Range-Based Set Reconciliation

Aljoscha Meyer

Set Reconciliation

- set union over a network
- between (exactly) two machines
- unstructured data
- no shared state or history

Trivial Reconciliation

Sam Gwilym

Model and Analysis

- Alfie and Betty talk over a network
- reliable communication, rounds of unit length, unlimited bandwidth
- probabilistic solutions

Model and Analysis

- Alfie and Betty talk over a network
- reliable communication, rounds of unit length, unlimited bandwidth
- probabilistic solutions
- n: size of the union
- $n_\triangle$: size of the symmetric difference

Model and Analysis

- roundtrips
- communicated bytes
- computation time per reconciliation session
- computation space per reconciliation session
- computation time per item
- computation space per item

P2P Reconciliation

Peer-to-peer systems:

- iterating over local set infeasible
- loading local set into memory infeasible
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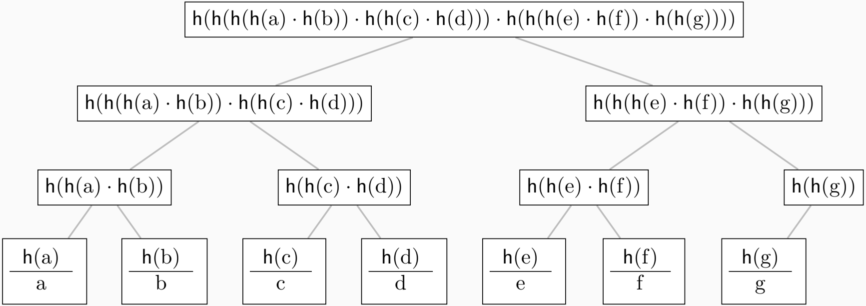
$\implies$ traditional approaches don't work

Reducing Computation Times

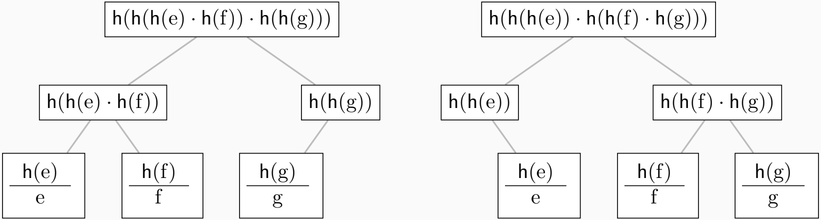
- Step 1: Put a Merkle tree on it
- Step 2: ???
- Step 3: Profit

Auvolat, Alex, and François Taïani. "Merkle search trees: Efficient state-based CRDTs in open networks." 2019 38th Symposium on Reliable Distributed Systems (SRDS). IEEE, 2019.

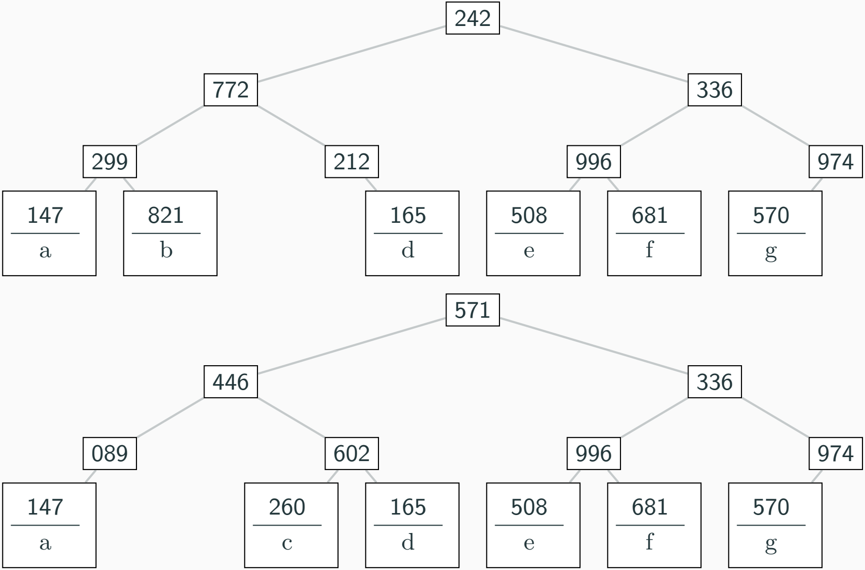
Merkle Trees



Merkle Trees



Merkle Tree Reconciliation



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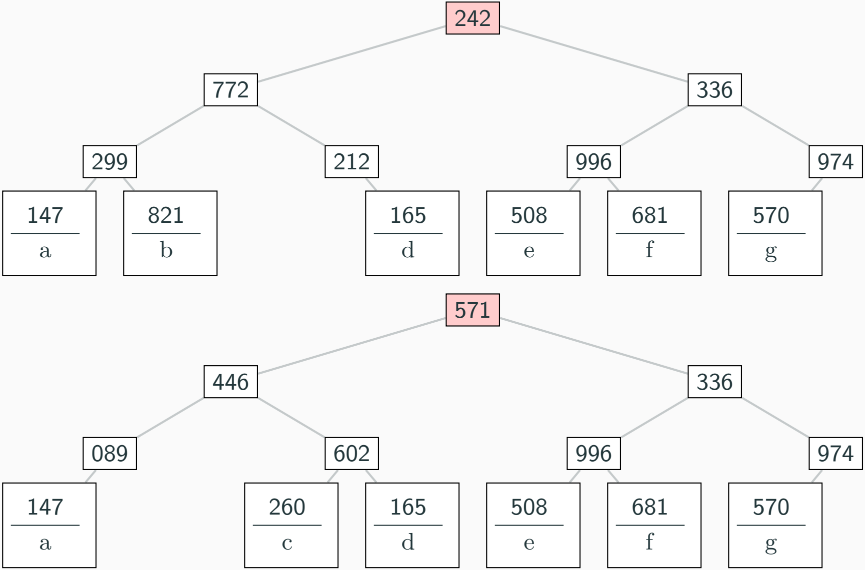

