

Range-Based Set Reconciliation

Aljoscha Meyer

Set Reconciliation

- set union over a network
- between (exactly) two machines
- unstructured data
- no shared state or history

Trivial Reconciliation



Sam Gwilym

Model and Analysis

- Alfie and Betty talk over a network
- reliable communication, rounds of unit length, unlimited bandwidth
- probabilistic solutions

Model and Analysis

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- reliable communication, rounds of unit length, unlimited bandwidth
- probabilistic solutions
- n : size of the union
- n_{\triangle} : size of the symmetric difference

Model and Analysis

- roundtrips
- communicated bytes
- computation time per reconciliation session
- computation space per reconciliation session
- computation time per item
- computation space per item

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- iterating over local set infeasible
- loading local set into memory infeasible
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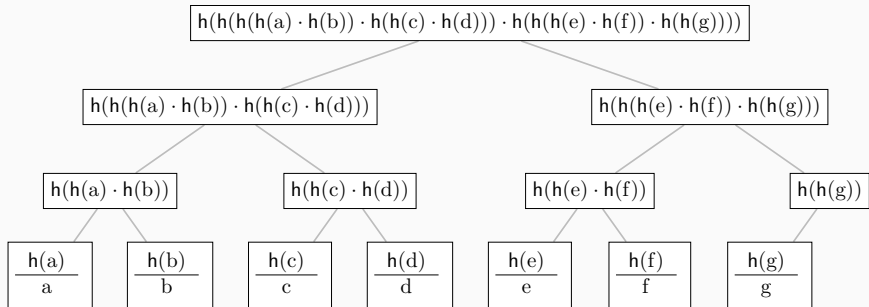
⇒ traditional approaches don't work

Reducing Computation Times

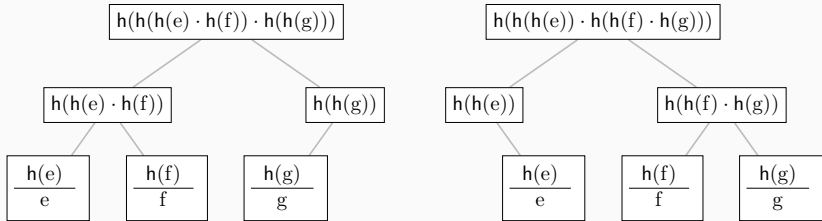
- Step 1: Put a Merkle tree on it
- Step 2: ???
- Step 3: Profit

Aurolat, Alex, and François Taïani. "Merkle search trees: Efficient state-based CRDTs in open networks." 2019 38th Symposium on Reliable Distributed Systems (SRDS). IEEE, 2019.

