Range-Based Set Reconciliation

Aljoscha Meyer

Set Reconciliation

- set union over a network
- between (exactly) two machines
- unstructured data
- no shared state or history

Trivial Reconciliation

Sam Gwilym

Model and Analysis

- Alfie and Betty talk over a network
- reliable communication, rounds of unit length, unlimited bandwidth
- probabilistic solutions

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- reliable communication, rounds of unit length, unlimited bandwidth
- probabilistic solutions
- n: size of the union
- $n_\triangle$: size of the symmetric difference

Model and Analysis

- roundtrips
- communicated bytes
- computation time
- computation space

Traditional Approaches

- obtain approximation of $n_{\triangle}$
- compute message of size $\mathcal{O}(n_{\triangle})$ by iterating over all n items
- exchange messenges
- recover symmetric difference from those messages using at least $\mathcal{O}(n_{\triangle})$ time and memory

P2P Reconciliation

Peer-to-peer systems:

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- loading local set into memory infeasible
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$\implies$ traditional approaches don't work

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- binary-search for differences
    - collaboratively
    - over the wire
    - in parallel
    - ok, the analogy is not perfect

Range-Based Set Reconciliation

$X_\alpha := \{\mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}, \mathrm{h}\}$

$X_\beta := \{\mathrm{a}, \mathrm{e}, \mathrm{f}, \mathrm{g}\}$

$\leftarrow$ $\big(\mathrm{a}, \mathsf{fp}(\{\mathrm{a}, \mathrm{e}, \mathrm{f}, \mathrm{g}\}), \mathrm{i}\big)$

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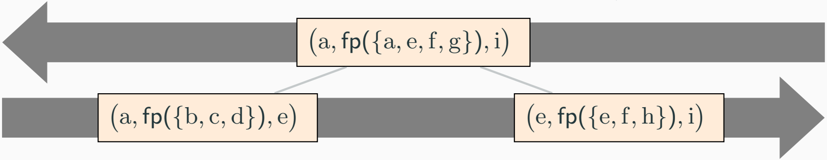

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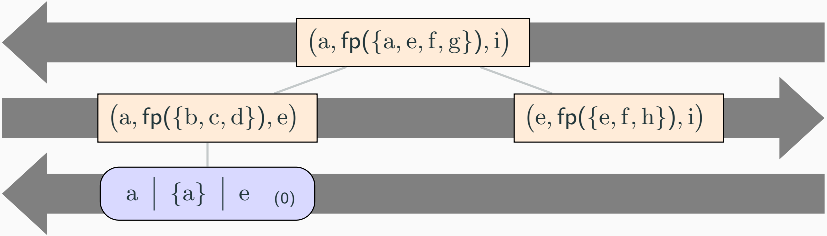

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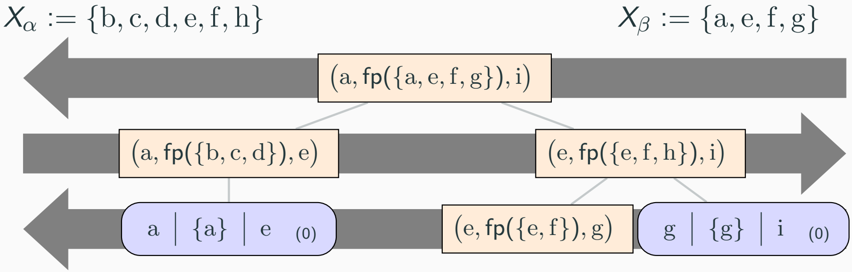

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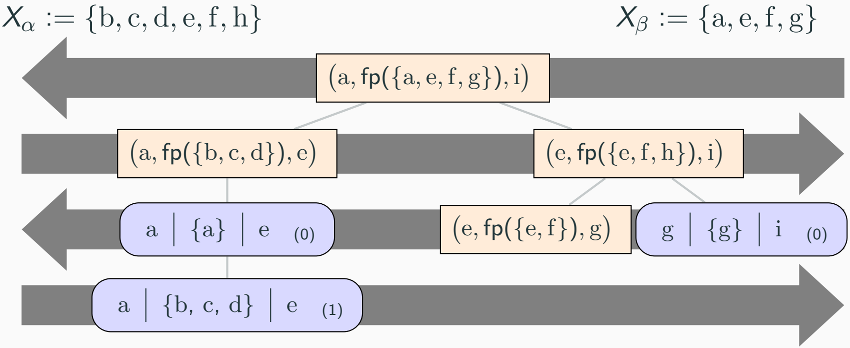

