	395	321	383	275	375	308	400
v_5	v_1	v_3	v_4	v_2	v_2	v_3	v_4	v_2	v_1

Perform the analysis of variance.

Theory:

Model for CRD: $y_{ij} = \mu + \tau_i + e_{ij}$ $\quad i = 1, 2, \ldots, t$; $j = 1, 2, \ldots, r_i$

where y_{ij} = observation for j^{th} replications of i^{th} treatment

μ = General mean

τ_i = effect due to i^{th} treatment

e_{ij} - Random error associated to i^{th} replication of j^{th} treatment

and $e_{ij} \sim N(0, \sigma_e^2)$ Also $\sum_i \sum_j \tau_i = 0$

Hypothesis:

H_0: $\tau_1 = \tau_2 = \ldots = \tau_v = 0$

v/s

H_1: atleast one $\tau_i \neq 0$

Row Sum of squares, $R.S.S = \sum_j \sum_i y_{ij}^2$

$C.F = \frac{G^2}{n}$ where $G = \sum_i \sum_j y_{ij}$ and $n = \sum_{i=1}^{t} r_i$

Total sum of squares, $T.S.S = RSS - C.F$

Sum of Square due to treatment, $SST = \sum_i \frac{T_i^2}{r_i} - CF$

then

Sum of square due to error, $SSE = TSS - SST$

ANOVA Table

Sources of variation	degrees of freedom	Sum of Squares	Mean sum of squares	variance Ratio
Treatments	$v-1$	SST	$MST = \frac{SST}{v-1}$	$\frac{MST}{MSE} \sim F_{v-1, n-v}$
Error	$n-v$	SSE	$MSE = \frac{SSE}{n-v}$	
Total	$n-1$	TSS		

Decision rule:

Reject H_0 at $\alpha\%$ l.o.s if $F_{cal} > F_{tab}$ at $(v-1, n-v)$ d.f otherwise accept H_0

Calculation:

$n = 30$ $\quad v = 5$

Remaining calculations in excel table.

we get,

$F_{cal} =$

tabulated $F_{(4,25)} =$

Result :

- Since calculated value of $F_{(4, 25)}$ is less than tabulated value, we accept H_0 and conclude that the treatments do not differ significantly.

PRACTICAL-3

Aim: To analyse the data and draw conclusions of Completely randomised data (C.R.D).

Problem: A set of data involving four "tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects expect the feeding treatments and each feeding treatment is given to 5 chicks. Analyse the data.

Feed	Gain in weight					Total T_i
A	55	49	42	21	52	219
B	61	112	30	89	63	355
C	42	97	81	95	92	407
D	169	137	169	85	154	714

$G = 1695$

Theory and Formula:

Model for CRD is given as: $y_{ij} = \mu + \tau_i + e_{ij}$

where y_{ij} = observation for j^{th} replication of i^{th} treatment

μ = General mean effect

τ_i = effect due to i^{th} treatment

e_{ij} = Random error associated to i^{th} replication of j^{th} treatment

and $e_{ij} \sim N(0, \sigma_e^2)$ Also $\sum\limits_i \sum\limits_j \tau_{ij} = 0$

Hypothesis:

$H_0: \tau_1 = \tau_2 = \tau_3 = \dots = \tau_v = 0$

v/s H_1: atleast one $\tau_i \neq 0$

Row sum of squares, $R.S.S = \sum_j \sum_i y_{ij}^2$

$C.F = \frac{G^2}{n}$ where $G = \sum_i \sum_j y_{ij}$ and $n = \sum_{i=1}^{t} r_i$

Total sum of squares, $T.S.S = R.S.S - C.F$

Sum of squares due to treatment, $SST = \sum_i \frac{T_i^2}{r_i} - C.F$

the Sum of square due to error, $SSE = TSS - SST$

ANOVA TABLE

Source of variation	degrees of freedom	Sum of Squares	Mean sum of square	Variance ratio
Treatments	$v-1$	SST	$MST = \frac{SST}{v-1}$	$\frac{MST}{MSE} \sim F_{v-1, n-v}$
error	$n-v$	SSE	$MSE = \frac{SSE}{n-v}$	
Total	$n-1$	TSS		

Decision Rule: Reject H_0 at $\alpha\%$ l.o.s if $F_{cal} > F_{tab}$ at $(t-1, n-t)$ degrees of freedom otherwise accept H_0.

If H_0 is rejected, we perform post-hoc analysis.

we find, the least difference between any two means to be significant

$$C.D = \sqrt{(2S_E^2/r)} \times [t_{0.025} \text{ (for error d.f)}]$$

$$= \sqrt{(2S_E^2/r)} \times t_{(n-v)}(0.025)$$

If Difference > C.D then treatment differ significantly

If Difference < C.D then treatments difference are not significant.

Calculation :

$n = 20$ $t = 4$

Rest calculations in Excel

Result :

	Practical No. - 4	5/3/24

Aim :- To analyse a randomised block design (RBD).

Problem :- A varietal trial was conducted at a Research Station. The design adopted for the same was five randomised blocks of 6 plots each. The yields in lb. per plot (of 1/20th of an acre) obtained from the experiment are as given in the table.

Blocks	Varieties					
	V_1	V_2	V_3	V_4	V_5	V_6
I	30	23	34	25	20	13
II	39	22	28	25	28	32
III	56	43	43	31	49	17
IV	38	45	36	35	32	20
V	44	51	23	58	40	30

Analyse the design and comment on your findings.