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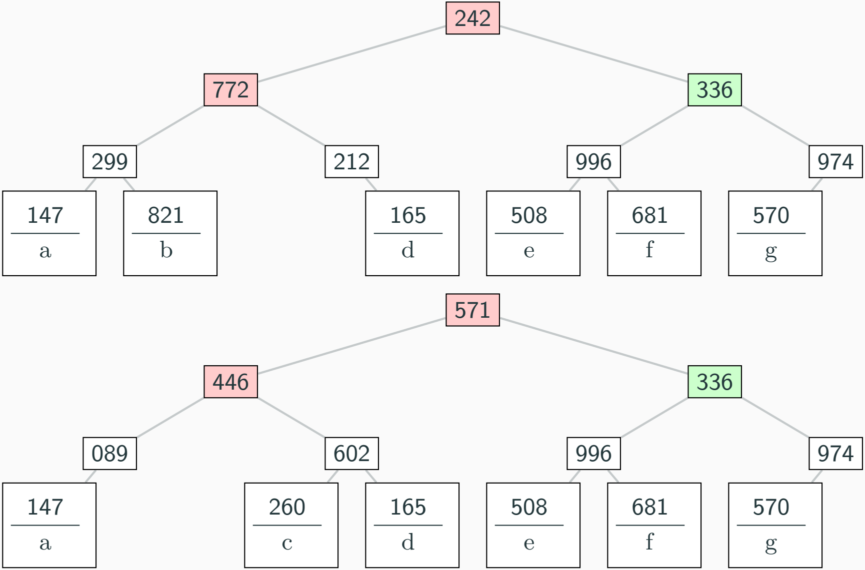

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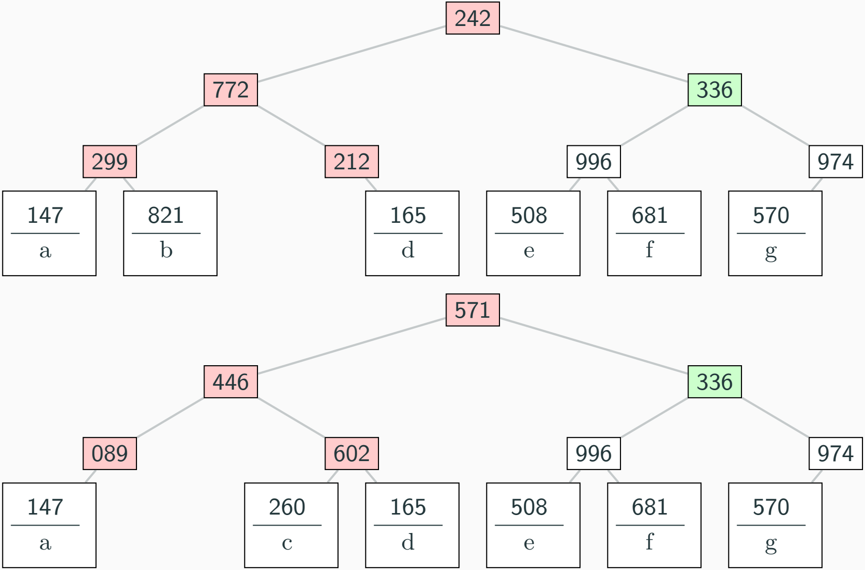

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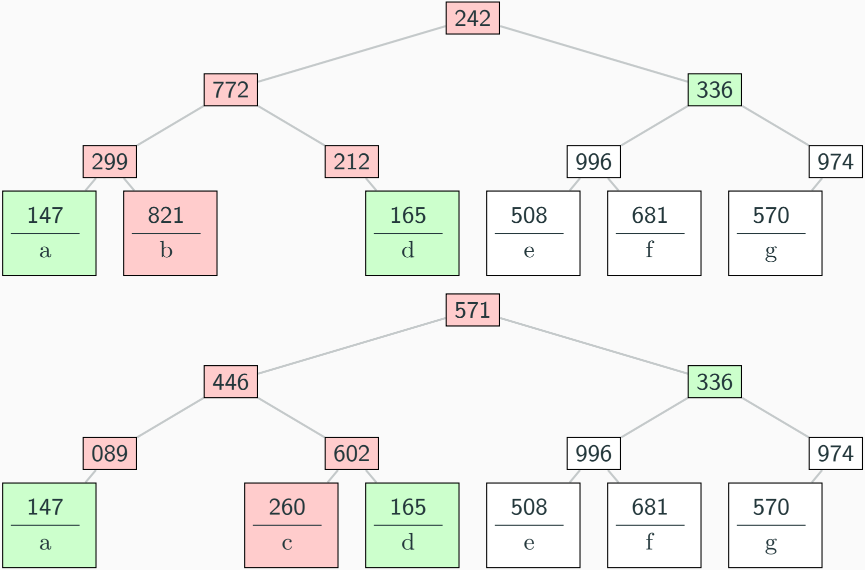
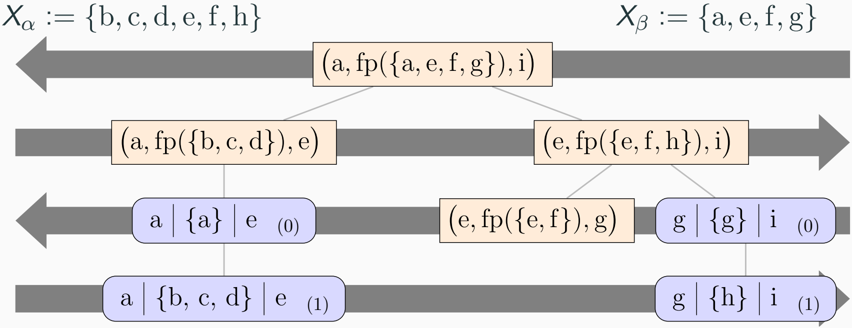