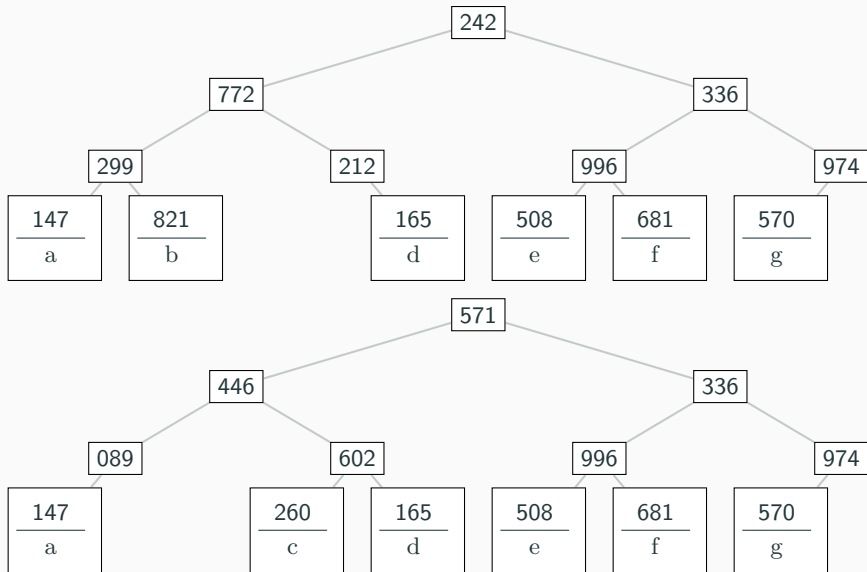
Merkle Trees



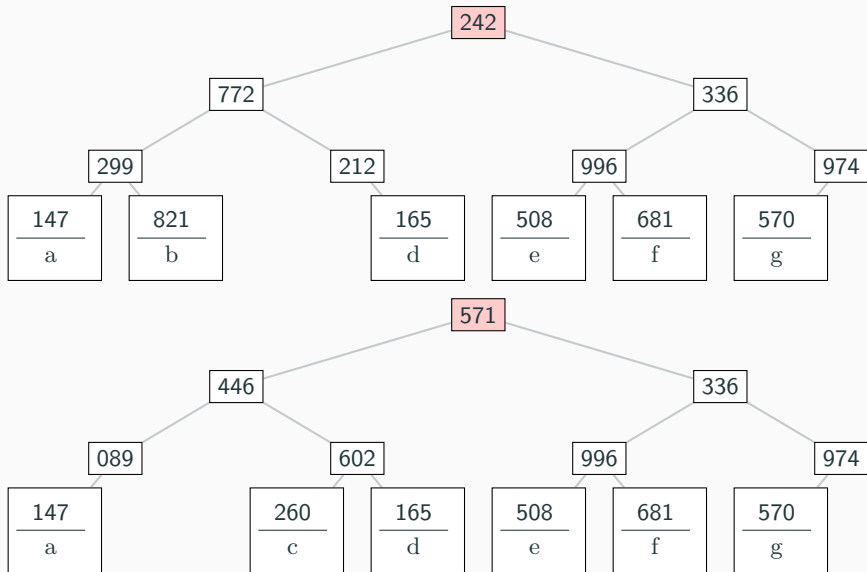
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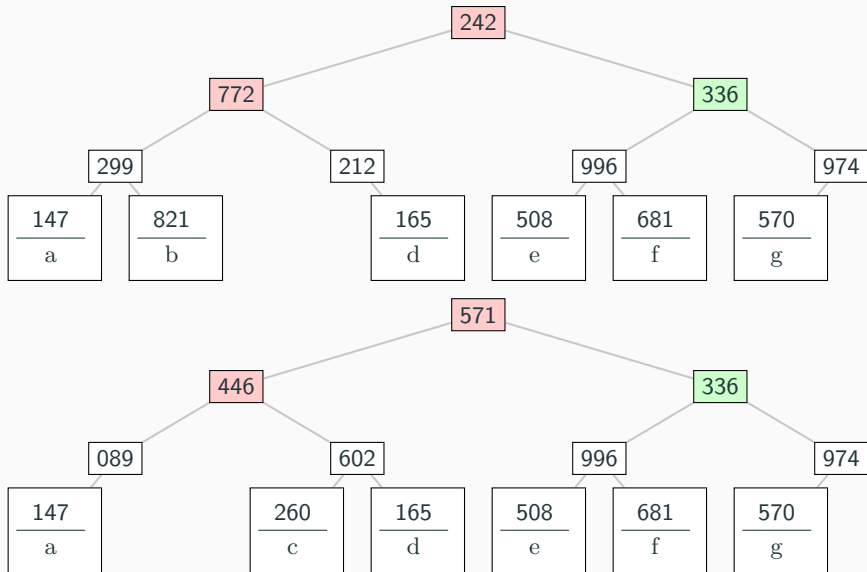
Merkle Tree Reconciliation