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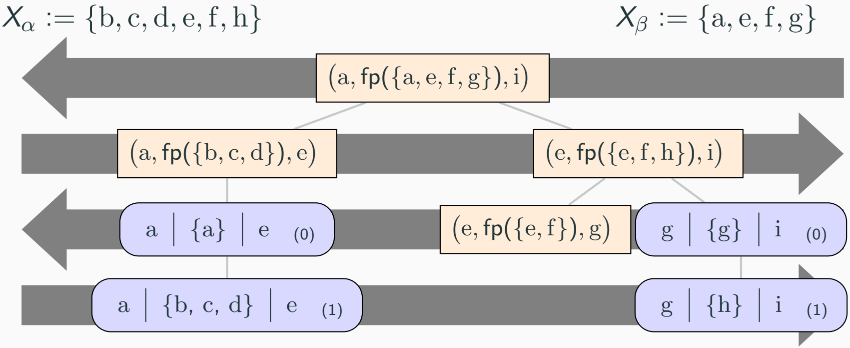

Some Nice Properties

- reasonably efficient: $\mathcal{O}(\min(n_\triangle \cdot \log(n), n))$ bytes communication, $\mathcal{O}(1)$ working memory
- can tune bandwidth vs roundtrip minimization
- arbitrary recursion anchor protocols
- arbitrary partition techniques

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- but: linear computation times

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- $\binom{n}{2}$ possible subranges $\implies$ cannot precompute all fingerprints
- free space budget of $\mathcal{O}(n)$
- labeled trees!

Intermission — Merkle Search Trees

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- use that shape for a Merkle tree of your set
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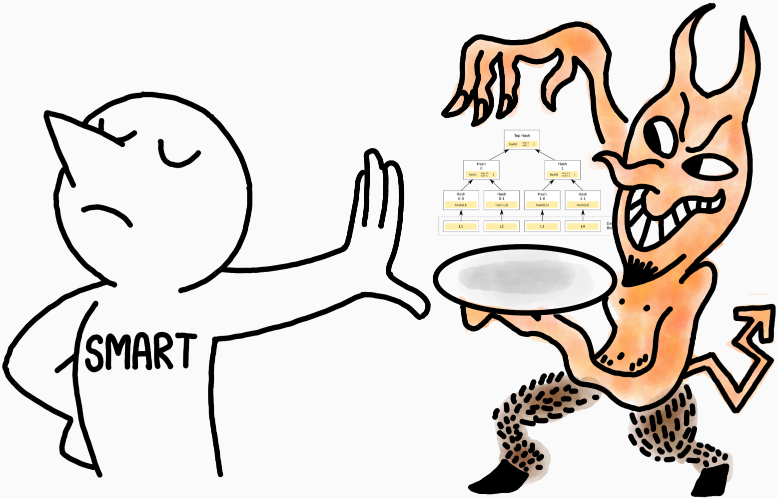
Problems:

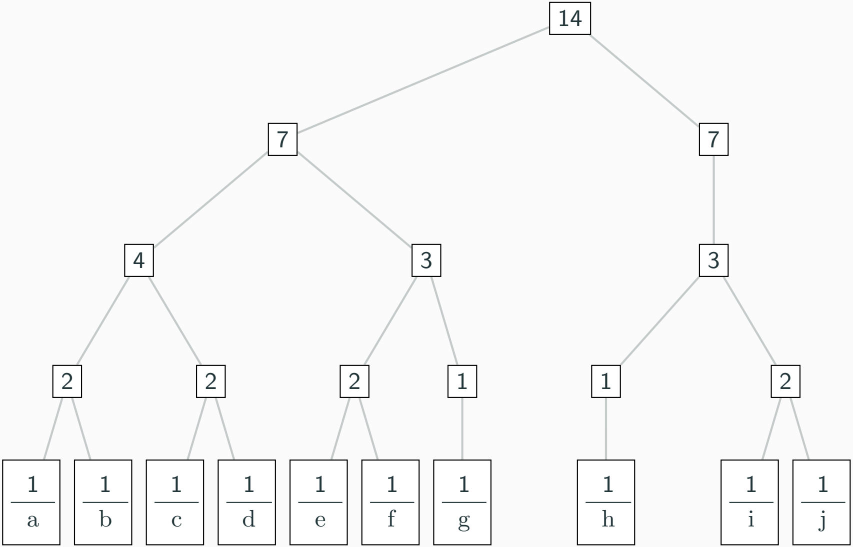
- easily attacked with degenerate tree shapes
- peers cannot optimize data representation for their use-case