Merkle Tree Reconciliation

- inflexible data representation
- inacceptable worst-case complexity
  - remember, some peers are out to get us

Range-Based Set Reconciliation



Some Nice Properties

- reasonably efficient: $\mathcal{O}(\min(n_\triangle \cdot \log(n), n))$ bytes communication, $\mathcal{O}(1)$ working memory
- can interpolate toward trivial
- arbitrary recursion anchor protocols
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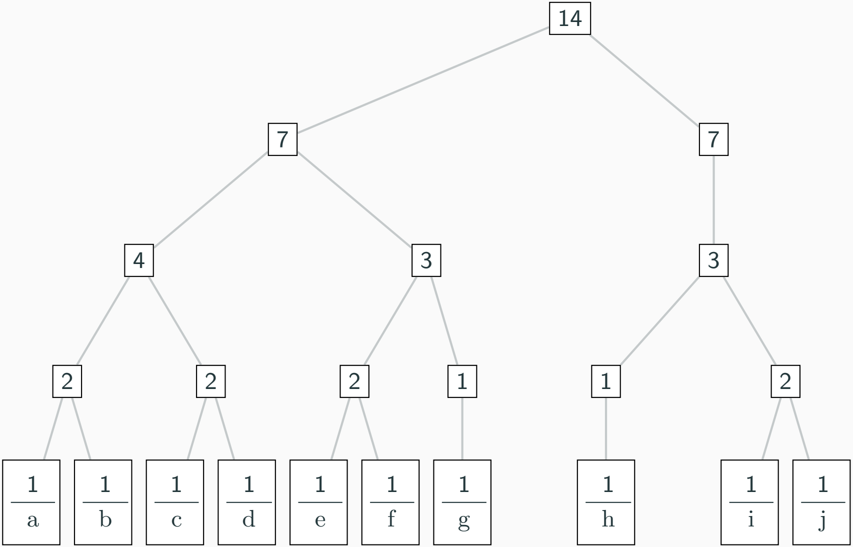
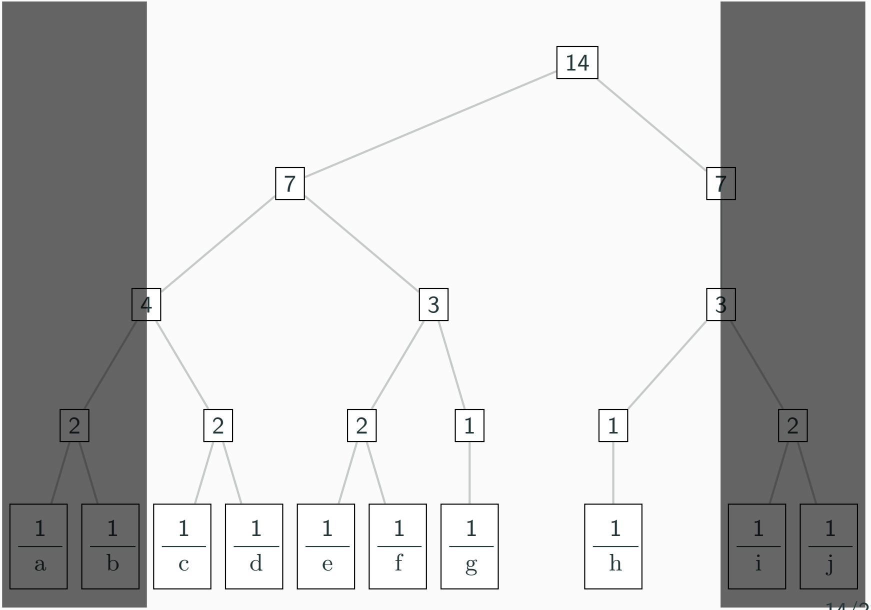
- reasonably efficient: $\mathcal{O}(\min(n_{\triangle} \cdot \log(n), n))$ bytes communication, $\mathcal{O}(1)$ working memory
- can interpolate toward trivial
- arbitrary recursion anchor protocols
- arbitrary partition techniques
- but: linear computation times