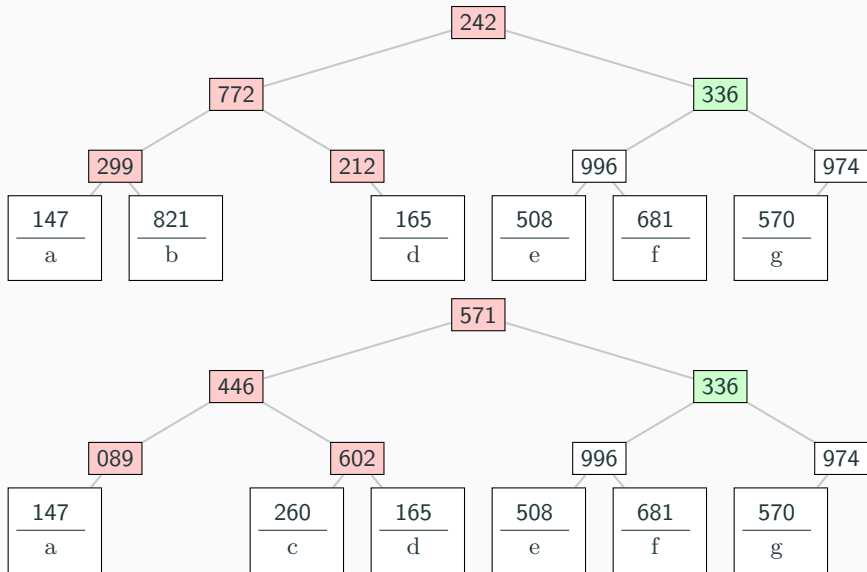
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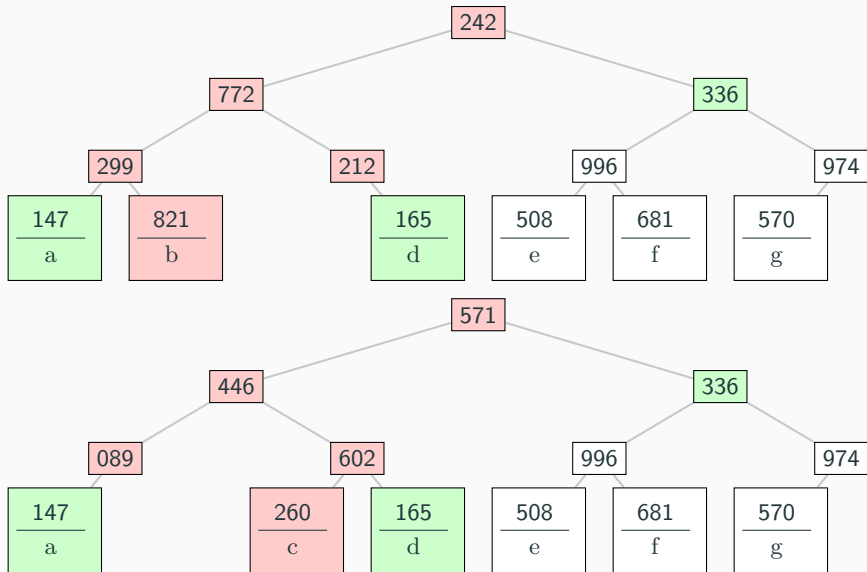
Merkle Tree Reconciliation



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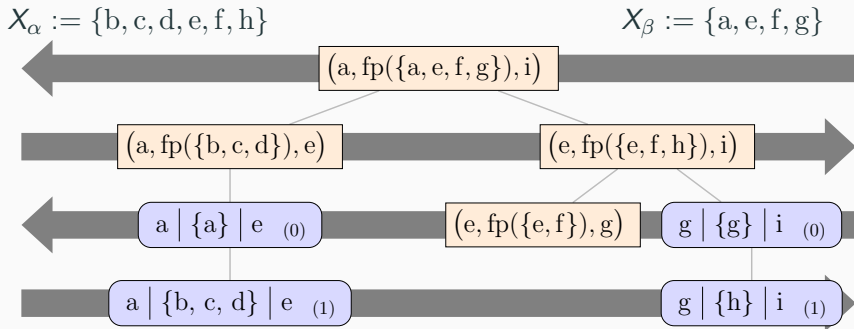
Merkle Tree Reconciliation



Merkle Tree Reconciliation

- inflexible data representation
- unacceptable worst-case complexity
 - remember, some peers are out to get us

Range-Based Set Reconciliation



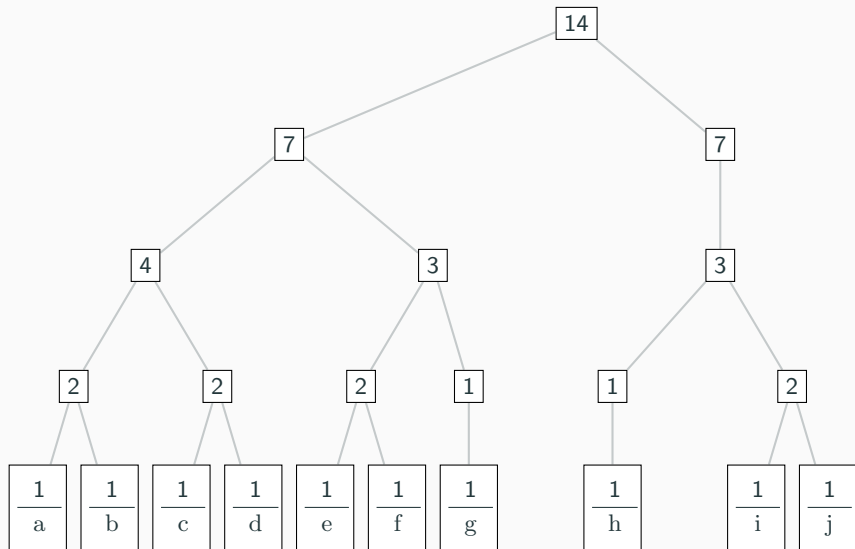
Some Nice Properties

- reasonably efficient: $\mathcal{O}(\min(n_{\Delta} \cdot \log(n), n))$ bytes communication, $\mathcal{O}(1)$ working memory
- can interpolate toward trivial
- arbitrary recursion anchor protocols
- arbitrary partition techniques