Theory and formula :-

The linear model for RBD for one observation per experimental unit is given as - $y_{ij} = \mu + \tau_i + b_j + \varepsilon_{ij}$; $(i = 1, 2, \ldots t ; j = 1, 2, \ldots r)$

where y_{ij} is the response of experimental unit ~~resolving~~ receiving the ith treatment in jth block

μ is general mean effect

τ_i is effect due to ith treatment

b_j is effect due to jth block or replicate

and $\varepsilon_{ij} \sim N(0, \sigma_e^2)$

where, μ, τ_i's and b_j's are constants so that $\sum_{i=1}^{t} \tau_i = 0$ & $\sum_{j=1}^{r} b_j = 0$

Now, we setup the null hypothesis $H_{01}: \tau_1 = \tau_2 = \ldots = \tau_t$ against H_{11}: all τ's are not equal &

$H_{02}: b_1 = b_2 = \ldots b_r$ against H_{12}: all b's are not equal

For the test statistic, follow following steps -

(i) Calculate $G = \sum_i \sum_j y_{ij}$ and $N = rt$

Raw S.S $= \sum_i \sum_j y_{ij}^2$, Correction Factor (C.F.) $= \frac{G^2}{N}$

Total S.S = R.S.S − C.F

(ii) S.S.T (due to treatments) $= \sum_i \frac{T_i^2}{r} - CF$

$T_i = \sum_{j=1}^{r} y_{ij}$, the total yield from ith treatment

S.S.B (due to blocks) $= \sum_j \frac{B_j^2}{t} - C.F.$

$B_j = \sum_i y_{ij}$, total yield due to jth block

Now, S.S due to Error = S.S.E = T.S.S − S.S.T − S.S.t

(iii) ANOVA Table for RBD is given as

Source of Variation	d.f	S.S	M.S.S	Variance Ratio
Treatments	$t-1$	S_T^2	$s_T^2 = S_T^2/t-1$	$F_T = \frac{s_T^2}{s_E^2} \sim F_{(t-1, (t-1)(r-1))}$
Blocks	$r-1$	S_B^2	$s_B^2 = S_B^2/r-1$	$F_B = \frac{s_B^2}{s_E^2} \sim F_{(r-1, (r-1)(t-1))}$
Error	$(t-1)(r-1)$	S_E^2	$s_E^2 = \frac{S_E^2}{(t-1)(r-1)}$	
Total	$rt-1$			

<u>Decision Rule</u>: If F_T is greater than F tabulated for $[(t-1), (t-1)(r-1)]$ d.f at certain L.O.S, usually 5%, then we reject H_0 and conclude treatment differ significantly otherwise accept & data don't provide any evidence against H_0

Similarly, under $H_{02}: b_1 = b_2 = \ldots b_r$

if $F_B > F_{tab}$ at $[(r-1), (r-1)(t-1)]$ d.f then, reject H_{02} & conclude blocks differ significantly otherwise accept

<u>Calculation</u>: In Excel sheet

Result :- (i) $F_{tab}(5,20) = 2.71089$ and $F_{cal} = 2.4704$.
here, $F_{cal} < F_{tab}$ thus, H_{01} is accepted at 5% LOS
Therefore, treatments do not differ significantly
(ii) for blocks, $F_{tab}(4,20) = 2.866$ and $F_{cal} = 3.8113$., here,
$F_{cal} > F_{tab}$. Thus, H_{02} is rejected i.e, blocks differ significantly.

8/3/24

Practical No - 5

Aim:- To analyse randomised complete block design for the harnessing testing experiment

Problem:- Consider the harness testing experiment. There are four tips and four available metal coupons. Each tip is tested once on each coupon, resulting in a Randomised complete block design. The data obtained are repeated for convenience in table. Remember that the order in which tips were tested on a particular coupon was determined randomly.

	Coupon (Block)			
Type of Tip	1	2	3	4
1	9.3	9.4	9.6	10.0
2	9.4	9.3	9.8	9.9
3	9.2	9.4	9.5	9.7
4	9.7	9.6	10.0	10.2

Theory and formula :- The linear model for RBD is :

$$y_{ij} = \mu + \tau_i + b_j + \varepsilon_{ij}, \quad (i = 1, 2, \dots t ; j = 1, 2, \dots r)$$

where, y_{ij} is yield of experimental unit receiving ith treatment in jth block.

μ is general mean effect

τ_i is effect due to ith treatment

b_j is effect due to jth block or replicate & $\varepsilon_{ij} \sim N(0, \sigma e^2)$.

where, μ, τ_i's & b_j's are constants so that $\sum_{i=1}^{t} \tau_i = 0$ & $\sum_j b_j = 0$

We setup the null hypothesis as :

$H_{0t} : \tau_1 = \tau_2 = \dots = \tau_t = 0$ $\qquad$ $H_{10} : b_1 = b_2 = \dots = b_r = 0$

v/s H_{01} : all τ_i's are not equal $\qquad$ vs H_{11} : all b_j's are not equal

For test statistic, the ANOVA table is given as -

Source of Variation	df	S.S.	M.S.S.	Variance ratio
Treatments	$t-1$	S_T^2	$s_T^2 = \frac{S_T^2}{t-1}$	$F_T = \frac{s_T^2}{s_E^2} \sim F_{(t-1),(r-1)(t-1))}$
Blocks	$r-1$	S_B^2	$s_B^2 = \frac{S_B^2}{r-1}$	$F_B = \frac{s_B^2}{s_E^2} \sim F_{(r-1),(r-1)(t-1))}$
Error	$(t-1)(r-1)$	S_E^2	$s_E^2 = \frac{S_E^2}{(t-1)(r-1)}$	
Total	$rt-1$			

Where $T.S.S = R.S.S - CF$

$$R.S.S = \sum_i \sum_j y_{ij}^2, \quad C.F. = \frac{G^2}{N} \quad (N = rt)$$

$$S.S.T = \sum_i \frac{T_i^2}{r} - CF, \quad SSB = \sum_j \frac{B_j^2}{t} - CF$$

Now, $SSE = TSS - SST - SSB$

Decision rule: (1) If $F_{cal_T} \geq F_{tab}(t-1, (r-1)(t-1))$, then reject H_{01} i.e. treatments differ significantly otherwise accept H_{01}.