Order-Statistic Trees



Order-Statistic Trees $[c, i)$



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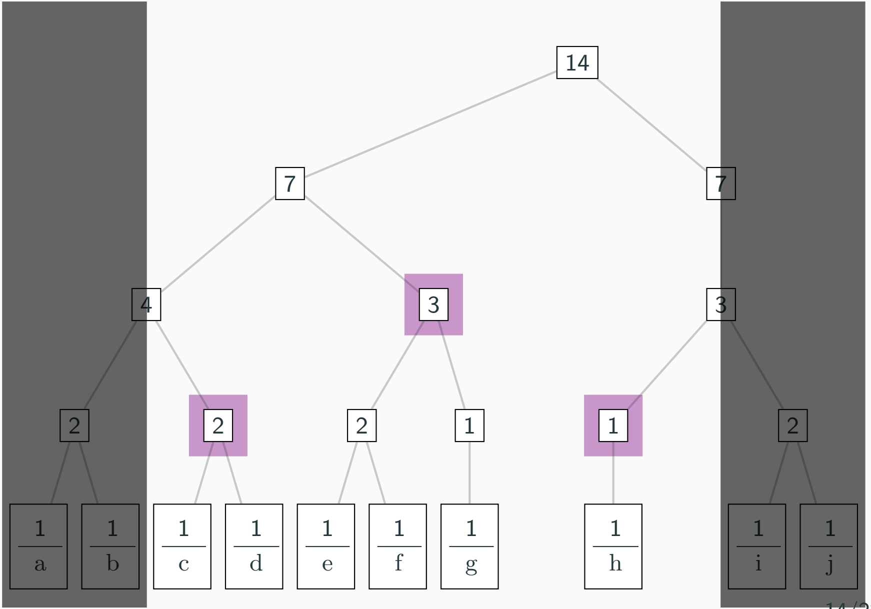

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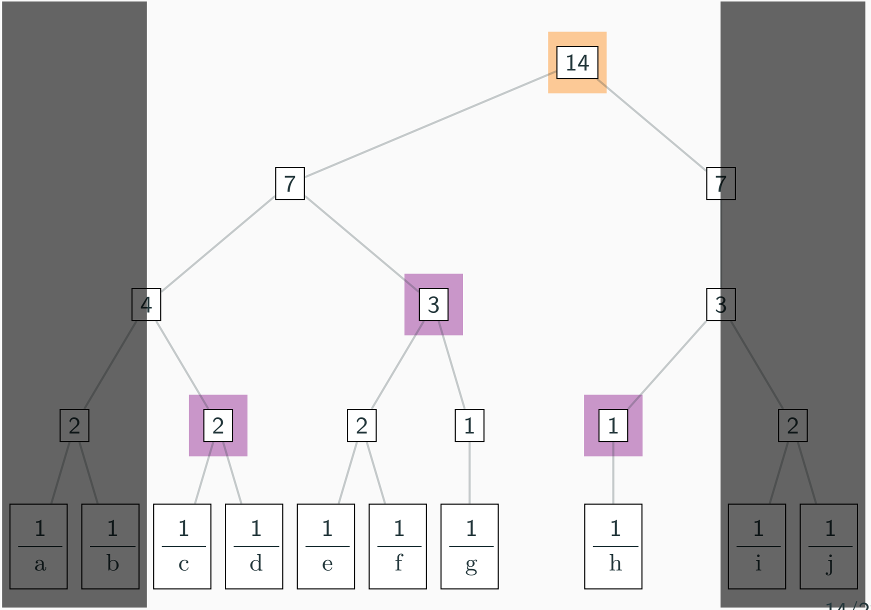

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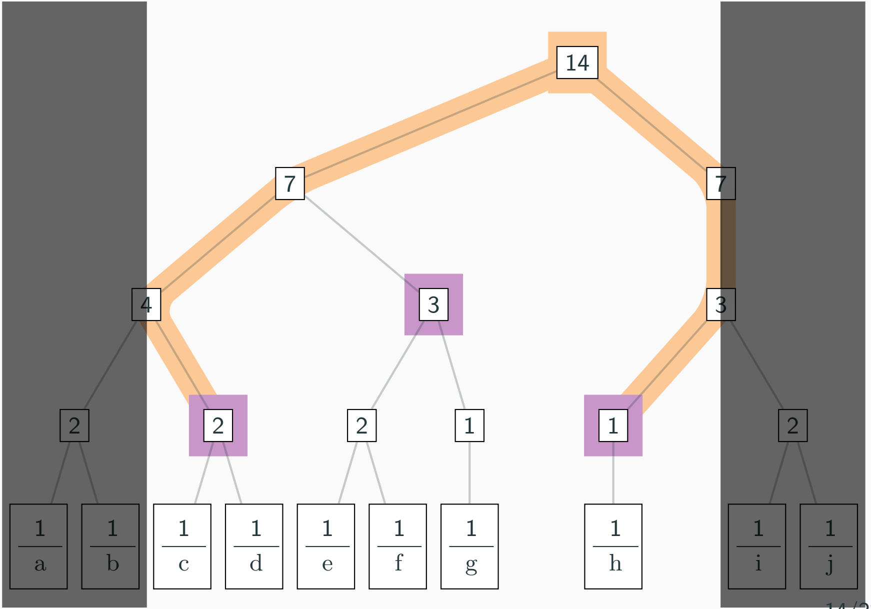

Monoid Trees

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- set of labels: $\{n : 0 <= n < 2^{256} - 1\}$
- binary associative function: xor
- neutral element: 0
- lifting into the monoid: sha256

Resulting Hash Functions

Let U be a set, $\preceq$ a linear order on U, $\mathcal{M} = (M, \oplus, \mathbb{0})$ a monoid, and $\mathsf{f} : U \to M$.