Say No to Merkle Trees

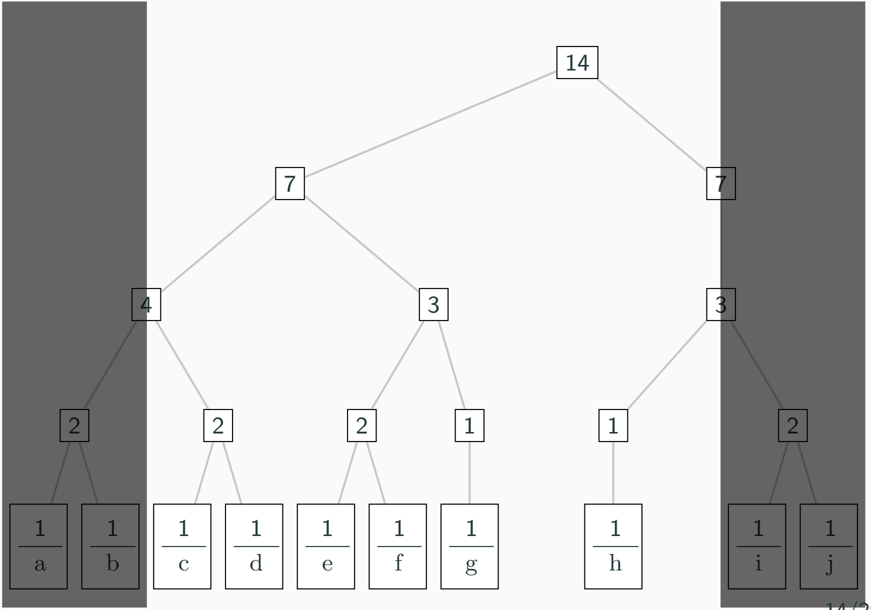


Drawing: Sam Gwilym

Order-Statistic Trees



Order-Statistic Trees $[c, i)$



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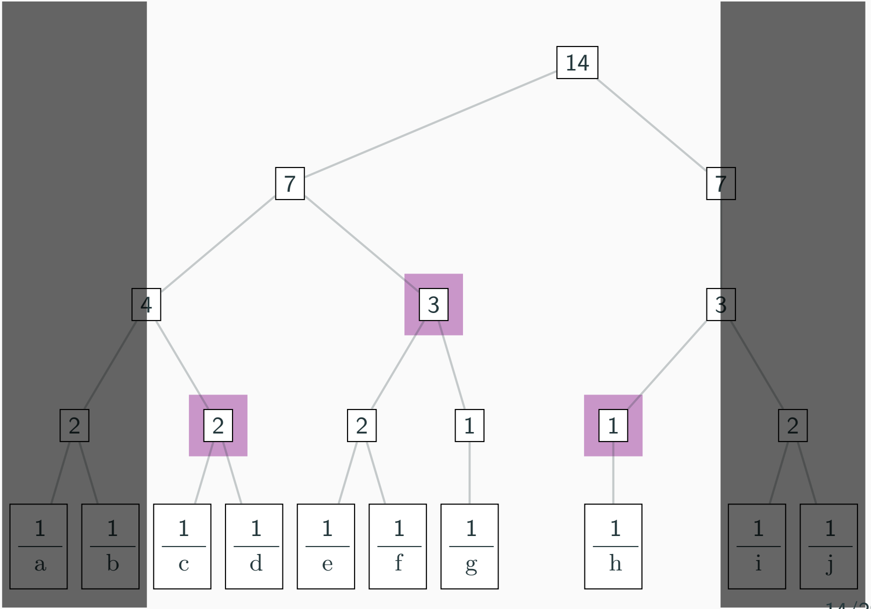

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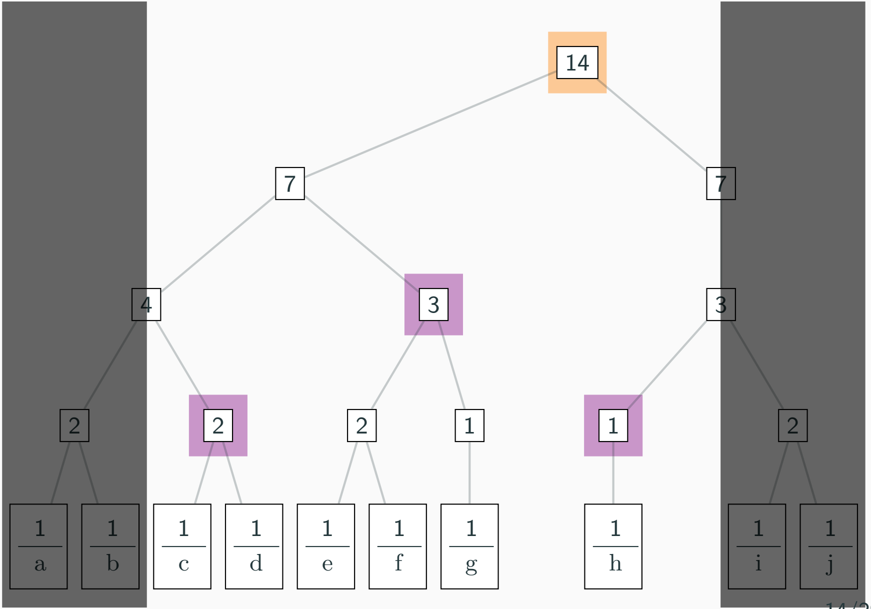