(2) If $F_B > F_{tab}(r-1, (r-1)(t-1))$ d.f., at α% LOS, then reject H_{02} i.e. blocks differ significantly otherwise accept H_{02}.

If the treatments show significant effect, we might be interested to test for significance of difference between any two treatment.

We arrange their mean yields in descending order of magnitude and then test for the significance of the pairwise difference by comparing them with critical difference:

$$CD = [t_{\alpha/2}(r-1)(t-1)] \times 2[s_E^2/r)^{1/2}$$

If Difference $> CD$, then treatment differ significantly o/w not.

Result - Here, $F_T > F_{tab}(3, 9)$, we reject H_{01} thus, treatments difference significantly.

By comparing the difference between mean yields for different treatments with C.D. we find that treatments 3,2,1 are alike in giving yields while treatment 4 differ significantly from all other treatments as difference b/w treatment 4 and other > C.D.

	Practical No - 6	10/3/24

Aim - To analyse randomised block design after eliminating the missing value

Problem - Analyse the following randomised block design after estimating the missing value. Also compare the treatments T_1 and T_2:

Treatments	Blocks I	II	III	IV
T_1	19.1	–	22.5	25.5
T_2	26.0	28.0	27.0	33.0
T_3	20.5	28.5	21.5	25.5

Theory and formula:

Estimation of missing value in RBD - Let the observation $y_{ij} = x$ in the jth block and recieving ith treatment be missing as given.

		Treatments 1	2	--	i	---	t	Total
Blocks	1	y_{11}	y_{21}	...	y_{i1}	...	y_{t1}	$y_{\cdot 1}$
	2	y_{21}	y_{22}	...	y_{i2}	...	y_{t2}	$y_{\cdot 2}$
	⋮							
	j	y_{1j}	y_{2j}	...	$y_{ij} = x$	...	y_{tj}	$y_{\cdot j}' + x$
	⋮							⋮
	r	y_{1r}	y_{2r}	...	y_{ir}	...	y_{tr}	
Total		$y_{1\cdot}$	$y_{2\cdot}$		$(y_{i\cdot}' + x)$	..	$y_{t\cdot}$	$y_{\cdot\cdot}' + x$

where, y_i' is total of known observation of getting ith treatment

$y_{\cdot j}'$ is total of known observation in jth block &

$y_{\cdot\cdot}'$ is total of all known observation

then,
$$x = \frac{r y_{\cdot j}' + t y_{i\cdot}' - y'}{(r-1)(t-1)}$$

Analysis of design Null Hypothesis (H_0): H_{0t} : $\tau_1 = \tau_2 = \dots = \tau_t$

H_{0b} : $b_1 = b_2 = \dots = b_r$

i.e. all treatments as well as replicates are homogeneous

Alternative hypothesis : H_1 H_{1t} : at least two τ_i's are different

H_{2b} at least two blocks are different

Substituting the value of x in given table.

Calculate Treatment totals and block totals

$G = \sum_i \sum_j y_{ij}$, $N = rt$

$C.F. = \frac{G^2}{N}$, $RSS = \sum_i \sum_j y_{ij}^2$

Total S.S $= RSS - CF$

$SST = \frac{1}{r} \sum_i T_i^2 - C.F.$, $SSB = \frac{1}{t} \sum_j B_j^2 - CF$

Error S.S. $= T.S.S - S.S.T - S.S.B.$

The bias or adjustment factor for treatment S.S is given by :

$$\frac{(y_{.j}' + t y_{i.}' - y_{..}')^2}{t(r-1)(t-1)^2}$$

$\therefore$ adjusted value of treatment S.S = S.S.T − Bias

ANOVA Table

Source of variation	df	S.S.	MSS	Variance ratio
Treatments (adjusted)	$t-1$	S_T^2	$s_T^2 = \frac{S_T^2}{t-1}$	$F_T = \frac{s_T^2}{s_E^2}$
Blocks	$r-1$	S_B^2	$s_B^2 = \frac{S_B^2}{r-1}$	$F_B = \frac{s_B^2}{s_E^2}$
Error	$rt-r-t$	S_E^2	$s_E^2 = \frac{S_E^2}{rt-r-t}$	
Total	$rt-2$			

Decision rule: If F_T Cal $> F_{tab}[(t-1),(rt-r-t)]$ d.f , then we reject H_{0t} and thus, difference b/w treatment is significant otherwise accept

__Result__: Here, $F_T\ cal = 7.145$ and $F_{tab}\ (2,5) = 5.786$.

and $F_{cal} > F_{tab}$. thus H_{0t} is rejected

i.e. Treatments T_1 and T_2 differ significantly.

	Practical No. 7	2/4/24

Aim - Set up the analysis for adjoining results of a latin Square design

Problem - Set up the analysis for adjoining results of a latin square design

A 12	C 19	B 10	D 8
C 18	B 12	D 6	A 7
B 22	D 10	A 5	C 21
D 12	A 7	C 27	B 17

Theory and Formula - Let y_{ij} $(j, k = 1, 2, \ldots m)$ denote the response from the unit in the ith row, jth column and recieving the kth treatment. If a single observation is made per experimental unit, then the linear additive model is:

$$y_{ij} = \mu + \alpha_i + \beta_j + \tau_k + \varepsilon_{ijk} \quad ; (i,j,k) \in S.$$

S denotes the set of m^2 values.