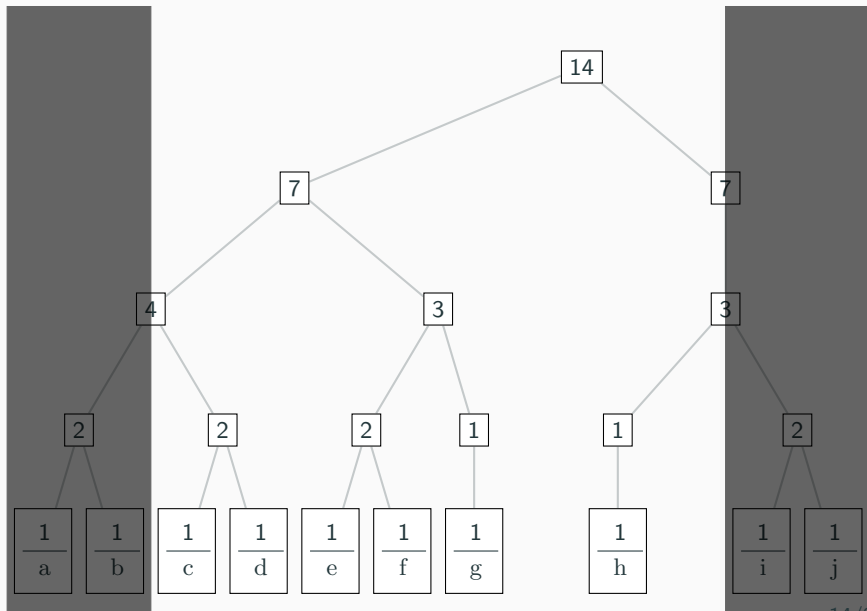
Some Nice Properties

- reasonably efficient: $\mathcal{O}(\min(n_{\Delta} \cdot \log(n), n))$ bytes communication, $\mathcal{O}(1)$ working memory
- can interpolate toward trivial
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- but: linear computation times