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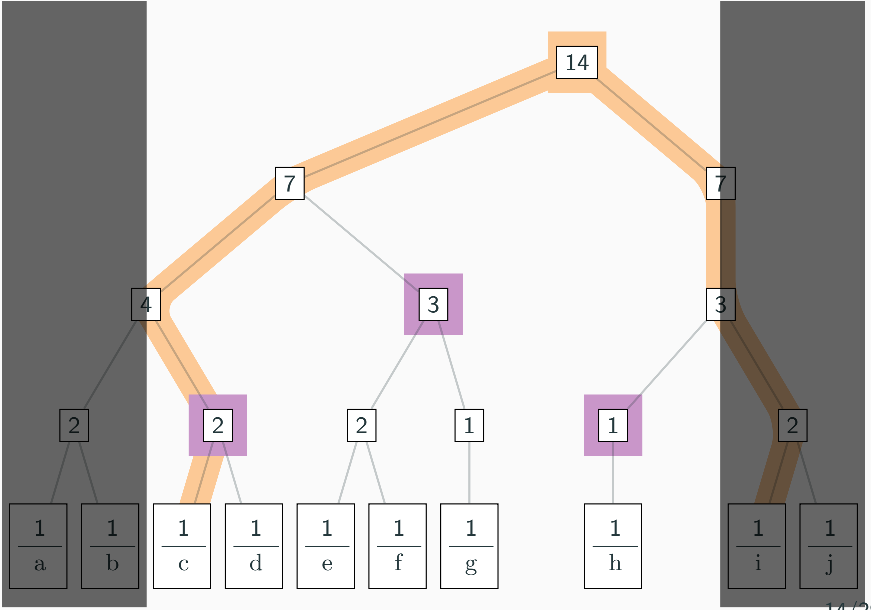

Monoid Trees

- set of labels: $\mathbb{N}$
- binary associative function: $+$
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- set of labels: $\{n : 0 <= n < 2^{256} - 1\}$
- binary associative function: xor
- neutral element: 0
- transforming into the monoid: sha256

Resulting Functions

Let U be a set, $\preceq$ a linear order on U, $\mathcal{M} = (M, \oplus, \mathbb{0})$ a monoid, and $\mathsf{h} : U \to M$.

We *lift* h *to finite sets via* $\mathcal{M}$ to obtain $\mathsf{lift}_{\mathsf{h}}^{\mathcal{M}} : \mathcal{P}(U) \rightharpoonup M$ with:

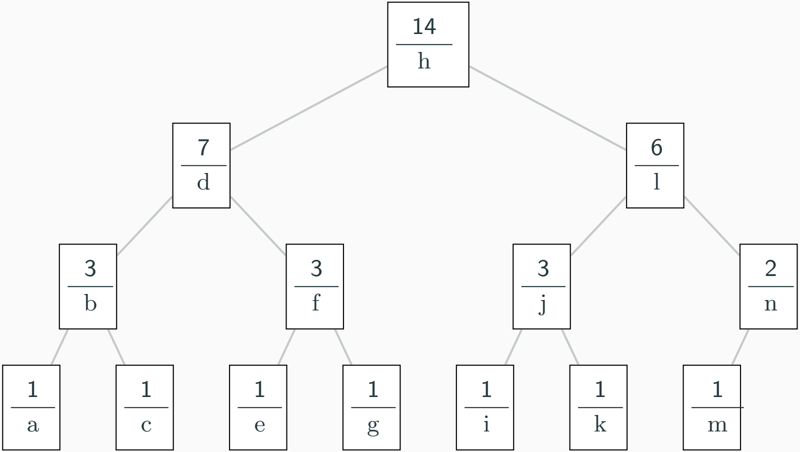
$$\begin{aligned} \mathsf{lift}_{\mathsf{h}}^{\mathcal{M}}(\emptyset) &:= \mathbb{0}, \\ \mathsf{lift}_{\mathsf{h}}^{\mathcal{M}}(S) &:= \mathsf{h}\big(\min(S)\big) \oplus \mathsf{lift}_{\mathsf{h}}^{\mathcal{M}}\big(S \setminus \{\min(S)\}\big). \end{aligned}$$

That is: $\mathsf{lift}_{\mathsf{h}}^{\mathcal{M}}(S) = \mathsf{h}(s_1) \oplus \mathsf{h}(s_2) \oplus \cdots \oplus \mathsf{h}(s_{|S|})$.