Where, μ is constant mean effect

α_i, β_j and τ_k are constant effect due to ith row, jth column and kth treatment respectively and

ε_{ijk} is error due to random component. $\varepsilon_{ijk} \overset{iid}{\sim} N(0, \sigma_e^2)$

Now, $G = \sum_{i,j,k} y_{ijk}$, $CF = \frac{G^2}{N}$, $N = m^2$

$Raw\ SS = \sum_{i,j,k} y_{ijk}^2$

$TSS = RSS - CF$, SSR (due to row) $= \frac{1}{m} \sum_i y_{i..}^2 - CF$

SSC (due to column) $= \frac{1}{m} \sum_j y_{.j.}^2 - CF$

SS (due to treatment) $= \frac{1}{m} \sum_k y_{..k}^2 - CF$

$SSE = TSS - SSR - SSC - SST$

Let us set up null hypothesis :

For row effects $H_{0\alpha}: \alpha_1 = \alpha_2 = \cdots = \alpha_m = 0$

For column effects $H_{0\beta}: \beta_1 = \beta_2 = \cdots = \beta_m = 0$ and

For treatment effects $H_{0\tau}: \tau_1 = \tau_2 = \cdots = \tau_m = 0$.

v/s H_1: $H_{1\alpha}$: at least two α_i's are different

$H_{1\beta}$: at least two β_i's are different

$H_{1\tau}$: at least two τ_i's are different

ANOVA Table

Source of variation	df	S.S	MSS	Variance ratio
Rows	$m-1$	S_R^2	$s_R^2 = \frac{S_R^2}{m-1}$	$F_R = s_R^2/s_E^2$
Columns	$m-1$	S_C^2	$s_C^2 = \frac{S_C^2}{m-1}$	$F_C = s_C^2/s_E^2$
Treatments	$m-1$	S_T^2	$s_T^2 = \frac{S_T^2}{m-1}$	$F_T = s_T^2/s_E^2$
Error	$(m-1)(m-2)$	S_E^2	$s_E^2 = \frac{S_E^2}{(m-1)(m-2)}$	
Total	m^2-1			

Decision rule: Let $F_\alpha[(m-1), (m-1)(m-2)]$ be tabulated value of F for $[(m-1), (m-1)(m-2)]$ d.f at α LOS. Thus, if $F_R > F_\alpha$, we reject $H_{0\alpha}$ and if $F_R \leq F_\alpha$ we fail to reject $H_{0\alpha}$.

Similarly, for $H_{0\beta}$ and $H_{0\tau}$

Result: Here, F_R cal $= 1.5165 < F_{tab}(3,6) = 4.76$

F_C cal $= 1.0755 < F_{tab}(3,6) = 4.76$

F_T cal $= 113.7212 > F_{tab}(3,6) = 4.76$

Thus, $H_{0\alpha}$ and $H_{0\beta}$ are accepted but $H_{0\tau}$ is rejected i.e. we conclude that variation due to rows and columns is not significant but the treatment have significant effect on the response.

3/4/24

Practical No - 8

Aim:- To eliminate the missing observation and analyse the latin square design.

Problem:- The given table gives the yield of wheat (kgs/plot) as observed in an experiment carried out in a 5X5 latin square. The five manurial treatments are indicated by A, B, C, D & E.

Row	Columns 1	2	3	4	5
1	B 57.8	C 48.6	A 33.4	D 53.5	E 41.8
2	D 50.5	E 45.5	C 51.8	B 52.6	A 31.9
3	A 46.1	D 47.9	B 55.6	E x	C 53.3
4	C 58.2	B 55.1	E 43.2	A 38.8	D 53.3
5	E 53.0	A 41.0	D 48.7	C 54.6	B 55.7

Theory and formula:

Estimation of missing value in RLSD - Let us suppose that in $m \times m$ latin square, the observation occuring in ith row, jth column and receiving the kth treatment is missing. Let us assume the value is x i.e. $y_{ijk} = x$

R = Total of known obs. in row containing 'x'

C = Total of known obs. in jth column containing 'x'

T = total of known obs. ~~or~~ treatment values containing 'x'

S = Total of all known observation

Then, $$\hat{x} = \frac{m(R+C+T) - 2S}{(m-1)(m-2)}$$

Statistical Analysis: After inserting the estimated value for missing observation, we perform the usual analysis of variance, Subtracting one d.f for total S.S. and consequently for Error SS Adjusted treatment SS is obtained by subtracting the quantity

$$\left[\frac{(m-1)T + R + C - S}{(m-1)(m-2)}\right]^2$$ from treatment S.S.

<u>Result</u>: F_R cal $= 2.377 < F_{tab}(4, 11) = 3.356$.

F_C cal $= 4.479 > F_{tab}(4, 11) = 3.356$

F_T cal $= 28.7126 > F_{tab}(4, 11) = 3.356$.

Thus, $H_{0\alpha}$ ~~and~~ is accepted but $H_{0\beta}$ and $H_{0\tau}$ are rejected i.e variations due to row is not significant but variations due to column and treatment is significant.

Practical No - 9

Aim: To perform Analysis of LSD

Problem: The following table gives the results of the LS exp. on the effect of five manurial treatments A, B, C, D, E on the yield of sugarcane. Test whether the treatments are equally effective, and if they are not so, compare treatments A and B.

B 405	A 525	E 463	D 441	C 481
C 325	D 445	B 424	A 413	E 493
E 471	B 492	A 472	C 381	D 410
A 552	C 431	D 425	E 572	B 451
D 430	E 469	C 432	B 457	A 460

Theory and formula

Let y_{ijk} be the response from the unit in i^{th} row, j^{th} column, receiving k^{th} treatment.