We *lift* f *to finite sets via* $\mathcal{M}$ to obtain $\mathsf{lift}_{\mathsf{f}}^{\mathcal{M}} : \mathcal{P}(U) \rightharpoonup M$ with:

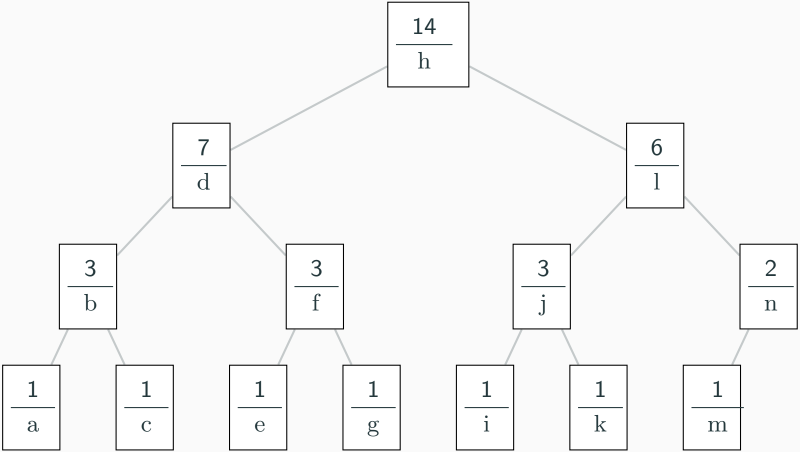
$$\begin{aligned} \mathsf{lift}_{\mathsf{f}}^{\mathcal{M}}(\emptyset) &:= \mathbb{0}, \\ \mathsf{lift}_{\mathsf{f}}^{\mathcal{M}}(S) &:= \mathsf{f}\big(\min(S)\big) \oplus \mathsf{lift}_{\mathsf{f}}^{\mathcal{M}}\big(S \setminus \{\min(S)\}\big). \end{aligned}$$

That is: $\mathsf{lift}_{\mathsf{f}}^{\mathcal{M}}(S) = \mathsf{f}(s_1) \oplus \mathsf{f}(s_2) \oplus \cdots \oplus \mathsf{f}(s_{|S|})$.

Advantages

- solid worst-case communication complexity
- implementation independence

Alternative Datastructures



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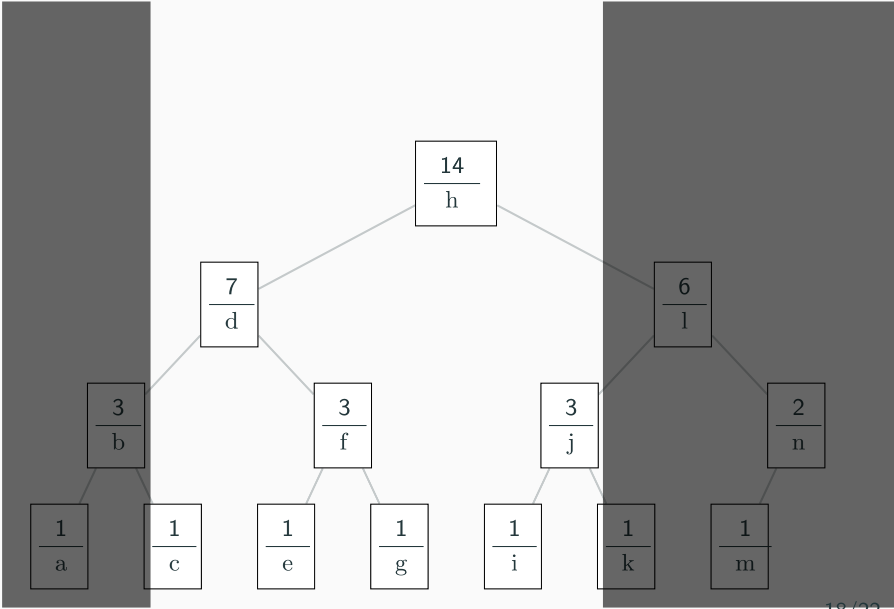

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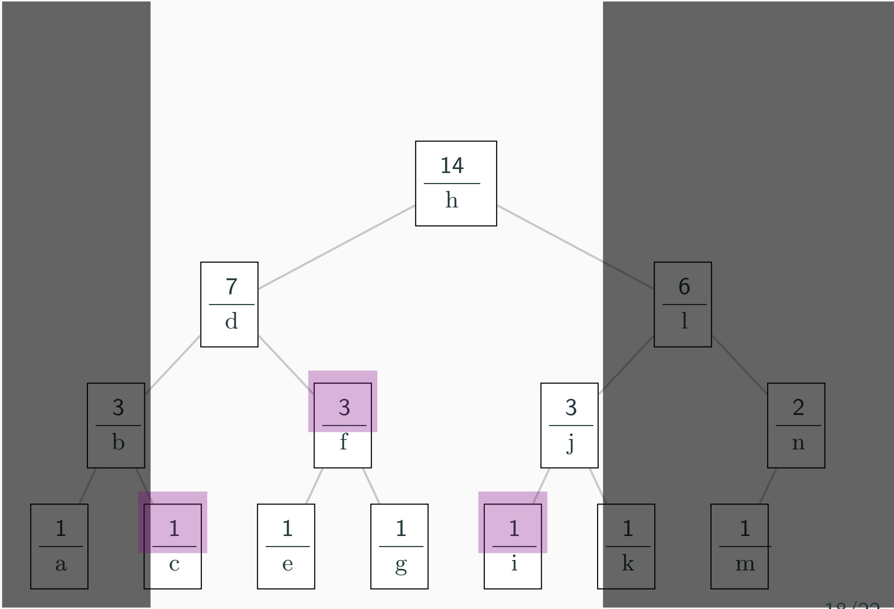