Advantages

- solid worst-case communication complexity
- implementation independence

Alternative Datastructures



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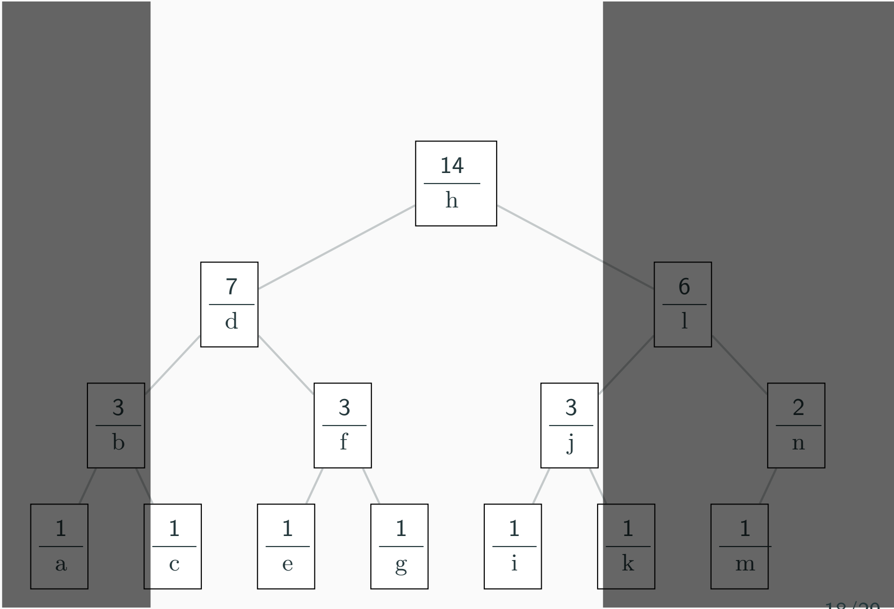

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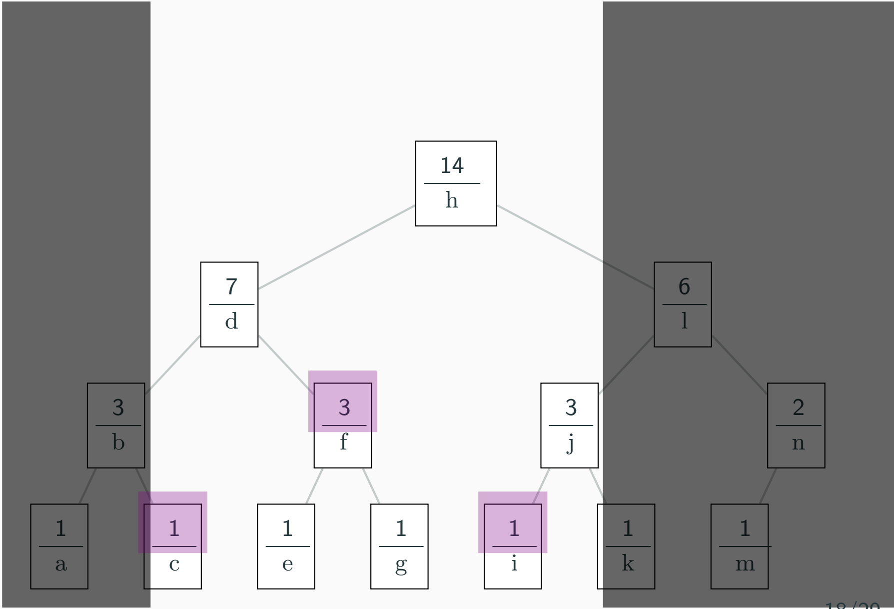

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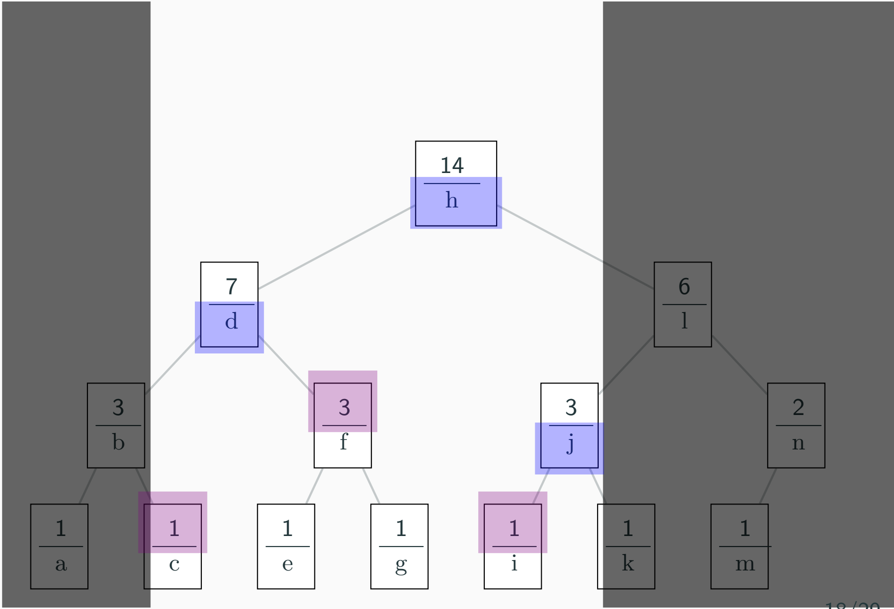