If a sample observation is

$$y_{ijk} = \mu + \alpha_i + \beta_j + \tau_k + \epsilon_{ijk} \quad (i,j,k \in S)$$

where S denotes the set of m^2 values

where μ - Constant mean effect

α_i - Effect due to i^{th} row

β_j - Effect due to j^{th} column

τ_k - Effect due to k^{th} trmt

and $\epsilon_{ijk} \sim N(0, \sigma_e^2)$

Now, $G = \sum\sum_{(i,j)} y_{ij}$

$CF = G^2/m^2$

$RSS = \sum\sum_{(i,j)} y_{ij}^2$, $TSS = RSS - CF$

$SSR = \frac{1}{m}\sum_i R_i^2 - CF$

$SSC = \frac{1}{m}\sum_j C_j^2 - CF$

$SST = \frac{1}{m}\sum_k T_k^2 - CF$

$SSE = TSS - SSR - SSC - SST$

Let us set up $H_{0\alpha}: \alpha_1 = \dots = \alpha_m = 0$
$H_{0\beta}: \beta_1 = \dots = \beta_m = 0$
$H_{0\gamma}: \gamma_1 = \dots = \gamma_m = 0$

v/s $H_{1\alpha}$: Atleast one $\alpha_i \neq 0$
$H_{1\beta}$: Atleast one $\beta_j \neq 0$
$H_{1\gamma}$: Atleast one $\gamma_k \neq 0$

ANOVA Table

SV	df	SS	MSS	VR
Rows	$m-1$	SSR	MSR	$F_R = MSR/MSE$
Col.	$m-1$	SSC	MSC	$F_C = MSC/MSE$
Trtts	$m-1$	SST	MST	$F_T = MST/MSE$
Error	$(m-1)(m-2)$	SSE	MSE	
Total	m^2-1	TSS		

If $F_{cal} > F_{tab}(m-1, (m-1, m-2))$ then reject H_0 otherwise accept H_0

Calculations : As done in excel sheet

$F_R = 1.41 < F_{tab} \Rightarrow$ Accept H_{0A}

$F_C = 0.46 < F_{tab} \Rightarrow$ Accept H_{0B}

$F_T = 3.094 < F_{tab} \Rightarrow$ Accept H_{0T}

Result : We can say that all rows, columns and treatments are equally effective.

Practical No-10

Aim: To perform intrablock analysis of BIBD

Theory and Formula:

Problem: The adjoining table gives the results of an exp involving seven trmts in seven blocks each of three plots, with a balanced layout

Blocks	Trmts and yields		
1	(1) 2.10	(2) 2.67	(4) 2.91
2	(2) 1.14	(3) 3.00	(5) 2.18
3	(3) 2.92	(4) 3.14	(6) 2.99
4	(4) 3.15	(5) 2.63	(7) 2.75
5	(5) 2.84	(6) 3.13	(1) 1.85
6	(6) 3.01	(7) 2.99	(2) 1.82
7	(7) 2.86	(1) 3.54	(3) 3.74

Theory and Formula

The BIBD model is given by :

$$y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}$$

$$E = \sum_{(i,j)}\sum e_{ij}^2 = \sum_i\sum_j n_{ij}(y_{ij} - \mu - \alpha_i - \beta_j)^2$$

Where μ = General mean effect
α_i = effect due to i^{th} trmt
β_j = effect due to j^{th} block
e_{ij} = Intra block error

$$n_{ij} = \begin{cases} 1 & \text{(When } i^{th} \text{ trmt occurs in } j^{th} \text{ block)} \\ 0 & \text{(o.w)} \end{cases}$$

We know, $G = \sum_i\sum_j y_{ij}$

$RSS = \sum_i\sum_j y_{ij}^2$

$CF = G^2/N$, $TSS = RSS - CF$

2nd order Interaction

ABC	1	$S_{ABC}^2 = [ABC]^2/8r$	$s_{ABC}^2 = S_{ABC}^2$	$F_{ABC} = s_{ABC}^2/s_E^2 \sim F[1, 7(r-1)]$
Error	$7(r-1)$	S_E^2 = By subtraction	$s_E^2 = \frac{S_E^2}{7(r-1)}$	
Total	$r.2^3-1 = 8r-1$			

The hypothesis of the presence of a factorial effect is rejected at α% LOS if the corresponding calculated F statistic in the above table is greater than tabulated $F_{\alpha,1,7(r-1)}$ otherwise the hypothesis may be accepted.

YATE'S METHOD FOR A 2^3 EXPERIMENT.

Treatment Combination (1)	Treatment Totals (2)	(3)	(4)	(5)	Effect Totals
'1'	[1]	$[1]+[a]=u_1$	$u_1+u_2=v_1$	$v_1+v_2=w_1$	Grand Total
a	[a]	$[b]+[ab]=u_2$	$u_3+u_4=v_2$	$v_3+v_4=w_2$	[A]
b	[b]	$[c]+[ac]=u_3$	$u_5+u_6=v_3$	$v_5+v_6=w_3$	[B]
ab	[ab]	$[bc]+[abc]=u_4$	$u_7+u_8=v_4$	$v_7+v_8=w_4$	[AB]
c	[c]	$[a]-[1]=u_5$	$u_2-u_1=v_5$	$v_2-v_1=w_5$	[C]
ac	[ac]	$[ab]-[b]=u_6$	$u_4-u_3=v_6$	$v_4-v_3=w_6$	[AC]
bc	[bc]	$[ac]-[c]=u_7$	$u_6-u_5=v_7$	$v_6-v_5=w_7$	[BC]
abc	[abc]	$[abc]-[bc]=u_8$	$u_8-u_7=v_8$	$v_8-v_7=w_8$	[ABC]

PRACTICAL → 12

- **AIM** → To analyse 2^3 factorial experiment.

- **EXPERIMENT** → The results of a 2^3 factorial experiment replicated 4 times are given below. The factors are A, B, C each at two levels. Analyse the data.