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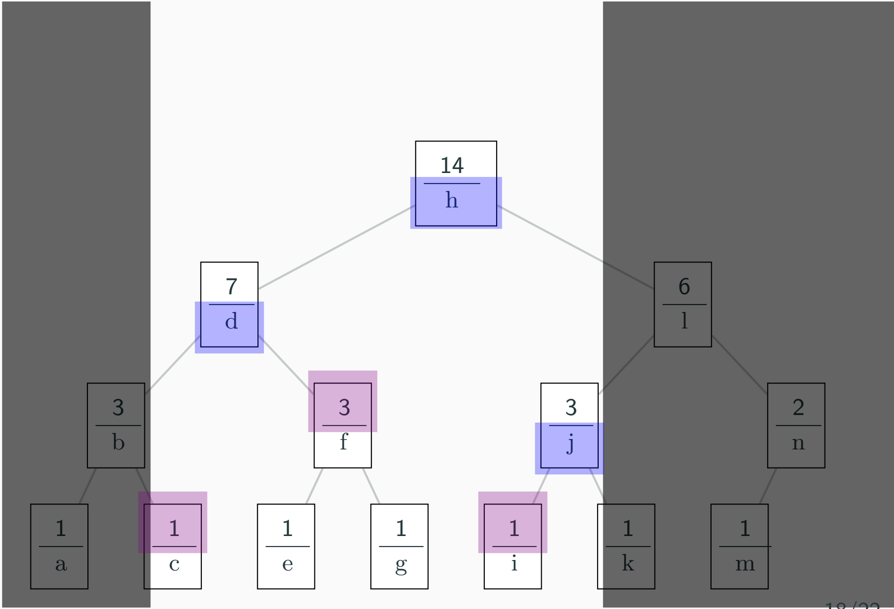

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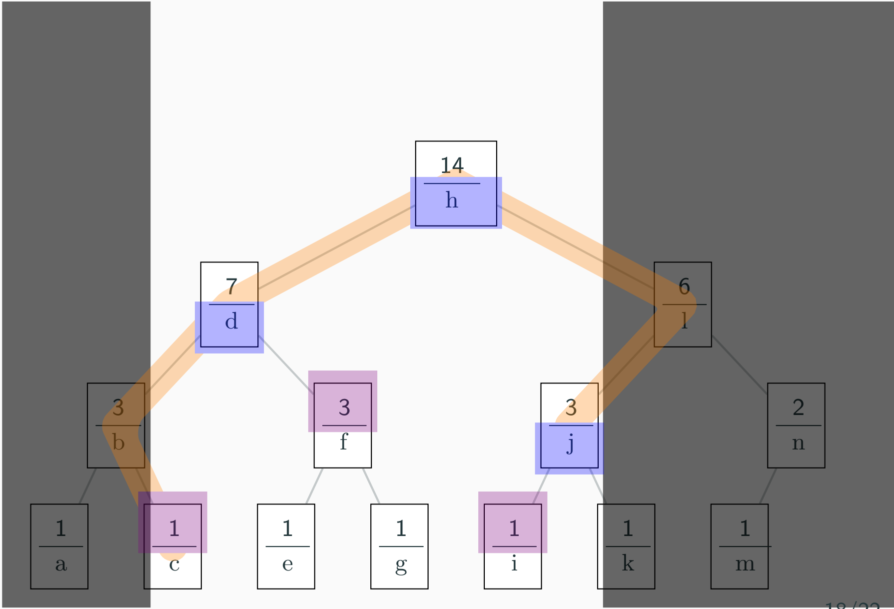
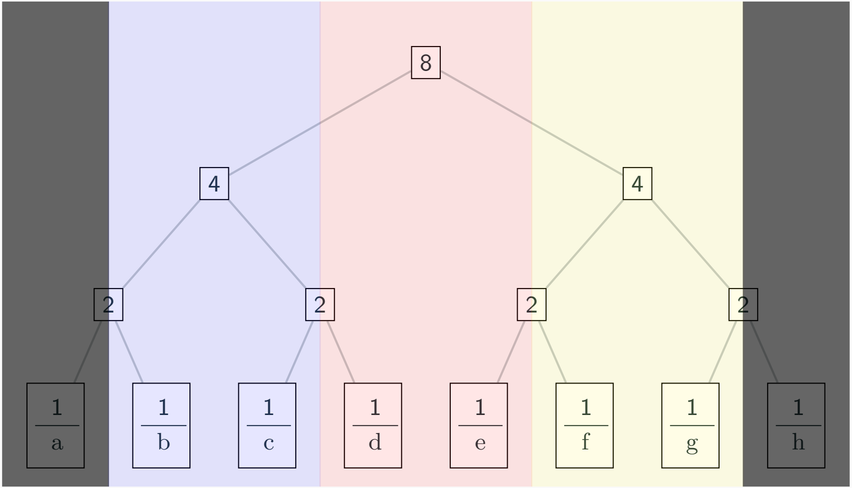