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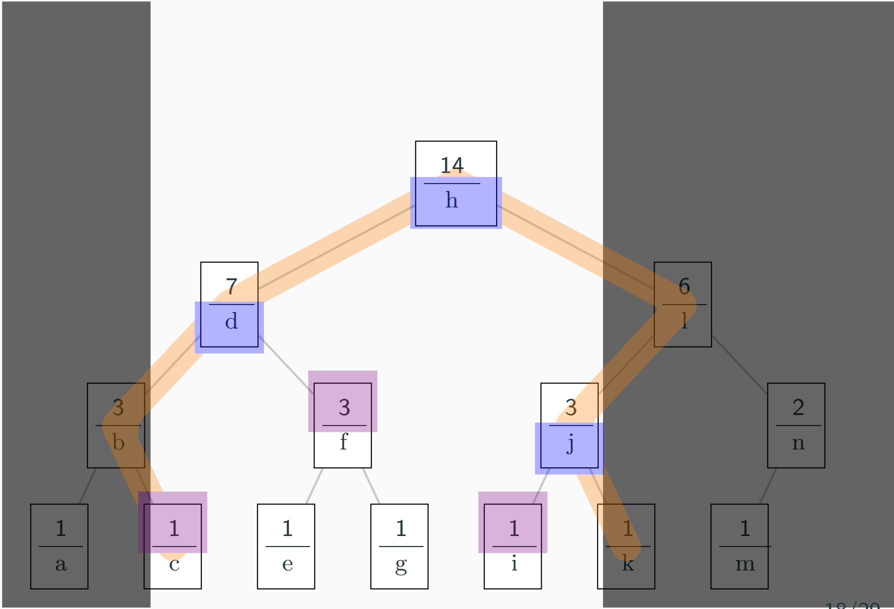
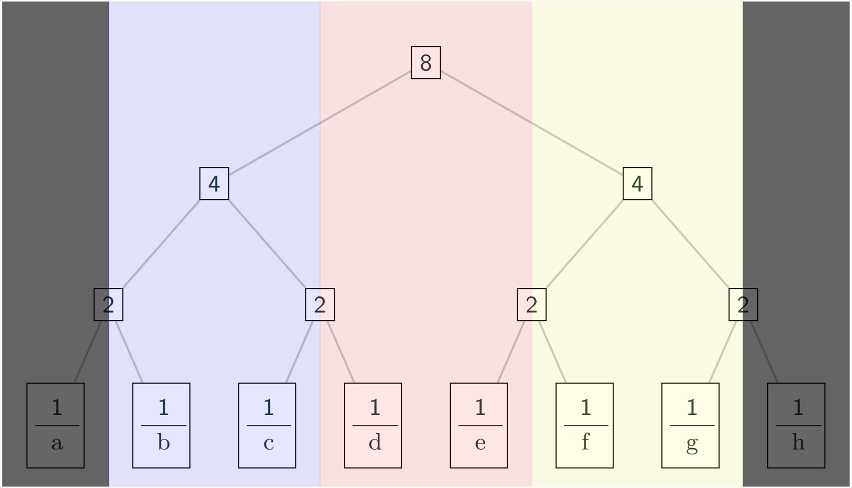


Alternative Datastructures

- monoid B-Tree
- monoid prefix tree
- monoid skip list
- monoid zip-tree
- no datastructure at all
- ...