Factor combination	Replications 1	2	3	4
1	28	24	27	23
A	42	39	32	44
B	31	27	28	33
C	38	33	35	41
AB	37	34	33	32
AC	30	42	36	34
AC	23	41	27	38
ABC	47	32	37	43

- **THEORY & FORMULA** →

for a 2^3-factorial experiment,

Main Effect :-

$$A = \frac{1}{4}\left[(a-1)(b+1)(c+1)\right]$$

$$B = \frac{1}{4}\left[(a+1)(b-1)(c+1)\right]$$

$$C = \frac{1}{4}\left[(a+1)(b+1)(c-1)\right]$$

Interaction Effect :- (first order)

$$AB = \frac{1}{4}\left[(a-1)(b-1)(c+1)\right]$$

$$BC = \frac{1}{4}\left[(a+1)(b-1)(c-1)\right]$$

$$AC = \frac{1}{4}\left[(a-1)(b+1)(c-1)\right]$$

Interaction Effect (second order)

$$ABC = \frac{1}{4}\left[(a-1)(b-1)(c-1)\right]$$

ANOVA TABLE for 2^3 Design

S.V.	d.f.	SS	MSS	VR
Replications	$(r-1)$	S_R^2	$s_R^2 = \frac{S_R^2}{(r-1)}$	$F_R = \frac{s_R^2}{s_e^2} \sim F[r-1, 7(r-1)]$
Main effects				
A	1	$S_A^2 = [A^2]/8r$	$s_A^2 = S_A^2$	$F_A = s_A^2/s_e^2 \sim F[1, 7(r-1)]$
B	1	$S_B^2 = [B^2]/8r$	$s_B^2 = S_B^2$	$F_B = s_B^2/s_e^2 \sim F[1, 7(r-1)]$
C	1	$S_C^2 = [C^2]/8r$	$s_C^2 = S_C^2$	$F_C = s_C^2/s_e^2 \sim F[1, 7(r-1)]$
Int. Effects (1st Order)				
AB	1	$S_{AB}^2 = [AB^2]/8r$	$s_{AB}^2 = S_{AB}^2$	$F_{AB} = s_{AB}^2/s_e^2 \sim F[1, 7(r-1)]$
AC	1	$S_{AC}^2 = [AC^2]/8r$	$s_{AC}^2 = S_{AC}^2$	$F_{AC} = s_{AC}^2/s_e^2 \sim F[1, 7(r-1)]$
BC	1	$S_{BC}^2 = [BC^2]/8r$	$s_{BC}^2 = S_{BC}^2$	$F_{BC} = s_{BC}^2/s_e^2 \sim F[1, 7(r-1)]$
Int. Effects (2nd Order)				
ABC	1	$S_{ABC}^2 = \frac{[ABC^2]}{8r}$		$F_{ABC} = s_{ABC}^2/s_e^2 \sim F(1, 7(r-1))$
Error	$7(r-1)$	$S_e^2 =$ By subtraction		
Total	$(8r-1)$			

Yate's Table

Treatment	Yield	[3]	[4]	[5]
1	[1]	[1]+[a]	[1]+[a]+[b]+[ab]	G
A	[a]	[b]+[ab]	bc+abc+c+ac	[A]
B	[b]	[c]+[ac]	-b+a-b+ab+a-1	[B]
AB	[ab]	[bc]+[abc]	[a]-[1]+[ab]-b	[AB]
C ~~AB~~	[c]	[a]-[1]	ac-c+abc-bc	[C]
AC	[ac]	[ab]-[b]	b+ab-1-a	[AC]
BC	[bc]	[ac]-[c]	abc+bc-ac-c	[BC]
ABC	[abc]	[abc]-[bc]	abc-bc-ac+c	[ABC]

- **CALCULATIONS:-** As in Excel sheet alongside.
- **RESULTS:-** The table is plotted in sheet alongside.

—— x —— x ——

- **AIM** → To analyse the given design for 2^2 factor experiment.

- **EXPERIMENT** → The field plan of yield of dry Aman paddy (in kg) for each plot are given below:

Block 1

(1)	s	rs	r
117	106	109	114

Block 2

rs	1	s	r
114	120	117	114

Block 3

(1)	rs	s	rs
111	117	114	106

Block 4

(rs)	r	s	(1)
93	121	112	108

Block 5

rs	s	(1)	r
75	97	73	88

Block 6

r	(1)	rs	s
58	81	106	117

Analyse the data to find out if there are any significant treatment.

- **THEORY & FORMULA** →

For a 2^2 - factor Design

Main Effect :-

$$A = \frac{1}{2}[(a-1)(b+1)]$$

$$B = \frac{1}{2}[(a+1)(b-1)]$$

Interaction Effect :-

$$AB = \frac{1}{2}[(a-1)(b-1)]$$

ANOVA Table :-

S.V.	d.f.	SS	MSS	V.R.
Blocks	$(r-1)$	S_R^2	$s_R^2 = \frac{S_R^2}{(r-1)}$	$F_R = s_R^2/s_e^2$
Main Effect				
A	1	$S_A^2 = [A^2]/4r$	$s_A^2 = S_A^2/1$	$F_A = s_A^2/s_e^2$
B	1	$S_B^2 = [B^2]/4r$	$s_B^2 = S_B^2/1$	$F_B = s_B^2/s_e^2$
Interaction				
AB	1	$S_{AB}^2 = [AB]^2/4r$	$s_{AB}^2 = S_{AB}^2$	$F_{AB} = s_{AB}^2/s$
Error	$3(r-1)$	S_e^2 = by substn	$s_e^2 = \frac{S_e^2}{3(r-1)}$	
Total	$4r-1$			

Yatt's Method

Treatment	Yields	[3]	[4]
1	[1]	[1] + [a]	(1)[1] + [a] + [b] + [ab]
a	[a]	[b] + [ab]	(a)[b] + [ab] + [a] - [1]
b	[b]	[a] - [1]	(b)[b] + [ab] - [1] - [a]
ab	[ab]	[ab] - [b]	(ab)[ab] - [b] - [a] + 1

- **CALCULATIONS:-** Table is plotted in excel sheet alongside
- **RESULTS:-** The values are plotted in sheet alongside

———×———×———

PRACTICAL-14

- **AIM:-** To analyze 2^3 design.
- **EXPERIMENT:-** Analyze the following 2^3-design