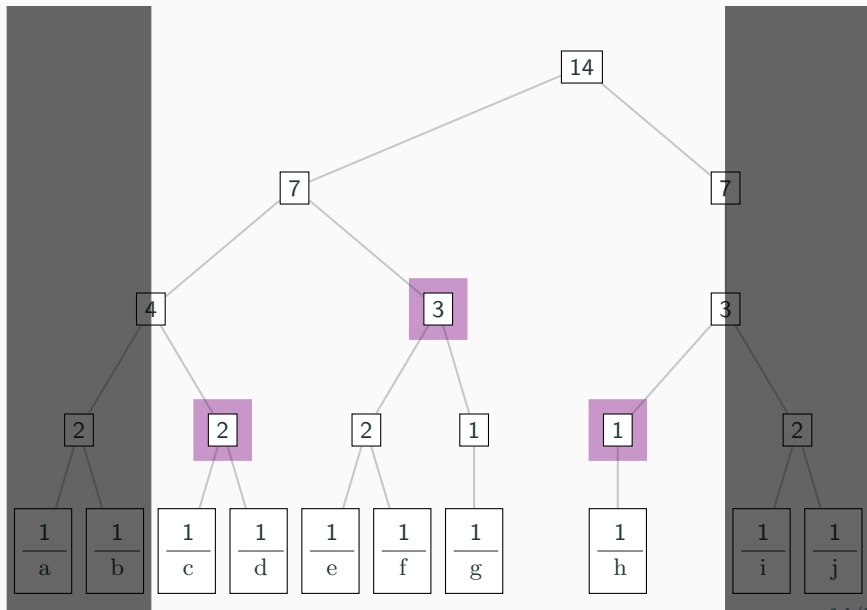
Order-Statistic Trees



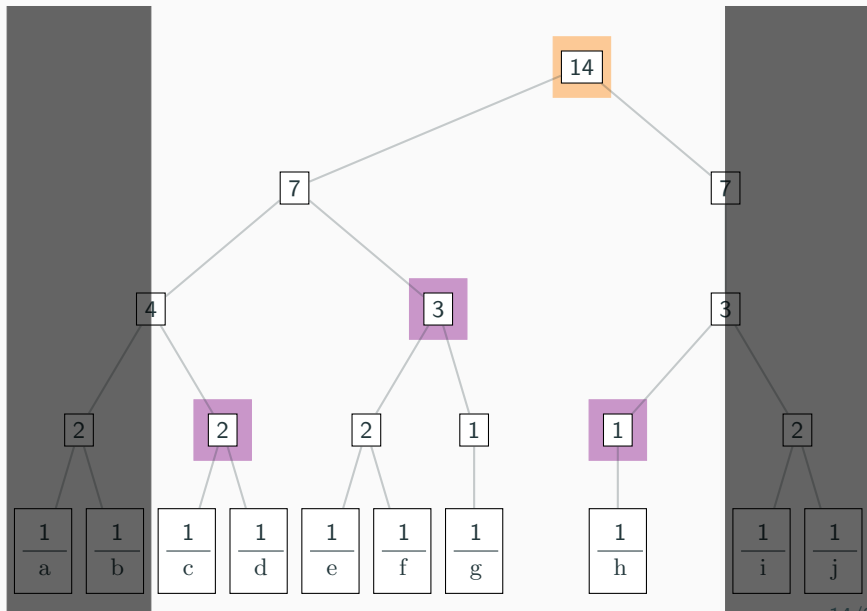
Order-Statistic Trees [c, i)



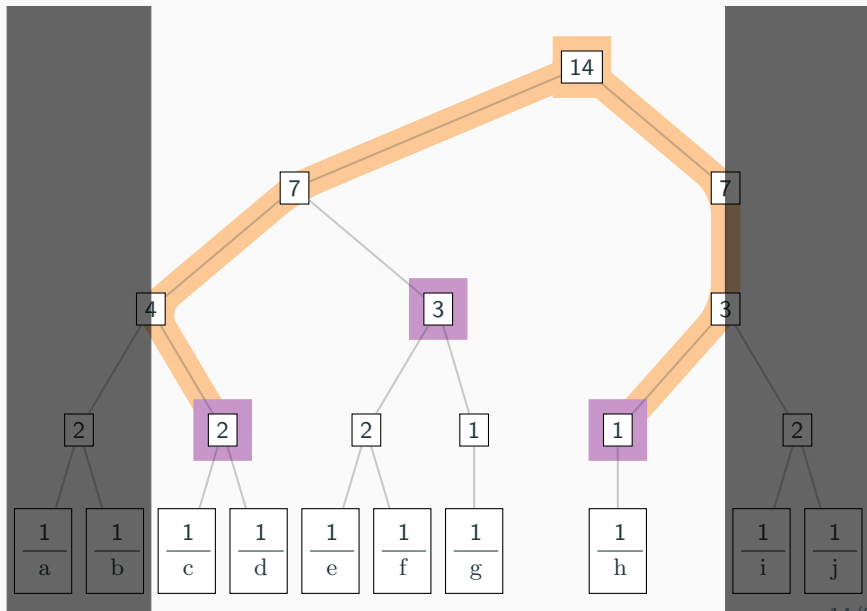
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- set of labels: $\{n : 0 \leq n < 2^{256} - 1\}$
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Monoid Trees

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- set of labels: $\{n : 0 \leq n < 2^{256} - 1\}$
 - binary associative function: `xor`
 - neutral element: 0
 - lifting into the monoid: `sha256`

Resulting Hash Functions

Let U be a set, \preceq a linear order on U , $\mathcal{M} = (M, \oplus, \mathbb{0})$ a monoid, and $f : U \rightarrow M$.

We *lift* f to finite sets via \mathcal{M} to obtain $\text{lift}_f^{\mathcal{M}} : \mathcal{P}(U) \rightarrow M$ with:

$$\text{lift}_f^{\mathcal{M}}(\emptyset) := \mathbb{0},$$

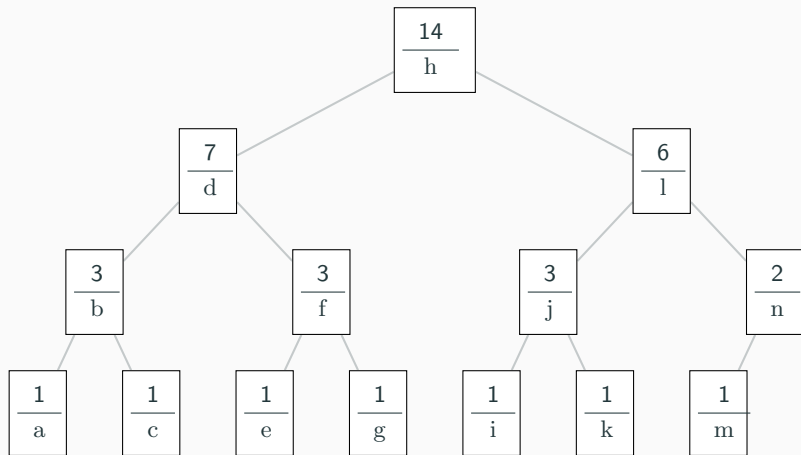
$$\text{lift}_f^{\mathcal{M}}(S) := f(\min(S)) \oplus \text{lift}_f^{\mathcal{M}}(S \setminus \{\min(S)\}).$$

That is: $\text{lift}_f^{\mathcal{M}}(S) = f(s_1) \oplus f(s_2) \oplus \cdots \oplus f(s_{|S|})$.