Successive Ranges



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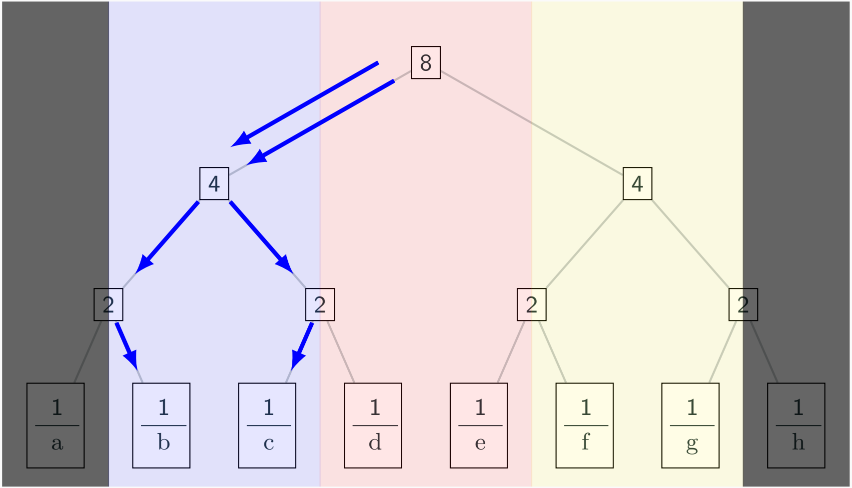

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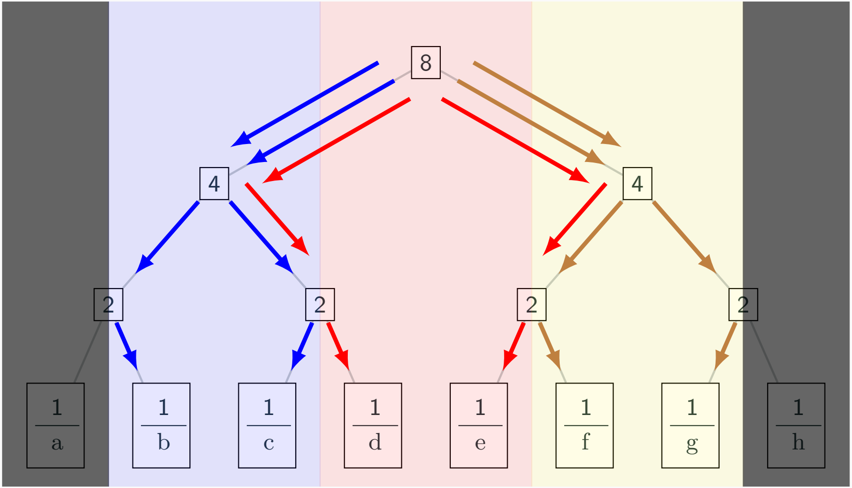

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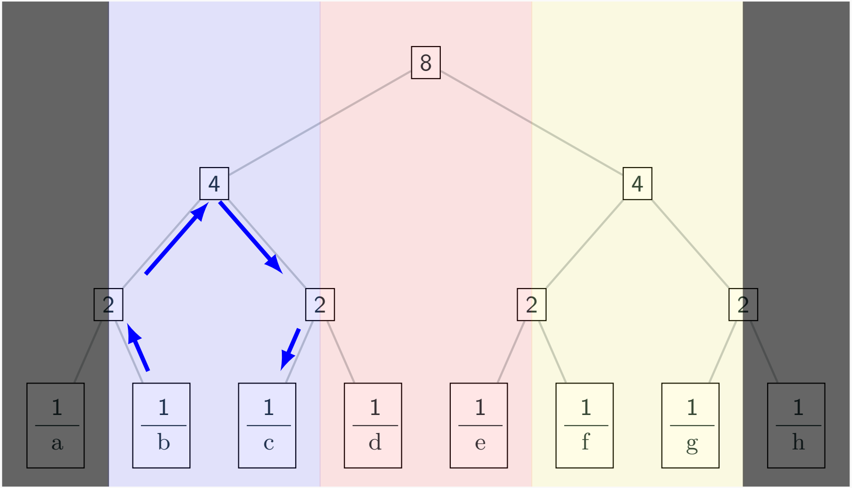

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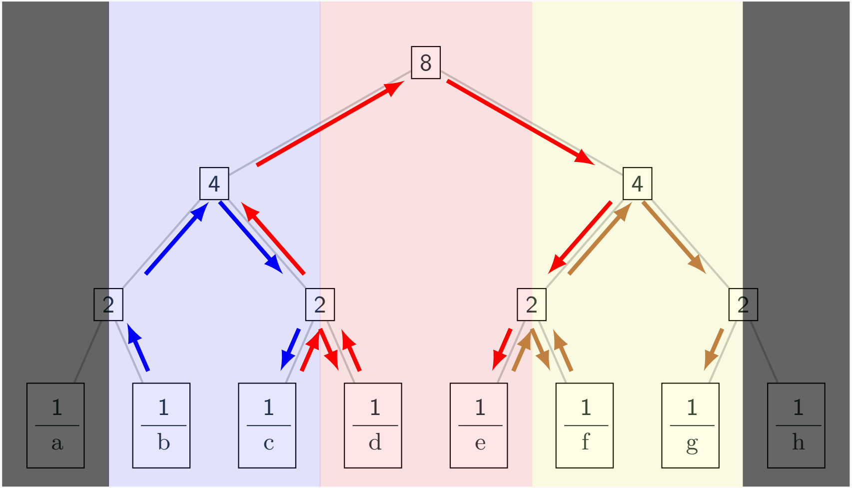