Blocks	Treatments							
	npk	(1)	k	np	p	n	nk	pk
B_1	49.3	47.2	45.3	53.4	44.7	62.7	57.3	50.6
	p	nk	k	np	(1)	npk	pk	n
B_2	56.6	50.1	55.8	52.3	54.2	63.8	47.7	60.3
	p	npk	k	(1)	n	k	pk	np
B_3	53.2	56.8	57.3	51.1	70.2	55.3	53.2	63.8
	(1)	np	npk	n	nk	pk	p	k
B_4	43.2	58.2	52.5	61.2	63.3	48.3	42.3	52.1

- **THEORY & FORMULA:-**

for a 2^3-design

$$\text{Main Effect } A = \frac{1}{8}\left[(a-1)(b+1)(c+1)\right]$$

$$B = \frac{1}{8}\left[(a+1)(b-1)(c+1)\right]$$

$$C = \frac{1}{8}\left[(a+1)(b+1)(c-1)\right]$$

Interaction Effect (1st Order):-

$$AB = \frac{1}{8}\left[(a-1)(b-1)(c+1)\right]$$

$$AC = \frac{1}{8}\left[(a-1)(b+1)(c-1)\right]$$

$$BC = \frac{1}{8}\left[(a+1)(b-1)(c-1)\right]$$

Interaction Effect (2nd Order):-

$$ABC = \frac{1}{8}\left[(a-1)(b-1)(c-1)\right]$$

Anova Table :-

S.V.	d.f.	SS	MSS	VR
Replication	$(r-1)$	S_R^2	$s_R^2 = \frac{S_R^2}{(r-1)}$	$F_R = \frac{s_R^2}{s_e^2} \sim F[r-1, 7(r-1)]$
Main Effect				
A	1	$S_A^2 = [A^2]/8r$	$s_A^2 = S_A^2$	$F_A = \frac{s_A^2}{s_e^2} \sim F[1, 7(r-1)]$
B	1	$S_B^2 = [B^2]/8r$	$s_B^2 = S_B^2$	$F_B = \frac{s_B^2}{s_e^2} \sim F[1, 7(r-1)]$
C	1	$S_C^2 = [C^2]/8r$	$s_C^2 = S_C^2$	$F_C = \frac{s_C^2}{s_e^2} \sim F[1, 7(r-1)]$
Int. Effect				
AB	1	$S_{AB}^2 = [AB^2]/8r$	$s_{AB}^2 = S_{AB}^2$	$F_{AB, AC, BC} = \frac{s_{AB}^2}{s_e^2}$
AC	1	$S_{AC}^2 = [AC^2]/8r$	$s_{AC}^2 = S_{AC}^2$	$\sim F[1, 7(r-1)]$
BC	1	$S_{BC}^2 = [BC^2]/8r$	$s_{BC}^2 = S_{BC}^2$	
Int. Effect (2)				
ABC	1	$S_{ABC}^2 = \frac{[ABC^2]}{8r}$	$s_{ABC}^2 = S_{ABC}^2$	$F_{ABC} = \frac{s_{ABC}^2}{s_e^2} \sim F[1, 7(r-1)]$
Error	$7(r-1)$	S_e^2	$s_e^2 = \frac{S_e^2}{7(r-1)}$	
Total	$8r-1$			

Yate's Table:-

Treatments	Yield	[3]	[4]	[5] Totals
1	[1]	$[1]+[a]=U_1$	$U_1+U_2=V_1$	V_1+V_2 [G]
a	[a]	$[b]+[ab]=U_2$	$U_3+U_4=V_2$	V_3+V_4 [A]
b	[b]	$[c]+[ac]=U_3$	$U_5+U_6=V_3$	V_5+V_6 [B]
ab	[ab]	$[bc]+[abc]=U_4$	$U_7+U_8=V_4$	V_7+V_8 [AB
c	[c]	$[a]-[1]=U_5$	$U_2-U_1=V_5$	V_2-V_1 [AC
ac	[ac]	$[ab]-[b]=U_6$	$U_4-U_3=V_6$	V_4-V_3 [BC
bc	[bc]	$[ac]-[c]=U_7$	$U_6-U_5=V_7$	V_6-V_5 [BC
abc	[abc]	$[abc]-[bc]=U_8$	$U_8-U_7=V_8$	V_8-V_7 [ABC

- CALCULATIONS:- As done in excel sheet alongside.
- RESULTS:- As shown in excel sheet.

——x——x——

PRACTICAL NO-15

Aim:- To analyse 2^3 completely confounded factorial design.

Experiment:- Analyze the following 2^3 completely confounded factorial design.

Replicate 1

Block I	'1'	(nk)	(np)	(kp)
	101	291	373	391
Block II	(nkp)	(n)	(k)	(p)
	450	106	265	312

Replicate 2

Block III	'1'	(nk)	(np)	(kp)
	106	306	338	407
Block IV	(nkp)	(n)	(k)	(p)
	449	89	272	324

Replicate 3

Block V	'1'	(nk)	(np)	(kp)
	87	334	324	423
Block VI	(nkp)	(n)	(k)	(p)
	471	128	279	323

Replicate 4

Block VII	'1'	(nk)	(np)	(kp)
	131	272	361	445
Block VIII	(nkp)	(n)	(k)	(p)
	437	103	302	324

Theory and formula:-

Confounding - The process by which unimportant comparisons are deliberately confused or mixed up with the block comparisons for the purpose of assessing more important comparisons with greater precision is called confounding.

Complete Confounding - when the same effect is confounded in all replications thereby leading to non-existing estimation of those treatment contrasts is called complete confounding.

In the above 2^3 factorial experiment replicate has been divided into blocks of 4 plots each. It is a 2^3 confounded design. A careful examination of the treatment combinations in diff blocks reveals that in each replicate the interaction NPK has been confounded.

Hence, the above design is a 2^3 factorial with the interaction NPK completely confounded with blocks.