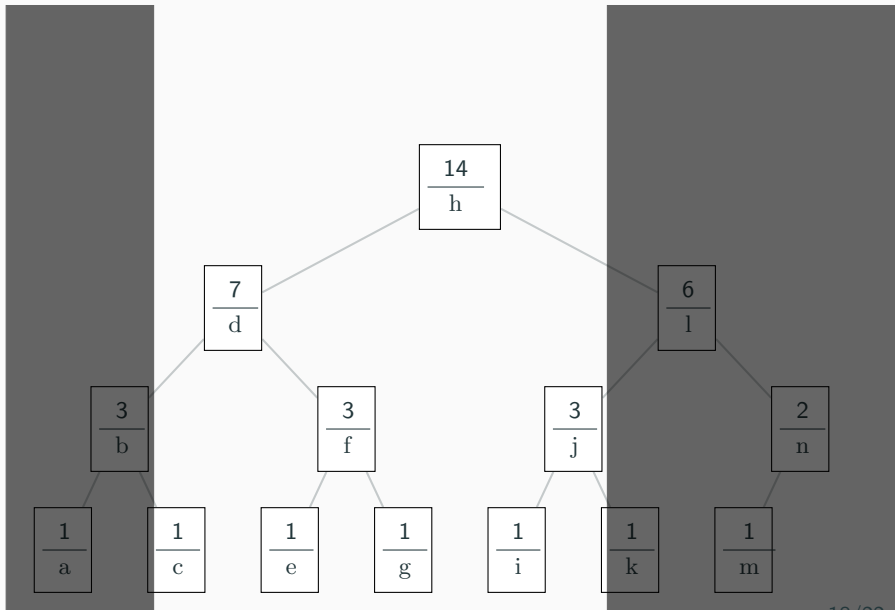
Advantages

- solid worst-case communication complexity
- implementation independence

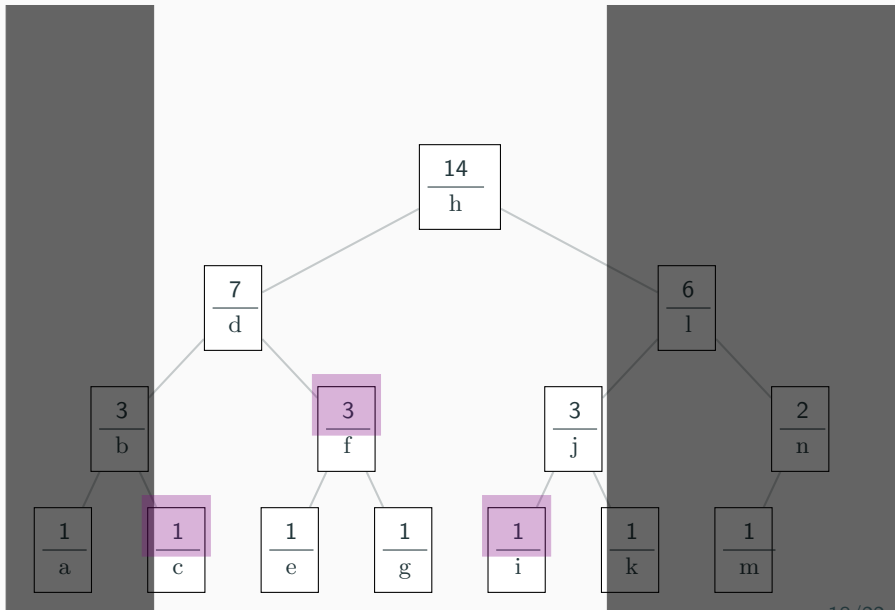
Alternative Datastructures



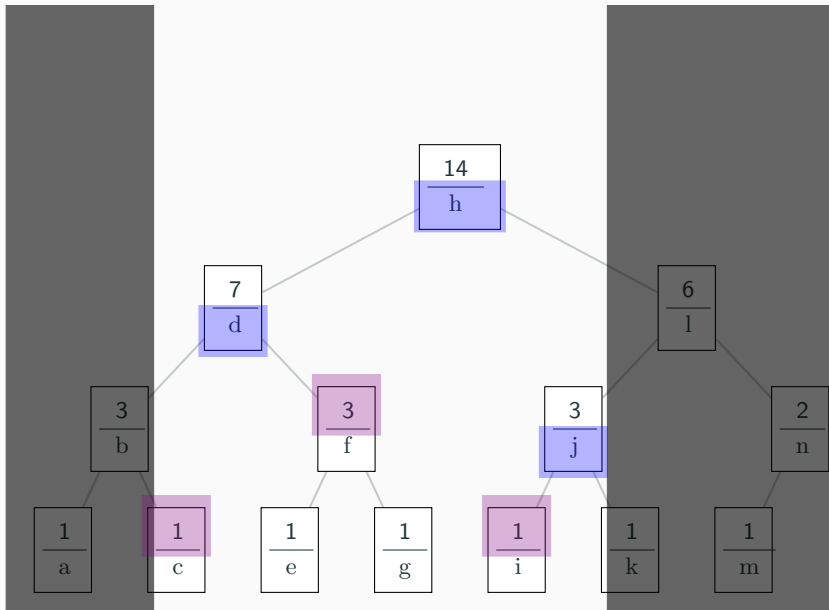
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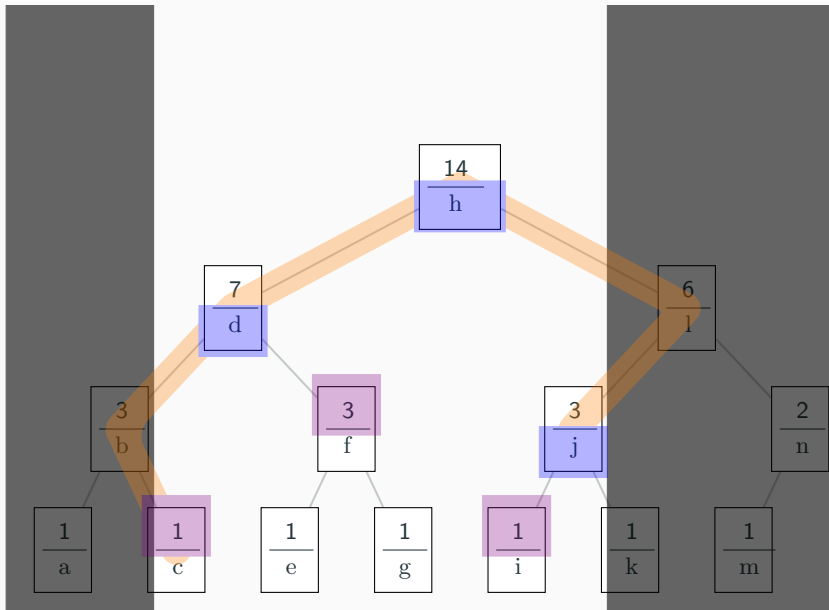