Successive Ranges



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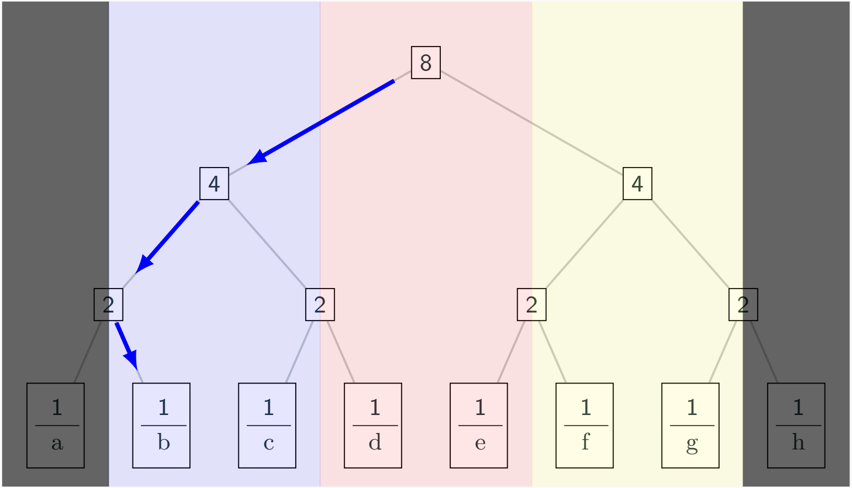

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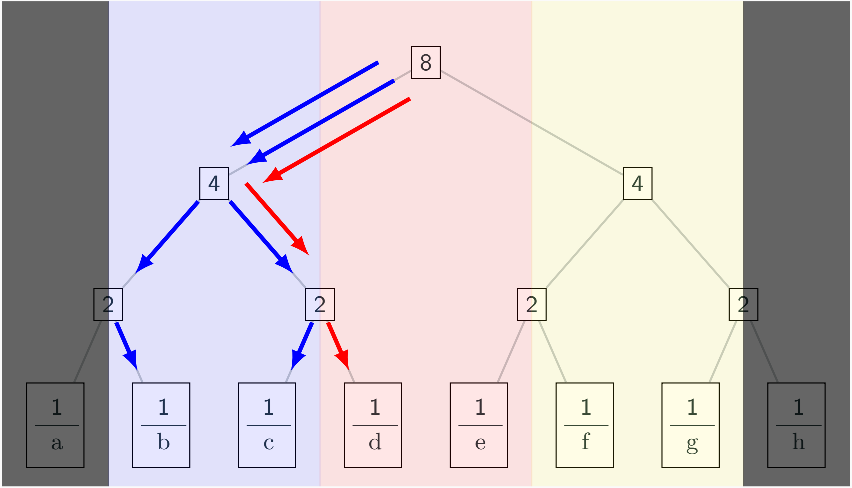

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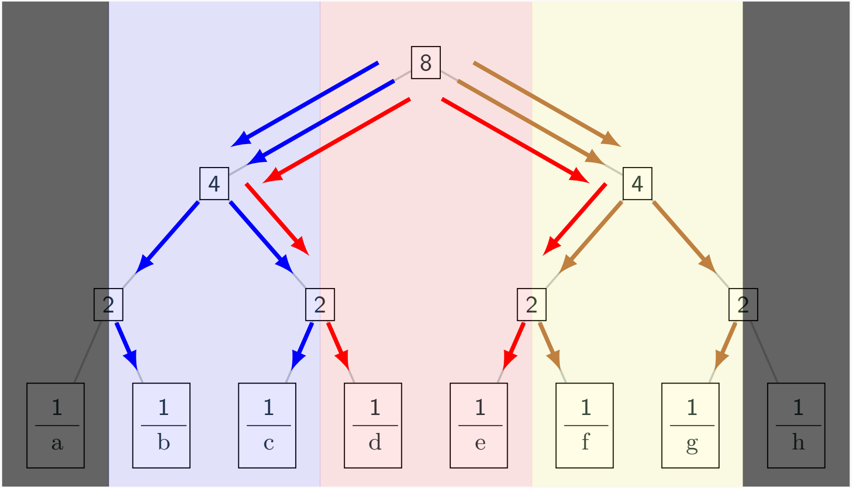

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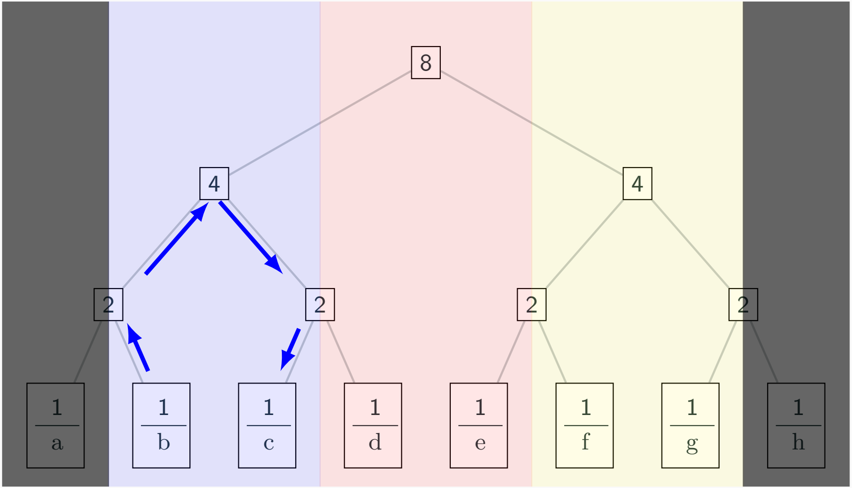