ANOVA Table with interaction NPK completely confounded

Sources of variation	d.f.	SS	MSS	Variance Ratio
Blocks	$2r-1$	S_B^2	$s_B^2 = S_B^2/(2r-1)$	$F_B = s_B^2/s_E^2$
Treatments				
N	1	$S_N^2 = [N^2]/32$	$s_N^2 = S_N^2$	$F_N = s_N^2/s_E^2$
K	1	$S_K^2 = [K^2]/32$	$s_K^2 = S_K^2$	$F_K = s_K^2/s_E^2$
P	1	$S_P^2 = [P^2]/32$	$s_P^2 = S_P^2$	$F_P = s_P^2/s_E^2$
NK	1	$S_{NK}^2 = [NK]^2/32$	$s_{NK}^2 = S_{NK}^2$	$F_{NK} = s_{NK}^2/s_E^2$
NP	1	$S_{NP}^2 = [NP]^2/32$	$s_{NP}^2 = S_{NP}^2$	$F_{NP} = s_{NP}^2/s_E^2$
KP	1	$S_{KP}^2 = [KP]^2/32$	$s_{KP}^2 = S_{KP}^2$	$F_{KP} = s_{KP}^2/s_E^2$
Error	$6(r-1)$	S_E^2	$s_E^2 = S_E^2/6(r-1)$	
Total	$8r-1$			

Yate's Table

Treatments	Yields Total				Effect Total
(1)	(2)	(3)	(4)	(5)	
'1'	(1)	$[1]+[N]=u_1$	$u_1+u_2=v_1$	$v_1+v_2=w_1$	Grand Total
N	[N]	$[K]+[NK]=u_2$	$u_3+u_4=v_2$	$v_3+v_4=w_2$	[N]
K	[K]	$[P]+[NP]=u_3$	$u_5+u_6=v_3$	$v_5+v_6=w_3$	[K]
NK	[NK]	$[KP]+[NKP]=u_4$	$u_7+u_8=v_4$	$v_7+v_8=w_4$	[NK]
P	[P]	$[N]-[1]=u_5$	$u_2-u_1=v_5$	$v_2-v_1=w_5$	[P]
NP	[NP]	$[NK]-[K]=u_6$	$u_4-u_3=v_6$	$v_4-v_3=w_6$	[NP]
KP	[KP]	$[NP]-[P]=u_7$	$u_6-u_5=v_7$	$v_6-v_5=w_7$	[KP]
NKP	[NKP]	$[NKP]-[KP]=u_8$	$u_8-u_7=v_8$	$v_8-v_7=w_8$	[NKP]

Result :-

$F(cal) < F(tab)$ for the d.f $(7, 3)$ at 5% LOS, we fail to reject the null hypothesis i.e H_0: Confounding is not effective.

Hence, we conclude that confounding is not effective.

PRACTICAL-16

Aim:- To analyze 2^2 factorial experiment conducted in C.R.D

Problem:- For the given data of yield for 2^2 factorial experiment conducted in C.R.D. Analyze the design.

(1) 20	(a) 28	(a) 24	(b) 10
(ab) 23	(b) 11	(ab) 22	(1) 17
(a) 24	(b) 15	(ab) 21	(1) 19

Theory and formula:-

Factorial effect total in 2^2 factorial experiment are given as

$$[A] = [ab] - [b] + [a] - [1]$$

$$[B] = [ab] + [b] - [a] - [1]$$

$$[AB] = [ab] - [a] - [b] - [1]$$

The S.S due to any factorial effect is obtained on multiplying the square of the effect total by the factor $(1/4r)$, where r is the common replication number. Thus

$$\text{S.S due to main effect of A} = \frac{[A]^2}{4r}$$

$$\text{S.S due to main effect of B} = \frac{[B]^2}{4r}$$

$$\text{S.S due to interaction AB} = \frac{[AB]^2}{4r}$$

and each with 1 d.f.

ANOVA Table for fixed effect Model two factor (2^2)

Source of variation	d.f	S.S	M.S.S	Variance Ratio (F)
Blocks (Replicates)	$r-1$	S_R^2	$s_R^2 = \frac{S_R^2}{r-1}$	$F_R = s_R^2 / s_E^2$
Main effect A	1	$S_A^2 = [A]^2/4r$	$s_A^2 = S_A^2$	$F_A = s_A^2 / s_E^2$
Main effect B	1	$S_B^2 = [B]^2/4r$	$s_B^2 = S_B^2$	$F_B = s_B^2 / s_E^2$
Interaction AB	1	$S_{AB}^2 = [AB]^2/4r$	$s_{AB}^2 = S_E^2/[3(r-1)]$	$F_{AB} = s_{AB}^2 / s_E^2$
Error	$3(r-1)$	S_E^2 = By subtraction	$s_E^2 = S_E^2/[3(r-1)]$	

Here, F_A, F_B and F_{AB} follows central F-distribution with $[1, 3(r-1)]$ df. If for any factorial effect, calculated F is greater than tabulated F for $[1, 3(r-1)]$ df and at certain LOS say α then the null hypothesis H_0 of the presence of the factorial effect is rejected, otherwise H_0 may be accepted.

→ Yate's Method for 2^2 Experiment

Treatment Combination	Total Yield from all replicants		Effect Totals	
(1)	(2)	(3)	(4)	(5)
1	[1]	[1] + [a]	[1]+[a]+[b]+[ab]	G
a	[a]	[b]+ [ab]	[ab]-[b]+[a]-[1]	[A]
b	[b]	[a] - [1]	[ab]+[b]-[a]-[1]	[B]
ab	[ab]	[ab] - [b]	[ab]-[b]-[a]+[1]	[AB]

Result :-

Main effect A = 208.333

Main effect B = 75

Interaction AB = 8.33

F_{cal} (24.82) > F_{tab} (4.066) at (3, 8) df and 5 % LOS.

Hence, we conclude that the presence of factorial effect is rejected.