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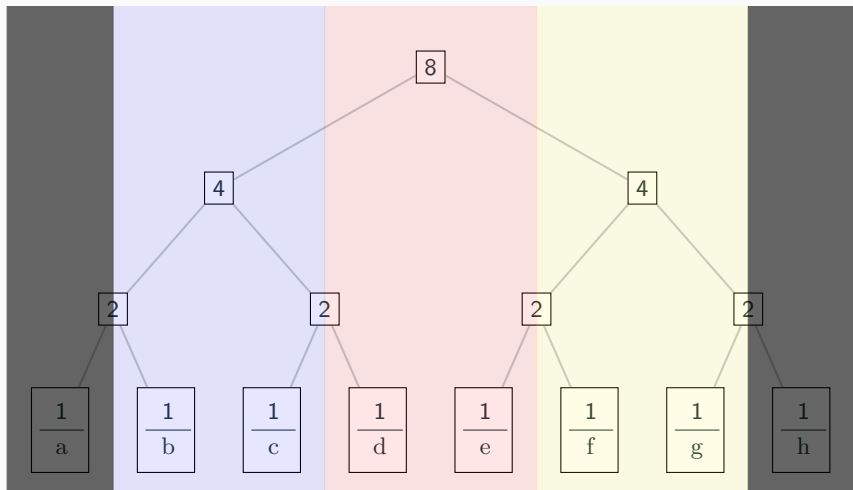
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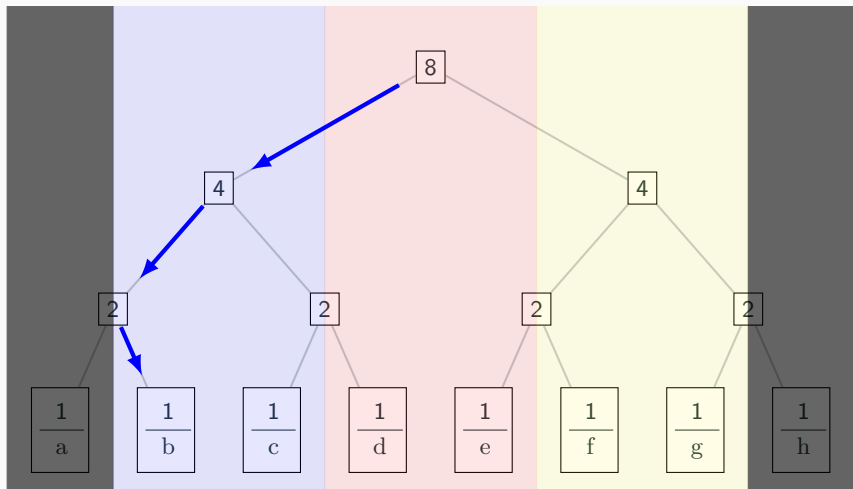
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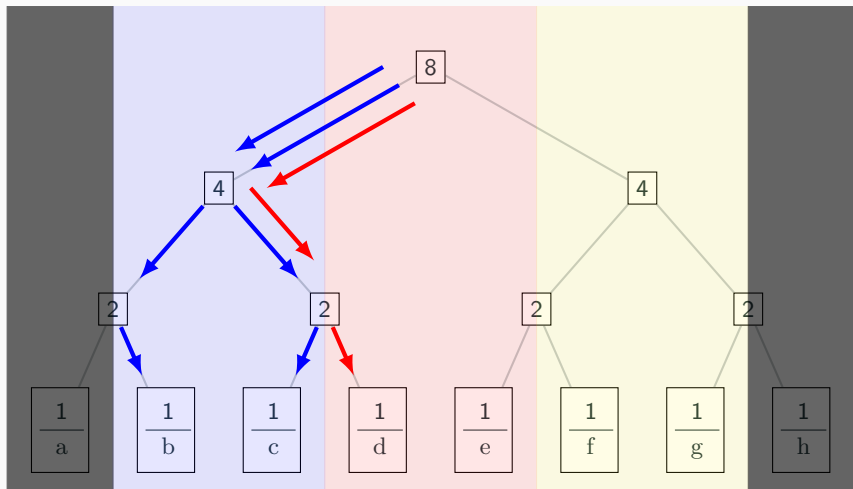
Successive Ranges