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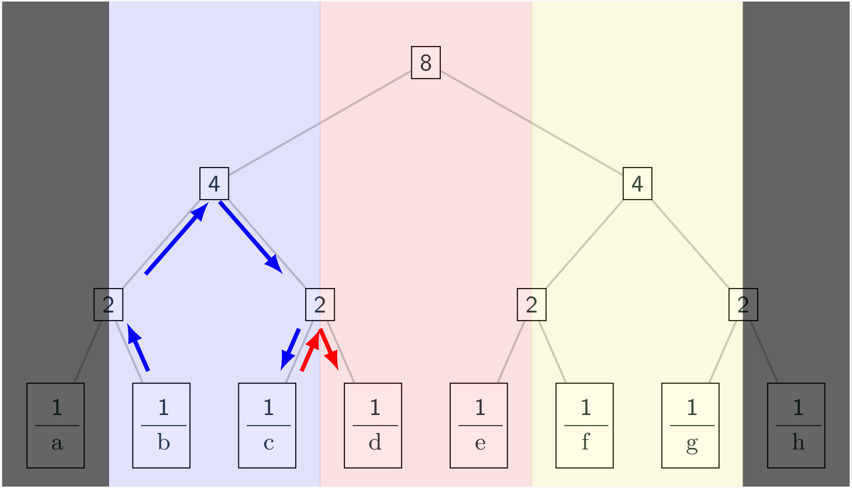

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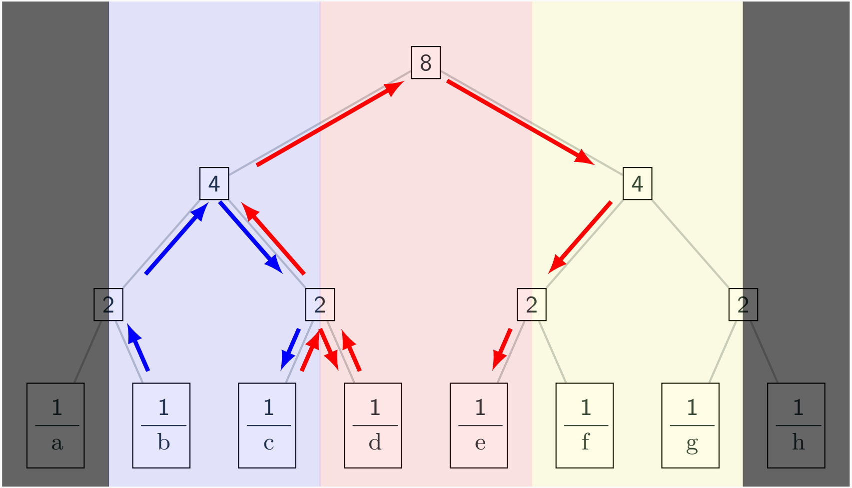

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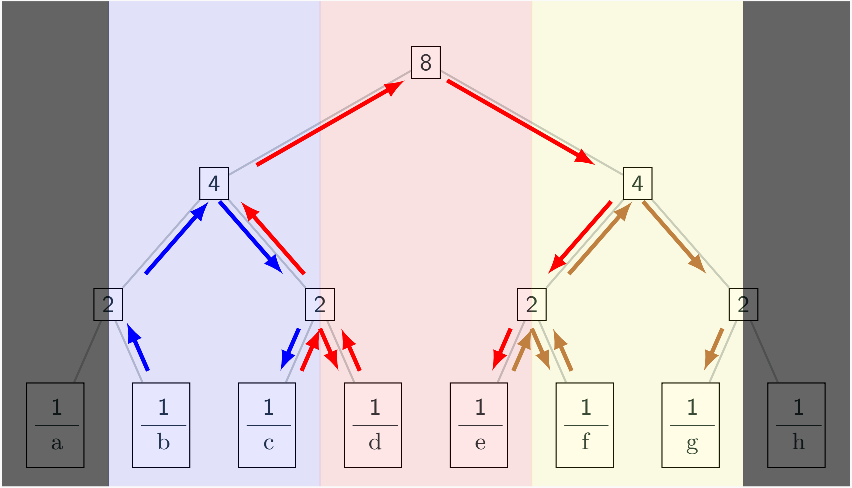

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- adversary must sabotage reconciliation
- active and passive adversaries
- randomized boundaries defeat individual collisions
- better protection: secure hash functions

Secure Hash Functions

Let U be a set, $\mathcal{M} = (M, \oplus, \mathbb{0})$ a monoid, and $\mathsf{f} : \mathcal{P}(U) \to M$.

f is *set-homomorphic* if $\mathsf{f}(U_1 \cup U_2) = \mathsf{f}(U_1) \oplus \mathsf{f}(U_2)$.

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XOR, addition, multiplication, lattices, RSA, eliptic curves.

Let further $\preceq$ a linear order on U.

f is a *tree-friendly function* if for $S_0, S_1 \in \mathcal{P}(U)$ with $\max(S_0) \prec \min(S_1)$, we have $\mathsf{f}(S_0 \cup S_1) = \mathsf{f}(S_0) \oplus \mathsf{f}(S_1)$.

No commutativity: Cayley hash functions

Conclusion

Unlike conventional reconciliation techniques:

- robust
- simple