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- active and passive adversaries
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- better protection: secure hash functions

Secure Hash Functions

Let U be a set, $\mathcal{M} = (M, \oplus, \mathbb{0})$ a monoid, and $\mathsf{f} : \mathcal{P}(U) \to M$.

f is *set-homomorphic* if $\mathsf{f}(U_1 \cup U_2) = \mathsf{f}(U_1) \oplus \mathsf{f}(U_2)$.

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h is a *tree-friendly function* if for $S_0, S_1 \in \mathcal{P}(U)$ with $\max(S_0) \prec \min(S_1)$, we have $\mathsf{h}(S_0 \cup S_1) = \mathsf{h}(S_0) \oplus \mathsf{h}(S_1)$.

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No commutativity: Cayley hash functions

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- divide-and-conquer to find differences by checking fingerprint equality
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- remember to say no to Merkle trees

Bonus Slides!

Reducing Computation Times

- Step 1: Put a Merkle tree on it
- Step 2: ???
- Step 3: Profit

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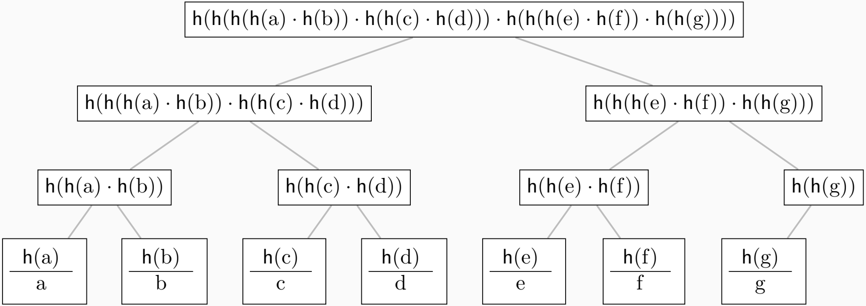
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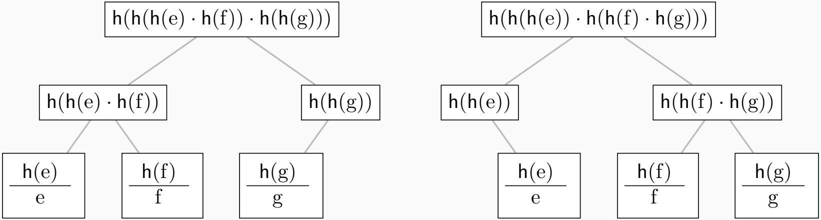
Merkle Trees



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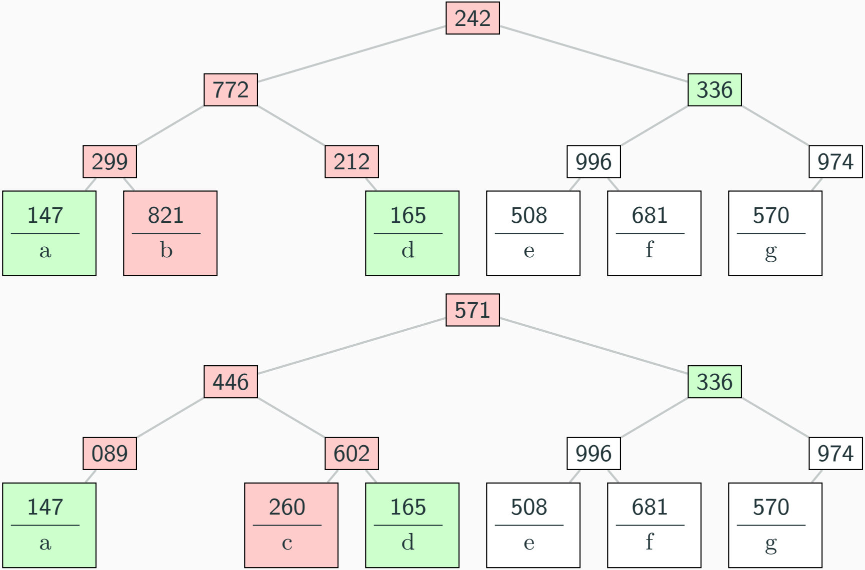

Merkle Tree Reconciliation

- inflexible data representation
- inacceptable worst-case complexity
    - remember, some peers are out to get us