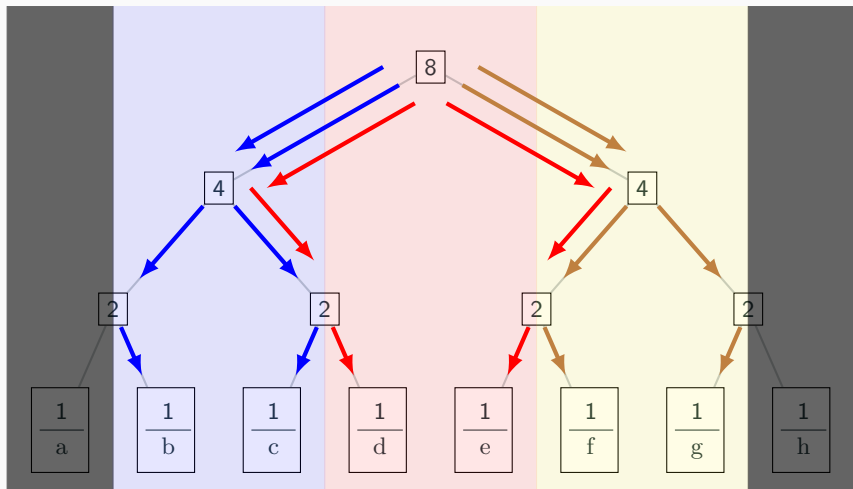
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- adversary must sabotage reconciliation
- active and passive adversaries
- randomized boundaries defeat individual collisions
- better protection: secure hash functions

Secure Hash Functions

Let U be a set, $\mathcal{M} = (M, \oplus, \mathbb{0})$ a monoid, and $f : \mathcal{P}(U) \rightarrow M$.

f is *set-homomorphic* if $f(U_1 \cup U_2) = f(U_1) \oplus f(U_2)$.

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Let further \preceq a linear order on U .

f is a *tree-friendly function* if for $S_0, S_1 \in \mathcal{P}(U)$ with $\max(S_0) \prec \min(S_1)$, we have $f(S_0 \cup S_1) = f(S_0) \oplus f(S_1)$.

No commutativity: Cayley hash functions

Unlike conventional reconciliation techniques:

- robust
- simple